Flavour Physics – Lecture 1

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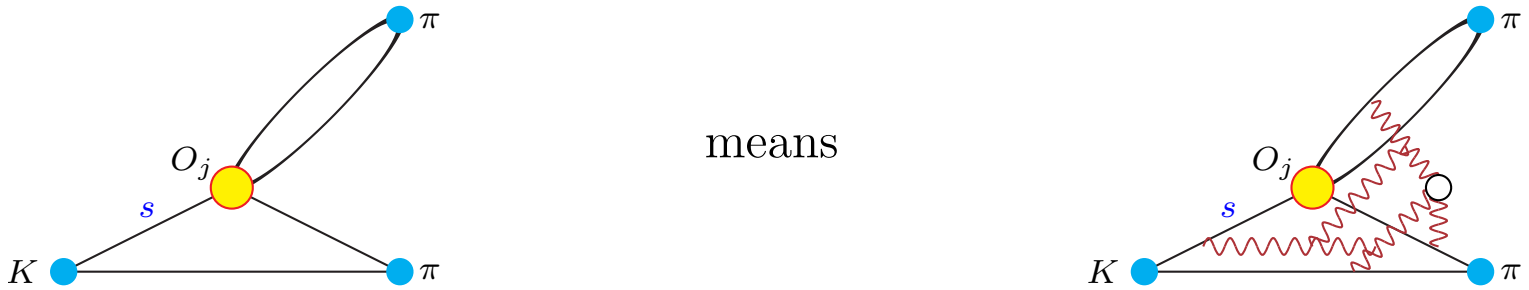
The rôle of flavour physics

- (Precision) Flavour physics, weak interaction processes in which the flavour (u, d, s, c, b, t) quantum number changes, is a key tool in exploring the limits of the Standard Model of Particle Physics and in searches for new physics.
- It is complementary to high-energy experiments (most notably the LHC).
  - If, as expected/hoped the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
  - The discovery potential of precision flavour physics should also not be underestimated. (In principle, the reach is about two-orders of magnitude deeper than the LHC!)
- Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.



- In fact, it is a major surprise to many of us that no unambiguous inconsistencies have arisen up to now.

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1. Introduction to *Flavour Physics*
2. Very brief introduction to lattice computations in Flavour Physics
3. Light-quark physics
4. Heavy-quark physics
5. New directions for improved precision

- Slides from an extended version of this course can be found on `http://www.ifsc.usp.br/ lattice/iipschool-sachrajda/`

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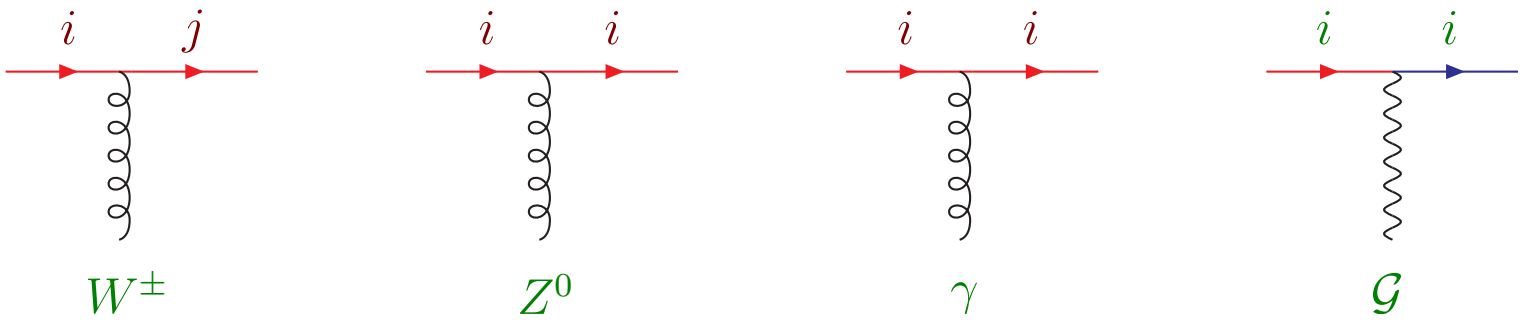
- In the Standard Model, the interaction of quarks with the gauge-bosons can be illustrated by the following vertices:



i,j represent the quark *flavour* $\{i,j = u,d,c,s,t,b\}$.
Colour is the charge of the strong interactions.

- In these lectures we will be particularly interested in the weak interactions. Feynman rule for W-vertex above is

$$i\frac{g_2}{2\sqrt{2}}V_{ij}\gamma_\mu(1-\gamma_5)\ ,$$

where g_2 is the coupling constant of the $SU(2)_L$ gauge group and V is the (unitary) Cabibbo-Kobayashi-Maskawa (CKM) matrix (see below).

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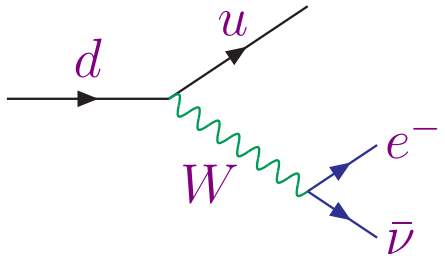
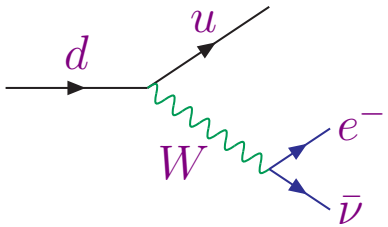
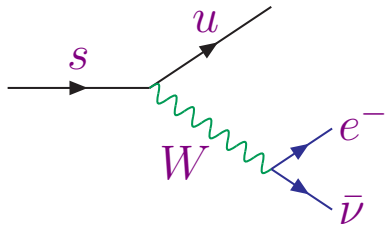
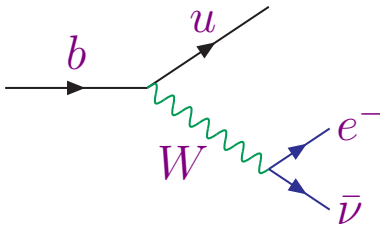
- At the level of quarks we understand nuclear β decay in terms of the fundamental process:



- With the 3 generations of quarks and leptons in the standard model this is generalized to other *charged current* processes, e.g.:

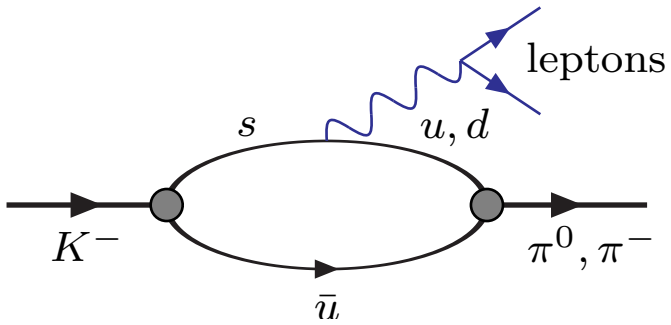






Two Experimental Numbers:

$$B(K^- \to \pi^0 e^- \nu_e) \simeq 5\% \ (K_{e3} \text{ Decay}) \quad \text{and} \quad B(K^- \to \pi^- e^+ e^-) < 3 \times 10^{-7}.$$



- Measurements like this show that $s \to u$ (charged-current) transitions are not very rare, but that *Flavour Changing Neutral Current* (FCNC) transitions, such as $s \to d$ are.
- From the above we see that we need to have a mechanism for transitions between quarks of different generations.
- The picture which has emerged is the Cabibbo-Kobayashi-Maskawa (CKM) theory of quark mixing which we now consider.

