Flavour Physics – Lectures 2 & 3

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Summer School on Symmetries and Fundamental Interactions,
Abtei Frauenwörth
September 1$^{\mathrm{st}}$ - 5$^{\mathrm{th}}$ 2014

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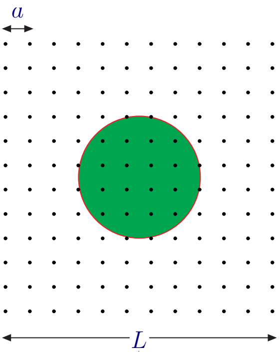
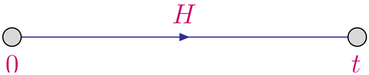


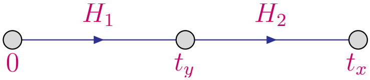
- Lattice phenomenology starts with the evaluation of correlation functions of the form:

$$\langle 0|\,O(x_1,x_2,\cdots,x_n)\,|0\rangle \quad = \quad \frac{1}{Z}\int [dA_\mu]\,[d\psi]\,[d\bar\psi]\,e^{-S}\,O(x_1,x_2,\cdots,x_n)\ ,$$

  where $O(x_1,x_2,\cdots,x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function.

- The physics which can be studied depends on the choice of the multilocal operator O.





- The functional integral is performed by discretising Euclidean space-time and using Monte-Carlo Integration.

The Scaling Trajectory

- In Lattice QCD, while it is natural to think in terms of the lattice spacing a, the input parameter is $\beta = 6/g^2(a)$.
- $g(a)$ is the bare coupling constant in the bare theory defined by the particular discretization of QCD used in the simulation. a^{-1} is the ultraviolet cut-off in momentum space.
- Imagine now that we are performing a simulation with $N_f = 2+1$ and that we are in an ideal world in which we can perform simulations with $m_{ud} = m_u = m_d$ around their “physical" values. The procedure for defining a physical scaling trajectory is then relatively simple.

The scaling trajectory (Cont.)

- At each β, choose two dimensionless quantities, e.g. m_π/m_Ω and m_K/m_Ω, and find the bare quark masses m_{ud} and m_s which give the corresponding physical values.
  These are then defined to be the physical (bare) quark masses at that β.
- Now consider a dimensionful quantity, e.g. m_Ω. The value of the lattice spacing is defined by

$$a^{-1} = \frac{1.672\,\mathrm{GeV}}{m_\Omega(\beta, m_{ud}, m_s)}$$

  where $m_\Omega(\beta, m_{ud}, m_s)$ is the measured value in lattice units.
- Other physical quantities computed at the physical bare-quark masses will now differ from their physical values by artefacts of $O(a^2)$.
- Repeating this procedure at different β defines a scaling trajectory. Other choices for the 3 physical quantities used to define different scaling trajectory are clearly possible.
- If the simulations are performed with m_c and/or $m_u \neq m_d$ then the procedure has to be extended accordingly.

- Lattice perturbation theory is not necessary to perform the renormalization of opertators.
- For illustration consider bare operators O, which do depend on the scale a, but which do not mix under renormalization:

$$O_R(\mu) = Z_O(\mu a)\, O_{\rm Latt}(a)\,.$$

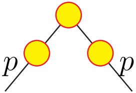
  The task is to determine Z_O .

- In the Rome-Southampton RI-Mom scheme, we impose that the matrix element of the operator between parton states, in the Landau gauge say, are equal to the tree level values for some specified external momenta.

  A General Method for Nonperturbative Renormalization of Lattice Operators

  G.Martinelli, C.Pittori, CTS, M.Testa and A.Vladikas, Nuclear Physics B445 (1995) 81

- These external momenta correspond to the renormalization scale.
- I will illustrate the idea by considering the scalar density $\bar{q}q$.



(i) Fix the gauge (to the Landau gauge say).

(ii) Evaluate the unamputated Green function:

$$G(x,y) = \langle 0 \,|u(x)\,[\bar{u}(0)d(0)]\,\bar{d}(y)\,|\,0\rangle$$

and Fourier transform to momentum space, at momentum p as in the diagram, $\Rightarrow\; G(p)$.

(iii) Amputate the Green function:

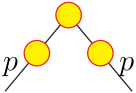
$$\Pi^{ij}_{S,\alpha\beta}(p) = S^{-1}(p)\,G(p)\,S^{-1}(p)\,,$$

where α,β (i,j) are spinor (colour) indices.

At tree level $\Pi^{ij}_{\alpha\beta}(p) = \delta_{\alpha,\beta}\,\delta^{i,j}$ and it is convenient to define

$$\Lambda_S(p) = \frac{1}{12}\,\mathrm{Tr}\,[\Pi_S(p)\,I]\,,$$

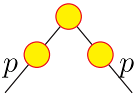
so that at tree-level $\Lambda_S = 1$.



- So far we have calculated the amputated Green function, in diagrammatic language, we have calculated the one-particle irreducible vertex diagrams.
- In order to determine the renormalization constant we need to multiply by $\sqrt{Z_q}$ for each external quark (i.e. there are two such factors).

(iv) We now evaluate Z_q. There are a number of ways of doing this, perhaps the best is to use the non-renormalization of the conserved vector current:

$$Z_q \Lambda_{V_C} = 1 \quad \text{where} \quad \Lambda_{V_C} = \frac{1}{48} \mathrm{Tr}\left[\Pi_{V_C^\mu}(p)\, \gamma^\mu\right].$$



- We now have all the ingredients necessary to impose the renormalization condition. We define the renormalized scalar density S_R by $S_R(\mu) = Z_S(\mu a) S_{\rm Latt}$ where
$$Z_S \, \frac{\Lambda_S(p)}{\Lambda_{V_C}(p)} = 1\,, \quad \text{for } p^2 = \mu^2\,.$$
- The scalar density has a non-zero anomalous dimension and therefore Z_S depends on the scale.
- The renormalization scheme here is a MOM scheme. We called it the RI-MOM scheme, where the RI stands for *Regularization Independent* to underline the fact that the renormalized operators do not depend on the bare theory (i.e. the lattice theory).

Perturbation Theory

- The precision of lattice calculations is now reaching the point where we need better interactions with the N^nLO QCD perturbation theory community.
- The traditional way of dividing responsibilities is:

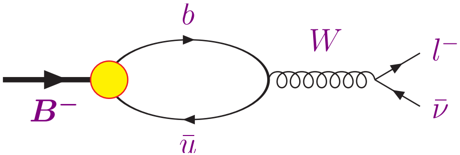
$$\text{Physics} \quad = \quad \underset{\substack{\uparrow \\ \text{Perturbative} \\ \text{QCD}}}{C} \quad \times \quad \underset{\substack{\uparrow \\ \text{Lattice} \\ \text{QCD}}}{\langle f\,|\,O\,|\,i\rangle}$$

- The two factors have to be calculated in the same scheme.
- Can we meet half way?

$$\begin{array}{c} \text{bare} \\ \text{lattice} \\ \text{operators} \end{array} \quad \longrightarrow \quad \textbf{?} \quad \longleftarrow \quad \begin{array}{c} \text{operators} \\ \text{renormalized} \\ \text{in } \overline{\text{MS}} \text{ scheme} \end{array}$$

- What is the best scheme for **?** (RI-SMOM, Schrödinger Functional, $\cdots$)?
- Recent examples of such collaborations following J.Gracey . . . :
  - two-loop matching factor for m_q between the RI-SMOM schemes and $\overline{\text{MS}}$. M.Gorbahn and S.Jager, arXiv:1004:3997, L.Almeida and C.Sturm, arXiv:1004:4613
  - HPQCD + Karlsruhe Group in determination of quark masses.

- The difficulty in making predictions for weak decays of hadrons is in controlling the non-perturbative strong interaction effects.
- As a particularly simple example consider the leptonic decays of pseudoscalar mesons in general and of the B-meson in particular.



