Precision QCD

Giulia Zanderighi (CERN & Oxford)

2nd Lecture

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The beta-function

$$\beta(\alpha_s^{\text{ren}}) \equiv \mu^2 \frac{d\alpha_s(\mu^2)}{d\mu^2}$$

The renormalized coupling is

$$\alpha_s(\mu) = \alpha_s^{\text{bare}} + b_0 \ln \frac{M_{UV}^2}{\mu^2} \left(\alpha_s^{\text{bare}}\right)^2$$

So, one immediately gets

$$\beta = -b_0 \alpha_s^2(\mu) + \ldots$$

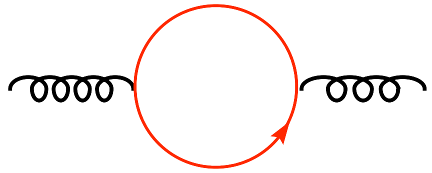
Integrating the differential equation one finds at lowest order

$$\frac{1}{\alpha_s(\mu)} = b_0 \ln \frac{\mu^2}{\mu_0^2} + \frac{1}{\alpha_s(\mu_0)} \qquad \Rightarrow \qquad \alpha_s(\mu) = \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda^2}}$$

More on the beta-function

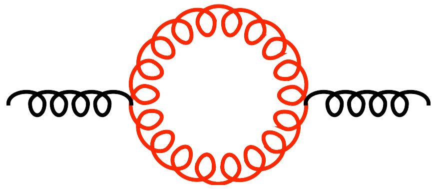
<u>Roughly speaking:</u>

(a) quark loop vacuum polarization diagram gives a negative contribution to $b_0 \sim n_f$



(a)

(b) gluon loop gives a positive contribution to $b_0 \sim N_c$



(b)

Since (b) > (a) $\Rightarrow$ $b_{0,QCD} > 0 \Rightarrow$ overall negative beta-function in QCD

While in QED (b) = 0 $\Rightarrow b_{0,QED} < 0$

$$\beta_{\rm QED} = \frac{1}{3\pi}\alpha^2 + \ldots$$

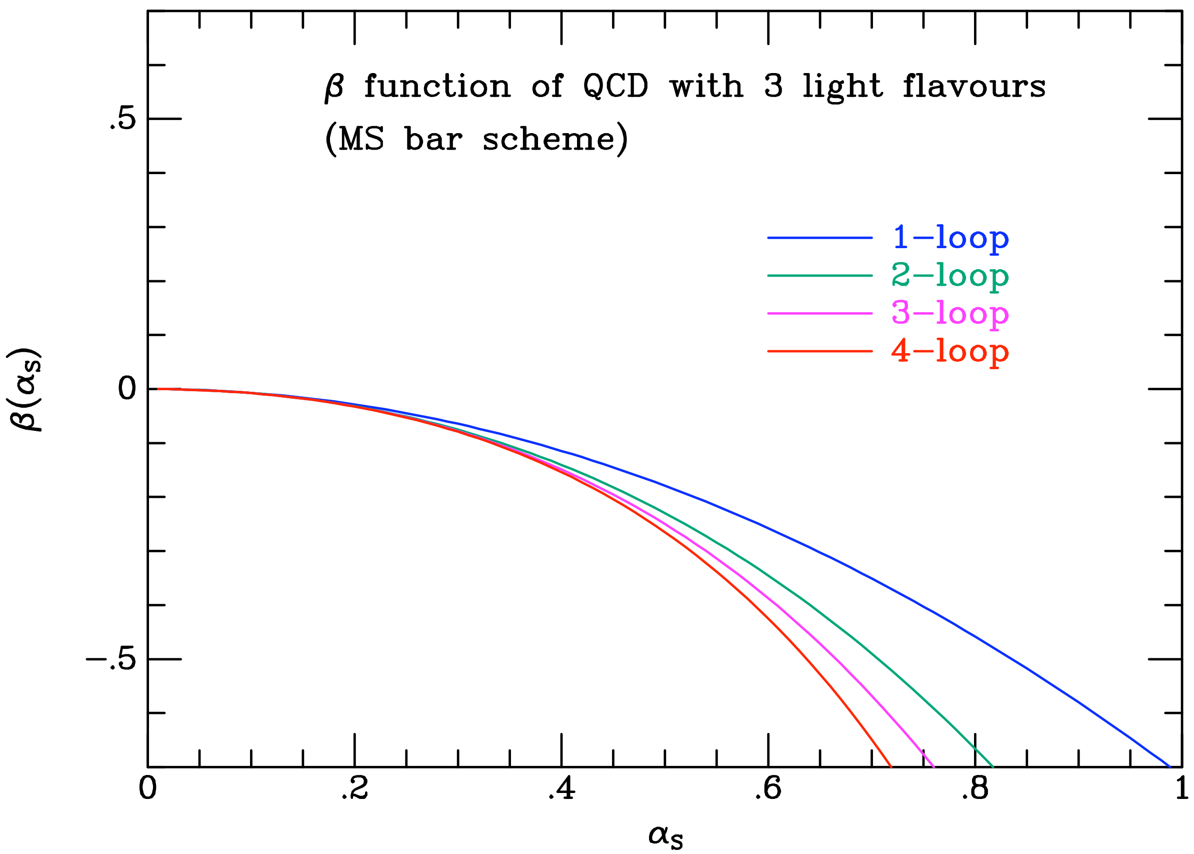
More on the beta-function

Perturbative expansion of the beta-function:

$$\beta = -\alpha_s^2(\mu) \sum_i b_i \alpha_s^i(\mu)$$

$$b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

$$b_1 = \frac{17N_c^2 - 5N_c n_f - 3C_F n_f}{24\pi^2}$$

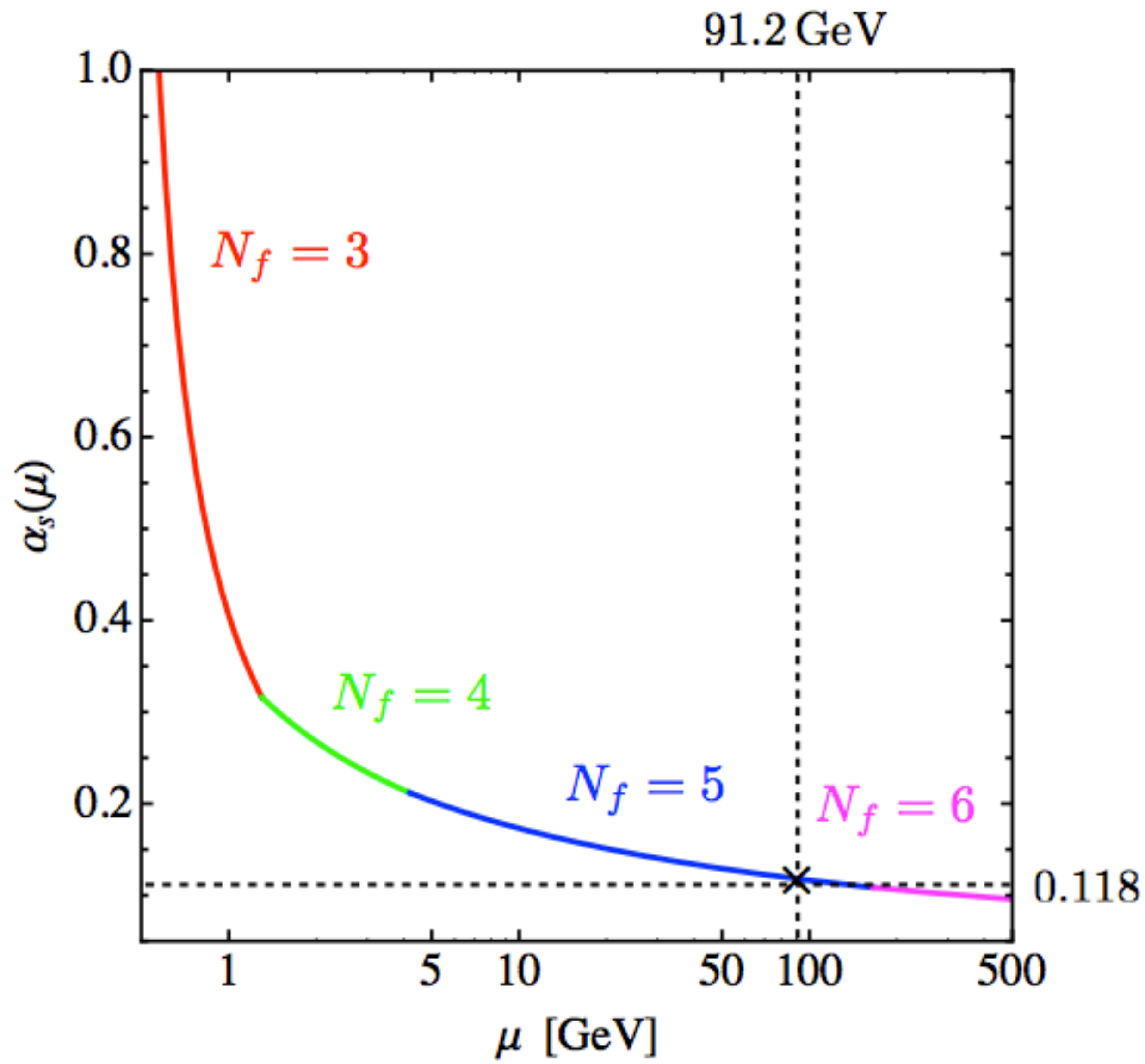


- n_f is the number of active flavours (depends on the scale)
- today, the beta-function known up to four loops, but only first two coefficients are independent of the renormalization scheme

Exercise: proof the above statement [hint: use the fact that at $O(\alpha_s)$ the coupling in two different schemes is related by a finite change]

Active flavours & running coupling

The active field content of a theory modifies the running of the couplings



Constrain New Physics by measuring the running at high scales?

Renormalization Group Equation

Consider a dimensionless quantity A, function of a single scale Q. The dimensionless quantity should be independent of Q. However in quantum field theory this is not true, as renormalization introduces a second scale μ

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So, for any observable A one can write a renormalization group equation

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s}\right] A\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

$$\alpha_s = \alpha_s(\mu^2) \qquad \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2}$$

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Scale dependence of A enters through the running of the coupling: knowledge of $A(1, \alpha_s(Q^2))$ allows one to compute the variation of A with Q given the beta-function

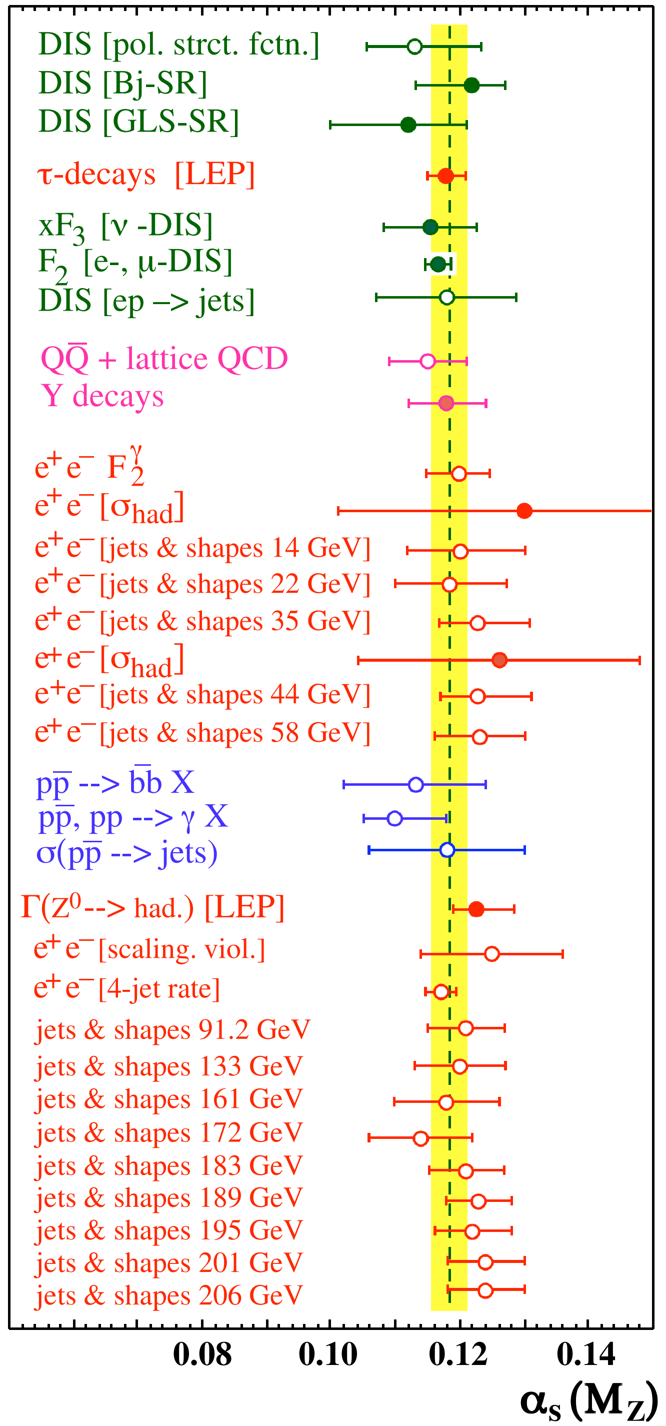
Measurements of the running coupling

<u>Summarizing:</u>

- overall consistent picture: α_s from very different observables compatible
- α_s is not so small at current scales
- α_s decreases slowly at higher energies (logarithmic only)
- higher order corrections are and will remain important

World average

$$\alpha_s(M_{Z^0}) = 0.1184 \pm 0.007$$

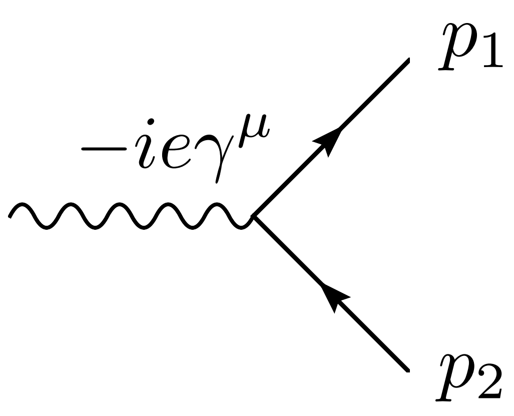


The soft approximation

Let's consider again the R-ratio. This is determined by $\gamma^* \to q\bar{q}$

At leading order:

$$M_0^\mu = \bar{u}(p_1)(-ie\gamma^\mu)v(p_2)$$



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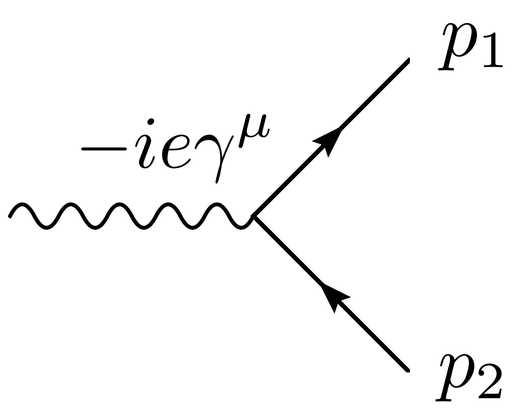
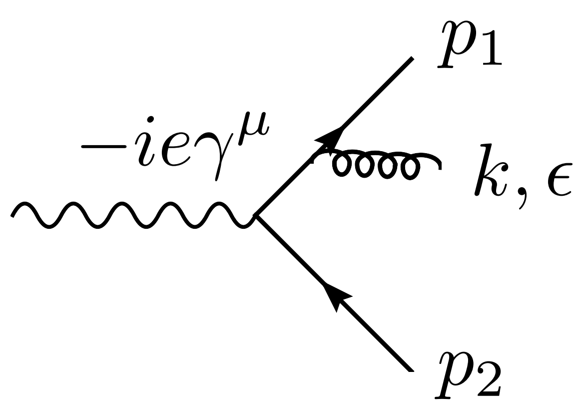


Emit one gluon:

$$\begin{aligned} M^\mu_{q\bar{q}g} &= \bar{u}(p_1)(-ig_s t^a \not{\epsilon})\frac{i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie\gamma^\mu)v(p_2) \\ &+ \bar{u}(p_1)(-ie\gamma^\mu)\frac{i(\not{p}_2-\not{k})}{(p_2-k)^2}(-ig_s t^a \not{\epsilon})v(p_2) \end{aligned}$$



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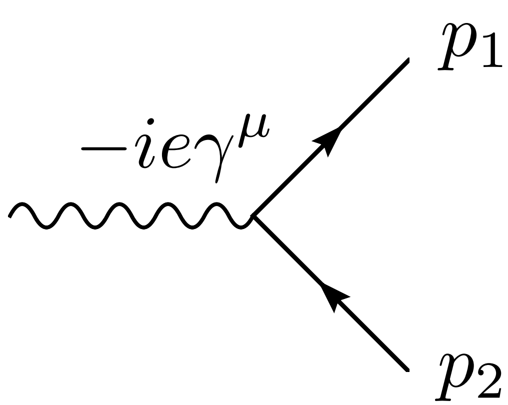

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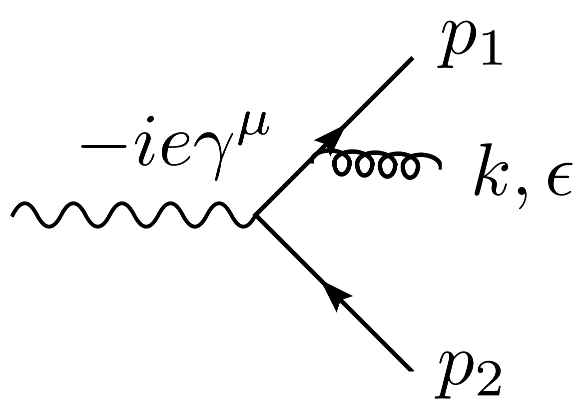

Consider the soft approximation: $k \ll p_1, p_2$

$$M^\mu_{q\bar{q}g} = \bar{u}(p_1)\left((-ie\gamma^\mu)(-ig_s t^a)v(p_2)\right)\left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k}\right)$$

$\Rightarrow$ factorization of soft part (crucial for resummed calculations)

Soft divergences

The squared amplitude becomes

$$|M^{\mu}_{q\bar{q}g}|^2 = \sum_{\text{pol}} \left| \bar{u}(p_1) \left((-ie\gamma^{\mu})(-ig_s t^a) v(p_2)\right) \left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k} \right) \right|^2$$

$$= \quad |M_{q\bar{q}}|^2 C_F g_s^2 \frac{2p_1 p_2}{(p_1 k)(p_2 k)}$$

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Including phase space

$$d\phi_{q\bar{q}g}|M_{q\bar{q}g}|^2 \quad = \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \frac{d^3k}{2\omega(2\pi)^3} C_F g_s^2 \frac{2p_1p_2}{(p_1k)(p_2k)}$$

$$= \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \omega d\omega d\cos\theta \frac{d\phi}{2\pi} \frac{2\alpha_s C_F}{\pi} \frac{1}{\omega^2(1-\cos^2\theta)}$$

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The differential cross section is

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi}\frac{d\omega}{\omega}\frac{d\theta}{\sin\theta}\frac{d\phi}{2\pi}$$

Soft & collinear divergences

Cross section for producing a $q\bar{q}$-pair and a gluon is infinite (IR divergent)

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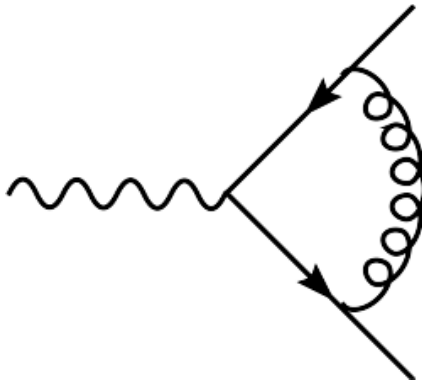
But the full $O(\alpha_s)$ correction to R is finite, because one must include a virtual correction which cancels the divergence of the real radiation

$$d\sigma_{q\bar{q},v} \sim -d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$



NB: here we kept only soft terms, if we do the full calculation one gets a finite correction of α_s/π

Soft & collinear divergences

$\underline{\omega \to 0}$ soft divergence: the four-momentum of the emitted particle approaches zero, typical of gauge theories, even if matter (radiating particle) is massive

$\underline{\theta \to 0}$ collinear divergence: particle emitted collinear to emitter. Divergence present only if all particles involved are massless

NB: the appearance of soft and collinear divergences discussed in the specific contect of $e^+e^- \to qq$ are a general property of QCD

Infrared safety (= finiteness)

So, the R-ratio is an infrared safe quantity.

In perturbation theory one can compute only IR-safe quantities, otherwise get infinities, which can not be renormalized away (why not?)

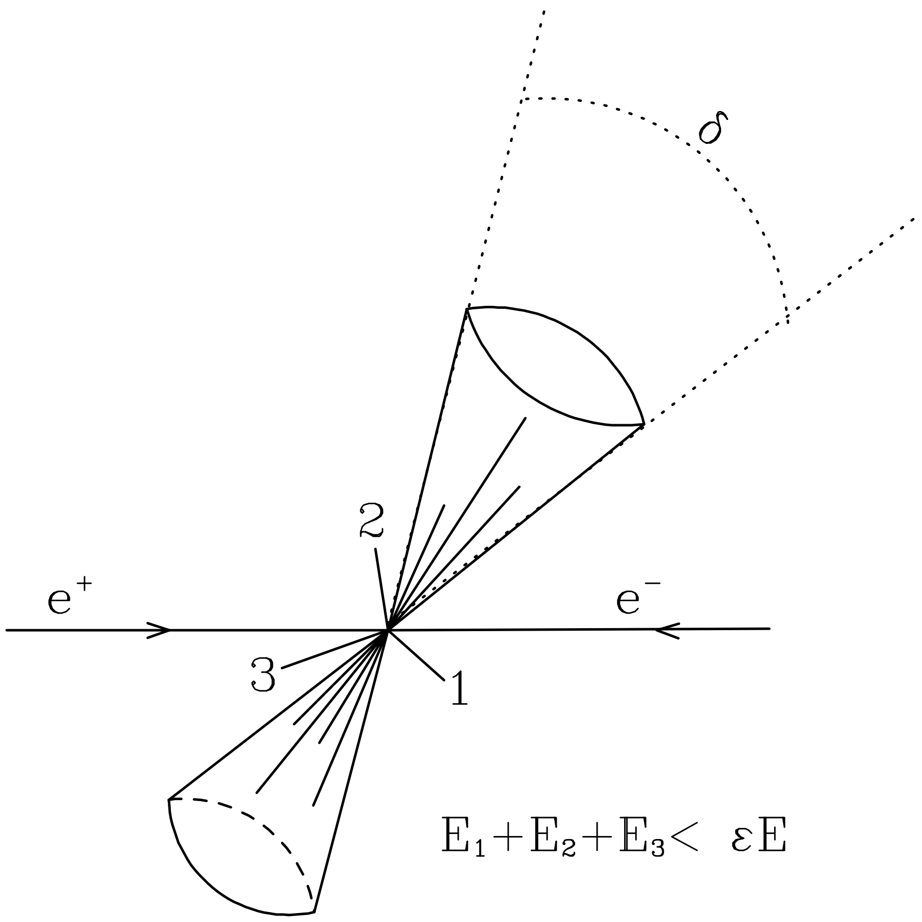
So, the natural questions are:

- are there other IR-safe quantities?
- what property of R guarantees its IR-safety?

Sterman-Weinberg jets

First formulation of cross-sections which are finite in perturbation theory and describe the hadronic final state

Introduce two parameters ε and δ: a pair of Sterman-Weinberg jets are two cones of opening angle δ that contain all the energy of the event excluding at most a fraction ε



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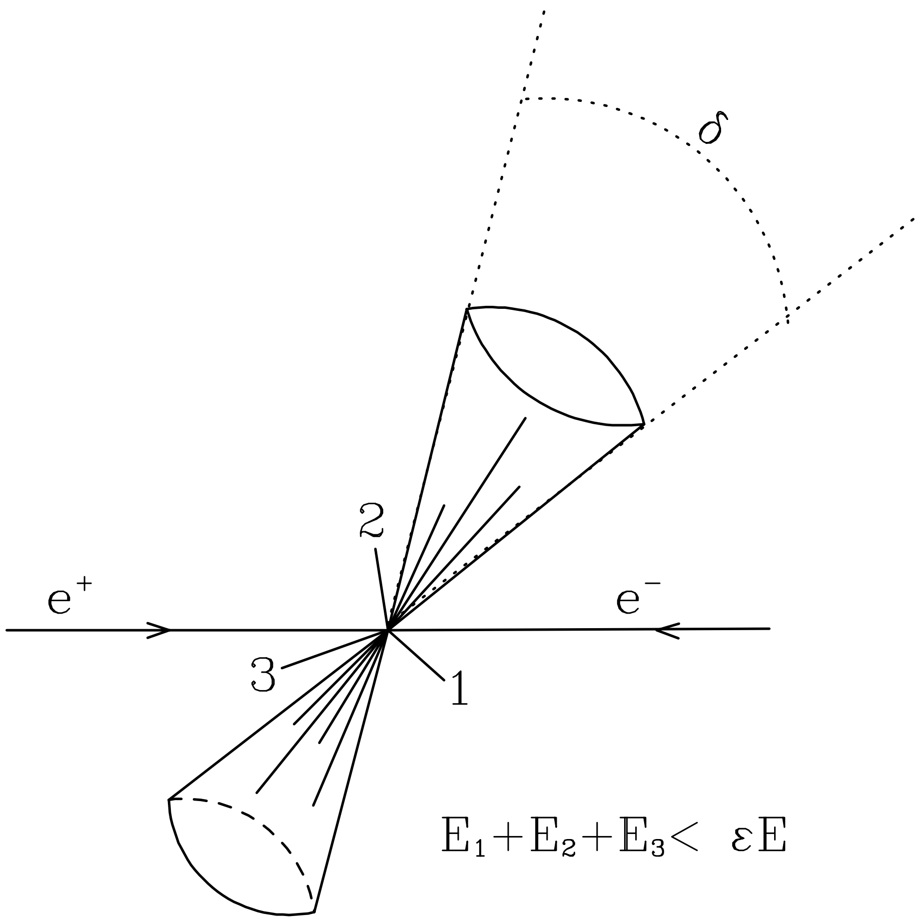

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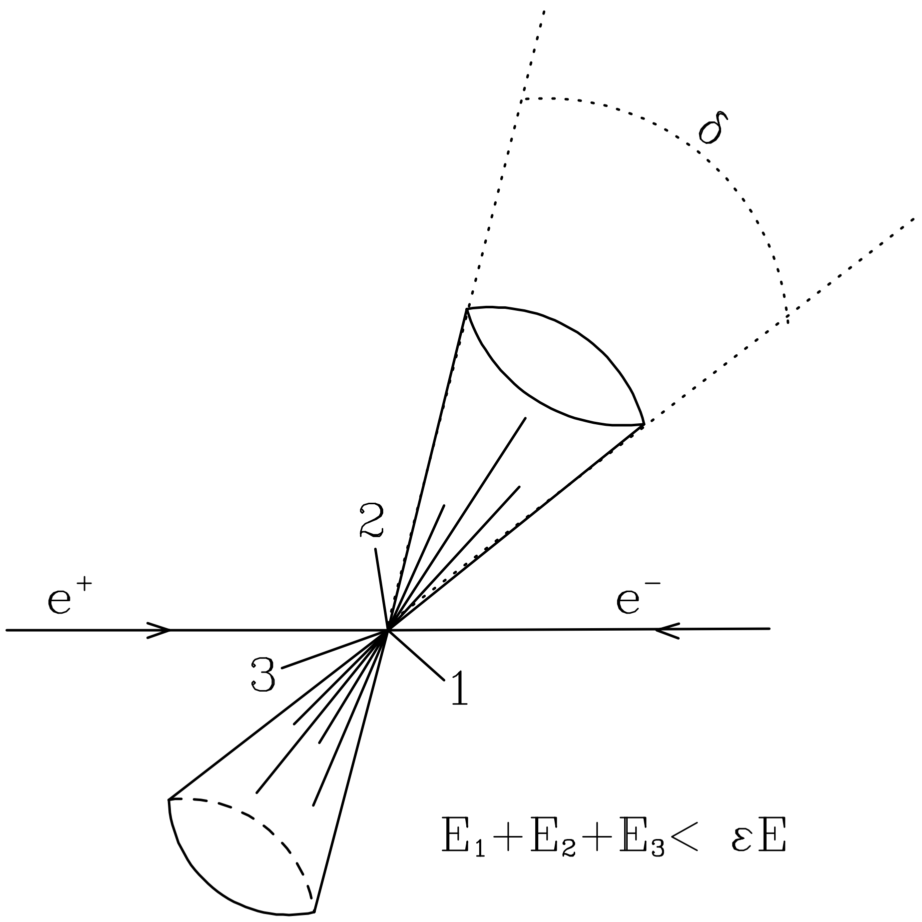

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Kinoshita-Lee-Nauenberg (KLN) theorem:
final-state infrared divergences cancel in measurable quantities (transition probabilities, cross-sections summed over indistinguishable states...)

Sterman-Weinberg jets

The Sterman-Weinberg jet cross-section up to $O(\alpha_s)$ is given by

$$\sigma_1 = \sigma_0 \left(1 + \frac{2\alpha_s C_F}{\pi} \ln\epsilon \ln\delta^2\right)$$

Effective expansion parameter in QCD is often $\alpha_s C_F/\pi$ not α_s

α_s-expansion enhanced by a double log: left-over from real-virtual cancellation

- if more gluons are emitted, one gets for each gluon
  - a power of $\alpha_s C_F/\pi$
  - a soft logarithm $\ln\varepsilon$
  - a collinear logarithm $\ln\delta$
- if ε and/or δ become too small the above result diverges
- if the logs are large, fixed order meaningless, one needs to resum large infrared and collinear logarithms to all orders in the coupling constant

Infrared safety: definition

An observable $\mathcal{O}$ is infrared and collinear safe if

$$\mathcal{O}_{n+1}(k_1, k_2, \ldots, k_i, k_j, \ldots k_n) \to \mathcal{O}_n(k_1, k_2, \ldots k_i + k_j, \ldots k_n)$$

whenever one of the k_i/k_j becomes soft or k_i and k_j are collinear

i.e. the observable is insensitive to emission of soft particles or to collinear splittings

Infrared safety: examples

Infrared safe ?

- energy of the hardest particle in the event
- multiplicity of gluons
- momentum flow into a cone in rapidity and angle
- cross-section for producing one gluon with $E > E_{min}$ and $\theta > \theta_{min}$
- jet cross-sections

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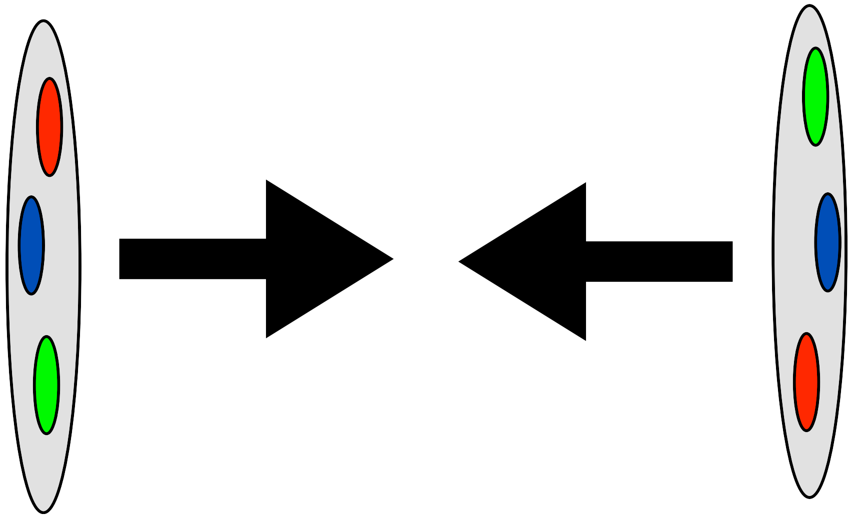
- jet cross-sections DEPENDS

Partons in the initial state

- We talked a lot about final state QCD effects
- This is the only thing to worry about at e^+e^- colliders (LEP)
- Hera/Tevatron/LHC involve protons in the initial state
- Proton are made of QCD constituents

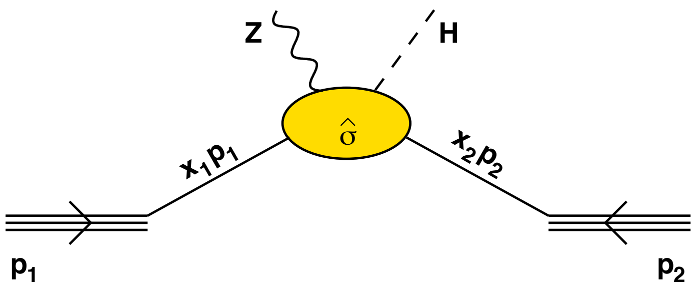
Next we will focus mainly on aspects related to initial state effects



The parton model

Basic idea of the parton model: intuitive picture where in a high transverse momentum scattering partons behave as quasi free in the collision
$\Rightarrow$ cross section is the incoherent sum of all partonic cross-sections

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \qquad \hat{s} = x_1 x_2 s$$



NB: This formula is wrong/incomplete (see later)

$f_i^{(P_j)}(x_i)$: parton distribution function (PDF) is the probability to find parton i in hadron j with a fraction x_i of the longitudinal momentum (transverse momentum neglected), extracted from data

$\hat{\sigma}(x_1 x_2 s)$: partonic cross-section for a given scattering process, computed in perturbative QCD

Sum rules

Momentum sum rule: conservation of incoming total momentum

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Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x) \right) = 1$$

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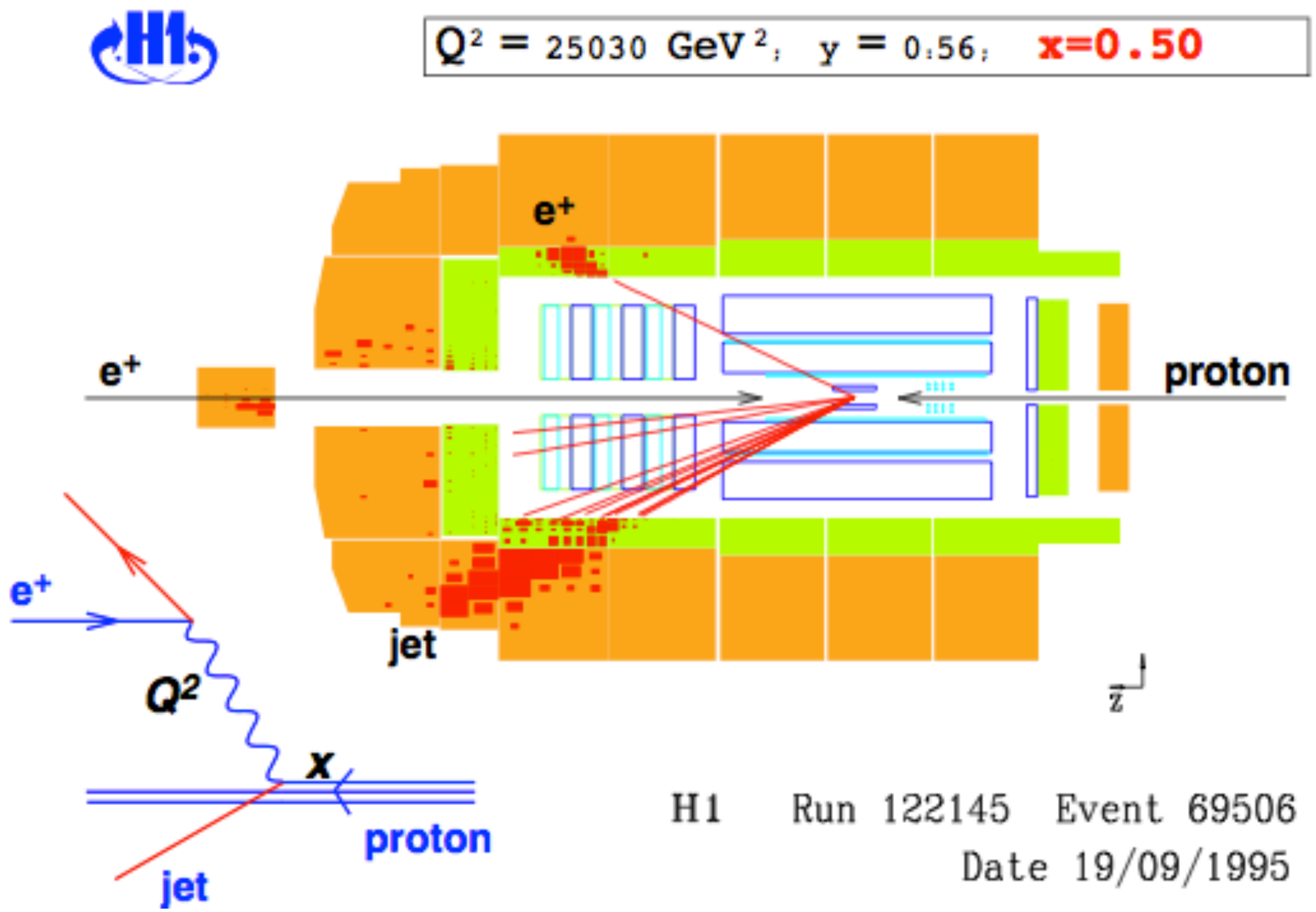
How can parton densities be extracted from data?