In the CKM theory the (quark) mass eigenstates are not the same as the weak-interaction eigenstates which we have been considering up to now.
Let

$$U' = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} = U_u \begin{pmatrix} u \\ c \\ t \end{pmatrix} = U_u U \quad \text{and} \quad D' = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U_d \begin{pmatrix} d \\ s \\ b \end{pmatrix} = U_d D$$

where the $'$s denote the weak interaction eigenstates and U_u and U_d are unitary matrices.

- For neutral currents:

$$\bar{U}' \cdots U' = \bar{U} \cdots U \quad \text{and} \quad \bar{D}' \cdots D' = \bar{D} \cdots D$$

  and no FCNC are induced. The $\cdots$ represent Dirac Matrices, but the identity in flavour.

- For charged currents:

$$J_W^{\mu +} = \frac{1}{\sqrt{2}} \bar{U}'_L \gamma^\mu D'_L = \frac{1}{\sqrt{2}} \bar{U}_L U_u^\dagger \gamma^\mu U_d D_L = \frac{1}{\sqrt{2}} \bar{U}_L \gamma^\mu (U_u^\dagger U_d) D_L \equiv \frac{1}{\sqrt{2}} \bar{U}_L \gamma^\mu V_{\mathrm{CKM}} D_L$$

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- The charged-current interactions are of the form

$$J^{+}_{\mu} = (\bar{u}, \bar{c}, \bar{t}\,)_L \gamma_\mu V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

- 2012 Particle Data Group summary for the magnitudes of the entries:

$$\begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix} .$$

- How many parameters are there?
  - Let N_g be the number of generations.
  - $N_g \times N_g$ unitary matrix has N_g^2 real parameters.
  - $(2N_g - 1)$ of them can be absorbed into unphysical phases of the quark fields.
  - $(N_g - 1)^2$ physical parameters to be determined.

- For $N_g = 2$ there is only one parameter, which is conventionally chosen to be the Cabibbo angle:

$$V_{\rm CKM} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}.$$

- For $N_g = 3$, there are 4 real parameters. Three of these can be interpreted as angles of rotation in three dimensions (e.g. the three Euler angles) and the fourth is a phase. The general parametrization recommended by the PDG is

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

  where c_{ij} and s_{ij} represent the cosines and sines respectively of the three angles θ_{ij}, $ij = 12$, 13 and 23. δ_{13} is the phase parameter.

- It is conventional to use approximate parametrizations, based on the hierarchy of values in $V_{\rm CKM}$ ($s_{12} \gg s_{23} \gg s_{13}$).

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The Wolfenstein parametrization is

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} .$$

- $\lambda = s_{12}$ is approximately the Cabibbo angle.
- A, ρ and η are real numbers that a priori were intended to be of order unity.
- Corrections are of $O(\lambda^4)$.

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Unitarity of the CKM-matrix we have a set of relations between the entries. A particularly useful one is:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ .$$

In terms of the Wolfenstein parameters, the components on the left-hand side are given by:

$$\begin{aligned} V_{ud}V_{ub}^* &= A\lambda^3[\bar{\rho} + i\bar{\eta}] + O(\lambda^7) \\ V_{cd}V_{cb}^* &= -A\lambda^3 + O(\lambda^7) \\ V_{td}V_{tb}^* &= A\lambda^3[1 - (\bar{\rho} + i\bar{\eta})] + O(\lambda^7) \ , \end{aligned}$$

where $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$.
The unitarity relation can be represented schematically by the famous “unitarity triangle" (obtained after scaling out a factor of $A\lambda^3$).

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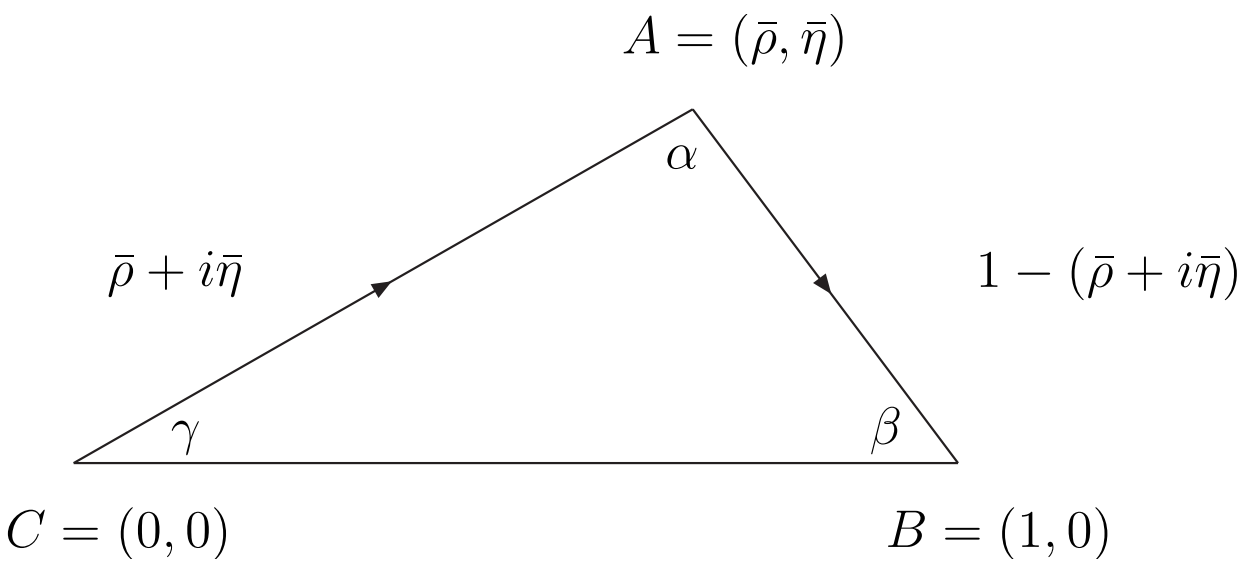
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\ .$$



- A particularly important approach to testing the *Limits of the SM* is to over-determine the position of the vertex A to check for consistency.

