Introduction to the Soft - Collinear Effective Theory

Lecture II

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Symmetry Breaking Summer School
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Outline (Lecture I)

- EFT concepts
  Intro to SCET
  SCET degrees of freedom $\Bigg\}$ Done on the Board
- SCET$_{\rm I}$, momentum scales and regions
- Field power counting in SCET
- Wilson lines, W, from off shell propagators
- Gauge Symmetry
- Hard-Collinear Factorization
- eg. Deep Inelastic Scattering

Outline (Lecture II)

- Review from Lecture I
- SCET Lagrangian
- Hard Operator Examples

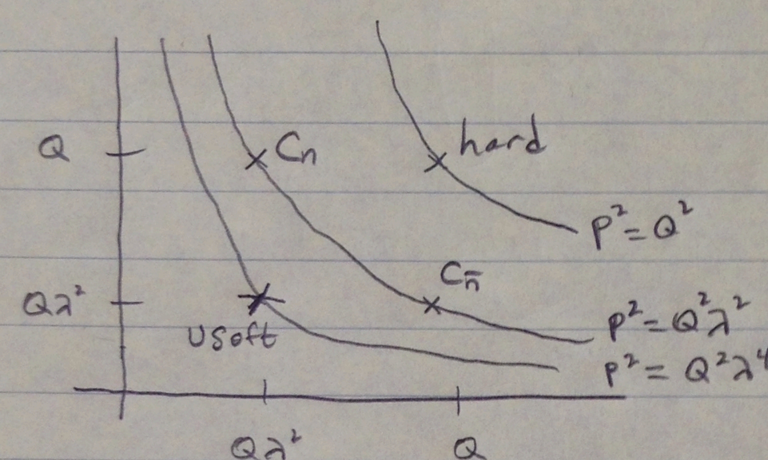
Done on the Board
(See the lecture notes below plus the summary slide.)

- Sudakov Resummation from RGE
- Soft-Collinear Factorization
- One-loop Matching Example
- $e^+e^- \to$ dijets & Factorization

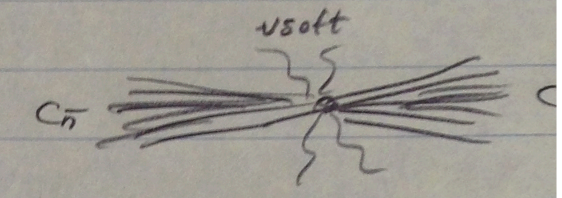
SCET Lecture II

Write ahead of time, review from L I :

SCET$_{I}$



$e^+e^- \to 2$ jets
with usoft radiati



Modes	$p^\mu = (+,-,\perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n \sim \lambda$, $A_n^\mu \sim (\lambda^2, 1, \lambda)$
$\bar n$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar n} \sim \lambda$, $A_{\bar n}^\mu \sim (1, \lambda^2, \lambda)$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us} \sim \lambda^3$, $A_{us}^\mu \sim \lambda^2$

And $\not{n}\, \xi_n = 0$

Quark Building Block: $\chi_n = W_n^\dagger \xi_n$, $\delta(\omega - i\bar n \cdot \partial_n)\, \chi_n$

Lecture II

$\boxed{\text{SCET } \mathcal{L}}$

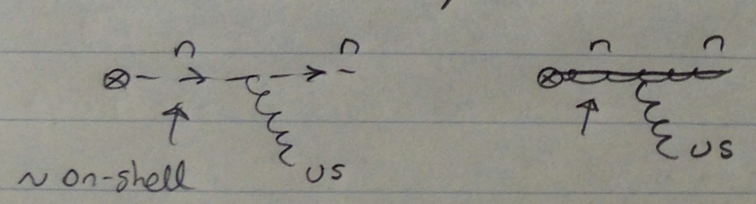
- for isolated interactions which are purely n-collinear or purely ultrasoft $\Rightarrow$ just full $\mathcal{L}_{QCD}$ for each sector

  usoft : nothing to expand, homogeneous

  n-collinear : boost everything $(\overset{+}{\lambda^2}, \overset{-}{1}, \overset{\perp}{\lambda}) \to (\lambda, \lambda, \lambda)$ then same arguement

- SCET describes interactions between sectors

  1) usoft interacting with collinear



2) hard interactions produce collinear quarks obeying

$\not{n}\, \xi_n = 0$ [boost arguement fails with hard int.]

$\mathcal{L}_{QCD} = \bar{\psi}\, i\not{D}\, \psi$ write $\psi = \left(\frac{\not{n}\not{\bar{n}}}{4} + \frac{\not{\bar{n}}\not{n}}{4}\right)\psi = \xi_n + \varphi_n$

$= \ldots = \bar{\xi}_n \frac{\not{\bar{n}}}{2}\, in\cdot D\, \xi_n + \bar{\varphi}_n \frac{\not{n}}{2} i\bar{n}\cdot D\, \varphi_n + \bar{\xi}_n i\not{D}_\perp \varphi_n + \bar{\varphi}_n i\not{D}_\perp \xi_n$

e.o.m $\delta/\delta\bar{\varphi}_n \Rightarrow \varphi_{\bar{n}} = \frac{1}{i\bar{n}\cdot D}\, i\not{D}_\perp \frac{\not{\bar{n}}}{2}\xi_n$

$\mathcal{L}_{QCD} = \bar{\xi}_n \left(in\cdot D + i\not{D}_\perp \frac{1}{i\bar{n}\cdot D} i\not{D}_\perp \right) \frac{\not{\bar{n}}}{2} \xi_n$ [still QCD]

Expand
- couple only to ξ_n [in path integral, $J\cdot\xi_n$]
- $\underset{\lambda^2}{in\cdot D} = \underset{\lambda^2}{in\cdot\partial} + \underset{\lambda^2}{g n\cdot A_n} + \underset{\lambda^2}{g n\cdot A_{us}}$
- $iD_\perp = \underset{\lambda}{iD_{n\perp}} + \ldots$ $\qquad \partial_{us}^\perp \ll i\partial_n^\perp$, $A_{us}^\perp \ll A_n^\perp$ multipole expansion
- $i\bar{n}\cdot D = \underset{\lambda^0}{i\bar{n}\cdot D_n} + \ldots$ $\qquad i\bar{n}\cdot\partial_{us} \ll i\bar{n}\cdot\partial_n$, $\bar{n}\cdot A_{us} \ll \bar{n}\cdot A_n$