Deep inelastic scattering

Easier than processes with two incoming hadrons is the scattering of a lepton on a (anti)-proton



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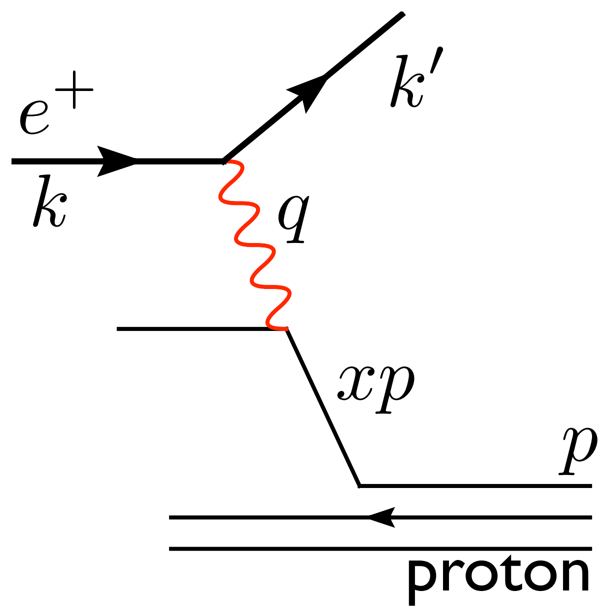

Kinematics:

$$Q^2 = -q^2 \quad s = (k+p)^2 \quad x_{Bj} = \frac{Q^2}{2p\cdot q} \quad y = \frac{p\cdot q}{k\cdot p}$$

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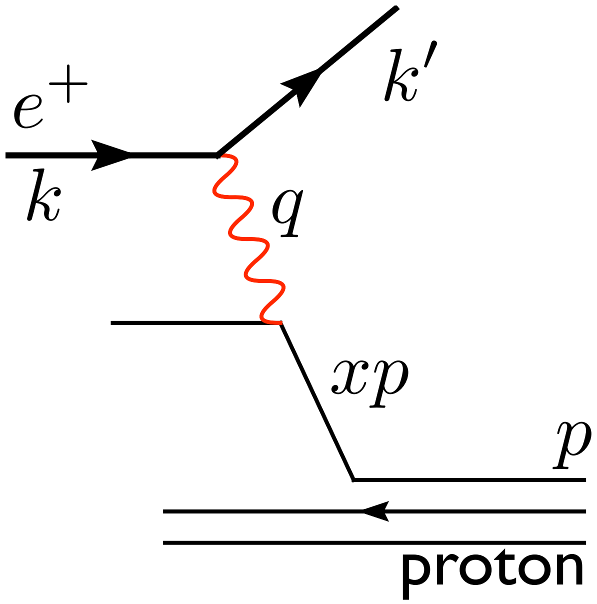

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Partonic variables:

$$\hat{p} = xp \quad \hat{s} = (k+\hat{p})^2 = 2k\cdot\hat{p} \quad \hat{y} = \frac{\hat{p}\cdot q}{k\cdot\hat{p}} = y \quad (\hat{p}+q)^2 = 2\hat{p}\cdot q - Q^2 = 0$$

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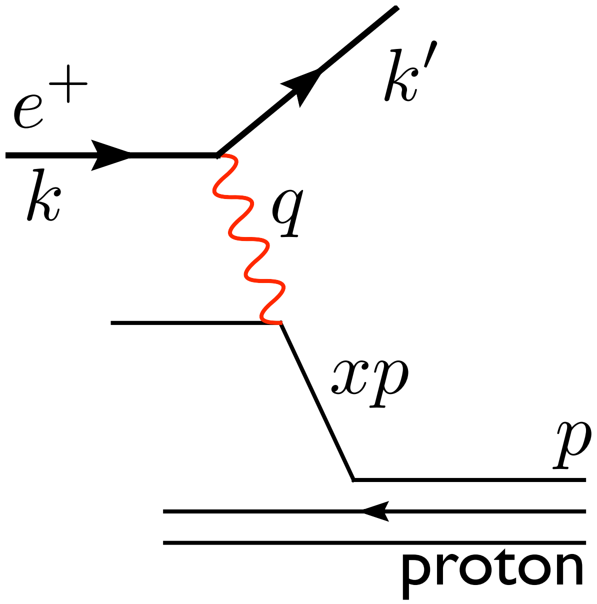

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Partonic cross section:

(just apply QED Feynman rules and add phase space)

$$\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\,\pi\,\alpha_{em} \left(1 + (1-\hat{y})^2\right)$$

Deep inelastic scattering

Hadronic cross section:

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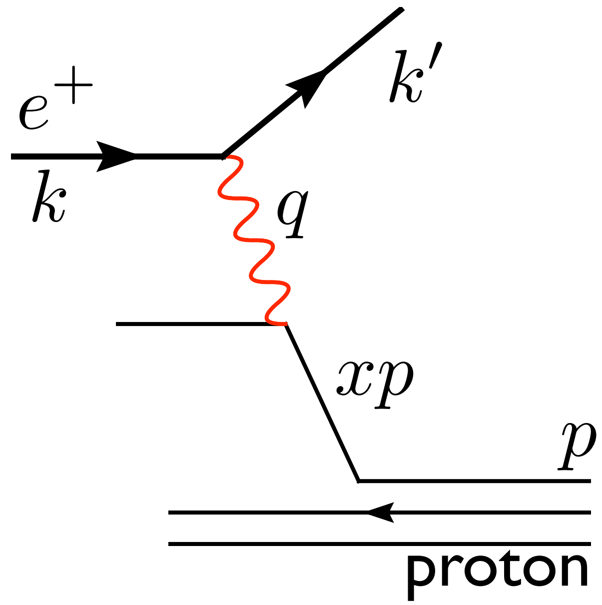
Using x = x_{BJ}

$$\frac{d\sigma}{dy\, dx_{Bj}} = \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}}$$

$$= \frac{2\pi\, \alpha_{em}^2 s x_{Bj}}{Q^4} \left(1 + (1-y)^2\right) \sum_l q_l^2 f_l^{(p)}(x_{Bj})$$



Deep inelastic scattering

<u>Hadronic cross section:</u>

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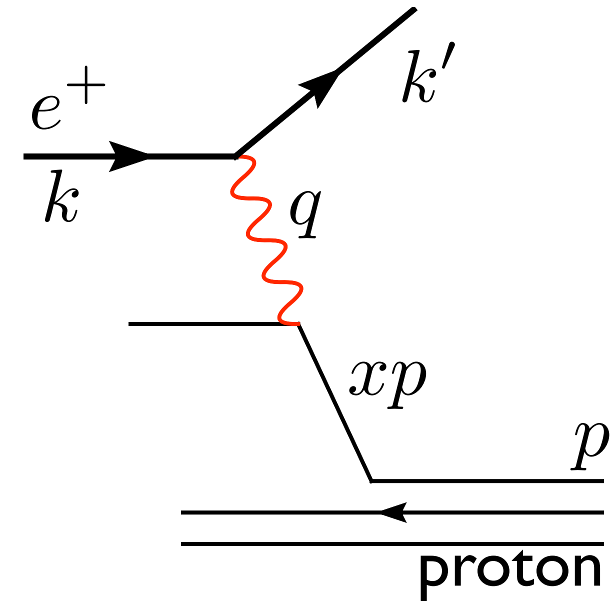

1. at fixed x_{Bj} and y the cross-section scales with s
2. the y-dependence of the cross-section is fully predicted and is typical of vector interaction with fermions ⇒Callan-Gross relation
3. can access (sums of) parton distribution functions
4. Bjorken scaling: pdfs depend on x and not on Q^2

The structure function F_2

$$\frac{d\sigma}{dydx} = \frac{2\pi\alpha_{em}^2 s}{Q^4}\left(1+(1-y^2\right)F_2(x) \qquad F_2(x) = \sum_l x q_l^2 f_l^{(p)}(x)$$

F_2 is called structure function (describes structure/constituents of nucleus)

For electron scattering on proton

$$F_2(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

NB: use perturbative language of quarks and gluons despite the fact that parton distribution are non-perturbative

Question: F_2 gives only a linear combination of u and d. How can they be extracted separately?

Isospin

Neutron is like a proton with u & d exchanged

Isospin

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For electron scattering on a proton

$$F_2^p(x) = x\left(\frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)\right)$$

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For electron scattering on a neutron

$$F_2^n(x) = x\left(\frac{1}{9}d_n(x) + \frac{4}{9}u_n(x)\right) = x\left(\frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)\right)$$

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F_2^n and F_2^p allow determination of u_p and d_p separately

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F_2^n and F_2^p allow determination of u_p and d_p separately

NB: experimentally get F_2^n from deuteron: $F_2^d(x) = F_2^p(x) + F_2^n(x)$

Sea quark distributions

Inside the proton there are fluctuations, and pairs of $u\bar{u}$,$d\bar{d}$,$c\bar{c}$,$s\bar{s}$ ... can be created

An infinite number of pairs can be created as long as they have very low momentum, because of the momentum sum rules.

We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx\,(u_p(x) - \bar{u}_p(x)) = 2 \qquad \int_0^1 dx\,\left(d_p(x) - \bar{d}_p(x)\right) = 1$$

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Photons interact in the same way with u(d) and $\bar{u}$($\bar{d}$)

How can one measure the difference?

Question: What interacts differently with particle and antiparticle?

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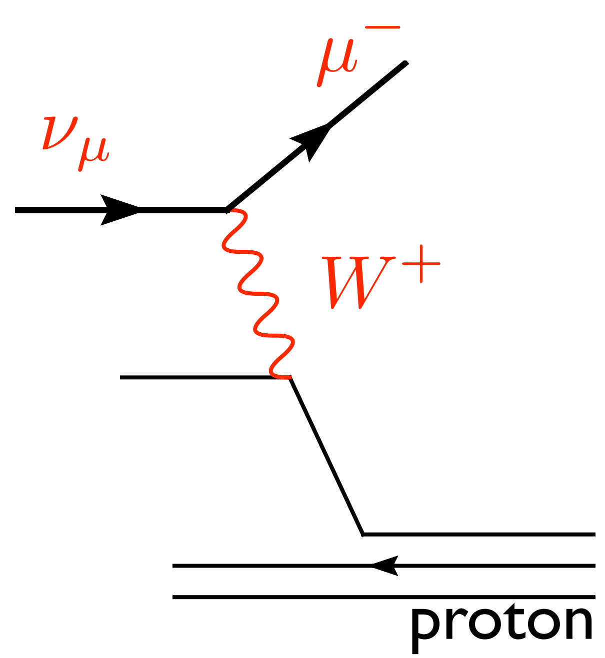
Question: What interacts differently with particle and antiparticle? W^+/W^- from neutrino scattering



Check of the momentum sum rule

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u_v	0.267
d_v	0.111
u_s	0.066
d_s	0.053
s_s	0.033
c_c	0.016
total	0.546

➟ *half of the longitudinal momentum carried by gluons*

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$\gamma/W^{+/-}$ don't interact with gluons

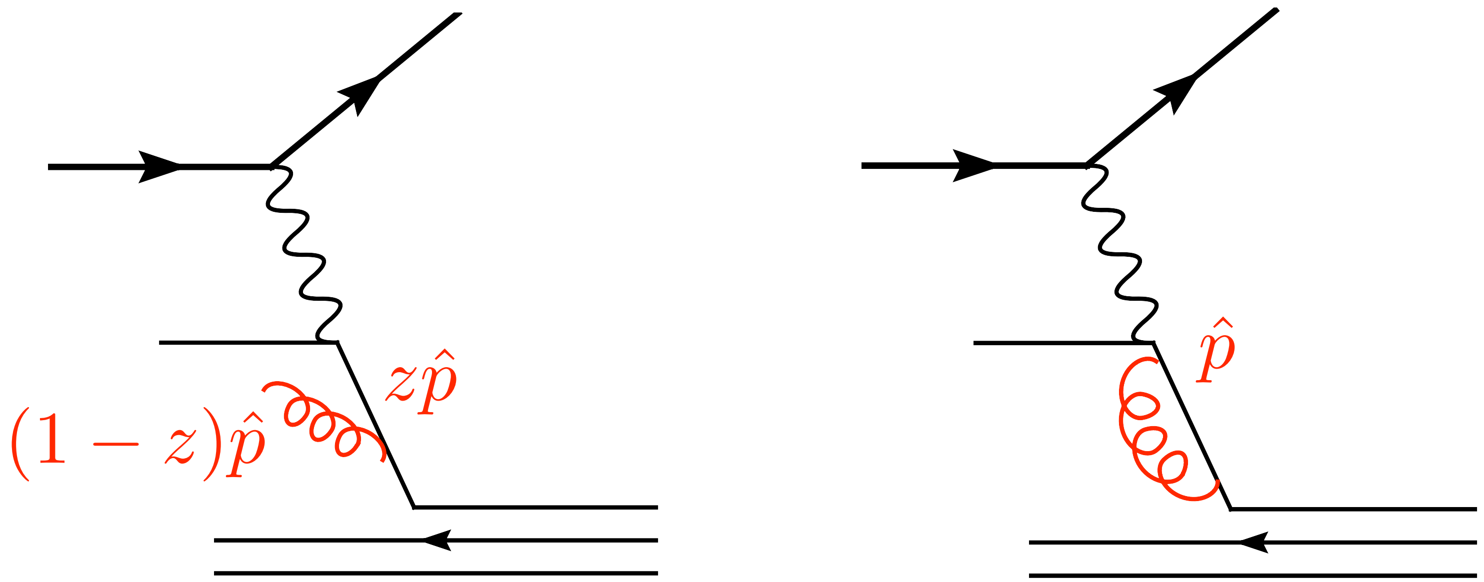
How can one measure gluon parton densities?

We need to discuss radiative effects first

Radiative corrections

To first order in the coupling:

need to consider the emission of one real gluon and a virtual one



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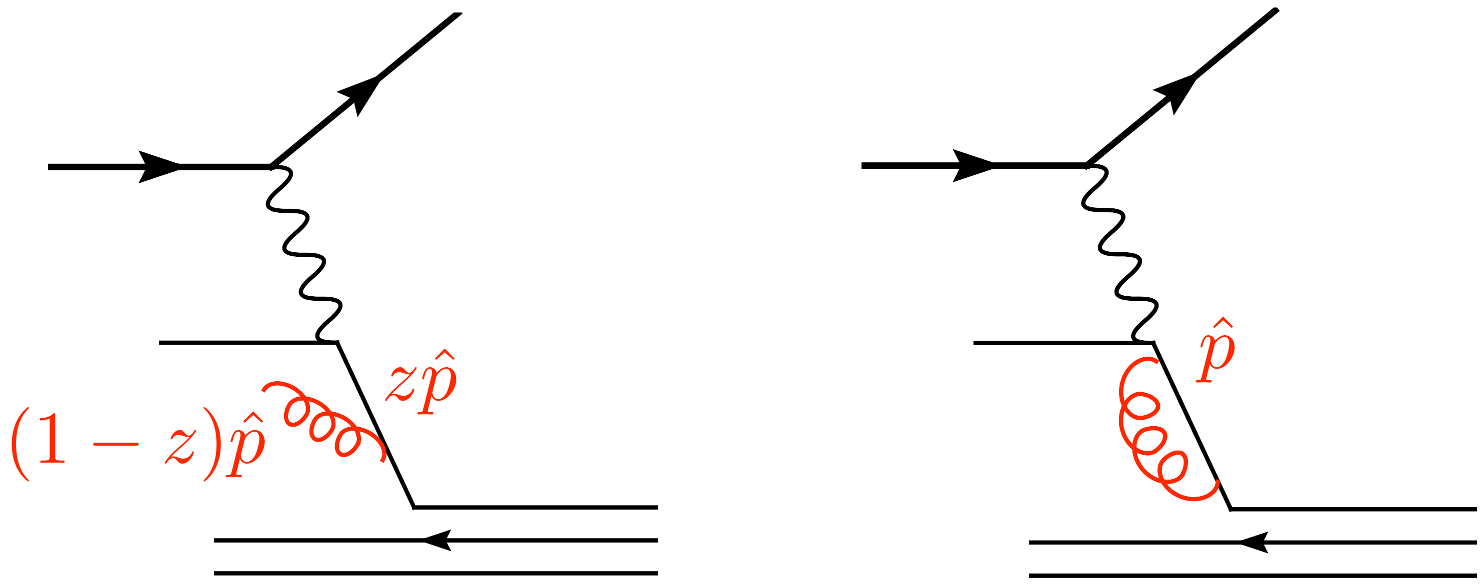

Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dk_\perp^2}{k_\perp^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right), \quad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at z=1 cancels between real and virtual terms

Collinear singularity: $k_\perp \to 0$ with finite z. Collinear singularity does not cancel because partonic scatterings occur at different energies

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$\Rightarrow$ naive parton model does not survive radiative corrections

Similarly to what is done when renormalizing UV divergences, collinear divergences from initial state emissions are absorbed into parton distribution functions

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

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Plus prescription makes the universal cancelation of singularities explicit

$$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 f(z) \left(g(z) - g(1) \right)$$

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Collinear singularities still there, but they factorize.

Factorization scale

Schematically use $$\ln \frac{Q^2}{\lambda^2} = \ln \frac{Q^2}{\mu_F^2} + \ln \frac{\mu_F^2}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_+\right) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_+\right) \sigma^{(0)}$$

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So we define

$$f_q(x,\mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{\lambda^2}P_{qq}^{(0)}\right) \qquad \hat{\sigma}(p,\mu_F) = \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu_F^2}P_{qq}^{(0)}\right)\sigma^{(0)}(p)$$

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NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently
- the factorization scale acts as a cut-off, it allows to move the divergent contribution into non-pertubative parton distribution functions

Improved parton model

Naive parton model:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \qquad \hat{s} = x_1 x_2 s$$

After radiative corrections:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

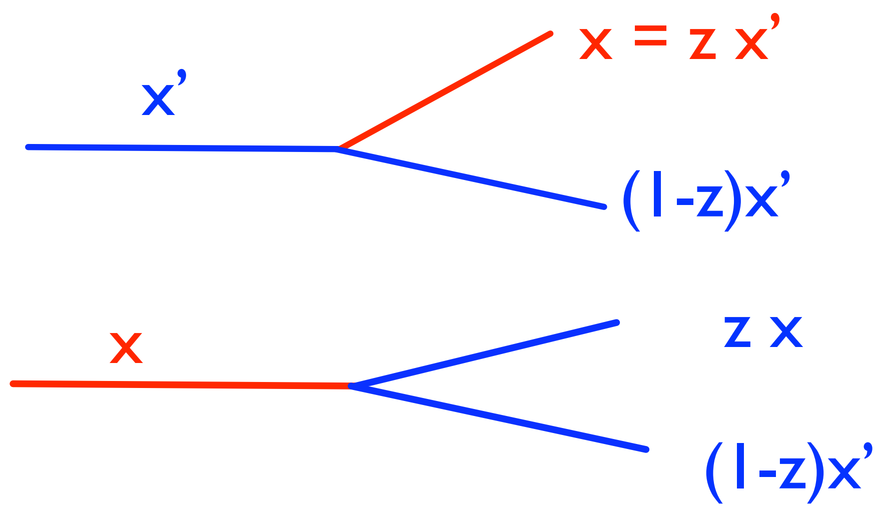
Intermediate recap

- With initial state parton collinear singularities don't cancel
- Initial state emissions with $k_\perp$ below a given scale are included in PDFs
- This procedure introduces a scale μ_F, the so-called factorization scale which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The dependence on μ_F becomes milder when including higher orders

Evolution of PDFs

A parton distribution changes when

- a different parton splits and produces it
- the parton itself splits



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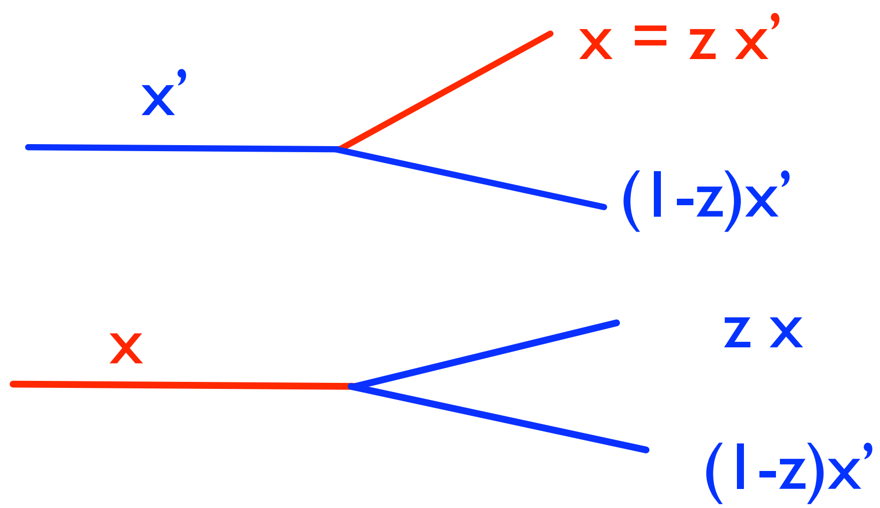


$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} P(z) f(x',\mu^2) \delta(zx'-x) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x,\mu^2)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f\left(x,\mu^2\right)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z},\mu^2\right)$$

The plus prescription $\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 dz f(z) \left(g(z) - g(1)\right)$

DGLAP equation

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

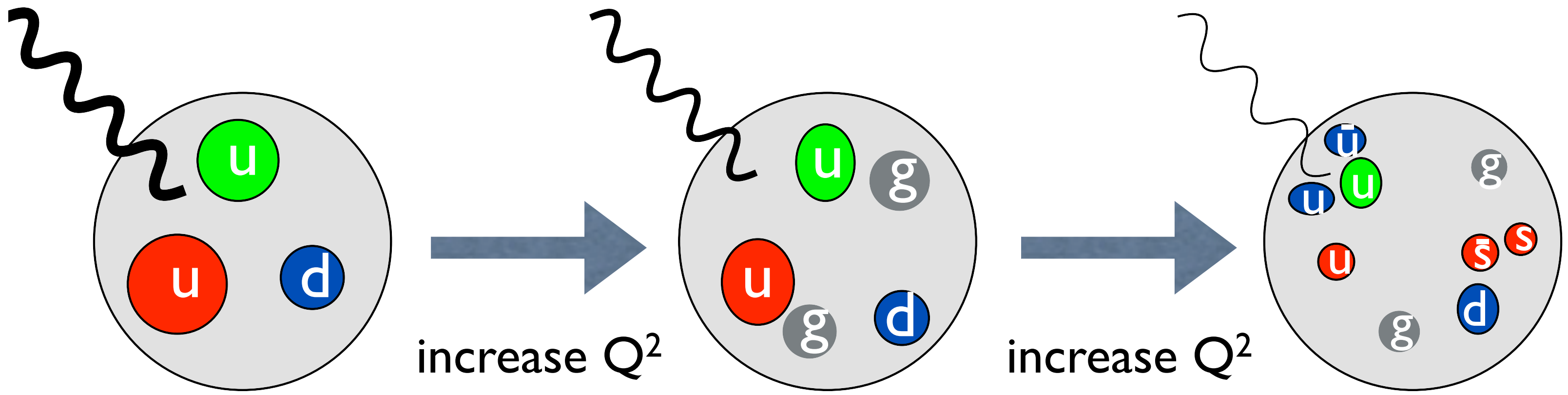
Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Evolution

So, in perturbative QCD we can not predict values for

- the coupling
- the masses
- the parton densities
- ...



What we can predict is the evolution with the Q^2 of those quantities. These quantities must be extracted at some scale from data.

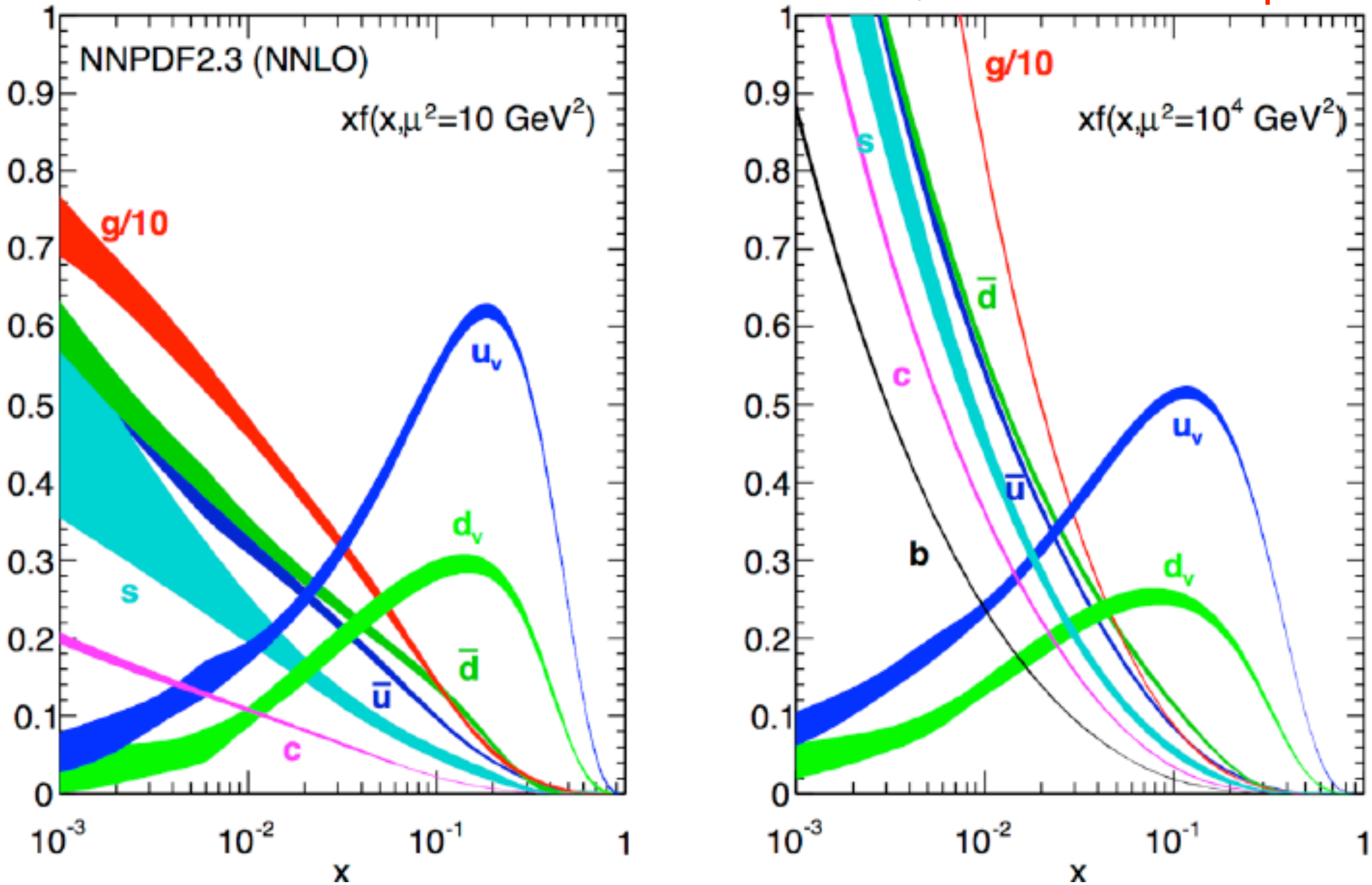
- not only is the coupling scale-dependent, but partons have a scale dependent sub-structure
- we started with the question of how one can access the gluon pdf: In DIS: because of the DGLAP evolution, we can access the gluon pdf indirectly, through the way it changes the evolution of quark pdfs. Today also direct measurements using Tevatron jet data and LHC tt production

DGLAP Evolution

The DGLAP evolution is a key to precision LHC phenomenology: it allows to measure PDFs at some scale (say in DIS) and evolve upwards to make LHC (7, 8, 13, 14, 33, 100.... TeV) predictions

Measure PDFs at 10 GeV

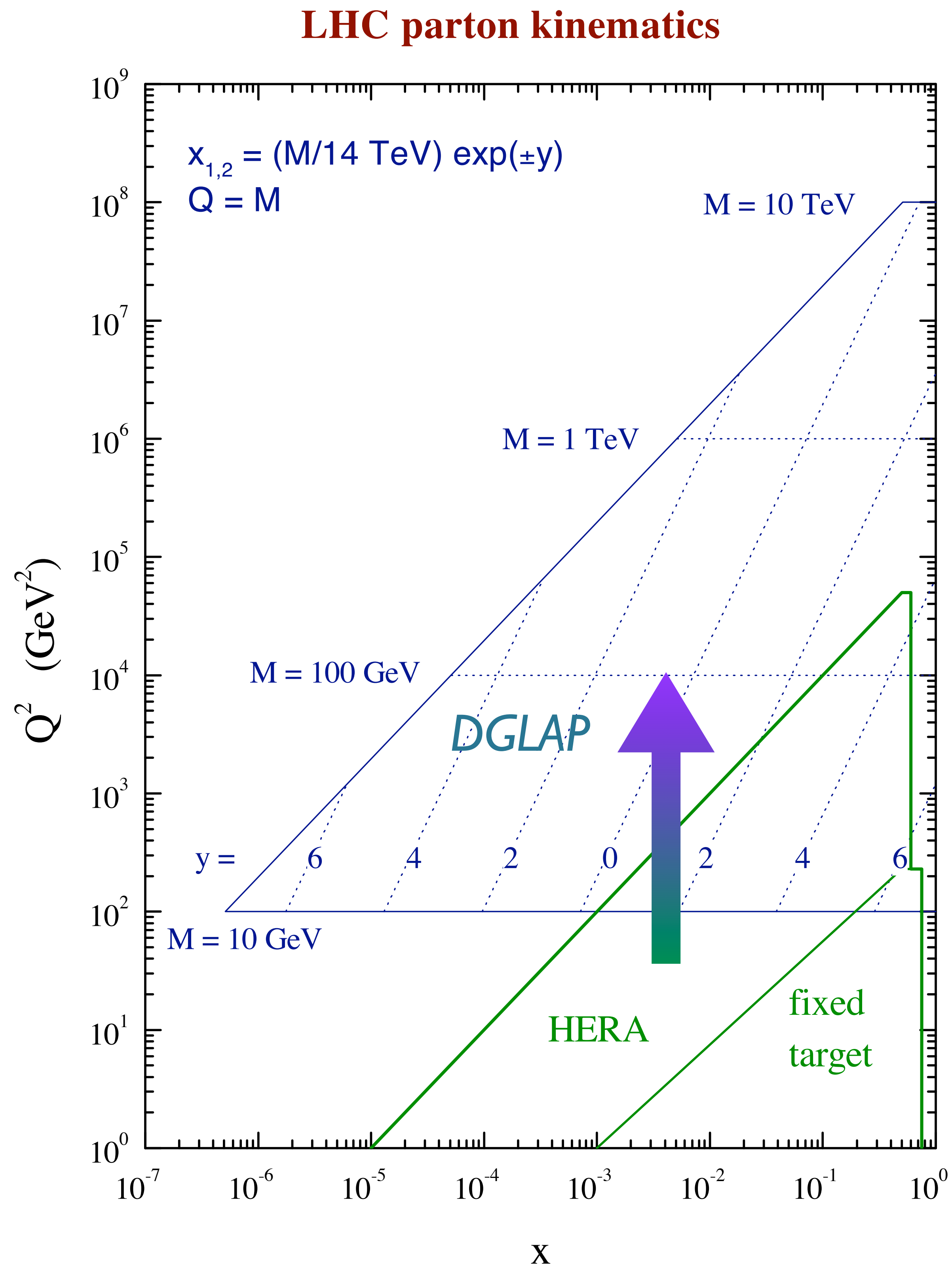
Evolve in Q^2 and make LHC predictions



Different PDFs evolve in different ways (different equations + unitarity constraint)

Parton density coverage

- most of the LHC x-range covered by Hera
- need 2-3 orders of magnitude Q^2-evolution
- rapidity distributions probe extreme x-values
- 100 GeV physics at LHC: small-x, sea partons
- TeV physics: large x



Parton densities: recent progress

Recent major progress:

- full NNLO evolution (previous approximate NNLO)
- improved treatment of heavy flavors near the quark mass
- more systematic use of uncertainties/correlations (e.g. dynamic tolerance, combinations of PDF + α_s uncertainty)
- Neural Network (NN) PDFs

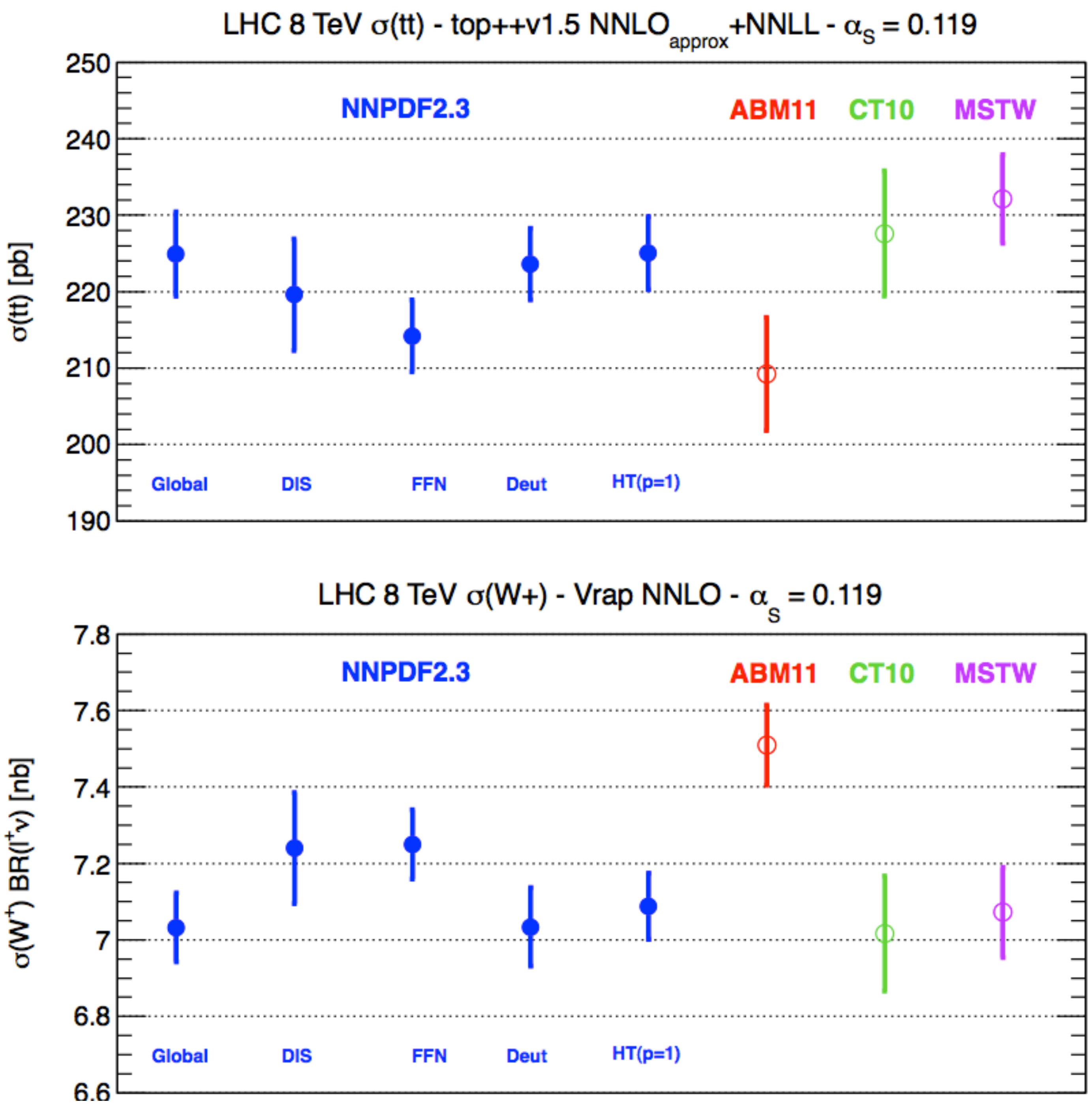
ABM, CTEQ, MSTW, NN collaboration

Still, considerable differences in predictions for benchmark process.