PDG2012 Unitarity Triangle

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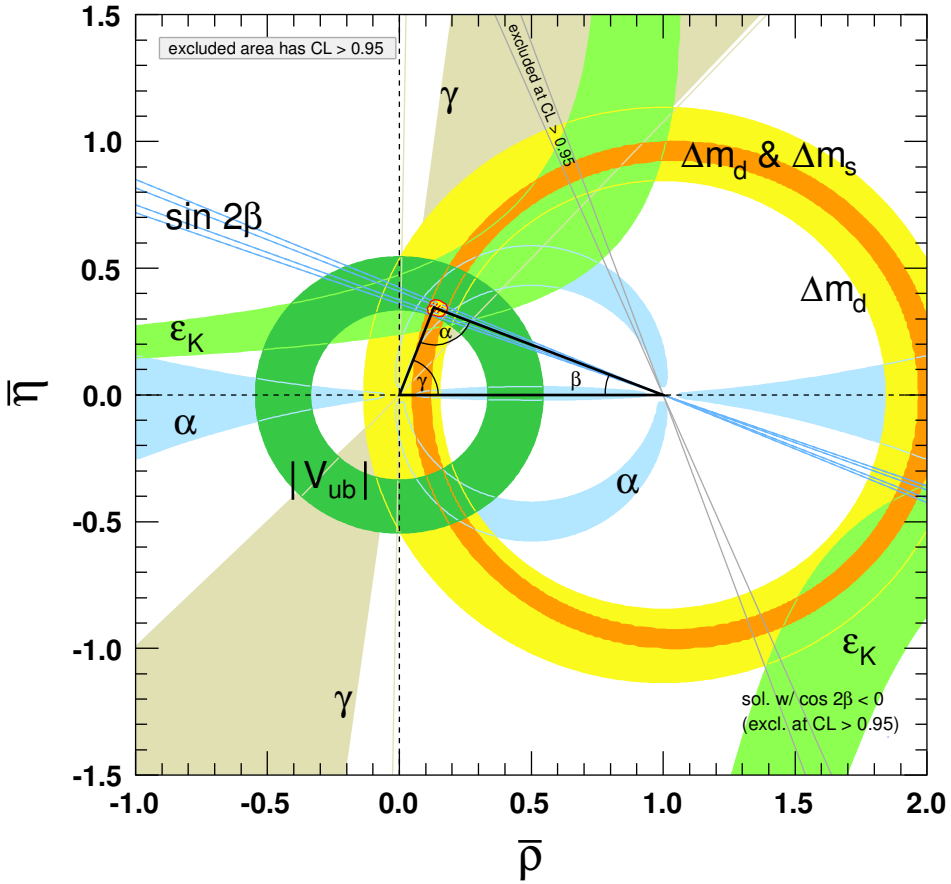

PDG2006 & 2012 Unitarity Triangle Comparison

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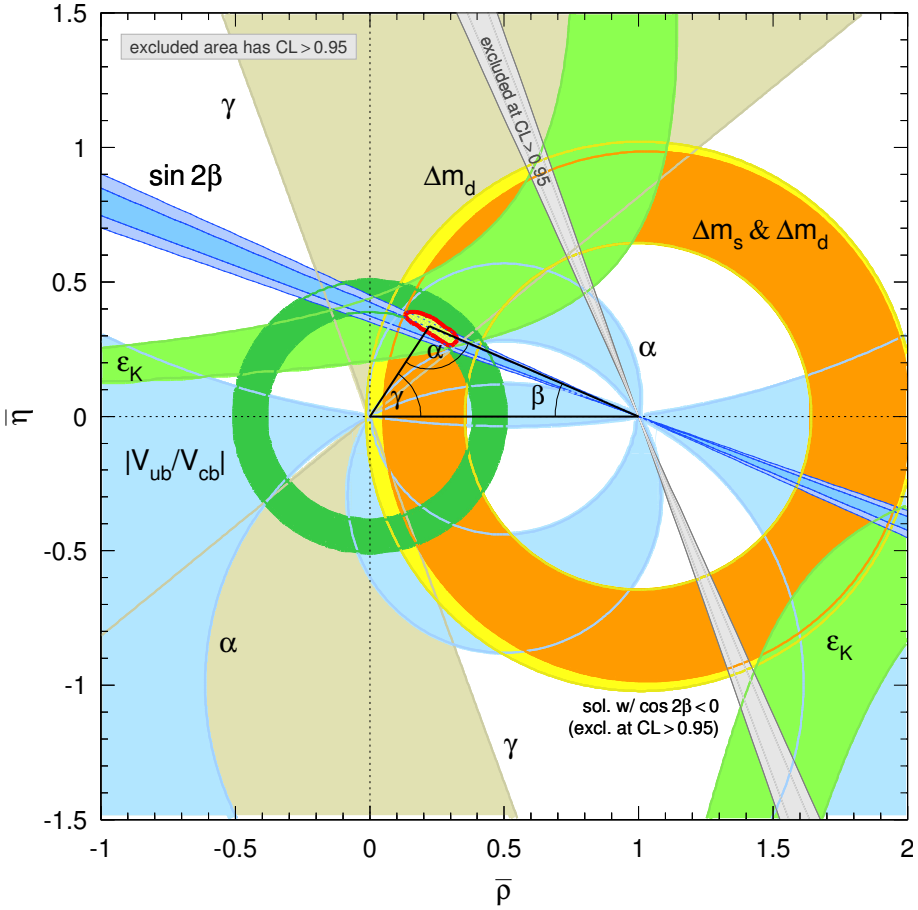
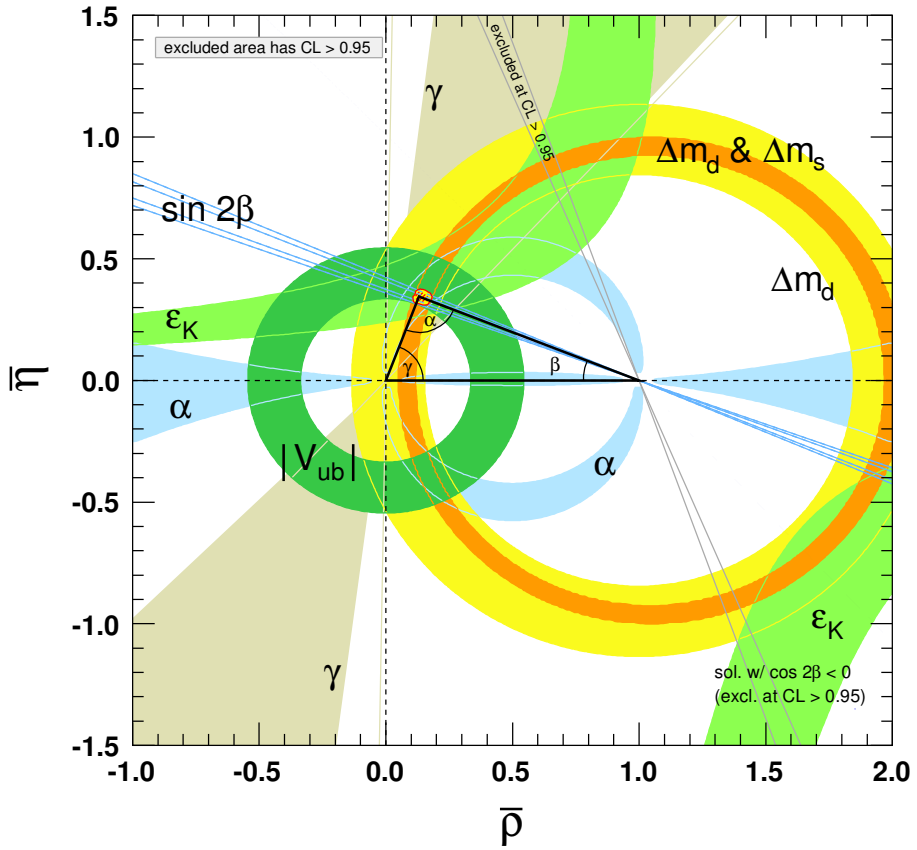


