Introduction to the Soft - Collinear Effective Theory

An effective field theory for energetic hadrons & jets

Lecture 1

Symmetry Breaking Summer School
on Lake Chiemsee, Germany
September, 2014

Outline (Lecture I)

- EFT concepts
  Intro to SCET
  SCET degrees of freedom

Done on the Board
(See the lecture notes below.)

- SCETI , momentum scales and regions
- Field power counting in SCET
- Wilson lines, W, from off shell propagators
- Gauge Symmetry
- Hard-Collinear Factorization
- eg. Deep Inelastic Scattering

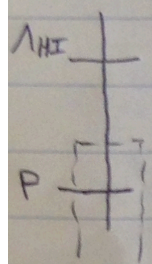
Soft - Collinear Effective Theory (SCET)

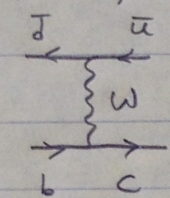
Iain Stewart

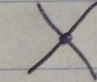
Symm Breaking 20

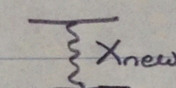
EFT Concepts

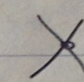
Decoupling heavy or offshell particles → decouple



• 

→  $1/M_W^2$ $M_W^2 \gg p_i^2$

or 

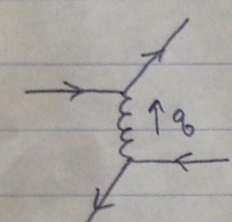
→  $\frac{1}{M_X^2}$

SM as an EFT

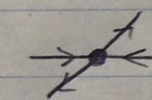
$\mathcal{L} = \mathcal{L}^{(0)}_{SM} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$

$\frac{\mathcal{O}}{M_X}$

(power counting $\frac{1}{m} \leftrightarrow$ mass dim $\mathcal{O}$)

• 

large mom. exchange

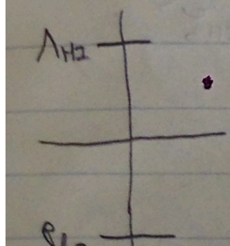
$q^2 \gg p_i^2$ 

(power counting not in mass dim)

Any EFT

- degrees of freedom? → what fields
  (low energy / on-shell modes)
- symmetries → constrain interactions / operators
- expansions, leading description → power counting

Renormalization



Split HI from LO (cutoff or renormalization scale)

$$\mathcal{L}_{EFT} = \sum_i C_i(\mu)\, \mathcal{O}_i(\mu)$$

short dist. couplings   long dist. operators

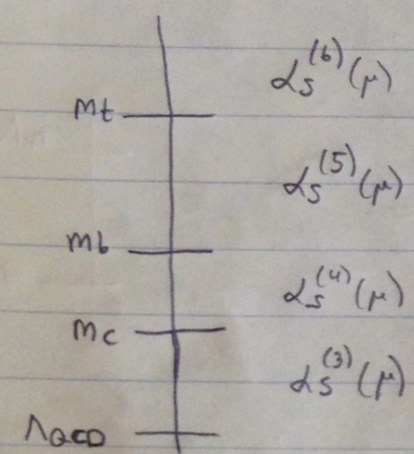
- $\mathcal{L}_{HI}$ & $\mathcal{L}_{EFT}$ have same IR, differ in UV
- $C_i(\mu)$ indep. of IR scale

"top-down EFT" (including SCET)

$$\boxed{\text{Know } \mathcal{L}_{HI}} \downarrow \boxed{\mathcal{L}_{EFT}}$$

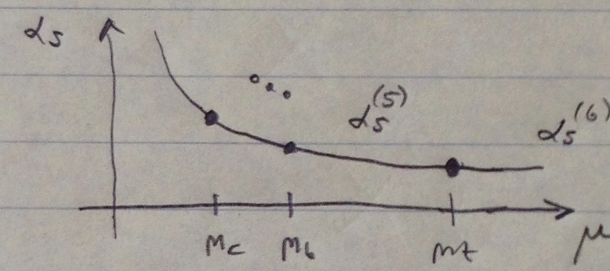
Calculate C [perturbatively
Construct $\mathcal{O}$

eg. $\alpha_s^{(n_f)}(\mu)$ in QCD



$$\mu \frac{d}{d\mu} \alpha_s^{(n_f)}(\mu) = -\frac{\beta_0^{(n_f)}}{2\pi} \left[\alpha_s^{(n_f)}(\mu)\right]^2$$

$$\beta_0^{(n_f)} = 11 - \frac{2}{3} n_f$$



Homework

$\boxed{\text{SCET}}$ describe soft & collinear infrared physics of hard collisions in QCD

QCD → SCET

$\Lambda_{QCD}^2, p_{IR}^2 \ll Q^2$ → jets, energetic hadrons, soft partons / hadrons

[$e^+e^- \to 2$ jets, DIS $e^-p \to e^-X$, $pp \to H + 1\text{-jet}$, $B \to \pi\pi, \ldots$]

Goals: Factorization → calculate σ, understand how nonperturbative effects (eg. PDFs) enter

Sum Sudakov Double Logs with RGE → higher precision

Wilson Lines, Operator Universality

Power Corrections

Coordinates

$n^\mu = (1, \hat{n})$ $\qquad \hat{n}^2 = 1$

$\bar{n}^\mu = (1, -\hat{n})$ $\qquad n^2 = \bar{n}^2 = 0$

$n \cdot \bar{n} = 2$

$$g^{\mu\nu} = \frac{n^\mu \bar{n}^\nu}{2} + \frac{\bar{n}^\mu n^\nu}{2} + g_\perp^{\mu\nu},$$