SCET $\mathcal{L}_{nq}^{(0)} = \bar{\xi}_n \left(in\cdot D + i\not{D}_{n\perp} \frac{1}{i\bar{n} D_n} i\not{D}_{n\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n$

gluon $\mathcal{L}_{ng}^{(0)}$, same D's

$$\mathcal{L}_{SCET}^{(0)} = \mathcal{L}_{us}^{(0)} + \sum_n \left(\mathcal{L}_{nq}^{(0)} + \mathcal{L}_{ng}^{(0)} \right)$$

$\boxed{\text{Gluon Building Block}}$

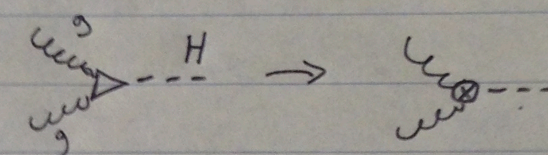
$g\,\mathcal{B}_{n\perp}^\mu = [W_n^\dagger\, iD_{n\perp}^\mu W_n] = \left[\frac{1}{i\bar{n}\partial_n} W_n^\dagger \underbrace{[i\bar{n}\cdot D_n, iD_{n\perp}^\mu]}_{\text{field strength}} W_n \right]$

$\mathcal{B}_{n\perp}^\mu = A_{n\perp}^\mu - \frac{k_\perp^\mu}{\bar{n}\cdot k}\bar{n}\cdot A_n + \ldots$

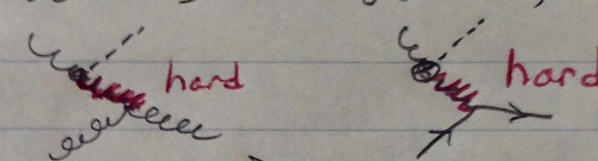
using eqtns of motion: building blocks $\{ \chi_n, \mathcal{B}_{n\perp}^\mu, \mathcal{P}_\perp^\mu \}$ suffice

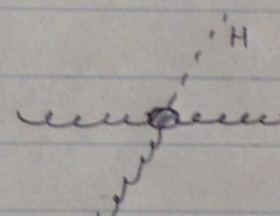
More Hard Operators

	$\mathcal{O}$	
$e^+e^- \to$ dijets	$\bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}$	$\leftarrow$ Ampl ✓
gluon PDF	$\mathcal{B}_{n\perp}^\mu \ \mathcal{B}_{n\perp\mu}$	$\leftarrow |\text{Ampl}|^2$
$e^+e^- \to \pi^+\pi^-$	$(\bar{\chi}_n \cdots \chi_n)(\bar{\chi}_{\bar{n}} \cdots \chi_{\bar{n}})$	

$pp \to H + 1$-jet   remove top 

remove <u>hard physics</u> $Q \sim M_H$ $gg \to Hg$ $gq \to Hq$, $q\bar{q} \to Hg$



$\Longrightarrow$  3 n_i's

focus on gluon case here

$$\mathcal{B}_{n_1\perp}^{a_1\mu_1} \ \mathcal{B}_{n_2\perp}^{a_2\mu_2} \ \mathcal{B}_{n_3\perp}^{a_3\mu_3} \ H \ \underbrace{T_{\mu_1\mu_2\mu_3}}_{\text{how many operators ?}} \ (if^{a_1a_2a_3}) \quad \leftarrow \text{no } d^{a_1a_2a_3} \text{ by charge conj.}$$

Use helicity basis (Cachazo's lectures later this week)

$$\mathcal{B}_{n\pm}^a = -\epsilon_\mp^\mu(n,\bar{n})\, \mathcal{B}_{n\mu}^\perp \quad , \qquad \epsilon_\pm(n,\bar{n}) = \frac{1}{\sqrt{2}}(0,1,\mp i,0)$$

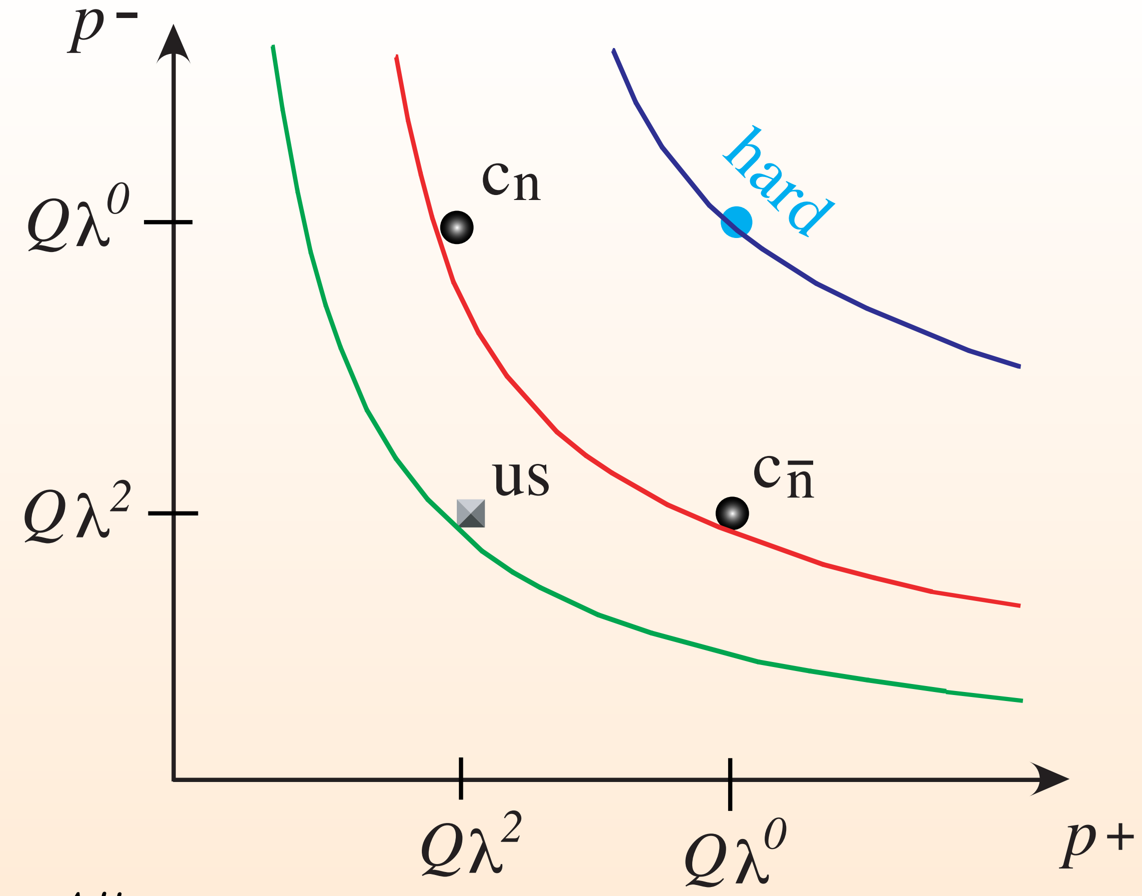
Allowed

$\mathcal{B}$	$\mathcal{B}$	$\mathcal{B}$
+	+	+
+	+	−
−	−	+
−	−	−

} Wilson Coeffs determined by Parity

2 non-trivial $C(\mu)$'s in SCET

$\mathrm{SCET_I}$ summary



usoft & collinear modes

$q_{us} \sim \lambda^3$ $\qquad$ $\xi_n \sim \lambda$

$A^\mu_{us} \sim \lambda^2$ $\qquad$ $(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p_c^\mu$