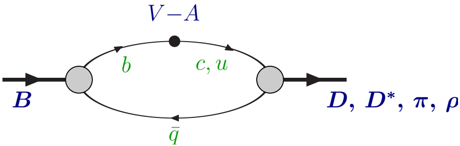
- Non-perturbative QCD effects are contained in the matrix element

$$\langle 0|\,\bar{b}\gamma^\mu(1-\gamma^5)u\,|B(p)\rangle\ .$$

- Lorentz Inv. + Parity $\Rightarrow$ $\langle 0|\,\bar{b}\gamma^\mu u\,|B(p)\rangle = 0.$
- Similarly $\quad \langle 0|\,\bar{b}\gamma^\mu\gamma^5 u\,|B(p)\rangle = if_B p^\mu\ .$

All QCD effects are contained in a single constant, f_B, the B-meson's *(leptonic) decay constant.* ($f_\pi \simeq 132\,$MeV)

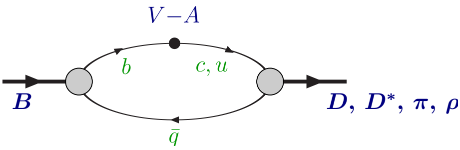
- These can be determined from either inclusive or exclusive decays. I start with a discussion of exclusive decays.



- Space-Time symmetries allow us to parametrise the non-perturbative strong interaction effects in terms of invariant form-factors. For example, for decays into a pseudoscalar meson P ($=\pi, D$ for example)

$$\langle P(k)|V^\mu|B(p)\rangle \quad = \quad f^+(q^2)\left[(p+k)^\mu - \frac{m_B^2-m_P^2}{q^2}\,q^\mu\right] + f^0(q^2)\,\frac{m_B^2-m_P^2}{q^2}\,q^\mu\,,$$

where $q=p-k$.



- For decays into a vector V $(=\rho, D^*$ for example), a conventional decomposition is

$$\langle V(k,\varepsilon)|V^\mu|B(p)\rangle = \frac{2V(q^2)}{m_B+m_V}\varepsilon^{\mu\gamma\delta\beta}\varepsilon^*_\beta p_\gamma k_\delta$$

$$\langle V(k,\varepsilon)|A^\mu|B(p)\rangle = i(m_B+m_V)A_1(q^2)\varepsilon^{*\mu} - i\frac{A_2(q^2)}{m_B+m_V}\varepsilon^*\cdot p\,(p+k)^\mu + i\frac{A(q^2)}{q^2}2m_V\varepsilon^*\cdot p\,q^\mu\ ,$$

where ε is the polarization vector of the final-state meson, and $q=p-k$.

$$\{A_3 = \frac{m_B+m_V}{2m_V}A_1 - \frac{m_B-m_V}{2m_V}A_2\ .\}$$

Flavour Physics Lattice Averaging Group (FLAG)

- Most of the compilations in this talk are taken from the current results of the FLAG collaboration: "Review of lattice results concerning low energy particle physics,"
  S. Aoki, Y. Aoki, C. Bernard, T. Blum, G. Colangelo, M. Della Morte, S. Dürr, A. El Khadra, H. Fukaya, A. Jüttner, R. Horsley, T. Kaneko, J. Laiho, L. Lellouch, H. Leutwyler, V. Lubicz, E. Lunghi, S. Necco, T. Onogi, C. Pena, C. Sachrajda, J. Shigemitsu, S. Simula, S. Sharpe, R. Sommer, R. Van de Water, A. Vladikas, U. Wenger, H. Wittig. arXiv:1310.8555, (255 pages!)

- This is an extension and continuation of the work of the Flavianet Lattice Averaging Group:
  G. Colangelo, S. Durr, A. Juttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. T. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig arXiv:1011.4408

- Motivation - to present to the wider community an *average* of lattice results for important quantities obtained after a critical expert review.
- Danger - original papers (particularly those which pioneer new techniques) do not get cited appropriately by the community.
- The closing date for arXiv:1310.8555v2 was Nov 30th 2013.

FLAG summary in light-quark physics

Quantity	■	$N_f=2+1+1$	■	$N_f=2+1$	■	$N_f=2$
m_s(MeV)			3	$93.8(2.4)$	2	$101(3)$
m_{ud}(MeV)			3	$3.42(9)$	1	$3.6(2)$
m_s/m_{ud}			3	$27.5(4)$	1	$28.1(1.2)$
m_d(MeV)				$4.68(14)(7)$		$4.8(15)(17)$
m_u(MeV)				$2.16(9)(7)$		$2.40(15)(17)$
m_u/m_d				$0.46(2)(2)$		$0.50(2)(3)$
$f_+^{K\pi}(0)$			1	$0.9667(23)(33)$	1	$0.9560(57)(62)$
f_{K^+}/f_{π^+}	1	$1.195(3)(4)$	4	$1.192(5)$	1	$1.205(6)(17)$
f_K(MeV)			3	$156.3(0.8)$	1	$158.1(2.5)$
f_π(MeV)			3	$130.2(1.4)$		
Σ(MeV)			2	$265(17)$	1	$270(7)$
F_π/F	1	$1.0760(28)$	2	$1.0620(34)$	1	$1.0733(73)$
$\bar{\ell}_3$	1	$3.70(27)$	3	$2.77(1.27)$	1	$3.45(26)$
$\bar{\ell}_4$	1	$4.67(10)$	3	$3.95(35)$	1	$4.59(26)$
$\hat{B}_K$			4	$0.766(10)$	1	$0.729(25)(17)$
$B_K^{\overline{\rm MS}}$(2 GeV)			4	$0.560(7)$	1	$0.533(18)(12)$

FLAG summary in heavy-quark physics

Quantity	■	$N_f = 2+1+1$	■	$N_f = 2+1$	■	$N_f = 2$
f_D(MeV)			2	$209.2(3.3)$	1	$212(8)$
f_{D_s}(MeV)			2	$248.6(2.7)$	1	$248(6)$
f_{D_s}/f_D			2	$1.187(12)$	1	$1.17(5)$
$f_+^{D\pi}(0)$			1	$0.666(29)$		
$f_+^{DK}(0)$			1	$0.747(19)$		
f_B(MeV)	1	$186(4)$	3	$190.5(4.2)$	1	$197(10)$
f_{B_s}(MeV)	1	$224(5)$	3	$227.7(4.5)$	1	$234(6)$
f_{B_s}/f_B	1	$1.205(7)$	2	$1.202(22)$	1	$1.19(5)$
$f_{B_d}\sqrt{\hat{B}_{B_d}}$(MeV)			1	$216(15)$		
$f_{B_s}\sqrt{\hat{B}_{B_s}}$(MeV)			1	$266(18)$		
$\hat{B}_{B_d}$			1	$1.27(10)$		
$\hat{B}_{B_s}$			1	$1.33(6)$		
ξ			1	$1.268(63)$		
$\hat{B}_{B_s}/\hat{B}_{B_d}$			1	$1.06(11)$		

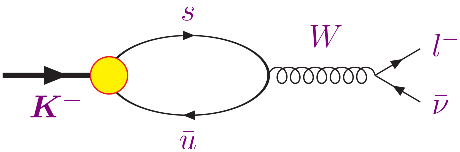
FLAG summary in heavy-quark physics (cont.) and α_s

Quantity	■	$N_f=2+1+1$	■	$N_f=2+1$	■	$N_f=2$
$\Delta\zeta^{B\pi}(\mathrm{ps}^{-1})$			2	$2.16(50)$		
$f_+^{B\pi}(q^2):\ a_0^{\mathrm{BCL}}$			2	$0.453(33)$		
a_1^{BCL}			2	$-0.43(33)$		
a_2^{BCL}			2	$0.9(3.9)$		
$\mathcal{F}^{B\to D^*}(1)$			1	$0.9017(51)(156)$		
$R(D)$			1	$0.316(12)(7)$		
$\alpha_{\overline{\mathrm{MS}}}^{(5)}(M_Z)$			4	$0.1184(12)$		

(i) Determination of V_{us}

(ii) ε_K

(iii) $K \to \pi\pi$ Decays



- All QCD effects are contained in a single constant, f_K, the kaon's *(leptonic) decay constant.*

$$\langle 0|\bar{s}\gamma^\mu\gamma^5 u\,|K(p)\rangle = if_K\,p^\mu\;. \qquad (f_\pi \simeq 132\,\mathrm{MeV})$$

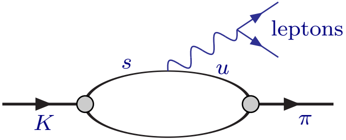
$$\frac{\Gamma(K\to\mu\bar{\nu}(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2}\,\frac{m_K\left(1-\frac{m_\mu^2}{m_K^2}\right)}{m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)}\times 0.9930(35)$$

- From the experimental ratio of the widths we get:

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2} = 0.07602(23)_{\mathrm{exp}}(27)_{\mathrm{RC}}\,, \qquad \text{PDG2006}$$

so that a precise determination of f_K/f_π will yield V_{us}/V_{ud} .