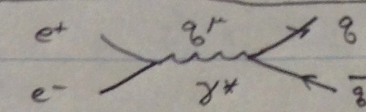
$$p^\mu = \underbrace{\bar{n}\cdot p}_{p^-} \frac{n^\mu}{2} + \underbrace{n \cdot p}_{p^+} \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

$$p^2 = (\bar{n}\cdot p)(n \cdot p) + \underbrace{p_\perp^2}_{-\vec{p}_\perp^{\,2}}$$

Sideboard

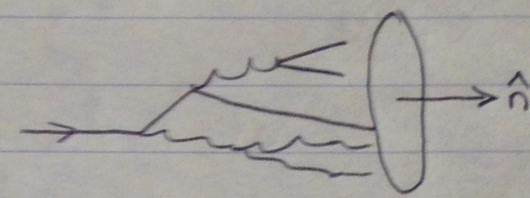
Degrees of freedom

$\boxed{e^+e^- \to 2 \text{ jets}}$



$Q^2 = q^2$

$\boxed{\text{Jet}}$:



has collimated radiation in $\hat{n}$

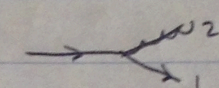
$E_{jet} \sim Q$

collinear?

1 massless

$p^\mu = \bar{n}\cdot p \, \frac{n^\mu}{2}$

2 massless



$p_i^\mu = \underbrace{\bar{n}\cdot p_i}_{\sim Q} \frac{n^\mu}{2} + \underbrace{p_{i\perp}}_{\sim \lambda Q \text{ say}} + \underbrace{n\cdot p_i}_{\sim \lambda^2 Q} \frac{\bar{n}^\mu}{2}$ (i=1,

$\lambda \ll 1$ (collimated)

$n\cdot p_i \sim \lambda^2$ since on-shell $p_i^2 = \underbrace{\bar{n}\cdot p_i}_{\lambda^0} \times \underbrace{n\cdot p_i}_{\lambda^2} + \underbrace{p_{\perp i}^2}_{\lambda^2} = 0$

n massless: same

Collinear Fields

quark ξ_n

gluon A_n^μ

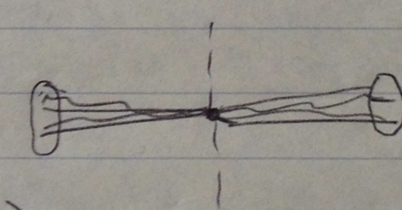
$p^\mu \sim Q(\overset{n\cdot p}{\lambda^2}, \overset{\bar{n}\cdot p}{1}, \overset{p_\perp}{\lambda})$

$\boxed{\text{2 back-to-back jets}}$

$\xi_{\bar{n}}$

$A_{\bar{n}}$

$(1, \lambda^2, \lambda)$

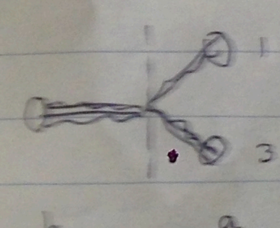


ξ_n

A_n

$(\lambda^2, 1, \lambda)$

3 hard jets? Measure something to forbid it



$(p_1+p_3)^2 \sim 2p_1\cdot p_3 \sim Q^2$

eg. measure hemisphere mass $M_a^2 = \left(\sum_{i\in a} p_i^\mu\right)^2$

$M_b^2 = \dots$

take $M_{a,b}^2 \sim Q^2\lambda^2 \ll Q^2$

↑ allows splittings $(p_{n1}+p_{n2})^2$

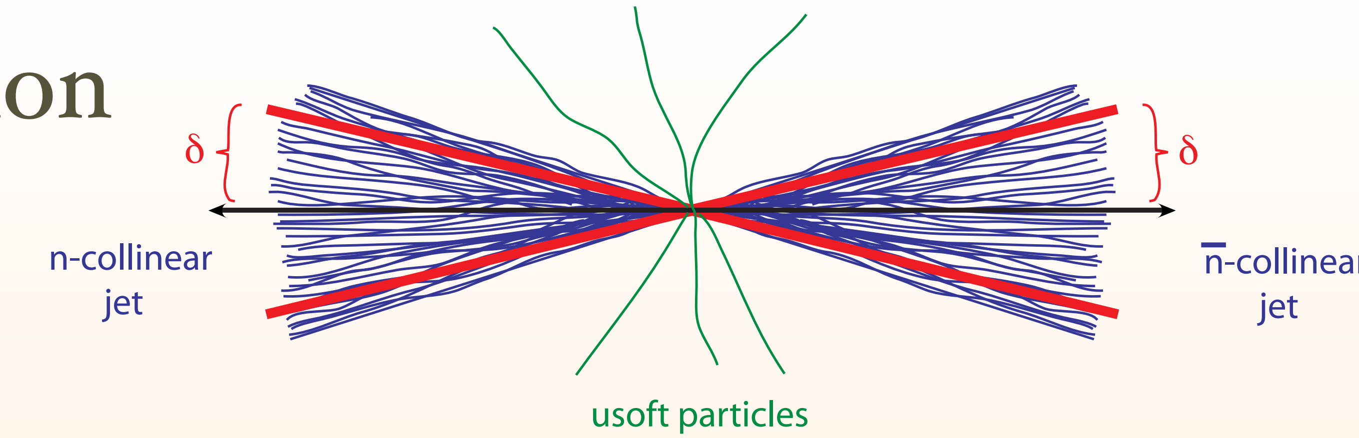
$\boxed{\text{Soft}}$ $p_s^\mu \sim Q(\lambda^\alpha, \lambda^\alpha, \lambda^\alpha)$ homogeneous

