Effective Field Theory

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Outline

hep-ph/9806303

1) General Aspects of Effective Field Theory

- Dimensional Analysis
- Relevant, Irrelevant and Marginal
- Quantum Loops
- Decoupling. Matching. Scaling

2) Chiral Perturbation Theory

- Goldstone Theorem
- Chiral Symmetry
- Effective Goldstone Theory
- Chiral Symmetry Breaking

3) Electroweak Effective Theory

- Higgs Mechanism
- Custodial Symmetry
- Equivalence Theorem
- Electroweak Effective Theory

Euler-Heisenberg Lagrangian

Light-by-light scattering in QED at very low energies $(\mathbf{E_\gamma \ll m_e})$

- Gauge, Lorentz, Charge Conjugation & Parity constraints
- Energy expansion (E_γ / m_e)

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{m_e^4} (F^{\mu\nu} F_{\mu\nu})^2 + \frac{b}{m_e^4} F^{\mu\nu} F_{\nu\sigma} F^{\sigma\rho} F_{\rho\mu} + \mathcal{O}(F^6/m_e^8)$$

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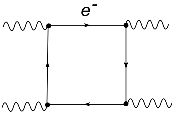

$$\longrightarrow \qquad a = -\frac{1}{36}\,\alpha^2 \quad , \quad b = \frac{7}{90}\,\alpha^2$$

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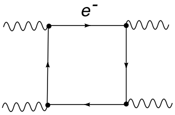

$$\Rightarrow \quad a = -\frac{1}{36}\alpha^2 \quad , \quad b = \frac{7}{90}\alpha^2$$

$$\sigma(\gamma\gamma \to \gamma\gamma) \propto \frac{\alpha^4 E^6}{m_e^8}$$

Why the sky looks blue?

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Rayleigh scattering

Low-energy scattering of photons with neutral atoms

$$E_\gamma \;\ll\; \Delta E \sim \alpha^2 m_e \;\ll\; a_0^{-1} \sim \alpha m_e \;\ll\; M_A$$

- Neutral atom + gauge invariance ➡ $F^{\mu\nu} = (\vec{E}, \vec{B})$
- Non-relativistic description

$$\mathcal{L}_{\rm int} \;=\; a_0^3\;\psi^\dagger\psi\;\left(c_1\vec{E}^2 + c_2\vec{B}^2\right) + \ldots \qquad , \qquad c_i \sim \mathcal{O}(1)$$

$$\mathcal{M} \;\sim\; c_i\, a_0^3\, E_\gamma^2 \qquad \longrightarrow \qquad \boldsymbol{\sigma} \;\propto\; \mathbf{a_0^6\, E_\gamma^4}$$

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Blue light is scattered more strongly than red one

Dimensions

$$S = \int d^4x \, \mathcal{L}(x) \quad \longrightarrow \quad [\mathcal{L}] = E^4$$

$$\mathcal{L}_{\mathrm{KG}} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \, \phi^\dagger \phi \quad \longrightarrow \quad [\phi] = [V^\mu] = [A^\mu] = E$$

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi} \, (i\gamma^\mu \partial_\mu - m) \, \psi \quad \longrightarrow \quad [\psi] = E^{3/2}$$

$$[\sigma] = E^{-2} \qquad , \qquad [\Gamma] = E$$

Scalar Field Theory

- $\mathcal{L}_{\mathbf{I}} = -\frac{\lambda}{3!}\,\phi^3 \quad \Longrightarrow \quad [\lambda] = E$

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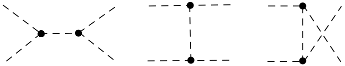

$$\sigma(1+2\to 3+4) \;\sim\; \frac{\lambda^4}{s^3}\,\left\{1+\mathcal{O}\left(\frac{\lambda^2}{s}\right)+\cdots\right\} \qquad (s\gg m^2)$$

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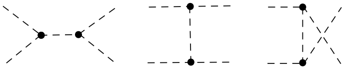

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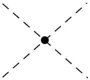

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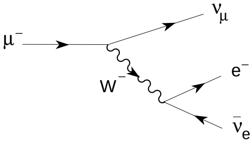
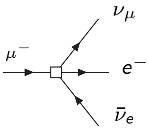
Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\, M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \quad \xrightarrow{q^2 \ll M_W^2} \quad \frac{g_{\mu\nu}}{M_W^2}$$



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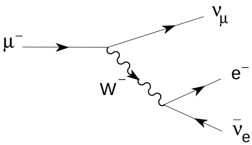

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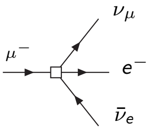

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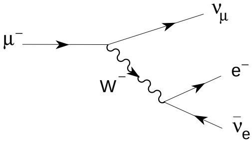

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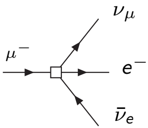

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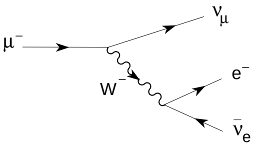

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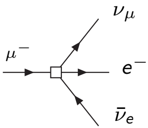

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$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$

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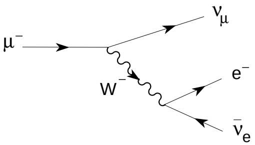

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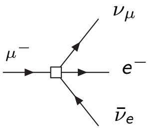

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Exp: $(17.83 \pm 0.04)\%$

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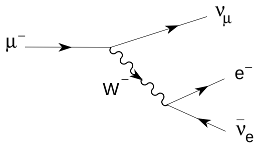

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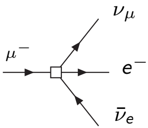

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- $\sigma(\nu_\mu e^- \to \mu^- \nu_e) \sim G_F^2\, s$

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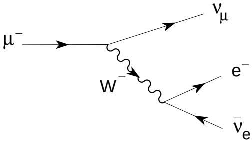

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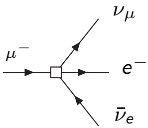

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- $\sigma(\nu_\mu e^- \to \mu^- \nu_e) \sim G_F^2\, s$ $\qquad$ **Violates unitarity at high energies**

Relevant, Irrelevant & Marginal

$$\mathcal{L} = \sum_i c_i\, O_i \qquad , \qquad [O_i] = d_i \quad \longrightarrow \quad c_i \sim \frac{1}{\Lambda^{d_i-4}}$$

Low-energy behaviour:

- **Relevant** **($d_i < 4$):** $\quad I\,,\ \phi^2\,,\ \phi^3\,,\ \bar\psi\psi$

  Enhanced by $(\Lambda/E)^{4-d_i}$

- **Marginal** **($d_i = 4$):** $\quad m^2\phi^2\,,\ m\,\bar\psi\psi\,,\ \phi^4\,,\ \phi\,\bar\psi\psi\,,\ V_\mu\,\bar\psi\gamma^\mu\psi$

- **Irrelevant** **($d_i > 4$):** $\quad \bar\psi\psi\,\bar\psi\psi\,,\ \partial_\mu\phi\,\bar\psi\gamma^\mu\psi\,,\ \phi^2\,\bar\psi\psi\,,\ \cdots$

  Suppressed by $(E/\Lambda)^{d_i-4}$

$$\alpha(Q^2) \; = \; \frac{\alpha(Q_0^2)}{1 - \beta_1 \, \frac{\alpha(Q_0^2)}{2\pi} \, \log\left(Q^2/Q_0^2\right)}$$

QED: $\beta_1 \; = \; \frac{2}{3} \sum_f Q_f^2 \, N_f \; > \; 0$ ➡ $\lim_{Q^2 \to 0} \alpha(Q^2) \; = \; 0$

Quantum corrections make QED irrelevant at low energies

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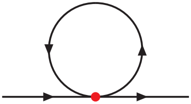
QCD: $\beta_1 = \frac{2\, N_F \, - \, 11\, N_C}{6} \; < \; 0$ ➡ $\lim_{Q^2\to 0} \alpha_s(Q^2) \; = \; \infty$

Quantum corrections make QCD relevant at low energies

Quantum Loops

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi - \frac{a}{\Lambda^2}\,(\bar{\psi}\psi)^2 - \frac{b}{\Lambda^4}\,(\bar{\psi}\,\Box\psi)(\bar{\psi}\psi) + \cdots$$



$$\delta m \sim 2i\,\frac{a}{\Lambda^2}\,m\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2}$$

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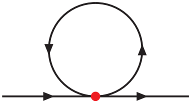

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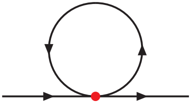

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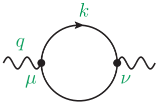
- **Dimensional regularization:** $\Delta_\infty = \frac{2\,\mu^{D-4}}{D-4} + \gamma_E - \log{(4\pi)}$

$$\delta m \;\sim\; 2a\,m\,\frac{m^2}{16\pi^2\Lambda^2}\,\left\{\Delta_\infty + \log\left(\frac{m^2}{\mu^2}\right) - 1 + \mathcal{O}(D-4)\right\}$$

Well-defined expansion **(Mass independent)**

Vacuum Polarization $(m_f \neq 0)$



$$i\,\Pi^{\mu\nu}(q) \;=\; i\,\left(-q^2 g^{\mu\nu} + q^\mu q^\nu\right)\,\Pi(q^2)$$

$$\Pi(q^2) \;=\; -\frac{\alpha Q_f^2}{3\pi}\left\{\Delta_\infty + 6\int_0^1 dx\; x(1-x)\,\log\left(\frac{m_f^2 - q^2 x(1-x)}{\mu^2}\right)\right\}$$

$$\equiv\; \Delta\Pi_\epsilon(\mu^2) \;+\; \Pi_R(q^2/\mu^2)$$

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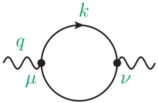

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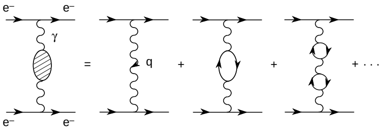
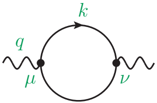


$$\alpha_0\,\left\{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\right\}$$

$$\equiv\;\; \alpha_R(\mu^2)\;\left\{1 - \Pi_R(q^2/\mu^2)\right\}$$

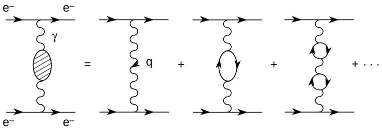
Vacuum Polarization $(m_f \neq 0)$



$$i\,\Pi^{\mu\nu}(q) \;=\; i\,\left(-q^2 g^{\mu\nu} + q^\mu q^\nu\right)\,\Pi(q^2)$$

$$\Pi(q^2) \;=\; -\frac{\alpha Q_f^2}{3\pi}\left\{\Delta_\infty + 6\int_0^1 dx\; x(1-x)\,\log\left(\frac{m_f^2 - q^2 x(1-x)}{\mu^2}\right)\right\}$$

$$\equiv\; \Delta\Pi_\epsilon(\mu^2) \;+\; \Pi_R(q^2/\mu^2)$$



$$\alpha_0\,\left\{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\right\}$$

$$\equiv\; \alpha_R(\mu^2)\;\left\{1 - \Pi_R(q^2/\mu^2)\right\}$$

$$\frac{\mu}{\alpha}\,\frac{d\alpha}{d\mu} \equiv \beta(\alpha) = \beta_1\,\frac{\alpha}{\pi} + \cdots \quad \longrightarrow \quad \alpha(Q^2) \;\approx\; \frac{\alpha(Q_0^2)}{1 - \beta_1\,\frac{\alpha(Q_0^2)}{2\pi}\;\log\left(Q^2/Q_0^2\right)}$$

Mass-Dependent Scheme: $\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2)$

$$\Pi_R(q^2/\mu^2) \; = \; -Q_f^2 \, \frac{\alpha}{3\pi} \, 6 \int_0^1 dx \, x(1-x) \, \log \left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2 + \mu^2 x(1-x)} \right]$$

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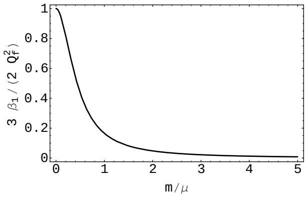

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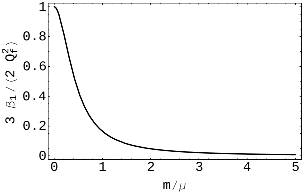

- $\mathbf{m_f^2 \ll \mu^2}$, $\mathbf{q^2}$: $\beta_1 = \frac{2}{3} Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(-q^2/\mu^2)$

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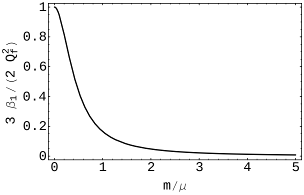

- $\mathbf{m_f^2 \ll \mu^2, q^2}$: $\beta_1 = \frac{2}{3} Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(-q^2/\mu^2)$
- $\mathbf{m_f^2 \gg \mu^2, q^2}$: $\beta_1 \sim \frac{2}{15} Q_f^2 \frac{\mu^2}{m_f^2}$, $\Pi_R(q^2/\mu^2) \sim Q_f^2 \frac{\alpha}{15\pi} \frac{q^2+\mu^2}{m_f^2}$

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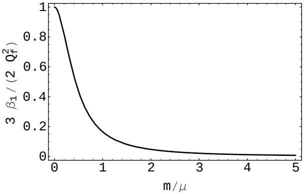

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→ **DECOUPLING** **(Appelquist-Carazzone Theorem)**

$\overline{\text{MS}}$ Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \;\equiv\; -Q_f^2\,\frac{\alpha_0}{3\pi}\,\Delta_\infty$$

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- $\beta_1 = \frac{2}{3}\,Q_f^2$ **Independent of $\mathbf{m_f}$**

Heavy fermions do not decouple

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Perturbation theory breaks down

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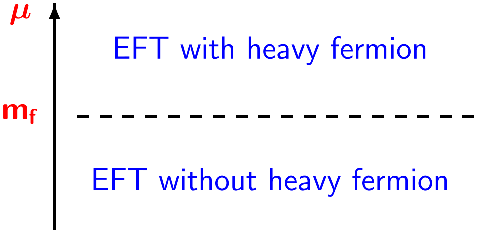
Perturbation theory breaks down

SOLUTION: Integrate Out Heavy Particles

Matching



$$\beta_1 = \frac{2}{3}\, Q_f^2 + \beta_1^{\rm light}$$

$$\beta_1 = \beta_1^{\rm light}$$

- **Two different EFTs (with and without the heavy fermion f)**
- **Same S-matrix elements for light-particle scattering at** $\mu = \mathbf{m_f}$