Parton densities: benchmark processes

Uncertainty from PDFs (no α_s) on benchmark processes

NN col. 1303.1189



In general differences due to:

1) different data in fits
2) different methodology [parametrization, theory]
3) treatment of heavy quarks
4) different α_s

Next: Perturbative calculations

Next, we will focus on perturbative calculations

- LO, NLO, NLO+MC, NNLO
- techniques, issues with divergences
- current status, sample results

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Perturbative calculations rely on the idea of an order-by-order expansion in the small coupling

$$\sigma \sim A + B\alpha_s + C\alpha_s^2 + D\alpha_s^3 + \dots$$

LO NLO NNLO NNNLO

Perturbative calculations

- Perturbative calculations = fixed-order expansion in the coupling constant, or more refined expansions that include terms to all orders
- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale)
- So changing scale the result changes. By how much? What does this dependence mean?
- Let's consider some examples

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

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So the change of scale is a NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

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- Notice that at Leading Order the normalization is not under control:

$$\frac{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu)}{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')}\right)^n$$

NLO n-jet cross-section

Now consider n-jet cross-section at NLO. At scale μ the result reads

$$\sigma^{\mathrm{NLO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots) + \alpha_s(\mu)^{n+1}\left(B(p_i, \epsilon_i, \dots) - n b_0 \ln\frac{\mu^2}{Q_0^2}\right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO
- Scale dependence and normalization start being under control only at NLO, since a compensation mechanism kicks in
- Notice also that a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the PT expansion
- Scale variation is conventionally used to estimate the theory uncertainty, but the validity of this procedure should not be overrated (see later)

Leading order with Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

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Bottlenecks

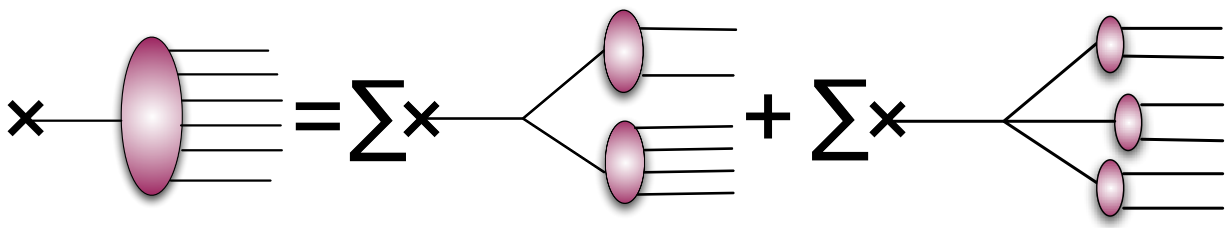
a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Techniques beyond Feynman diagrams

✓ Berends-Giele relations: compute helicity amplitudes **recursively** using off-shell currents



Berends, Giele '88

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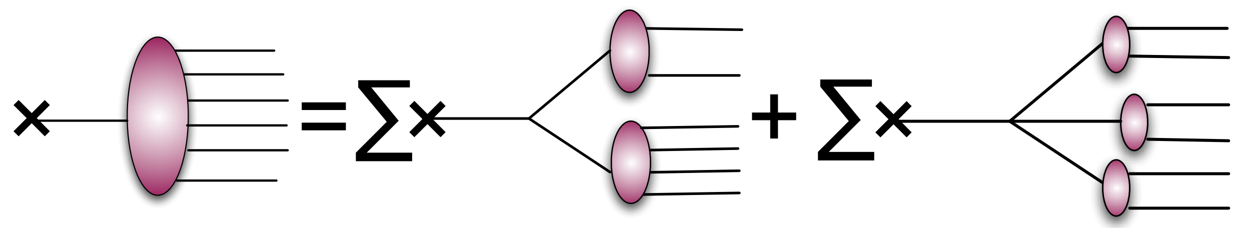
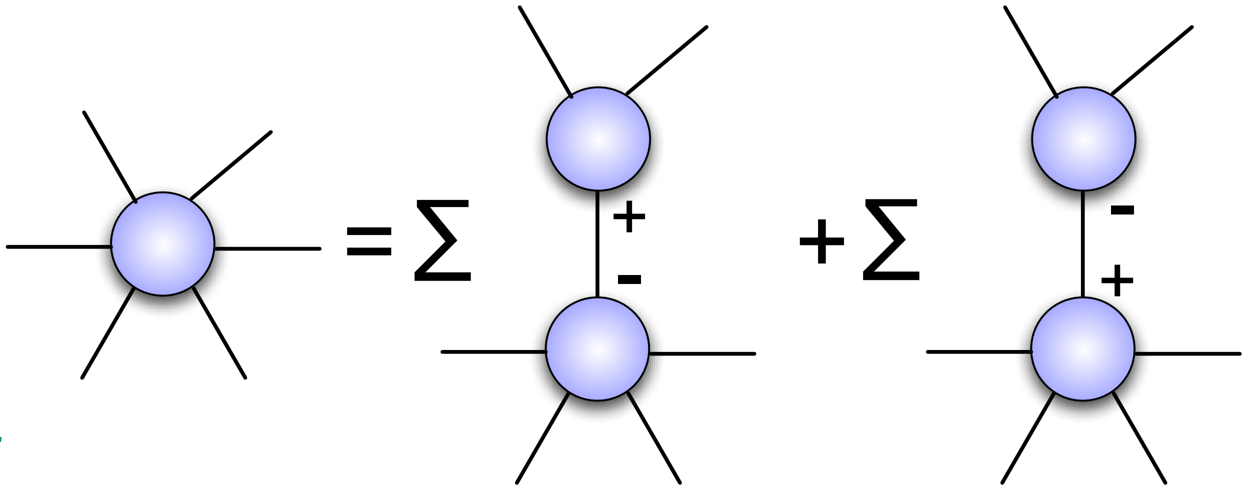


Berends, Giele '88

✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)



Britto, Cachazo, Feng '04

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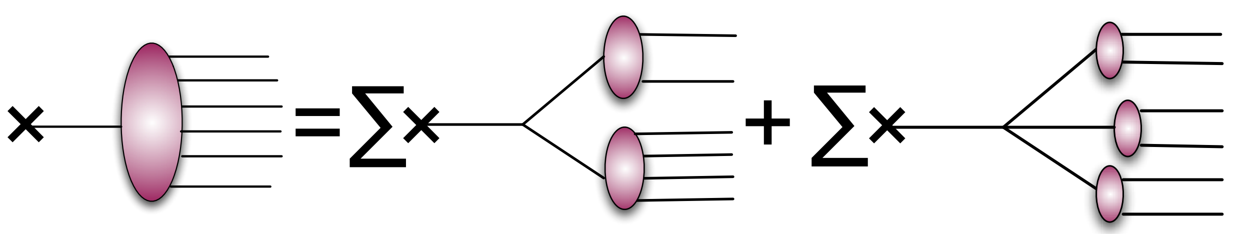

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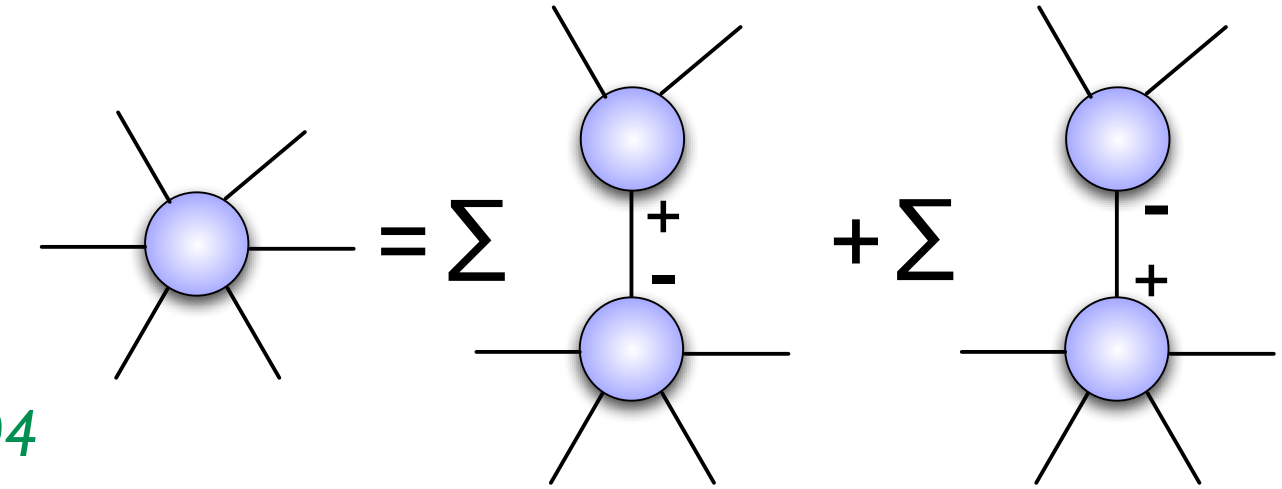
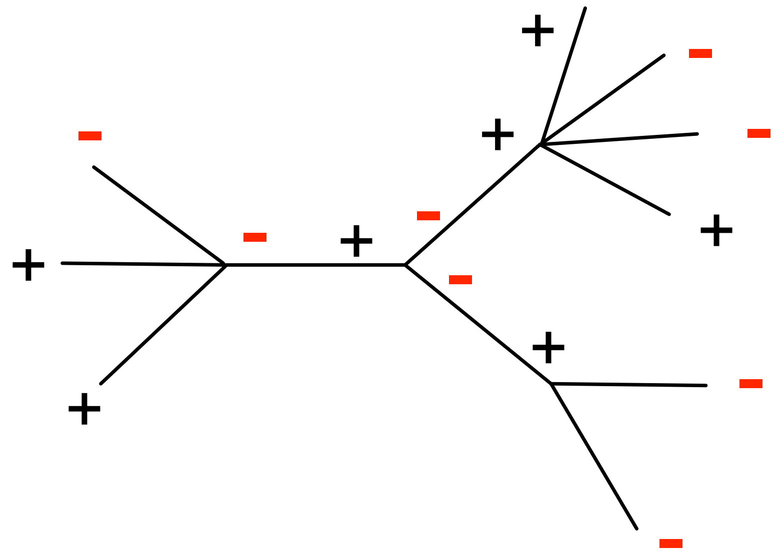


Britto, Cachazo, Feng '04

✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten '04

Benefits and drawbacks of LO

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- fastest option; often the only one
- test quickly new ideas with fully exclusive description
- many working, well-tested approaches
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Drawbacks of LO:

- large scale dependences, reflecting large theory uncertainty
- no control on normalization
- poor control on shapes
- poor modeling of jets

Example: W+4 jet cross-section $\propto \alpha_s(Q)^4$

Vary $\alpha_s(Q)$ by ±10% via change of Q ⇒ cross-section varies by ±40%

Next-to-leading order

Benefits of next-to-leading order (NLO)

- reduce dependence on unphysical scales
- establish normalization and shape of cross-sections
- small scale dependence at LO can be very misleading, small dependence at NLO robust sign that PT is under control
- large NLO correction or large dependence at NLO robust sign that neglected other higher order are important
- through loop effects get indirect information about sectors not directly accessible

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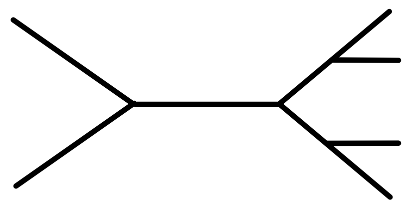

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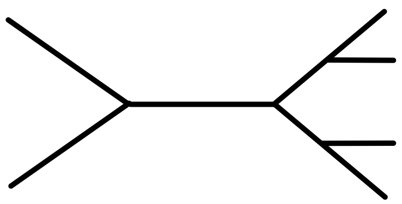

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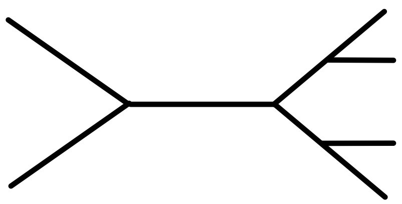

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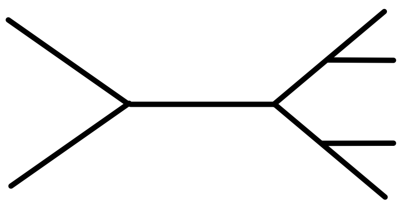

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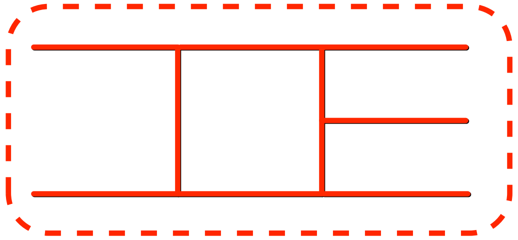

Bottleneck for a long time. Now understood how to compute this automatically

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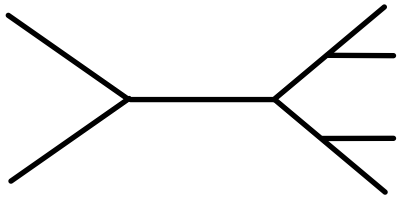

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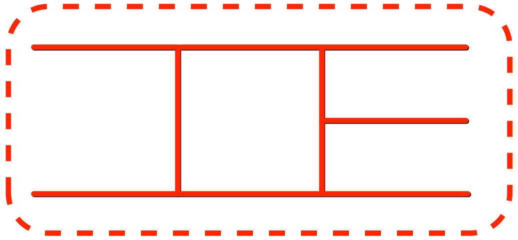

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- ☑ set of subtraction terms to cancel divergences

We won't have time to do detailed NLO calculations, but let's look a bit more in detail at the issue of divergences/subtraction

Regularization in QCD

Regularization: a way to make intermediate divergent quantities meaningful

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- In QCD dimensional regularization is today the standard procedure, based on the fact that d-dimensional integrals are more convergent if one reduces the number of dimensions.

$$\int \frac{d^4 l}{(2\pi)^4} \to \mu^{2\epsilon} \int \frac{d^d l}{(2\pi)^d}\,, \quad d = 4 - 2\epsilon < 4$$

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Alternative regularization schemes: photon mass (EW), cut-offs, Pauli-Villard ...
Compared to those methods, dimensional regularizatiom has the big virtue that it leaves the regularized theory Lorentz invariant, gauge invariant, unitary etc.

Subtraction and slicing methods

- Consider e.g. an n-jet cross-section with some arbitrary infrared safe jet definition. At NLO, two divergent integrals, but the sum is finite

$$\sigma^J_{\rm NLO} = \int_{n+1} d\sigma^J_{\rm R} + \int_n d\sigma^J_{\rm V}$$

- Since one integrates over a different number of particles in the final state, real and virtual need to be evaluated first, and combined then

- This means that one needs to find a way of removing divergences before evaluating the phase space integrals

- Two main techniques to do this
  - *phase space slicing* ⇒ obsolete because of practical/numerical issues
  - *subtraction method* ⇒ most used in recent applications

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1}|\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1, \ldots, p_{n+1})$$

where F^J is the arbitrary jet-definition

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- IR divergences in the loop integration regularized by taking D = 4-2ε

$$2\,\mathrm{Re}\{\mathcal{M}_V \cdot \mathcal{M}_0^*\} = \frac{1}{\epsilon} \mathcal{V}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma_{\mathrm{NLO}}^{J} = \int_{0}^{1} \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_{n+1}^{J}(x) + \frac{1}{\epsilon} \mathcal{V} F_{n}^{J}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

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- One can then add and subtract the analytically computed divergent part

$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F^J_n + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F^J_n + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

Subtraction method

- This can be rewritten exactly as

$$\sigma_{\rm NLO}^{J} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) \left(F_1^J(x) - \mathcal{V} F_0^J\right) + \mathcal{O}(1) \mathcal{V} F_0^J$$

$\Rightarrow$ Now both terms are finite and can be evaluated numerically

- Subtracted cross-section must be calculated separately for each process (but mostly automated now). It must be valid everywhere in phase space
- Systematized in the seminal papers of Catani-Seymour (dipole subtraction, '96) and Frixione-Kunszt-Signer (FKS method, '96)
- Subtraction used in all recent NLO applications and public codes (Event2, Disent, MCFM, NLOjet++, MC@NLO, POWHEG ...)

Approaches to virtual (loop) part of NLO

Two complementary approaches:

- Numerical/traditional Feynman diagram methods:
  use robust computational methods [integration by parts, reduction techniques...], then let the computer do the work for you

  Bottleneck:
  factorial growth, 2 → 4 doable, difficult to go beyond

- Analytical approaches:
  improve understanding of field theory [e.g. generalized unitarity, recursions, OPP, Open Loops ...]

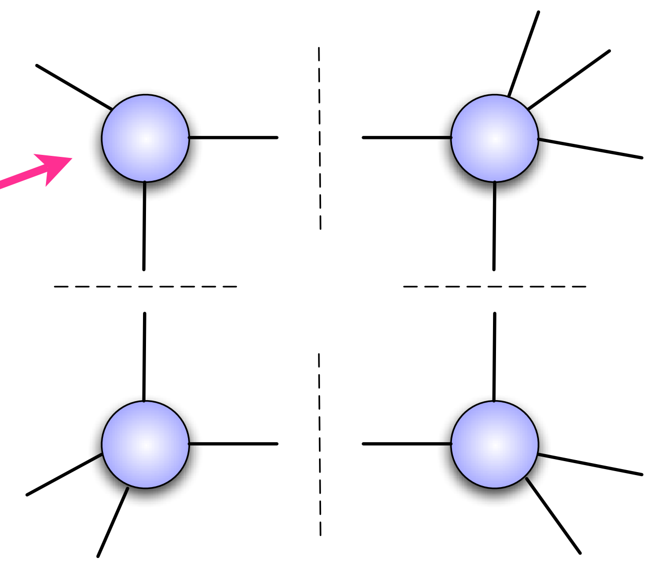
  Status:
  moving towards more legs (5 or 6 in the final state) + towards full automation [GoSam, MadLoop]

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

1) "... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ..."

NB: non-zero because cut gives complex momenta



Britto, Cachazo, Feng '04

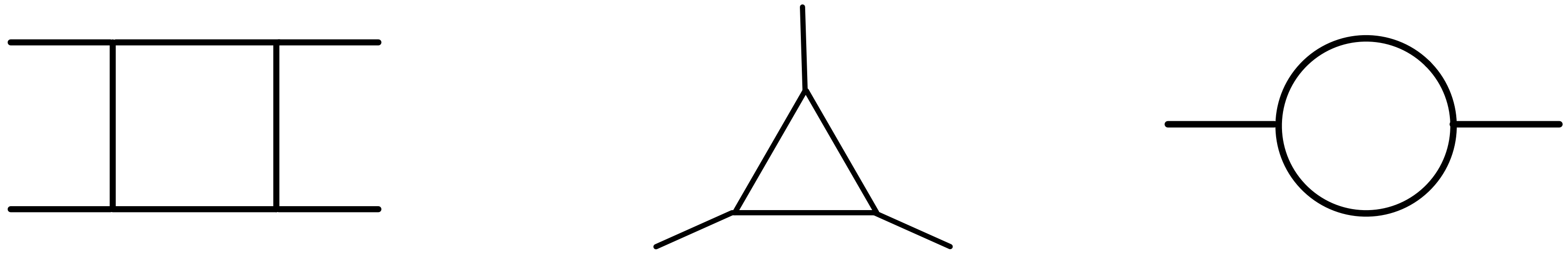
Quadrupole cuts: 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But rational part of the amplitude, coming from D=4-2ε not 4, computed separately

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

2) The OPP method: "We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals...."

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right)$$



Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving system of equations: no loops, no twistors, just algebra!

The 2007 Les Houches wishlist

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress)
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6, 7]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$; VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
11. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
12. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

CLOSED

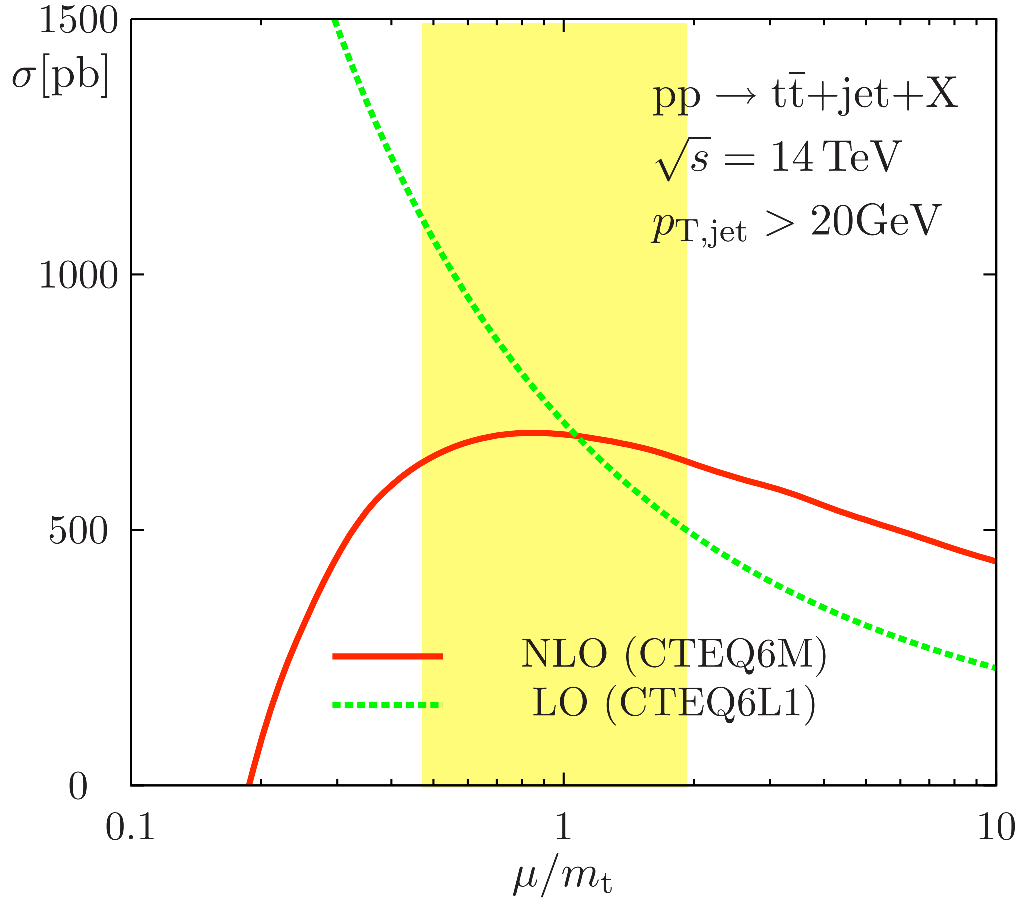
with Feynman diagrams

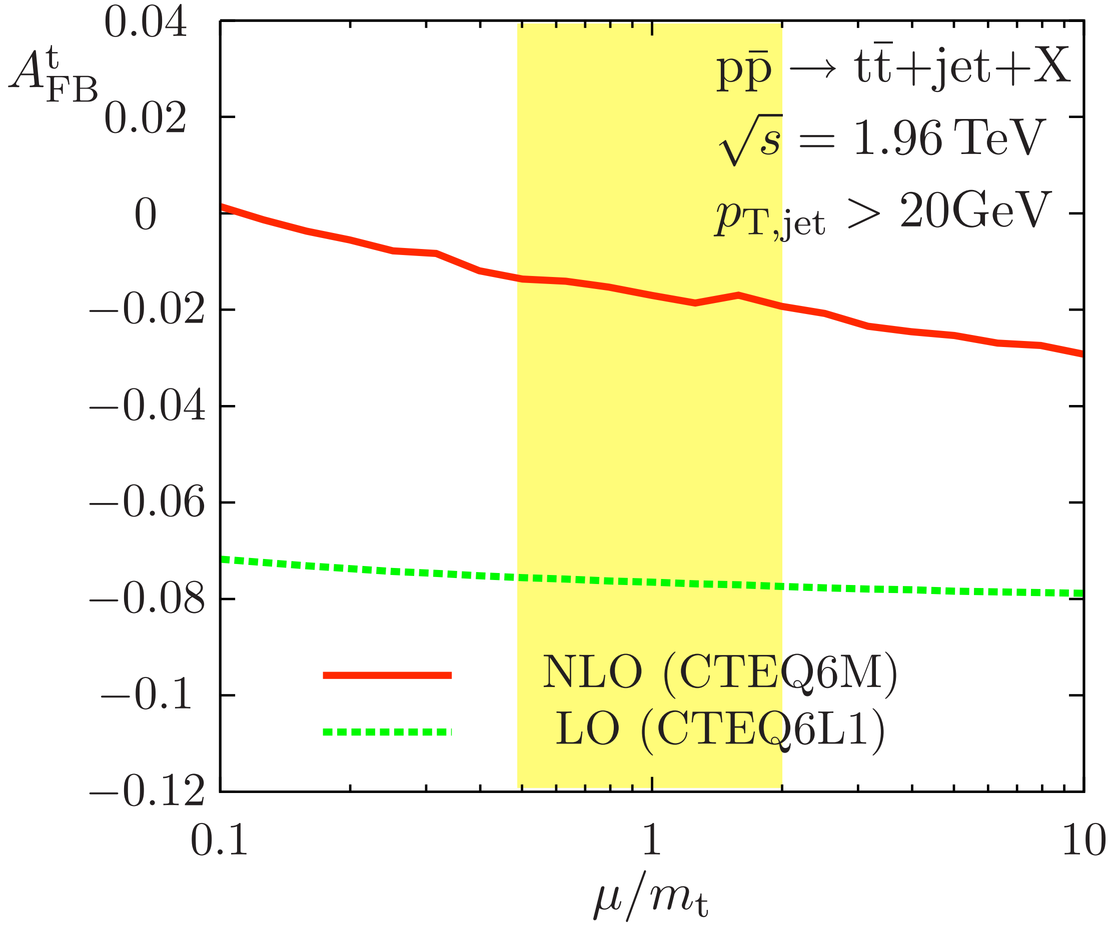
with Feynman diagrams or unitarity/onshell methods

The NLO multi-leg Working group report 0803.0494

Example of NLO result: tt+1jet

Dittmaier, Kallweit, Uwer '07-'08





- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- essential ingredient of NNLO tt production

Automated NLO

Alwall et al '14

Process		Syntax	Cross section (pb)			
Vector boson +jets			LO 13 TeV		NLO 13 TeV	
a.1	$pp \to W^{\pm}$	`p p > wpm`	$1.375 \pm 0.002 \cdot 10^5$	$^{+15.4\%}_{-16.6\%}$ $^{+2.0\%}_{-1.6\%}$	$1.773 \pm 0.007 \cdot 10^5$	$^{+5.2\%}_{-9.4\%}$ $^{+1.9\%}_{-1.6\%}$
a.2	$pp \to W^{\pm}j$	`p p > wpm j`	$2.045 \pm 0.001 \cdot 10^4$	$^{+19.7\%}_{-17.2\%}$ $^{+1.4\%}_{-1.1\%}$	$2.843 \pm 0.010 \cdot 10^4$	$^{+5.9\%}_{-8.0\%}$ $^{+1.3\%}_{-1.1\%}$
a.3	$pp \to W^{\pm}jj$	`p p > wpm j j`	$6.805 \pm 0.015 \cdot 10^3$	$^{+24.5\%}_{-18.6\%}$ $^{+0.8\%}_{-0.7\%}$	$7.786 \pm 0.030 \cdot 10^3$	$^{+2.4\%}_{-6.0\%}$ $^{+0.9\%}_{-0.8\%}$
a.4	$pp \to W^{\pm}jjj$	`p p > wpm j j j`	$1.821 \pm 0.002 \cdot 10^3$	$^{+41.0\%}_{-27.1\%}$ $^{+0.5\%}_{-0.5\%}$	$2.005 \pm 0.008 \cdot 10^3$	$^{+0.9\%}_{-6.7\%}$ $^{+0.6\%}_{-0.5\%}$
a.5	$pp \to Z$	`p p > z`	$4.248 \pm 0.005 \cdot 10^4$	$^{+14.6\%}_{-15.8\%}$ $^{+2.0\%}_{-1.6\%}$	$5.410 \pm 0.022 \cdot 10^4$	$^{+4.6\%}_{-8.6\%}$ $^{+1.9\%}_{-1.5\%}$
a.6	$pp \to Zj$	`p p > z j`	$7.209 \pm 0.005 \cdot 10^3$	$^{+19.3\%}_{-17.0\%}$ $^{+1.2\%}_{-1.0\%}$	$9.742 \pm 0.035 \cdot 10^3$	$^{+5.8\%}_{-7.8\%}$ $^{+1.2\%}_{-1.0\%}$
a.7	$pp \to Zjj$	`p p > z j j`	$2.348 \pm 0.006 \cdot 10^3$	$^{+24.3\%}_{-18.5\%}$ $^{+0.6\%}_{-0.6\%}$	$2.665 \pm 0.010 \cdot 10^3$	$^{+2.5\%}_{-6.0\%}$ $^{+0.7\%}_{-0.7\%}$
a.8	$pp \to Zjjj$	`p p > z j j j`	$6.314 \pm 0.008 \cdot 10^2$	$^{+40.8\%}_{-27.0\%}$ $^{+0.5\%}_{-0.5\%}$	$6.996 \pm 0.028 \cdot 10^2$	$^{+1.1\%}_{-6.8\%}$ $^{+0.5\%}_{-0.5\%}$
a.9	$pp \to \gamma j$	`p p > a j`	$1.964 \pm 0.001 \cdot 10^4$	$^{+31.2\%}_{-26.0\%}$ $^{+1.7\%}_{-1.8\%}$	$5.218 \pm 0.025 \cdot 10^4$	$^{+24.5\%}_{-21.4\%}$ $^{+1.4\%}_{-1.6\%}$
a.10	$pp \to \gamma jj$	`p p > a j j`	$7.815 \pm 0.008 \cdot 10^3$	$^{+32.8\%}_{-24.2\%}$ $^{+0.9\%}_{-1.2\%}$	$1.004 \pm 0.004 \cdot 10^4$	$^{+5.9\%}_{-10.9\%}$ $^{+0.8\%}_{-1.2\%}$