2006



2012

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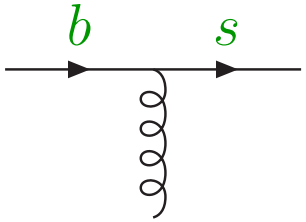
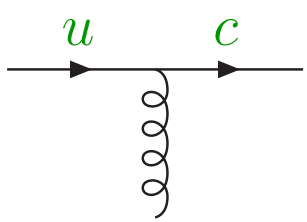
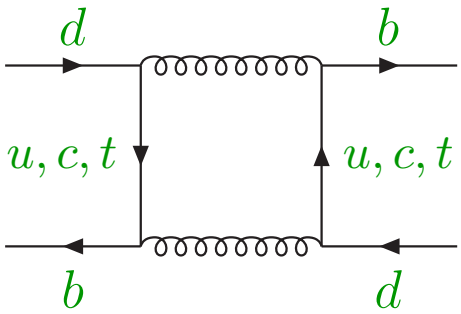
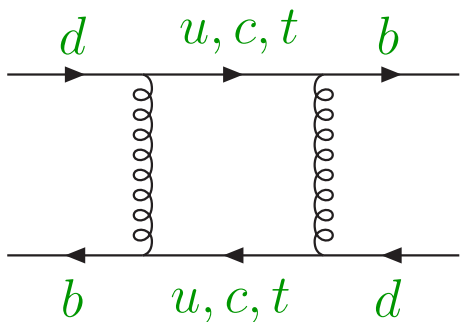
We have seen that in the SM, unitarity implies that there are no FCNC reactions at tree level, i.e. there are no vertices of the type:



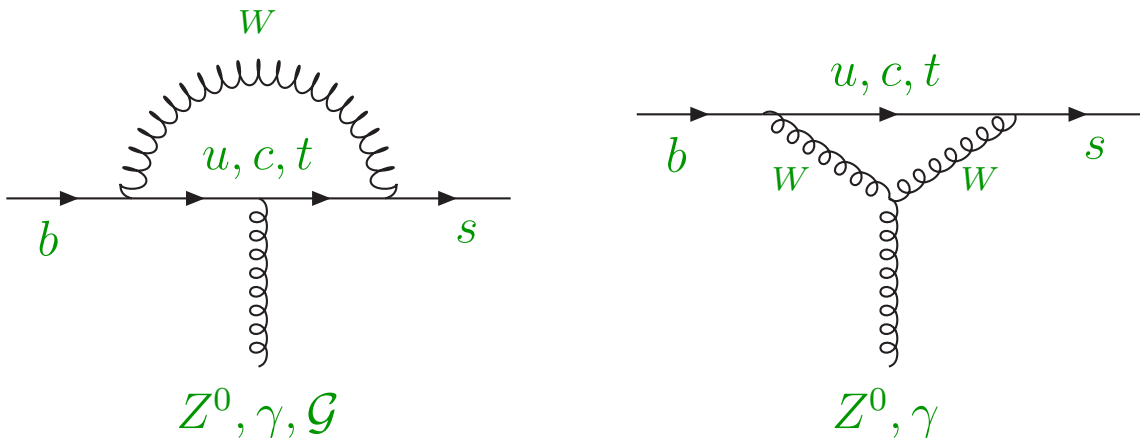


Quantum loops, however, can generate FCNC reactions, through *box* diagrams or *penguin* diagrams.
Example relevant for $\bar{B}^0 - B^0$ mixing:





Examples of penguin diagrams relevant for $b \to s$ transitions:



We will discuss several of the physical processes induced by these loop-effects. The Glashow-Illiopoulos-Maiani (GIM) mechanism $\Rightarrow$ FCNC effects vanish for degenerate quarks ($m_u = m_c = m_t$). For example unitarity implies

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$

$\Rightarrow$ each of the above penguin vertices vanish.

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- The property of asymptotic freedom $\Rightarrow$ quark and gluon interactions become weak at short distances, i.e. distances $\ll 1$ fm.

  Nobel prize in 2004 to Gross, Politzer and Wilczek.

- Thus at short distances we can use perturbation theory.
- Schematically weak decay amplitudes are organized as follows:

$$\mathscr{A}_{i\to f} = \sum_j C_j(\mu)\,\langle f\,|\,O_j(0)\,|\,i\rangle_\mu$$

  where

  - The C_j contain the short-distance effects and are calculable in perturbation theory;
  - the long-distance *non-perturbative* effects are contained in the matrix elements of composite local operators $\{O_i(0)\}$ which are the quantities which are computed in lattice QCD simulations;
  - the renormalization scale μ can be viewed as the scale at which we separate the short-distances from long-distances.

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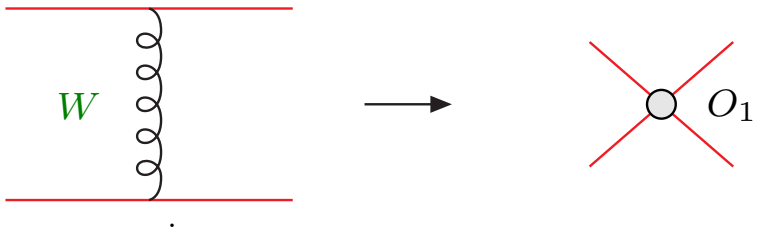
- Quarks interact strongly $\Rightarrow$ we have to consider QCD effects even in weak processes.
- Our inability to control (non-perturbative) QCD Effects is frequently the largest systematic error in attempts to obtain fundamental information from experimental studies of weak processes!
- Tree-Level:



- Since $M_W \simeq 80\,\text{GeV}$, at low energies the momentum in the W-boson is much smaller than its mass $\Rightarrow$ the four quark interaction can be approximated by the local Fermi β-decay vertex with coupling

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} \,.$$

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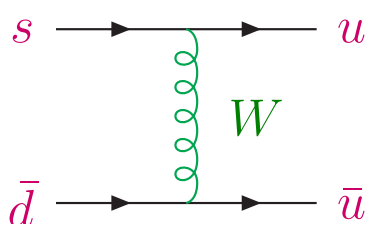
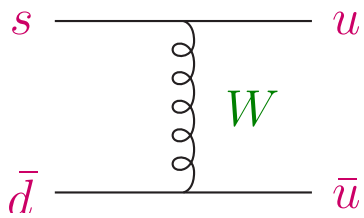
- *Asymptotic Freedom* $\Rightarrow$ we can treat QCD effects at short distances, $|x| \ll \Lambda_{QCD}^{-1}$ ($|x| < 0.1$ fm say) or corresponding momenta $|p| \gg \Lambda_{QCD}$ ($|p| > 2$ GeV say), using perturbation theory.
- The natural scale of strong interaction physics is of $O(1\ \mathrm{fm})$ however, and so in general, and for most of the processes discussed here, non-perturbative techniques must be used.
- For illustration consider $K \to \pi\pi$ decays, for which the tree-level amplitude is proportional to

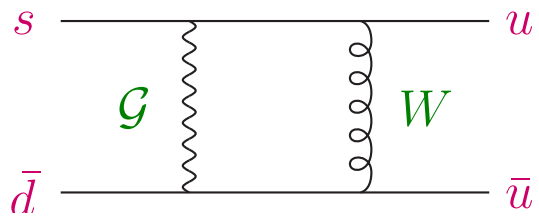
$$\frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \langle \pi\pi | (\bar{d}\gamma^\mu (1-\gamma^5) u)\, (\bar{u}\gamma_\mu (1-\gamma^5) s) | K \rangle \ .$$



- We therefore need to determine the matrix element of the operator

$$O_1 = (\bar{d}\gamma^\mu (1-\gamma^5) u)\, (\bar{u}\gamma_\mu (1-\gamma^5) s) \ .$$





- Gluonic corrections generate a second operator $(\bar{d}T^a\gamma^\mu(1-\gamma^5)u)\,(\bar{u}T^a\gamma_\mu(1-\gamma^5)s)$, which by using Fierz Identities can be written as a linear combination of O_1 and O_2 where

$$O_2 = (\bar{d}\gamma^\mu(1-\gamma^5)s)\,(\bar{u}\gamma_\mu(1-\gamma^5)u)\,.$$

- OPE $\Rightarrow$ the amplitude for a weak decay process can be written as

$$A_{if} = \frac{G_F}{\sqrt{2}}\,V_{CKM}\sum_i C_i(\mu)\langle f|\,O_i(\mu)\,|i\rangle\ .$$

- μ is the renormalization scale at which the operators O_i are defined.
- Non-perturbative QCD effects are contained in the matrix elements of the O_i, which are independent of the large momentum scale, in this case of M_W.
- The Wilson coefficient functions $C_i(\mu)$ are independent of the states i and f and are calculated in perturbation theory.
- Since physical amplitudes manifestly do not depend on μ, the μ-dependence in the operators $O_i(\mu)$ is cancelled by that in the coefficient functions $C_i(\mu)$.