- Every collaboration calculates f_K and f_π.



$\Rightarrow \quad V_{us}$

$$\langle\, \pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|\,K(p_K)\,\rangle = f_0(q^2)\,\frac{M_K^2-M_\pi^2}{q^2}q_\mu + f_+(q^2)\,\left[(p_\pi+p_K)_\mu - \frac{M_K^2-M_\pi^2}{q^2}q_\mu\right]$$

where $q \equiv p_K - p_\pi$.

$$\Gamma_{K\to\pi\ell\nu} = C_K^2\,\frac{G_F^2 m_K^5}{192\pi^3}\,I\,S_{\mathrm{EW}}[1+2\Delta_{\mathrm{SU}(2)}+\Delta_{EM}]\,|V_{us}|^2\,|f_+(0)|^2$$

From the experimental measurement of the width we get:

$$|V_{us}|f_+(0) = 0.2169(9)\,, \qquad \text{PDG2006}$$

so that a precise determination of $f_+(0)$ will yield V_{us}.

FLAG

- We have the two precise results:

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599(59) \quad \text{and} \quad |V_{us} f_+(0)| = 0.21661(47)$$

Flavianet – arXiv:0801.1817

- We can view these as two equation for the four unknowns f_K/f_π, $f_+(0)$, V_{us} and V_{ud} .
- Within the Standard Model we also have the unitarity constraint:

$$|V_{ud}|^2 + |V_{us}|^2 \cancel{+ |V_{ub}|^2} = 1$$

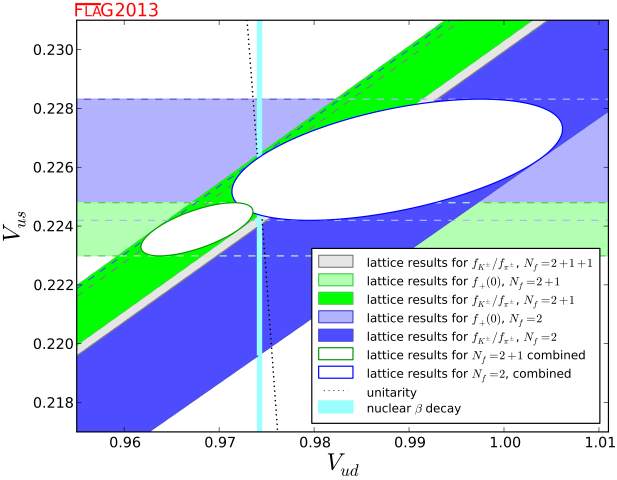
- Thus we now have 3 equations for four unknowns.
- There has been considerable work recently in updating the determination of V_{ud} based on 20 different superallowed transitions. Hardy and Towner, arXiV:0812.1202

$$|V_{ud}| = 0.97425(22)\,.$$

- If we accept this value then we are able to determine the remaining 3 unknowns:

$$|V_{us}| = 0.22544(95), \quad f_+(0) = 0.9608(46), \quad \frac{f_K}{f_\pi} = 1.1927(59)\,.$$

V_{us} from Lattice Simulations



Flavianet Lattice Averaging Group - arXiv:1310.8555v2

FLAG

Lattice results are consistent with the unitarity of the CKM Matrix

- For $N_f = 2+1$ simulations FLAG quotes the following current values:

$$\frac{f_K}{f_\pi} = 1.192(5) \quad \text{and} \quad f_+(0) = 0.9677(23)(33)\,.$$

- Taking the experimental results for $K_{\ell 2}$ and $K_{\ell 3}$ decays and dividing by the $N_f = 2+1$ lattice values of f_K/f_π and $f_+(0)$ gives:

$$V_{ud}^2 + V_{us}^2 = 0.987(10)\,.$$

- If we combine the experimental results with the value of V_{ud} and the lattice values of $f_+(0)$ or f_K/f_π we find:

$$V_{ud}^2 + V_{us}^2 = 0.9993(5) \quad \text{or} \quad V_{ud}^2 + V_{us}^2 = 1.0000(6)\,.$$

- Very significant test of universality of coupling of "W"-like bosons to quarks and leptons.
- Private question: At such level of precision, are there still continuum (and perhaps chiral) effects to be controlled fully?

$$\varepsilon_K = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = e^{i\phi_\varepsilon} \sin\phi_\varepsilon \left[\frac{\mathrm{Im}\langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle}{\Delta m_K} + \text{L.D. effects} \right]$$

where

Buras, Guadagnoli, arXiv:0805.3887

$$\begin{aligned} |\varepsilon_K| &= 2.228(11) \times 10^{-3} \\ \phi_\varepsilon &= \arctan \frac{\Delta m_K}{\Delta\Gamma_K/2} = 43.52(5)^\circ \\ \Delta m_K &= m_{K_L} - m_{K_S} = 3.4839(59) \times 10^{-12}\,\text{MeV} \\ \Delta\Gamma_K &= \Gamma_S - \Gamma_L = 7.3382(33) \times 10^{-15}\,\text{GeV}. \end{aligned}$$

- It is conventional to present the short-distance contribution in terms of the B_K parameter:

$$\langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle \propto \langle \bar{K}^0 | (\bar{s}\gamma^\mu(1-\gamma^5)d)\,(\bar{s}\gamma_\mu(1-\gamma^5)d) | K^0 \rangle \equiv \frac{8}{3} f_K^2 m_K^2 B_K(\mu)\,.$$

- Lattice calculations of B_K have been performed since the mid 1980s. The precision is now such that the $O(5\%)$ long-distance (LD) effects have to be considered.

Buras, Guadagnoli, Isidori arXiv:1002.3612

- FLAG-2 quote from simulations with $N_f = 2+1$:
$$\hat{B}_K = 0.766(10) \text{ corresponding to } B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.560(7)\,.$$
- The FLAG-1 result was $\hat{B}_K = 0.738(20)$ and at EPS 1993 I quoted a summary $\hat{B}_K = 0.8(2)$. M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Nucl.Phys. B369 (1992) 139
- The dominant contribution to $\varepsilon_K \propto |V_{cb}|^4$ and PDG(2012) quote $|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$ error on B_K is no longer the dominant one.
- Among our (RBC-UKQCD) main projects are the evaluation of ΔM_K and the long-distance contributions to ε_K.

Neutral Kaon Mixing BSM

- Beyond the standard model there are in general 5 independent operators which contribute neutral Kaon mixing:

$$\mathscr{H}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu)\, Q_i^{\Delta S=2}(\mu)\,.$$

- The five operators are:

$$\begin{aligned}
Q_1^{\Delta S=2} &= [\bar{s}^i\gamma_\mu(1-\gamma_5)d^i]\,[\bar{s}^j\gamma_\mu(1-\gamma_5)d^j] \\
Q_2^{\Delta S=2} &= [\bar{s}^i(1-\gamma_5)d^i]\,[\bar{s}^j(1-\gamma_5)d^j] \\
Q_3^{\Delta S=2} &= [\bar{s}^i(1-\gamma_5)d^j]\,[\bar{s}^j(1-\gamma_5)d^i] \\
Q_4^{\Delta S=2} &= [\bar{s}^i(1-\gamma_5)d^i]\,[\bar{s}^j(1+\gamma_5)d^j] \\
Q_5^{\Delta S=2} &= [\bar{s}^i(1-\gamma_5)d^j]\,[\bar{s}^j(1+\gamma_5)d^i]
\end{aligned}$$

  - i,j are colour indices.