Effective Field Theory

$$\mathcal{L}(\phi,\Phi) \;=\; \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{\lambda}{2}\phi^2\Phi$$

Effective Field Theory

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$$\sigma(\phi\phi \to \phi\phi) \;\sim\; \frac{1}{E^2} \;\times\; \begin{cases} (\lambda/E)^4 \quad , & \qquad (m \ll M \ll E) \\ (\lambda/M)^4 \quad , & \qquad (m, E \ll M) \end{cases}$$

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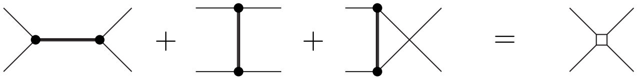

$\mathbf{E, m \ll M}$

$$\frac{\lambda^2}{s-M^2} = -\frac{\lambda^2}{M^2}\sum_{n=0}\frac{s}{M^2} \quad\longrightarrow\quad \mathcal{L}_{\text{eff}}(\phi) \;=\; \sum_i c_i\, O_i(\phi)$$

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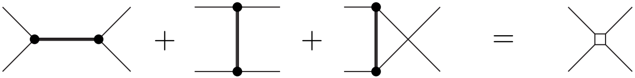

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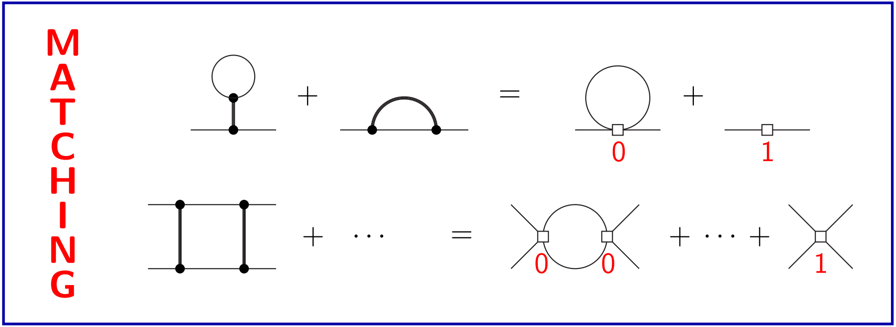
$$[O_i] = d_i \qquad ; \qquad c_i \sim \frac{\lambda^2}{M^2}\,\frac{1}{M^{d_i-4}}$$

One Loop:

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2}\, a\, (\partial\phi)^2 - \frac{1}{2}\, b\, \phi^2 + c\, \frac{\lambda^2}{8M^2}\, \phi^4 + \cdots$$



$$a = 1 + a_1\, \frac{\lambda^2}{16\pi^2 M^2} + \cdots \qquad ; \qquad b = m^2 + b_1\, \frac{\lambda^2}{16\pi^2} + \cdots$$

$$c = 1 + c_1\, \frac{\lambda^2}{16\pi^2 M^2} + \cdots \qquad ; \qquad \cdots$$

Principles of Effective Field Theory

- **Low-energy dynamics** independent of details at high energies
- Appropriate physics description at the analyzed scale **(degrees of freedom)**
- **Energy gaps:** $\quad \mathbf{0 \leftarrow m \ll E \ll M \rightarrow \infty}$
- Non-local heavy-particle exchanges replaced by a **tower of local interactions** among the light particles
- **Accuracy:** $\quad (E/M)^{(d_i-4)} \gtrsim \epsilon \quad \Longleftrightarrow \quad d_i \lesssim 4 + \frac{\log(1/\epsilon)}{\log(M/E)}$
- **Same infrared** (but different ultraviolet) **behaviour** than the underlying fundamental theory
- The only remnants of the high-energy dynamics are in the **low-energy couplings** and in the **symmetries** of the EFT

Evolution from High to Low Scales

Large μ

$\mathcal{L}(\phi_i) + \mathcal{L}(\phi_i, \Phi)$ — ϕ_i, Φ

↓ **Renormalization Group**

$\mu = M$ — **Matching**

$\mathcal{L}(\phi_i) + \delta\mathcal{L}(\phi_i)$ — ϕ_i

↓ **Renormalization Group**

Small μ

Wilson Coefficients: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$

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$$\langle O_i \rangle_B = Z_i(\epsilon,\mu)\, \langle O_i(\mu)\rangle_R \qquad ; \qquad \mu\,\frac{d}{d\mu}\,\langle O_i\rangle_B = 0$$

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$$\left(\mu \frac{d}{d\mu} + \gamma_{O_i} \right) \langle O_i \rangle_R = 0 \qquad ; \qquad \gamma_{O_i} \equiv \frac{\mu}{Z_i} \frac{dZ_i}{d\mu} = \gamma_{O_i}^{(1)} \frac{\alpha}{\pi} + \gamma_{O_i}^{(2)} \left(\frac{\alpha}{\pi} \right)^2 + \cdots$$

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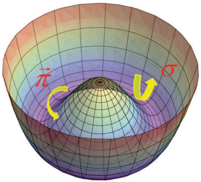
$$\longrightarrow \quad \begin{aligned} c_i(\mu) &= c_i(\mu_0)\, \exp\left\{\int_{\alpha_0}^{\alpha} \frac{d\alpha}{\alpha} \frac{\gamma_{O_i}(\alpha)}{\beta(\alpha)}\right\} \\ &= c_i(\mu_0) \left[\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right]^{\gamma_{O_i}^{(1)}/\beta_1} \left\{1 + \cdots\right\} \end{aligned}$$

2. Chiral Perturbation Theory

- **Goldstone Theorem**
- **Chiral Symmetry**
- **Effective Goldstone Theory**
- **Chiral Symmetry Breaking**

Sigma Model

$\mathbf{\Phi}^\mathsf{T} \equiv (\sigma, \vec{\pi})$



$$\mathcal{L}_\sigma = \frac{1}{2}\,\partial_\mu \mathbf{\Phi}^\mathsf{T} \partial^\mu \mathbf{\Phi} \;-\; \frac{\lambda}{4}\,\left(\mathbf{\Phi}^\mathsf{T}\mathbf{\Phi} - v^2\right)^2$$

Global Symmetry: $\quad O(4) \sim SU(2) \otimes SU(2)$

- $\mathbf{v^2 < 0}$**:** $\quad m_\Phi^2 = -\lambda\, v^2$
- $\mathbf{v^2 > 0}$**:** $\quad \langle 0|\sigma|0\rangle = v \quad , \quad \langle 0|\vec{\pi}|0\rangle = 0$

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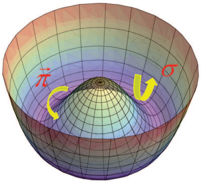

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SSB: $\quad O(4) \to O(3)$ $\quad [\frac{4\times 3}{2} - \frac{3\times 2}{2} \;=\; 3$ broken generators]

$$\mathcal{L}_\sigma = \frac{1}{2}\left\{\partial_\mu\hat{\sigma}\,\partial^\mu\hat{\sigma} + \partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} - M^2\hat{\sigma}^2\right\} - \frac{M^2}{2v}\,\hat{\sigma}\,\left(\hat{\sigma}^2 + \vec{\pi}^2\right) - \frac{M^2}{8v^2}\,\left(\hat{\sigma}^2 + \vec{\pi}^2\right)^2$$

$$\hat{\sigma} \equiv \sigma - v \qquad ; \qquad M^2 = 2\,\lambda\, v^2$$

3 Massless Goldstone Bosons

1) $\boldsymbol{\Sigma}(x) \equiv \sigma(x)\, \mathbf{I}_2 + i\, \vec{\tau}\, \vec{\pi}(x)$; $\langle \mathbf{A} \rangle \equiv \mathrm{Tr}\,(\mathbf{A})$

$$\mathcal{L}_\sigma = \frac{1}{4}\, \langle \partial_\mu \boldsymbol{\Sigma}^\dagger\, \partial^\mu \boldsymbol{\Sigma} \rangle - \frac{\lambda}{16}\, \left(\langle \boldsymbol{\Sigma}^\dagger \boldsymbol{\Sigma} \rangle - 2\, v^2 \right)^2$$

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O(4) $\sim$ SU(2)$_\mathbf{L} \otimes$ SU(2)$_\mathbf{R}$ Symmetry: $\boldsymbol{\Sigma} \to g_R\, \boldsymbol{\Sigma}\, g_L^\dagger$; $g_{L,R} \in SU(2)_{L,R}$

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2) $\boldsymbol{\Sigma}(x) \equiv [v + S(x)]\; \mathbf{U}(x)$; $\mathbf{U} \equiv \exp\left\{ \frac{i}{v}\, \vec{\tau}\, \vec{\phi} \right\} \to g_R\, \mathbf{U}\, g_L^\dagger$

$$\mathcal{L}_\sigma = \frac{v^2}{4}\, \left(1 + \frac{S}{v}\right)^2 \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle + \frac{1}{2}\, \left(\partial_\mu S\, \partial^\mu S - M^2 S^2 \right) - \frac{M^2}{2v}\, S^3 - \frac{M^2}{8v^2}\, S^4$$

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Derivative Golstone Couplings

1) $\mathbf{\Sigma}(x) \equiv \sigma(x)\, \mathbf{I}_2 + i\, \vec{\tau}\, \vec{\pi}(x)$; $\langle \mathbf{A} \rangle \equiv \mathrm{Tr}(\mathbf{A})$

$$\mathcal{L}_\sigma = \frac{1}{4} \langle \partial_\mu \mathbf{\Sigma}^\dagger \partial^\mu \mathbf{\Sigma} \rangle - \frac{\lambda}{16} \left(\langle \mathbf{\Sigma}^\dagger \mathbf{\Sigma} \rangle - 2 v^2 \right)^2$$

$O(4) \sim SU(2)_L \otimes SU(2)_R$ Symmetry: $\mathbf{\Sigma} \to g_R\, \mathbf{\Sigma}\, g_L^\dagger$; $g_{L,R} \in SU(2)_{L,R}$

2) $\mathbf{\Sigma}(x) \equiv [v + S(x)]\; \mathbf{U}(x)$; $\mathbf{U} \equiv \exp\left\{ \frac{i}{v}\, \vec{\tau}\, \vec{\phi} \right\} \to g_R\, \mathbf{U}\, g_L^\dagger$

$$\mathcal{L}_\sigma = \frac{v^2}{4} \left(1 + \frac{S}{v}\right)^2 \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle + \frac{1}{2} \left(\partial_\mu S\, \partial^\mu S - M^2 S^2 \right) - \frac{M^2}{2v} S^3 - \frac{M^2}{8v^2} S^4$$

Derivative Golstone Couplings

3) $E \ll M \sim v$:

$$\boxed{\mathcal{L}_\sigma \approx \frac{v^2}{4} \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle}$$

Symmetry Realizations

Symmetry **G** $\{T_a\}$ ⟺ **Conserved charges** $\mathcal{Q}_a$

Noether Theorem: $$\partial_\mu j_a^\mu = 0 \quad ; \quad \mathcal{Q}_a = \int d^3x \, j_a^0(x) \quad ; \quad \frac{d}{dt}\,\mathcal{Q}_a = 0$$

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Wigner–Weyl

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- Exact Symmetry
- Degenerate Multiplets
- Linear Representation

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Wigner–Weyl	**Nambu–Goldstone**
$\mathcal{Q}_a \mid 0 \rangle = 0$	$\mathcal{Q}_a \mid 0 \rangle \neq 0$
• Exact Symmetry	• Spontaneously Broken Symmetry
• Degenerate Multiplets	• Massless Goldstone Bosons
• Linear Representation	• Non-Linear Representation

Chiral Symmetry $\mathbf{m_q = 0}$ **(Chiral Limit)**

$$\mathcal{L}^0_{QCD} = -\frac{1}{4}\, G^{\mu\nu}_a G^a_{\mu\nu} + \bar{\mathbf{q}}_L\, i\gamma^\mu D_\mu \mathbf{q}_L + \bar{\mathbf{q}}_R\, i\gamma^\mu D_\mu \mathbf{q}_R$$

$\mathbf{q}^\mathrm{T} \equiv (u\, ,\, d\, ,\, s)$

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$$q = \left(\frac{1-\gamma_5}{2}\right) q + \left(\frac{1+\gamma_5}{2}\right) q \equiv q_L + q_R$$

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- $\mathcal{L}_{QCD}^{0}$ **invariant under** $\mathbf{G \equiv SU(3)_L \otimes SU(3)_R}$**:**

$$\bar{\mathbf{q}}_L \to g_L\, \bar{\mathbf{q}}_L \quad ; \quad \bar{\mathbf{q}}_R \to g_R\, \bar{\mathbf{q}}_R \quad ; \quad (g_L, g_R) \in \mathbf{G}$$

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- **Only $\mathbf{SU(3)_V}$ in the hadronic spectrum:** $(\pi, K, \eta)_{0^-}$; $(\rho, K^*, \omega)_{1^-}$; $\cdots$

$$M_{0^-} < M_{0^+} \qquad ; \qquad M_{1^-} < M_{1^+}$$

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- **The vacuum is not invariant (SSB):** $\langle 0|\,(\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L)\,|0\rangle \neq 0$

8 Massless $\mathbf{0^-}$ Goldstone Bosons

Effective Goldstone Theory

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$$\mathbf{\Phi} \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix}$$

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Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\downarrow$ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\downarrow$ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

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- **Expansion in powers of momenta $\Leftrightarrow$ derivatives**

Parity $\Rightarrow$ even dimension ; $\mathbf{U}\,\mathbf{U}^\dagger = 1 \Rightarrow 2n \geq 2$

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➡ $$\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle$$ **Derivative Coupling**

Goldstones become free at zero momenta

$$\mathcal{L}_2 = \frac{f^2}{4}\langle\partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U}\rangle = \partial_\mu\pi^-\partial^\mu\pi^+ + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 + \cdots$$

$$+ \frac{1}{6f^2}\left\{\left(\pi^+ \overleftrightarrow{\partial}_\mu \pi^-\right)\left(\pi^+ \overleftrightarrow{\partial}^\mu \pi^-\right) + 2\left(\pi^0 \overleftrightarrow{\partial}_\mu \pi^+\right)\left(\pi^- \overleftrightarrow{\partial}^\mu \pi^0\right) + \cdots\right\}$$

$$+ O\left(\pi^6/f^4\right)$$

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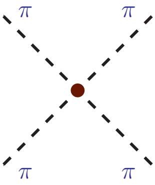
Chiral Symmetry Determines the Interaction:



$$T\left(\pi^+\pi^0 \to \pi^+\pi^0\right) = \frac{t}{f^2}$$

$t \equiv (p'_+ - p_+)^2$

Weinberg

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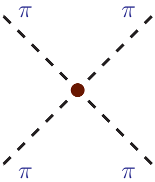