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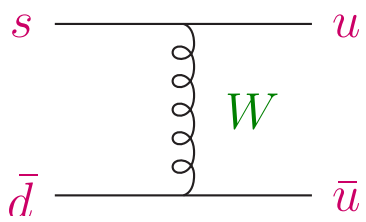
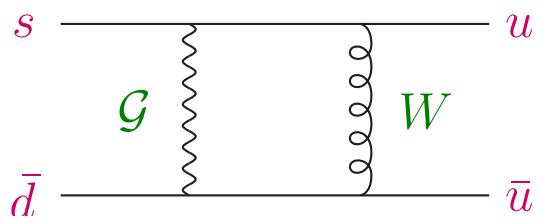


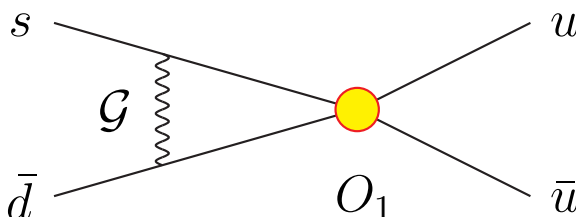


- For large loop-momenta k the right-hand graph is ultra-violet convergent:

$$\int_{k \text{ large}} \frac{1}{k}\,\frac{1}{k}\,\frac{1}{k^2}\,\frac{1}{k^2 - M_W^2}\,d^4k\ ,$$

  ($1/k$ for each quark propagator and $1/k^2$ for the gluon propagator.)
  We see that there is a term $\sim \log(M_W^2/p^2)$, where p is some infra-red scale.

- In the OPE we do not have the W-propagator.



Power Counting : $\int_{k \text{ large}} \frac{1}{k}\,\frac{1}{k}\,\frac{1}{k^2}\,d^4k \quad \Rightarrow \quad$ **divergence** $\quad \Rightarrow \quad \mu-$dependence.

- Infra-dependence is the same as in the full field-theory.

$$\log\left(\frac{M_W^2}{p^2}\right) = \log\left(\frac{M_W^2}{\mu^2}\right) + \log\left(\frac{\mu^2}{p^2}\right)$$

- The ir physics is contained in the matrix elements of the operators and the uv physics in the coefficient functions:

$$\log\left(\frac{M_W^2}{\mu^2}\right) \quad \to \quad C_i(\mu)$$

$$\log\left(\frac{\mu^2}{p^2}\right) \quad \to \quad \text{matrix element of } O_i$$

- In practice, the matrix elements are computed in lattice simulations with an ultraviolet cut-off of 2 – 4 GeV. Thus we have to resum *large logarithms* of the form $\alpha_s^n \log^n(M_W^2/\mu^2)$ in the coefficient functions $\Rightarrow$ factors of the type

$$\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\gamma_0/2\beta_0} .$$

$$\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\gamma_0/2\beta_0}$$

- γ_0 is the one-loop contribution to the *anomalous dimension* of the operator (proportional to the coefficient of $\log(\mu^2/p^2)$ in the evaluation of the one-loop graph above) and β_0 is the first term in the β-function, ($\beta \equiv \partial g/\partial \ln(\mu) = -\beta_0 g^3/16\pi^2$).
- In general when there is more than one operator contributing to the right hand side of the OPE, the mixing of the operators $\Rightarrow$ matrix equations.
- The factor above represents the sum of the *leading logarithms*, i.e. the sum of the terms $\alpha_s^n \log^n(M_W^2/\mu^2)$. For almost all the important processes, the first (or even higher) corrections have also been evaluated.
- These days, for most processes of interest, the perturbative calculations have been performed to several loops (2,3,4), $\mathrm{N}^n\mathrm{LO}$ calculations.

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- The *effective Hamiltonian* for weak decays takes the form

$$\mathscr{H}_{\rm eff} \equiv \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu)\, O_i(\mu) \ .$$

- We shall see below that for some important physical quantities (e.g. ε'/ε), there may be as many as ten operators, whose matrix elements have to be estimated.
- Lattice simulations enable us to evaluate the matrix elements non-perturbatively.
- In weak decays the large scale, M_W, is of course fixed. For other processes, most notably for deep-inelastic lepton-hadron scattering, the OPE is useful in computing the behaviour of the amplitudes with the large scale (e.g. with the momentum transfer).

- **Parity**

$$(\vec{x}, t) \to (-\vec{x}, t).$$

The vector and axial-vector fields transform as:

$$V_\mu(\vec{x}, t) \to V^\mu(-\vec{x}, t) \quad \text{and} \quad A_\mu(\vec{x}, t) \to -A^\mu(-\vec{x}, t).$$

  - The vector and axial-vector currents transform similarly.

Left-handed components of fermions $\psi_L = (\frac{1}{2}(1-\gamma^5)\psi)$ transform into right-handed ones $\psi_R = (\frac{1}{2}(1+\gamma^5)\psi)$, and vice-versa.

  - Since CC weak interactions in the SM only involve the left-handed components, parity is not a good symmetry of the weak force.
  - QCD and QED are invariant under parity transformations.

- **Charge Conjugation** – Charge conjugation is a transformation which relates each complex field ϕ with $\phi^\dagger$.

  Under C the currents transform as follows:

$$\bar{\psi}_1 \gamma_\mu \psi_2 \to -\bar{\psi}_2 \gamma_\mu \psi_1 \quad \text{and} \quad \bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2 \to \bar{\psi}_2 \gamma_\mu \gamma_5 \psi_1 ,$$

  where ψ_i represents a spinor field of type (flavour or lepton species) i.

- **CP** – Under the combined CP-transformation, the currents transform as:

$$\bar{\psi}_1 \gamma_\mu \psi_2 \to -\bar{\psi}_2 \gamma^\mu \psi_1 \quad \text{and} \quad \bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2 \to -\bar{\psi}_2 \gamma^\mu \gamma_5 \psi_1 .$$

  The fields on the left (right) hand side are evaluated at $(\vec{x}, t)$ ($(-\vec{x}, t)$).

- Consider now a charged current interaction:

$$(W_\mu^1 - iW_\mu^2)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V_{ij}D^j + (W_\mu^1 + iW_\mu^2)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V_{ij}^*U^i,$$

  U^i and D^j are up and down type quarks of flavours i and j respectively.

- Under a CP transformation, the interaction term transforms to:

$$(W_\mu^1 + iW_\mu^2)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V_{ij}U^i + (W_\mu^1 - iW_\mu^2)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V_{ij}^*D^j$$

- CP-invariance requires V to be real
  (or more strictly that any phases must be able to be absorbed into the definition of the quark fields).
- For CP-violation in the quark sector we therefore require 3 generations.