- The matrix elements can be calculated in a similar way to B_K. For a recent study and references to the original literature see Boyle, Garron and Hudspith.

arXiv:1206.5737

- $Q_1^{\Delta S=2}$ transforms as (27,1) under SU(3)$_L\times$SU(3)$_R$, $Q_2^{\Delta S=2}$ and $Q_3^{\Delta S=2}$ as $(6,\bar{6})$ and $Q_4^{\Delta S=2}$ and $Q_5^{\Delta S=2}$ as (8,8) $\Rightarrow$ Renormalization matrix is block diagonal.

Direct Evaluation of $K \to \pi\pi$ Decays

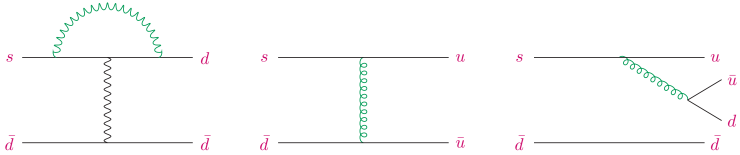
- $K \to \pi\pi$ decays are a very important class of processes for standard model phenomenology.
- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.
- Among the interesting issues are the origin of the $\Delta I = 1/2$ rule (ReA_0/Re$A_2 \simeq 22.5$) and an understanding of the experimental value of ε'/ε, the parameter which was the first experimental evidence of direct CP-violation.
- The evaluation of $K \to \pi\pi$ matrix elements requires an extension of the standard computations of $\langle 0 \,|\, O(0) \,|\, h \rangle$ and $\langle h_2 \,|\, O(0) \,|\, h_1 \rangle$ matrix elements with a single hadron in the initial and/or final state.

- Directly CP-violating decays are those in which a CP-even (-odd) state decays into a CP-odd (-even) one: $K_L \;\propto\; K_2 \;+\; \bar{\epsilon} K_1\,.$

  $K_2 \to \pi\pi$ Direct (ϵ')

  $K_1 \to \pi\pi$ Indirect (ϵ_K)

- Consider the following contributions to $K \to \pi\pi$ decays:



$I = 0$, Complex (a) $\quad$ $I = 0$, Real (b) $\quad$ $I = 0$ or 2, Real (c)

  Direct CP-violation in kaon decays manifests itself as a non-zero relative phase between the $I = 0$ and $I = 2$ amplitudes.

- We also have *strong phases*, δ_0 and δ_2 which are independent of the form of the weak Hamiltonian.

Effective Hamiltonian for $K \to \pi\pi$ Decays

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} \left[z_i(\mu) + \tau\, y_i(\mu)\right] Q_i\,, \text{ where } \tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \text{ and}$$

Current – Current Operators

$$Q_1 = (\bar{s}d)_L(\bar{u}u)_L \qquad Q_2 = (\bar{s}^i d^j)_L(\bar{u}^j u^i)_L$$

QCD Penguin Operators

$$Q_3 = (\bar{s}d)_L \textstyle\sum_{q=u,d,s}(\bar{q}q)_L \qquad Q_4 = (\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s}(\bar{q}^j q^i)_L$$

$$Q_5 = (\bar{s}d)_L \textstyle\sum_{q=u,d,s}(\bar{q}q)_R \qquad Q_6 = (\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s}(\bar{q}^j q^i)_R$$

Electroweak Penguin Operators

$$Q_7 = \tfrac{3}{2}(\bar{s}d)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}q)_L \qquad Q_8 = \tfrac{3}{2}(\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}^j q^i)_L$$

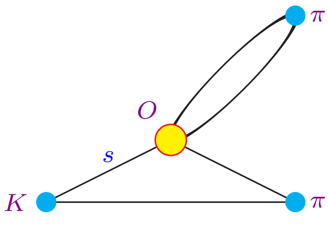
$$Q_9 = \tfrac{3}{2}(\bar{s}d)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}q)_R \qquad Q_{10} = \tfrac{3}{2}(\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}^j q^i)_R$$

This 10 operator basis is very natural but over-complete:

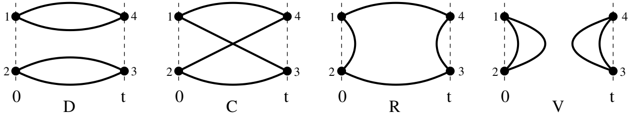
$$\begin{aligned} Q_{10} - Q_9 &= Q_4 - Q_3 \\ Q_4 - Q_3 &= Q_2 - Q_1 \\ 2Q_9 &= 3Q_1 - Q_3\,. \end{aligned}$$

The original material on this topic is taken from the following RBC-UKQCD papers:

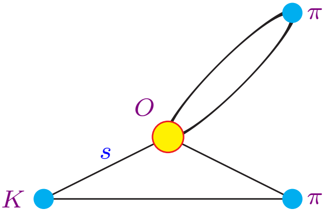
1. "K to $\pi\pi$ Decay amplitudes from Lattice QCD,"
   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D.Mawhinney, C.T.Sachrajda, A.Soni, C.Sturm, H.Yin and R.Zhou,
   Phys. Rev. D **84** (2011) 114503 (arXiv:1106.2714 [hep-lat]).
2. "The $K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"
   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.Sachrajda, A.Soni and C.Sturm,
   Phys. Rev. Lett. 108 (2012) 141601, (arXiv:1111.1699 [hep-lat]).
3. "Lattice Determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude A_2,"
   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.Sachrajda, A.Soni and C.Sturm,
   Phys.Rev. D86 (2012) 074513, (arXiv:1206.5142 [hep-lat]).
4. "Emerging understanding of the $\Delta I = 1/2$ rule from Lattice QCD,"
   P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Janowski, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, C.T.Sachrajda, A.Soni and D.Zhang,
   Phys. Rev. Lett. 110 (2013) 152001, (arXiv:1212.1474 [hep-lat]).



- We need to evaluate correlation functions as in the diagram above.
- In order to divide by $\langle 0|J_\pi J_\pi|\pi\pi\rangle$, we also need to evaluate the two-pion correlation functions.



- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.



- In the physical decay, in the centre-of-mass frame, $E_{\pi\pi} = m_K$.
- In lattice calculations, in order to eliminate excited states we do not integrate over time, and so, in general, energy is not conserved.
- In the centre-of-mass frame the ground-state is the two-pion state with $E_{\pi\pi} \simeq 2m_\pi$.
- Therefore the correlation function is dominated by the unphysical transition of a kaon at rest into two pions at rest. Maiani-Testa Problem
- The Lellouch-Lüscher solution is to tune the volume so that one of the excited states corresponds to $E_{\pi\pi} = m_K$. (Loss of precision.) hep-lat/0003023

- The operators whose matrix elements have to be calculated are:

$$\begin{aligned}
O^{3/2}_{(27,1)} &= (\bar{s}^i d^i)_L \{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_L \\
O^{3/2}_{7} &= (\bar{s}^i d^i)_L \{(\bar{u}^j u^j)_R - (\bar{d}^j d^j)_R\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_R \\
O^{3/2}_{8} &= (\bar{s}^i d^j)_L \{(\bar{u}^j u^i)_R - (\bar{d}^j d^i)_R\} + (\bar{s}^i u^j)_L (\bar{u}^j d^i)_R
\end{aligned}$$

- It is convenient to use the Wigner-Eckart Theorem: (Notation - $O^{\Delta I}_{\Delta I_z}$)

$$_{I=2}\langle \pi^+(p_1)\pi^0(p_2)\,|O^{3/2}_{1/2}|K^+\rangle = \frac{\sqrt{3}}{2}\langle \pi^+(p_1)\pi^+(p_2)\,|O^{3/2}_{3/2}|K^+\rangle\,,$$

where
  - $O^{3/2}_{3/2}$ has the flavour structure $(\bar{s}d)\,(\bar{u}d)$.
  - $O^{3/2}_{1/2}$ has the flavour structure $(\bar{s}d)\,(\,(\bar{u}u) - (\bar{d}d)\,) + (\bar{s}u)\,(\bar{u}d)$.