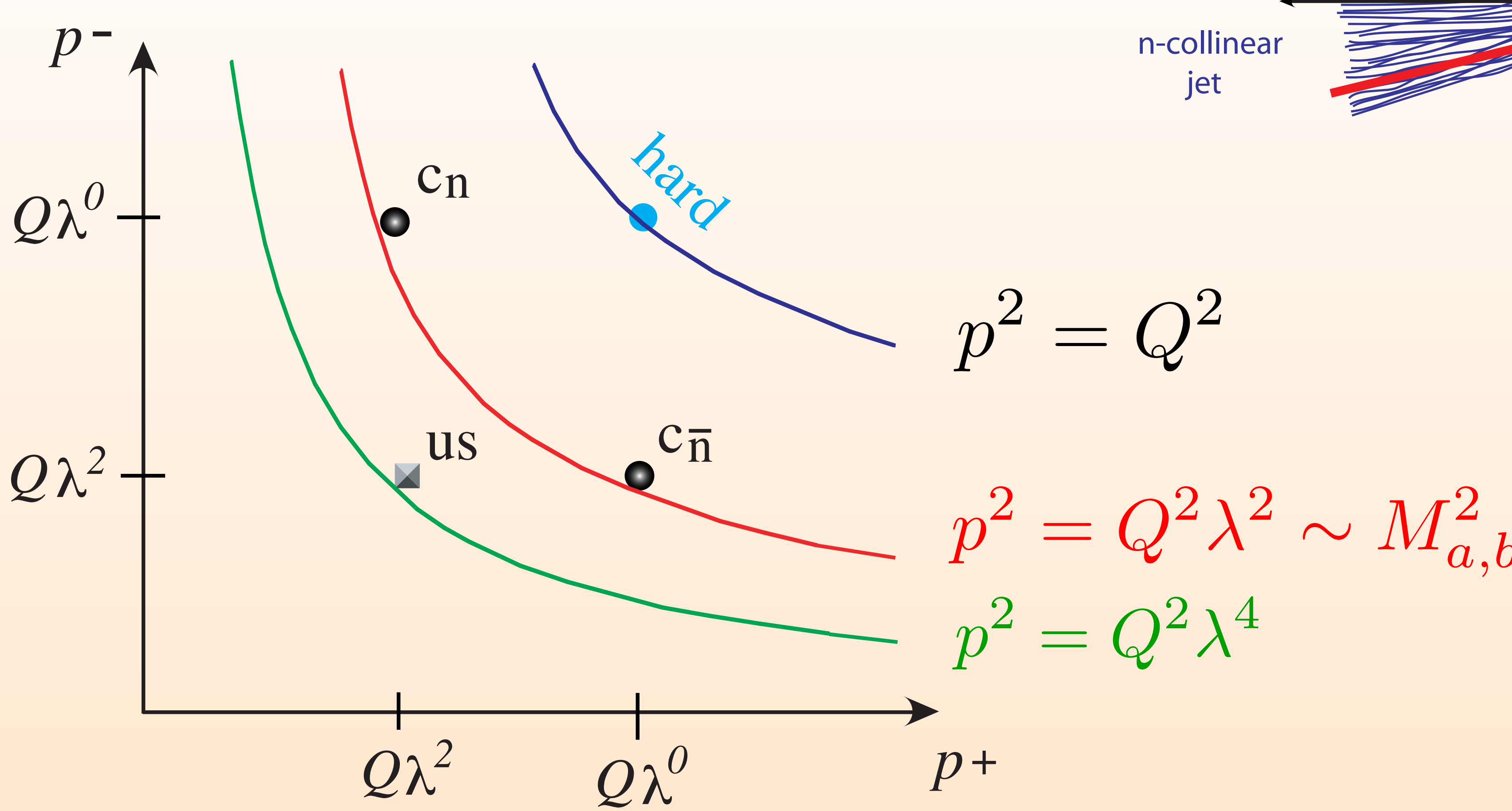
$M_a^2 = (p_n+p_s)^2 \sim p_n\cdot p_s \sim \underbrace{\bar{n}\cdot p_n}_{\lambda^0} \, \underbrace{n\cdot p_s}_{\lambda^\alpha} + \dots$ (suppressed)

$\Rightarrow \alpha = 2$ "ultrasoft"

$\boxed{\text{continue with slides}}$

Two jets and ultrasoft radiation





$p^2 = p^+ p^-$
for picture

Comments:

1) multiple modes for IR

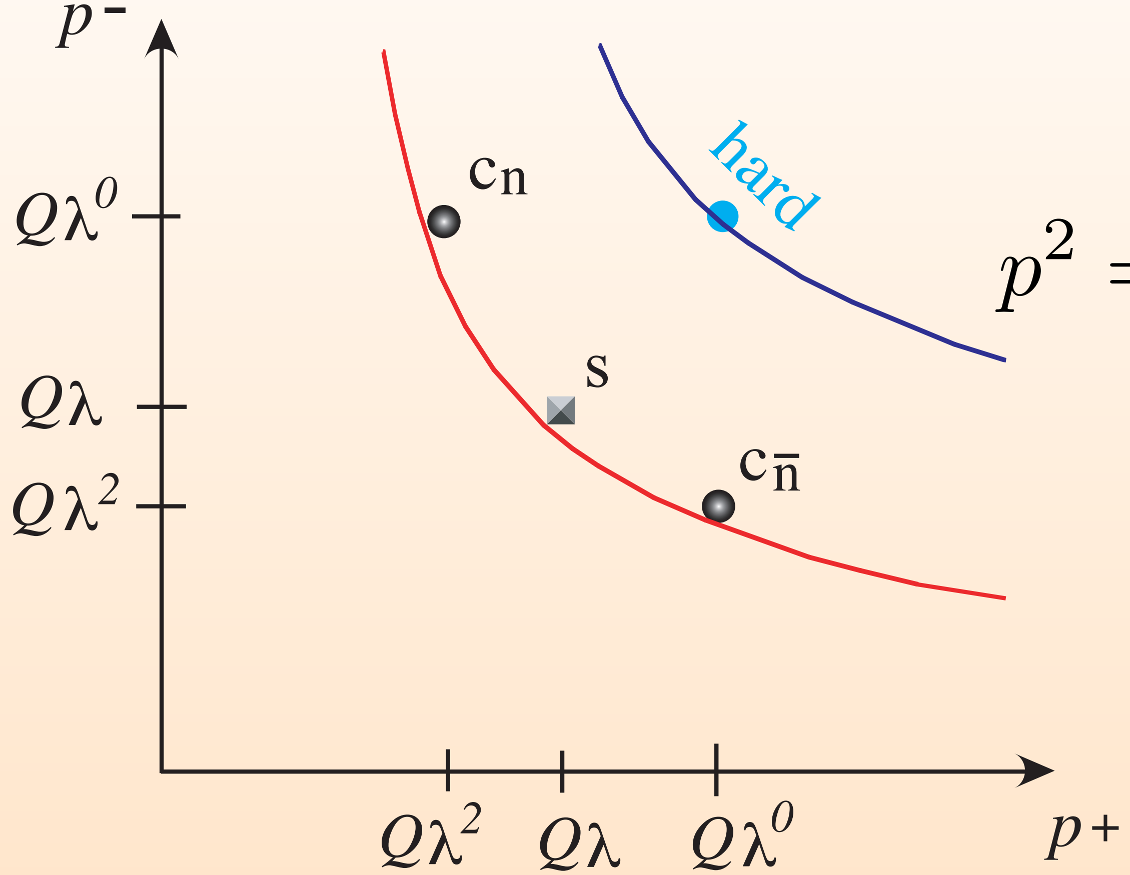
2) integrate out modes above a hyperbola

SCET$_{\mathrm{I}}$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

Another SCET: $\mathrm{SCET_{II}}$ (not covered here)

Two jets and soft radiation with $p_\perp$-type measurement



$$p^2 = Q^2$$

$$p^2 = p^+ p^-$$
for picture

soft $\quad p_s^\mu \sim Q\lambda$

instead of ultrasoft $\; p_{us}^\mu \sim Q\lambda^2$

$\mathrm{SCET_{II}}$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

n-Collinear Propagators

$$p^2 + i\epsilon = \bar{n}\cdot p \; n\cdot p - \vec{p}_\perp^{\,2} + i\epsilon$$
$$\sim \; \lambda^0 * \lambda^2 \; - (\lambda)^2$$

same size

Collinear Fermions

$$\frac{i\,\not p}{p^2+i\epsilon} = \frac{i\,\not n}{2}\,\frac{\bar{n}\cdot p}{p^2+i\epsilon} + \ldots$$
$$= \frac{i\,\not n}{2}\,\frac{1}{n\cdot p - \frac{\vec{p}_\perp^{\,2}}{\bar{n}\cdot p} + i\epsilon\,\mathrm{sign}(\bar{n}\cdot p)} + \ldots$$

thus we expect

$$\underbrace{\int d^4x}_{\lambda^{-4}}\, e^{ip\cdot x}\, \langle 0|T\underbrace{\xi_n(x)\bar{\xi}_n(0)}_{\text{must be }\lambda^2}|0\rangle = \frac{i\,\not n}{2}\,\underbrace{\frac{\bar{n}\cdot p}{p^2+i\epsilon}}_{\lambda^{-2}}$$

so $\boxed{\xi_n \sim \lambda}$

power counting for the field

$$d^4x \sim (dp^+ dp^- d^2p_\perp)^{-1}$$
$$\lambda^2 \quad \lambda^0 \quad (\lambda)^2$$

This also implies: $\not n\,\xi_n = 0$ since $\not n^2 = n^2 = 0$

Projection:

Take $\xi_n = \frac{\not{n}\not{\bar{n}}}{4}\psi$ for spin

$$\boxed{\frac{\not{n}\not{\bar{n}}}{4}\xi_n = \xi_n \quad , \quad \not{n}\xi_n = 0}$$

For spinors:

QCD SCET $p^+ \ll p^-$

$$u_n = \frac{\not{n}\not{\bar{n}}}{4}\, u^{\mathrm{QCD}}$$

$$u_+(p) = |p+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{p^-}\\ \sqrt{p^+}e^{i\,\phi_p}\\ \sqrt{p^-}\\ \sqrt{p^+}e^{i\,\phi_p}\end{pmatrix} \Longrightarrow u_n^+ = \sqrt{\frac{p^-}{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

$$e^{\pm i\phi_p} = \frac{p_\perp^1 \pm i p_\perp^2}{\sqrt{p^+p^-}}$$

$$u_-(p) = |p-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{p^+}e^{i\,\phi_p}\\ -\sqrt{p^-}\\ -\sqrt{p^+}e^{i\,\phi_p}\\ \sqrt{p^-}\end{pmatrix} \Longrightarrow u_n^- = \sqrt{\frac{p^-}{2}}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

Check:

$$\sum_s u_n^s \bar{u}_n^s = \frac{\not{n}\not{\bar{n}}}{4}\sum_s u^s\bar{u}^s\,\frac{\not{\bar{n}}\not{n}}{4} = \frac{\not{n}\not{\bar{n}}}{4}\,\not{p}\,\frac{\not{\bar{n}}\not{n}}{4} = \frac{\not{n}}{2}\,\bar{n}\cdot p$$

agrees with numerator of propagator $\quad i\frac{\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2 + i\epsilon}$

Gauge Fields for SCET$_\mathrm{I}$

Collinear Gluons - same propagator as QCD

covariant gauges

$$\int d^4x\, e^{ip\cdot x}\, \langle 0|T A_n^\mu(x) A_n^\nu(0)|0\rangle = \frac{-i}{p^2}\Big(g^{\mu\nu} - \tau \frac{p^\mu p^\nu}{p^2}\Big)$$

components scale differently

solution $\boxed{(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p^\mu}$

Usoft Gluon $A_{us}^\mu \sim (\lambda^2, \lambda^2, \lambda^2) \sim p_{us}^\mu$

Usoft Quark $q_{us} \sim \lambda^3$

Power Counting Summary

Type	$(p^+, p^-, p^\perp)$	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}$	λ
		$(A^+_{n,p}, A^-_{n,p}, A^\perp_{n,p})$	$(\lambda^2, 1, \lambda)$
soft	$(\lambda, \lambda, \lambda)$	$q_{s,p}$	$\lambda^{3/2}$
		$A^\mu_{s,p}$	λ
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	q_{us}	λ^3
		A^μ_{us}	λ^2

Power counting of fields and derivatives gives a power counting for operators

Power counting of operators yields a power counting for any Feynman graph

The power counting can be associated entirely to vertices and is then gauge invariant

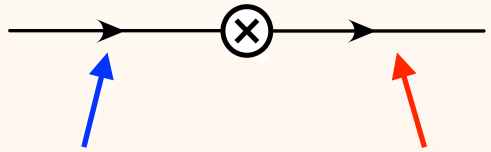
$$(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p^\mu$$

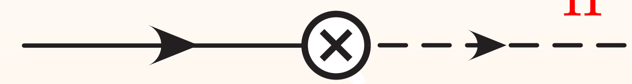
Currents

QCD | **SCET**

$\bar{\psi}\,\Gamma\,\psi$

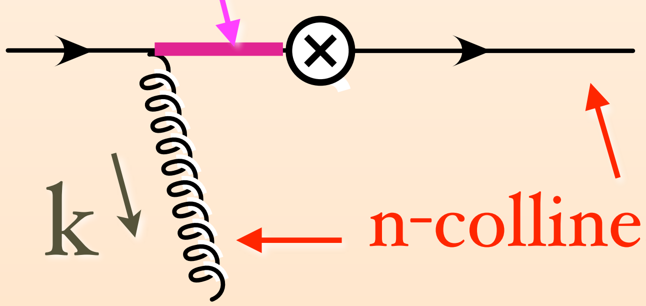
no gluons

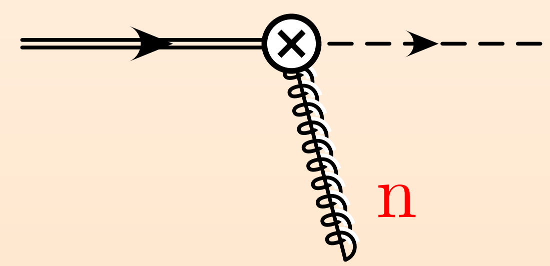




$\bar{\xi}_n\ \Gamma\ \psi$

one gluon





$$\bar{\xi}_n \frac{(-g\bar{n}\cdot A_n)}{-\bar{n}\cdot k}\Gamma\psi$$

offshell $\quad (q-k)^2 = q^2 + k^2 - 2q\cdot k = -\bar{n}\cdot k\, n\cdot q + \ldots$