Nobel prize in 2008 to Kobayashi and Maskawa.

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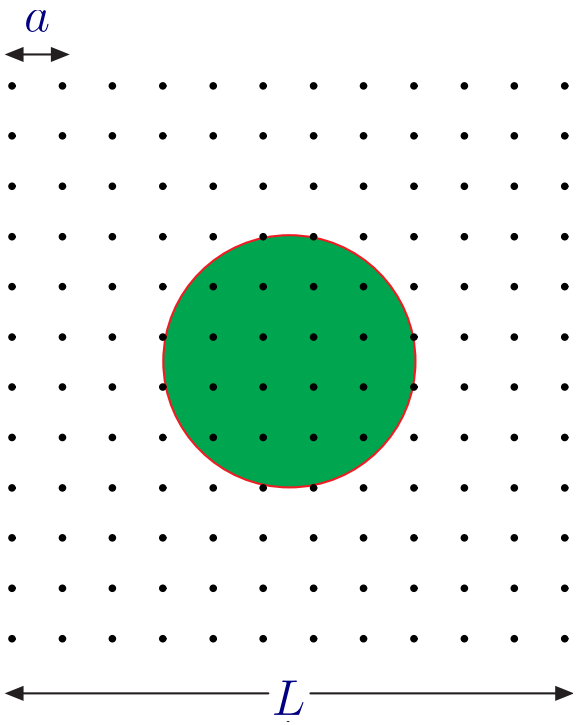
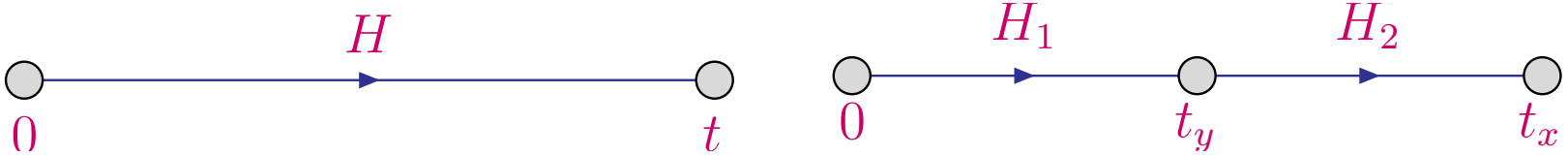


- Lattice phenomenology starts with the evaluation of correlation functions of the form:

$$\langle 0|\, O(x_1, x_2, \cdots, x_n)\, |0\rangle \quad = \quad \frac{1}{Z}\int [dA_\mu]\,[d\psi]\,[d\bar{\psi}]\, e^{-S}\, O(x_1, x_2, \cdots, x_n)\ ,$$

where $O(x_1, x_2, \cdots, x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function.

- The physics which can be studied depends on the choice of the multilocal operator O.



- The functional integral is performed by discretising Euclidean space-time and using Monte-Carlo Integration.

Consider two-point correlation functions of the form:

$$C_2(t) = \int d^3x \, e^{i\vec{p}\cdot\vec{x}} \, \langle 0 | J(\vec{x},t) J^\dagger(\vec{0},0) | 0 \rangle \ ,$$

where J and $J^\dagger$ are any interpolating operators for the hadron H which we wish to study and the time t is taken to be positive.

- We assume that H is the lightest hadron which can be created by $J^\dagger$.
- We take $t > 0$, but it should be remembered that lattice simulations are frequently performed on periodic lattices, so that both time-orderings contribute.

$$C_2(t) = \int d^3x\, e^{i\vec{p}\cdot\vec{x}} \,\langle 0|\, J(\vec{x},t)\, J^\dagger(\vec{0},0)\,|0\rangle \ ,$$

Inserting a complete set of states $\{|n\rangle\}$:

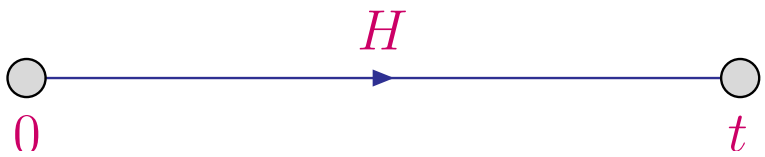
$$\begin{aligned} C_2(t) &= \sum_n \int d^3x\, e^{i\vec{p}\cdot\vec{x}} \,\langle 0|\, J(\vec{x},t)\,|n\rangle \,\langle n|J^\dagger(\vec{0},0)\,|0\rangle \\ &= \int d^3x\, e^{i\vec{p}\cdot\vec{x}} \,\langle 0|\, J(\vec{x},t)\,|H\rangle \,\langle H|J^\dagger(\vec{0},0)\,|0\rangle + \cdots \end{aligned}$$

where the $\cdots$ represent contributions from heavier states with the same quantum numbers as H.
Finally using translational invariance:

$$\boxed{C_2(t) = \frac{1}{2E}\, e^{-iEt} \left|\langle 0|J(\vec{0},0)|H(p)\rangle\right|^2 + \cdots \ ,}$$

where $E = \sqrt{m_H^2 + \vec{p}^{\,2}}$.

$$C_2(t) = \frac{1}{2E}\, e^{-iEt} \left| \langle 0|J(\vec{0},0)|H(p)\rangle \right|^2 + \cdots \ .$$



- In Euclidean space $\exp(-iEt) \to \exp(-Et)$.
- By fitting $C(t)$ to the form above, both the energy (or, if $\vec{p} = 0$, the mass) and the modulus of the matrix element

$$\left| \langle 0|J(\vec{0},0)|H(p)\rangle \right|$$

  can be evaluated.
- Example: if $J = \bar{u}\gamma^\mu\gamma^5 d$ then the decay constant of the π-meson can be evaluated,

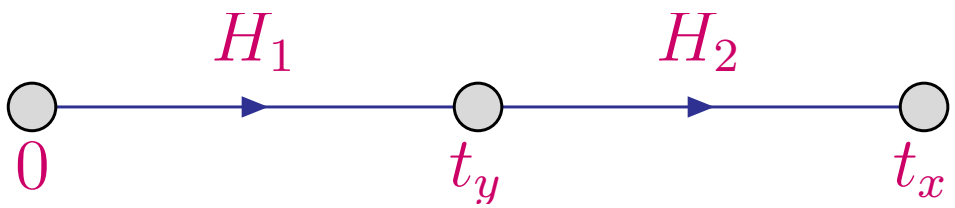
$$\left| \langle 0|\, \bar{u}\gamma^\mu\gamma^5 d\, |\pi^+(p)\rangle \right| = f_\pi\, p^\mu \ ,$$

  (the physical value of $f_\pi \simeq$ is 132 MeV).

Consider now a three-point correlation function of the form:

$$C_3(t_x,t_y) = \int d^3x\, d^3y\; e^{i\vec{p}\cdot\vec{x}}\; e^{i\vec{q}\cdot\vec{y}}\; \langle 0|\, J_2(\vec{x},t_x)\, O(\vec{y},t_y)\, J_1^\dagger(\vec{0},0)\,|0\rangle \;,$$

where $J_{1,2}$ may be interpolating operators for different particles and we assume that $t_x > t_y > 0$.



