Quantum Simulation of Abelian and non-Abelian Gauge Theories

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UNIVERSITÄT BERN
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

Summer School: Symmetries, Fundamental Interactions and Cosmology
Chiemsee, September 2014

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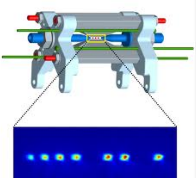
erc European Research Council



Outline

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The first “digital computer” in Babylonia about 2400 b.c.

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The first "analog computer": Antikythera for determining the position of celestial bodies, Crete, about 100 b.c.

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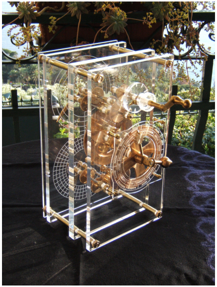
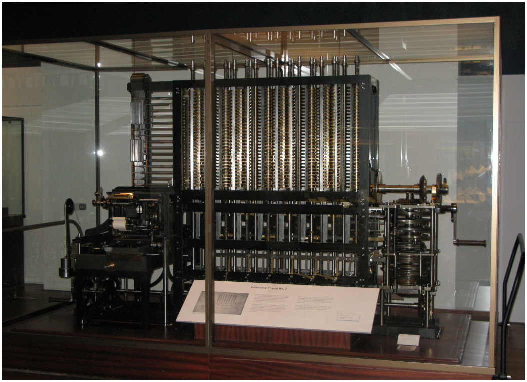
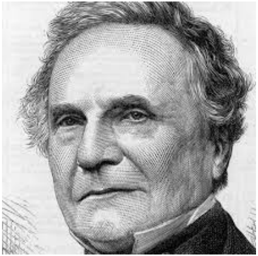


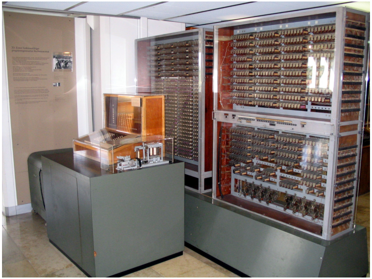
The first programmable computer:
mechanical "difference engine"
Charles Babbage (1791-1871)





was realized by his son after Babagge's death.

Konrad Zuse's (1910-1992) relay-driven computer Z3



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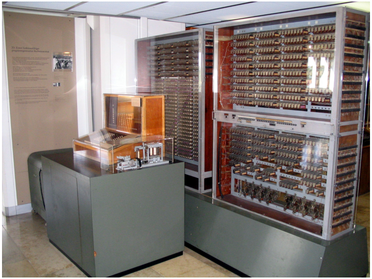
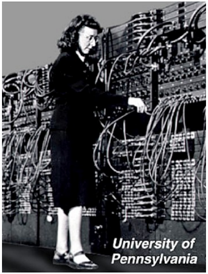


From the vacuum-tube ENIAC to the IBM Blue Gene



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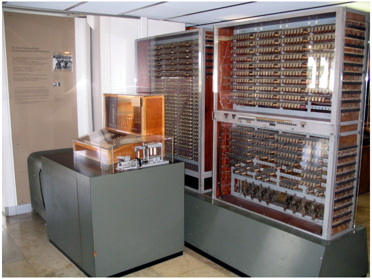

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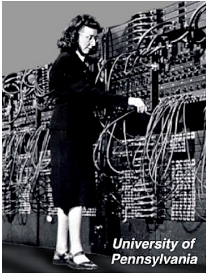
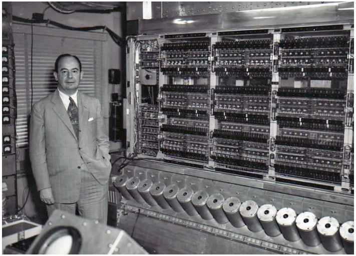
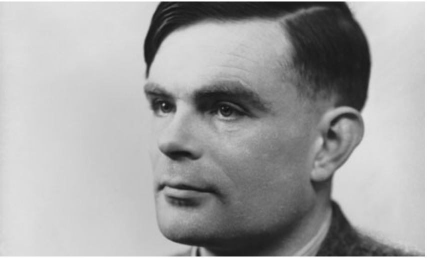


Pioneers of theoretical computer science: John von Neumann (1903-1992) and Alan Turing (1912-1954)





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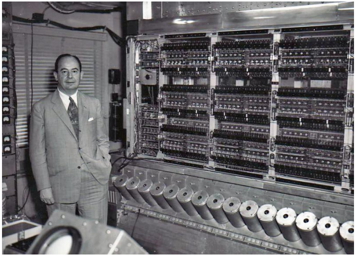
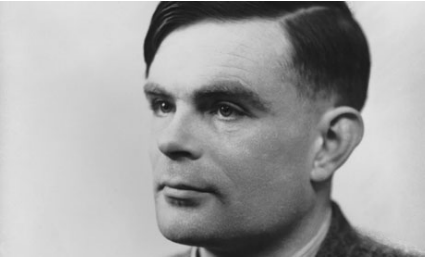
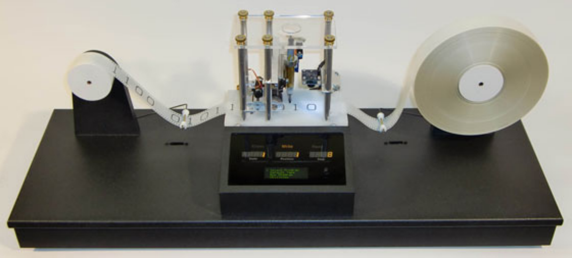




Model of a universal Turing machine



RSA encryption: multiplication is easy, factorization is hard.

RSA decryption challenge in 1991:
factorize the following 174-digit number with 576 bits

$$
\begin{aligned}
RSA576 \quad = \quad & 188198812920607963838697239461650439807163563 \\
& 379417382700763356422988859715234665485319060 \\
& 606504743045317388011303396716199692321205 \\
& 734031879550656996221305168759307650257059
\end{aligned}
$$

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factorize the following 174-digit number with 576 bits

$$
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RSA576 &= 18819881292060796383869723946165043980716356 \\
& 33794173827007633564229888597152346654853190 \\
& 60606504743045317388011303396716199692321205 \\
& 734031879550656996221305168759307650257059 \\
&= 39807508642406493739712550055038649119906436 \\
& 2342526708406385189575946388957261768583317 \\
&* \; 47277214610743530253622307197304822463291469 \\
& 5302097116459852171130520711256363590397527
\end{aligned}
$$

This problem was solved only in 2003 by two mathematicians in Bonn using very large computer resources.