- We can then use antiperiodic boundary conditions for the u-quark say, so that the $\pi\pi$ ground-state is $\langle \pi^+(\pi/L)\,\pi^+(-\pi/L)\,|$. C-h Kim, Ph.D. Thesis
  - • Do not have to isolate an excited state. •
  - Size (L) needed for physical $K \to \pi\pi$ decay halved.

- The main theoretical ingredients of the *infrared* problem with two-pions in the s-wave are understood.
- Two-pion quantization condition in a finite-volume

$$\delta(q^*) + \phi^P(q^*) = n\pi\,,$$

where $E^2 = 4(m_\pi^2 + q^{*\,2})$, δ is the s-wave $\pi\pi$ phase shift and ϕ^P is a kinematic function.

M.Lüscher, 1986, 1991, $\cdots$.

- The relation between the physical $K \to \pi\pi$ amplitude A and the finite-volume matrix element M

$$|A|^2 = 8\pi V^2 \frac{m_K E^2}{q^{*\,2}} \left\{\delta'(q^*) + \phi^{P\prime}(q^*)\right\} |M|^2\,,$$

where $'$ denotes differentiation w.r.t. q^* .

L.Lellouch and M.Lüscher, hep-lat/0003023; C.h.Kim, CTS and S.Sharpe, hep-lat/0507006;
N.H.Christ, C.h.Kim and T.Yamazaki hep-lat/0507009

- Computation of $K \to (\pi\pi)_{I=2}$ matrix elements does not require the subtraction of power divergences or the evaluation of disconnected diagrams.
- In 2011-2012, we evaluated the $\Delta I = 3/2$ $K \to \pi\pi$ matrix elements for the first time and at physical kinematics.

- The calculations were performed on a $32^3 \times 64 \times 32$ ($L = 4.58$ fm, $a^{-1} = 0.14$ fm lattice using Domain Wall Fermions and the IDSDR gauge action.

Systematic Error Budget	$\mathrm{Re}A_2$	$\mathrm{Im}A_2$
lattice artefacts	15%	15%
finite-volume corrections	6.0%	6.5%
partial quenching	3.5%	1.7%
renormalization	1.8%	5.6%
unphysical kinematics	0.4%	0.8%
derivative of the phase shift	0.97%	0.97%
Wilson coefficients	6.6%	6.6%
Total	18%	19%

- The dominant error is due to lattice artefacts and the fact that out lattice is coarse. This will be eliminated when the calculation is repeated at a second lattice spacing. T.Janowski et al., arXiv:1311.3844 and paper in preparation
- The 15% estimate, intended to be conservative, is obtained by
  1. Studying the dependence on a of quantities which have been calculated at several lattice spacings.
  2. In particular by determining the a dependence of B_K, which is also given by the matrix element of a (27,1) operator.

Our results for the amplitude A_2 are:

$$\mathrm{Re}A_2 = (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})\,10^{-8}\,\mathrm{GeV}$$
$$\mathrm{Im}A_2 = -(6.54 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}})\,10^{-13}\,\mathrm{GeV}.$$

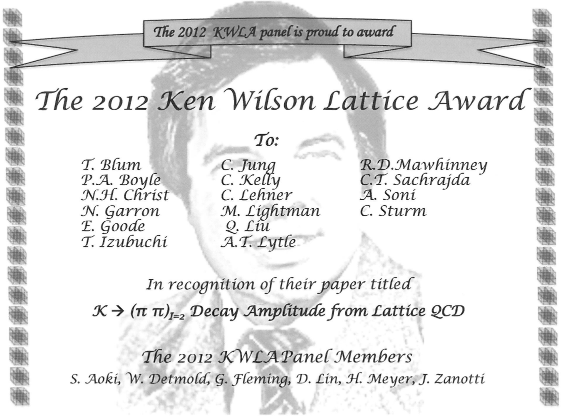
- The result for Re A_2 agrees well with the experimental value of $1.479(4) \times 10^{-8}$ GeV obtained from K^+ decays and $1.573(57) \times 10^{-8}$ GeV obtained from K_S decays .
- Im A_2 is unknown so that our result provides its first direct determination.
- For the phase of A_2 we find Im A_2/Re$A_2 = -4.42(31)_{\mathrm{stat}}(89)_{\mathrm{syst}}\,10^{-5}$.
- Combining our result for Im A_2 with the experimental results for Re A_2, Re $A_0 = 3.3201(18) \cdot 10^{-7}$ GeV and ε'/ε we obtain:

$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = -1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}} \times 10^{-4}\,.$$

(Of course, we wish to confirm this directly.)

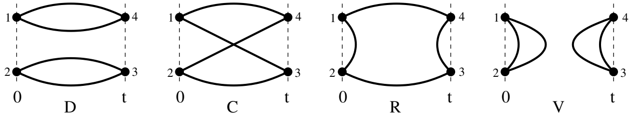
$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\sqrt{2}\,|\varepsilon|}{\omega}\frac{\varepsilon'}{\varepsilon}$$

$$-1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}} \times 10^{-4} = -4.42(31)_{\mathrm{stat}}(89)_{\mathrm{syst}} \times 10^{-5} - 1.16(18) \times 10^{-4}\,.$$



- For this work we received the 2012 Ken Wilson Lattice award at Lattice 2012.
- Criteria: The paper must be important research beyond the existing state of the art. $\cdots$

- The $I = 0$ final state has vacuum quantum numbers.
- Vacuum contribution must be subtracted; disconnected diagrams require statistical cancelations to obtain the $e^{-2m_\pi t}$ behaviour.
- Consider first the two-pion correlation functions, which are an important ingredient in the evaluation of $K \to \pi\pi$ amplitudes.

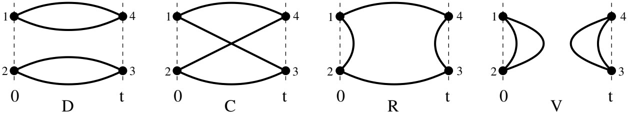


- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.
- For I=0 $\pi\pi$ states the correlation function is proportional to 2D+C-6R+3V.

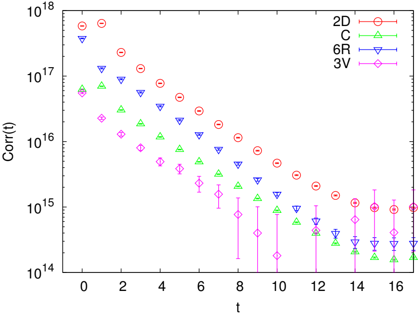
The major practical difficulty is to subtract the vacuum contribution with sufficient precision.

- In the paper we report on high-statistics experiments on a $16^3 \times 32$ lattice, $a^{-1} = 1.73$ GeV, $m_\pi = 420$ MeV, with the propagators evaluated from each time-slice.

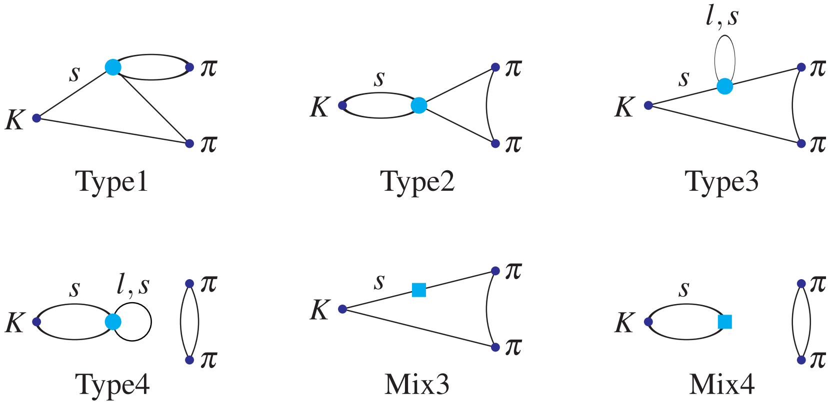
Diagrams contributing to two-pion correlation functions



- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.
- For I=0 $\pi\pi$ states the correlation function is proportional to 2D+C-6R+3V.