Automated NLO

Alwall et al '14

Process		Syntax	Cross section (pb)					
Vector-boson pair +jets			LO 13 TeV			NLO 13 TeV		
b.1	$pp \to W^+W^-$ (4f)	p p > w+ w-	$7.355 \pm 0.005 \cdot 10^1$	$^{+5.0\%}_{-6.1\%}$	$^{+2.0\%}_{-1.5\%}$	$1.028 \pm 0.003 \cdot 10^2$	$^{+4.0\%}_{-4.5\%}$	$^{+1.9\%}_{-1.4\%}$
b.2	$pp \to ZZ$	p p > z z	$1.097 \pm 0.002 \cdot 10^1$	$^{+4.5\%}_{-5.6\%}$	$^{+1.9\%}_{-1.5\%}$	$1.415 \pm 0.005 \cdot 10^1$	$^{+3.1\%}_{-3.7\%}$	$^{+1.8\%}_{-1.4\%}$
b.3	$pp \to ZW^\pm$	p p > z wpm	$2.777 \pm 0.003 \cdot 10^1$	$^{+3.6\%}_{-4.7\%}$	$^{+2.0\%}_{-1.5\%}$	$4.487 \pm 0.013 \cdot 10^1$	$^{+4.4\%}_{-4.4\%}$	$^{+1.7\%}_{-1.3\%}$
b.4	$pp \to \gamma\gamma$	p p > a a	$2.510 \pm 0.002 \cdot 10^1$	$^{+22.1\%}_{-22.4\%}$	$^{+2.4\%}_{-2.1\%}$	$6.593 \pm 0.021 \cdot 10^1$	$^{+17.6\%}_{-18.8\%}$	$^{+2.0\%}_{-1.9\%}$
b.5	$pp \to \gamma Z$	p p > a z	$2.523 \pm 0.004 \cdot 10^1$	$^{+9.9\%}_{-11.2\%}$	$^{+2.0\%}_{-1.6\%}$	$3.695 \pm 0.013 \cdot 10^1$	$^{+5.4\%}_{-7.1\%}$	$^{+1.8\%}_{-1.4\%}$
b.6	$pp \to \gamma W^\pm$	p p > a wpm	$2.954 \pm 0.005 \cdot 10^1$	$^{+9.5\%}_{-11.0\%}$	$^{+2.0\%}_{-1.7\%}$	$7.124 \pm 0.026 \cdot 10^1$	$^{+9.7\%}_{-9.9\%}$	$^{+1.5\%}_{-1.3\%}$
b.7	$pp \to W^+W^-j$ (4f)	p p > w+ w- j	$2.865 \pm 0.003 \cdot 10^1$	$^{+11.6\%}_{-10.0\%}$	$^{+1.0\%}_{-0.8\%}$	$3.730 \pm 0.013 \cdot 10^1$	$^{+4.9\%}_{-4.9\%}$	$^{+1.1\%}_{-0.8\%}$
b.8	$pp \to ZZj$	p p > z z j	$3.662 \pm 0.003 \cdot 10^0$	$^{+10.9\%}_{-9.3\%}$	$^{+1.0\%}_{-0.8\%}$	$4.830 \pm 0.016 \cdot 10^0$	$^{+5.0\%}_{-4.8\%}$	$^{+1.1\%}_{-0.9\%}$
b.9	$pp \to ZW^\pm j$	p p > z wpm j	$1.605 \pm 0.005 \cdot 10^1$	$^{+11.6\%}_{-10.0\%}$	$^{+0.9\%}_{-0.7\%}$	$2.086 \pm 0.007 \cdot 10^1$	$^{+4.9\%}_{-4.8\%}$	$^{+0.9\%}_{-0.7\%}$
b.10	$pp \to \gamma\gamma j$	p p > a a j	$1.022 \pm 0.001 \cdot 10^1$	$^{+20.3\%}_{-17.7\%}$	$^{+1.2\%}_{-1.5\%}$	$2.292 \pm 0.010 \cdot 10^1$	$^{+17.2\%}_{-15.1\%}$	$^{+1.0\%}_{-1.4\%}$
b.11*	$pp \to \gamma Zj$	p p > a z j	$8.310 \pm 0.017 \cdot 10^0$	$^{+14.5\%}_{-12.8\%}$	$^{+1.0\%}_{-1.0\%}$	$1.220 \pm 0.005 \cdot 10^1$	$^{+7.3\%}_{-7.4\%}$	$^{+0.9\%}_{-0.9\%}$
b.12*	$pp \to \gamma W^\pm j$	p p > a wpm j	$2.546 \pm 0.010 \cdot 10^1$	$^{+13.7\%}_{-12.1\%}$	$^{+0.9\%}_{-1.0\%}$	$3.713 \pm 0.015 \cdot 10^1$	$^{+7.2\%}_{-7.1\%}$	$^{+0.9\%}_{-1.0\%}$
b.13	$pp \to W^+W^+jj$	p p > w+ w+ j j	$1.484 \pm 0.006 \cdot 10^{-1}$	$^{+25.4\%}_{-18.9\%}$	$^{+2.1\%}_{-1.5\%}$	$2.251 \pm 0.011 \cdot 10^{-1}$	$^{+10.5\%}_{-10.6\%}$	$^{+2.2\%}_{-1.6\%}$
b.14	$pp \to W^-W^-jj$	p p > w- w- j j	$6.752 \pm 0.007 \cdot 10^{-2}$	$^{+25.4\%}_{-18.9\%}$	$^{+2.4\%}_{-1.7\%}$	$1.003 \pm 0.003 \cdot 10^{-1}$	$^{+10.1\%}_{-10.4\%}$	$^{+2.5\%}_{-1.8\%}$
b.15	$pp \to W^+W^-jj$ (4f)	p p > w+ w- j j	$1.144 \pm 0.002 \cdot 10^1$	$^{+27.2\%}_{-19.9\%}$	$^{+0.7\%}_{-0.5\%}$	$1.396 \pm 0.005 \cdot 10^1$	$^{+5.0\%}_{-6.8\%}$	$^{+0.7\%}_{-0.6\%}$
b.16	$pp \to ZZjj$	p p > z z j j	$1.344 \pm 0.002 \cdot 10^0$	$^{+26.6\%}_{-19.6\%}$	$^{+0.7\%}_{-0.6\%}$	$1.706 \pm 0.011 \cdot 10^0$	$^{+5.8\%}_{-7.2\%}$	$^{+0.8\%}_{-0.6\%}$
b.17	$pp \to ZW^\pm jj$	p p > z wpm j j	$8.038 \pm 0.009 \cdot 10^0$	$^{+26.7\%}_{-19.7\%}$	$^{+0.7\%}_{-0.5\%}$	$9.139 \pm 0.031 \cdot 10^0$	$^{+3.1\%}_{-5.1\%}$	$^{+0.7\%}_{-0.5\%}$
b.18	$pp \to \gamma\gamma jj$	p p > a a j j	$5.377 \pm 0.029 \cdot 10^0$	$^{+26.2\%}_{-19.8\%}$	$^{+0.6\%}_{-1.0\%}$	$7.501 \pm 0.032 \cdot 10^0$	$^{+8.8\%}_{-10.1\%}$	$^{+0.6\%}_{-1.0\%}$
b.19*	$pp \to \gamma Zjj$	p p > a z j j	$3.260 \pm 0.009 \cdot 10^0$	$^{+24.3\%}_{-18.4\%}$	$^{+0.6\%}_{-0.6\%}$	$4.242 \pm 0.016 \cdot 10^0$	$^{+6.5\%}_{-7.3\%}$	$^{+0.6\%}_{-0.6\%}$
b.20*	$pp \to \gamma W^\pm jj$	p p > a wpm j j	$1.233 \pm 0.002 \cdot 10^1$	$^{+24.7\%}_{-18.6\%}$	$^{+0.6\%}_{-0.6\%}$	$1.448 \pm 0.005 \cdot 10^1$	$^{+3.6\%}_{-5.4\%}$	$^{+0.6\%}_{-0.7\%}$

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	Process Three vector bosons +jet	Syntax	Cross section (pb) LO 13 TeV		NLO 13 TeV	
c.1	$pp \to W^+W^-W^\pm$ (4f)	`p p > w+ w- wpm`	$1.307 \pm 0.003 \cdot 10^{-1}$	$^{+0.0\%}_{-0.3\%}$ $^{+2.0\%}_{-1.5\%}$	$2.109 \pm 0.006 \cdot 10^{-1}$	$^{+5.1\%}_{-4.1\%}$ $^{+1.6\%}_{-1.2\%}$
c.2	$pp \to ZW^+W^-$ (4f)	`p p > z w+ w-`	$9.658 \pm 0.065 \cdot 10^{-2}$	$^{+0.8\%}_{-1.1\%}$ $^{+2.1\%}_{-1.6\%}$	$1.679 \pm 0.005 \cdot 10^{-1}$	$^{+6.3\%}_{-5.1\%}$ $^{+1.6\%}_{-1.2\%}$
c.3	$pp \to ZZW^\pm$	`p p > z z wpm`	$2.996 \pm 0.016 \cdot 10^{-2}$	$^{+1.0\%}_{-1.4\%}$ $^{+2.0\%}_{-1.6\%}$	$5.550 \pm 0.020 \cdot 10^{-2}$	$^{+6.8\%}_{-5.5\%}$ $^{+1.5\%}_{-1.1\%}$
c.4	$pp \to ZZZ$	`p p > z z z`	$1.085 \pm 0.002 \cdot 10^{-2}$	$^{+0.0\%}_{-0.5\%}$ $^{+1.9\%}_{-1.5\%}$	$1.417 \pm 0.005 \cdot 10^{-2}$	$^{+2.7\%}_{-2.1\%}$ $^{+1.9\%}_{-1.5\%}$
c.5	$pp \to \gamma W^+W^-$ (4f)	`p p > a w+ w-`	$1.427 \pm 0.011 \cdot 10^{-1}$	$^{+1.9\%}_{-2.6\%}$ $^{+2.0\%}_{-1.5\%}$	$2.581 \pm 0.008 \cdot 10^{-1}$	$^{+5.4\%}_{-4.3\%}$ $^{+1.4\%}_{-1.1\%}$
c.6	$pp \to \gamma\gamma W^\pm$	`p p > a a wpm`	$2.681 \pm 0.007 \cdot 10^{-2}$	$^{+4.4\%}_{-5.6\%}$ $^{+1.9\%}_{-1.6\%}$	$8.251 \pm 0.032 \cdot 10^{-2}$	$^{+7.6\%}_{-7.0\%}$ $^{+1.0\%}_{-1.0\%}$
c.7	$pp \to \gamma ZW^\pm$	`p p > a z wpm`	$4.994 \pm 0.011 \cdot 10^{-2}$	$^{+0.8\%}_{-1.4\%}$ $^{+1.9\%}_{-1.6\%}$	$1.117 \pm 0.004 \cdot 10^{-1}$	$^{+7.2\%}_{-5.9\%}$ $^{+1.2\%}_{-0.9\%}$
c.8	$pp \to \gamma ZZ$	`p p > a z z`	$2.320 \pm 0.005 \cdot 10^{-2}$	$^{+2.0\%}_{-2.9\%}$ $^{+1.9\%}_{-1.5\%}$	$3.118 \pm 0.012 \cdot 10^{-2}$	$^{+2.8\%}_{-2.7\%}$ $^{+1.8\%}_{-1.4\%}$
c.9	$pp \to \gamma\gamma Z$	`p p > a a z`	$3.078 \pm 0.007 \cdot 10^{-2}$	$^{+5.6\%}_{-6.8\%}$ $^{+1.9\%}_{-1.6\%}$	$4.634 \pm 0.020 \cdot 10^{-2}$	$^{+4.5\%}_{-5.0\%}$ $^{+1.7\%}_{-1.3\%}$
c.10	$pp \to \gamma\gamma\gamma$	`p p > a a a`	$1.269 \pm 0.003 \cdot 10^{-2}$	$^{+9.8\%}_{-11.0\%}$ $^{+2.0\%}_{-1.8\%}$	$3.441 \pm 0.012 \cdot 10^{-2}$	$^{+11.8\%}_{-11.6\%}$ $^{+1.4\%}_{-1.5\%}$
c.11	$pp \to W^+W^-W^\pm j$ (4f)	`p p > w+ w- wpm j`	$9.167 \pm 0.010 \cdot 10^{-2}$	$^{+15.0\%}_{-12.2\%}$ $^{+1.0\%}_{-0.7\%}$	$1.197 \pm 0.004 \cdot 10^{-1}$	$^{+5.2\%}_{-5.6\%}$ $^{+1.0\%}_{-0.8\%}$
c.12*	$pp \to ZW^+W^-j$ (4f)	`p p > z w+ w- j`	$8.340 \pm 0.010 \cdot 10^{-2}$	$^{+15.6\%}_{-12.6\%}$ $^{+1.0\%}_{-0.7\%}$	$1.066 \pm 0.003 \cdot 10^{-1}$	$^{+4.5\%}_{-5.3\%}$ $^{+1.0\%}_{-0.7\%}$
c.13*	$pp \to ZZW^\pm j$	`p p > z z wpm j`	$2.810 \pm 0.004 \cdot 10^{-2}$	$^{+16.1\%}_{-13.0\%}$ $^{+1.0\%}_{-0.7\%}$	$3.660 \pm 0.013 \cdot 10^{-2}$	$^{+4.8\%}_{-5.6\%}$ $^{+1.0\%}_{-0.7\%}$
c.14*	$pp \to ZZZj$	`p p > z z z j`	$4.823 \pm 0.011 \cdot 10^{-3}$	$^{+14.3\%}_{-11.8\%}$ $^{+1.4\%}_{-1.0\%}$	$6.341 \pm 0.025 \cdot 10^{-3}$	$^{+4.9\%}_{-5.4\%}$ $^{+1.4\%}_{-1.0\%}$
c.15*	$pp \to \gamma W^+W^-j$ (4f)	`p p > a w+ w- j`	$1.182 \pm 0.004 \cdot 10^{-1}$	$^{+13.4\%}_{-11.2\%}$ $^{+0.8\%}_{-0.7\%}$	$1.233 \pm 0.004 \cdot 10^{3}$	$^{+18.9\%}_{-19.9\%}$ $^{+1.0\%}_{-1.5\%}$
c.16	$pp \to \gamma\gamma W^\pm j$	`p p > a a wpm j`	$4.107 \pm 0.015 \cdot 10^{-2}$	$^{+11.8\%}_{-10.2\%}$ $^{+0.6\%}_{-0.8\%}$	$5.807 \pm 0.023 \cdot 10^{-2}$	$^{+5.8\%}_{-5.5\%}$ $^{+0.7\%}_{-0.7\%}$
c.17*	$pp \to \gamma ZW^\pm j$	`p p > a z wpm j`	$5.833 \pm 0.023 \cdot 10^{-2}$	$^{+14.4\%}_{-12.0\%}$ $^{+0.7\%}_{-0.6\%}$	$7.764 \pm 0.025 \cdot 10^{-2}$	$^{+5.1\%}_{-5.5\%}$ $^{+0.8\%}_{-0.6\%}$
c.18*	$pp \to \gamma ZZj$	`p p > a z z j`	$9.995 \pm 0.013 \cdot 10^{-3}$	$^{+12.5\%}_{-10.6\%}$ $^{+1.2\%}_{-0.9\%}$	$1.371 \pm 0.005 \cdot 10^{-2}$	$^{+5.6\%}_{-5.5\%}$ $^{+1.2\%}_{-0.9\%}$
c.19*	$pp \to \gamma\gamma Zj$	`p p > a a z j`	$1.372 \pm 0.003 \cdot 10^{-2}$	$^{+10.9\%}_{-9.4\%}$ $^{+1.0\%}_{-0.9\%}$	$2.051 \pm 0.011 \cdot 10^{-2}$	$^{+7.0\%}_{-6.3\%}$ $^{+1.0\%}_{-0.9\%}$
c.20*	$pp \to \gamma\gamma\gamma j$	`p p > a a a j`	$1.031 \pm 0.006 \cdot 10^{-2}$	$^{+14.3\%}_{-12.6\%}$ $^{+0.9\%}_{-1.2\%}$	$2.020 \pm 0.008 \cdot 10^{-2}$	$^{+12.8\%}_{-11.0\%}$ $^{+0.8\%}_{-1.2\%}$

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	Process	Syntax	Cross section (pb)					
	Four vector bosons		LO 13 TeV			NLO 13 TeV		
c.21*	$pp \to W^+W^-W^+W^-$ (4f)	`p p > w+ w- w+ w-`	$5.721 \pm 0.014 \cdot 10^{-4}$	$^{+3.7\%}_{-3.5\%}$	$^{+2.3\%}_{-1.7\%}$	$9.959 \pm 0.035 \cdot 10^{-4}$	$^{+7.4\%}_{-6.0\%}$	$^{+1.7\%}_{-1.2\%}$
c.22*	$pp \to W^+W^-W^\pm Z$ (4f)	`p p > w+ w- wpm z`	$6.391 \pm 0.076 \cdot 10^{-4}$	$^{+4.4\%}_{-4.1\%}$	$^{+2.4\%}_{-1.8\%}$	$1.188 \pm 0.004 \cdot 10^{-3}$	$^{+8.4\%}_{-6.8\%}$	$^{+1.7\%}_{-1.2\%}$
c.23*	$pp \to W^+W^-W^\pm \gamma$ (4f)	`p p > w+ w- wpm a`	$8.115 \pm 0.064 \cdot 10^{-4}$	$^{+2.5\%}_{-2.5\%}$	$^{+2.2\%}_{-1.7\%}$	$1.546 \pm 0.005 \cdot 10^{-3}$	$^{+7.9\%}_{-6.3\%}$	$^{+1.5\%}_{-1.1\%}$
c.24*	$pp \to W^+W^-ZZ$ (4f)	`p p > w+ w- z z`	$4.320 \pm 0.013 \cdot 10^{-4}$	$^{+4.4\%}_{-4.1\%}$	$^{+2.4\%}_{-1.7\%}$	$7.107 \pm 0.020 \cdot 10^{-4}$	$^{+7.0\%}_{-5.7\%}$	$^{+1.8\%}_{-1.3\%}$
c.25*	$pp \to W^+W^-Z\gamma$ (4f)	`p p > w+ w- z a`	$8.403 \pm 0.016 \cdot 10^{-4}$	$^{+3.0\%}_{-2.9\%}$	$^{+2.3\%}_{-1.7\%}$	$1.483 \pm 0.004 \cdot 10^{-3}$	$^{+7.2\%}_{-5.8\%}$	$^{+1.6\%}_{-1.2\%}$
c.26*	$pp \to W^+W^-\gamma\gamma$ (4f)	`p p > w+ w- a a`	$5.198 \pm 0.012 \cdot 10^{-4}$	$^{+0.6\%}_{-0.9\%}$	$^{+2.1\%}_{-1.6\%}$	$9.381 \pm 0.032 \cdot 10^{-4}$	$^{+6.7\%}_{-5.3\%}$	$^{+1.4\%}_{-1.1\%}$
c.27*	$pp \to W^\pm ZZZ$	`p p > wpm z z z`	$5.862 \pm 0.010 \cdot 10^{-5}$	$^{+5.1\%}_{-4.7\%}$	$^{+2.4\%}_{-1.8\%}$	$1.240 \pm 0.004 \cdot 10^{-4}$	$^{+9.9\%}_{-8.0\%}$	$^{+1.7\%}_{-1.2\%}$
c.28*	$pp \to W^\pm ZZ\gamma$	`p p > wpm z z a`	$1.148 \pm 0.003 \cdot 10^{-4}$	$^{+3.6\%}_{-3.5\%}$	$^{+2.2\%}_{-1.7\%}$	$2.945 \pm 0.008 \cdot 10^{-4}$	$^{+10.8\%}_{-8.7\%}$	$^{+1.3\%}_{-1.0\%}$
c.29*	$pp \to W^\pm Z\gamma\gamma$	`p p > wpm z a a`	$1.054 \pm 0.004 \cdot 10^{-4}$	$^{+1.7\%}_{-1.9\%}$	$^{+2.1\%}_{-1.7\%}$	$3.033 \pm 0.010 \cdot 10^{-4}$	$^{+10.6\%}_{-8.6\%}$	$^{+1.1\%}_{-0.8\%}$
c.30*	$pp \to W^\pm \gamma\gamma\gamma$	`p p > wpm a a a`	$3.600 \pm 0.013 \cdot 10^{-5}$	$^{+0.4\%}_{-1.0\%}$	$^{+2.0\%}_{-1.6\%}$	$1.246 \pm 0.005 \cdot 10^{-4}$	$^{+9.8\%}_{-8.1\%}$	$^{+0.9\%}_{-0.8\%}$
c.31*	$pp \to ZZZZ$	`p p > z z z z`	$1.989 \pm 0.002 \cdot 10^{-5}$	$^{+3.8\%}_{-3.6\%}$	$^{+2.2\%}_{-1.7\%}$	$2.629 \pm 0.008 \cdot 10^{-5}$	$^{+3.5\%}_{-3.0\%}$	$^{+2.2\%}_{-1.7\%}$
c.32*	$pp \to ZZZ\gamma$	`p p > z z z a`	$3.945 \pm 0.007 \cdot 10^{-5}$	$^{+1.9\%}_{-2.1\%}$	$^{+2.1\%}_{-1.6\%}$	$5.224 \pm 0.016 \cdot 10^{-5}$	$^{+3.3\%}_{-2.7\%}$	$^{+2.1\%}_{-1.6\%}$
c.33*	$pp \to ZZ\gamma\gamma$	`p p > z z a a`	$5.513 \pm 0.017 \cdot 10^{-5}$	$^{+0.0\%}_{-0.3\%}$	$^{+2.1\%}_{-1.6\%}$	$7.518 \pm 0.032 \cdot 10^{-5}$	$^{+3.4\%}_{-2.6\%}$	$^{+2.0\%}_{-1.5\%}$
c.34*	$pp \to Z\gamma\gamma\gamma$	`p p > z a a a`	$4.790 \pm 0.012 \cdot 10^{-5}$	$^{+2.3\%}_{-3.1\%}$	$^{+2.0\%}_{-1.6\%}$	$7.103 \pm 0.026 \cdot 10^{-5}$	$^{+3.4\%}_{-3.2\%}$	$^{+1.6\%}_{-1.5\%}$
c.35*	$pp \to \gamma\gamma\gamma\gamma$	`p p > a a a a`	$1.594 \pm 0.004 \cdot 10^{-5}$	$^{+4.7\%}_{-5.7\%}$	$^{+1.9\%}_{-1.7\%}$	$3.389 \pm 0.012 \cdot 10^{-5}$	$^{+7.0\%}_{-6.7\%}$	$^{+1.3\%}_{-1.3\%}$

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Process		Syntax	Cross section (pb)			
Heavy quarks and jets			LO 13 TeV		NLO 13 TeV	
d.1	$pp \to jj$	`p p > j j`	$1.162 \pm 0.001 \cdot 10^{6}$	$^{+24.9\%}_{-18.8\%}$ $^{+0.8\%}_{-0.9\%}$	$1.580 \pm 0.007 \cdot 10^{6}$	$^{+8.4\%}_{-9.0\%}$ $^{+0.7\%}_{-0.9\%}$
d.2	$pp \to jjj$	`p p > j j j`	$8.940 \pm 0.021 \cdot 10^{4}$	$^{+43.8\%}_{-28.4\%}$ $^{+1.2\%}_{-1.4\%}$	$7.791 \pm 0.037 \cdot 10^{4}$	$^{+2.1\%}_{-23.2\%}$ $^{+1.1\%}_{-1.3\%}$
d.3	$pp \to b\bar{b}$ (4f)	`p p > b b~`	$3.743 \pm 0.004 \cdot 10^{3}$	$^{+25.2\%}_{-18.9\%}$ $^{+1.5\%}_{-1.8\%}$	$6.438 \pm 0.028 \cdot 10^{3}$	$^{+15.9\%}_{-13.3\%}$ $^{+1.5\%}_{-1.7\%}$
d.4*	$pp \to b\bar{b}j$ (4f)	`p p > b b~ j`	$1.050 \pm 0.002 \cdot 10^{3}$	$^{+44.1\%}_{-28.5\%}$ $^{+1.6\%}_{-1.8\%}$	$1.327 \pm 0.007 \cdot 10^{3}$	$^{+6.8\%}_{-11.6\%}$ $^{+1.5\%}_{-1.8\%}$
d.5*	$pp \to b\bar{b}jj$ (4f)	`p p > b b~ j j`	$1.852 \pm 0.006 \cdot 10^{2}$	$^{+61.8\%}_{-35.6\%}$ $^{+2.1\%}_{-2.4\%}$	$2.471 \pm 0.012 \cdot 10^{2}$	$^{+8.2\%}_{-16.4\%}$ $^{+2.0\%}_{-2.3\%}$
d.6	$pp \to b\bar{b}b\bar{b}$ (4f)	`p p > b b~ b b~`	$5.050 \pm 0.007 \cdot 10^{-1}$	$^{+61.7\%}_{-35.6\%}$ $^{+2.9\%}_{-3.4\%}$	$8.736 \pm 0.034 \cdot 10^{-1}$	$^{+20.9\%}_{-22.0\%}$ $^{+2.9\%}_{-3.4\%}$
d.7	$pp \to t\bar{t}$	`p p > t t~`	$4.584 \pm 0.003 \cdot 10^{2}$	$^{+29.0\%}_{-21.1\%}$ $^{+1.8\%}_{-2.0\%}$	$6.741 \pm 0.023 \cdot 10^{2}$	$^{+9.8\%}_{-10.9\%}$ $^{+1.8\%}_{-2.1\%}$
d.8	$pp \to t\bar{t}j$	`p p > t t~ j`	$3.135 \pm 0.002 \cdot 10^{2}$	$^{+45.1\%}_{-29.0\%}$ $^{+2.2\%}_{-2.5\%}$	$4.106 \pm 0.015 \cdot 10^{2}$	$^{+8.1\%}_{-12.2\%}$ $^{+2.1\%}_{-2.5\%}$
d.9	$pp \to t\bar{t}jj$	`p p > t t~ j j`	$1.361 \pm 0.001 \cdot 10^{2}$	$^{+61.4\%}_{-35.6\%}$ $^{+2.6\%}_{-3.0\%}$	$1.795 \pm 0.006 \cdot 10^{2}$	$^{+9.3\%}_{-16.1\%}$ $^{+2.4\%}_{-2.9\%}$
d.10	$pp \to t\bar{t}t\bar{t}$	`p p > t t~ t t~`	$4.505 \pm 0.005 \cdot 10^{-3}$	$^{+63.8\%}_{-36.5\%}$ $^{+5.4\%}_{-5.7\%}$	$9.201 \pm 0.028 \cdot 10^{-3}$	$^{+30.8\%}_{-25.6\%}$ $^{+5.5\%}_{-5.9\%}$
d.11	$pp \to t\bar{t}b\bar{b}$ (4f)	`p p > t t~ b b~`	$6.119 \pm 0.004 \cdot 10^{0}$	$^{+62.1\%}_{-35.7\%}$ $^{+2.9\%}_{-3.5\%}$	$1.452 \pm 0.005 \cdot 10^{1}$	$^{+37.6\%}_{-27.5\%}$ $^{+2.9\%}_{-3.5\%}$

Automated NLO

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Process		Syntax	Cross section (pb)			
Heavy quarks+vector bosons			LO 13 TeV		NLO 13 TeV	
e.1	$pp \to W^\pm b\bar{b}$ (4f)	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^2$	${}^{+42.3\%}_{-29.2\%}$ ${}^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^2$	${}^{+29.8\%}_{-23.6\%}$ ${}^{+1.5\%}_{-1.2\%}$
e.2	$pp \to Z b\bar{b}$ (4f)	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^2$	${}^{+33.5\%}_{-24.4\%}$ ${}^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^3$	${}^{+19.9\%}_{-17.4\%}$ ${}^{+1.0\%}_{-1.4\%}$
e.3	$pp \to \gamma b\bar{b}$ (4f)	`p p > a b b~`	$1.731 \pm 0.001 \cdot 10^3$	${}^{+51.9\%}_{-34.8\%}$ ${}^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^3$	${}^{+33.7\%}_{-27.1\%}$ ${}^{+1.4\%}_{-1.9\%}$
e.4*	$pp \to W^\pm b\bar{b}j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^2$	${}^{+42.5\%}_{-27.7\%}$ ${}^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^2$	${}^{+27.0\%}_{-21.0\%}$ ${}^{+0.7\%}_{-0.6\%}$
e.5*	$pp \to Z b\bar{b}j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^2$	${}^{+42.4\%}_{-27.6\%}$ ${}^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^2$	${}^{+21.0\%}_{-17.6\%}$ ${}^{+0.8\%}_{-1.0\%}$
e.6*	$pp \to \gamma b\bar{b}j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^2$	${}^{+51.2\%}_{-32.0\%}$ ${}^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^3$	${}^{+18.9\%}_{-19.9\%}$ ${}^{+1.0\%}_{-1.5\%}$
e.7	$pp \to t\bar{t}W^\pm$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}$	${}^{+23.9\%}_{-18.0\%}$ ${}^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}$	${}^{+11.2\%}_{-10.6\%}$ ${}^{+1.7\%}_{-1.3\%}$
e.8	$pp \to t\bar{t}Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}$	${}^{+30.5\%}_{-21.8\%}$ ${}^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}$	${}^{+9.7\%}_{-11.1\%}$ ${}^{+1.9\%}_{-2.2\%}$
e.9	$pp \to t\bar{t}\gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^0$	${}^{+29.6\%}_{-21.3\%}$ ${}^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^0$	${}^{+9.8\%}_{-11.0\%}$ ${}^{+1.7\%}_{-2.0\%}$
e.10*	$pp \to t\bar{t}W^\pm j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}$	${}^{+40.9\%}_{-27.1\%}$ ${}^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}$	${}^{+11.2\%}_{-14.0\%}$ ${}^{+1.2\%}_{-0.9\%}$
e.11*	$pp \to t\bar{t}Zj$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}$	${}^{+46.2\%}_{-29.5\%}$ ${}^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}$	${}^{+7.0\%}_{-12.3\%}$ ${}^{+2.5\%}_{-2.9\%}$
e.12*	$pp \to t\bar{t}\gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{-1}$	${}^{+45.4\%}_{-29.1\%}$ ${}^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^0$	${}^{+7.5\%}_{-12.2\%}$ ${}^{+2.2\%}_{-2.5\%}$
e.13*	$pp \to t\bar{t}W^-W^+$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.006 \cdot 10^{-3}$	${}^{+30.9\%}_{-21.9\%}$ ${}^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}$	${}^{+10.9\%}_{-11.8\%}$ ${}^{+2.1\%}_{-2.1\%}$
e.14*	$pp \to t\bar{t}W^\pm Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}$	${}^{+26.6\%}_{-19.6\%}$ ${}^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}$	${}^{+10.6\%}_{-10.8\%}$ ${}^{+2.3\%}_{-1.6\%}$
e.15*	$pp \to t\bar{t}W^\pm\gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}$	${}^{+25.4\%}_{-18.9\%}$ ${}^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}$	${}^{+10.3\%}_{-10.4\%}$ ${}^{+2.0\%}_{-1.5\%}$
e.16*	$pp \to t\bar{t}ZZ$	`p p > t t~ z z`	$1.349 \pm 0.014 \cdot 10^{-3}$	${}^{+29.3\%}_{-21.1\%}$ ${}^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}$	${}^{+7.9\%}_{-9.9\%}$ ${}^{+1.7\%}_{-1.5\%}$
e.17*	$pp \to t\bar{t}Z\gamma$	`p p > t t~ z a`	$2.548 \pm 0.003 \cdot 10^{-3}$	${}^{+30.1\%}_{-21.5\%}$ ${}^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}$	${}^{+9.7\%}_{-11.0\%}$ ${}^{+1.8\%}_{-1.9\%}$
e.18*	$pp \to t\bar{t}\gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}$	${}^{+28.4\%}_{-20.6\%}$ ${}^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}$	${}^{+7.8\%}_{-9.7\%}$ ${}^{+1.4\%}_{-1.4\%}$

Automated NLO

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	Process Single-top	Syntax	Cross section (pb) LO 13 TeV		NLO 13 TeV	
f.1	$pp \to tj$ (t-channel)	`p p > tt j $$ w+ w-`	$1.520 \pm 0.001 \cdot 10^{2}$	$^{+9.4\%}_{-11.9\%}$ $^{+0.4\%}_{-0.6\%}$	$1.563 \pm 0.005 \cdot 10^{2}$	$^{+1.4\%}_{-1.8\%}$ $^{+0.4\%}_{-0.6\%}$
f.2	$pp \to t\gamma j$ (t-channel)	`p p > tt a j $$ w+ w-`	$9.956 \pm 0.014 \cdot 10^{-1}$	$^{+6.4\%}_{-8.8\%}$ $^{+0.9\%}_{-1.0\%}$	$1.017 \pm 0.003 \cdot 10^{0}$	$^{+1.3\%}_{-1.2\%}$ $^{+0.8\%}_{-0.9\%}$
f.3	$pp \to tZj$ (t-channel)	`p p > tt z j $$ w+ w-`	$6.967 \pm 0.007 \cdot 10^{-1}$	$^{+3.5\%}_{-5.5\%}$ $^{+0.9\%}_{-1.0\%}$	$6.993 \pm 0.021 \cdot 10^{-1}$	$^{+1.6\%}_{-1.1\%}$ $^{+0.9\%}_{-1.0\%}$
f.4	$pp \to tbj$ (t-channel, 4f)	`p p > tt bb j $$ w+ w-`	$1.003 \pm 0.000 \cdot 10^{2}$	$^{+13.8\%}_{-11.5\%}$ $^{+0.4\%}_{-0.5\%}$	$1.319 \pm 0.003 \cdot 10^{2}$	$^{+5.8\%}_{-5.2\%}$ $^{+0.4\%}_{-0.5\%}$
f.5*	$pp \to tbj\gamma$ (t-channel, 4f)	`p p > tt bb j a $$ w+ w-`	$6.293 \pm 0.006 \cdot 10^{-1}$	$^{+16.8\%}_{-13.5\%}$ $^{+0.8\%}_{-0.9\%}$	$8.612 \pm 0.025 \cdot 10^{-1}$	$^{+6.2\%}_{-6.6\%}$ $^{+0.8\%}_{-0.9\%}$
f.6*	$pp \to tbjZ$ (t-channel, 4f)	`p p > tt bb j z $$ w+ w-`	$3.934 \pm 0.002 \cdot 10^{-1}$	$^{+18.7\%}_{-14.7\%}$ $^{+1.0\%}_{-0.9\%}$	$5.657 \pm 0.014 \cdot 10^{-1}$	$^{+7.7\%}_{-7.9\%}$ $^{+0.9\%}_{-0.9\%}$
f.7	$pp \to tb$ (s-channel, 4f)	`p p > w+ > t b~, p p > w- > t~ b`	$7.489 \pm 0.007 \cdot 10^{0}$	$^{+3.5\%}_{-4.4\%}$ $^{+1.9\%}_{-1.4\%}$	$1.001 \pm 0.004 \cdot 10^{1}$	$^{+3.7\%}_{-3.9\%}$ $^{+1.9\%}_{-1.5\%}$
f.8*	$pp \to tb\gamma$ (s-channel, 4f)	`p p > w+ > t b~ a, p p > w- > t~ b a`	$1.490 \pm 0.001 \cdot 10^{-2}$	$^{+1.2\%}_{-1.8\%}$ $^{+1.9\%}_{-1.5\%}$	$1.952 \pm 0.007 \cdot 10^{-2}$	$^{+2.6\%}_{-2.3\%}$ $^{+1.7\%}_{-1.4\%}$
f.9*	$pp \to tbZ$ (s-channel, 4f)	`p p > w+ > t b~ z, p p > w- > t~ b z`	$1.072 \pm 0.001 \cdot 10^{-2}$	$^{+1.3\%}_{-1.5\%}$ $^{+2.0\%}_{-1.6\%}$	$1.539 \pm 0.005 \cdot 10^{-2}$	$^{+3.9\%}_{-3.2\%}$ $^{+1.9\%}_{-1.5\%}$