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Non-Linear Lagrangian: $2\pi \to 2\pi, 4\pi, \cdots$ related

Explicit Symmetry Breaking

$$\mathcal{L}_{QCD} \equiv \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}\,(\not{\mathbf{v}} + \not{\mathbf{a}}\,\gamma_5)\,\mathbf{q} - \bar{\mathbf{q}}\,(\mathbf{s} - i\,\gamma_5\,\mathbf{p})\,\mathbf{q}$$

$$= \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}_L\,\not{\mathbf{l}}\,\mathbf{q}_L + \bar{\mathbf{q}}_R\,\not{\mathbf{r}}\,\mathbf{q}_R - \bar{\mathbf{q}}_R\,(\mathbf{s} + i\,\mathbf{p})\,\mathbf{q}_L - \bar{\mathbf{q}}_L\,(\mathbf{s} - i\,\mathbf{p})\,\mathbf{q}_R$$

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$$\mathbf{l}_\mu \equiv \mathbf{v}_\mu - \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots \qquad\qquad \mathcal{Q} \equiv \tfrac{1}{3}\,\mathrm{diag}(2,-1,-1)$$

$$\mathbf{r}_\mu \equiv \mathbf{v}_\mu + \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots$$

$$\mathbf{s} = \mathcal{M} + \cdots \qquad ; \qquad \mathcal{M} \equiv \mathrm{diag}(m_u, m_d, m_s)$$

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Local $SU(3)_L \otimes SU(3)_R$ Symmetry:

$$\mathbf{q}_L \to g_L\,\mathbf{q}_L \qquad \mathbf{l}_\mu \to g_L\,\mathbf{l}_\mu\,g_L^\dagger + i\,g_L\,\partial_\mu g_L^\dagger$$

$$\mathbf{r}_\mu \to g_R\,\mathbf{r}_\mu\,g_R^\dagger + i\,g_R\,\partial_\mu g_R^\dagger$$

$$\mathbf{q}_R \to g_R\,\mathbf{q}_R \qquad (\mathbf{s} + i\,\mathbf{p}) \to g_R\,(\mathbf{s} + i\,\mathbf{p})\,g_L^\dagger$$

Lowest-Order Effective Lagrangian:

$$\mathcal{L} = \frac{f^2}{4} \langle D_\mu \mathbf{U} \, D^\mu \mathbf{U}^\dagger + \chi \, \mathbf{U}^\dagger + \mathbf{U} \, \chi^\dagger \rangle$$

$$D_\mu \mathbf{U} = \partial_\mu \mathbf{U} - i \, \mathbf{r}_\mu \, \mathbf{U} + i \, \mathbf{U} \, \mathbf{l}_\mu$$

$$\chi \equiv 2 \, B_0 \, (\mathbf{s} + i \, \mathbf{p})$$

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Currents:

$$\mathbf{J}^\mu_{L} = \frac{\partial}{\partial\, \mathbf{l}_\mu} \mathcal{L}_2 = \frac{i}{2} f^2\, D^\mu \mathbf{U}^\dagger\, \mathbf{U} = \frac{f}{\sqrt{2}}\, D^\mu \mathbf{\Phi} + \cdots$$

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$$\langle 0 |\, (J^\mu_A)_{12}\, | \pi^+(p) \rangle = i \sqrt{2}\, f\, p^\mu \quad \Longrightarrow \quad f = f_\pi \approx 92.4\ \mathrm{MeV}$$

$(\pi^+ \to \mu^+ \nu_\mu)$

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$$\bar{\mathbf{q}}^j_L \, \mathbf{q}^i_R = -\frac{\partial \, \mathcal{L}_2}{\partial (\mathbf{s} - i \, \mathbf{p})^{ji}} = -\frac{f^2}{2} \, B_0 \, \mathbf{U}^{ij} \quad \longrightarrow$$

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$$\langle 0 |\, (J^\mu_A)_{12}\, | \pi^+(p) \rangle = i\, \sqrt{2}\, f\; p^\mu \quad \longrightarrow \quad \boxed{f = f_\pi \approx 92.4 \text{ MeV}}$$

$(\pi^+ \to \mu^+ \nu_\mu)$

$$\bar{\mathbf{q}}^j_L\, \mathbf{q}^i_R = -\frac{\partial\, \mathcal{L}_2}{\partial(\mathbf{s} - i\, \mathbf{p})^{ji}} = -\frac{f^2}{2}\, B_0\, \mathbf{U}^{ij} \quad \longrightarrow \quad \boxed{\langle 0 | \bar{\mathbf{q}}^j \mathbf{q}^i | 0 \rangle = -f^2\, B_0\, \delta_{ij}}$$

Quark Masses: $\frac{f^2}{4}\,\langle \chi\, \mathbf{U}^\dagger + \mathbf{U}\,\chi^\dagger \rangle \quad \rightarrow \quad \mathcal{L}_m = -B_0\,\langle \mathcal{M}\,\Phi^2 \rangle$

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$$\mathcal{L}_m \,=\, -B_0 \left\{ (m_u + m_d) \left[\pi^+\pi^- + \frac{1}{2}\,\pi^0\pi^0 \right] \,+\, (m_u + m_s)\, K^+ K^- \right.$$

$$\left. +\, (m_d + m_s)\, K^0 \bar{K}^0 \,+\, \frac{1}{6}\,(m_u + m_d + 4\, m_s)\,\eta^2 \,+\, \frac{1}{\sqrt{3}}\,(m_u - m_d)\,\pi^0\eta \right\}$$

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Isospin limit: $\mathbf{m_u = m_d = \hat{m}}$

$$\frac{M_\pi^2}{2\,\hat{m}} = \frac{M_K^2}{\hat{m} + m_s} = \frac{3\, M_\eta^2}{2\,\hat{m} + 4\, m_s} = B_0$$

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- **Gell-Mann–Okubo:** $4\,M_K^2 = M_\pi^2 + 3\,M_\eta^2$

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$$\mathcal{L}_m = -B_0 \Big\{ (m_u + m_d) \left[\pi^+\pi^- + \frac{1}{2}\, \pi^0\pi^0 \right] + (m_u + m_s)\, K^+ K^-$$

$$+ (m_d + m_s)\, K^0 \bar{K}^0 + \frac{1}{6}\, (m_u + m_d + 4\, m_s)\, \eta^2 + \frac{1}{\sqrt{3}}\, (m_u - m_d)\, \pi^0 \eta \Big\}$$

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- **Gell-Mann–Okubo:** $4\, M_K^2 = M_\pi^2 + 3\, M_\eta^2$
- **Gell-Mann–Oakes–Renner:** $f^2\, M_\pi^2 = -\hat{m}\, \langle 0 | \bar{u}\, u + \bar{d}\, d | 0 \rangle$

Quark Mass Ratios:

Dashen Theorem

$$\left(M^2_{K^0} - M^2_{K^\pm}\right)_{\mathrm{em}} = \left(M^2_{\pi^0} - M^2_{\pi^\pm}\right)_{\mathrm{em}} + \mathcal{O}(e^2 p^2)$$

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Dashen Theorem

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Proof: $e^2 \langle \mathcal{Q}_{\mathrm{R}} \mathrm{U} \mathcal{Q}_{\mathrm{L}} \mathrm{U}^\dagger \rangle = -\frac{2e^2}{f^2} \left(\pi^+\pi^- + K^+K^-\right) + \mathcal{O}(\phi^4)$; $\mathcal{Q}_X \to g_X \, \mathcal{Q}_X \, g_X^\dagger$ □

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$$\frac{m_d - m_u}{m_d + m_u} = \frac{\left(M^2_{K^0} - M^2_{K^\pm}\right) - \left(M^2_{\pi^0} - M^2_{\pi^\pm}\right)}{M^2_{\pi^0}} \approx 0.29$$

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Proof: $e^2 \langle \mathcal{Q}_{\rm R} {\rm U} \mathcal{Q}_{\rm L} {\rm U}^\dagger \rangle = -\frac{2e^2}{f^2}\left(\pi^+\pi^- + K^+K^-\right) + \mathcal{O}(\phi^4) \quad ; \quad \mathcal{Q}_X \to g_X\, \mathcal{Q}_X\, g_X^\dagger \qquad \square$

$$\frac{m_d - m_u}{m_d + m_u} = \frac{\left(M^2_{K^0} - M^2_{K^\pm}\right) - \left(M^2_{\pi^0} - M^2_{\pi^\pm}\right)}{M^2_{\pi^0}} \approx 0.29$$

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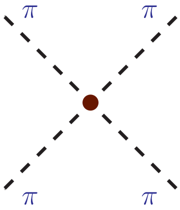
$\longrightarrow$ $$\mathbf{m_u : m_d : m_s = 0.55 : 1 : 20.3}$$ Weinberg

$$\frac{f^2}{4}\,\langle \chi\, \mathbf{U}^\dagger + \mathbf{U}\,\chi^\dagger \rangle \;=\; -B_0\,\langle\, \mathcal{M}\,\Phi^2\rangle \;+\; \frac{B_0}{6\,f^2}\,\langle\, \mathcal{M}\,\Phi^4\rangle \;+\;\cdots$$

$$\frac{f^2}{4}\,\langle\chi\,\mathbf{U}^\dagger + \mathbf{U}\,\chi^\dagger\rangle \;=\; -B_0\,\langle\,\mathcal{M}\,\Phi^2\rangle \;+\; \frac{B_0}{6\,f^2}\,\langle\,\mathcal{M}\,\Phi^4\rangle \;+\;\cdots$$



$$T\left(\pi^+\pi^0\to\pi^+\pi^0\right) \;=\; \frac{t-M_\pi^2}{f_\pi^2}$$

$t \equiv (p'_+ - p_+)^2$

Weinberg

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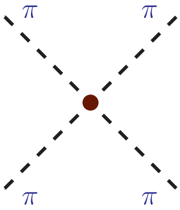

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$t \equiv (p'_+ - p_+)^2$

Weinberg

$\mathcal{L}_2$ ⟷ Current Algebra 60's

Chiral Power Counting

$\mathbf{U}$	$\mathcal{O}(p^0)$
$D_\mu \mathbf{U}\,,\ \mathbf{l}_\mu\,,\ \mathbf{r}_\mu$	$\mathcal{O}(p^1)$
$\chi\,,\ \mathbf{F}_{L,R}^{\mu\nu}$	$\mathcal{O}(p^2)$

$$\mathbf{F}_L^{\mu\nu} \equiv \partial^\mu \mathbf{l}^\nu - \partial^\nu \mathbf{l}^\mu - i\ [\mathbf{l}^\mu, \mathbf{l}^\nu]$$

$$\mathbf{F}_R^{\mu\nu} \equiv \partial^\mu \mathbf{r}^\nu - \partial^\nu \mathbf{r}^\mu - i\ [\mathbf{r}^\mu, \mathbf{r}^\nu]$$

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General connected diagram with N_d vertices of $\mathcal{O}(p^d)$ and L loops:

$$D \,=\, 2\,L + 2 + \sum_d N_d\,(d-2)$$

Weinberg

Chiral Power Counting

$\mathbf{U}$	$\mathcal{O}(p^0)$
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$$\mathbf{F}^{\mu\nu}_L \equiv \partial^\mu \mathbf{l}^\nu - \partial^\nu \mathbf{l}^\mu - i\ [\mathbf{l}^\mu, \mathbf{l}^\nu]$$

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General connected diagram with N_d vertices of $\mathcal{O}(p^d)$ and L loops:

$$D = 2L + 2 + \sum_d N_d\,(d-2)$$

Weinberg

- $D = 2:$ $\quad L = 0\ ,\ d = 2$
- $D = 4:$ $\quad L = 0\ ,\ d = 4\ ,\ N_4 = 1$
  $\quad L = 1\ ,\ d = 2$

$\mathcal{O}(\mathbf{p}^4)$ χPT

i) $\mathcal{L}_4$ **at tree level** (Gasser–Leutwyler)

$$\begin{aligned}
\mathcal{L}_4 \;=\;& L_1 \langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U\rangle \langle D^\mu U^\dagger D^\nu U\rangle \\
&+ L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\rangle + L_4 \langle D_\mu U^\dagger D^\mu U\rangle \langle U^\dagger \chi + \chi \dagger U\rangle \\
&+ L_5 \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U\right)\rangle + L_6 \langle U^\dagger \chi + \chi^\dagger U\rangle^2 \\
&+ L_7 \langle U^\dagger \chi - \chi^\dagger U\rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi\rangle \\
&- i\, L_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle
\end{aligned}$$

$\mathcal{O}(\mathbf{p}^4)$ χPT

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&- i\, L_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle
\end{aligned}$$

ii) $\mathcal{L}_2$ **at one loop (unitarity):** $T_4 \sim p^4 \left\{ a \log(p^2/\mu^2) + b(\mu) \right\}$

- Chiral Logarithms unambiguously predicted

$\mathcal{O}(\mathbf{p}^4)$ χPT

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- L_i's fixed by QCD dynamics. 1-loop divergences ➡ $L_i^r(\mu)$

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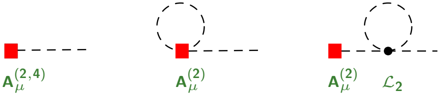
- L_i's fixed by QCD dynamics. 1-loop divergences ➡ $L_i^r(\mu)$

iii) **Wess–Zumino–Witten term (chiral anomaly):** $\pi^0, \eta \to \gamma\gamma$

Meson Decay Constants:



$$\mu_P \equiv \frac{M_P^2}{32\pi^2 f^2} \log\left(\frac{M_P^2}{\mu^2}\right)$$

$$f_\pi = f \left\{1 - 2\mu_\pi - \mu_K + \frac{4M_\pi^2}{f^2} L_5^r(\mu) + \frac{8M_K^2 + 4M_\pi^2}{f^2} L_4^r(\mu)\right\}$$