RBC/UKQCD, Qi Liu – Lattice 2010



- There are 48 different contractions and we classify the contributions into the 6 different types illustrated above.
- Mix3 and Mix4 are needed to subtract the power divergences which are proportional to matrix elements of $\bar{s}\gamma_5 d$.

- These results are for the $K\to\pi\pi$ (almost) on-shell amplitudes with 420 MeV pions at rest: RBC/UKQCD arXiv:1106.2714

Re A_0	$(3.80\pm0.82)\,10^{-7}$ GeV	Im A_0	$-(2.5\pm2.2)\,10^{-11}$ GeV
Re A_2	$(4.911\pm0.031)\,10^{-8}$ GeV	Im A_2	$-(5.502\pm0.0040)\,10^{-13}$ GeV

- This was an exploratory exercise in which we are learning how to do the calculation.
- We, along with the rest of the world, continue to develop techniques with the aim of enhancing the signal for disconnected diagrams.
- The exploratory results for $K\to(\pi\pi)_{I=0}$ decays are very encouraging.
- For $(\pi\pi)_I=0$ states the Wigner-Eckart theorem and the use of antiperiodic boundary conditions for the d-quark does not help.

  C.Sachrajda and G.Villadoro hep-lat/0411033

  We are currently developing and testing the use of G-parity boundary conditions.

  C.-h Kim, hep-lat/0311003

  $\Rightarrow$ a quantitative understanding of the $\Delta I=1/2$ rule and the value of ε'/ε.

- The evaluation of disconnected diagram has allowed us to study the η and η' mesons and their mixing. RBC-UKQCD – arXiV:1002.2999 arXiv:1212.1474

- In his thesis Qi Liu extended the above study to the $24^3 \times 64$ ensembles.
  - Larger $T \Rightarrow$ suppression of around-the-world effects.
  - Two-pion sources separated in time $\Rightarrow$ better plateaus.
  - Faster algorithm for the inversions.

1. $16^3 \times 32$ ensembles; 877 MeV kaon decaying into two 422 MeV pions at rest:

$$\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 9.1 \pm 2.1\,.$$

2. $24^3 \times 64$ ensembles; 662 MeV kaon decaying into two 329 MeV pions at rest:

$$\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 12.0 \pm 1.7\,.$$

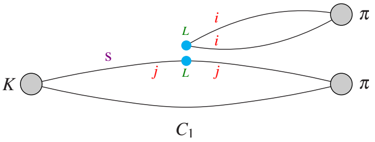
- Whilst both these results are obtained at unphysical kinematics and are different from the physical value of 22.5, it is nevertheless interesting to understand the origin of these enhancements.
- 99% of the contribution to the real part of A_0 and A_2 come from the matrix elements of the current-current operators.

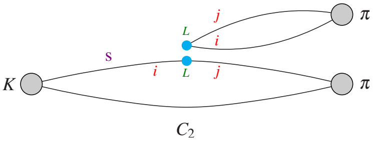
Contributions from Individual Matrix Elements

i	Q_i^{lat} [GeV]	$Q_i^{\overline{\text{MS}}\text{-NDR}}$ [GeV]
1	8.1(4.6) 10^{-8}	6.6(3.1) 10^{-8}
2	2.5(0.6) 10^{-7}	2.6(0.5) 10^{-7}
3	-0.6(1.0) 10^{-8}	5.4(6.7) 10^{-10}
4	–	2.3(2.1) 10^{-9}
5	-1.2(0.5) 10^{-9}	4.0(2.6) 10^{-10}
6	4.7(1.7) 10^{-9}	-7.0(2.4) 10^{-9}
7	1.5(0.1) 10^{-10}	6.3(0.5) 10^{-11}
8	-4.7(0.2) 10^{-10}	-3.9(0.1) 10^{-10}
9	–	2.0(0.6) 10^{-14}
10	–	1.6(0.5) 10^{-11}
ReA_0	3.2(0.5) 10^{-7}	3.2(0.5) 10^{-7}

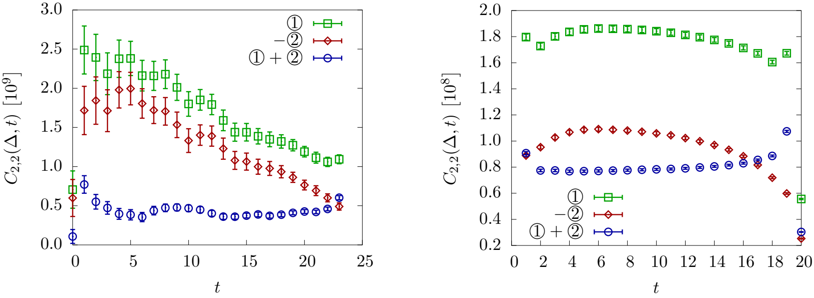
- Contributions from each operator to ReA_0 for $m_K = 662$ MeV and $m_\pi = 329$ MeV. The second column contains the contributions from the 7 linearly independent lattice operators with $1/a = 1.73(3)$ GeV and the third column those in the 10-operator basis in the $\overline{\text{MS}}$-NDR scheme at $\mu = 2.15$ GeV. Numbers in parentheses represent the statistical errors.

Emerging understanding of the $\Delta I = 1/2$ Rule (Cont.)





- $\mathrm{Re}\,A_2$ is proportional to $C_1 + C_2$.
- The contribution to $\mathrm{Re}\,A_0$ from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3}C_1$.
  - Much continuum phenomenology has been done in the vacuum insertion hypothesis.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- A_2 has a larger kinematic dependence than A_0.
- **We believe that the strong suppression of Re A_2 and the (less-strong) enhancement of Re A_0 is a major factor in the $\Delta I = 1/2$ rule.**
  - Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we need to compute $\mathrm{Re}\,A_0$ at physical kinematics and reproduce the experimental value of 22.5.



Physical Kinematics $\qquad$ $m_\pi \simeq$ 330 MeV at threshold.

- Notation ⓘ $\equiv C_i,\ i = 1,2.$

UNIVERSITY OF Southampton
School of Physics and Astronomy

- We are completing a paper updating our results of A_2, computed at two finer lattice spacings and at physical quark masses. (Errors are significantly reduced.)
- Development and testing of G-parity boundary conditions with the primary aim of computing the $K \to (\pi\pi)_{I=0}$ decay amplitude A_0.
- Evaluation of long-distance effects in ΔM_K and ε_K.
- Beginning to perform the exploratory work to study the rare kaon decays $K \to \pi \ell^+ \ell^-$ and $K \to \pi \nu \bar{\nu}$.
- These last two quantities are an extension of lattice calculations to matrix elements of the form:

$$\int d^4x \int d^4y \, \langle h_2 \,|\, T\{O_1(x)\, O_2(y)\} \,|\, h_1 \rangle \, .$$

4. B-Physics

- The b-quark is light-enough to be produced copiously and heavy enough to have a huge number of possible decay channels.
- In addition to the lattice systematics already discussed, we now have to deal with the fact that $m_b a \gtrsim 1$.
- Most approaches rely on effective theories and invest a considerable effort in matching the effective theory to QCD.
  - Heavy Quark Effective Theory (expansion in $\frac{\Lambda_{\text{QCD}}}{m_B}$).
  - Nonrelativistic QCD (expansion in the quark's velocity).
  - Relativistic Heavy Quarks ("Fermilab Approach" and extensions).