covariant derivatives:

$$iD_\perp^{n\mu} = i\partial^\mu_{n\perp} + gA_n^{\perp\mu} \qquad iD^\mu_{us} = i\partial^\mu + gA^\mu_{us}$$

$$i\bar{n}\cdot D_n = i\bar{n}\cdot\partial_n + g\bar{n}\cdot A_n$$

LO $\mathrm{SCET_I}$ Lagrangian:

$$\mathcal{L}^{(0)} = \mathcal{L}^{(0)}_{us} + \sum_n \left(\mathcal{L}^{(0)}_{n\xi} + \mathcal{L}^{(0)}_{ng}\right)$$

$$\mathcal{L}^{(0)}_{n\xi} = \bar{\xi}_n \left\{ n\cdot iD_{us} + gn\cdot A_n + i\not{D}^n_\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}^n_\perp \right\} \frac{\not{\bar{n}}}{2}\xi_n$$

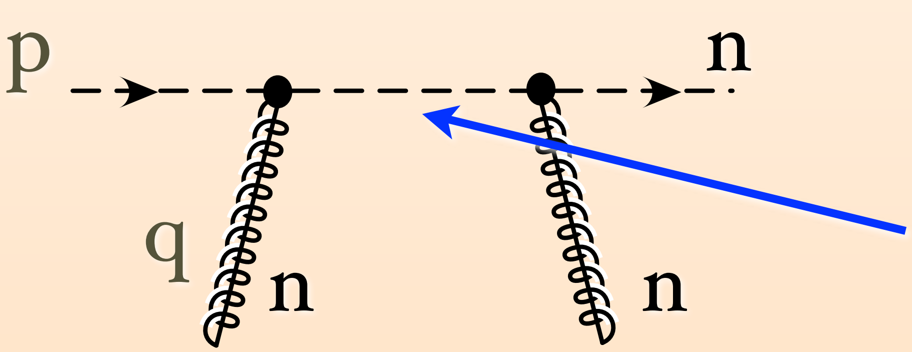
$$\mathcal{L}^{(0)}_{ng} = \mathcal{L}^{(0)}_{ng}(D^\mu_{n\perp}, \bar{n}\cdot D_n, in\cdot D_{us} + gn\cdot A_n)\,, \qquad \mathcal{L}^{(0)}_{us} = \mathcal{L}^{\rm QCD}(q_{us}, A^\mu_{us})$$

Properties of $\mathcal{L}^{(0)}_{n\xi} = \bar{\xi}_n \Big\{ n\cdot iD_{us} + g n\cdot A_n + i \not{D}^n_\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}^n_\perp \Big\} \frac{\bar{\not{n}}}{2} \xi_n$

1) has particles and antiparticles, pair creation & annihilation

$$\frac{i\not{n}}{2}\frac{\theta(\bar{n}\cdot p)}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} + i\epsilon} + \frac{i\not{n}}{2}\frac{\theta(-\bar{n}\cdot p)}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} - i\epsilon} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{n\cdot p\,\bar{n}\cdot p + p_\perp^2 + i\epsilon} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{p^2 + i\epsilon}$$

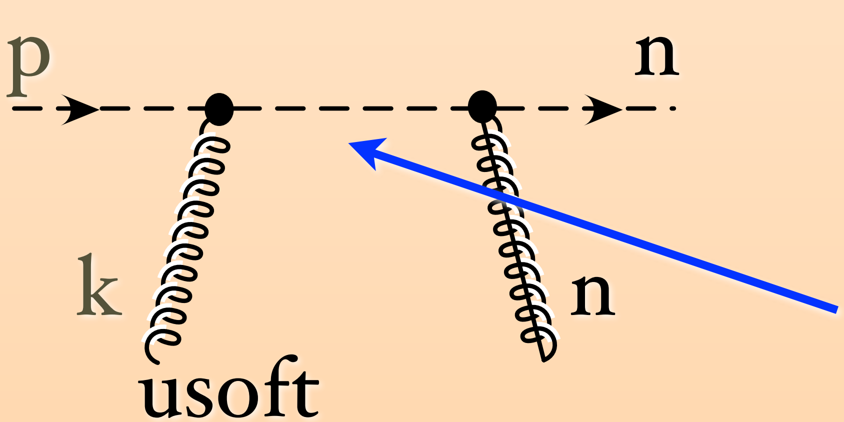
2) all components of A_n^μ couple to ξ_n



$$\frac{i\not{n}}{2}\frac{\bar{n}\cdot(p+q)}{(p+q)^2 + i\epsilon}$$

all components of p & q appear

3) only $n\cdot A_{us}$ couple at LO, only depends on $n\cdot k_{us}$ momentum



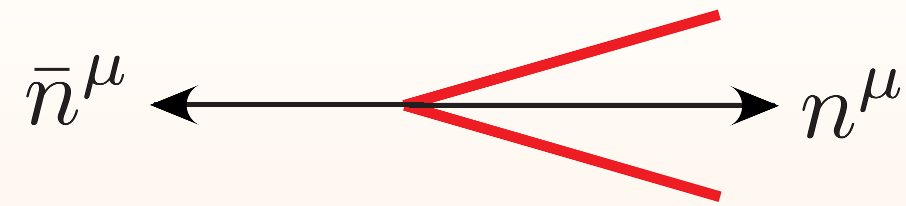
$$\frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{\bar{n}\cdot p\, n\cdot(p+k) + p_\perp^2 + i\epsilon} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{\bar{n}\cdot p\, n\cdot k + p^2 + i\epsilon}$$