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= \quad & 398075086424064937397125500550386491199064362342526708406385189575946388957261768583317 \\
* \quad & 472772146107435302536223071973048224632914695302097116459852171130520711256363590397527
\end{aligned}
$$

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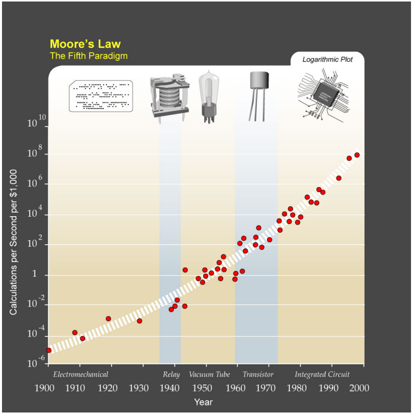
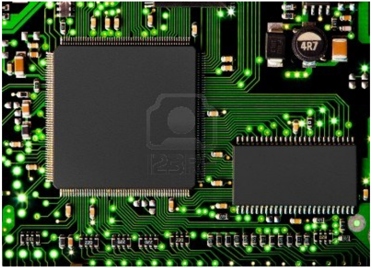
Only in 2009, when the challenge was no longer active, the 232-digit number RSA768 with 768 bits has finally been factorized.

Moore's law: "Every two years the number of transistors per area increases by a factor of 2."





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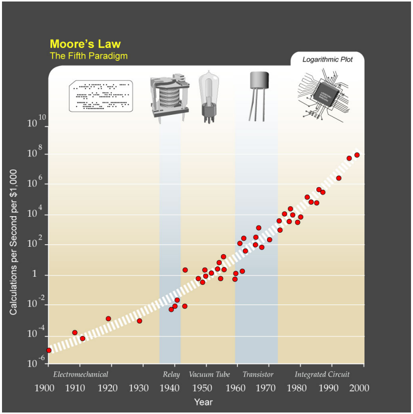

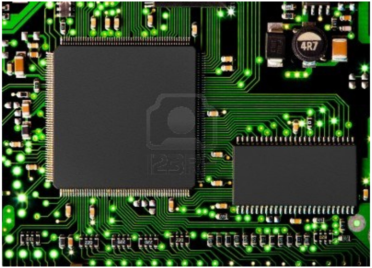

Modern micro chips consist of several billions of transistors, each about 10^{-8} m in size. This is already close to the quantum mechanical limit set by the size of individual atoms.

From bits to qubits

$$|\Psi\rangle = a|1\rangle + b|0\rangle, \quad |a|^2 + |b|^2 = 1$$

Entangled state of two qubits

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$$

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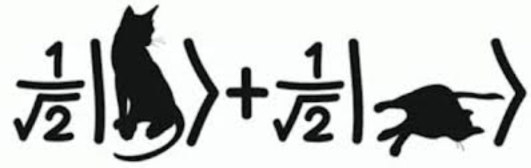

Outline

Richard Feynman's vision of 1982

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

A universal quantum computer (David Deutsch's quantum analog of a classical Turing machine) could use Peter Shor's algorithm to solve the factorization problem.

David Deutsch

Peter Shor

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David Deutsch

Peter Shor

Does the NSA have a quantum computer?

Is the D-wave machine a quantum computer?

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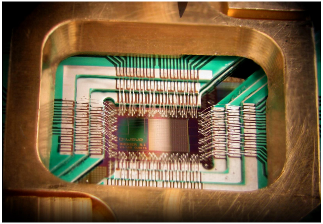
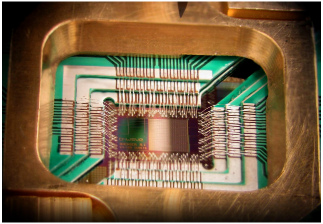


Tests have confirmed certain quantum mechanical features, but the machine is clearly inferior to classical computers.

Is the D-wave machine a quantum computer?



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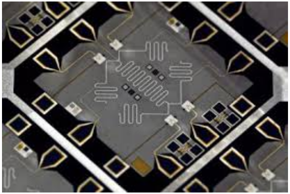
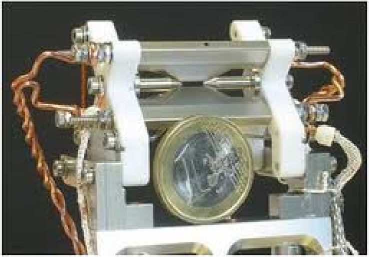
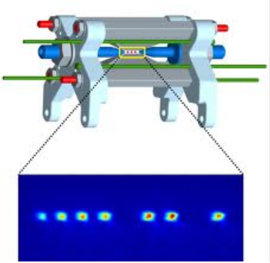
Until today, only $15 = 3 \cdot 5$ has been correctly factorized by a quantum computer, at least in about 50 % of all trials.



Ion traps as a digital quantum computer?





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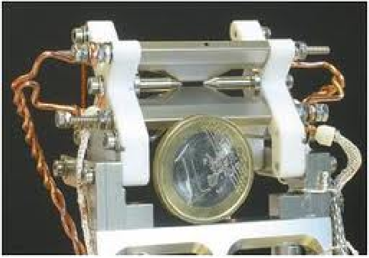
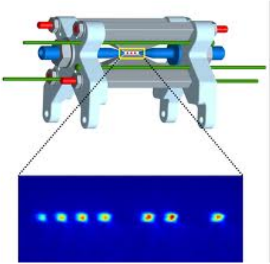
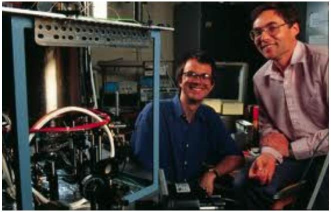


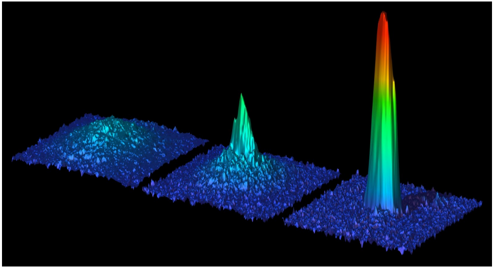


Franklin Medal 2010: I. Cirac, D. Wineland, P. Zoller

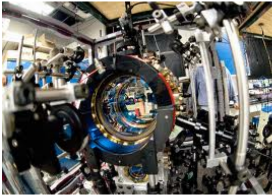
Bose-Einstein condensation in ultra-cold atomic gases