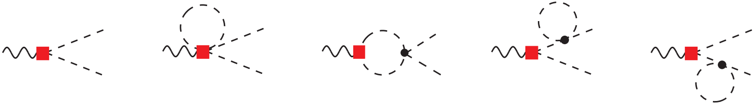
$$f_K = f \left\{1 - \frac{3}{4}\mu_\pi - \frac{3}{2}\mu_K - \frac{3}{4}\mu_{\eta_8} + \frac{4M_K^2}{f^2} L_5^r(\mu) + \frac{8M_K^2 + 4M_\pi^2}{f^2} L_4^r(\mu)\right\}$$

$$f_{\eta_8} = f \left\{1 - 3\mu_K + \frac{4M_{\eta_8}^2}{f^2} L_5^r(\mu) + \frac{8M_K^2 + 4M_\pi^2}{f^2} L_4^r(\mu)\right\}$$

$$\frac{f_K}{f_\pi} = 1.22 \pm 0.01 \;\longrightarrow\; L_5^r(M_\rho) = (1.4 \pm 0.5) \cdot 10^{-3} \;\longrightarrow\; \frac{f_{\eta_8}}{f_\pi} = 1.3 \pm 0.05$$

Vector Form Factor:

$$\langle \pi^+\pi^- | J^\mu_{\rm em} | 0 \rangle \; = \; (p_+ - p_-)^\mu \; F^V_\pi(s)$$

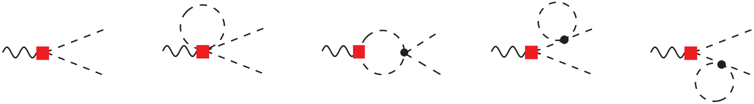


$$F^V_\pi(s) \; = \; 1 \; + \; \frac{2 L_9^r(\mu)}{f^2}\, s \; - \; \frac{s}{96\pi^2 f^2} \left[A\left(\frac{m_\pi^2}{s}, \frac{m_\pi^2}{\mu^2}\right) + \frac{1}{2}\, A\left(\frac{m_K^2}{s}, \frac{m_K^2}{\mu^2}\right)\right]$$

$$= \; 1 \; + \; \frac{1}{6}\, \langle r^2 \rangle^V_\pi \; s \; + \; \cdots$$

$$A\left(\frac{m_P^2}{s}, \frac{m_P^2}{\mu^2}\right) \; = \; \log\left(\frac{m_P^2}{\mu^2}\right) + \frac{8 m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \; \log\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \qquad , \qquad \sigma_P \equiv \sqrt{1 - \frac{4 m_P^2}{s}}$$

Vector Form Factor: $\langle \pi^+\pi^- | J^\mu_{\rm em} | 0\rangle = (p_+ - p_-)^\mu \, F^V_\pi(s)$



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$$\langle r^2\rangle^V_\pi = \frac{12\, L_9^r(\mu)}{f^2} - \frac{1}{32\pi^2 f^2}\left\{2\,\log\left(\frac{M_\pi^2}{\mu^2}\right) + \log\left(\frac{M_K^2}{\mu^2}\right) + 3\right\}$$

$\langle r^2\rangle^V_\pi = (0.439 \pm 0.008)\ \mathrm{fm}^2$ ➡ $L_9^r(M_\rho) = (6.9 \pm 0.7)\cdot 10^{-3}$

$O(p^4)$ χPT COUPLINGS

i	$L_i^r(M_\rho) \times 10^3$	Source	Γ_i
1	0.4 ± 0.3	$K_{e4},\ \pi\pi \to \pi\pi$	$3/32$
2	1.4 ± 0.3	$K_{e4},\ \pi\pi \to \pi\pi$	$3/16$
3	-3.5 ± 1.1	$K_{e4},\ \pi\pi \to \pi\pi$	0
4	-0.3 ± 0.5	Zweig rule	$1/8$
5	1.4 ± 0.5	F_K/F_π	$3/8$
6	-0.2 ± 0.3	Zweig rule	$11/144$
7	-0.4 ± 0.2	GMO, $L_{5,8}$	0
8	0.9 ± 0.3	$M_{K^0} - M_{K^+},\ L_5,\ (m_s - \hat{m})/(m_d - m_u)$	$5/48$
9	6.9 ± 0.7	$\langle r^2 \rangle_V^\pi$	$1/4$
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$	$-1/4$

- $$L_i \;=\; L_i^r(\mu) + \Gamma_i\, \frac{\mu^{D-4}}{32\pi^2} \left\{ \frac{2}{D-4} + \gamma_E - \log(4\pi) - 1 \right\}$$

$O(p^4)$ χPT COUPLINGS

i	$L_i^r(M_\rho) \times 10^3$	Source	Γ_i
1	0.4 ± 0.3	K_{e4}, $\pi\pi \to \pi\pi$	$3/32$
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3	-3.5 ± 1.1	K_{e4}, $\pi\pi \to \pi\pi$	0
4	-0.3 ± 0.5	Zweig rule	$1/8$
5	1.4 ± 0.5	F_K/F_π	$3/8$
6	-0.2 ± 0.3	Zweig rule	$11/144$
7	-0.4 ± 0.2	GMO, $L_{5,8}$	0
8	0.9 ± 0.3	$M_{K^0} - M_{K^+}$, L_5, $(m_s - \hat{m})/(m_d - m_u)$	$5/48$
9	6.9 ± 0.7	$\langle r^2 \rangle_V^\pi$	$1/4$
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$	$-1/4$

- $$L_i = L_i^r(\mu) + \Gamma_i \frac{\mu^{D-4}}{32\pi^2} \left\{ \frac{2}{D-4} + \gamma_E - \log(4\pi) - 1 \right\}$$
- $\Lambda_\chi \sim 1\,\mathrm{GeV}$ ➡ $L_i \sim \frac{f_\pi^2/4}{\Lambda_\chi^2} \sim 2 \times 10^{-3}$

$O(p^4)$ χPT COUPLINGS

i	$L_i^r(M_\rho) \times 10^3$	Source	Γ_i
1	0.4 ± 0.3	K_{e4}, $\pi\pi \to \pi\pi$	$3/32$
2	1.4 ± 0.3	K_{e4}, $\pi\pi \to \pi\pi$	$3/16$
3	-3.5 ± 1.1	K_{e4}, $\pi\pi \to \pi\pi$	0
4	-0.3 ± 0.5	Zweig rule	$1/8$
5	1.4 ± 0.5	F_K/F_π	$3/8$
6	-0.2 ± 0.3	Zweig rule	$11/144$
7	-0.4 ± 0.2	GMO, $L_{5,8}$	0
8	0.9 ± 0.3	$M_{K^0} - M_{K^+}$, L_5, $(m_s - \hat{m})/(m_d - m_u)$	$5/48$
9	6.9 ± 0.7	$\langle r^2 \rangle_V^\pi$	$1/4$
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$	$-1/4$

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- χPT Loops $\sim$ $1/(4\pi f_\pi)^2$

$\mathcal{O}(p^6)$ χPT

i) $\mathcal{L}_6 = \sum_i C_i\, O_i^{p^6}$ **at tree level** Bijnens-Colangelo-Ecker, Fearing-Scherer

$90 + 4$ $[53 + 4]$ terms in SU(3) [SU(2)] χPT (even-intrinsic parity only)

ii) $\mathcal{L}_4$ **at one loop,** $\mathcal{L}_2$ **at two loops** Bijnens-Colangelo-Ecker

Double chiral logarithms

Many Calculations: M_ϕ, f_ϕ, $\gamma\gamma \to \pi\pi$, $\pi\pi \to \pi\pi$, $\pi K \to \pi K$, K_{l4}, $\pi \to e\,\bar{\nu}_e\gamma$, $F_V(s)$, $F_S(s)$, $\Pi_{V,A}(s)$, $\cdots$

Amoros-Bijnens-Dhonte-Talavera, Ananthanarayan-Colangelo-Gasser-Leutwyler, Bellucci-Gasser-Sainio, Bürgui, Bijnens et al, Descotes-Genon et al, Golowich-Kambor, Post-Schilcher...

Theoretical Challenge: **QCD calculation of the χPT couplings**

$K^+ \to \pi^0 \ell^+ \nu_\ell$, $K^0 \to \pi^- \ell^+ \nu_\ell$:

$C_{K^+\pi^0} = \frac{1}{\sqrt{2}}$, $C_{K^0\pi^-} = 1$

$$\langle \pi | \bar{s}\gamma^\mu u | K \rangle = C_{K\pi} \left[(P_K + P_\pi)^\mu \; f_+^{K\pi}(t) + (P_K - P_\pi)^\mu \; f_-^{K\pi}(t) \right]$$

- **Lowest order** $[\mathcal{O}(p^2)]$**:** $f_+^{K\pi}(t) = 1$, $f_-^{K\pi}(t) = 0$

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- $\mathcal{O}(\mathbf{p^4})$**:** $\quad f_+^{K^0\pi^-}(0) \,=\, 0.977 \quad , \quad \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} \,=\, 1.022$

Gasser-Leutwyler '85

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Gasser-Leutwyler '85

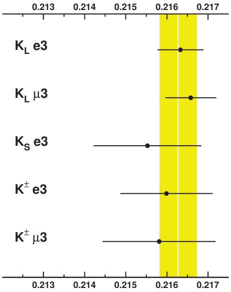
Needed to determine $\mathbf{V_{us}}$

$K \to \pi \, \ell \, \nu_\ell$

$$|\mathbf{V_{us}\, f_+(0)}| = \mathbf{0.2163 \pm 0.0005}$$

Flavianet Kaon WG, arXiv:1005.2323 [hep-ph]

$$\langle \pi^- | \bar{s}\gamma_\mu u | K^0 \rangle = (p_\pi + p_K)_\mu \; f_+(t) + (p_K - p_\pi)_\mu \; f_-(t)$$



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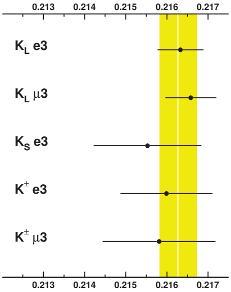

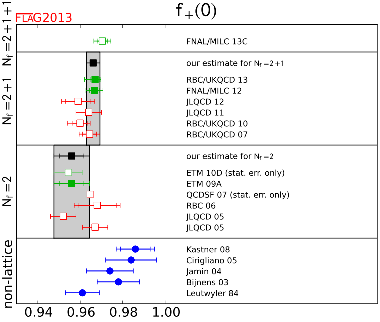

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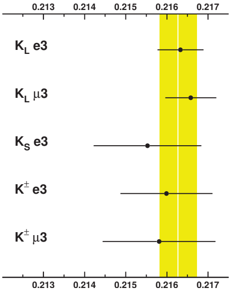

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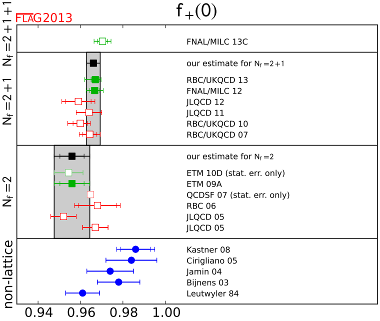


$$f_+(0) = 0.9661\,(32)$$

$$\longrightarrow \quad |V_{us}| = 0.2239\,(9)$$

$$f_+(0) = 1 + f_2 + f_4 + \cdots$$

Large $\mathcal{O}(p^6)$ χPT correction

3. Electroweak Effective Theory

- **Higgs Mechanism**
- **Custodial Symmetry**
- **Equivalence Theorem**
- **Goldstone Electroweak Effective Theory**
- **Fermions, Higgs**
- **Linear Realization**

Energy Scale	Fields	Effective Theory
$\Lambda_{NP} \sim$ TeV	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Underlying Dynamics
······ Energy Gap ······	↓	
M_W	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model

Effective Field Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(4)} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}} \mathcal{O}_i^{(D)}$$

- **Most general Lagrangian with the SM gauge symmetries**
- **Light ($m \ll \Lambda_{NP}$) fields only**
- **The SM Lagrangian corresponds to $D = 4$**
- **$c_i^{(D)}$ contain information on the underlying dynamics:**

$$\mathcal{L}_{\rm NP} \doteq g_X \,(\bar{q}_L\gamma^\mu q_L)\, X_\mu \quad \longrightarrow \quad \frac{g_X^2}{M_X^2}\,(\bar{q}_L\gamma^\mu q_L)\,(\bar{q}_L\gamma_\mu q_L)$$

- **Options for H(126):**
  - $SU(2)_L$ doublet (SM)
  - Scalar singlet
  - Additional light scalars

Higgs Mechanism:

Gauge invariance

Massless $W^{\pm}, Z$ (spin 1)

3×2 polarizations $= 6$

Higgs Mechanism: 3 additional degrees of freedom $\theta_i(x)$

Gauge invariance

Massless $W^\pm$, Z (spin 1)

3×2 polarizations $= 6$

$+$

3 Goldstones $\theta_i(x)$

SSB $\Downarrow$

Massive $W^\pm$, Z

3×3 polarizations $= 9$

Higgs Mechanism: 3 additional degrees of freedom $\theta_i(x)$

Gauge invariance

Massless $W^\pm$, Z (spin 1)

3×2 polarizations $= 6$

$+$

3 Goldstones $\theta_i(x)$

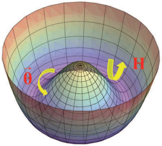
SSB ↓

Massive $W^\pm$, Z

3×3 polarizations $= 9$

Spontaneous Symmetry Breaking

$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - \mu^2\Phi^\dagger\Phi - \lambda\,(\Phi^\dagger\Phi)^2$$



$\mu^2 < 0$

$$\Phi(x) = \exp\left\{i\,\frac{\vec{\sigma}}{2}\cdot\vec{\theta}(x)\right\}\,\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

Higgs Mechanism: 3 additional degrees of freedom $\theta_i(x)$

Gauge invariance

Massless $W^\pm$, Z (spin 1)

3×2 polarizations $=$ 6

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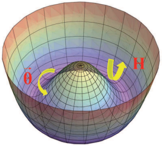

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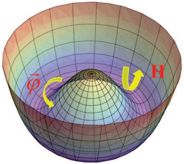
$$\Phi(x) = \exp\left\{i\,\frac{\vec{\sigma}}{2}\cdot\vec{\theta}(x)\right\}\,\frac{1}{\sqrt{2}}\begin{bmatrix}0\\ v+H(x)\end{bmatrix}$$

$D_\mu\Phi = (\partial_\mu + \frac{i}{2}\,g\,\vec{\sigma}\cdot\vec{W}_\mu + \frac{i}{2}\,g'\,B_\mu)\,\Phi$; $v^2 = -\mu^2/\lambda$