$$= \frac{i\not{n}}{2}\frac{1}{n\cdot k + i\epsilon}$$

eikonal

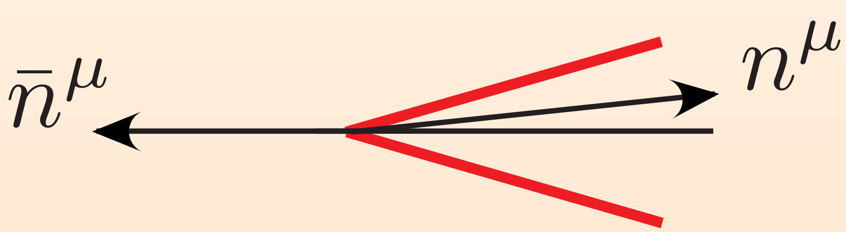
onshell $p^2 = 0$

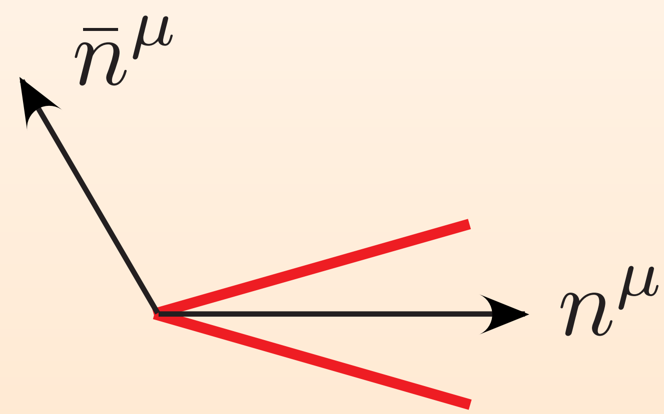
Symmetries: Gauge Invariance ✓



Reparameterization Invariance (RPI)

$n\ ,\ \ \bar{n}$ break Lorentz invariance, restored within collinear cone by RPI, three types



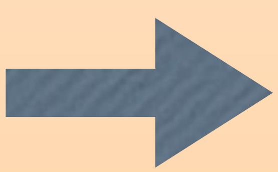


longitudinal boost

$$\text{(I)} \begin{cases} n_\mu \to n_\mu + \Delta^\perp_\mu \\ \bar{n}_\mu \to \bar{n}_\mu \end{cases} \qquad \text{(II)} \begin{cases} n_\mu \to n_\mu \\ \bar{n}_\mu \to \bar{n}_\mu + \varepsilon^\perp_\mu \end{cases} \qquad \text{(III)} \begin{cases} n_\mu \to (1+\alpha)\, n_\mu \\ \bar{n}_\mu \to (1-\alpha)\, \bar{n}_\mu \end{cases}$$

$\Delta^\perp_\mu \sim \lambda$ $\qquad$ $\varepsilon^\perp_\mu \sim \lambda^0$ $\qquad$ $\alpha \sim \lambda^0$

unique



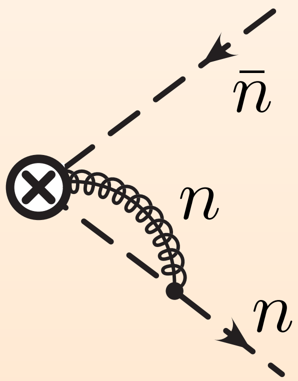
$$\mathcal{L}^{(0)}_{n\xi} = \bar{\xi}_n \Big\{ n\cdot iD_{us} + g n\cdot A_n + i\not{D}^n_\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}^n_\perp \Big\} \frac{\bar{\not{n}}}{2} \xi_n$$

Sudakov Logs & RGE (Renormalization Group Equations)

UV renormalization in SCET

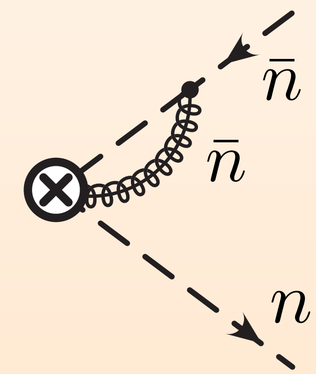
eg. $e^+e^- \to$ dijets $\qquad \bar\chi_n \gamma_\perp^\mu \chi_{\bar n} = (\bar\xi_n W_n)\gamma_\perp^\mu (W_{\bar n}^\dagger \xi_{\bar n})$

(Feynman gauge, UV: $d = 4-2\epsilon$, IR: $p^2 \neq 0$, $\bar p^2 \neq 0$)

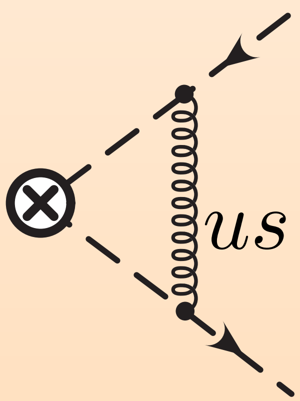


$$= \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon}\ln\left(\frac{-\cancel{p^2}}{\mu^2}\right) + \ldots\right]$$

$$\int \frac{d^dk}{(2\pi)^d}\frac{\bar n\cdot(k+p)}{\bar n\cdot k\,(p+k)^2k^2}$$ - (0-bin)

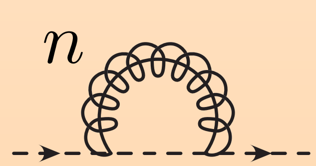


= same with $p^2 \leftrightarrow \bar p^2$

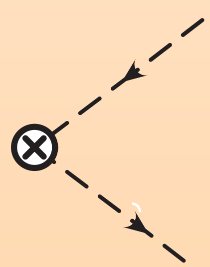


$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{(-\cancel{p^2})(-\bar p^2)}{(-Q^2)\mu^2}\right) + \ldots\right]$$

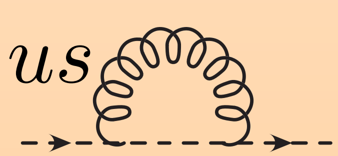
$$\int \frac{d^dk}{(2\pi)^d}\frac{n\cdot\bar n}{\left(n\cdot k + \frac{p^2}{Q}\right)\left(\bar n\cdot k + \frac{\bar p^2}{Q}\right)k^2}$$



$\times$



$$= -\frac{\alpha_s C_F}{4\pi}\frac{1}{\epsilon}$$



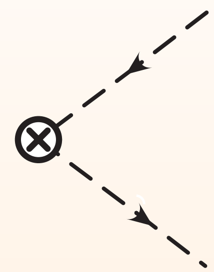
$= 0$

$$\text{sum} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) + \frac{3}{\epsilon} + \ldots\right]$$

$$\text{sum} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) + \frac{3}{\epsilon} + \dots\right]$$