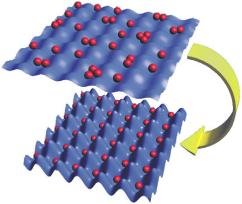
 

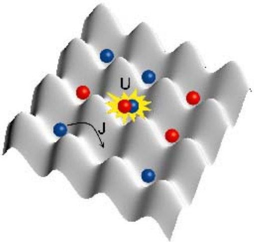
Eric Cornell, Carl Wieman, Wolfgang Ketterle, 1995



Ultra-cold atoms in optical lattices as analog quantum simulators







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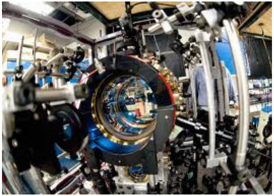
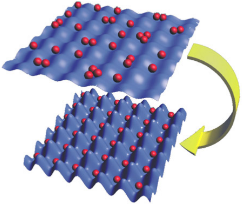
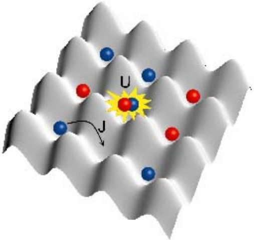
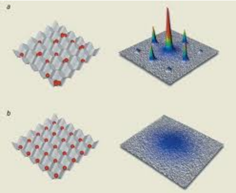


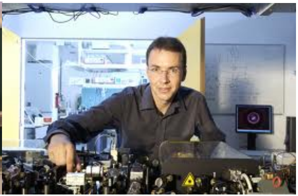




Transition from a superfluid to a Mott insulator

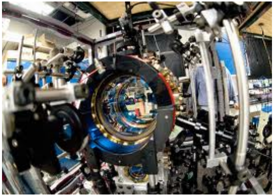


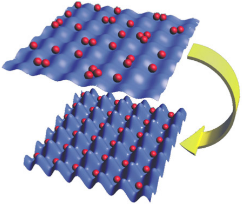
Theodor Hänsch

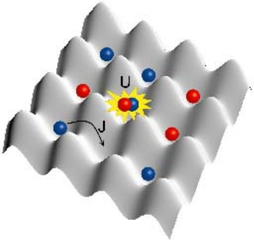


Immanuel Bloch

Ultra-cold atoms in optical lattices as analog quantum simulators







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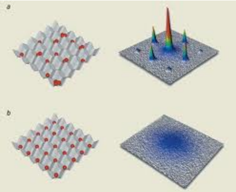
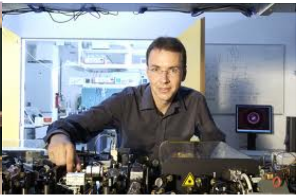


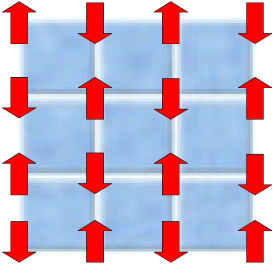


Theodor Hänsch Immanuel Bloch

Can one understand high-T_c superconductivity in this way?

Outline

The spin $\frac{1}{2}$ quantum Heisenberg model



Quantum spins $[S_x^a, S_y^b] = i\delta_{xy}\varepsilon_{abc}S_x^c$ and their Hamiltonian

$$H = J\sum_{\langle xy\rangle} \vec{S}_x \cdot \vec{S}_y$$

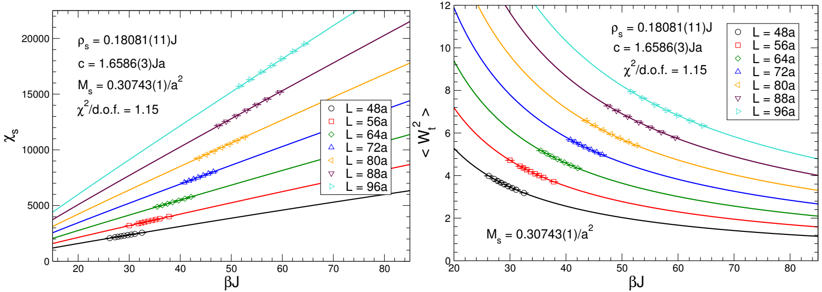
Partition function at inverse temperature $\beta = 1/T$

$$Z = \mathrm{Tr}\exp(-\beta H)$$

Low-energy effective action for antiferromagnetic magnons

$$S[\vec{e}] = \int_0^\beta dt \int d^2x \, \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right)$$

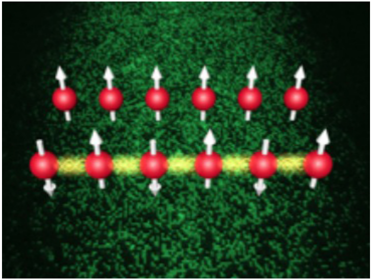
Fit to analytic predictions of effective theory

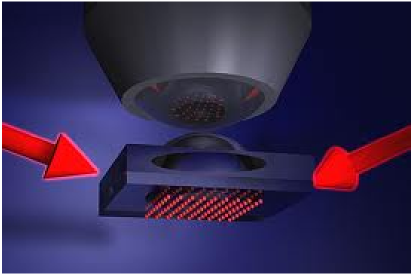


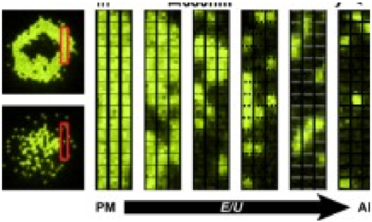
$$\mathcal{M}_s = 0.30743(1), \quad \rho_s = 0.18081(11)J, \quad c = 1.6586(3)Ja$$

UJW, H.-P. Ying (1994); F.-J. Jiang, UJW (2010)

Optical lattice quantum simulation of quantum spin systems







J. Simon, W. S. Bakir, R. Ma, M. E. Tai, P. M. Preis, M. Greiner, Nature 472 (2011) 307.

Homework 1:

Show that the Heisenberg Hamiltonian H commutes with the total spin $\vec{S}$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y, \quad \vec{S} = \sum_x \vec{S}_x.$$

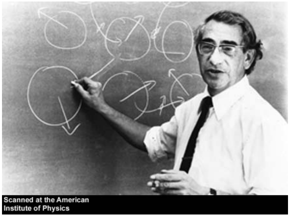
Some important lessons from lecture 1:

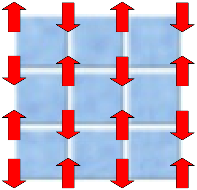
• Quantum computers or quantum simulators are potentially much more powerful than classical computers.