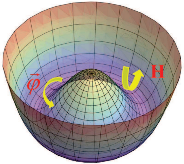
$$(D_\mu\Phi)^\dagger D^\mu\Phi \;\to\; M_W^2\,W_\mu^\dagger W^\mu + \frac{M_Z^2}{2}\,Z_\mu Z^\mu$$

$$\mathbf{M_W = M_Z\,\cos\theta_W = \frac{1}{2}\,g\,v}$$



$$\mathcal{L}_\Phi \;=\; (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix}$$



$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$$

$$= \frac{1}{2} \mathrm{Tr} \left[(D^\mu \Sigma)^\dagger D_\mu \Sigma \right] - \frac{\lambda}{4} \left(\mathrm{Tr} \left[\Sigma^\dagger \Sigma \right] - v^2 \right)^2$$

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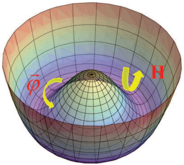

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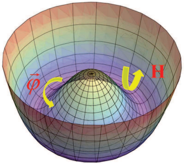
$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\Sigma \to g_L \, \Sigma \, g_R^\dagger$

Custodial Symmetry

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} (v + H)\, U(\vec{\theta}\,)$$

$$U(\vec{\varphi}\,) \equiv \exp\left\{ i\, \vec{\sigma} \cdot \frac{\vec{\varphi}}{v} \right\}$$



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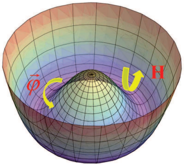

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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \qquad , \qquad \vec{\pi} \to \vec{\varphi} \to W_L^\pm, Z_L$$

EFFECTIVE LAGRANGIAN: $\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$

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- **Goldstone Bosons**

$\langle 0| \bar{q}^j_L q^i_R |0\rangle$ **(QCD)**, Φ **(SM)** $\longrightarrow$ $U_{ij}(\phi) = \{\exp(i\vec{\sigma}\cdot\vec{\varphi}/f)\}_{ij}$

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- **Expansion in powers of momenta** ⬌ **derivatives**

Parity ➡ even dimension ; $U\,U^\dagger = 1$ ➡ $2n \geq 2$

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Derivative Coupling

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Derivative Coupling

Goldstones become free at zero momenta

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \quad \xrightarrow{U=1} \quad \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2\, Z_\mu Z^\mu$$

$$\mathbf{M_W = M_Z \cos\theta_W = \frac{1}{2}\, g\, v}$$

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$$D^\mu U = \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i\,U^\dagger \hat{W}^\mu - i\,\hat{B}^\mu U^\dagger$$

$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i\,[\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i\,[\hat{B}^\mu, \hat{B}^\nu]$$

$$\hat{W}^\mu = -\tfrac{g}{2}\,\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu = -\tfrac{g'}{2}\,\sigma_3\, B^\mu$$ **(explicit symmetry breaking)**

Electroweak Symmetry Breaking

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$$\mathbf{M_W = M_Z \cos\theta_W = \tfrac{1}{2}\, g\, v}$$

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$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i\,[\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i\,[\hat{B}^\mu, \hat{B}^\nu]$$

$$\hat{W}^\mu = -\tfrac{g}{2}\,\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu = -\tfrac{g'}{2}\,\sigma_3\, B^\mu$$ **(explicit symmetry breaking)**

- **EW Goldstones are responsible for $M_{W,Z}$** **(not the Higgs!)**

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \quad \xrightarrow{U=1} \quad \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2\, Z_\mu Z^\mu$$

$$\mathbf{M_W = M_Z \cos\theta_W = \tfrac{1}{2}\, g\, v}$$

$$D^\mu U = \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \qquad , \qquad D^\mu U^\dagger = \partial^\mu U^\dagger + i\,U^\dagger \hat{W}^\mu - i\,\hat{B}^\mu U^\dagger$$

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- **EW Goldstones are responsible for $M_{W,Z}$** **(not the Higgs!)**
- **QCD pions also generate small W, Z masses:** $\delta_\pi M_W = \dfrac{1}{2}\, g\, f_\pi$

Goldstone interactions are determined by the underlying symmetry

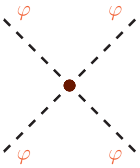
$$
\begin{aligned}
\frac{v^2}{4}\,\langle \partial_\mu U^\dagger \partial^\mu U\rangle &= \partial_\mu \varphi^- \partial^\mu \varphi^+ + \frac{1}{2}\,\partial_\mu \varphi^0 \partial^\mu \varphi^0 \\
&+ \frac{1}{6f^2}\left\{ \left(\varphi^+ \overleftrightarrow{\partial}_\mu \varphi^-\right)\left(\varphi^+ \overleftrightarrow{\partial}^\mu \varphi^-\right) + 2\left(\varphi^0 \overleftrightarrow{\partial}_\mu \varphi^+\right)\left(\varphi^- \overleftrightarrow{\partial}^\mu \varphi^0\right)\right\} \\
&+ O\left(\varphi^6/f^4\right)
\end{aligned}
$$

Goldstone interactions are determined by the underlying symmetry

$$\frac{v^2}{4}\langle\partial_\mu U^\dagger \partial^\mu U\rangle \;=\; \partial_\mu\varphi^-\partial^\mu\varphi^+ + \frac{1}{2}\,\partial_\mu\varphi^0\partial^\mu\varphi^0$$

$$+\;\; \frac{1}{6f^2}\left\{\left(\varphi^+\overleftrightarrow{\partial}_\mu\,\varphi^-\right)\left(\varphi^+\overleftrightarrow{\partial}^\mu\varphi^-\right) + 2\left(\varphi^0\overleftrightarrow{\partial}_\mu\,\varphi^+\right)\left(\varphi^-\overleftrightarrow{\partial}^\mu\varphi^0\right)\right\}$$

$$+\;\; O\left(\varphi^6/f^4\right)$$



$$T\left(\varphi^+\varphi^-\to\varphi^+\varphi^-\right) \;=\; \frac{s+t}{f^2}$$

Goldstone interactions are determined by the underlying symmetry

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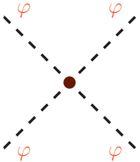
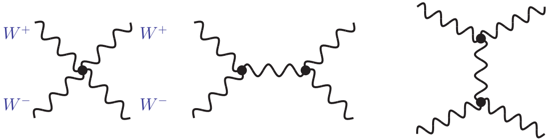


$$T\left(\varphi^+\varphi^-\to\varphi^+\varphi^-\right) \;=\; \frac{s+t}{f^2}$$

Non-Linear Lagrangian: $2\varphi\to 2\varphi,\ 4\varphi\ \cdots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$
\begin{aligned}
T(W_L^+ W_L^- \to W_L^+ W_L^-) &= \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right) \\
&= T(\varphi^+\varphi^- \to \varphi^+\varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)
\end{aligned}
$$

The scattering amplitude grows with energy

Goldstone dynamics ⟷ **derivative interactions**

Tree-level violation of unitarity

Longitudinal Polarizations

$$k^\mu = \left(k^0, 0, 0, |\vec{k}|\right) \qquad \longrightarrow \qquad \epsilon_L^\mu(\vec{k}) = \frac{1}{M_W} \left(|\vec{k}|, 0, 0, k^0\right) = \frac{k^\mu}{M_W} + O\left(\frac{M_W}{|\vec{k}|}\right)$$

Longitudinal Polarizations

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One naively expects $$T(W_L^+ W_L^- \to W_L^+ W_L^-) \;\sim\; g^2\,\frac{|\vec{k}|^4}{M_W^4}$$

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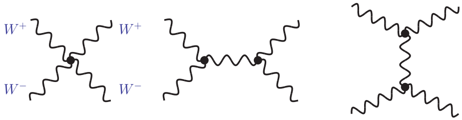
One naively expects $\quad T(W_L^+ W_L^- \to W_L^+ W_L^-) \sim g^2 \frac{|\vec{k}|^4}{M_W^4}$

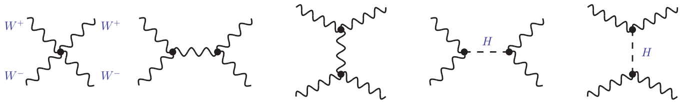


Gauge Cancelation

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$

$$= T(\varphi^+ \varphi^- \to \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

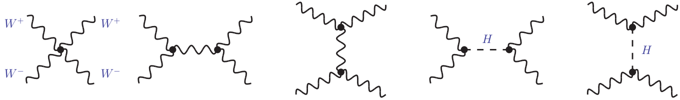
$W_L^+ W_L^- \to W_L^+ W_L^-$:



$$T_{\rm SM} = \frac{1}{v^2}\left\{ s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2} \right\} = -\frac{M_H^2}{v^2}\left\{ \frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

$W_L^+ W_L^- \to W_L^+ W_L^-$:



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Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

When $s \gg M_H^2$, $\quad T_{\rm SM} \approx -\frac{2M_H^2}{v^2} \quad , \quad a_0 \equiv \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \; T_{\rm SM} \approx -\frac{M_H^2}{8\pi v^2}$

Unitarity:

Lee–Quigg–Thacker

$$|a_0| \leq 1 \quad \Rightarrow \quad \mathbf{M_H < \sqrt{8\pi}\, v} \; \underbrace{\sqrt{2/3}}_{W^+W^-,\, ZZ,\, HH} \approx \mathbf{1\ TeV}$$

What happens in QCD?

- **QCD satisfies unitarity** **(it is a renormalizable theory)**
- **Pion scattering unitarized by exchanges of resonances** **(composite objects):**
  - **P-wave $(J = 1)$ unitarized by ρ exchange**
  - **S-wave $(J = 0)$ unitarized by σ exchange**
- **The σ meson is the QCD equivalent of the SM Higgs**
- **BUT, the σ is an 'effective' object generated through π rescattering (summation of pion loops)**

Does not seem to work this way in the EW case, but . . .

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\rm EW}^{(4)}\Big|_{\rm CP-even} = \sum_{i=0}^{14} a_i\, \mathcal{O}_i$$

(Appelquist, Longhitano)

$$\mathcal{O}_0 = v^2\, \langle T_L V_\mu\rangle^2$$

$\mathcal{O}_1 = \langle U\, \hat{B}_{\mu\nu}\, U^\dagger\, \hat{W}^{\mu\nu}\rangle$	$\mathcal{O}_2 = i\,\langle U\, \hat{B}_{\mu\nu}\, U^\dagger\, [V^\mu, V^\nu]\rangle$
$\mathcal{O}_3 = i\,\langle \hat{W}_{\mu\nu}\, [V^\mu, V^\nu]\rangle$	$\mathcal{O}_4 = \langle V_\mu V_\nu\rangle\, \langle V^\mu V^\nu\rangle$
$\mathcal{O}_5 = \langle V_\mu V^\mu\rangle^2$	$\mathcal{O}_6 = 4\,\langle V_\mu V_\nu\rangle\, \langle T_L V^\mu\rangle\, \langle T_L V^\nu\rangle$
$\mathcal{O}_7 = 4\,\langle V_\mu V^\mu\rangle\, \langle T_L V_\nu\rangle^2$	$\mathcal{O}_8 = \langle T_L \hat{W}_{\mu\nu}\rangle^2$
$\mathcal{O}_9 = -2\,\langle T_L \hat{W}_{\mu\nu}\rangle\, \langle T_L\, [V^\mu, V^\nu]\rangle$	$\mathcal{O}_{10} = 16\, \{\langle T_L V_\mu\rangle\, \langle T_L V_\nu\rangle\}^2$
$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2\rangle$	$\mathcal{O}_{12} = 4\,\langle T_L D_\mu D_\nu V^\nu\rangle\, \langle T_L V^\mu\rangle$
$\mathcal{O}_{13} = 2\,\langle T_L D_\mu V_\nu\rangle^2$	$\mathcal{O}_{14} = -2i\, \varepsilon^{\mu\nu\rho\sigma}\, \langle \hat{W}_{\mu\nu} V_\rho\rangle\, \langle T_L V_\sigma\rangle$

$$V_\mu \equiv D_\mu U\, U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i\,[\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \to g_L\, (V_\mu, D_\mu V_\nu, T_L)\, g_L^\dagger$$

Symmetry breaking: $T_L \equiv U\, \frac{\sigma_3}{2}\, U^\dagger \quad , \quad \hat{B}_{\mu\nu} \equiv -g'\, \frac{\sigma_3}{2}\, B_{\mu\nu}$

Low-Energy Effective Theory ➡ Power Counting

- **Momentum expansion:** $\Lambda \sim 4\pi v\, ,\, M_X$

$$T \;=\; \sum_n T_n \left(\frac{p}{\Lambda}\right)^n$$

- $U \sim O(p^0) \quad , \quad D_\mu U,\, \hat{W}_\mu,\, \hat{B}_\mu \sim O(p^1) \quad , \quad \hat{W}_{\mu\nu},\, \hat{B}_{\mu\nu} \sim O(p^2)$

- **A general connected diagram with N_d vertices of $O(p^d)$ and L Goldstone loops has a power dimension:**

$$D = 2L + 2 + \sum_d N_d\,(d-2)$$

➡ **Finite number of divergences / counterterms**

NLO Predictions

- $\mathcal{L}_{\mathbf{EW}}^{(2)}$ **at one loop:** **Unitarity**

  Non-local (logarithmic) dependences unambiguously predicted

- $\mathcal{L}_{\mathbf{EW}}^{(4)}$ **at tree level:** **Local (polynomic) amplitude**

  Short-distance information encoded in the a_i couplings

  Loop divergences reabsorbed through renormalized a_i

$$a_i \;=\; a_i^r(\mu) + \frac{\gamma_i}{16\pi^2}\left[\frac{2\,\mu^{D-4}}{4-D} + \log\left(4\pi\right) - \gamma_E\right]$$