gluon field $\quad \not{A}^a = \bar{n}\cdot A^a\,\frac{\not{n}}{2} + \ldots$

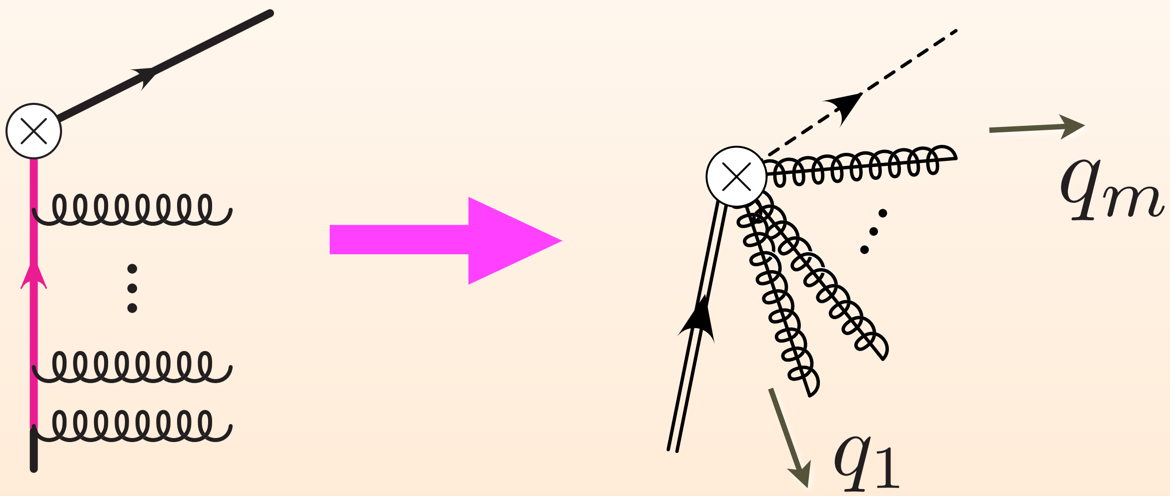
graph = $\bar{u}_n\Gamma\frac{i(\not{q}-\not{k}+m)}{(q-k)^2-m^2}ig\not{A}^aT^a\gamma^\mu u(q)$ = ... (Homework #2) ...

$$= \frac{(-g\bar{n}\cdot A_n^a)}{-\bar{n}\cdot k}\,\bar{u}_n\Gamma T^a u(q)$$

Currents

add any number of gluons

$\Rightarrow$ $\bar{\xi}_n W_n \, \Gamma \, \psi$ get a Wilson line W



$$= \; g^m \sum_{\text{perms}} \frac{(\bar{n}^{\mu_m} T^{A_m}) \cdots (\bar{n}^{\mu_1} T^{A_1})}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)] \cdots [\bar{n}\cdot \sum_{i=1}^m q_i]}$$

$\sim \lambda^0$ no cost to add these gluons

momentum space Wilson line $\quad W_n = \sum_k \sum_{\text{perms}} \frac{(-g)^k}{k!} \left(\frac{\bar{n}\cdot A_{\bar{n},q_1} \cdots \bar{n}\cdot A_{\bar{n},q_k}}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)] \cdots [\bar{n}\cdot \sum_{i=1}^k q_i]} \right)$

position space Wilson line $\quad W_n(y,-\infty) = P \exp\left(ig \int_{-\infty}^0 ds \, \bar{n}\cdot A_n(y+s\bar{n}) \right)$

$W_n \sim \lambda^0$

(more on Homework #2)

SCET Operator: $\quad \underbrace{\bar{\xi}_n W_n}_{\bar{\chi}_n} \, \Gamma \, \psi$

this is generic, gives an operator "building block" like a "parton" or "jet" field

Gauge symmetry

$$U(x) = \exp\left[i\alpha^A(x)T^A\right]$$

need to consider U's which leave us in the EFT

collinear $\quad i\partial^\mu \mathcal{U}_c(x) \sim p_c^\mu \mathcal{U}_c(x) \leftrightarrow A^\mu_{n,q}$

usoft $\quad i\partial^\mu U_{us}(x) \sim p_{us}^\mu U_{us}(x) \leftrightarrow A^\mu_{us}$

Object	Collinear $\mathcal{U}_c$	Usoft U_{us}
ξ_n	$\mathcal{U}_c\, \xi_n$	$U_{us}\, \xi_n$
gA_n^μ	$\mathcal{U}_c\, gA_n^\mu\, \mathcal{U}_c^\dagger + \mathcal{U}_c\left[i\mathcal{D}^\mu, \mathcal{U}_c^\dagger\right]$	$U_{us}\, gA_n^\mu\, U_{us}^\dagger$
W	$\mathcal{U}_c\, W$	$U_{us}\, W\, U_{us}^\dagger$
q_{us}	q_{us}	$U_{us}\, q_{us}$
gA_{us}^μ	gA_{us}^μ	$U_{us} gA_{us}^\mu U_{us}^\dagger + U_{us}[i\partial^\mu, U_{us}^\dagger]$
Y	Y	$U_{us}\, Y$

our current is invariant:

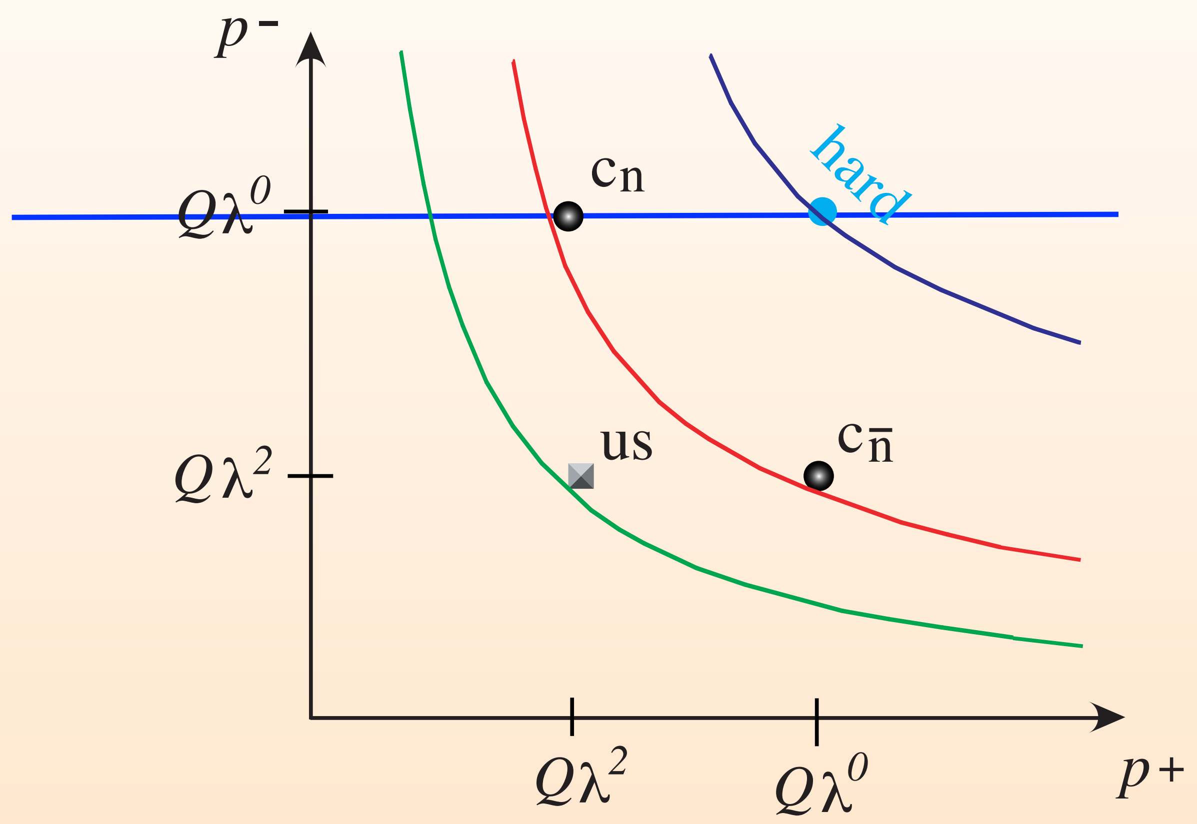
$$(\bar\xi_n W)\Gamma\psi \quad \rightarrow (\bar\xi_n \mathcal{U}_c^\dagger \mathcal{U}_c W)\Gamma\psi = (\bar\xi_n W)\Gamma\psi$$

$$\rightarrow (\bar\xi_n U_{us}^\dagger U_{us} W) U_{us}^\dagger \Gamma U_{us}\psi = (\bar\xi_n W)\Gamma\psi$$

Wilson Coefficients and Hard - Collinear Factorization

$$\begin{array}{lll} & & \underline{(+,-,\perp)} \\ \text{hard:} & p^\mu \sim & (1\,,\,1\,,\,1) \\ \text{collinear:} & p^\mu \sim & (\lambda^2, 1, \lambda) \end{array}$$

can exchange momenta



$i\bar{n}\cdot\partial_n \sim \lambda^0$

Constrained by gauge invariance:

$C(i\bar{n}\cdot\partial_n)$ coefficients depend on large collinear momenta

eg. $$C(i\bar{n}\cdot\partial_n)\ \chi_n = \int d\omega\, C(\omega)\, \delta(\omega - i\bar{n}\cdot\partial_n)\chi_n$$

only the product is gauge invariant $\chi_n = W_n^\dagger\, \xi_n$

implies convolutions between coefficients and operators $\displaystyle \int d\omega\ C(\omega,\mu)\ O(\omega,\mu)$

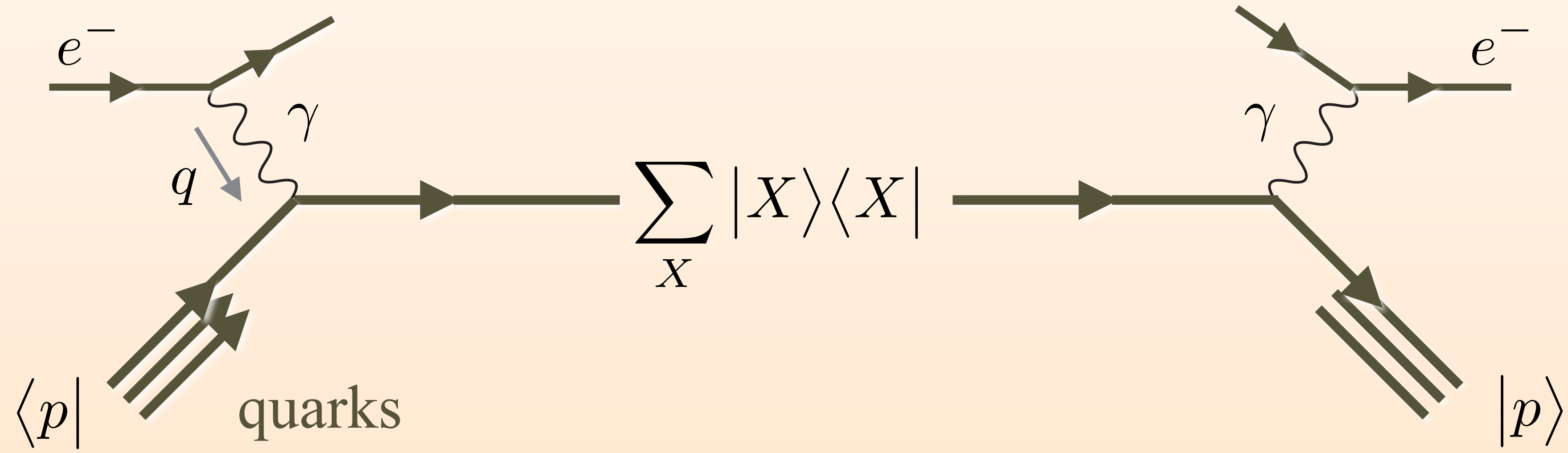
Deep Inelastic Scattering $\quad e^- p \to e^- X$