• The Heisenberg quantum spin model in thermal equilibrium can be simulated very efficiently using classical computers.

• The collective dynamics of discrete quantum spin degrees of freedom can give rise to an emergent quantum field theory for the low-energy spin wave Goldstone boson excitations.

The Hubbard Model for doped antiferromagnets





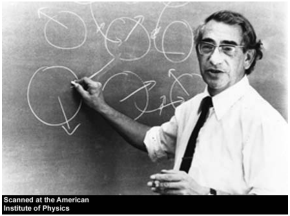
$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

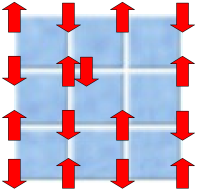
reduces to the Heisenberg model at half-filling for $U \gg t$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





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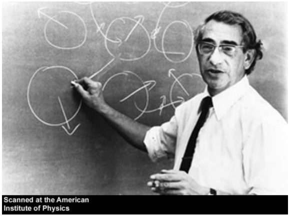

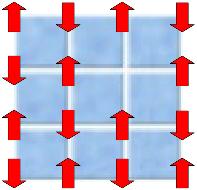

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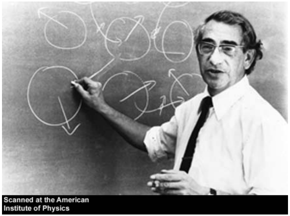
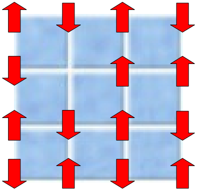
$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





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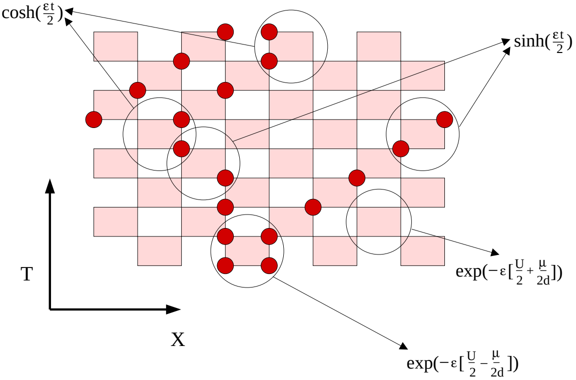
Does the Hubbard model explain high-T_c superconductivity?

Path integral

$$
\begin{aligned}
Z_f &= \mathrm{Tr}[\exp(-\varepsilon H_1)\exp(-\varepsilon H_2)\ldots\exp(-\varepsilon H_M)]^N \\
&= \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])
\end{aligned}
$$



Sign problem of fermionic path integrals

$$Z_f = \mathrm{Tr}\exp(-\beta H) = \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n]) \ , \quad \mathrm{Sign}[n] = \pm 1$$

Average sign is exponentially small

$$\langle \mathrm{Sign} \rangle = \frac{\sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])}{\sum_{[n]} \exp(-S[n])} = \frac{Z_f}{Z_b} = \exp(-\beta V \Delta f)$$

The statistical error is exponentially large

$$\frac{\sigma_{\mathrm{Sign}}}{\langle \mathrm{Sign} \rangle} = \frac{\sqrt{\langle \mathrm{Sign}^2 \rangle - \langle \mathrm{Sign} \rangle^2}}{\sqrt{N}\langle \mathrm{Sign} \rangle} = \frac{\exp(\beta V \Delta f)}{\sqrt{N}} \ .$$

Some very hard sign problems are NP complete

M. Troyer, UJW, Phys. Rev. Lett. 94 (2005) 170201.

Homework 2:

Show that the anti-commutation relations $\{c^\dagger_{x,s}, c_{y,s'}\} = \delta_{xy}\delta_{ss'}$ of fermionic creation and annihilation operators imply angular momentum commutation relations

$$[S^a_x, S^b_y] = i\delta_{xy}\varepsilon_{abc}S^c_x, \quad \vec{S}_x = \sum_x c^\dagger_x \frac{\vec{\sigma}}{2} c_x, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}.$$

Homework 3:

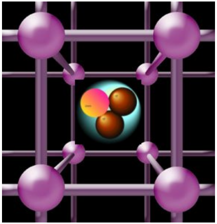
Show that the Hubbard Hamiltonian H commutes with the total spin $\vec{S}$ and with the particle number N

$$H = -t \sum_{\langle xy \rangle} (c^\dagger_x c_y + c^\dagger_y c_x) + U \sum_x (c^\dagger_x c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

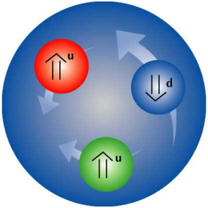
$$\vec{S} = \sum_x \vec{S}_x = \sum_x c^\dagger_x \frac{\vec{\sigma}}{2} c_x, \quad N = \sum_x n_x = \sum_x c^\dagger_x c_x.$$

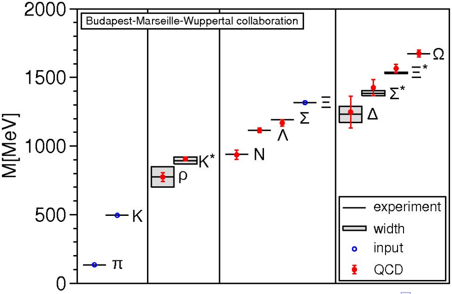
Outline

Kenneth Wilson's lattice QCD describes confinement of quarks and gluons inside protons und neutrons

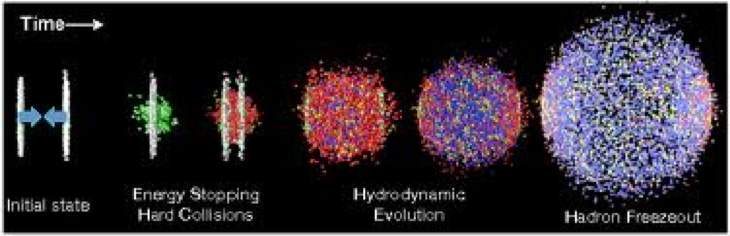


and confirms the experimentally measured mass spectrum





Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?