Automated NLO

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Process		Syntax	Cross section (pb)			
Single Higgs production			LO 13 TeV		NLO 13 TeV	
g.1	$pp \to H$ (HEFT)	`p p > h`	$1.593 \pm 0.003 \cdot 10^{1}$	$^{+34.8\%}_{-26.0\%}$ $^{+1.2\%}_{-1.7\%}$	$3.261 \pm 0.010 \cdot 10^{1}$	$^{+20.2\%}_{-17.9\%}$ $^{+1.1\%}_{-1.6\%}$
g.2	$pp \to Hj$ (HEFT)	`p p > h j`	$8.367 \pm 0.003 \cdot 10^{0}$	$^{+39.4\%}_{-26.4\%}$ $^{+1.2\%}_{-1.4\%}$	$1.422 \pm 0.006 \cdot 10^{1}$	$^{+18.5\%}_{-16.6\%}$ $^{+1.1\%}_{-1.4\%}$
g.3	$pp \to Hjj$ (HEFT)	`p p > h j j`	$3.020 \pm 0.002 \cdot 10^{0}$	$^{+59.1\%}_{-34.7\%}$ $^{+1.4\%}_{-1.7\%}$	$5.124 \pm 0.020 \cdot 10^{0}$	$^{+20.7\%}_{-21.0\%}$ $^{+1.3\%}_{-1.5\%}$
g.4	$pp \to Hjj$ (VBF)	`p p > h j j $$ w+ w- z`	$1.987 \pm 0.002 \cdot 10^{0}$	$^{+1.7\%}_{-2.0\%}$ $^{+1.9\%}_{-1.4\%}$	$1.900 \pm 0.006 \cdot 10^{0}$	$^{+0.8\%}_{-0.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.5	$pp \to Hjjj$ (VBF)	`p p > h j j j $$ w+ w- z`	$2.824 \pm 0.005 \cdot 10^{-1}$	$^{+15.7\%}_{-12.7\%}$ $^{+1.5\%}_{-1.0\%}$	$3.085 \pm 0.010 \cdot 10^{-1}$	$^{+2.0\%}_{-3.0\%}$ $^{+1.5\%}_{-1.1\%}$
g.6	$pp \to HW^{\pm}$	`p p > h wpm`	$1.195 \pm 0.002 \cdot 10^{0}$	$^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.5\%}$	$1.419 \pm 0.005 \cdot 10^{0}$	$^{+2.1\%}_{-2.6\%}$ $^{+1.9\%}_{-1.4\%}$
g.7	$pp \to HW^{\pm}j$	`p p > h wpm j`	$4.018 \pm 0.003 \cdot 10^{-1}$	$^{+10.7\%}_{-9.3\%}$ $^{+1.2\%}_{-0.9\%}$	$4.842 \pm 0.017 \cdot 10^{-1}$	$^{+3.6\%}_{-3.7\%}$ $^{+1.2\%}_{-1.0\%}$
g.8*	$pp \to HW^{\pm}jj$	`p p > h wpm j j`	$1.198 \pm 0.016 \cdot 10^{-1}$	$^{+26.1\%}_{-19.4\%}$ $^{+0.8\%}_{-0.6\%}$	$1.574 \pm 0.014 \cdot 10^{-1}$	$^{+5.0\%}_{-6.5\%}$ $^{+0.9\%}_{-0.6\%}$
g.9	$pp \to HZ$	`p p > h z`	$6.468 \pm 0.008 \cdot 10^{-1}$	$^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.4\%}$	$7.674 \pm 0.027 \cdot 10^{-1}$	$^{+2.0\%}_{-2.5\%}$ $^{+1.9\%}_{-1.4\%}$
g.10	$pp \to HZj$	`p p > h z j`	$2.225 \pm 0.001 \cdot 10^{-1}$	$^{+10.6\%}_{-9.2\%}$ $^{+1.1\%}_{-0.8\%}$	$2.667 \pm 0.010 \cdot 10^{-1}$	$^{+3.5\%}_{-3.6\%}$ $^{+1.1\%}_{-0.9\%}$
g.11*	$pp \to HZjj$	`p p > h z j j`	$7.262 \pm 0.012 \cdot 10^{-2}$	$^{+26.2\%}_{-19.4\%}$ $^{+0.7\%}_{-0.6\%}$	$8.753 \pm 0.037 \cdot 10^{-2}$	$^{+4.8\%}_{-6.3\%}$ $^{+0.7\%}_{-0.6\%}$
g.12*	$pp \to HW^{+}W^{-}$ (4f)	`p p > h w+ w-`	$8.325 \pm 0.139 \cdot 10^{-3}$	$^{+0.0\%}_{-0.3\%}$ $^{+2.0\%}_{-1.6\%}$	$1.065 \pm 0.003 \cdot 10^{-2}$	$^{+2.5\%}_{-1.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.13*	$pp \to HW^{\pm}\gamma$	`p p > h wpm a`	$2.518 \pm 0.006 \cdot 10^{-3}$	$^{+0.7\%}_{-1.4\%}$ $^{+1.9\%}_{-1.5\%}$	$3.309 \pm 0.011 \cdot 10^{-3}$	$^{+2.7\%}_{-2.0\%}$ $^{+1.7\%}_{-1.4\%}$
g.14*	$pp \to HZW^{\pm}$	`p p > h z wpm`	$3.763 \pm 0.007 \cdot 10^{-3}$	$^{+1.1\%}_{-1.5\%}$ $^{+2.0\%}_{-1.6\%}$	$5.292 \pm 0.015 \cdot 10^{-3}$	$^{+3.9\%}_{-3.1\%}$ $^{+1.8\%}_{-1.4\%}$
g.15*	$pp \to HZZ$	`p p > h z z`	$2.093 \pm 0.003 \cdot 10^{-3}$	$^{+0.1\%}_{-0.6\%}$ $^{+1.9\%}_{-1.5\%}$	$2.538 \pm 0.007 \cdot 10^{-3}$	$^{+1.9\%}_{-1.4\%}$ $^{+2.0\%}_{-1.5\%}$
g.16	$pp \to Ht\bar{t}$	`p p > h t t~`	$3.579 \pm 0.003 \cdot 10^{-1}$	$^{+30.0\%}_{-21.5\%}$ $^{+1.7\%}_{-2.0\%}$	$4.608 \pm 0.016 \cdot 10^{-1}$	$^{+5.7\%}_{-9.0\%}$ $^{+2.0\%}_{-2.3\%}$
g.17	$pp \to Htj$	`p p > h tt j`	$4.994 \pm 0.005 \cdot 10^{-2}$	$^{+2.4\%}_{-4.2\%}$ $^{+1.2\%}_{-1.3\%}$	$6.328 \pm 0.022 \cdot 10^{-2}$	$^{+2.9\%}_{-1.8\%}$ $^{+1.5\%}_{-1.6\%}$
g.18	$pp \to Hb\bar{b}$ (4f)	`p p > h b b~`	$4.983 \pm 0.002 \cdot 10^{-1}$	$^{+28.1\%}_{-21.0\%}$ $^{+1.5\%}_{-1.8\%}$	$6.085 \pm 0.026 \cdot 10^{-1}$	$^{+7.3\%}_{-9.6\%}$ $^{+1.6\%}_{-2.0\%}$
g.19	$pp \to Ht\bar{t}j$	`p p > h t t~ j`	$2.674 \pm 0.041 \cdot 10^{-1}$	$^{+45.6\%}_{-29.2\%}$ $^{+2.6\%}_{-2.9\%}$	$3.244 \pm 0.025 \cdot 10^{-1}$	$^{+3.5\%}_{-8.7\%}$ $^{+2.5\%}_{-2.9\%}$
g.20*	$pp \to Hb\bar{b}j$ (4f)	`p p > h b b~ j`	$7.367 \pm 0.002 \cdot 10^{-2}$	$^{+45.6\%}_{-29.1\%}$ $^{+1.8\%}_{-2.1\%}$	$9.034 \pm 0.032 \cdot 10^{-2}$	$^{+7.9\%}_{-11.0\%}$ $^{+1.8\%}_{-2.2\%}$

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Process		Syntax	Cross section (pb)			
Higgs pair production			LO 13 TeV		NLO 13 TeV	
h.1	$pp \to HH$ (Loop improved)	`p p > h h`	$1.772 \pm 0.006 \cdot 10^{-2}$	$^{+29.5\%}_{-21.4\%}$ $^{+2.1\%}_{-2.6\%}$	$2.763 \pm 0.008 \cdot 10^{-2}$	$^{+11.4\%}_{-11.8\%}$ $^{+2.1\%}_{-2.6\%}$
h.2	$pp \to HHjj$ (VBF)	`p p > h h j j $$ w+ w- z`	$6.503 \pm 0.019 \cdot 10^{-4}$	$^{+7.2\%}_{-6.4\%}$ $^{+2.3\%}_{-1.6\%}$	$6.820 \pm 0.026 \cdot 10^{-4}$	$^{+0.8\%}_{-1.0\%}$ $^{+2.4\%}_{-1.7\%}$
h.3	$pp \to HHW^\pm$	`p p > h h wpm`	$4.303 \pm 0.005 \cdot 10^{-4}$	$^{+0.9\%}_{-1.3\%}$ $^{+2.0\%}_{-1.5\%}$	$5.002 \pm 0.014 \cdot 10^{-4}$	$^{+1.5\%}_{-1.2\%}$ $^{+2.0\%}_{-1.6\%}$
h.4*	$pp \to HHW^\pm j$	`p p > h h wpm j`	$1.922 \pm 0.002 \cdot 10^{-4}$	$^{+14.2\%}_{-11.7\%}$ $^{+1.5\%}_{-1.1\%}$	$2.218 \pm 0.009 \cdot 10^{-4}$	$^{+2.7\%}_{-3.3\%}$ $^{+1.6\%}_{-1.1\%}$
h.5*	$pp \to HHW^\pm \gamma$	`p p > h h wpm a`	$1.952 \pm 0.004 \cdot 10^{-6}$	$^{+3.0\%}_{-3.0\%}$ $^{+2.2\%}_{-1.6\%}$	$2.347 \pm 0.007 \cdot 10^{-6}$	$^{+2.4\%}_{-2.0\%}$ $^{+2.1\%}_{-1.6\%}$
h.6	$pp \to HHZ$	`p p > h h z`	$2.701 \pm 0.007 \cdot 10^{-4}$	$^{+0.9\%}_{-1.3\%}$ $^{+2.0\%}_{-1.5\%}$	$3.130 \pm 0.008 \cdot 10^{-4}$	$^{+1.6\%}_{-1.2\%}$ $^{+2.0\%}_{-1.5\%}$
h.7*	$pp \to HHZj$	`p p > h h z j`	$1.211 \pm 0.001 \cdot 10^{-4}$	$^{+14.1\%}_{-11.7\%}$ $^{+1.4\%}_{-1.1\%}$	$1.394 \pm 0.006 \cdot 10^{-4}$	$^{+2.7\%}_{-3.2\%}$ $^{+1.5\%}_{-1.1\%}$
h.8*	$pp \to HHZ\gamma$	`p p > h h z a`	$1.397 \pm 0.003 \cdot 10^{-6}$	$^{+2.4\%}_{-2.5\%}$ $^{+2.2\%}_{-1.7\%}$	$1.604 \pm 0.005 \cdot 10^{-6}$	$^{+1.7\%}_{-1.4\%}$ $^{+2.3\%}_{-1.7\%}$
h.9*	$pp \to HHZZ$	`p p > h h z z`	$2.309 \pm 0.005 \cdot 10^{-6}$	$^{+3.9\%}_{-3.8\%}$ $^{+2.2\%}_{-1.7\%}$	$2.754 \pm 0.009 \cdot 10^{-6}$	$^{+2.3\%}_{-2.0\%}$ $^{+2.3\%}_{-1.7\%}$
h.10*	$pp \to HHZW^\pm$	`p p > h h z wpm`	$3.708 \pm 0.013 \cdot 10^{-6}$	$^{+4.8\%}_{-4.5\%}$ $^{+2.3\%}_{-1.7\%}$	$4.904 \pm 0.029 \cdot 10^{-6}$	$^{+3.7\%}_{-3.2\%}$ $^{+2.2\%}_{-1.6\%}$
h.11*	$pp \to HHW^+W^-$ (4f)	`p p > h h w+ w-`	$7.524 \pm 0.070 \cdot 10^{-6}$	$^{+3.5\%}_{-3.4\%}$ $^{+2.3\%}_{-1.7\%}$	$9.268 \pm 0.030 \cdot 10^{-6}$	$^{+2.3\%}_{-2.1\%}$ $^{+2.3\%}_{-1.7\%}$
h.12	$pp \to HHt\bar{t}$	`p p > h h t t~`	$6.756 \pm 0.007 \cdot 10^{-4}$	$^{+30.2\%}_{-21.6\%}$ $^{+1.8\%}_{-1.8\%}$	$7.301 \pm 0.024 \cdot 10^{-4}$	$^{+1.4\%}_{-5.7\%}$ $^{+2.2\%}_{-2.3\%}$
h.13	$pp \to HHtj$	`p p > h h tt j`	$1.844 \pm 0.008 \cdot 10^{-5}$	$^{+0.0\%}_{-0.6\%}$ $^{+1.8\%}_{-1.8\%}$	$2.444 \pm 0.009 \cdot 10^{-5}$	$^{+4.5\%}_{-3.1\%}$ $^{+2.8\%}_{-3.0\%}$
h.14*	$pp \to HHb\bar{b}$	`p p > h h b b~`	$7.849 \pm 0.022 \cdot 10^{-8}$	$^{+34.3\%}_{-23.9\%}$ $^{+3.1\%}_{-3.7\%}$	$1.084 \pm 0.012 \cdot 10^{-7}$	$^{+7.4\%}_{-10.8\%}$ $^{+3.1\%}_{-3.7\%}$

Automated NLO

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Process		Syntax	Cross section (pb)			
Heavy quarks and jets			LO 1 TeV		NLO 1 TeV	
i.1	$e^+e^- \to jj$	`e+ e- > j j`	$6.223 \pm 0.005 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$6.389 \pm 0.013 \cdot 10^{-1}$	$^{+0.2\%}_{-0.2\%}$
i.2	$e^+e^- \to jjj$	`e+ e- > j j j`	$3.401 \pm 0.002 \cdot 10^{-1}$	$^{+9.6\%}_{-8.0\%}$	$3.166 \pm 0.019 \cdot 10^{-1}$	$^{+0.2\%}_{-2.1\%}$
i.3	$e^+e^- \to jjjj$	`e+ e- > j j j j`	$1.047 \pm 0.001 \cdot 10^{-1}$	$^{+20.0\%}_{-15.3\%}$	$1.090 \pm 0.006 \cdot 10^{-1}$	$^{+0.0\%}_{-2.8\%}$
i.4	$e^+e^- \to jjjjj$	`e+ e- > j j j j j`	$2.211 \pm 0.006 \cdot 10^{-2}$	$^{+31.4\%}_{-22.0\%}$	$2.771 \pm 0.021 \cdot 10^{-2}$	$^{+4.4\%}_{-8.6\%}$
i.5	$e^+e^- \to t\bar{t}$	`e+ e- > t t~`	$1.662 \pm 0.002 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$1.745 \pm 0.006 \cdot 10^{-1}$	$^{+0.4\%}_{-0.4\%}$
i.6	$e^+e^- \to t\bar{t}j$	`e+ e- > t t~ j`	$4.813 \pm 0.005 \cdot 10^{-2}$	$^{+9.3\%}_{-7.8\%}$	$5.276 \pm 0.022 \cdot 10^{-2}$	$^{+1.3\%}_{-2.1\%}$
i.7*	$e^+e^- \to t\bar{t}jj$	`e+ e- > t t~ j j`	$8.614 \pm 0.009 \cdot 10^{-3}$	$^{+19.4\%}_{-15.0\%}$	$1.094 \pm 0.005 \cdot 10^{-2}$	$^{+5.0\%}_{-6.3\%}$
i.8*	$e^+e^- \to t\bar{t}jjj$	`e+ e- > t t~ j j j`	$1.044 \pm 0.002 \cdot 10^{-3}$	$^{+30.5\%}_{-21.6\%}$	$1.546 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-11.6\%}$
i.9*	$e^+e^- \to t\bar{t}t\bar{t}$	`e+ e- > t t~ t t~`	$6.456 \pm 0.016 \cdot 10^{-7}$	$^{+19.1\%}_{-14.8\%}$	$1.221 \pm 0.005 \cdot 10^{-6}$	$^{+13.2\%}_{-11.2\%}$
i.10*	$e^+e^- \to t\bar{t}t\bar{t}j$	`e+ e- > t t~ t t~ j`	$2.719 \pm 0.005 \cdot 10^{-8}$	$^{+29.9\%}_{-21.3\%}$	$5.338 \pm 0.027 \cdot 10^{-8}$	$^{+18.3\%}_{-15.4\%}$
i.11	$e^+e^- \to b\bar{b}$ (4f)	`e+ e- > b b~`	$9.198 \pm 0.004 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$9.282 \pm 0.031 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$
i.12	$e^+e^- \to b\bar{b}j$ (4f)	`e+ e- > b b~ j`	$5.029 \pm 0.003 \cdot 10^{-2}$	$^{+9.5\%}_{-8.0\%}$	$4.826 \pm 0.026 \cdot 10^{-2}$	$^{+0.5\%}_{-2.5\%}$
i.13*	$e^+e^- \to b\bar{b}jj$ (4f)	`e+ e- > b b~ j j`	$1.621 \pm 0.001 \cdot 10^{-2}$	$^{+20.0\%}_{-15.3\%}$	$1.817 \pm 0.009 \cdot 10^{-2}$	$^{+0.0\%}_{-3.1\%}$
i.14*	$e^+e^- \to b\bar{b}jjj$ (4f)	`e+ e- > b b~ j j j`	$3.641 \pm 0.009 \cdot 10^{-3}$	$^{+31.4\%}_{-22.1\%}$	$4.936 \pm 0.038 \cdot 10^{-3}$	$^{+4.8\%}_{-8.9\%}$
i.15*	$e^+e^- \to b\bar{b}b\bar{b}$ (4f)	`e+ e- > b b~ b b~`	$1.644 \pm 0.003 \cdot 10^{-4}$	$^{+19.9\%}_{-15.3\%}$	$3.601 \pm 0.017 \cdot 10^{-4}$	$^{+15.2\%}_{-12.5\%}$
i.16*	$e^+e^- \to b\bar{b}b\bar{b}j$ (4f)	`e+ e- > b b~ b b~ j`	$7.660 \pm 0.022 \cdot 10^{-5}$	$^{+31.3\%}_{-22.0\%}$	$1.537 \pm 0.011 \cdot 10^{-4}$	$^{+17.9\%}_{-15.3\%}$
i.17*	$e^+e^- \to t\bar{t}b\bar{b}$ (4f)	`e+ e- > t t~ b b~`	$1.819 \pm 0.003 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$2.923 \pm 0.011 \cdot 10^{-4}$	$^{+9.2\%}_{-8.9\%}$
i.18*	$e^+e^- \to t\bar{t}b\bar{b}j$ (4f)	`e+ e- > t t~ b b~ j`	$4.045 \pm 0.011 \cdot 10^{-5}$	$^{+30.5\%}_{-21.6\%}$	$7.049 \pm 0.052 \cdot 10^{-5}$	$^{+13.7\%}_{-13.1\%}$

Automated NLO

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Process Top quarks +bosons		Syntax	Cross section (pb) LO 1 TeV		NLO 1 TeV	
j.1	$e^+e^- \to t\bar{t}H$	`e+ e- > t t~ h`	$2.018 \pm 0.003 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$1.911 \pm 0.006 \cdot 10^{-3}$	$^{+0.4\%}_{-0.5\%}$
j.2*	$e^+e^- \to t\bar{t}Hj$	`e+ e- > t t~ h j`	$2.533 \pm 0.003 \cdot 10^{-4}$	$^{+9.2\%}_{-7.8\%}$	$2.658 \pm 0.009 \cdot 10^{-4}$	$^{+0.5\%}_{-1.5\%}$
j.3*	$e^+e^- \to t\bar{t}Hjj$	`e+ e- > t t~ h j j`	$2.663 \pm 0.004 \cdot 10^{-5}$	$^{+19.3\%}_{-14.9\%}$	$3.278 \pm 0.017 \cdot 10^{-5}$	$^{+4.0\%}_{-5.7\%}$
j.4*	$e^+e^- \to t\bar{t}\gamma$	`e+ e- > t t~ a`	$1.270 \pm 0.002 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$1.335 \pm 0.004 \cdot 10^{-2}$	$^{+0.5\%}_{-0.4\%}$
j.5*	$e^+e^- \to t\bar{t}\gamma j$	`e+ e- > t t~ a j`	$2.355 \pm 0.002 \cdot 10^{-3}$	$^{+9.3\%}_{-7.9\%}$	$2.617 \pm 0.010 \cdot 10^{-3}$	$^{+1.6\%}_{-2.4\%}$
j.6*	$e^+e^- \to t\bar{t}\gamma jj$	`e+ e- > t t~ a j j`	$3.103 \pm 0.005 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$4.002 \pm 0.021 \cdot 10^{-4}$	$^{+5.4\%}_{-6.6\%}$
j.7*	$e^+e^- \to t\bar{t}Z$	`e+ e- > t t~ z`	$4.642 \pm 0.006 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$4.949 \pm 0.014 \cdot 10^{-3}$	$^{+0.6\%}_{-0.5\%}$
j.8*	$e^+e^- \to t\bar{t}Zj$	`e+ e- > t t~ z j`	$6.059 \pm 0.006 \cdot 10^{-4}$	$^{+9.3\%}_{-7.8\%}$	$6.940 \pm 0.028 \cdot 10^{-4}$	$^{+2.0\%}_{-2.6\%}$
j.9*	$e^+e^- \to t\bar{t}Zjj$	`e+ e- > t t~ z j j`	$6.351 \pm 0.028 \cdot 10^{-5}$	$^{+19.4\%}_{-15.0\%}$	$8.439 \pm 0.051 \cdot 10^{-5}$	$^{+5.8\%}_{-6.8\%}$
j.10*	$e^+e^- \to t\bar{t}W^\pm jj$	`e+ e- > t t~ wpm j j`	$2.400 \pm 0.004 \cdot 10^{-7}$	$^{+19.3\%}_{-14.9\%}$	$3.723 \pm 0.012 \cdot 10^{-7}$	$^{+9.6\%}_{-9.1\%}$
j.11*	$e^+e^- \to t\bar{t}HZ$	`e+ e- > t t~ h z`	$3.600 \pm 0.006 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$3.579 \pm 0.013 \cdot 10^{-5}$	$^{+0.1\%}_{-0.0\%}$
j.12*	$e^+e^- \to t\bar{t}\gamma Z$	`e+ e- > t t~ a z`	$2.212 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$2.364 \pm 0.006 \cdot 10^{-4}$	$^{+0.6\%}_{-0.5\%}$
j.13*	$e^+e^- \to t\bar{t}\gamma H$	`e+ e- > t t~ a h`	$9.756 \pm 0.016 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$9.423 \pm 0.032 \cdot 10^{-5}$	$^{+0.3\%}_{-0.4\%}$
j.14*	$e^+e^- \to t\bar{t}\gamma\gamma$	`e+ e- > t t~ a a`	$3.650 \pm 0.008 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$3.833 \pm 0.013 \cdot 10^{-4}$	$^{+0.4\%}_{-0.4\%}$
j.15*	$e^+e^- \to t\bar{t}ZZ$	`e+ e- > t t~ z z`	$3.788 \pm 0.004 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$4.007 \pm 0.013 \cdot 10^{-5}$	$^{+0.5\%}_{-0.5\%}$
j.16*	$e^+e^- \to t\bar{t}HH$	`e+ e- > t t~ h h`	$1.358 \pm 0.001 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$1.206 \pm 0.003 \cdot 10^{-5}$	$^{+0.9\%}_{-1.1\%}$
j.17*	$e^+e^- \to t\bar{t}W^+W^-$	`e+ e- > t t~ w+ w-`	$1.372 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$1.540 \pm 0.006 \cdot 10^{-4}$	$^{+1.0\%}_{-0.9\%}$

Automated NLO

- few years ago: each item in each table resulted in a paper. Now, as for leading order, just run a code and get the results (also for distributions)
- possibility to do precise studies of signal and backgrounds using the same tool (very practical + avoid errors)
- what lead to this remarkable progress? the fact that

1. leading order can be computed automatically and efficiently (e.g. via recursion relations)
2. one can reduce the one-loop to product of tree-level amplitudes
3. it was well understood how to subtract singularities
4. the basis of master integrals was known

But for item 2. everything was there since the time of Passarino-Veltman (even item 2. was understood, but no efficient/practical method exited).
We will now compare this to the current status of NNLO

NNLO: when is NLO not good enough?

- when NLO corrections are large (NLO correction ~ LO)

  This may happens when

  - process involve very different scales → large logarithms of ratio of scales appear
  - new channels open up at NLO (at NLO they are effectively LO)
  - paramount example: Higgs production

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- when high precision is needed to match small experimental error

  - W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

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- when high precision is needed to match small experimental error

  - W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

- when a reliable error estimate is needed

Some history of NNLO

- first NNLO computation of a collider process was inclusive Drell-Yan production by Hamberg, van Neerven and Matsuura in '91
- second NNLO calculation: Higgs production in gluon-gluon fusion by Harlander and Kilgore in '02

  Both calculations refer to inclusive, total cross-sections that are not measurable

- first exclusive NNLO computation (for fiducial volume cross-sections) was Higgs $\to \gamma\gamma$ in '04 by Anastasiou, Melnikov and Petriello, followed by other exclusive calculations of Higgs and Drell-Yan processes
- only last year NNLO corrections to $2 \to 2$ processes also with QCD partons in the final state started to appear. This indicates a more complete understanding of NNLO

  Many things at NNLO are new and took a while to understand. Today's technology is likely not to be finalized yet

Ingredients for NNLO

Remember crucial steps for automated NLO were

1. leading order can be computed automatically and efficiently (e.g. via recursion relations)
2. one can reduce the one-loop to product of tree-level amplitudes
3. it was well understood how to subtract singularities
4. the basis of master integrals was known

At NNLO the situation is very different

1. leading order of very limited importance
2. no procedure to reduce two-loop to tree-level (unitarity approaches at two face still many outstanding issues)
3. subtraction of singularities far from trivial
4. basis set of master integrals not known, integrals not all/always known analytically

And all this for simple processes (no result exist, or has been attempted, for any $2 \to 3$ scattering process)

Ingredients for NNLO

What changed in the last years

1. technology to compute integrals
2. extension of systematic FKS subtraction to NNLO

Collider processes known at NNLO

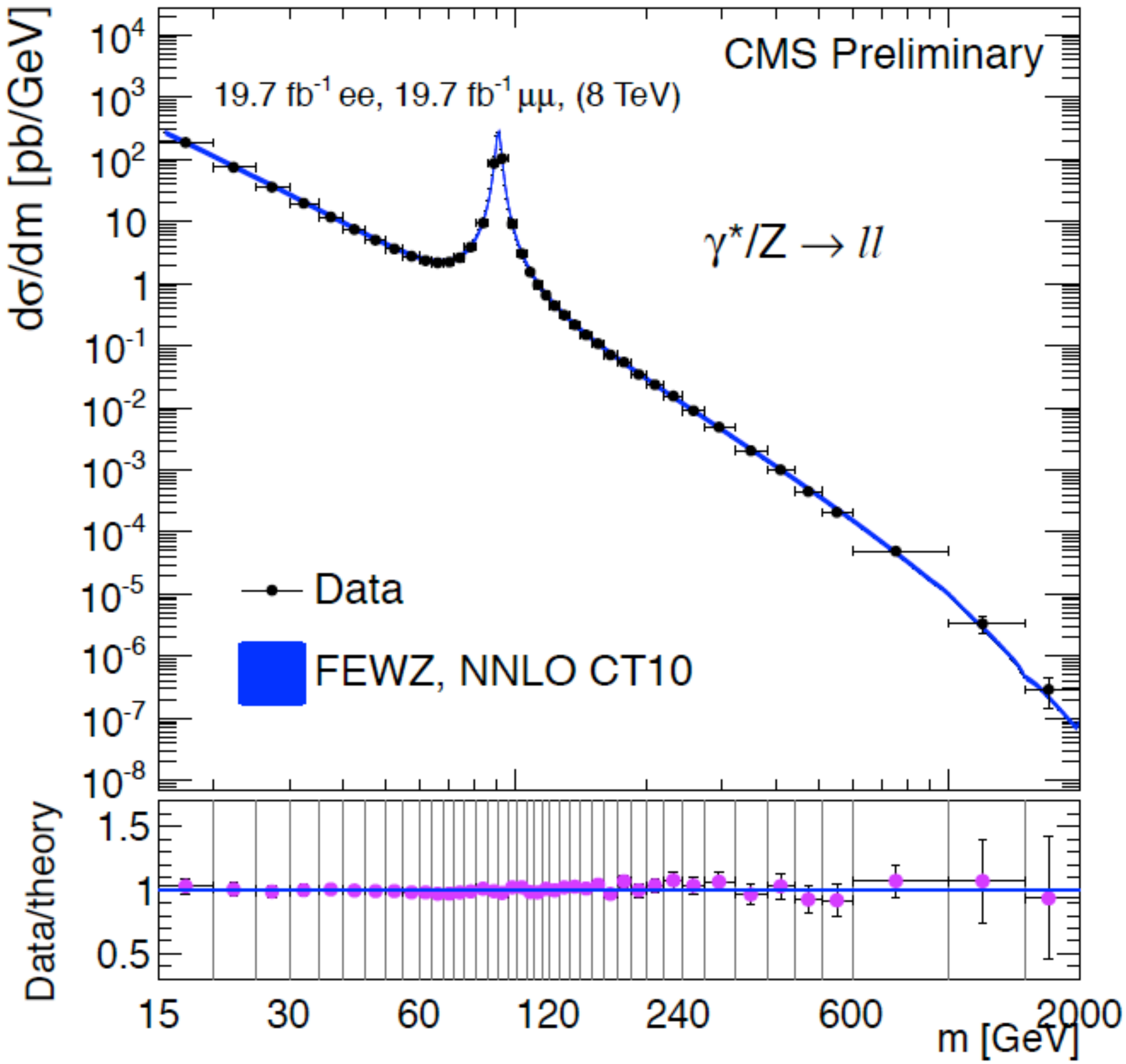
1. Drell-Yan (Z,W) (inclusive) — van Neerven '90
2. Higgs (inclusive) — Harlander et al '02; Anastasiou et al '02; Ravindran et al '03
3. Higgs differential — Anastasiou et al '04; Catani et al '07
4. WH/ZH total cross-section — Brein et al '04; Ferrera et al '11
5. di-photon production — Catani et al '11
6. H+1jet — Boughezal et al. '13
7. top-pair production — Czakon et al '13
8. inclusive jets — Currie et al. '13
9. Z/W + photon — Grazzini et al. '13-14
10. ZZ — Cascioli et al. '14
11. t-channel single top — Bruscherseifer '14

NB: this list is growing really quickly now ...