inclusive factorization

[full analysis requires bit more knowledge, eg. SCET Lagrangian, here we cover the key conceptual part, skipping softs, prefactors, tensor indices, etc.]

$$\sigma = (\text{prefactor})\ L^{\text{leptons}}_{\mu\nu} \sum_X (2\pi)^4 \delta^{(4)}(p_p + q - p_X)\, \langle p | J^\mu(0) | X \rangle \langle X | J^\nu(0) | p \rangle$$

$$J^\mu = \bar\psi \gamma^\mu \psi$$



$$q = (0,0,0,Q) = \frac{Q}{2}(\bar n - n)$$ picked a frame (Breit frame), Bjorken $x = \dfrac{Q^2}{2p_p \cdot q}$

$q^2 = -Q^2$ spacelike $\qquad Q^2 \gg \Lambda^2_{\text{QCD}} \qquad \lambda = \dfrac{\Lambda_{\text{QCD}}}{Q} \ll 1$

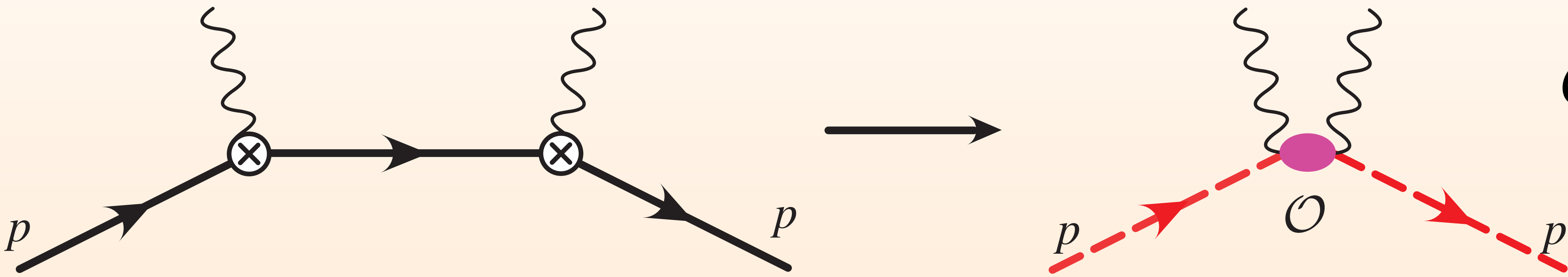
Proton $\quad p_p^\mu = \dfrac{n^\mu}{2} \bar n \cdot p_p + \dfrac{\bar n^\mu}{2} \dfrac{m_p^2}{\bar n \cdot p_p}$ $\quad$ n-collinear $\qquad \bar n \cdot p_p = \dfrac{Q}{x}$

big $\qquad$ small

X $\quad p_X^\mu = p_p^\mu - q^\mu = \dfrac{n^\mu}{2}\dfrac{Q(1-x)}{x} + \dfrac{\bar n^\mu}{2} Q$ $\quad$ hard, offshell

Deep Inelastic Scattering $\quad e^- p \to e^- X$

QCD $\qquad\qquad$ SCET



$$\mathcal{O} = \bar\chi_n \frac{\bar{n}\!\!\!/}{2} \chi_n$$

$$\mathcal{O} \sim \lambda^2$$

(twist 2)

Add arbitrary perturbative corrections (any order in α_s)

$$\mathcal{L}_{\rm int}^{\rm hard} = \int d\omega\, d\cancel{\omega'}\; C(\omega, \cancel{\omega'})\; \bar\chi_n \frac{\bar{n}\!\!\!/}{2} \delta(\cancel{w'} + i\bar{n}\cdot \overleftarrow{\partial}_n)\delta(w - i\bar{n}\cdot\partial_n)\chi_n$$

forward matrix element fixes $\omega' = \omega$

$$\sigma = (\text{prefactor}) \int d\omega\; \mathrm{Im}C(\omega,\mu)\; \langle p|\bar\chi_n \frac{\bar{n}\!\!\!/}{2}\delta(\omega - i\bar{n}\cdot\partial_n)\,\chi_n|p\rangle(\mu)$$

$$= (\text{prefactor}) \int \frac{d\xi}{\xi}\; H\Big(\frac{\xi}{x}, Q, \mu\Big)\; f(\xi,\mu)$$

$$\omega = \xi\, \bar{n}\cdot p_p$$

hard perturbative corrections $\qquad$ parton distribution function

A more detailed set of SCET lecture notes can be found under “textbooks” in the 8.EFTx course.

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