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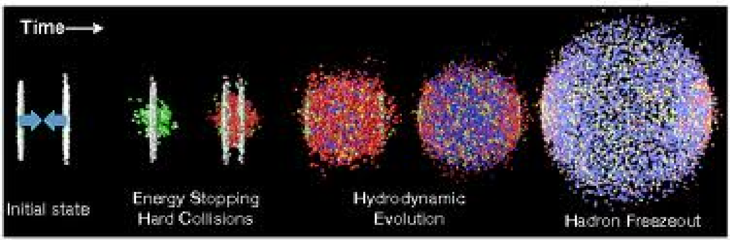

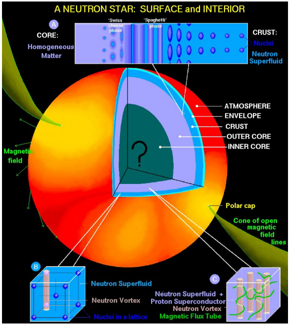

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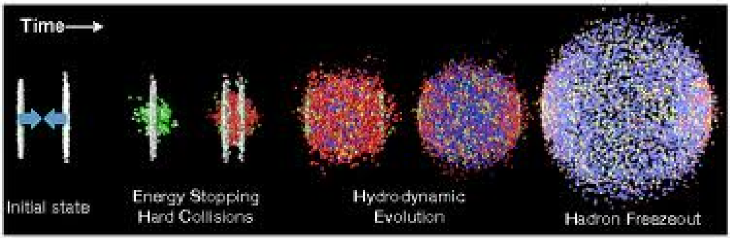

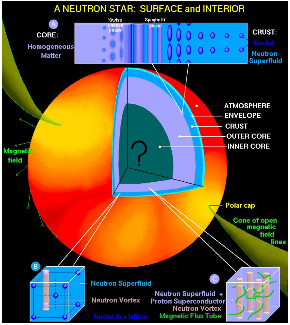

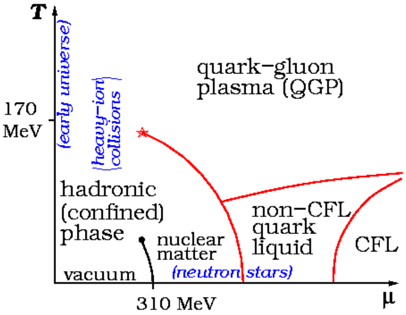

Outline

Different descriptions of dynamical Abelian gauge fields:

Maxwell's classical electromagnetic gauge fields

$$\vec{\nabla} \cdot \vec{E}(\vec{x}, t) = \rho(\vec{x}, t), \quad \vec{\nabla} \cdot \vec{B}(\vec{x}, t) = 0, \quad \vec{B}(\vec{x}, t) = \vec{\nabla} \times \vec{A}(\vec{x}, t)$$

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Quantum Electrodynamics (QED) for perturbative treatment

$$E_i(\vec{x}, t) = -i \frac{\partial}{\partial A_i(\vec{x}, t)}, \quad [E_i, A_j] = i\delta_{ij}, \quad \left[\vec{\nabla} \cdot \vec{E} - \rho\right] |\Psi[A]\rangle = 0$$

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Wilson's $U(1)$ lattice gauge theory for classical simulation

$$U_{xy} = \exp\left(ie \int_x^y d\vec{l} \cdot \vec{A}\right) = \exp(i\varphi_{xy}) \in U(1), \quad E_{xy} = -i \frac{\partial}{\partial \varphi_{xy}},$$

$$[E_{xy}, U_{xy}] = U_{xy}, \quad \left[\sum_i (E_{x,x+\hat{i}} - E_{x-\hat{i},x}) - \rho\right] |\Psi[U]\rangle = 0$$

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$$[E_{xy}, U_{xy}] = U_{xy}, \quad \left[\sum_i (E_{x,x+\hat{i}} - E_{x-\hat{i},x}) - \rho\right] |\Psi[U]\rangle = 0$$

$U(1)$ quantum link models for quantum simulation

$$U_{xy} = S_{xy}^+, \quad U_{xy}^\dagger = S_{xy}^-, \quad E_{xy} = S_{xy}^3,$$

$$[E_{xy}, U_{xy}] = U_{xy}, \quad [E_{xy}, U_{xy}^\dagger] = -U_{xy}^\dagger, \quad [U_{xy}, U_{xy}^\dagger] = 2E_{xy}^\dagger$$

Hamiltonian formulation of Wilson's $U(1)$ lattice gauge theory

$$U = \exp(i\varphi), \ U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

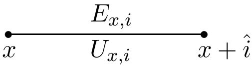
$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link

$U(1)$ quantum links from spins $\frac{1}{2}$

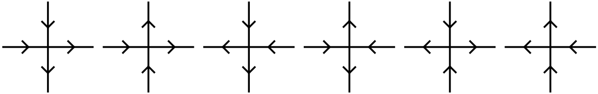


$$U = S_1 + iS_2 = S_+, \ \ U^\dagger = S_1 - iS_2 = S_-$$

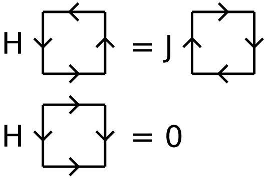
Electric flux operator E

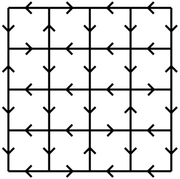
$$E = S_3, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 2E$$

Gauss law



Ring-exchange plaquette Hamiltonian



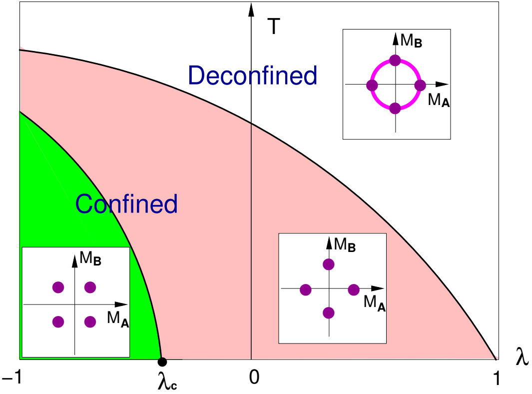


D. Horn, Phys. Lett. B100 (1981) 149
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Hamiltonian with Rokhsar-Kivelson term