NNLO vs LHC data

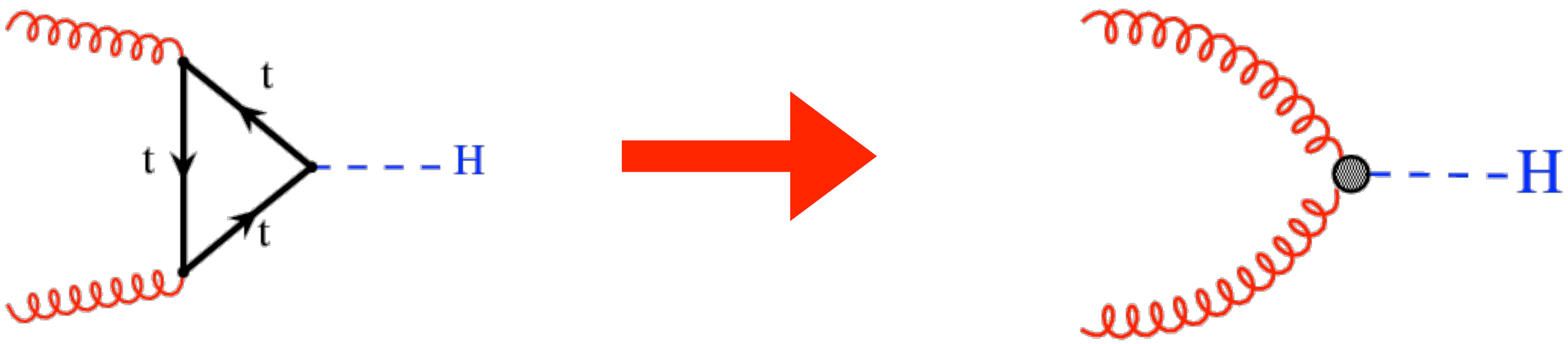
Impressive agreement between experiment and NNLO theory

CMS-PAS-SMP-14-003

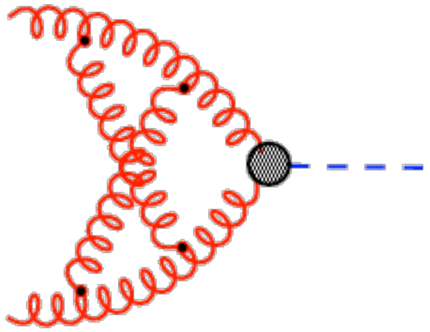


Inclusive NNLO Higgs production

Inclusive Higgs production via gluon-gluon fusion in the large m_t-limit:

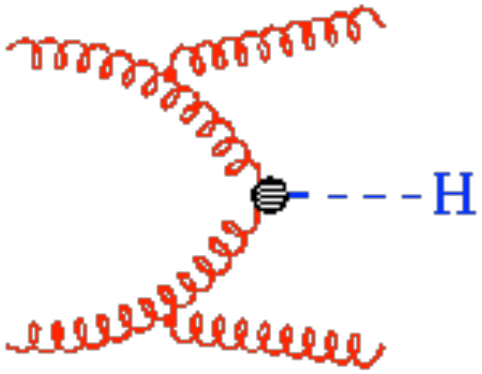


NNLO corrections known since many years now:



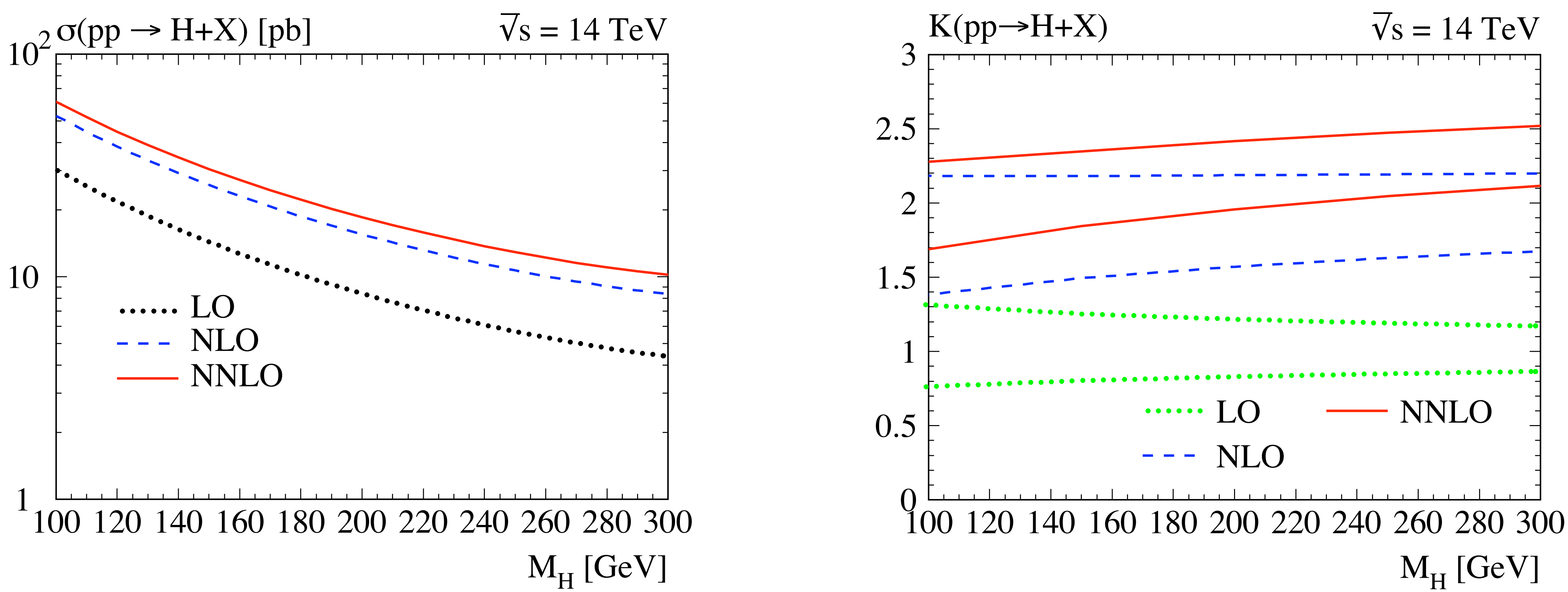
virtual-virtual

real-virtual



real-real

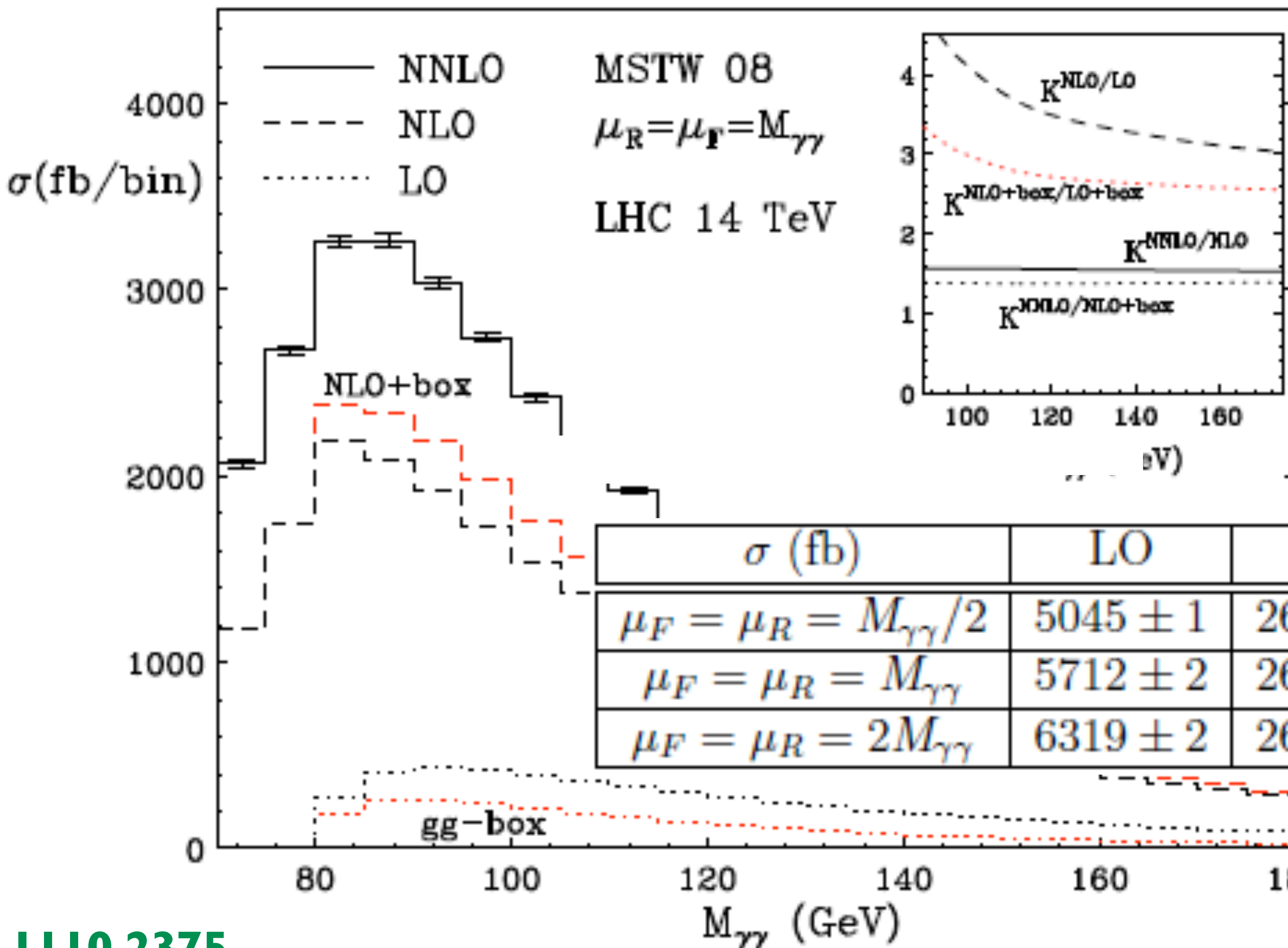
Inclusive NNLO Higgs production



Kilgore, Harlander '02
Anastasiou, Melnikov '02

Many improvements on this calculation over the last 10 years (EW corrections, NNLO+PS, resummations, exclusive decays...)

Recent NNLO highlights:γγ



σ (fb)	LO	NLO	NNLO
$\mu_F = \mu_R = M_{\gamma\gamma}/2$	5045 ± 1	26581 ± 23	45588 ± 97
$\mu_F = \mu_R = M_{\gamma\gamma}$	5712 ± 2	26402 ± 25	43315 ± 54
$\mu_F = \mu_R = 2M_{\gamma\gamma}$	6319 ± 2	26045 ± 24	41794 ± 77

Catani et al. 1110.2375

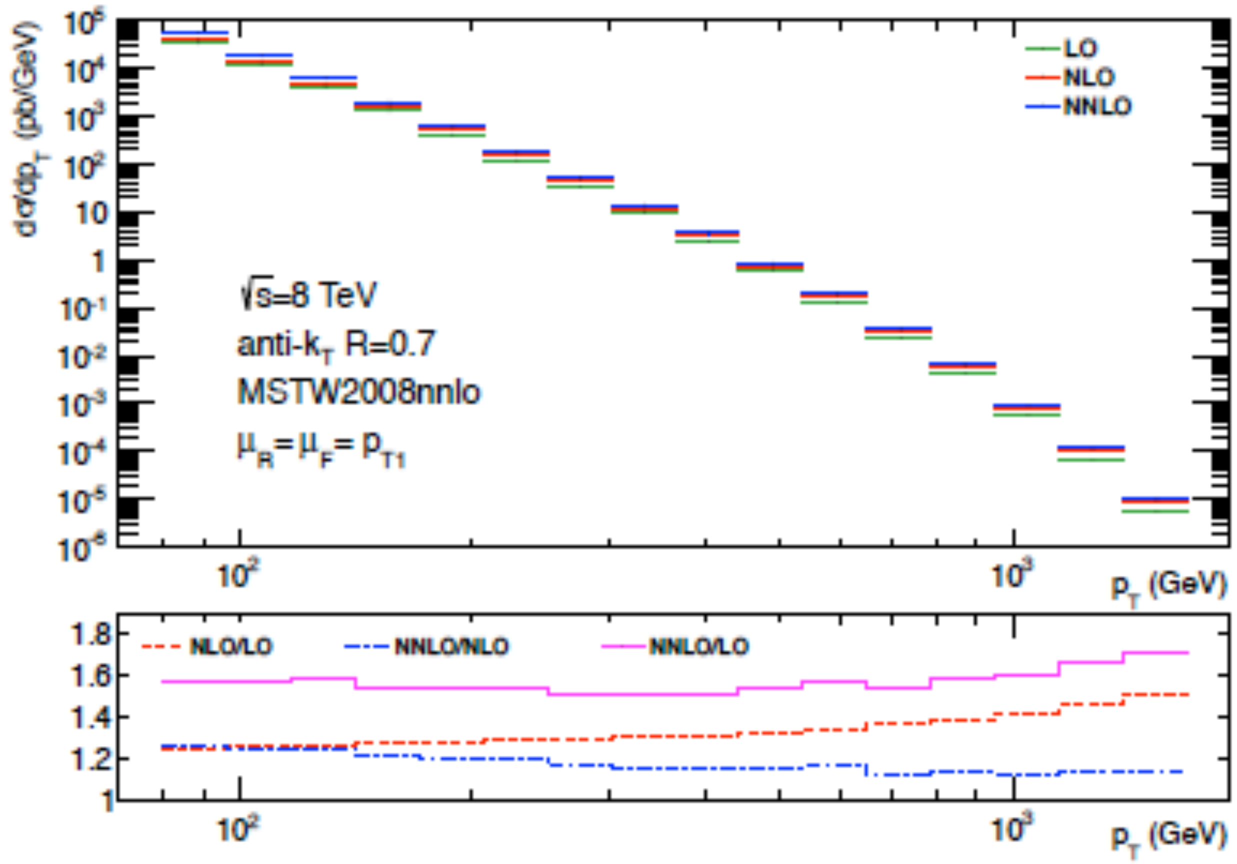
⇒ no good convergence of PT (asymmetric cuts + new channels)

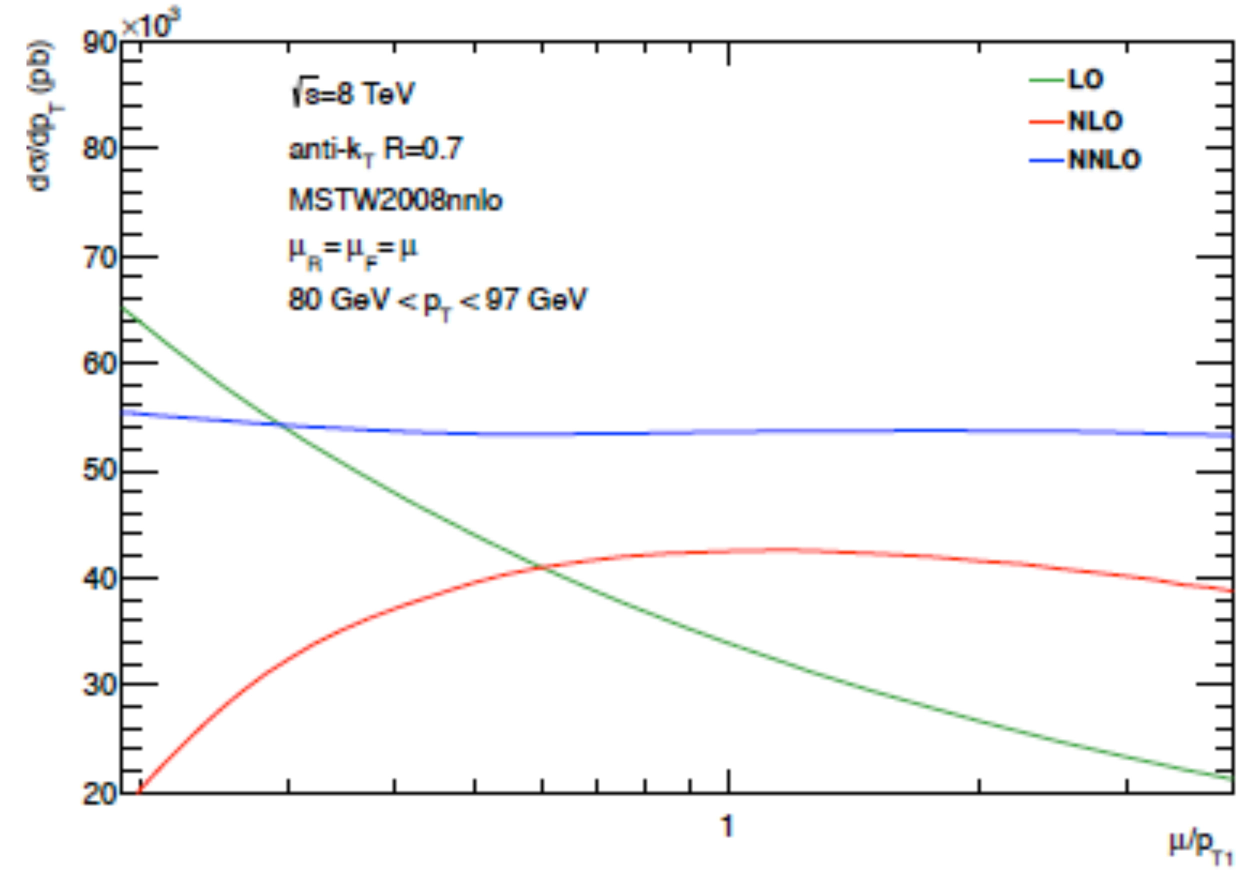
[similar to gg → H]

Recent NNLO highlights: dijets

gluon only contribution

Gehrmann et al. 1301.7310





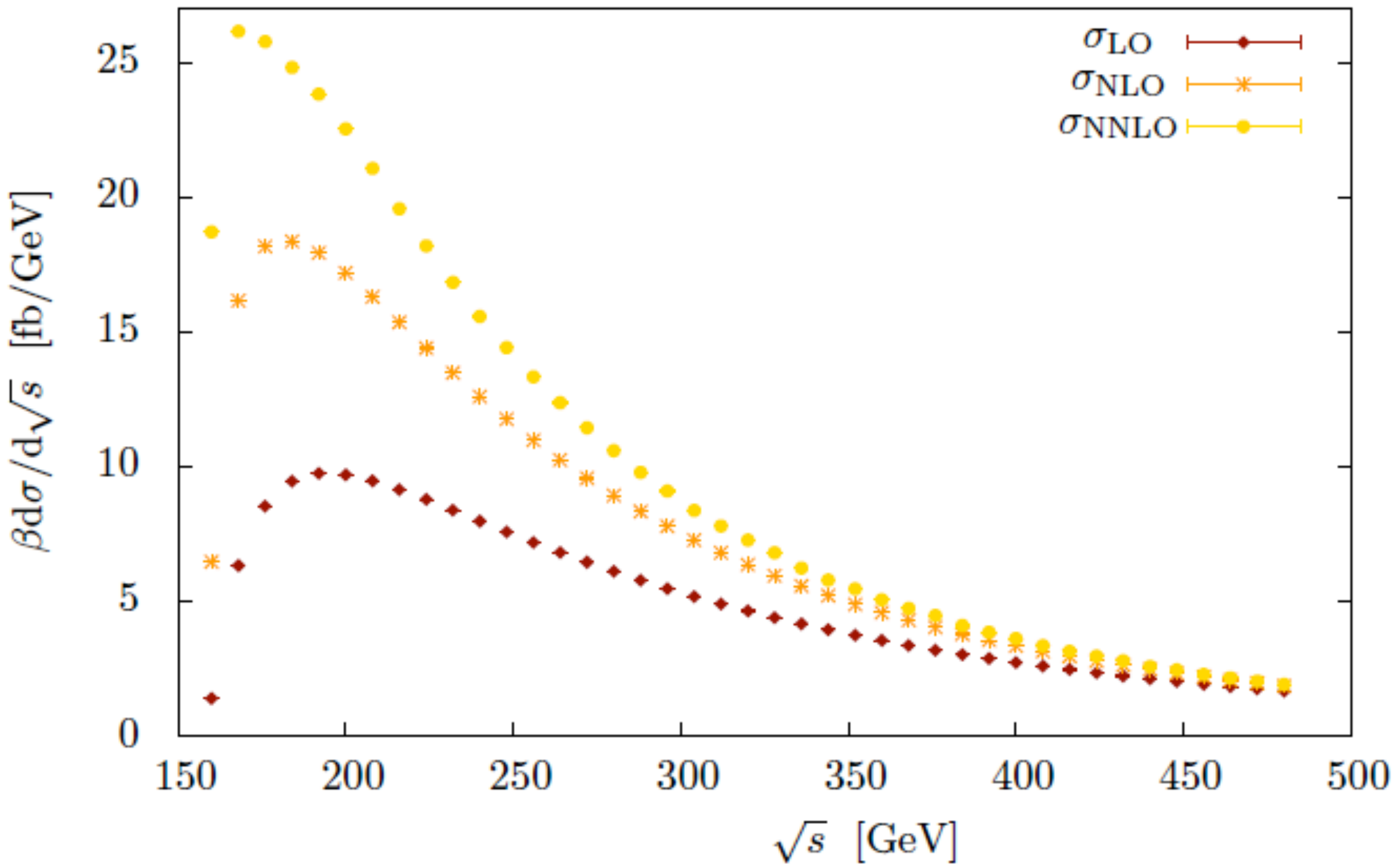
⇒ no good convergence of PT [similar to gg → H, pp → γγ]

Does this pattern survive once the full NNLO calculation is completed?

Recent NNLO highlights: H+1jet

Gluon fusion contribution to H+1jet

Bouzhegal et al. 1302.6216

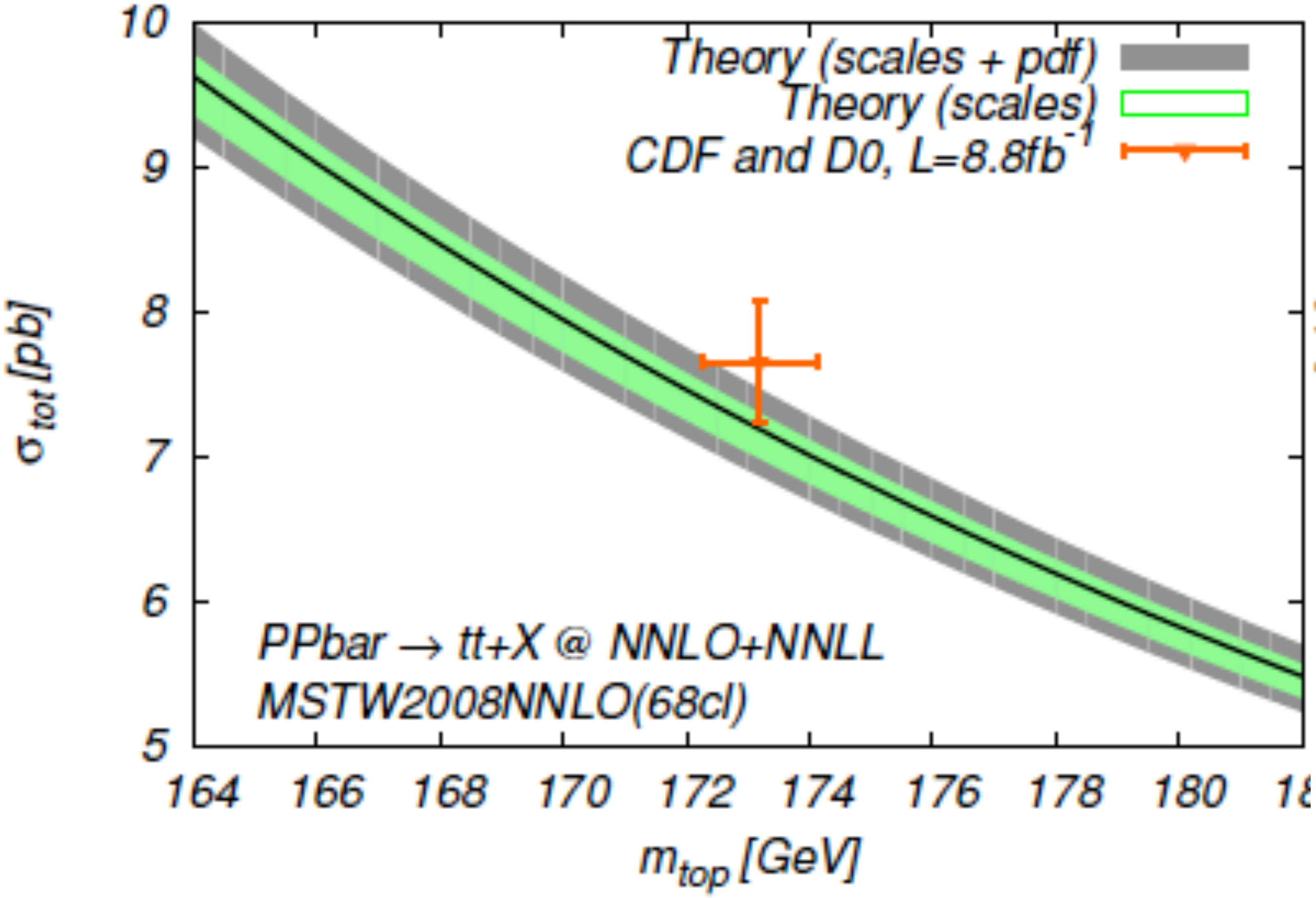


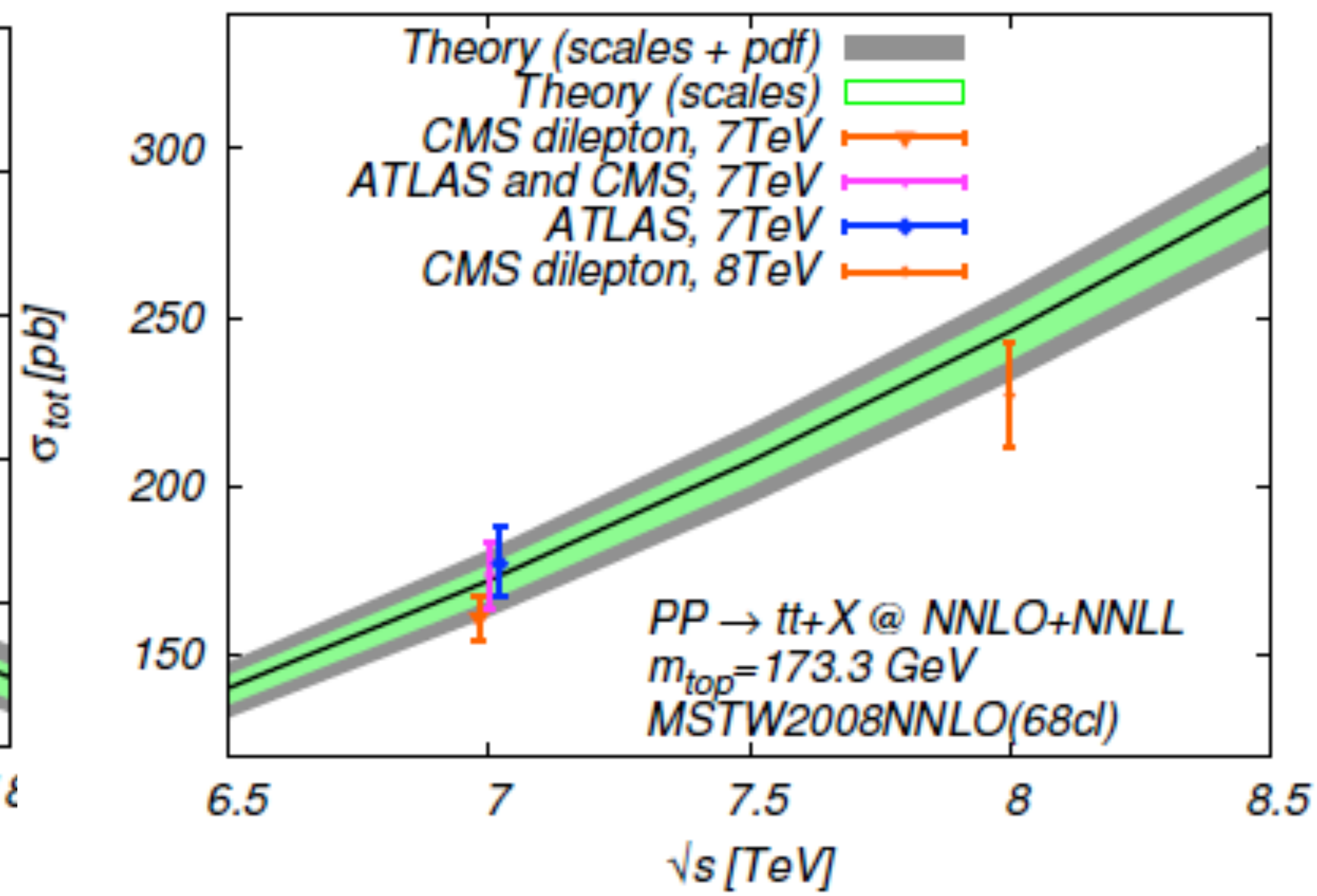
⇒ no good convergence of PT [similar to $gg \to H$, $pp \to \gamma\gamma$, $pp \to$ dijets]

Does this pattern survive once the full NNLO calculation is completed?

Recent NNLO highlights: tt

First full NNLO calculation with colored particles in the initial and final state. Paves the way to a number of other calculations





Czakon et al. 1303.6254
[+ previous refs...]

Beyond NNLO

Anastasiou et al 1403.4616

First approximate N^{3}LO calculation of inclusive Higgs production

$$\hat{\sigma}_{ij}(\hat{s}, m_H) = \frac{\pi C(\mu^2)^2}{8v^2} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k \eta_{ij}^{(k)}(z)$$

where $C(\mu^2)/(4v)$ is the effective Hgg coupling and $z = m_H^2/\hat{s}$

New! Result for delta and plus terms at N^{3}LO in the threshold expansion

$$\begin{aligned}
\hat{\eta}^{(3)}(z) \simeq\ & \delta(1-z)\, 1124.308887\ldots & (\to 5.1\%) \\
& + \left[\frac{1}{1-z}\right]_+ 1466.478272\ldots & (\to -5.85\%) \\
& - \left[\frac{\log(1-z)}{1-z}\right]_+ 6062.086738\ldots & (\to -22.88\%) \\
& + \left[\frac{\log^2(1-z)}{1-z}\right]_+ 7116.015302\ldots & (\to -52.45\%) \\
& - \left[\frac{\log^3(1-z)}{1-z}\right]_+ 1824.362531\ldots & (\to -39.90\%) \\
& - \left[\frac{\log^4(1-z)}{1-z}\right]_+ 230 & (\to 20.01\%) \\
& + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 216\,. & (\to 93.72\%)
\end{aligned}$$

large cancellations between different terms lead to:

$$\hat{\eta}^{(3)}(z) \sim -2.2\%$$

Reminder:

$$\int_0^1 dz \left[\frac{g(z)}{1-z}\right]_+ f(z) \equiv \int_0^1 dz\, \frac{g(z)}{1-z}\,[f(z) - f(1)]$$

Beyond NNLO

Anastasiou et al 1403.4616

Problem threshold expansion ambiguous (can multiply and divide out by any function that goes to 1 for $z \to 1$)

$$\int dx_1\, dx_2\, [f_i(x_1)\, f_j(x_2) z g(z)] \lim_{z\to 1} \left[\frac{\hat{\sigma}_{ij}(s,z)}{z g(z)}\right]$$

Take different form for g(z) and look at the N^3LO correction relative to the fixed order

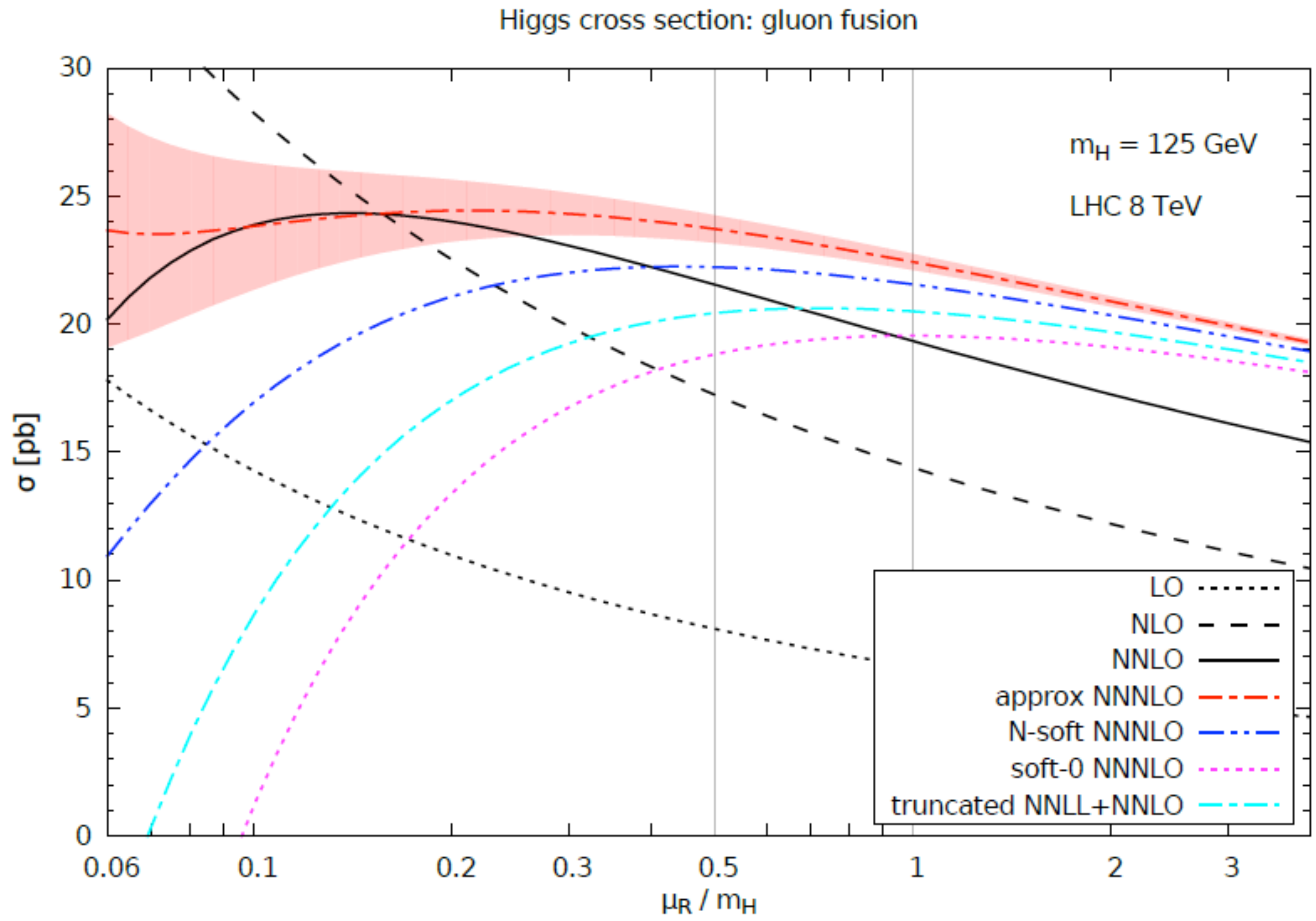
g(z)	1	z	z^2	1/z
$\delta N^3LO/LO$	-2.2%	8.2%	30.2%	7.7%

Too premature for phenomenology ... ?

Beyond NNLO

Bonvini et al 1404.3204

Comparison of several approximate N^3LO



Large N^3LO corrections + large spread in the predictions

Exact NNNLO may not be that far ...

Recap of fixed order

- Leading order
  - everything can be computed in principle today (practical edge: 8 particles in the final state), many public codes
  - techniques: standard Feynman diagrams or recursive methods (Berends-Giele, BCF, CSW, ...)
- Next-to-leading order
  - automation realized for QCD corrections
  - next: NLO EW corrections and NLO for BSM
- Next-to-next-to-leading order
  - $2 \to 1$ processes available since a while (Higgs, Drell-Yan)
  - a number of new results for $2 \to 2$ processes. More to come soon.
- Next-to-next-to-next-to-leading order
  - very first steps ...