$\overline{\text{MS}}$
counterterm

$$(Z_C - 1) \times$$



$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) - \frac{3}{\epsilon} + \dots\right]$$

$$C^{\text{bare}} = Z_C\, C(\mu)$$

RGE:

$$0 = \mu\frac{d}{d\mu}\, C^{\text{bare}} = \left[\mu\frac{d}{d\mu}\, Z_C\right] C(\mu) + Z_C\left[\mu\frac{d}{d\mu}\, C(\mu)\right] \Rightarrow \mu\frac{d}{d\mu}\, C(\mu) = \gamma_C\, C(\mu)$$

$$\gamma_C = \left(-Z_C^{-1}\right)\mu\frac{d}{d\mu}\, Z_C = (-1)\frac{C_F}{4\pi}\left[(-2\epsilon\,\alpha_s)\left(\cancel{\frac{-2}{\epsilon^2}} - \frac{2}{\epsilon}\ln\frac{\mu^2}{-Q^2} - \frac{3}{\epsilon}\right) + \alpha_s\left(\cancel{\frac{-4}{\epsilon}}\right)\right]$$

$$\mu\frac{d}{d\mu}\alpha_s = -2\epsilon\,\alpha_s + \dots$$

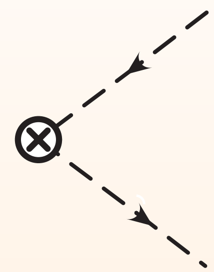
$$= -\frac{\alpha_s(\mu)}{4\pi}\left[4C_F\ln\frac{\mu^2}{-Q^2} + 6C_F\right]$$

finite

$$\text{sum} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) + \frac{3}{\epsilon} + \ldots\right]$$

$\overline{\text{MS}}$
counterterm

$$(Z_C - 1) \times$$



$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) - \frac{3}{\epsilon} + \ldots\right]$$

$$C^{\text{bare}} = Z_C\, C(\mu)$$

RGE:

square the amplitude: $\quad H = \left|C(\mu)\right|^2$

$$\mu \frac{d}{d\mu} H(Q,\mu) = \left(\gamma_C + \gamma_C^*\right) H(Q,\mu) = -\frac{\alpha_s(\mu)}{2\pi}\left[8C_F \ln\frac{\mu}{Q} + 6C_F\right] H(Q,\mu)$$

Solve This on Homework #3

$$H(Q,\mu_1) = H(Q,\mu_0)\exp\left[-\#\,\alpha_s \ln^2\frac{\mu_1}{Q} + \ldots\right]$$ frozen coupling

$$H(Q,\mu_1) = H(Q,\mu_0)\exp\left[-\#\,\frac{1}{\alpha_s(\mu_0)} f\left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right) + \ldots\right]$$ running coupling

Sudakov Form Factor

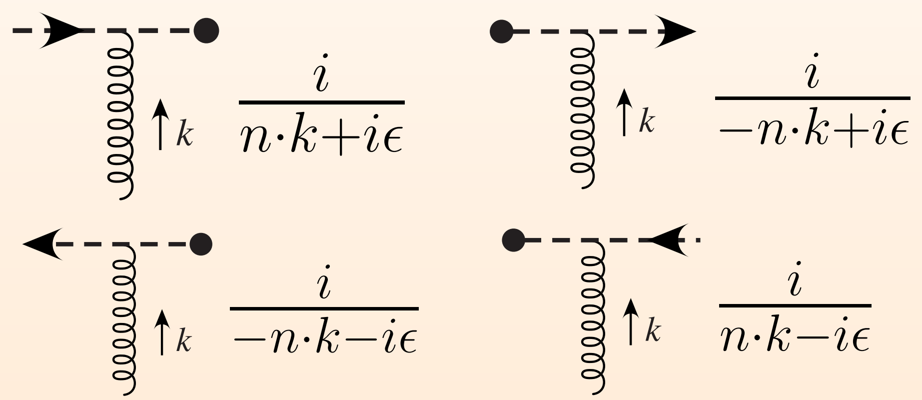
$\bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}$ restricts radiation, Sudakov = no emission probability

Ultrasoft - Collinear Factorization

Multipole Expansion:

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \Big\{ n \cdot iD_{us} + g n \cdot A_n + i \not{D}_\perp^c \frac{1}{i\bar{n} \cdot D_c} i \not{D}_\perp^c \Big\} \frac{\not{\bar{n}}}{2} \xi_n$$

usoft gluons have eikonal Feynman rules and induce eikonal propagators



Field Redefinition:

$$\xi_n \longrightarrow Y_n \xi_n\,, \quad A_n \longrightarrow Y_n A_n Y_n^\dagger$$

$$Y_n(x) = P \exp\Big(ig \int_{-\infty}^{0} ds\, n\cdot A_{us}(x+ns) \Big)$$

$$n\cdot D_{us} Y_n = 0, \quad Y_n^\dagger Y_n = 1$$

gives $\mathcal{L}_{n\xi}^{(0)} = \bar{\xi}_n \Big\{ n \cdot iD_{us} + \ldots \Big\} \frac{\not{\bar{n}}}{2} \xi_n \implies \bar{\xi}_n \Big\{ n \cdot iD_n + i \not{D}_{n\perp} \frac{1}{i\bar{n} \cdot D_n} i \not{D}_{n\perp} \Big\} \frac{\not{\bar{n}}}{2} \xi_n$

similar for $\mathcal{L}_{ng}^{(0)}$

Moves all usoft gluons to operators, simplifies cancellations

Field Theory gives the same results pre- and post- field redefinition, but the organization is different

$$\xi_n \longrightarrow Y_n \xi_n$$

also $W_n \to Y_n W_n Y_n^\dagger$

Ultrasoft - Collinear Factorization:

eg1. $$\bar\chi_n \gamma_\perp^\mu \chi_{\bar n} \Longrightarrow \bar\chi_n \big(Y_n^\dagger Y_{\bar n}\big) \gamma_\perp^\mu \chi_{\bar n}$$

usoft-collinear factorization is simple in SCET

eg2. $$\bar\chi_n \frac{\bar{n}\!\!\!/}{2} \chi_n \Longrightarrow \bar\chi_n \big(Y_n^\dagger Y_n\big) \frac{\bar{n}\!\!\!/}{2} \chi_n = \bar\chi_n \frac{\bar{n}\!\!\!/}{2} \chi_n$$

color transparency

note: not upset by $\delta(\omega - i\bar n \cdot \partial_n)$
since ultrasoft gluons carry no $i\bar n \cdot \partial_n \sim \lambda^0$ momenta