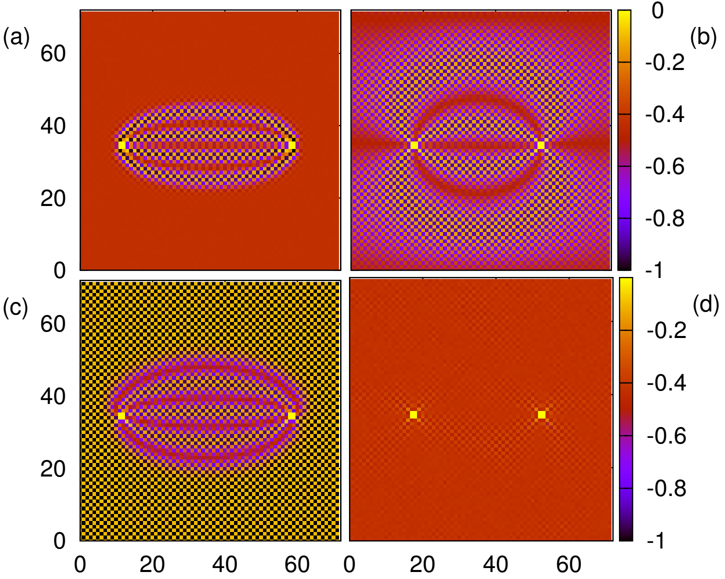
$$H = -J\left[\sum_{\square}(U_{\square} + U_{\square}^{\dagger}) - \lambda\sum_{\square}(U_{\square} + U_{\square}^{\dagger})^2\right]$$

Phase diagram



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

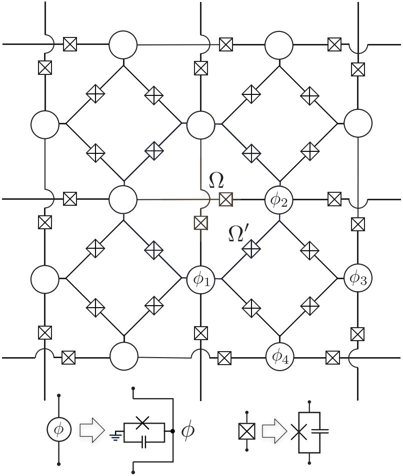
Energy density of charge-anti-charge pair $Q = \pm 2$



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

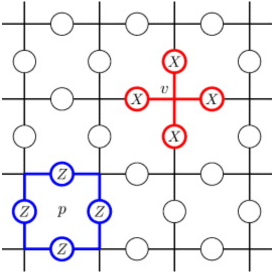
Outline

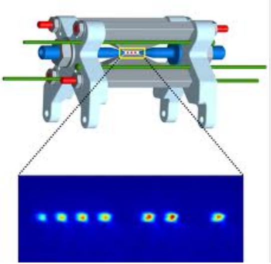
“String theory on a chip” with superconducting circuits



D. Marcos, P. Rabl, E. Rico, P. Zoller,
Phys. Rev. Lett. 111 (2013) 110504 (2013).
D. Marcos, P. Widmer, E. Rico, M. Hafezi, P. Rabl, UJW, P. Zoller,
arXiv:1407.6066.

Digital quantum simulation of Kitaev's toric code (a $\mathbb{Z}(2)$ quantum link model) with trapped ions





• Precisely controllable many-body quantum device, which can execute a prescribed sequence of quantum gate operations.

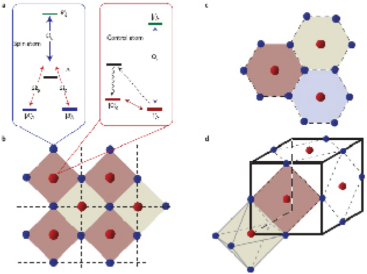
• State of simulated system encoded as quantum information.

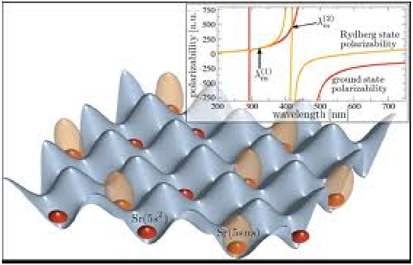
• Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

A. Y. Kitaev, Ann. Phys. 303 (2003) 2.

B. P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J. T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, C. F. Roos, Science 334 (2011) 6052.

$U(1)$ quantum link models can also be simulated with Rydberg atoms in an optical lattice





- Lasers can excite atoms to high-lying Rydberg states.
- Rydberg atoms are large and have collective interactions.
- Ensemble Rydberg atoms represent qubits at link centers.
- Control atoms at lattice sites ensure the Gauss' law.

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502.
H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H. P. Büchler, Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

$$U_{x,x+1} = b_x b_{x+1}^\dagger, \;\; E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

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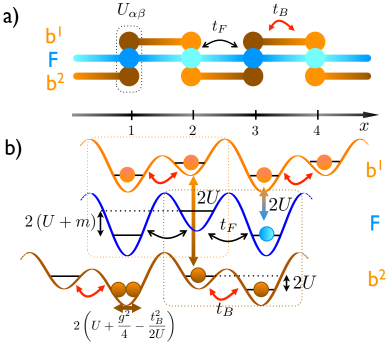
Optical lattice with Bose-Fermi mixture of ultra-cold atoms



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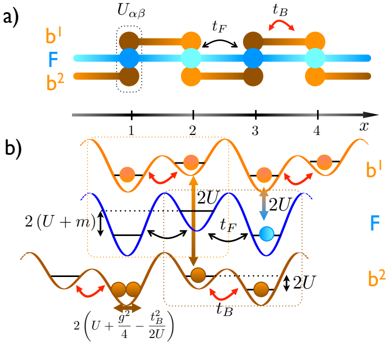
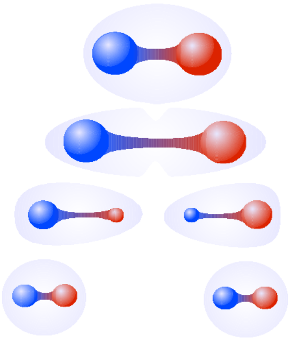
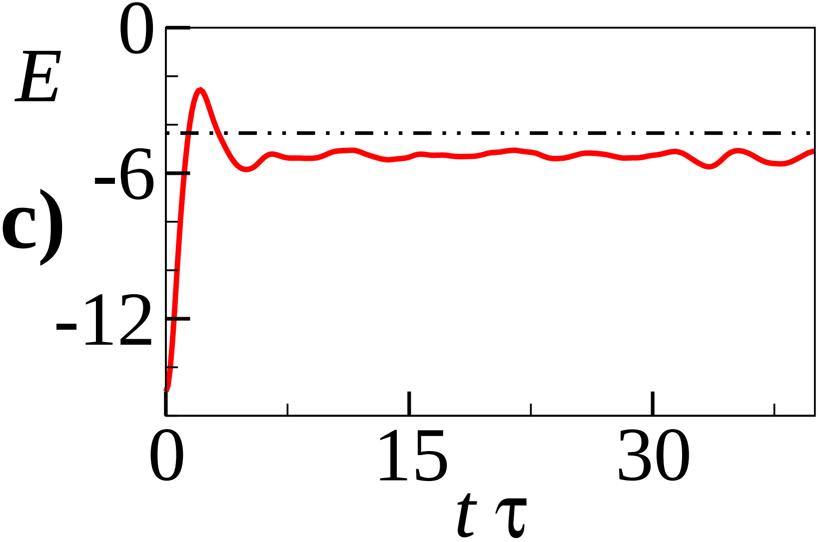