Parton shower & Monte Carlo methods

- the probability for emitting a gluon above k_t is given by

$$P(\text{emission above } k_t) \sim \frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \int \frac{d\theta}{\theta} \Theta(E\theta - k_t)$$

NB: based on soft-collinear approximation

- useful to look at the probability of not emitting a gluon

$$P(\text{no emission above } k_t) \sim 1 - \frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \int \frac{d\theta}{\theta} \Theta(E\theta - k_t)$$

- the probability of nothing happening to all orders is the exponential of the first order result -- this is called Sudakov form factor

$$\Delta(k_t, Q) \sim exp\left\{ -\frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \int \frac{d\theta}{\theta} \Theta(E\theta - k_t) \right\}$$

Done properly: α_s in the integration and use full splitting function

Parton shower: use above to generate many emissions in the soft-collinear approximation + add hadronization model

NLO + parton shower

NLO + parton shower combines the best features of the two methods: correct rates (NLO) and hadron-level description of events (PS)

Difficult because need to avoid double counting

Two main working examples:

1. MC@NLO (aMC@NLO)

   Frixione&Webber '02 and later refs.

   - explicitly subtract double counting

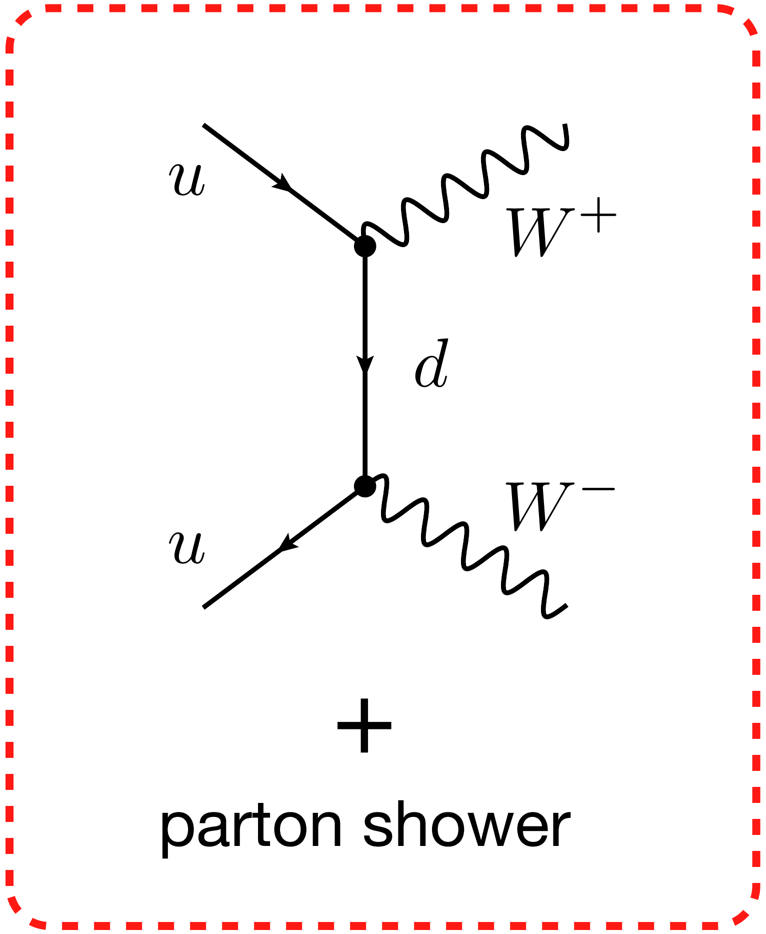
2. POWHEG (POWHEG-BOX)

   Nason '04 and later refs.

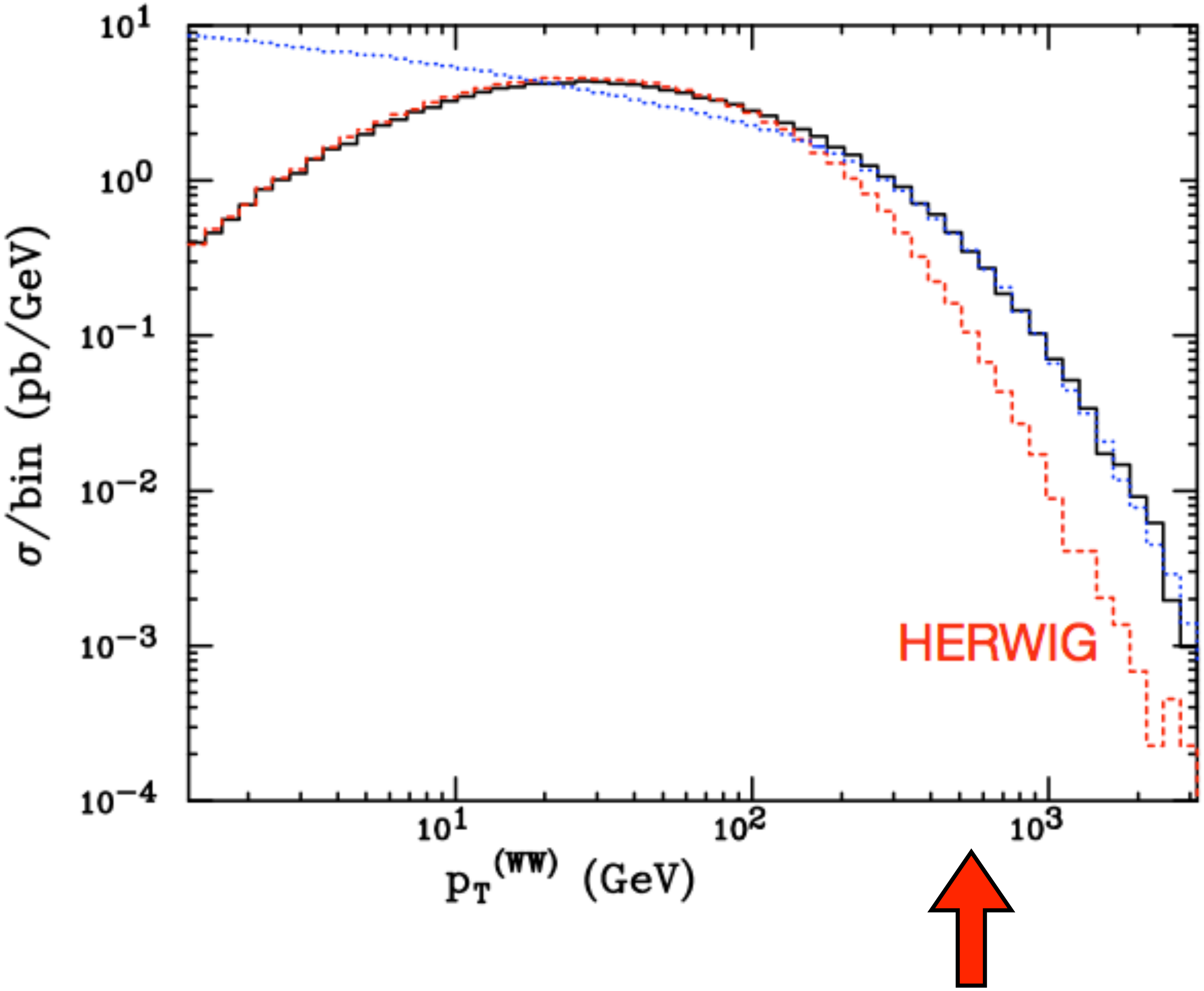
   - hardest emission from NLO (good for p_t ordered shower)

First only processes with no light jets in the final state, now large number of processes implemented. In fact, almost automated procedures reached in the POWHEG BOX and in aMC@NLO

MC@NLO: W^+W^- production (LHC)

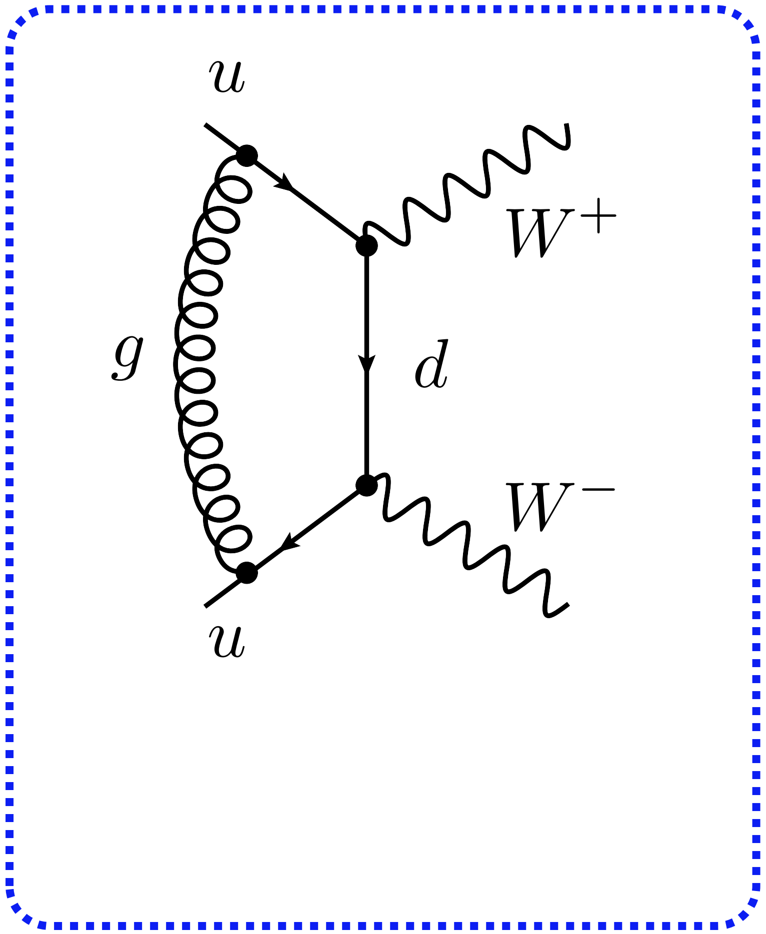


HERWIG

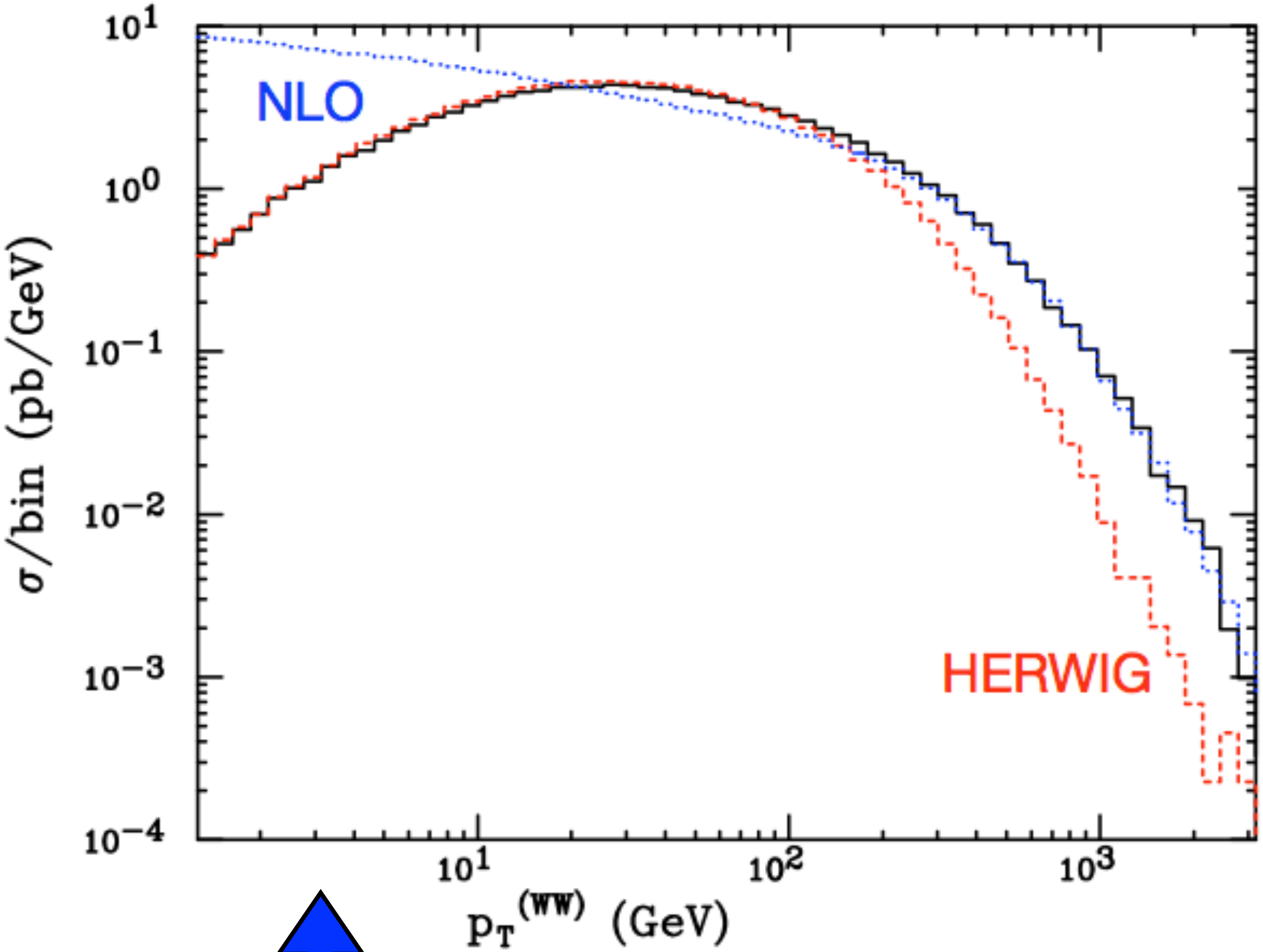


Herwig too soft in the high-p_t region

MC@NLO: W^+W^- production (LHC)

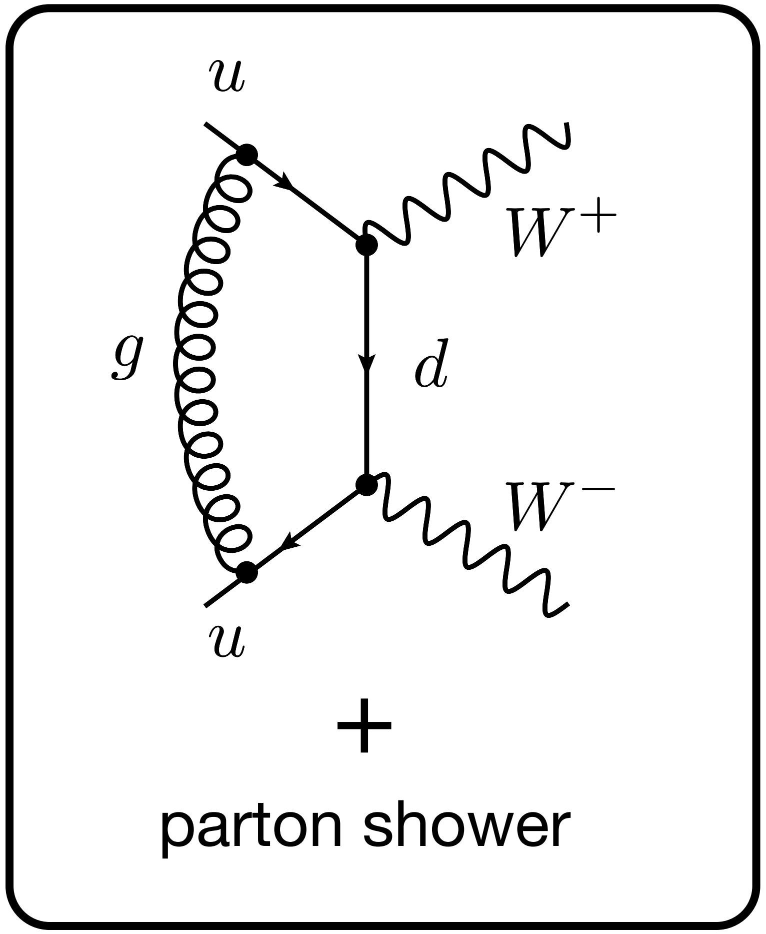


NLO

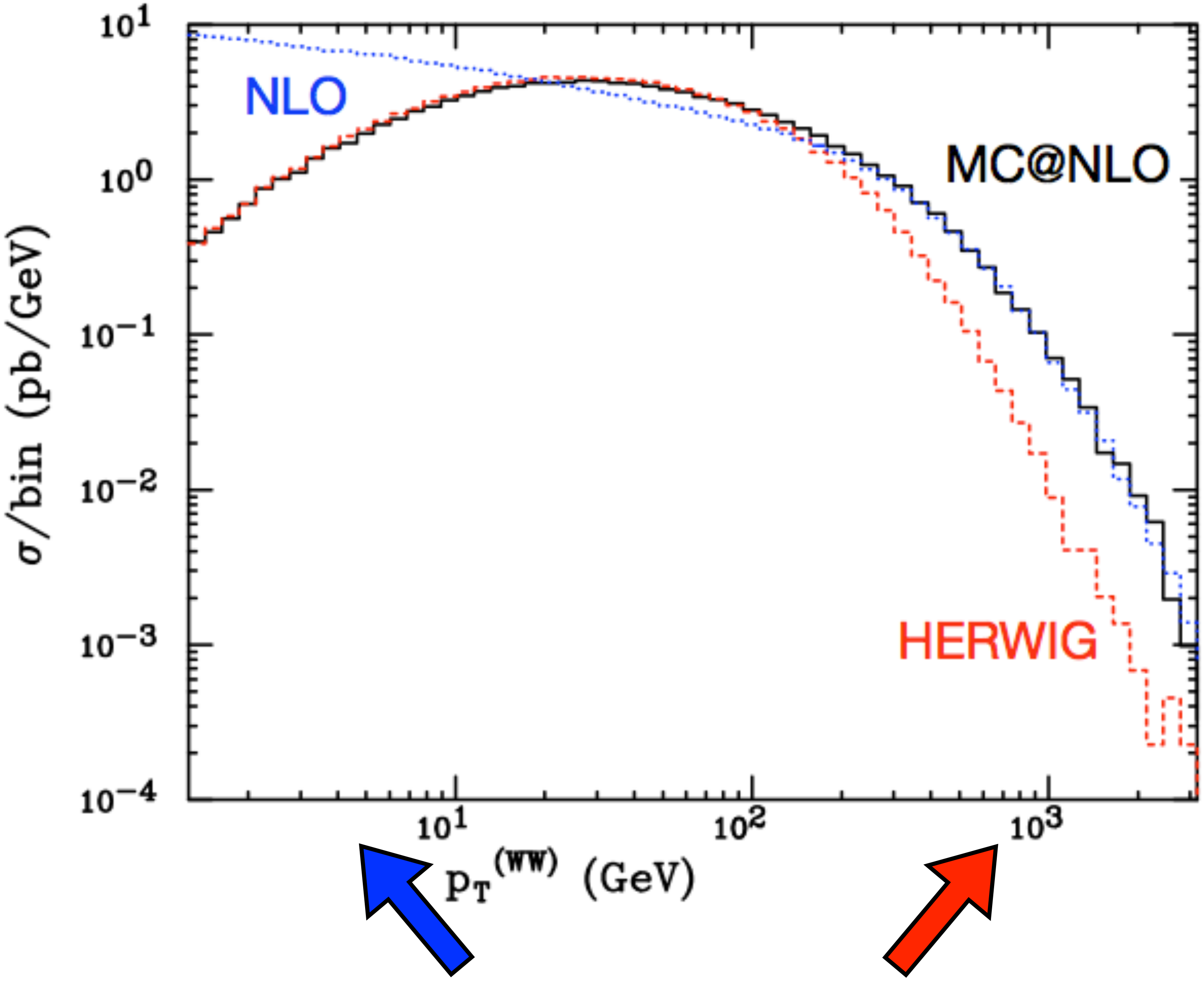


NLO divergent
in the soft region

MC@NLO: W^+W^- production (LHC)



MC@NLO



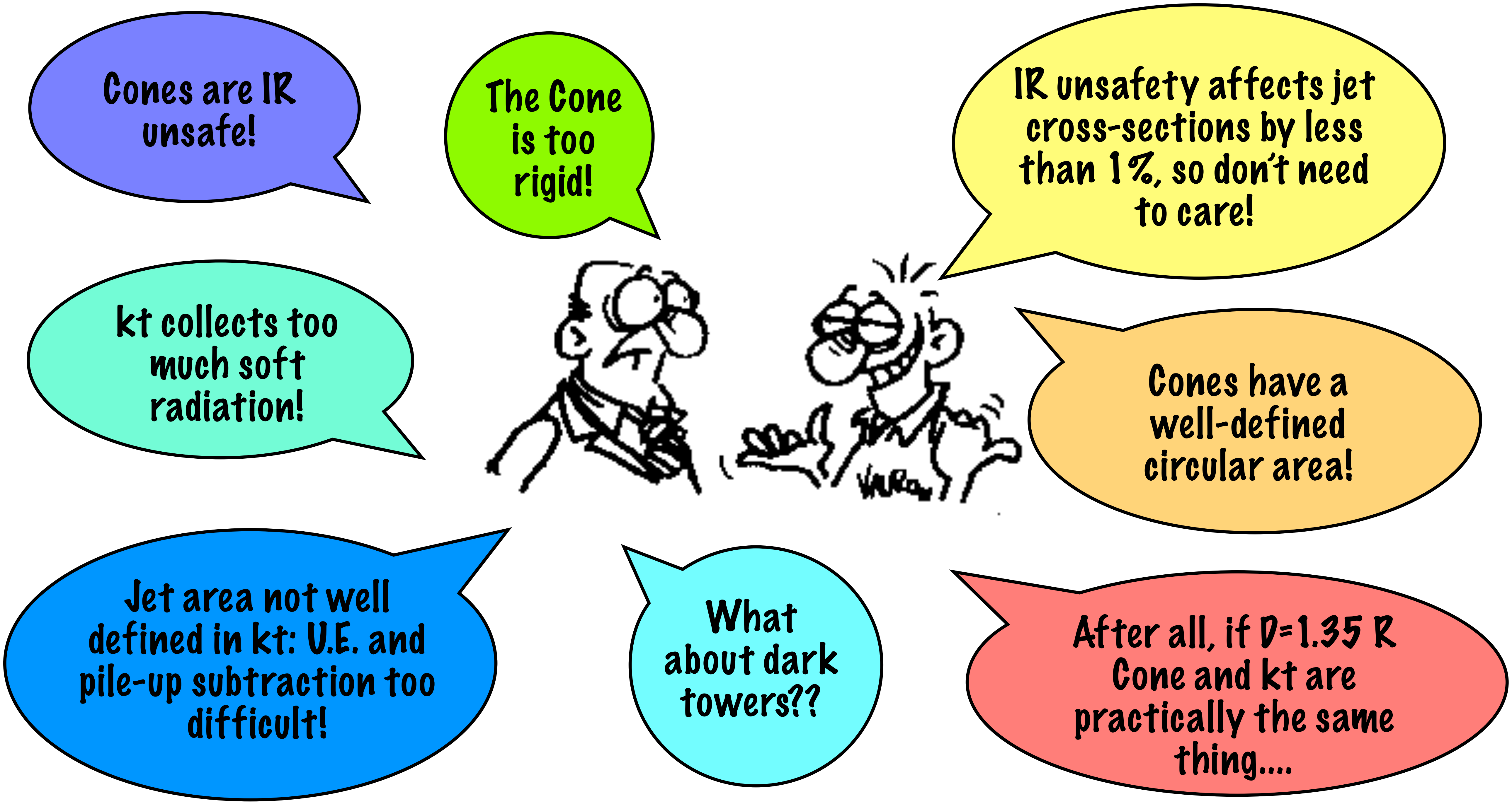
MC@NLO correctly interpolates between the two regimes

NNLO+PS

New challenge given the many recent NNLO results, natural to look for matching NNLO and parton shower

It turns out that this problem is intimately related to merging of NLO+PS for different jet multiplicities. Lots of activity in this direction recently.

Jets: about 10 years ago...



Where do jets enter ?

Essentially everywhere at colliders!

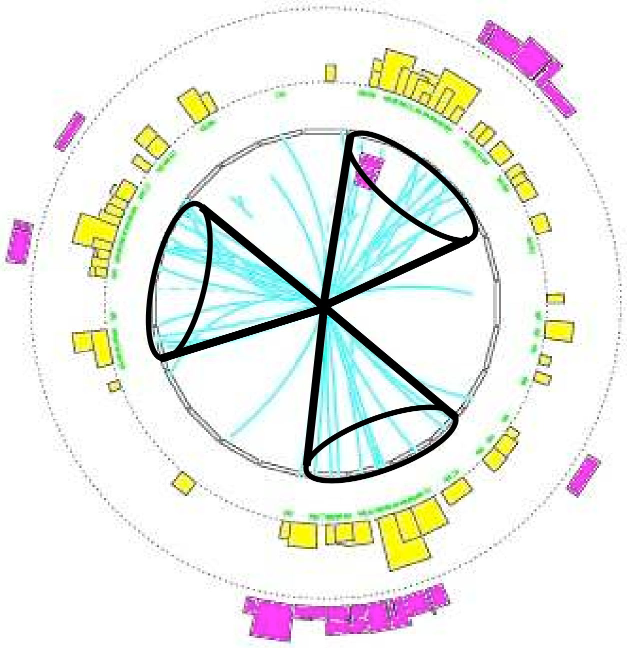
Jets are an essential tool for a variety of studies:

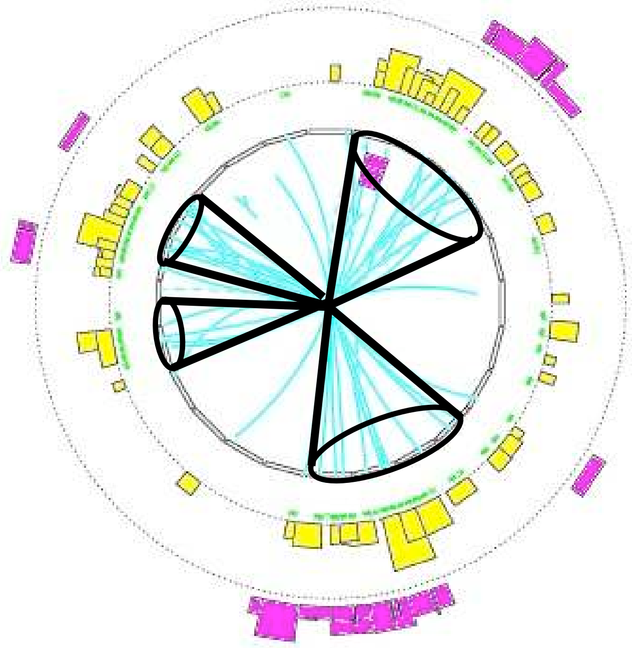
- top reconstruction
- mass measurements
- most Higgs and New Physics searches
- general tool to attribute structure to an event
- instrumental for QCD studies, e.g. inclusive-jet measurements
  $\Rightarrow$ important input for PDF determinations

Jets

Jets provide a way of projecting away the multiparticle dynamics of an event ⇒ leave a simple quasi-partonic picture of the hard scattering

The projection is fundamentally ambiguous ⇒ jet physics is a rich subject





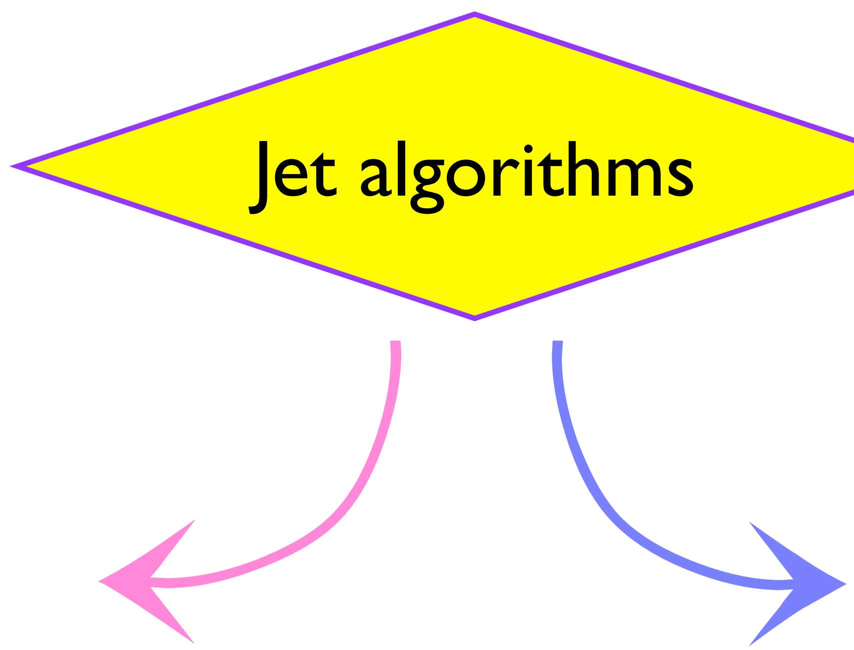
Ambiguities:

1) Which particles should belong to a same jet ?

2) How does recombine the particle momenta to give the jet-momentum?

Two broad classes of jet algorithms

Today many extensions of the original Sterman-Weinberg jets.
Modern jet-algorithms divided into two broad classes



Cone type

(UA1,JetCLU, Midpoint, SISCone..)

top down approach:
cluster particles according to distance in coordinate-space

Idea: put cones along dominant direction of energy flow

Sequential

(kt-type, Jade, Cambridge/ Aachen...)

bottom up approach: cluster particles according to distance in momentum-space

Idea: undo branchings occurred in the PT evolution

Inclusive k_t/Durham-algorithm

Catani et. al '92-'93; Ellis&Soper '93

<u>Inclusive algorithm:</u>

1. For any pair of final state particles i,j define the distance

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \min\{k_{ti}^2, k_{tj}^2\}$$

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2. For each particle i define a distance with respect to the beam

$$d_{iB} = k_{ti}^2$$

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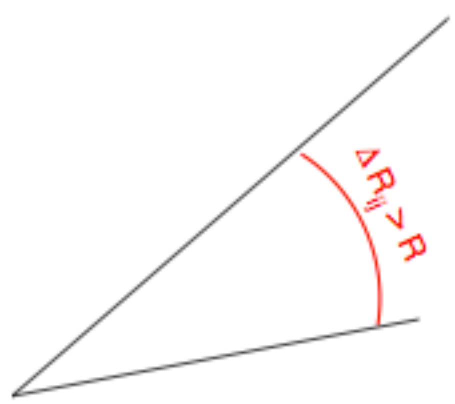
$$d_{iB} = k_{ti}^2$$

3. Find the smallest distance. If it is a d_{ij} recombine i and j into a new particle ($\Longrightarrow$ recombination scheme); if it is d_{iB} declare i to be a jet and remove it from the list of particles

   NB: if $\Delta R_{ij}^2 \equiv \Delta y_{ij}^2 + \Delta\phi_{ij}^2 < R^2$ then partons (ij) are always recombined, so R sets the minimal interjet angle



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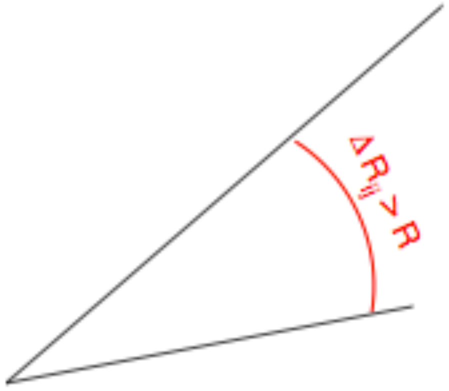
2. For each particle i define a distance with respect to the beam

$$d_{iB} = k_{ti}^2$$

3. Find the smallest distance. If it is a d_{ij} recombine i and j into a new particle (⇒ recombination scheme); if it is d_{iB} declare i to be a jet and remove it from the list of particles

   NB: if $\Delta R_{ij}^2 \equiv \Delta y_{ij}^2 + \Delta\phi_{ij}^2 < R^2$ then partons (ij) are always recombined, so R sets the minimal interjet angle



4. repeat the procedure until no particles are left

Exclusive k_t/Durham-algorithm

Inclusive algorithm gives a variable number of jets per event, according to the specific event topology

Exclusive k_t/Durham-algorithm

Inclusive algorithm gives a variable number of jets per event, according to the specific event topology

Exclusive version: run the inclusive algorithm but stop when either

- all d_{ij}, $d_{iB} > d_{cut}$ or
- when reaching the desired number of jets n

The CA and the anti-k_t algorithm

The Cambridge/Aachen: sequential algorithm like k_t, but uses only angular properties to define the distance parameters

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = 1 \qquad \Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$$

Dotshitzer et. al '97;Wobisch and Wengler '99

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The anti-kt algorithm: designed not to recombine soft particles together

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Cacciari, Salam, Soyez '08

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Cacciari, Salam, Soyez '08

anti-kt is the default algorithm for ATLAS and CMS

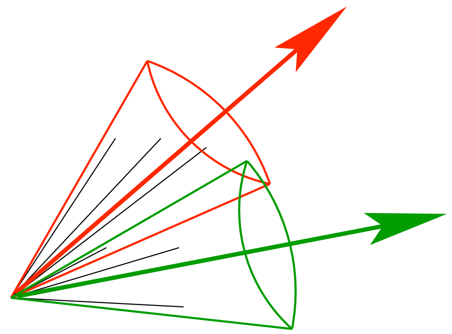
unfortunately with different default R 0.4 & 0.6 [ATLAS] 0.5 & 0.7 [CMS]

First time only IR-safe algorithms are used systematically at a collider

Cone algorithms

1. A particle i at rapidity and azimuthal angle (y_i, Φ_i) ⊂ cone C iff

$$\sqrt{(y_i - y_C)^2 + (\phi_i - \phi_C)^2} \leq R_{\rm cone}$$



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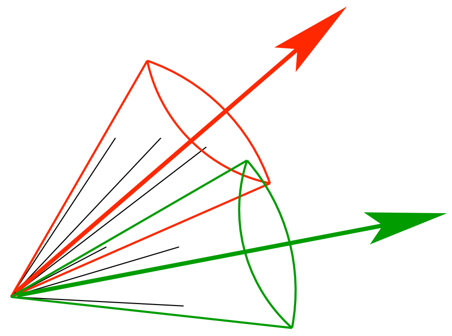
$$\bar{y}_C \equiv \frac{\sum_{i \in C} y_i \cdot p_{T,i}}{\sum_{i \in C} p_{T,i}} \qquad \bar{\phi}_C \equiv \frac{\sum_{i \in C} \phi_i \cdot p_{T,i}}{\sum_{i \in C} p_{T,i}}$$



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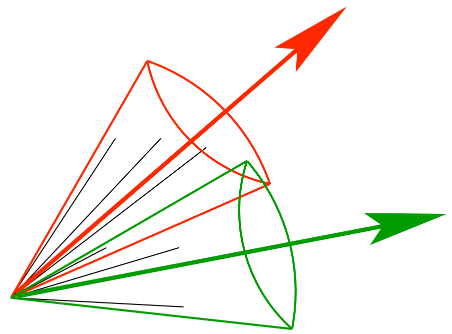

3. If weighted and geometrical averages coincide $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ a stable cone ($\Rightarrow$ jet) is found, otherwise set $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ & iterate

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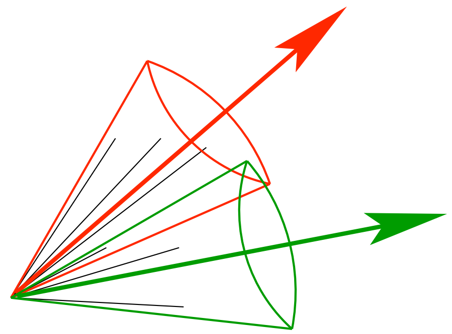

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4. Stable cones can overlap. Run a split-merge on overlapping jets: merge jets if they share more than an energy fraction f, else split them and assign the shared particles to the cone whose axis they are closer to.
Remark: too small f (<0.5) creates large jets, not recommended

Cone algorithms

- The question is where does one start looking for stable cone ?
- The direction of these trial cones are called seeds
- Ideally, place seeds everywhere, so as not to miss any stable cone
- Practically, this is unfeasible. Speed of recombination grows fast with the number of seeds. So place only some seeds, e.g. at the (y, Φ)-location of particles.