For sufficiently large times t_y and $t_x - t_y$

$$\begin{aligned} C_3(t_x,t_y) \quad \simeq \quad & \frac{e^{-E_1 t_y}}{2E_1}\; \frac{e^{-E_2(t_x-t_y)}}{2E_2}\; \langle 0|J_2(0)|H_2(\vec{p})\rangle \\ & \times \langle H_2(\vec{p})|O(0)|H_1(\vec{p}+\vec{q})\rangle\; \langle H_1(\vec{p}+\vec{q})|J_1^\dagger(0)|0\rangle \;, \end{aligned}$$

where $E_1^2 = m_1^2 + (\vec{p}+\vec{q})^2$ and $E_2^2 = m_1^2 + \vec{p}^{\,2}$.

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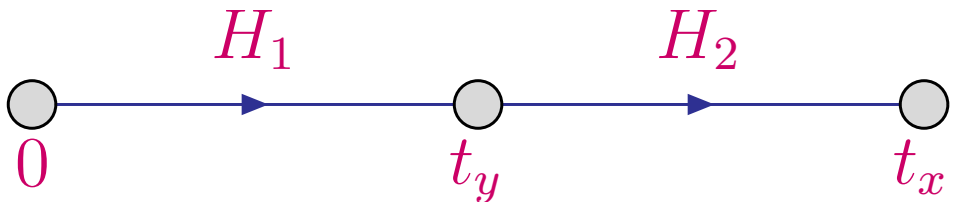

- From the evaluation of two-point functions we have the masses and the matrix elements of the form $|\langle 0|J|H(\vec{p})\rangle|$. Thus, from the evaluation of three-point functions we obtain matrix elements of the form $|\langle H_2|O|H_1\rangle|$.
- Important examples include:
  - $K^0-\bar{K}^0$ ($B^0-\bar{B}^0$) mixing. In this case

$$O = \bar{s}\gamma^\mu(1-\gamma^5)d\ \bar{s}\gamma_\mu(1-\gamma^5)d\,.$$

  - Semileptonic and rare radiative decays of hadrons of the form $B \to \pi, \rho$ + leptons or $B \to K^*\gamma$. Now O is a quark bilinear operator such as $\bar{b}\gamma^\mu(1-\gamma^5)u$ or an *electroweak penguin* operator.

- In Lattice QCD, while it is natural to think in terms of the lattice spacing a, the input parameter is $\beta = 6/g^2(a)$.
- $g(a)$ is the bare coupling constant in the bare theory defined by the particular discretization of QCD used in the simulation. a^{-1} is the ultraviolet cut-off in momentum space.
- Imagine now that we are performing a simulation with $N_f = 2+1$ and that we are in an ideal world in which we can perform simulations with $m_{ud} = m_u = m_d$ around their “physical" values. The procedure for defining a physical scaling trajectory is then relatively simple.

- At each β, choose two dimensionless quantities, e.g. m_π/m_Ω and m_K/m_Ω, and find the bare quark masses m_{ud} and m_s which give the corresponding physical values.
  These are then defined to be the physical (bare) quark masses at that β.
- Now consider a dimensionful quantity, e.g. m_Ω. The value of the lattice spacing is defined by
$$a^{-1} = \frac{1.672\,\mathrm{GeV}}{m_\Omega(\beta, m_{ud}, m_s)}$$
  where $m_\Omega(\beta, m_{ud}, m_s)$ is the measured value in lattice units.
- Other physical quantities computed at the physical bare-quark masses will now differ from their physical values by artefacts of $O(a^2)$.
- Repeating this procedure at different β defines a scaling trajectory. Other choices for the 3 physical quantities used to define different scaling trajectory are clearly possible.
- If the simulations are performed with m_c and/or $m_u \neq m_d$ then the procedure has to be extended accordingly.

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- The precision of lattice calculations is now reaching the point where we need better interactions with the N^nLO QCD perturbation theory community.
- The traditional way of dividing responsibilities is:

$$\text{Physics} = \underset{\substack{\uparrow \\ \text{Perturbative} \\ \text{QCD}}}{C} \times \underset{\substack{\uparrow \\ \text{Lattice} \\ \text{QCD}}}{\langle f | O | i \rangle}$$

- The two factors have to be calculated in the same scheme.
- Can we meet half way?

$$\begin{array}{c}\text{bare} \\ \text{lattice} \\ \text{operators}\end{array} \longrightarrow \; ? \; \longleftarrow \begin{array}{c}\text{operators} \\ \text{renormalized} \\ \text{in } \overline{\text{MS}} \text{ scheme}\end{array}$$

- What is the best scheme for **?** (RI-SMOM, Schrödinger Functional, $\cdots$)?
- Recent examples of such collaborations following J.Gracey ...:
  - two-loop matching factor for m_q between the RI-SMOM schemes and $\overline{\text{MS}}$.
    M.Gorbahn and S.Jager, arXiv:1004:3997, L.Almeida and C.Sturm, arXiv:1004:4613
  - HPQCD + Karlsruhe Group in determination of quark masses.

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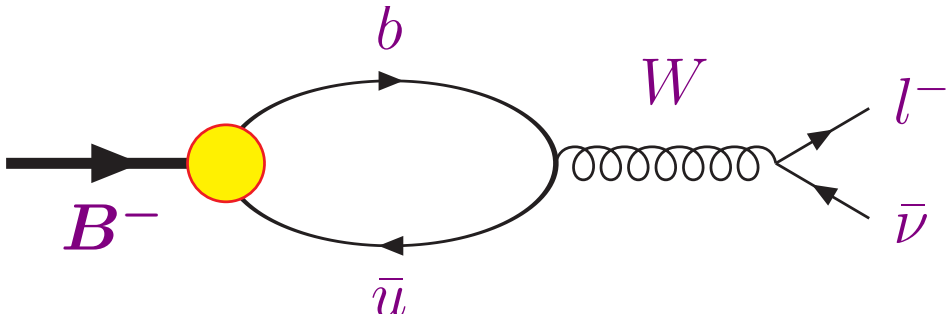
- The difficulty in making predictions for weak decays of hadrons is in controlling the non-perturbative strong interaction effects.
- As a particularly simple example consider the leptonic decays of pseudoscalar mesons in general and of the B-meson in particular.



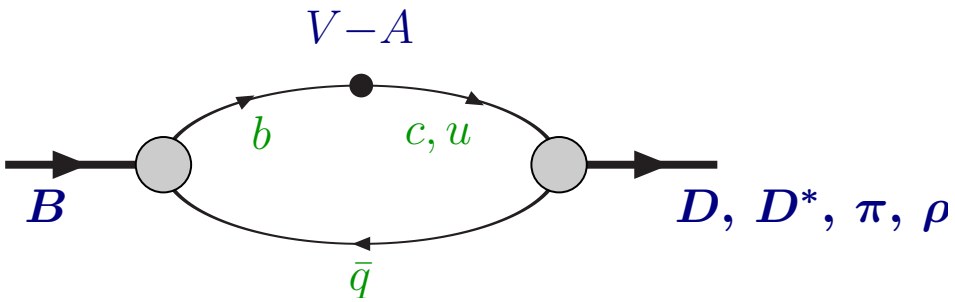
- Non-perturbative QCD effects are contained in the matrix element

$$\langle 0|\,\bar{b}\gamma^{\mu}(1-\gamma^{5})u\,|B(p)\rangle\ .$$

- Lorentz Inv. + Parity $\Rightarrow\ \langle 0|\,\bar{b}\gamma^{\mu}u\,|B(p)\rangle = 0.$
- Similarly $\langle 0|\,\bar{b}\gamma^{\mu}\gamma^{5}u\,|B(p)\rangle = if_{B}p^{\mu}\ .$