A. El Khadra, A. Kronfeld and P. Mackenzie, hep-lat/9604004

- Some groups also extrapolate results from the charm to the bottom region, using scaling laws where applicable and possibly using results in the static limit.
- There are far fewer calculations in heavy-quark physics, so less opportunity to check for consistency of different approaches.
  This is not a criticism of those who have done the calculations but of those of us who have not!
- Unfortunately we do not know (yet?) how to compute non-leptonic B-decays ($B \to \pi\pi$, $B \to \pi K$ etc).

- For many years the experimental upper bound for this FCNC decay has been several orders of magnitude above the SM prediction.
- Most extensions of the SM give loop corrections which enhance the width and hence this was viewed as a good channel for the discovery of new physics.
- The LHC experiments observed this decay in 2012 and recent results are

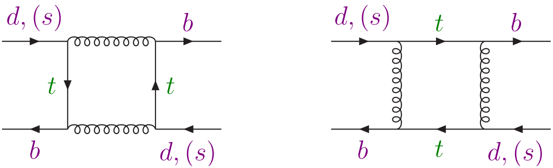
$$\begin{aligned}
\mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{LHCb}} &= 2.9^{+1.1}_{-1.0} \times 10^{-9} && \mathrm{LHCb, arXiv: 1307.5024} \\
\mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{CMS}} &= 3.0^{+1.0}_{-0.9} \times 10^{-9} && \mathrm{CMS, arXiv: 1307.5025} \\
\mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{Combined}} &= (2.9 \pm 0.7) \times 10^{-9} && \mathrm{LHCb + CMS,\ Conf.\ Presentation}
\end{aligned}$$

- Unfortunately(!?), these results are fully consistent with the Standard Model e.g.

$$\begin{aligned}
\mathrm{Br}(B_s \to \mu^+\mu^-) &= (3.65 \pm 0.23) \times 10^{-9} && \mathrm{Bobeth\ et\ al.,\ arXiv: 1311.0903} \\
\mathrm{Br}(B_s \to \mu^+\mu^-) &= (3.23 \pm 0.27) \times 10^{-9} && \mathrm{Buras\ et\ al.,\ arXiv: 1208.0934}
\end{aligned}$$

- Lattice input is the evaluation of f_{B_s} ($f_{B_s} = 227.7 \pm 4.5$ MeV).
- For the corresponding branching fraction of B_d decays, the combined result is $(3.6^{+1.6}_{-1.4}) \times 10^{-10}$ compared to the theoretical prediction of $(1.06 \pm 0.09) \times 10^{-10}$.

Neutral B-meson mixing



- For the SU(3)-breaking parameter ξ, FLAG take the result of the FNAL/MILC collaboration as currently the best result: FNAL/MILC, arXiv:1205.7013

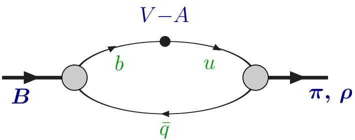
$$\xi^2 \equiv \frac{\langle \bar{B}^0_s \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)s)(\bar{b}\gamma_\mu(1-\gamma^5)s)\,|B^0_s\rangle}{\langle \bar{B}^0 \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)d)(\bar{b}\gamma_\mu(1-\gamma^5)d)\,|B^0\rangle} = 1.268(63)\,.$$

- Combining this result with experimental values of Δm_d and $\Delta m_s \Rightarrow$

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.216 \pm 0.011\,.$$

FNAL/MILC, arXiv:1205.7013

- For generic BSM theories, there are 5 $\Delta B = 2$ operators (and 5 $\Delta S = 2$ operators for neutral kaon mixing) whose matrix elements can be computed in a similar way.



QCD effects are contained in form factors e.g. for $B \to \pi$ decays:

$$\langle \pi(p_\pi) | \bar{b}\gamma_\mu u | B(p_B) \rangle = f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu + f_+(q^2) \left[(p_\pi + p_B)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right]$$

where $q \equiv p_B - p_\pi$.

- For B-decays, in order to avoid lattice artefacts, the momentum of the π or ρ is limited $\Rightarrow$ get results only at large values of q^2.
- Thus V_{ub} can only be obtained directly by combining the lattice results with a subset of the experimental data:

$$\Delta\zeta(q_1^2, q_2^2) = \frac{1}{|V_{ub}|^2} \int_{q_1^2}^{q_2^2} dq^2\, \frac{d\Gamma}{dq^2}\,.$$

- The lattice results can be combined with theoretically motivated parametrisations for the form factors, including perhaps constrains from analyticity and other general properties of field theory, to extend the range of the predictions. (Not discussed here.)

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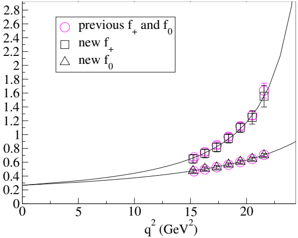
School of Physics and Astronomy

The (peer-reviewed) published values for the form factors are relatively old:

Collaboration	Reference	$\Delta\zeta$ ps^{-1}
FNAL/MILC	arXiv:0811.3640	$2.21^{+0.47}_{-0.42}$
HPQCD	hep-lat/0601021	2.07(41)(39)



HPQCD

- The two collaborations use overlapping sets of rooted staggered ensembles, but different treatments of the heavy quarks (HPQCD use NRQCD and FNAL/MILC use the FNAL approach). Assuming (conservatively) a 100% correlation FLAG quote

$$\Delta\zeta(16\,\mathrm{GeV}^2, q^2_{\mathrm{max}}) = 2.16(50)\,\mathrm{ps}^{-1}$$

- FLAG, perform a detailed analysis, finding a preferred parametrization and quote

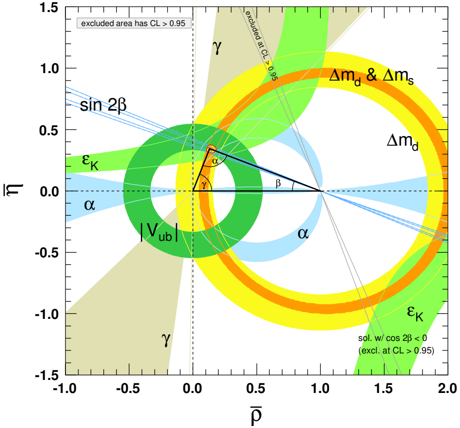
$$\text{Lattice + BABAR} \qquad |V_{ub}| = 3.37(21)\times 10^{-3}$$
$$\text{Lattice + Belle} \qquad |V_{ub}| = 3.47(22)\times 10^{-3}.$$

- FLAG, perform a detailed analysis, finding a preferred parametrization and quote

$$\begin{array}{rl} \text{Lattice} + \text{BaBar} & |V_{ub}| = 3.37(21) \times 10^{-3} \\ \text{Lattice} + \text{Belle} & |V_{ub}| = 3.47(22) \times 10^{-3}. \end{array}$$

- Assuming (not assuming) unitarity PDG quote $|V_{ub}| = 3.51^{+0.15}_{-0.14} \times 10^{-3}$ ($|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$).
- The issue is the tension with the inclusive determination $|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.19}) \times 10^{-3}$. This has very different systematics and cannot be studied in lattice simulations.
- The evaluation of $f_+(q^2)$ and $f_0(q^2)$ and the subsequent determination of V_{ub} is clearly a major priority for lattice simulations and is now a priority of several collaborations.

PDG2012 Unitarity Triangle



5. Electromagnetic corrections to weak matrix elements

N.Carrasco, V.Lubicz, G.Martinelli, CTS, F.Sanfillipo, N.Tantalo, C.Tarantino, M.Testa (in preparation)

- For a review of electromagnetic mass-splittings see the talk by A.Portelli at Lattice 2014.
- The evaluation of (some) weak matrix elements are now being quoted with $O(1\%)$ precision e.g. FLAG Collaboration, arXiv:1310.8555

f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}
130.2(1.4)	156.3(0.8)	209.2(3.3)	248.6(2.7)	190.5(4.2)	227.7(4.5)

(results given in MeV)

- We therefore need to start considering electromagnetic (and other isospin breaking) effects if we are to use these results to extract CKM matrix elements at a similar precision.
- For illustration, I consider f_π but the discussion is general. I do not use ChPT. For a ChPT based discussion of f_π, see J.Gasser & G.R.S.Zarnauskas, arXiv:1008.3479
- At $O(\alpha^0)$

$$\Gamma(\pi^+ \to \ell^+ \nu_\ell) = \frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\, m_\pi\, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2 .$$

- At $O(\alpha)$ infrared divergences are present and we have to consider

$$\begin{aligned}\Gamma(\pi^+ \to \ell^+\nu_\ell(\gamma)) &= \Gamma(\pi^+ \to \ell^+\nu_\ell) + \Gamma(\pi^+ \to \ell^+\nu_\ell\gamma) \\ &\equiv \Gamma_0 + \Gamma_1\,,\end{aligned}$$

  where the suffix denotes the number of photons in the final state.