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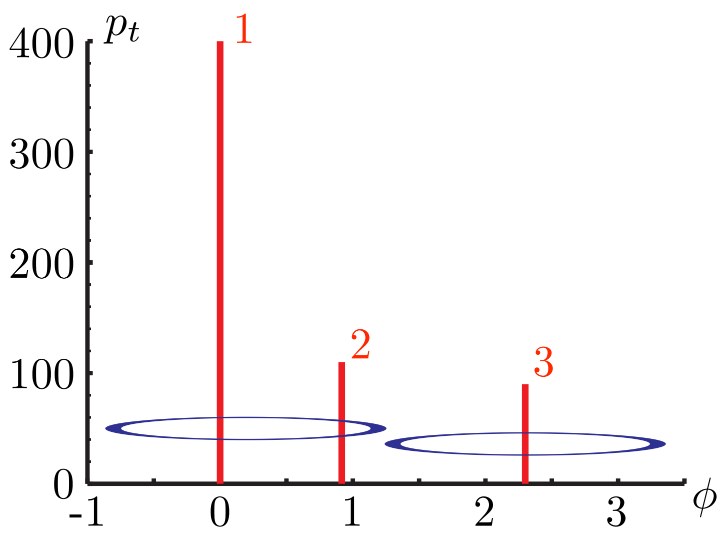
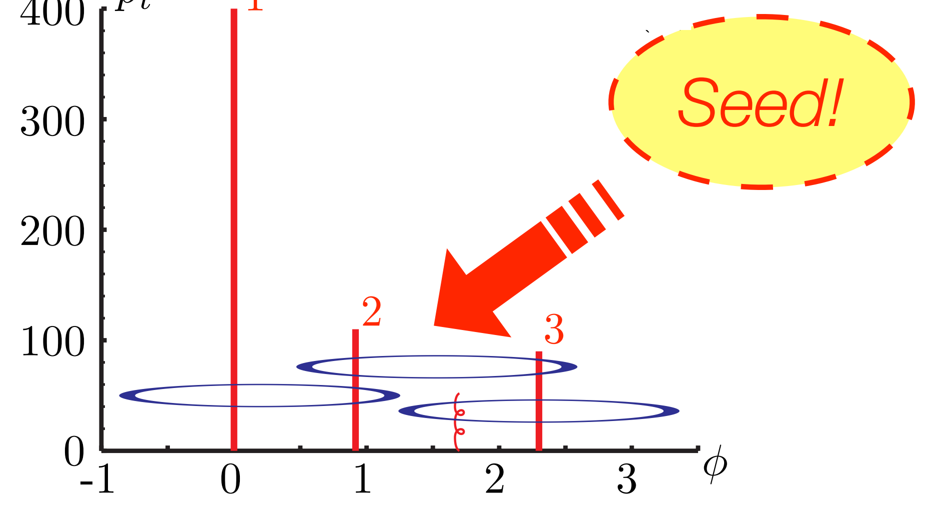
Seeds make cone algorithms infrared unsafe

Jets: infrared unsafety of cones



3 hard ⇒ 2 stable cones



3 hard + 1 soft ⇒ 3 stable cones

Soft emission changes the hard jets ⇒ algorithm is IR unsafe

Midpoint algorithm: take as seed position of emissions and midpoint between two emissions (postpones the infrared safety problem)

Seedless cones

Solution:
use a seedless algorithm, i.e. consider all possible combinations of particles as candidate cones, so find all stable cones [$\Rightarrow$ jets]

Blazey '00

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The problem:
clustering time growth as $N2^N$. So for an event with 100 particles need 10^{17} ys to cluster the event ⇒ prohibitive beyond PT (N=4,5)

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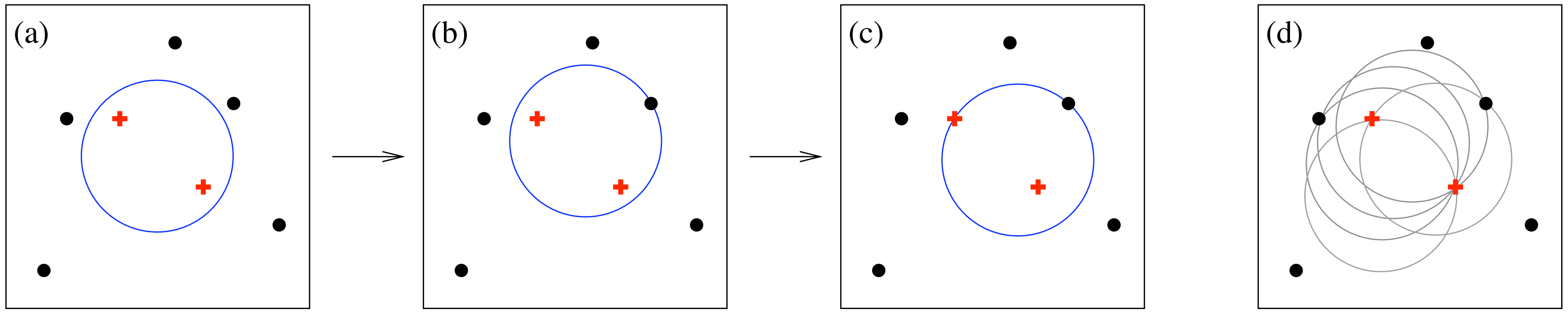
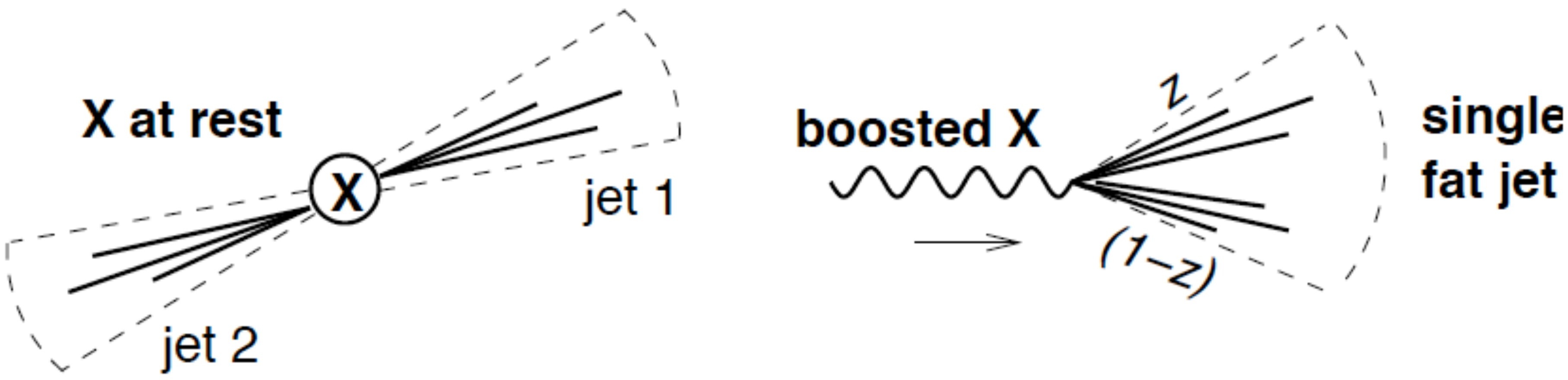
Better solution:

SISCone recasts the problem as a computational geometry problem, the identification of all distinct circular enclosures for points in 2D and finds a solution to that ⇒ $N^2 \ln N$ time IR safe algorithm



Salam, Soyez '07

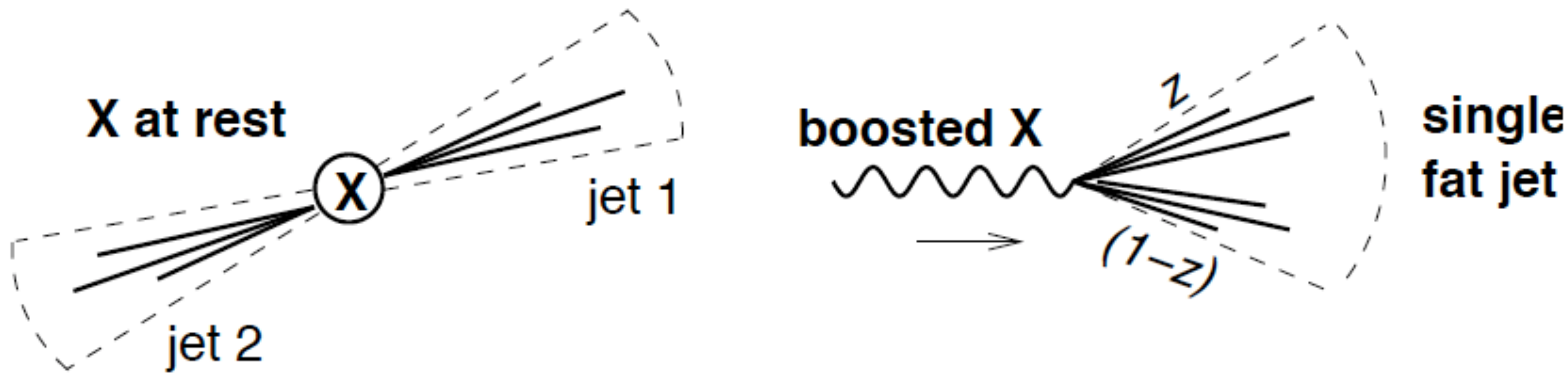
Jet-substructure at the LHC



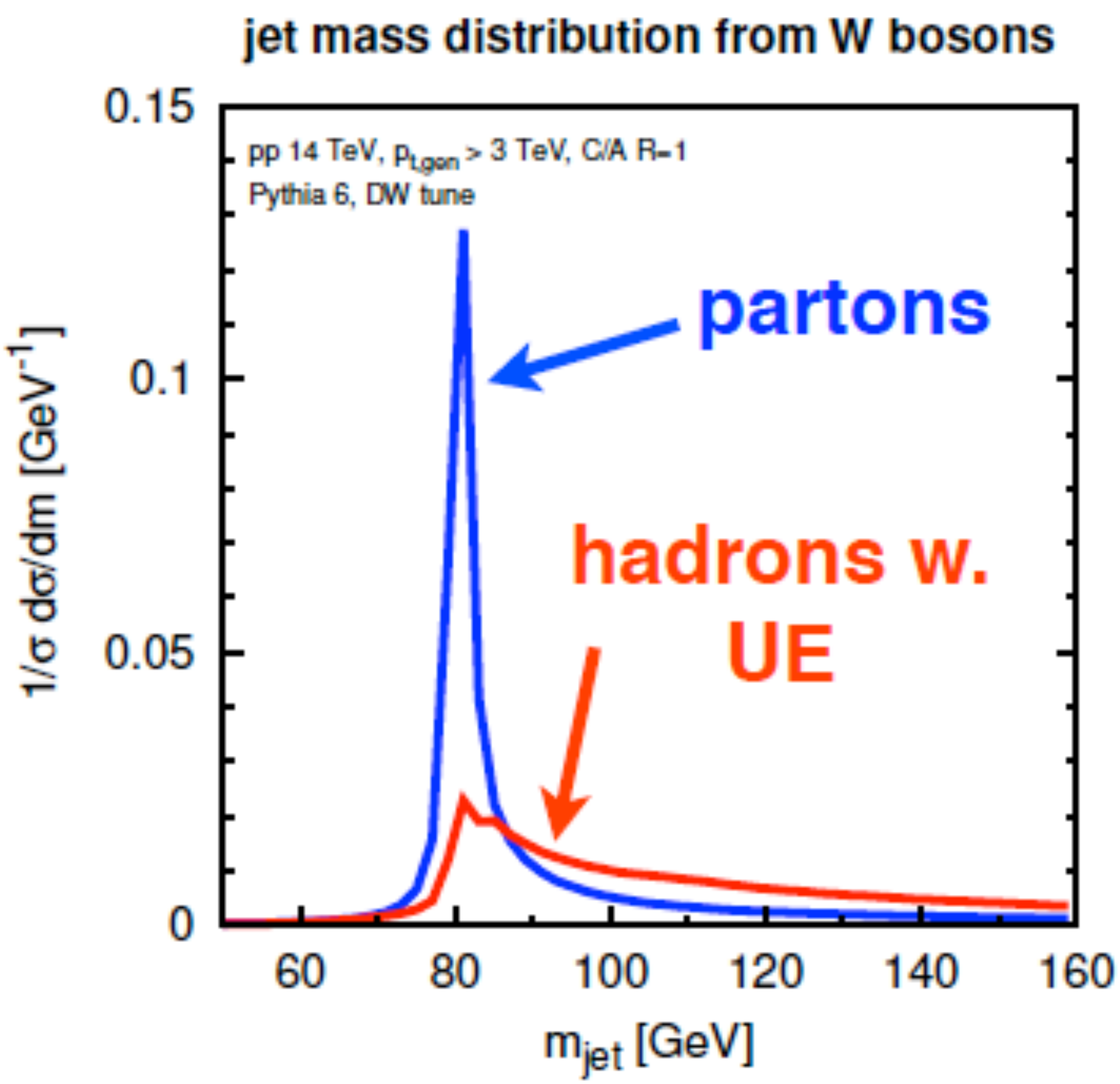
Triggered by a paper in 2008 by Butterworth, Davison Rubin, Salam ["Jet substructure as a new Higgs search channel at the LHC"] vibrant new sub-field emerged using jet-substructure to discover boosted heavy new particles

- well over 100 papers in the past 5 years
- dedicated conferences and write-ups (see e.g. 1012.5412, 1311.2708 or 1312.2708)
- upcoming BOOST2014 conference in August at UCL
- new nomenclature (trimming, pruning, filtering, mass-drop, N subjettiness, shower deconstruction ...)

Jet-substructure at the LHC



Jet-mass is a natural variable to look for massive particles, but very large smearing from QCD radiation, hadronization, underlying event/pileup ...



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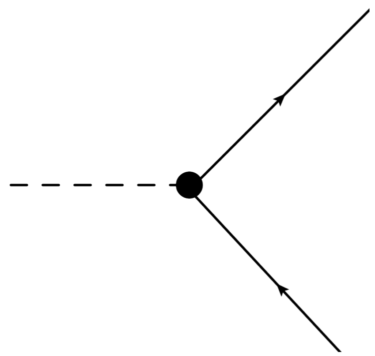
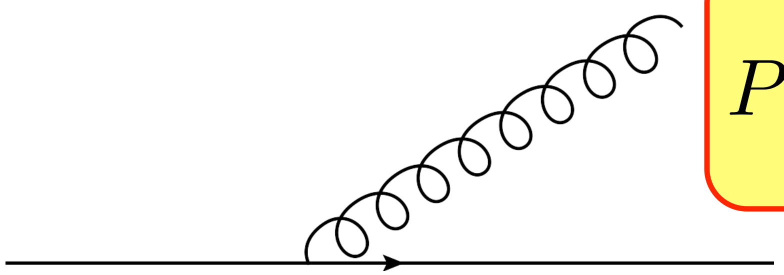


$P(z) \propto 1$

BSM signal



$$P(z) \propto \frac{1+z^2}{1-z}$$

QCD background

Two main handles to

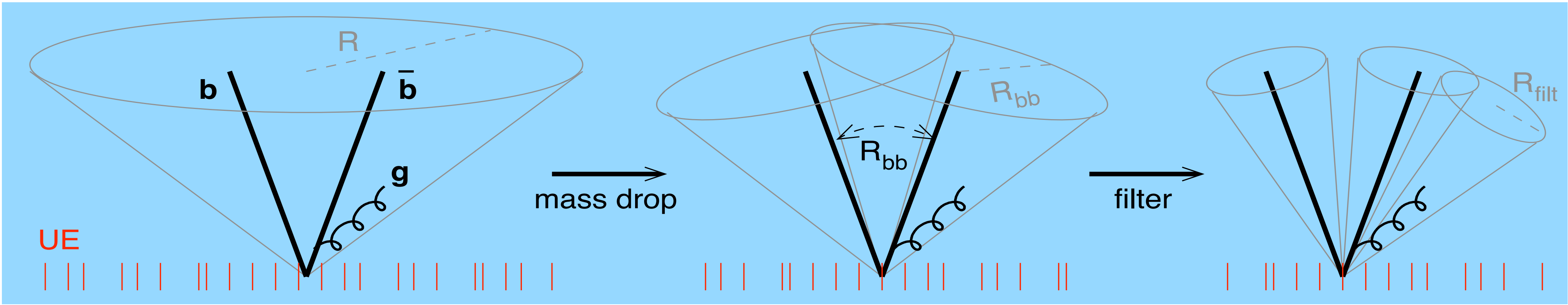
- signal prefer symmetric splittings, while background (QCD) prefers soft radiation, i.e. asymmetric splitting
- large angle radiation from color singlet is suppressed (angular ordering) $\rightarrow$ cutting wide angle radiation kills the background and does not affect much the signal

A large variety of methods (10-20?) to achieve these goals.

Typically: performance of new method tested with Monte Carlo

Mass-drop tagger for $H \to bb$

Butterworth, Davison, Rubin, Salam '08



1. cluster the event with e.g. CA algo and large-ish R

2. undo last recomb: large mass drop + symmetric + b tags

3. filter away the UE: take only the 3 hardest sub-jets

Exploit the specific pattern of $H \to bb$ vs $g \to gg$, $q \to gg$

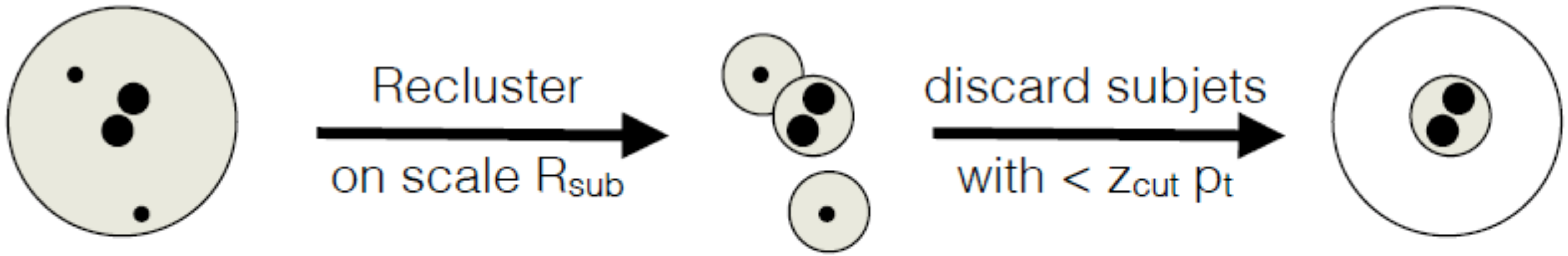
- QCD partons prefer soft emissions (hard → hard + soft)
- Higgs decay prefers symmetric splitting
- try to beat down contamination from underlying event
- try to capture most of the perturbative QCD radiation

Subsequently changed (modified mass-drop tagger) to follow the higher p_t branch

Dasgupta, Marzani, Fergoso, Salam '13

Pruning and trimming

Pruning fixes a radius $R=m/p_t$ and reclusters the jet such that if two object are separated by angles larger then this and the branching is asymmetric, i.e. $\min(p_{t,a}, p_{t,b}) < z_{cut}\, p_{t,a+b}$, then the softer object is discarded.



Trimming uses a fixed radius R_{trim}

Jet-substructure at the LHC

Typical procedure:

introduce a way to analyze/deconstruct the event . Methods introduce energy/angular constraints, cuts (fixed or dynamical)

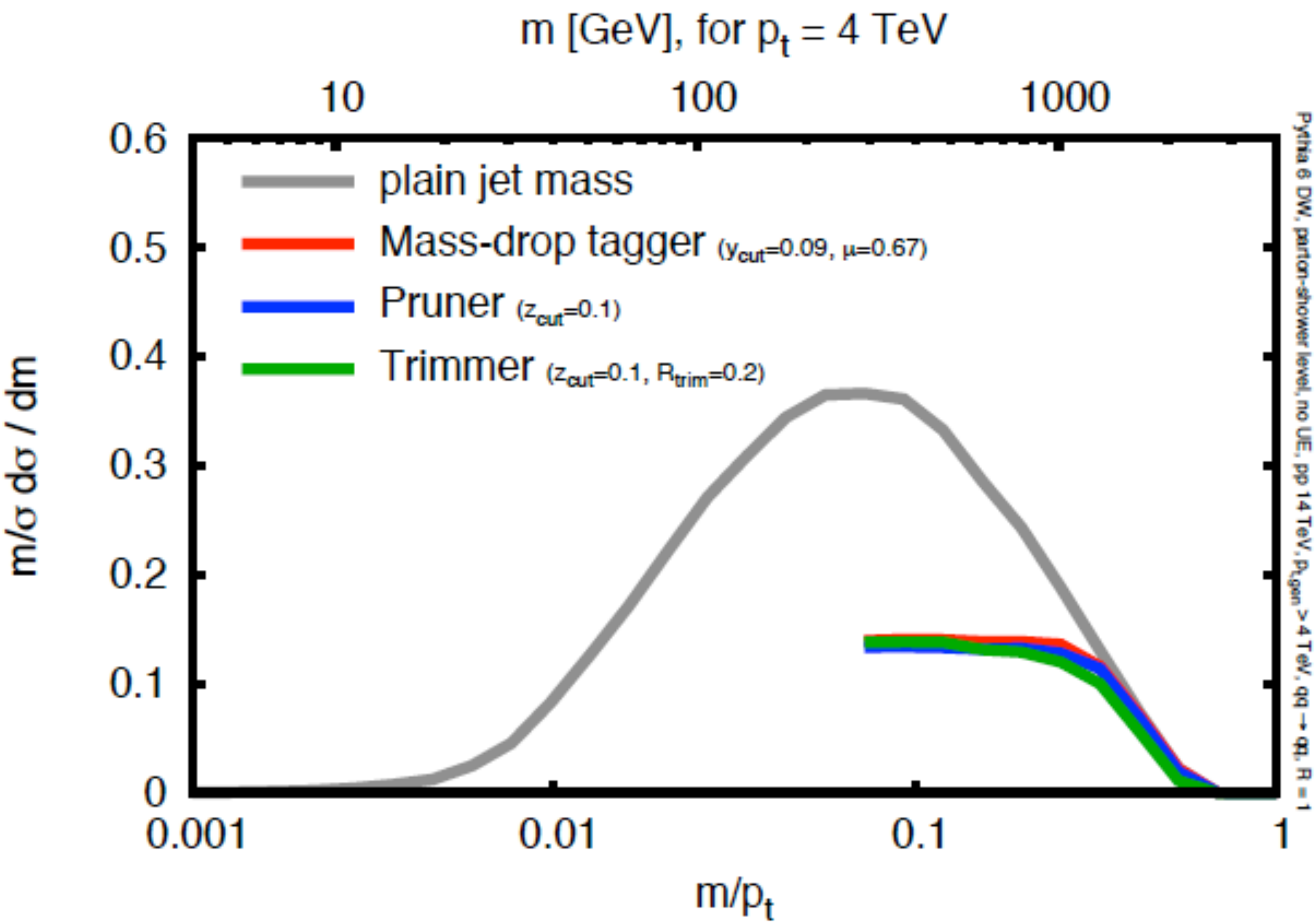
As a consequence:

- many parameters, complicated procedure, transparency lost
- potential of duplication/redundancy

Important questions

- how to judge/optimize performance? obvious answer: run Monte Carlo. But only a limited number of studies can be performed
- robustness: how much do results depend on parameters?
- how can one chose parameters a priori (without knowing where/what BSM physics might show up?)

Monte Carlo comparison of taggers



Taggers look quite similar ...

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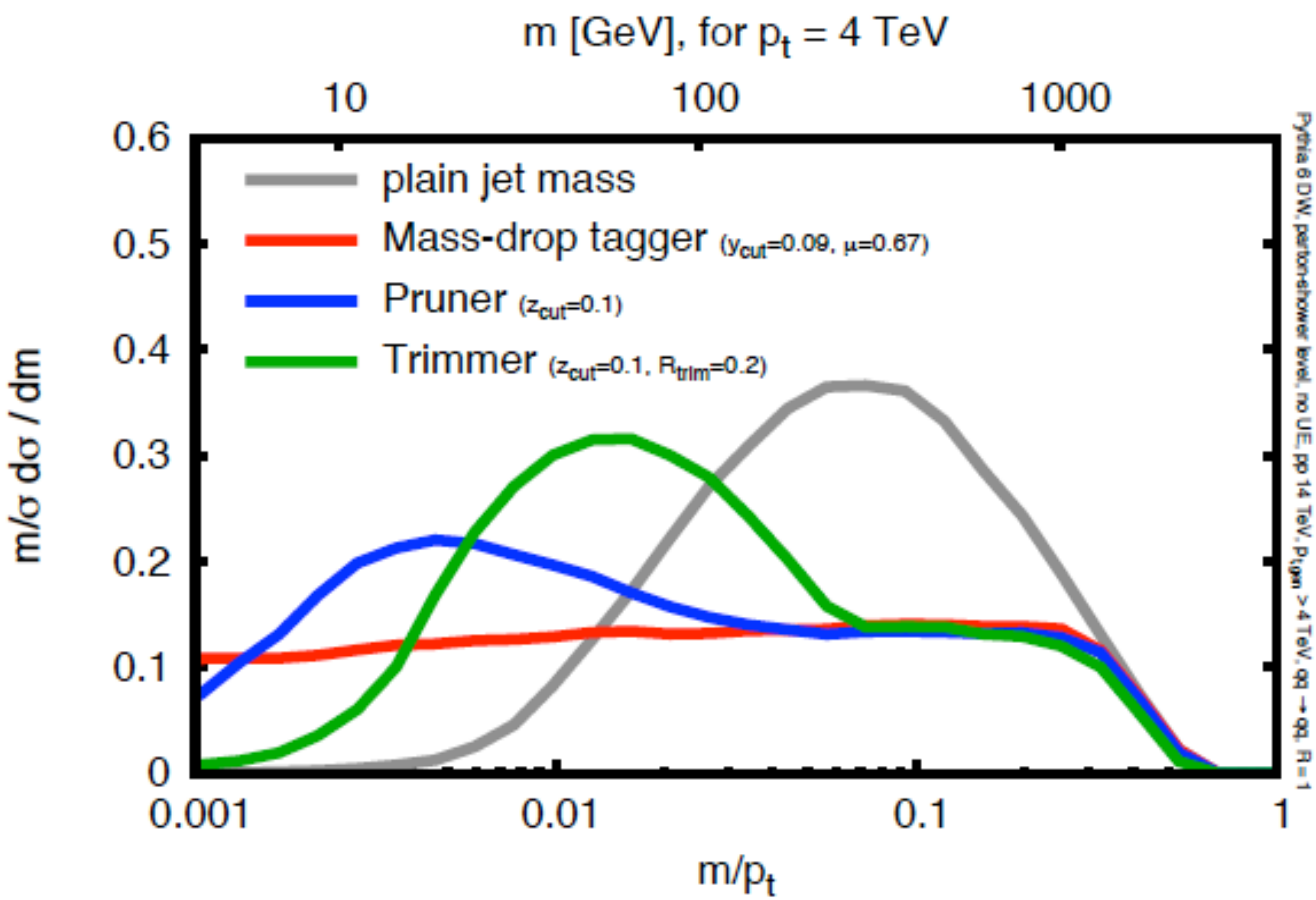


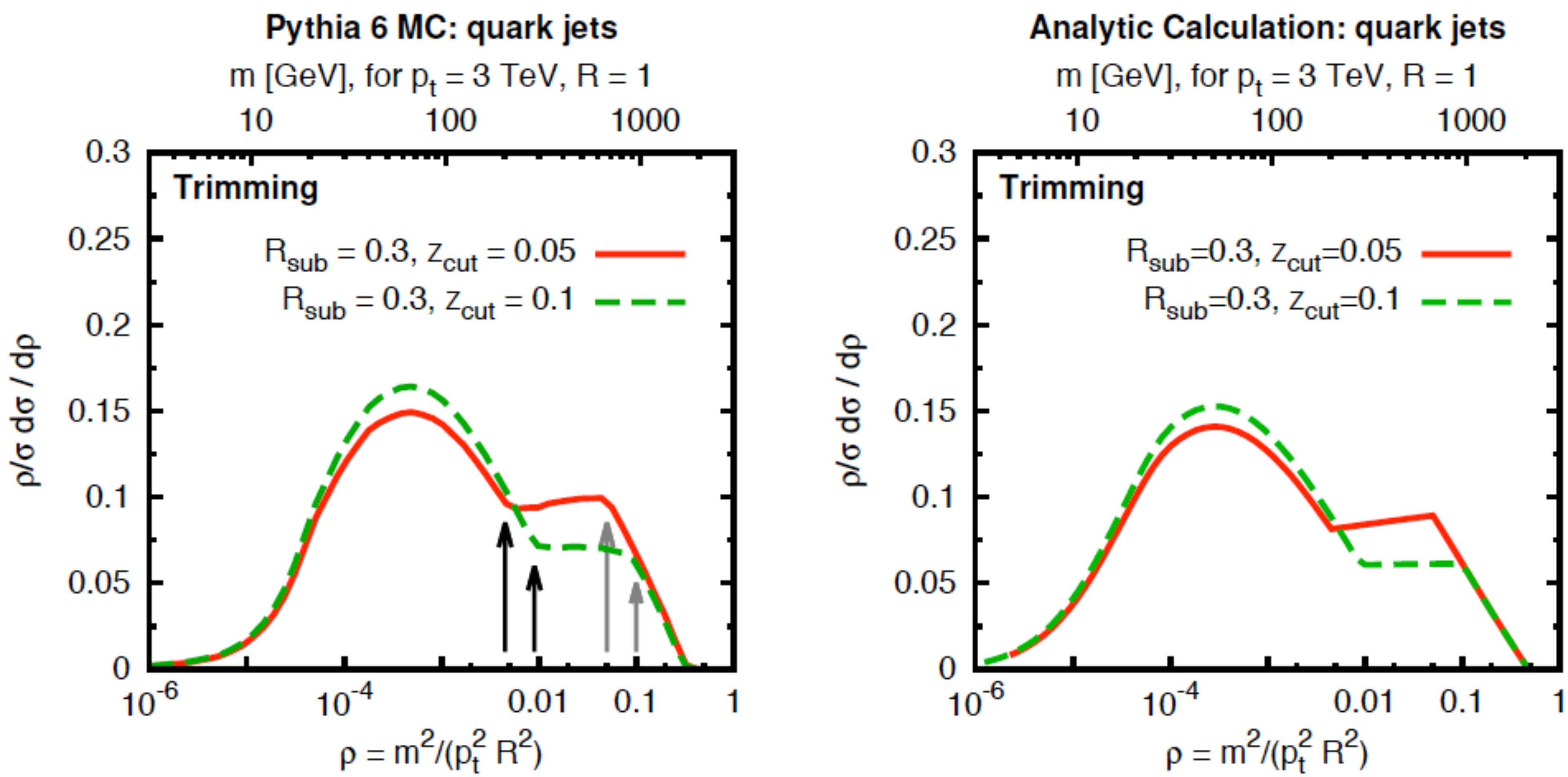
Taggers look quite similar ... but only in a limited region

Can one understand the shapes, kinks, peaks analytically ?

NB: kinks particularly dangerous for data-driven background estimate

First analytic approaches ...

Dasgupta, Fregoso, Marzani, Salam, Powling 1307.007



$$\rho \frac{\partial}{\partial \rho} \exp \left[-C_F \frac{\alpha_s}{2\pi} \left(-\frac{3}{2} \ln \frac{1}{\rho} + \Theta(\rho - z) \ln^2 \frac{1}{\rho} + \Theta(z - \rho) \, 2 \ln \frac{z}{\rho} \ln \frac{1}{z} + \Theta(zr^2 - \rho) \ln^2 \frac{zr^2}{\rho} \right) \right]$$

Simple analytic calculation allows to understand these features !

This means: have control and predict. Then use MC only to check/validate ...

Much more to come in the next years ...

My top ten QCD theory challenges

Theory challenge	Status
1. automated NLO	(✓)
2. reliable PDF error	(✓)
3. PDF with EW effects	✗
4. NNLO for generic $2 \to 2$ processes	4-5 years?
5. analytic understanding of jet-substructure	first results
6. NNLO + parton shower	Higgs, Drell Yan
7. N^3LO for Higgs and Drell Yan (differential?)	partial results
8. multi-jet merging	2-3 years?
9. automated NNLL resummations	✓ at NLL
10. improve Monte Carlo (+reliable error estimate)	only some ideas