$\hat{a}_i \equiv a_i/(16\pi)^2$ for different limits of the SM

	$M_H \to \infty$	$M_{t',b'} \to \infty$	$M_t \to \infty$
$\hat{a}_0$	$-\frac{3}{4}g'^2\left[\log(M_H/\mu) - \frac{5}{12}\right]$	0	$\frac{3}{2}\frac{M_t^2}{v^2}$
$\hat{a}_1$	$-\frac{1}{6}\log(M_H/\mu) + \frac{5}{72}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{1}{4}$
$\hat{a}_2$	$-\frac{1}{12}\log(M_H/\mu) + \frac{17}{144}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{3}{4}$
$\hat{a}_3$	$\frac{1}{12}\log(M_H/\mu) - \frac{17}{144}$	$\frac{1}{2}$	$\frac{3}{8}$
$\hat{a}_4$	$\frac{1}{6}\log(M_H/\mu) - \frac{17}{72}$	$\frac{1}{4}$	$\log(M_t/\mu) - \frac{5}{6}$
$\hat{a}_5$	$\frac{2\pi^2 v^2}{M_H^2} + \frac{1}{12}\log(M_H/\mu) - \frac{79}{72} + \frac{9\pi}{16\sqrt{3}}$	$-\frac{1}{8}$	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_6$	0	0	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_7$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_8$	0	0	$\log(M_t/\mu) - \frac{7}{12}$
$\hat{a}_9$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_{10}$	0	0	$-\frac{1}{64}$
$\hat{a}_{11}$	—	$-\frac{1}{2}$	$-\frac{1}{2}$
$\hat{a}_{12}$	—	0	$-\frac{1}{8}$
$\hat{a}_{13}$	—	0	$-\frac{1}{4}$
$\hat{a}_{14}$	0	0	$\frac{3}{8}$

Equations of Motion ➡ Redundant Operators

Equivalent to field redefinitions which only affect higher-order terms

For massless fermions: $\partial_\mu \langle T_L V^\mu \rangle = 0 \quad , \quad D_\mu V^\mu = 0$

$$\Longrightarrow \begin{cases} \mathcal{O}_{11} = \mathcal{O}_{12} = 0 \\ \mathcal{O}_{13} = -\frac{g'^2}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{O}_1 - \mathcal{O}_4 + \mathcal{O}_5 - \mathcal{O}_6 + \mathcal{O}_7 + \mathcal{O}_8 \end{cases}$$

Equations of Motion $\longrightarrow$ Redundant Operators

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Heavy top: $\mathcal{O}_{11} \doteq \frac{g^4}{8M_W^4}\, m_t^2 \left\{ (\bar{t}\gamma_5 t)^2 - 4 \sum_{i,j} (\bar{d}_{iL} t_R)(\bar{t}_R d_{jL})\, V_{tj} V_{ti}^* \right\}$

Unitary Gauge: $\mathbf{U = 1}$

All invariants reduce to polynomials of gauge fields

- **Bilinear terms:** $\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_8, \mathcal{O}_{11}, \mathcal{O}_{12}, \mathcal{O}_{13}$

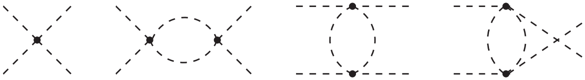
  ➡ **Oblique corrections** $(\Delta r, \Delta\rho, \Delta k \longleftrightarrow S, T, U)$

- **Trilinear terms:** $\mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_9, \mathcal{O}_{14}$

- **Quartic terms:** $\mathcal{O}_4, \mathcal{O}_5, \mathcal{O}_6, \mathcal{O}_7, \mathcal{O}_{10}$

- $\mathcal{O}_{11} \sim m_t^2\, (\bar\psi\psi)(\bar\psi\psi)$**:** $Z\bar{b}b,\ B^0\text{–}\bar{B}^0,\ \varepsilon_K \ldots$

$\varphi^a \varphi^b \to \varphi^c \varphi^d$:



$$\mathcal{A}(\varphi^a \varphi^b \to \varphi^c \varphi^d) \;=\; A(s,t,u)\,\delta_{ab}\,\delta_{cd} + A(t,s,u)\,\delta_{ac}\,\delta_{bd} + A(u,t,s)\,\delta_{ad}\,\delta_{bc}$$

$$\begin{aligned} A(s,t,u) \;=\; & \frac{s}{v^2} + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\,a_5^r(\mu)\,s^2\right] \\ & + \frac{1}{16\pi^2 v^2}\left\{\frac{5}{9}\,s^2 + \frac{13}{18}\,(t^2+u^2) + \frac{1}{12}\,(s^2-3t^2-u^2)\,\log\left(\frac{-t}{\mu^2}\right)\right. \\ & \left. + \frac{1}{12}\,(s^2-t^2-3u^2)\,\log\left(\frac{-u}{\mu^2}\right) - \frac{1}{2}\,s^2\,\log\left(\frac{-s}{\mu^2}\right)\right\} \end{aligned}$$

$$a_i \;=\; a_i^r(\mu) + \frac{\gamma_i}{16\pi^2}\left[\frac{2\,\mu^{D-4}}{4-D} + \log\,(4\pi) - \gamma_E\right] \quad , \quad \gamma_4 = -\frac{1}{12} \quad , \quad \gamma_5 = -\frac{1}{24}$$

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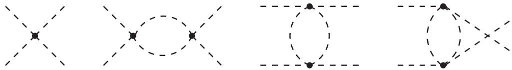


+ Higgs (tree + 1-loop) contributions

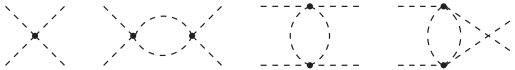
$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2\, a\, \frac{H}{v} + b\, \frac{H^2}{v^2} \right]$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned} A(s,t,u) \;=\; & \frac{s}{v^2}\,(1-a^2) + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\, a_5^r(\mu)\, s^2 \right] \\ & + \frac{1}{16\pi^2 v^2} \left\{ \frac{1}{9}\,(14\,a^4 - 10\,a^2 - 18\,a^2 b + 9\,b^2 + 5)\, s^2 + \frac{13}{18}\,(1-a^2)^2\,(t^2+u^2) \right. \\ & \qquad - \frac{1}{2}\,(2\,a^4 - 2\,a^2 - 2\,a^2 b + b^2 + 1)\, s^2 \log\left(\frac{-s}{\mu^2}\right) \\ & \qquad \left. + \frac{1}{12}\,(1-a^2)^2 \left[(s^2 - 3t^2 - u^2)\log\left(\frac{-t}{\mu^2}\right) + (s^2 - t^2 - 3u^2)\log\left(\frac{-u}{\mu^2}\right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\tfrac{1}{12}\,(1-a^2)^2 \qquad , \qquad \gamma_5 = -\tfrac{1}{48}\,(2 + 5\,a^4 - 4\,a^2 - 6\,a^2 b + 3\,b^2)$$

$$\varphi^a \varphi^b \to \varphi^c \varphi^d$$



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Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

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$$\gamma_4 \; = \; -\tfrac{1}{12}\, (1 - a^2)^2 \qquad , \qquad \gamma_5 \; = \; -\tfrac{1}{48}\, (2 + 5\, a^4 - 4\, a^2 - 6\, a^2 b + 3\, b^2)$$

SM: $a = b = 1$, $a_4 = a_5 = 0$ ⟶ $A(s,t,u) \sim \mathcal{O}(M_H^2/v^2)$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L\, U(\varphi) \left[\hat{\mathbf{Y}}_{\mathbf{u}}\, \mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}}\, \mathcal{P}_- \right] Q_R + \bar{L}_L\, U(\varphi)\, \hat{\mathbf{Y}}_{\ell}\, \mathcal{P}_+\, L_R + \text{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger \quad , \quad Q_L \to g_L\, Q_L \quad , \quad Q_R \to g_R\, Q_R \quad , \quad \mathcal{P}_\pm \to g_R\, \mathcal{P}_\pm\, g_R^\dagger$$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L \, U(\varphi) \left[\hat{\mathbf{Y}}_{\mathbf{u}} \, \mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}} \, \mathcal{P}_- \right] Q_R + \bar{L}_L \, U(\varphi) \, \hat{\mathbf{Y}}_{\ell} \, \mathcal{P}_+ \, L_R + \mathrm{h.c.} \right\}$$

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$$U(\varphi) \to g_L \, U(\varphi) \, g_R^\dagger \quad , \quad Q_L \to g_L \, Q_L \quad , \quad Q_R \to g_R \, Q_R \quad , \quad \mathcal{P}_\pm \to g_R \, \mathcal{P}_\pm \, g_R^\dagger$$

Symmetry Breaking: $\mathcal{P}_\pm = \frac{1}{2} \left(\mathrm{I}_2 \pm \sigma_3 \right)$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L \, U(\varphi) \left[\hat{\mathbf{Y}}_{\mathbf{u}} \, \mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}} \, \mathcal{P}_- \right] Q_R + \bar{L}_L \, U(\varphi) \, \hat{\mathbf{Y}}_{\ell} \, \mathcal{P}_+ \, L_R + \text{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L \, U(\varphi) \, g_R^\dagger \quad , \quad Q_L \to g_L \, Q_L \quad , \quad Q_R \to g_R \, Q_R \quad , \quad \mathcal{P}_\pm \to g_R \, \mathcal{P}_\pm \, g_R^\dagger$$

Symmetry Breaking: $\mathcal{P}_\pm = \frac{1}{2} \left(\mathrm{I}_2 \pm \sigma_3 \right)$

Flavour Structure: $\hat{\mathbf{Y}}_{\mathbf{u},\mathbf{d},\ell}$ **3×3 matrices in flavour space**

NLO Operators

Buchalla–Catá

$$\mathcal{O}_{\psi V1} = i\,\bar{Q}_L\gamma^\mu Q_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V2} = i\,\bar{Q}_L\gamma^\mu T_L Q_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V3} = i\,\bar{Q}_L\gamma^\mu \tilde{P}_{12} Q_L\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V4} = i\,\bar{u}_R\gamma^\mu u_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V5} = i\,\bar{d}_R\gamma^\mu d_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V6} = i\,\bar{u}_R\gamma^\mu d_R\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V7} = i\,\bar{L}_L\gamma^\mu L_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V8} = i\,\bar{L}_L\gamma^\mu T_L L_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V9} = \bar{L}_L\gamma^\mu \tilde{P}_{12} L_L\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V10} = \bar{\ell}_R\gamma^\mu \ell_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}^\dagger_{\psi V3} \qquad \mathcal{O}^\dagger_{\psi V6}$$

$$\mathcal{O}_{\psi S1,2} = \bar{Q}_L\tilde{P}_\pm UQ_R\,\langle D_\mu U^\dagger D^\mu U\rangle \qquad \mathcal{O}_{\psi S3,4} = \bar{Q}_L\tilde{P}_\pm UQ_R\,\langle V_\mu T_L\rangle^2$$

$$\mathcal{O}_{\psi S5} = \bar{Q}_L\tilde{P}_{12} UQ_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\mu T_L\rangle \qquad \mathcal{O}_{\psi S6} = \bar{Q}_L\tilde{P}_{12} Q_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V^\mu T_L\rangle$$

$$\mathcal{O}_{\psi S7} = \bar{L}_L\tilde{P}_- UL_R\,\langle D_\mu U^\dagger D^\mu U\rangle \qquad \mathcal{O}_{\psi S8} = \bar{L}_L\tilde{P}_- UL_R\,\langle V^\mu T_L\rangle^2$$

$$\mathcal{O}_{\psi S9} = \bar{L}_L\tilde{P}_{12} UL_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V^\mu T_L\rangle$$

$$\mathcal{O}_{\psi T1} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_{12} UQ_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\nu T_L\rangle \qquad \mathcal{O}_{\psi T2} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_{21} UQ_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V^\nu T_L\rangle$$

$$\mathcal{O}_{\psi T3,4} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_\pm UQ_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V_\nu \tilde{P}_{21}\rangle \qquad \mathcal{O}_{\psi T5} = \bar{L}_L\sigma^{\mu\nu}\tilde{P}_{12} UL_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\nu T_L\rangle$$

$$\mathcal{O}_{\psi T6} = \bar{L}_L\sigma^{\mu\nu}\tilde{P}_- UL_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V_\nu \tilde{P}_{21}\rangle$$

$$V_\mu = D_\mu U\, U^\dagger \quad , \quad T_L = U\,\frac{\sigma_3}{2}\,U^\dagger \quad , \quad \tilde{P}_{12} = U\,\frac{\sigma_{1+i2}}{2}\,U^\dagger \quad , \quad \tilde{P}_{21} = U\,\frac{\sigma_{1-i2}}{2}\,U^\dagger \quad , \quad \tilde{P}_\pm = U\,P_\pm\,U^\dagger$$

NLO Operators (cont.)

$$\mathcal{O}_{LL6} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{Q}_L\gamma_\mu T_L Q_L \qquad \mathcal{O}_{LL7} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{Q}_L\gamma_\mu Q_L \qquad \mathcal{O}_{LL8} = \bar{q}_{L\alpha}\gamma^\mu T_L Q_{L\beta}\ \bar{Q}_{L\beta}\gamma_\mu T_L Q_{L\alpha}$$

$$\mathcal{O}_{LL10} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LL11} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{L}_L\gamma_\mu L_L \qquad \mathcal{O}_{LL9} = \bar{Q}_{L\alpha}\gamma^\mu T_L Q_{L\beta}\ \bar{Q}_{L\beta}\gamma_\mu Q_{L\alpha}$$

$$\mathcal{O}_{LL12} = \bar{Q}_L\gamma^\mu Q_L\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LL13} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu T_L Q_L \qquad \mathcal{O}_{LL14} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu Q_L$$

$$\mathcal{O}_{LL15} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LL16} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu L_L$$

$$\mathcal{O}_{LR10} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{u}_R\gamma_\mu u_R \qquad \mathcal{O}_{LR12} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{d}_R\gamma_\mu d_R \qquad \mathcal{O}_{LR11} = \bar{Q}_L\gamma^\mu t^a T_L Q_L\ \bar{u}_R\gamma_\mu t^a u_R$$

$$\mathcal{O}_{LR14} = \bar{u}_R\gamma^\mu u_R\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LR15} = \bar{d}_R\gamma^\mu d_R\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LR13} = \bar{Q}_L\gamma^\mu t^a T_L Q_L\ \bar{d}_R\gamma_\mu t^a d_R$$

$$\mathcal{O}_{LR16} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{\ell}_R\gamma_\mu \ell_R \qquad \mathcal{O}_{LR17} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{\ell}_R\gamma_\mu \ell_R \qquad \mathcal{O}_{LR18} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{\ell}_R\gamma_\mu d_R$$

$$\mathcal{O}_{ST5} = \bar{Q}_L\tilde{P}_+ UQ_R\ \bar{Q}_L\tilde{P}_- UQ_R \qquad \mathcal{O}_{ST6} = \bar{Q}_L\tilde{P}_{21} UQ_R\ \bar{Q}_L\tilde{P}_{12} UQ_R \qquad \mathcal{O}_{ST7} = \bar{Q}_L t^a\tilde{P}_+ UQ_R\ \bar{Q}_L t^a\tilde{P}_- UQ_R$$