- Each of the two terms on the rhs is infrared divergent, the divergences cancel in the sum.
- The cancelation of infrared divergences between contributions with virtual and real photons is an old and well understood issue.

  F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.
- (The evaluation of em corrections to the spectrum is free from UV divergences.)

- At this stage we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
  - A cut-off Δ of $O(10\,\mathrm{MeV})$ appears to be appropriate both experimentally and theoretically.
  - (In the future, as techniques and resources improve, it may be better to compute Γ_1 nonperturbatively over a larger range of photon energies.)
- We now write

$$\boxed{\Gamma_0 + \Gamma_1(\Delta) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\mathrm{pt}}) + \lim_{V\to\infty}(\Gamma_0^{\mathrm{pt}} + \Gamma_1(\Delta))\,.}$$

  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log\Delta$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated nonperturbatively and Γ_0^{pt} in perturbation theory. The subtraction in the first term is performed for each momentum and then the sum over momenta is performed (see below).

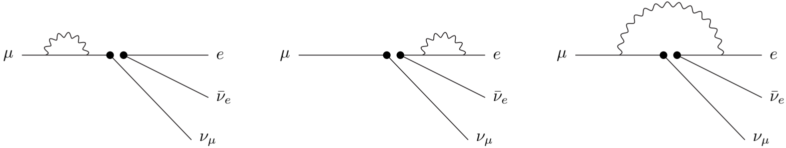
The procedure

1. The result for the width is expressed in terms of G_F, the Fermi constant $(G_F = 1.16632(2)\times 10^{-5}\,\mathrm{GeV}^{-2})$. This is obtained from the muon lifetime:

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1-\frac{8m_e^2}{m_\mu^2}\right]\left[1+\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right].$$

S.M.Berman, PR 112 (1958) 267; T.Kinoshita and A.Sirlin, PR 113 (1959) 1652

- This expression can be viewed as the definition of G_F. Many EW corrections are absorbed into the definition of G_F; the explicit $O(\alpha)$ corrections come from the following diagrams in the effective theory:



together with the diagrams with a real photon.

- The diagrams are evaluated in the W-regularisation in which the photon propagator is modified by: A.Sirlin, PRD 22 (1980) 971

$$\frac{1}{k^2} \to \frac{M_W^2}{M_W^2-k^2}\,\frac{1}{k^2}\,. \qquad \left(\frac{1}{k^2} = \frac{1}{k^2-M_W^2} + \frac{M_W^2}{M_W^2-k^2}\,\frac{1}{k^2}\right)$$

2 Most (but not all) of the EW corrections which are absorbed in G_F are common to other processes (including pion decay) $\Rightarrow$ factor in the amplitude of $(1+3\alpha/4\pi(1+2\bar{Q})\log M_Z/M_W)$, where $\bar{Q}=\frac{1}{2}(Q_u+Q_d)=1/6$.

A.Sirlin, NP B196 (1982) 83; E.Braaten & C.S.Li, PRD 42 (1990) 3888

3 We therefore need to calculate the pion-decay diagrams in the effective theory (with $H_{\rm eff}\propto(\bar{d}_L\gamma^\mu u_L)(\bar{\nu}_{\ell,L}\gamma_\mu\ell_L)$) in the W-regularization. These can be related to the lattice theory by perturbation theory, e.g. for Wilson fermions:

$$O_{LL}^{W-\rm reg}=\left(1+\frac{\alpha}{4\pi}\left(2\log a^2M_W^2-15.539\right)+O(\alpha\alpha_s)\right)O_{LL}^{\rm bare}\,.$$

4 We now return to the master formula:

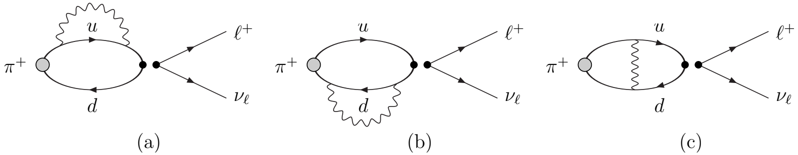
$$\boxed{\Gamma_0+\Gamma_1(\Delta)=\lim_{V\to\infty}(\Gamma_0-\Gamma_0^{\rm pt})+\lim_{V\to\infty}(\Gamma_0^{\rm pt}+\Gamma_1(\Delta))\,.}$$

- The term which is added and subtracted is not unique, but we require that both terms are free of ir divergences and independent of the ir regulator.
- Kinoshita performed the calculation for a pointlike pion, (i) integrating over all phase space and (ii) imposing a cut-off on the charged-lepton energy.

T.Kinoshita, PRL 2 (1959) 477

- We have reproduced these results and extended them to a cut-off on the photon energy.

5. Consider now the evaluation of the first term in the master formula.



- The correlation function for this set of diagrams is of the form:

$$C_1(t) = \frac{1}{2}\int d^3\vec{x}\, d^4x_1\, d^4x_2\, \langle 0|T\{J_W^\nu(0) j^\mu(x_1) j_\mu(x_2)\phi^\dagger(\vec{x},t)\}|0\rangle\, \Delta(x_1,x_2)\,,$$

where $j_\mu(x) = \sum_f Q_f \bar{f}(x)\gamma_\mu f(x)$, J_W is the weak current, ϕ is an interpolating operator for the pion and Δ is the photon propagator.

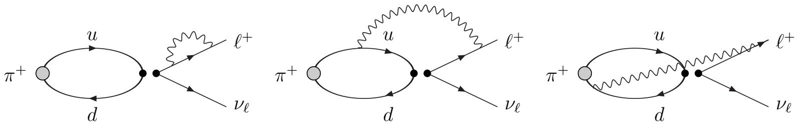
- Combining C_1 with the lowest order correlator:

$$C_0(t) + C_1(t) \simeq \frac{e^{-m_\pi t}}{2m_\pi} Z^\phi\, \langle 0|J_W^\nu(0)|\pi^+\rangle\,,$$

where now $O(\alpha)$ terms are included.

- $e^{-m_\pi t} \simeq e^{-m_\pi^0 t}(1-\delta m_\pi t)$ and Z^ϕ is obtained from the two-point function.

The procedure (cont.)



(d) (e) (f)

- Diagrams (e) and (f) are not simply generalisations of the evaluation of f_π. The leptonic part is treated using perturbative propagators.
  (There are also disconnected diagrams to be evaluated.)
- We have to be able to isolate the finite-volume ground state (pion).
- The Minkowski $\leftrightarrow$ Euclidean continuation can be performed (the time integrations are convergent).
- Finite volume effects, expected to be $O(1/(L\Lambda_{\rm QCD})^n)$, being investigated.
- The next step will be to start implementing this procedure.
- As we learn how to do such calculations it will be useful to consider simpler quantities such as $\Gamma(\pi\to\mu\nu_\mu(\gamma))/\Gamma(\pi\to e\nu_e(\gamma))$.

Conclusions

- Precision flavour physics is a complementary approach to the large $p_\perp$ studies at the LHC in exploring the limits of the standard model.
- The hugely improved precision of Lattice QCD simulations is making this approach truly viable.
- In addition to the improved precision in the evaluation of "standard" quantities, it is important to continue extending the range of physical quantities which can be studied.
- There is a huge amount of work to be done!