One-Loop Matching Calculation

IR: $p^2 = \bar{p}^2 \neq 0$

both theories

QCD $\qquad \mathcal{L}_{\rm QCD} \quad + \quad J = \bar{\psi}\gamma^\mu\psi$

$\downarrow$

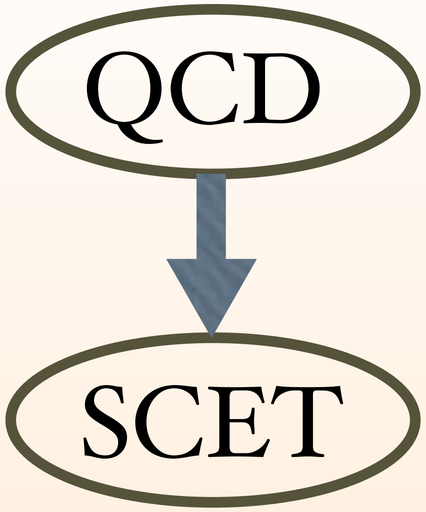
SCET $\qquad \mathcal{L}^{(0)}_{\rm SCET} \quad + \quad C\ \bar{\chi}_n \gamma^\mu_\perp \chi_{\bar{n}}$ $\qquad$ find C at $\mathcal{O}(\alpha_s)$

$$\boxed{\big(\text{1-loop ren. QCD}\big) - \big(\text{1-loop ren. SCET}\big) = C^{\rm 1loop} \langle \bar{\chi}_n \gamma^\mu_\perp \chi_{\bar{n}} \rangle_{\rm tree}}$$

One-Loop Matching Calculation

IR: $p^2 = \bar{p}^2 \neq 0$

both theories



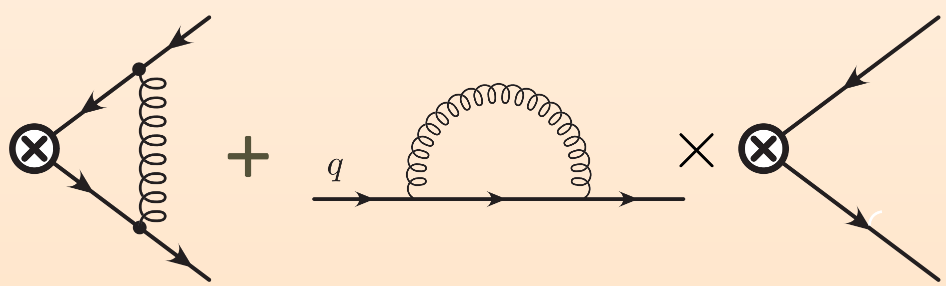
$$\mathcal{L}_{\rm QCD} \quad + \quad J = \bar{\psi}\gamma^\mu\psi$$

$$\mathcal{L}^{(0)}_{\rm SCET} \quad + \quad C\ \bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}$$

find C at $\mathcal{O}(\alpha_s)$

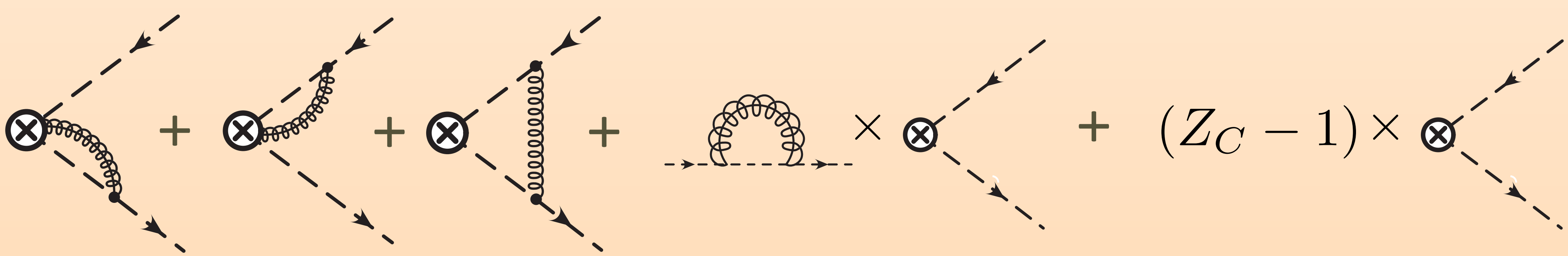
$$\boxed{(\text{1-loop ren. QCD}) - (\text{1-loop ren. SCET}) = C^{\rm 1loop}\langle \bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}\rangle_{\rm tree}}$$

QCD



$$= \frac{\alpha_s C_F}{4\pi}\left[\underline{-2\ln^2\frac{p^2}{Q^2} - 3\ln\frac{p^2}{Q^2}} - 1 - \frac{2\pi^2}{3}\right]$$

SCET



$$= \frac{\alpha_s C_F}{4\pi}\left[2\ln^2\frac{\mu^2}{-p^2} + 3\ln\frac{\mu^2}{-p^2} - \ln^2\left(\frac{\mu^2 Q^2}{-p^4}\right) + 7 - \frac{5\pi^2}{6}\right]$$

collinear

soft

$$= \frac{\alpha_s C_F}{4\pi}\left[\ln^2\frac{\mu^2}{-Q^2} \underline{- 2\ln^2\frac{p^2}{Q^2} - 3\ln\frac{p^2}{Q^2}} + 3\ln\frac{\mu^2}{-Q^2} + 7 - \frac{5\pi^2}{6}\right]$$

IR divergences match

One-Loop Matching Calculation

$$\mathrm{QCD} - \mathrm{SCET} = \frac{\alpha_s C_F}{4\pi}\left[-\ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

$$\Rightarrow \quad C(Q,\mu) = 1 + \frac{\alpha_s(\mu) C_F}{4\pi}\left[-\ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

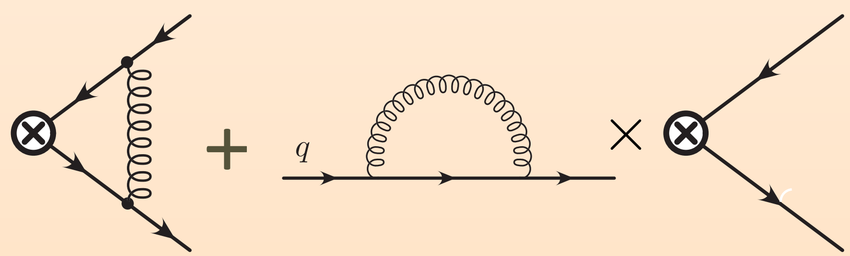
One-Loop Matching Calculation

$$\mathrm{QCD} - \mathrm{SCET} = \frac{\alpha_s C_F}{4\pi}\left[-\ln^2 \frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

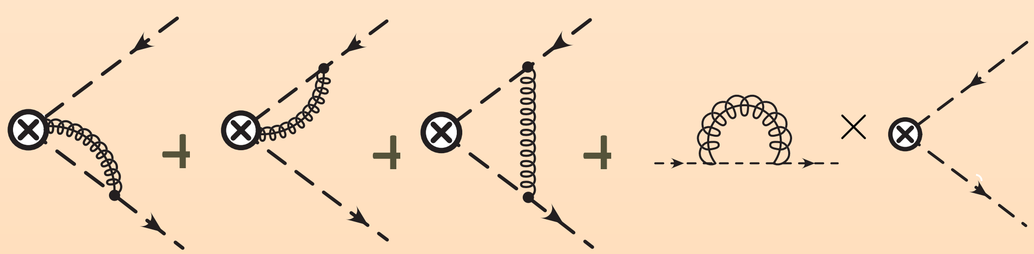
$$\Rightarrow \qquad C(Q,\mu) = 1 + \frac{\alpha_s(\mu) C_F}{4\pi}\left[-\ln^2 \frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

Once we know how this works, there is a much easier way to get this answer.