$$\mathcal{O}_{ST9} = \bar{Q}_L\tilde{P}_+ UQ_R\ \bar{L}_L\tilde{P}_- UL_R \qquad \mathcal{O}_{ST10} = \bar{Q}_L\tilde{P}_{21} UQ_R\ \bar{L}_L\tilde{P}_{12} UL_R \qquad \mathcal{O}_{ST8} = \bar{Q}_L t^a\tilde{P}_{21} UQ_R\ \bar{Q}_L t^a\tilde{P}_{12} UQ_R$$

$$\mathcal{O}_{ST11} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_+ UQ_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_- UL_R \qquad \mathcal{O}_{ST12} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_{21} UQ_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_{12} UL_R$$

$$\mathcal{O}_{FY4} = \bar{Q}_L t^a\tilde{P}_- UQ_R\ \bar{Q}_L t^a\tilde{P}_- UQ_R \qquad \mathcal{O}_{FY8} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_- UQ_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_- UL_R$$

$$\mathcal{O}_{FY1} = \bar{Q}_L\tilde{P}_+ UQ_R\ \bar{Q}_L\tilde{P}_+ UQ_R \qquad \mathcal{O}_{FY3} = \bar{Q}_L\tilde{P}_- UQ_R\ \bar{Q}_L\tilde{P}_- UQ_R \qquad \mathcal{O}_{FY2} = \bar{Q}_L t^a\tilde{P}_+ UQ_R\ \bar{Q}_L t^a\tilde{P}_+ UQ_R$$

$$\mathcal{O}_{FY5} = \bar{Q}_L\tilde{P}_- UQ_R\ \bar{Q}_R U^\dagger\tilde{P}_+ Q_L \qquad \mathcal{O}_{FY7} = \bar{Q}_L\tilde{P}_- UQ_R\ \bar{L}_L\tilde{P}_- UL_R \qquad \mathcal{O}_{FY6} = \bar{Q}_L t^a\tilde{P}_- UQ_R\ \bar{Q}_R t^a U^\dagger\tilde{P}_+ Q_L$$

$$\mathcal{O}_{FY9} = \bar{L}_L\tilde{P}_- UL_R\ \bar{Q}_R U^\dagger\tilde{P}_+ Q_R \qquad \mathcal{O}_{FY10} = \bar{L}_L\tilde{P}_- UL_R\ \bar{L}_L\tilde{P}_- UL_R \qquad \mathcal{O}_{FY11} = \bar{L}_L\tilde{P}_- UQ_R\ \bar{Q}_R U^\dagger\tilde{P}_+ L_L$$

Including a Light (singlet) Higgs in the EWET

Let us assume that h(126) is an $SU(2)_{L+R}$ scalar singlet

All Higgsless operators can be multiplied by an arbitrary function of **h**:

$$\mathcal{O}_X \quad \longrightarrow \quad \tilde{\mathcal{O}}_X \equiv \mathcal{F}_X(h)\, \mathcal{O}_X$$

$$\mathcal{F}_X(h) = \sum_{n=0} c_X^{(n)} \left(\frac{h}{v}\right)^n$$

In addition, the LO Lagrangian should include the **scalar potential**:

$$V(h) = v^4 \sum_{n=2} c_V^{(n)} \left(\frac{h}{v}\right)^n$$

Plus operators with derivatives $(\partial_\mu \mathbf{h})$: $F_X \equiv F_X(h)$

$$\mathcal{O}_{D7} = -\langle V_\mu V^\mu \rangle \frac{\partial_\nu h\, \partial^\nu h}{v^2} F_{D7} \qquad \mathcal{O}_{D8} = -\langle V_\mu V_\nu \rangle \frac{\partial^\mu h\, \partial^\nu h}{v^2} F_{D8} \qquad \mathcal{O}_{D11} = \frac{(\partial_\mu h\, \partial^\mu h)^2}{v^4} F_{D11}$$

$$\mathcal{O}_{D6} = -\langle T_L V_\mu V_\nu \rangle \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{D6} \qquad \mathcal{O}_{D9} = -\langle T_L V_\mu \rangle \langle T_L V^\mu \rangle \frac{\partial_\nu h\, \partial^\nu h}{v^2} F_{D9}$$

$$\mathcal{O}_{D10} = -\langle T_L V_\mu \rangle \langle T_L V_\nu \rangle \frac{\partial^\mu h\, \partial^\nu h}{v^2} F_{D10} \qquad \mathcal{O}_{\psi S18} = \bar{L}_L \tilde{P}_- U L_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S18}$$

$$\mathcal{O}_{\psi S10} = -i\, \bar{Q}_L \tilde{P}_+ U Q_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S10} \qquad \mathcal{O}_{\psi S11} = -i\, \bar{Q}_L \tilde{P}_- U Q_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S11}$$

$$\mathcal{O}_{\psi S12} = -i\, \bar{Q}_L \tilde{P}_{12} U Q_R \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S12} \qquad \mathcal{O}_{\psi S13} = -i\, \bar{Q}_L \tilde{P}_{21} U Q_R \langle \tilde{P}_{12} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S13}$$

$$\mathcal{O}_{\psi S14} = \bar{Q}_L \tilde{P}_+ U Q_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S14} \qquad \mathcal{O}_{\psi S15} = \bar{Q}_L \tilde{P}_- U Q_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S15}$$

$$\mathcal{O}_{\psi S16} = -i\, \bar{L}_L \tilde{P}_- U L_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S16} \qquad \mathcal{O}_{\psi S17} = -i\, \bar{L}_L \tilde{P}_{12} U L_R \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S17}$$

$$\mathcal{O}_{\psi T7} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_+ U Q_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T7} \qquad \mathcal{O}_{\psi T8} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_- U Q_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T8}$$

$$\mathcal{O}_{\psi T9} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{21} U Q_R \langle \tilde{P}_{12} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T9} \qquad \mathcal{O}_{\psi T10} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{12} U Q_R \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T10}$$

$$\mathcal{O}_{\psi T11} = -i\, \bar{L}_L \sigma_{\mu\nu} \tilde{P}_- U L_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T11} \qquad \mathcal{O}_{\psi T12} = -i\, \bar{L}_L \sigma_{\mu\nu} \tilde{P}_{12} U L_R \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T12}$$

Buchalla–Catà–Krause

Linear Realization: $SU(2)_L \otimes U(1)_Y$

Assumes that H(126) and $\vec{\varphi}$ combine into an $SU(2)_L$ doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{2}(v+H)\, U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The SM Lagrangian is the low-energy effective theory with $D = 4$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}}\, \mathcal{O}_i^{(D)}$$

- **1 operator with D = 5:** $\mathcal{O}^{(5)} = \bar{L}_L \tilde{\Phi} \tilde{\Phi}^T L_L^c$ **(violates L by 2 units)**

  Weinberg

- **59 independent $\mathcal{O}_i^{(6)}$ preserving B and L (for 1 generation)**

  Buchmuller–Wyler, Grzadkowski–Iskrzynski–Misiak–Rosiek

- **5 independent $\mathcal{O}_i^{(6)}$ violating B and L (for 1 generation)**

  Weinberg, Wilczek–Zee, Abbott–Wise,

$\mathbf{D = 6}$ Operators (other than 4-fermion ones)

Grzadkowski–Iskrzynski–Misiak–Rosiek

X^3		Φ^6 and $\Phi^4 D^2$		$\psi^2\Phi^3$	
$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_\Phi$	$(\Phi^\dagger\Phi)^3$	$\mathcal{O}_{e\Phi}$	$(\Phi^\dagger\Phi)(\bar{l}_p e_r \Phi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{\Phi\Box}$	$(\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$	$\mathcal{O}_{u\Phi}$	$(\Phi^\dagger\Phi)(\bar{q}_p u_r \widetilde{\Phi})$
$\mathcal{O}_W$	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{\Phi D}$	$(\Phi^\dagger D^\mu\Phi)^\star (\Phi^\dagger D_\mu\Phi)$	$\mathcal{O}_{d\Phi}$	$(\Phi^\dagger\Phi)(\bar{q}_p d_r \Phi)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\Phi^2$		$\psi^2 X\Phi$		$\psi^2\Phi^2 D$	
$\mathcal{O}_{\Phi G}$	$\Phi^\dagger\Phi\, G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu} e_r)\,\tau^I\Phi\, W_{\mu\nu}^I$	$\mathcal{O}_{\Phi l}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\Phi\widetilde{G}}$	$\Phi^\dagger\Phi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu} e_r)\,\Phi\, B_{\mu\nu}$	$\mathcal{O}_{\Phi l}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I \Phi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\Phi W}$	$\Phi^\dagger\Phi\, W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu} T^A u_r)\,\widetilde{\Phi}\, G_{\mu\nu}^A$	$\mathcal{O}_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\Phi\widetilde{W}}$	$\Phi^\dagger\Phi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu} u_r)\,\tau^I\widetilde{\Phi}\, W_{\mu\nu}^I$	$\mathcal{O}_{\Phi q}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\Phi B}$	$\Phi^\dagger\Phi\, B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu} u_r)\,\widetilde{\Phi}\, B_{\mu\nu}$	$\mathcal{O}_{\Phi q}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I \Phi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\Phi\widetilde{B}}$	$\Phi^\dagger\Phi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu} T^A d_r)\,\Phi\, G_{\mu\nu}^A$	$\mathcal{O}_{\Phi u}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\Phi WB}$	$\Phi^\dagger\tau^I\Phi\, W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu} d_r)\,\tau^I\Phi\, W_{\mu\nu}^I$	$\mathcal{O}_{\Phi d}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\Phi\widetilde{W}B}$	$\Phi^\dagger\tau^I\Phi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu} d_r)\,\Phi\, B_{\mu\nu}$	$\mathcal{O}_{\Phi ud}$	$i(\widetilde{\Phi}^\dagger D_\mu\Phi)(\bar{u}_p\gamma^\mu d_r)$

$q = q_L\,,\; l = l_L\,,\; u = u_R\,,\; d = d_R\,,\; e = e_R \qquad , \qquad p, r =$ generation indices

$D = 6$ Four-Fermion Operators

Grzadkowski–Iskrzynski–Misiak–Rosiek

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\,\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k u_t)$	$\mathcal{O}_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}\,(\tau^I\varepsilon)_{jk}\,(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

$q = q_L$, $l = l_L$, $u = u_R$, $d = d_R$, $e = e_R$, $p, r, s, t =$ generation indices

OUTLOOK

- **Effective Field Theory: powerful low-energy tool**
- **Mass Gap:** $E, m_{light} \ll \Lambda_{NP}$
- **Assumption: relevant symmetries (breakings) & light fields**
- **Most general $\mathcal{L}_{eff}(\phi_{light})$ allowed by symmetry**
- **Short-distance dynamics encoded in LECs**
- **LECs constrained phenomenologically**
- **Goal: get hints on the underlying fundamental dynamics**

$\longrightarrow$ **New Physics**

Learning from QCD experience. **EW problem more difficult**

Fundamental Underlying Theory unknown

QCD	?
↓	↑
χPT	Standard Model

Additional dynamical input (fresh ideas!) needed

Backup Slides

QCD Matching

$$(\mu > M) \qquad \mathcal{L}_{\mathrm{QCD}}^{(N_F)} \quad \Longleftrightarrow \quad \mathcal{L}_{\mathrm{QCD}}^{(N_F-1)} + \sum_{d_i>4} \frac{c_i}{M^{d_i-4}}\, O_i \qquad (\mu < M)$$

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$$\alpha_s^{(N_F)}(\mu^2) \; = \; \alpha_s^{(N_F-1)}(\mu^2) \left\{ 1 + \sum_{k=1}^{\infty} C_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

$$L \equiv \ln\left(\mu^2/m_q^2\right)$$

$$m_q^{(N_F)}(\mu^2) \; = \; m_q^{(N_F-1)}(\mu^2) \left\{ 1 + \sum_{k=1}^{\infty} H_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

QCD Matching

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- Matching conditions known to 4 (3) loops: $C_{1,2,3,4}$, $H_{1,2,3}$

  (Schroder-Steinhauser, Chetyrkin et al, Larin et al)

- L dependence known to 4 loops: $H_4(L)$
- **$\alpha_s(\mu^2)$ is not continuous at threshold**

Goldstone Theorem

$$\mathcal{Q} = \int d^3x\, j^0(x) \quad ; \quad \partial_\mu j_a^\mu = 0 \quad ; \quad \exists\, \mathcal{O} : \; v(t) \equiv \langle 0 | \left[\mathcal{Q}(t), \mathcal{O}\right] | 0 \rangle \neq 0$$

$$\Downarrow$$

$$\exists\, |n\rangle : \; \langle 0|\mathcal{O}|n\rangle\, \langle n|j^0|0\rangle \neq 0 \quad ; \quad E_n\, \delta^{(3)}\left(\vec{p}_n\right) = 0 \quad ; \quad M_n = 0$$

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Proof:

$$j^0(x) = \mathrm{e}^{iP\cdot x}\, j^0(0)\, \mathrm{e}^{-iP\cdot x} \qquad ; \qquad \sum_n |n\rangle\langle n| = 1$$

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$$=\, \sum_n \int d^3x\, \left\{\mathrm{e}^{-ip_n\cdot x}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle \,-\, \mathrm{e}^{ip_n\cdot x}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle\right\}$$

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$$= \sum_n \int d^3x \, \left\{ \mathrm{e}^{-ip_n\cdot x} \, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle - \mathrm{e}^{ip_n\cdot x} \, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle \right\}$$

$$= (2\pi)^3 \sum_n \delta^{(3)}(\vec{p}_n) \, \left\{ \mathrm{e}^{-iE_n t} \, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle - \mathrm{e}^{iE_n t} \, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle \right\} \neq 0$$

$$\frac{d}{dt} \, v(t) = 0$$

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$$\frac{d}{dt}\, v(t) \,=\, 0 \,=\, -i\,(2\pi)^3 \sum_n \delta^{(3)}(\vec{p}_n)\, E_n\, \left\{\, \mathrm{e}^{-iE_n t}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle \right.$$

$$\left. +\, \mathrm{e}^{iE_n t}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle\right\} \qquad \square$$