D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

Quantum simulation of the real-time evolution of string breaking



Homework 4:

Show that the Hamiltonian H of the 2-d $U(1)$ quantum link model commutes with the local generators of gauge transformations $G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i})$

$$H = -\frac{1}{2g^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.}).$$

Homework 5:

Show that the $SU(2)$ embedding algebra of the $U(1)$ quantum link model can be realized with bosonic rishons

$$U_{xy} = b_x b_y^\dagger, \ E_{xy} = \frac{1}{2}(b_y^\dagger b_y - b_x^\dagger b_x)$$

Some important lessons from lecture 2:

- Gauge theories with exact continuous gauge symmetry can be formulated in terms of discrete quantum link degrees of freedom.
- Thanks to their finite-dimensional Hilbert space per link, quantum links can be embodied by ultra-cold fermionic or bosonic atoms in an optical lattice or by superconducting flux cubits.

Outline

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \; U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \; i,j \in \{1,2,\dots,N\}, \; [U^{ij},(U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c, \; [R^a, R^b] = if_{abc}R^c, \; a,b,c \in \{1,2,\dots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

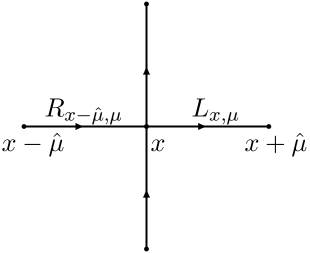
Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \; [R^a, U] = U\lambda^a, \; [E, U] = U$$

Algebraic structures of $U(N)$ quantum link models

$U^{ij}, \; L^a, \; R^a, E, \; 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502



Generator of $SU(N)$ gauge transformations

$$G_x^a = \sum_\mu (R^a_{x-\hat{\mu},\mu} + L^a_{x,\mu})$$

$U(N)$-invariant Hamiltonian "action" operator

$$H = -J \sum_{x,\mu<\nu} \mathrm{Tr}(U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu} + \text{h.c.}), \ [H, G_x^a] = 0$$

Functional integral of a quantum link model

$$Z = \mathrm{Tr} \exp(-\beta H)$$

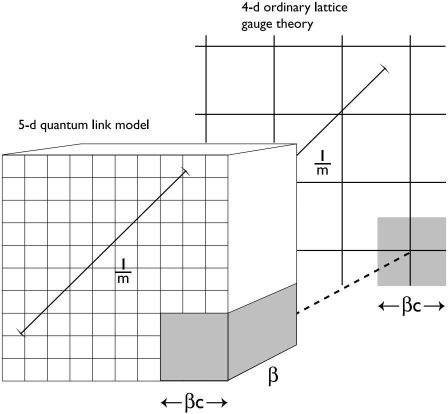
defines a quantum field theory using discrete variables

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\ \partial_5 G_\mu \partial_5 G_\mu \right), \ G_5 = 0$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

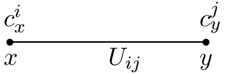
$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu}, \ \frac{1}{g^2} = \frac{\beta}{e^2}, \ \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$



Fermionic rishons at the two ends of a link

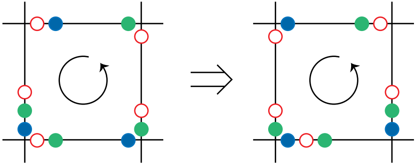
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

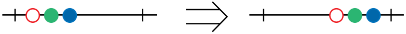


$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a "rishon abacus" implemented with ultra-cold atoms be used as a quantum simulator?



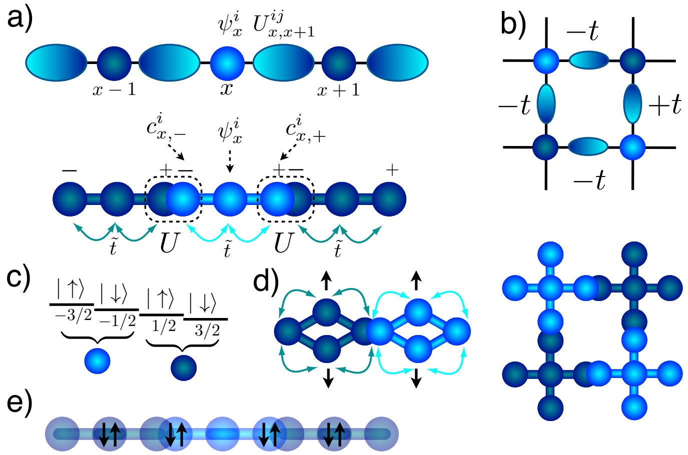
Tr Up



det $U_{x,\mu}$

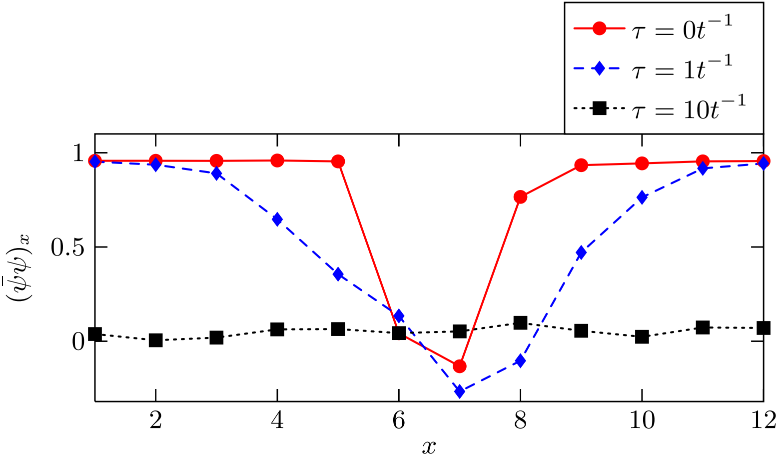
Outline

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin

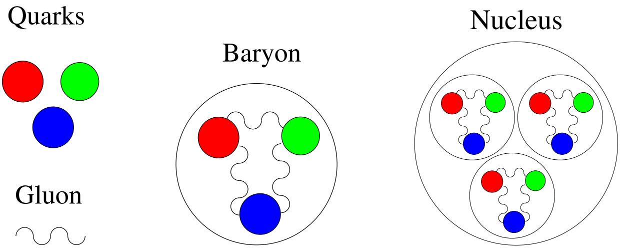


D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

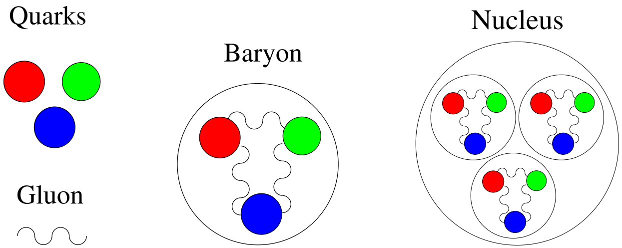
Expansion of a “fireball” mimicking a hot quark-gluon plasma