All QCD effects are contained in a single constant, f_B, the B-meson's *(leptonic) decay constant.* ($f_\pi \simeq 132\,\text{MeV}$)

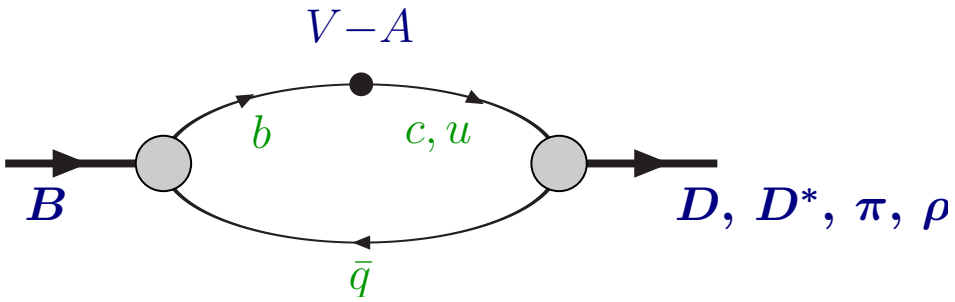
• These can be determined from either inclusive or exclusive decays. I start with a discussion of exclusive decays.



• Space-Time symmetries allow us to parametrise the non-perturbative strong interaction effects in terms of invariant form-factors. For example, for decays into a pseudoscalar meson P ($=\pi, D$ for example)

$$\langle P(k)|V^\mu|B(p)\rangle \quad = \quad f^+(q^2)\left[(p+k)^\mu - \frac{m_B^2-m_P^2}{q^2}\,q^\mu\right] + f^0(q^2)\,\frac{m_B^2-m_P^2}{q^2}\,q^\mu\,,$$

where $q=p-k$.



- For decays into a vector V ($=\rho, D^*$ for example), a conventional decomposition is

$$\langle V(k,\varepsilon)|V^\mu|B(p)\rangle = \frac{2V(q^2)}{m_B+m_V}\varepsilon^{\mu\gamma\delta\beta}\varepsilon^*_\beta p_\gamma k_\delta$$

$$\langle V(k,\varepsilon)|A^\mu|B(p)\rangle = i(m_B+m_V)A_1(q^2)\varepsilon^{*\mu} - i\frac{A_2(q^2)}{m_B+m_V}\varepsilon^*\cdot p\,(p+k)^\mu + i\frac{A(q^2)}{q^2}2m_V\varepsilon^*\cdot p\,q^\mu \ ,$$

where ε is the polarization vector of the final-state meson, and $q=p-k$.

$$\{A_3 = \frac{m_B+m_V}{2m_V}A_1 - \frac{m_B-m_V}{2m_V}A_2 \ .\}$$

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- Most of the compilations in this talk are taken from the current results of the FLAG collaboration: "Review of lattice results concerning low energy particle physics,"
  S. Aoki, Y. Aoki, C. Bernard, T. Blum, G. Colangelo, M. Della Morte, S. Dürr, A. El Khadra, H. Fukaya, A. Jüttner, R. Horsley, T. Kaneko, J. Laiho, L. Lellouch, H. Leutwyler, V. Lubicz, E. Lunghi, S. Necco, T. Onogi, C. Pena, C. Sachrajda, J. Shigemitsu, S. Simula, S. Sharpe, R. Sommer, R. Van de Water, A. Vladikas, U. Wenger, H. Wittig. arXiv:1310.8555, (255 pages!)

- This is an extension and continuation of the work of the Flavianet Lattice Averaging Group:
  G. Colangelo, S. Durr, A. Juttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. T. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig arXiv:1011.4408

- Motivation - to present to the wider community an *average* of lattice results for important quantities obtained after a critical expert review.
- Danger - original papers (particularly those which pioneer new techniques) do not get cited appropriately by the community.
- The closing date for arXiv:1310.8555v2 was Nov 30th 2013.

FLAG summary in light-quark physics

Quantity	■	N_f=2+1+1	■	$N_f = 2+1$	■	$N_f = 2$
m_s(MeV)			3	93.8(2.4)	2	101(3)
m_{ud}(MeV)			3	3.42(9)	1	3.6(2)
m_s/m_{ud}			3	27.5(4)	1	28.1(1.2)
m_d(MeV)				4.68(14)(7)		4.8(15)(17)
m_u(MeV)				2.16(9)(7)		2.40(15)(17)
m_u/m_d				0.46(2)(2)		0.50(2)(3)
$f_+^{K\pi}(0)$			1	0.9667(23)(33)	1	0.9560(57)(62)
f_{K^+}/f_{π^+}	1	1.195(3)(4)	4	1.192(5)	1	1.205(6)(17)
f_K(MeV)			3	156.3(0.8)	1	158.1(2.5)
f_π(MeV)			3	130.2(1.4)		
Σ(MeV)			2	265(17)	1	270(7)
F_π/F	1	1.0760(28)	2	1.0620(34)	1	1.0733(73)
$\bar{\ell}_3$	1	3.70(27)	3	2.77(1.27)	1	3.45(26)
$\bar{\ell}_4$	1	4.67(10)	3	3.95(35)	1	4.59(26)
$\hat{B}_K$			4	0.766(10)	1	0.729(25)(17)
$B_K^{\overline{\rm MS}}$(2 GeV)			4	0.560(7)	1	0.533(18)(12)

FLAG summary in heavy-quark physics

Quantity	■	N_f=2+1+1	■	$N_f = 2+1$	■	$N_f = 2$
f_D(MeV)			2	209.2(3.3)	1	212(8)
f_{D_s}(MeV)			2	248.6(2.7)	1	248(6)
f_{D_s}/f_D			2	1.187(12)	1	1.17(5)
$f_+^{D\pi}(0)$			1	0.666(29)		
$f_+^{DK}(0)$			1	0.747(19)		
f_B(MeV)	1	186(4)	3	190.5(4.2)	1	197(10)
f_{B_s}(MeV)	1	224(5)	3	227.7(4.5)	1	234(6)
f_{B_s}/f_B	1	1.205(7)	2	1.202(22)	1	1.19(5)
$f_{B_d}\sqrt{\hat{B}_{B_d}}$(MeV)			1	216(15)		
$f_{B_s}\sqrt{\hat{B}_{B_s}}$(MeV)			1	266(18)		
$\hat{B}_{B_d}$			1	1.27(10)		
$\hat{B}_{B_s}$			1	1.33(6)		
ξ			1	1.268(63)		
$\hat{B}_{B_s}/\hat{B}_{B_d}$			1	1.06(11)		

FLAG summary in heavy-quark physics (cont.) and α_s

Quantity	■	N_f=2+1+1	■	$N_f = 2+1$	■	$N_f = 2$
$\Delta\zeta^{B\pi}(\text{ps}^{-1})$			2	2.16(50)		
$f_+^{B\pi}(q^2)$: a_0^{BCL}			2	0.453(33)		
a_1^{BCL}			2	−0.43(33)		
a_2^{BCL}			2	0.9(3.9)		
$\mathscr{F}^{B\to D^*}(1)$			1	0.9017(51)(156)		
$R(D)$			1	0.316(12)(7)		
$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z)$			4	0.1184(12)		