Noether QCD Currents: $G \equiv SU(3)_L \otimes SU(3)_R$

$$J_X^{a\mu} = \bar{\mathbf{q}}_X \, \gamma^\mu \, \frac{\lambda^a}{2} \, \mathbf{q}_X \quad ; \quad \mathcal{Q}_X^a = \int d^3x \, J_X^{a0}(x) \qquad (a = 1, \cdots, 8 \; ; \; X = L, R)$$

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Current Algebra ('60) : $\left[\mathcal{Q}_X^a \, , \, \mathcal{Q}_Y^b\right] \, = \, i \, \delta_{XY} \, f^{abc} \, \mathcal{Q}_X^c$

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Dynamical Symmetry Breaking:

- 8 Pseudoscalar Goldstones $\pi^a = (\pi, K, \eta)$

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- $\mathcal{Q}_A^a = \mathcal{Q}_R - \mathcal{Q}_L \quad ; \quad \mathcal{O}^b = \bar{\mathbf{q}} \, \gamma_5 \, \lambda^b \, \mathbf{q}$

$$\langle 0| \left[\mathcal{Q}_A^a \, , \mathcal{O}^b\right] |0\rangle = -\frac{1}{2} \, \langle 0| \, \bar{\mathbf{q}} \, \left\{\lambda^a \, , \lambda^b\right\} \mathbf{q} \, |0\rangle = -\frac{2}{3} \, \langle 0| \, \bar{\mathbf{q}} \, \mathbf{q} \, |0\rangle$$

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$$\longrightarrow \qquad \langle 0 | \, \bar{u} \, u \, | 0 \rangle = \langle 0 | \, \bar{d} \, d \, | 0 \rangle = \langle 0 | \, \bar{s} \, s \, | 0 \rangle \neq 0$$

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$$\longrightarrow \qquad \langle 0| \, \bar{u} \, u \, |0\rangle = \langle 0| \, \bar{d} \, d \, |0\rangle = \langle 0| \, \bar{s} \, s \, |0\rangle \neq 0$$

- $\langle 0| \, J_A^{a\mu} \, |\pi^b(p)\rangle = i \, \delta^{ab} \, \sqrt{2} \, f_\pi \, p^\mu$

Chiral Anomaly: $\delta Z[v,a,s,p] = -\frac{N_C}{16\pi^2} \int d^4x \, \langle \delta\beta(x)\, \Omega(x) \rangle$

$$g_{L,R} \approx 1 + i\delta\alpha \mp i\delta\beta$$

$$\Omega(x) = \varepsilon^{\mu\nu\sigma\rho} \Big[v_{\mu\nu} v_{\sigma\rho} + \tfrac{4}{3} \nabla_\mu a_\nu \nabla_\sigma a_\rho + \tfrac{2}{3} i \{ v_{\mu\nu}, a_\sigma a_\rho \} + \tfrac{8}{3} i\, a_\sigma v_{\mu\nu} a_\rho + \tfrac{4}{3}\, a_\mu a_\nu a_\sigma a_\rho \Big]$$

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i\,[v_\mu, v_\nu] \quad , \qquad \nabla_\mu a_\nu = \partial_\mu a_\nu - i\,[v_\mu, a_\nu] \quad , \qquad \varepsilon_{0123} = 1$$

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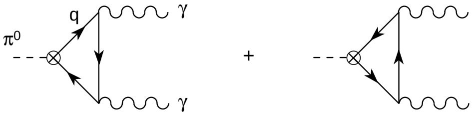
$$S[U,\ell,r]_{WZW} = -\frac{iN_C}{240\pi^2}\int d\sigma^{ijklm}\,\langle \Sigma^L_i \Sigma^L_j \Sigma^L_k \Sigma^L_l \Sigma^L_m\rangle - \frac{iN_C}{48\pi^2}\int d^4x\;\varepsilon_{\mu\nu\alpha\beta}\left(W(U,\ell,r)^{\mu\nu\alpha\beta} - W(\mathbf{1},\ell,r)^{\mu\nu\alpha\beta}\right)$$

Wess–Zumino–Witten

$$\begin{aligned} W(U,\ell,r)_{\mu\nu\alpha\beta} = {} & \Big\langle U\ell_\mu\ell_\nu\ell_\alpha U^\dagger r_\beta + \frac{1}{4}U\ell_\mu U^\dagger r_\nu U\ell_\alpha U^\dagger r_\beta + iU\partial_\mu\ell_\nu\ell_\alpha U^\dagger r_\beta \\ & + i\partial_\mu r_\nu U\ell_\alpha U^\dagger r_\beta - i\Sigma^L_\mu \ell_\nu U^\dagger r_\alpha U\ell_\beta + \Sigma^L_\mu U^\dagger \partial_\nu r_\alpha U\ell_\beta - \Sigma^L_\mu\Sigma^L_\nu U^\dagger r_\alpha U\ell_\beta \\ & + \Sigma^L_\mu \ell_\nu \partial_\alpha \ell_\beta + \Sigma^L_\mu \partial_\nu \ell_\alpha \ell_\beta - i\Sigma^L_\mu \ell_\nu\ell_\alpha\ell_\beta + \frac{1}{2}\Sigma^L_\mu\ell_\nu\Sigma^L_\alpha\ell_\beta - i\Sigma^L_\mu\Sigma^L_\nu\Sigma^L_\alpha\ell_\beta\Big\rangle \\ & - (L \leftrightarrow R) \end{aligned}$$

$$\Sigma^L_\mu = U^\dagger \partial_\mu U \qquad , \qquad \Sigma^R_\mu = U\partial_\mu U^\dagger$$

$\pi^0 \to \gamma\gamma$:



$A_3^\mu \equiv \bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d$

$$\Gamma(\pi^0 \to \gamma\gamma) \; = \; \left(\frac{N_c}{3}\right)^2 \frac{\alpha^2 M_\pi^3}{64\,\pi^3 f_\pi^2} \; = \; 7.73 \;\; \mathrm{eV}$$

Exp: $(7.7 \pm 0.6)\ \mathrm{eV}$

There are no QCD corrections

The chiral anomaly contributes to: $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$

$\gamma\, 3\pi$, $\gamma\, \pi^+\pi^-\eta$, $K\bar{K}3\pi$, $\cdots$

Goldstone Electroweak Effective Theory

$$\mathcal{L}_{\mathrm{EW}}^{(2)} \; = \; -\frac{1}{2g^2}\,\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle \; - \; \frac{1}{2g'^{\,2}}\,\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\,\langle D^\mu U^\dagger D_\mu U\rangle$$

$$U(\varphi) \; = \; \exp\left\{\frac{i\sqrt{2}}{v}\,\Phi\right\} \qquad , \qquad \Phi \;\equiv\; \frac{1}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{\varphi} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\,\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\,\varphi^0 \end{pmatrix}$$

$$D^\mu U \;=\; \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \qquad , \qquad D^\mu U^\dagger \;=\; \partial^\mu U^\dagger + i\,U^\dagger\,\hat{W}^\mu - i\,\hat{B}^\mu\,U^\dagger \qquad , \qquad \langle A\rangle \;\equiv\; \mathrm{Tr}(A)$$

$$\hat{W}^{\mu\nu} \;=\; \partial^\mu\hat{W}^\nu - \partial^\nu\hat{W}^\mu - i\,[\hat{W}^\mu,\hat{W}^\nu] \qquad , \qquad \hat{B}^{\mu\nu} \;=\; \partial^\mu\hat{B}^\nu - \partial^\nu\hat{B}^\mu - i\,[\hat{B}^\mu,\hat{B}^\nu]$$

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$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\quad U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger$

$$\hat{W}^\mu \to g_L\,\hat{W}^\mu g_L^\dagger + i\,g_L\,\partial^\mu g_L^\dagger \qquad , \qquad \hat{B}^\mu \to g_R\,\hat{B}^\mu g_R^\dagger + i\,g_R\,\partial^\mu g_R^\dagger$$

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$$\mathcal{L}_{\rm EW}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle$$

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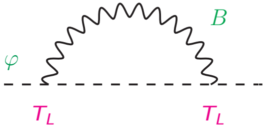
$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i\,[\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i\,[\hat{B}^\mu, \hat{B}^\nu]$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ Symmetry: $\quad U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger$

$$\hat{W}^\mu \to g_L\,\hat{W}^\mu g_L^\dagger + i\,g_L\,\partial^\mu g_L^\dagger \quad , \quad \hat{B}^\mu \to g_R\,\hat{B}^\mu g_R^\dagger + i\,g_R\,\partial^\mu g_R^\dagger$$

SM Symmetry Breaking: $\quad \hat{W}^\mu = -\frac{g}{2}\,\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu = -\frac{g'}{2}\,\sigma_3\,B^\mu$

Custodial Symmetry Breaking: $\hat{B}_\mu \equiv -g' \frac{\sigma_3}{2} B_\mu$



$$\longrightarrow \quad \langle V_\mu T_L V^\mu T_L \rangle$$

$$V_\mu \equiv D_\mu U\, U^\dagger = i\, \frac{\sqrt{2}}{v}\, D_\mu \Phi + \cdots \qquad , \qquad T_L \equiv U\, \frac{\sigma_3}{2}\, U^\dagger \qquad , \qquad T_L T_L = \frac{1}{4}\, I_2$$

$$\begin{aligned} \langle V_\mu T_L V^\mu T_L \rangle &= \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle - \frac{1}{2} \langle V_\mu V^\mu \rangle \langle T_L T_L \rangle \\ &= \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle - \frac{1}{4} \langle V_\mu V^\mu \rangle \end{aligned}$$

$$\longrightarrow \qquad \mathcal{O}_0 = v^2 \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle$$

Decoupling:

Appelquist–Carazzone

The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles

SM: $M_W = M_Z \cos\theta_W = \frac{1}{2}\, g\, v \quad , \quad M_H = \sqrt{2\lambda}\, v \quad , \quad M_f = \frac{1}{\sqrt{2}}\, y_f\, v$

- **Decoupling occurs when $v \to \infty$, keeping the couplings fixed**

- **There is no decoupling if some $M_i \to \infty$, keeping $v = 246$ GeV**
  $(g, \lambda, y_f \to \infty)$

$\varphi^a \varphi^b \to \varphi^c \varphi^d$:

- **Isospin:**

$$\begin{aligned} A_0(s,t,u) &= 3\,A(s,t,u) + A(t,s,u) + A(u,t,s) \\ A_1(s,t,u) &= A(t,s,u) - A(u,t,s) \\ A_2(s,t,u) &= A(t,s,u) + A(u,t,s) \end{aligned}$$

- **Partial Waves:**

$$\begin{aligned} A_{IJ}(s) &= \frac{1}{64\pi} \int_{-1}^{+1} d\cos\theta\; P_J(\cos\theta)\; A_I(s,t,u) \\ \sigma(s) &= \frac{64\pi}{s} \sum_{I,J} (2I+1)\,(2J+1)\; |A_{IJ}|^2 \end{aligned}$$

$$\begin{aligned} A_{00}(s) &= \frac{s}{16\pi v^2}\left\{1 + \frac{s}{16\pi^2 v^2}\left[\frac{101}{36} + \frac{64\pi^2}{3}\,(7\,a_4^r + 11\,a_5^r) - \frac{25}{18}\,\log\left(\frac{s}{\mu^2}\right) + i\,\pi\right] + \cdots\right\} \\ A_{11}(s) &= \frac{s}{96\pi v^2}\left\{1 + \frac{s}{16\pi^2 v^2}\left[\frac{1}{9} + 64\pi^2\,(a_4^r - 2\,a_5^r) + i\,\frac{\pi}{6}\right] + \cdots\right\} \\ A_{20}(s) &= \frac{-s}{32\pi v^2}\left\{1 + \frac{s}{16\pi^2 v^2}\left[-\frac{91}{36} - \frac{256\pi^2}{3}\,(2\,a_4^r + a_5^r) + \frac{10}{9}\,\log\left(\frac{s}{\mu^2}\right) - i\,\frac{\pi}{2}\right] + \cdots\right\} \end{aligned}$$

Power Counting

- **Momentum expansion:** $\Lambda \sim 4\pi v\, ,\, M_X$

$$T \;=\; \sum_n T_n \, \left(\frac{p}{\Lambda}\right)^n$$

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- **Loop Expansion:** A generic L-loop diagram $\mathcal{D}$ scales as

$$\mathcal{D} \sim \frac{(yv)^{\nu}(gv)^{m+2r+2x+u+z}}{v^{F_L+F_R-2-2\omega}}\,\frac{p^d}{\Lambda^{2L}}\;\bar\psi_L^{F_L^1}\psi_L^{F_L^2}\bar\psi_R^{F_R^1}\psi_R^{F_R^2}\left(\frac{X_{\mu\nu}}{v}\right)^V\left(\frac{\varphi}{v}\right)^B\left(\frac{h}{v}\right)^H$$

$$d \equiv 2L + 2 - \tfrac{1}{2}\,(F_L + F_R) - V - \nu - m - 2r - 2x - u - z - 2\omega$$

Buchalla–Catà–Krause

external fields: $F_L = F_L^1 + F_L^2$, $F_R = F_R^1 + F_R^2$, B , H , V (X_μ = gauge boson)

vertices: $m \equiv \sum_l m_l\ (X_\mu\varphi^l)$, $r \equiv \sum_s r_s\ (X_\mu^2\varphi^s)$, $u\ (X_\mu^3)$, $x\ (X_\mu^4)$, $\omega \equiv \sum_q \omega_q\ (h^q)$

$\nu \equiv \sum_k \nu_k\ (\bar\psi\psi\varphi^k) + \sum_{t,b}\tau_{tb}\ (\bar\psi\psi\varphi^t h^b)$, $z \equiv z_L + z_R\ (\bar\psi_\lambda\psi_\lambda X_\mu\,,\ \lambda = L, R)$

Too many operators/couplings ➡ **Further input needed**

$$\mathcal{F}_X(h) \;=\; \sum_{n=0} c_X^{(n)} \left(\frac{h}{v}\right)^n \;=\; \sum_{n=0} \tilde{c}_X^{(n)} \left(\frac{g_h\, h}{\Lambda_{\mathrm{NP}}}\right)^n$$

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- **Weak coupling:** $g_h \ll 1$
- **Strong coupling:** $g_h \sim 4\pi = \Lambda_{\mathrm{NP}}/f$ ➡ $\mathcal{F}_X(h/f)$
- **v ≪ f** ➡ $\xi \equiv \frac{v^2}{f^2}$, $c_X^{(n)} \;=\; \tilde{c}_X^{(n)}\, \xi^{n/2}$