Result for C is independent of our choice of IR regulator. Use dim.reg. for IR too.

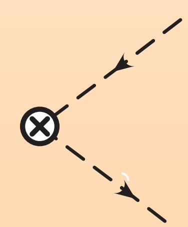


$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon_{\mathrm{IR}}^2} - \frac{2}{\epsilon_{\mathrm{IR}}}\ln\frac{\mu^2}{-Q^2} - \frac{3}{\epsilon_{\mathrm{IR}}} - \ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$



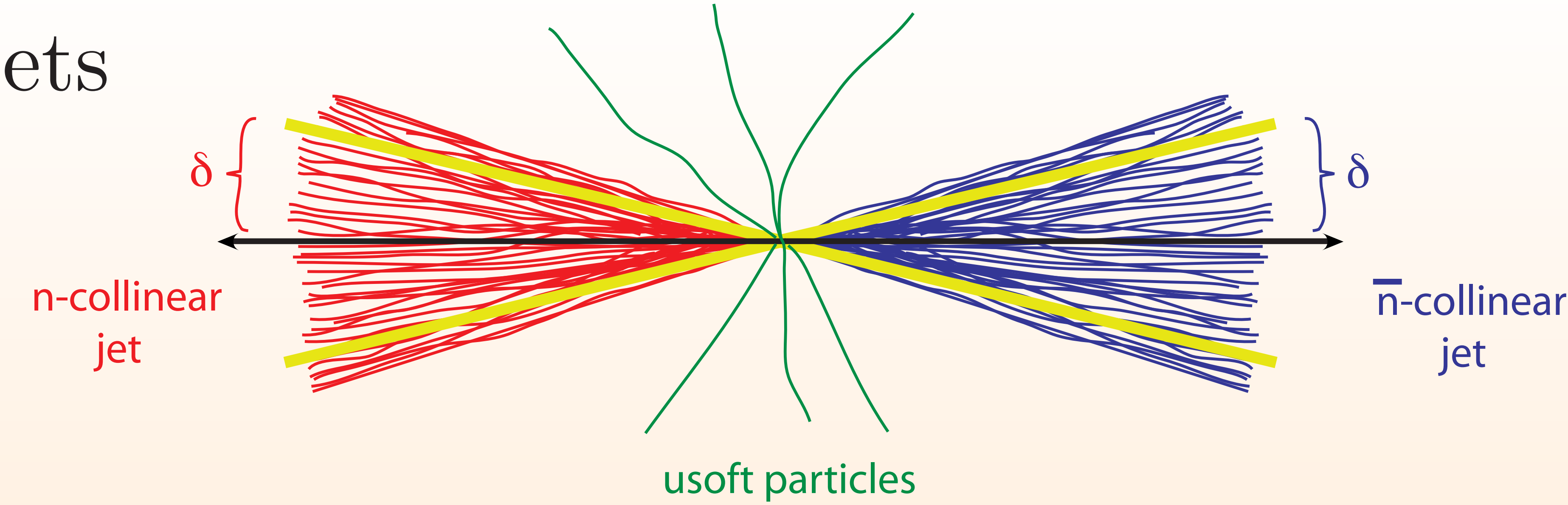
$$\propto \frac{1}{\epsilon_{\mathrm{UV}}^2} - \frac{1}{\epsilon_{\mathrm{IR}}^2}\,, \quad \frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}$$

vanish for $\epsilon_{\mathrm{UV}} = \epsilon_{\mathrm{IR}}$

$(Z_C - 1)\times$ 

$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon_{\mathrm{UV}}^2} - \frac{2}{\epsilon_{\mathrm{UV}}}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) - \frac{3}{\epsilon_{\mathrm{UV}}} + \ldots\right]$$

$\mathrm{QCD} - \mathrm{SCET} = \frac{\alpha_s C_F}{4\pi}\left[-\ln^2 \frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$ = same as above = IR finite part of QCD calculation with this regulator eg. $e^+e^- \to 2$ jets



event shape in two jet region

$$\frac{d\sigma}{de} = \frac{1}{Q^2}\sum_X Ł_{\mu\nu}\langle 0|J^{\dagger\nu}(0)|X\rangle\langle X|J^\mu(0)|0\rangle\delta(e-e(X))\delta^4(q-p_X)$$

$\mathrm{SCET_I}$

$$J^{(0)} = \int d\omega d\bar\omega\, C(\omega,\bar\omega)\,\bar\chi_{n,\omega}\Gamma\chi_{\bar n,\bar\omega}$$
$$= \int d\omega d\bar\omega\, C(\omega,\bar\omega)\,\bar\chi_{n,\omega}\, Y_n^\dagger\Gamma Y_{\bar n}\,\chi_{\bar n,\bar\omega}$$

$$\chi_{n,\omega} = \delta(\omega - i\bar n\cdot\partial_n)\chi_n$$

$$|X\rangle = |X_n X_{\bar n} X_{us}\rangle$$

$$\frac{d\sigma}{de} = \frac{1}{Q^2} \sum_{X_{us}, X_{\bar{n}}, X_n} L_{\mu\nu} \int [d\omega_i] C(\omega, \bar{\omega}) C(\omega', \bar{\omega}') \langle 0 | (\tilde{Y}_{\bar{n}}^\dagger \Gamma \tilde{Y}_n) | X_{us} \rangle \langle X_{us} | (Y_n^\dagger \Gamma Y_{\bar{n}}) | 0 \rangle$$

$$\langle 0 | \bar{\chi}_{\bar{n},\bar{\omega}'} | X_{\bar{n}} \rangle \langle X_{\bar{n}} | \chi_{\bar{n},\bar{\omega}} | 0 \rangle \langle 0 | \chi_{n,\omega'} | X_n \rangle \langle X_n | \bar{\chi}_{n,\omega} | 0 \rangle \delta(e - e(X)) \delta^4(q - p_X)$$