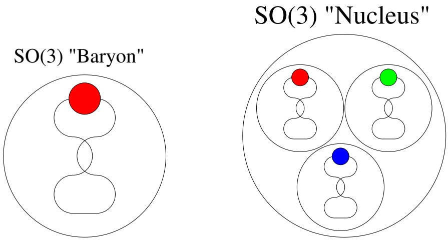
Nuclear Physics from $SU(3)$ QCD



Nuclear Physics from $SU(3)$ QCD



"Nuclear Physics" in an $SO(3)$ lattice gauge theory?

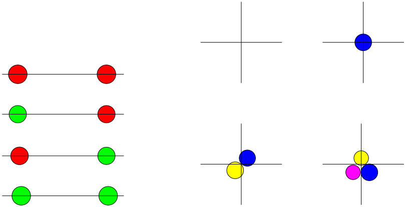


1-d $SO(3)$ quantum link model with adjoint triplet-fermions

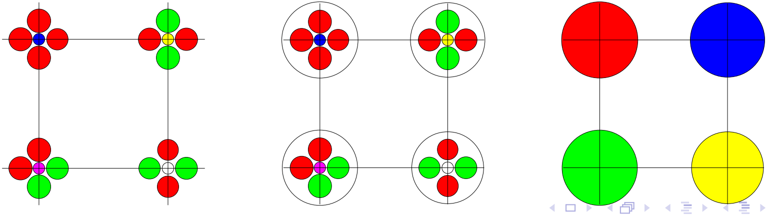
$$H = -t \sum_x \left[\psi_x^{i\dagger} O_{x,x+1}^{ij} \psi_{x+1}^j + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i$$

$SO(3)$ quantum links

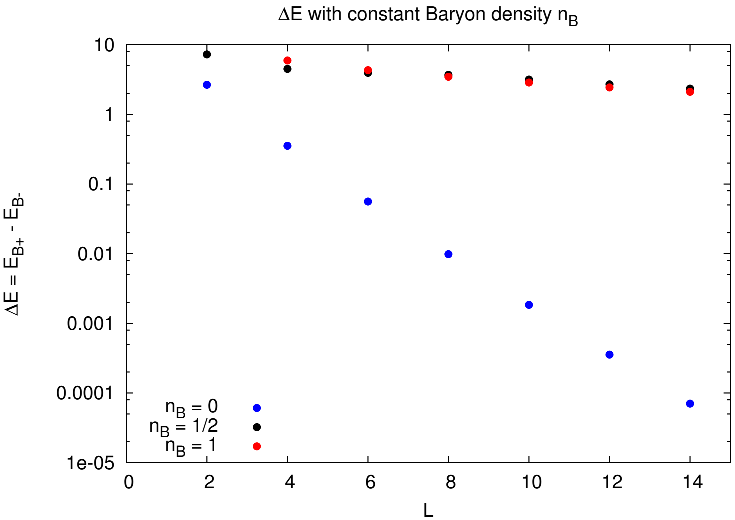
$$O_{x,x+1}^{ij} = \sigma_{x,L}^i \sigma_{x+1,R}^j$$



Encoding manifestly gauge invariant states obeying Gauss' law

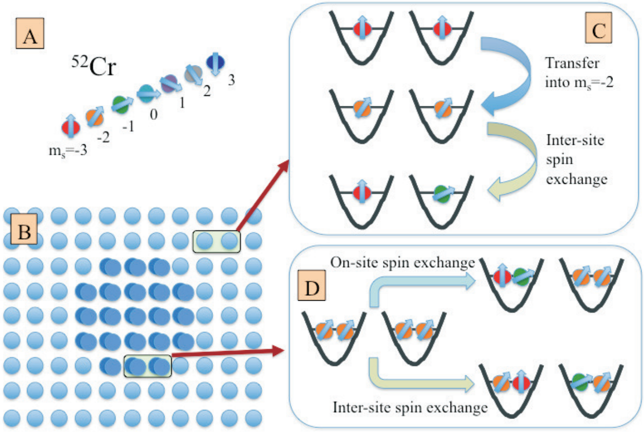


Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



M. Dalmonte, E. Rico, D. Banerjee, M. Bögli, P. Stebler, UJW, P. Zoller, in preparation

Implementation with magnetic atoms (e.g. Cr), whose dipolar interactions allow spin-spin interactions without superexchange



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Analog quantum simulator proposals

H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller, Phys. Rev. Lett. 95 (2005) 040402.

E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.

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Digital quantum simulator proposals

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502; Nat. Phys. 6 (2010) 382.

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L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

Review on quantum simulators for lattice gauge theories

UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

Homework 6:

Show that the $SU(2N)$ embedding algebra of the $U(N)$ quantum link model can be realized with fermionic rishons

$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \; L_{xy}^a = c_x^{i\dagger} \lambda_{ij}^a c_x^j, \; R_{xy}^a = c_y^{i\dagger} \lambda_{ij}^a c_y^j, \; E_{xy} = \frac{1}{2}(c_y^{i\dagger} c_y^i - c_x^{i\dagger} c_x^i)$$

Some important lessons from lecture 3:

- Non-Abelian quantum link models are formulated in terms of discrete quantum links, but have an exact $SU(N)$ gauge symmetry.
- Continuous gluon fields emerge as low-energy collective excitations of the discrete quantum link variables, just as magnetic spin wave fields emerge from discrete quantum spins.
- 4-d QCD emerges by dimensional reduction from a $(4+1)$-d $SU(3)$ quantum link model, with quarks as domain wall fermions.
- Quantum links have fermionic "rishon" constituents which can be embodied by alkaline-earth atoms.

Outline

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- Accessible effects may include chiral symmetry restoration, baryon superfluidity, or color superconductivity at high baryon density, as well as the quantum simulation of "nuclear" collisions.
- The path towards quantum simulation of QCD will be a long one.

However, with a lot of interesting physics along the way.