should specify "e" to go further, but generically we get

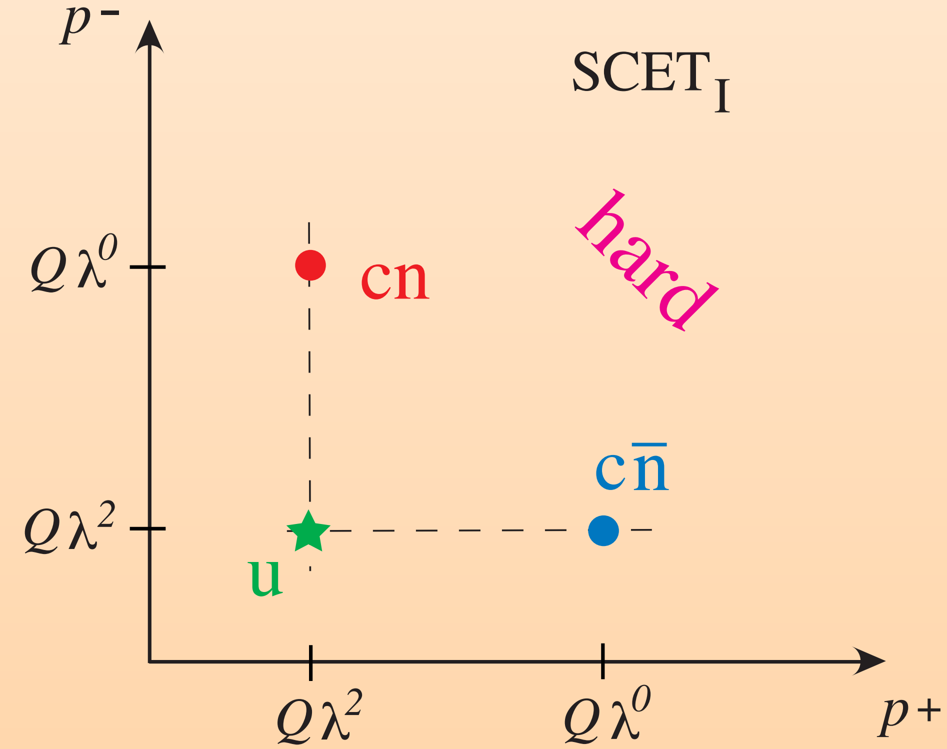
$$\frac{d\sigma}{de} \sim |C(Q, \mu)|^2 \int d\ell^+ \, d\ell^- \, J_{\bar{n}}(\ell^-, \mu) J_n(\ell^+, \mu) S(\ell^-, \ell^+, \mu)$$

hard perturbative corrections

perturbative jet functions

soft function

Homework #4:
Compute the jet function at one-loop



Non-perturbative Factorization:

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

parton distributions

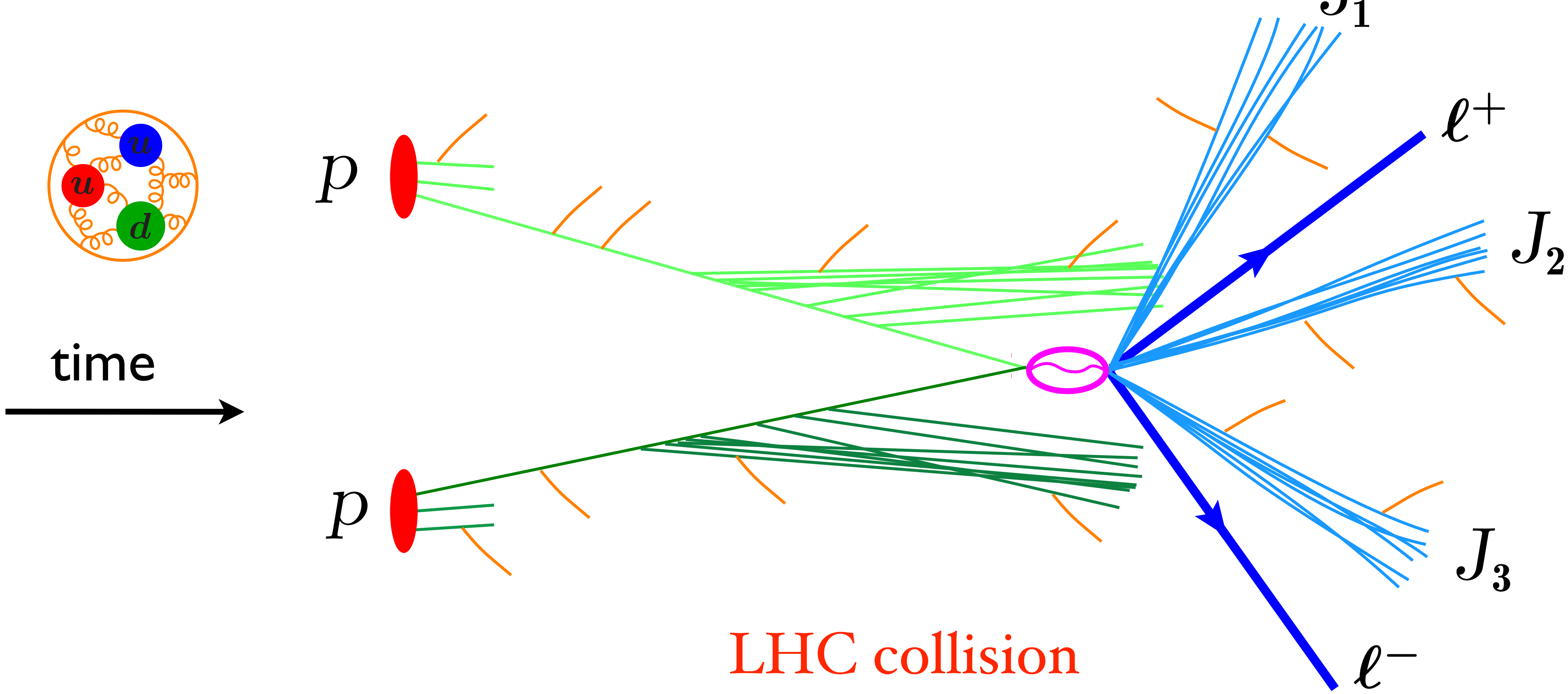
perturbative partonic cross section

hadronization

(known description with QFT operators in some cases)

$$\hat{\sigma} = \sigma_0 \left[1 + \alpha_s + \alpha_s^2 + \dots \right]$$

NLO NNLO



Perturbative Factorization: for multi-scale problems

beam hard jet pert. soft

$$\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

μ_B μ_H μ_J μ_S