NUCLEAR WEAK CHARGES AND WEAK RADII AT MESA

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Parity Violation and Related Topics Workshop, July 29th, 2020

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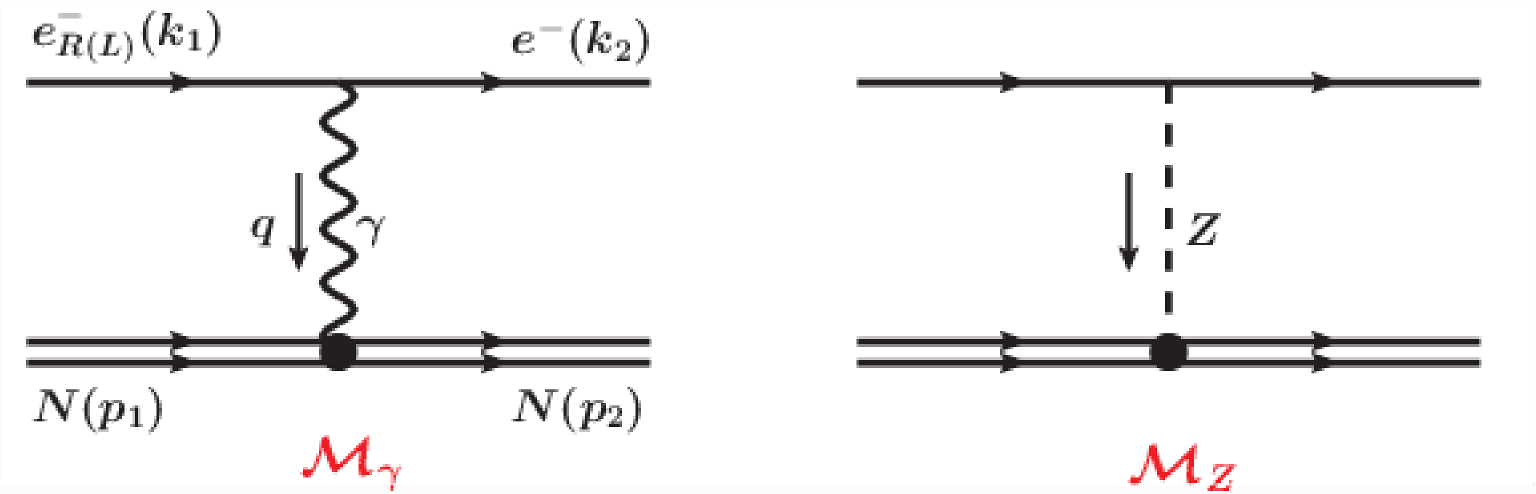
OUTLINE

- PV asymmetry on spin-0 nuclei and implications of future measurements of the asymmetry at MESA.
- Theory of calculation of PV asymmetry on nuclei and corresponding uncertainties.
- Feasibility study of a simultaneous sub-percent extraction of the weak charge and the weak radius of Carbon-12 at MESA.
- Conclusions.

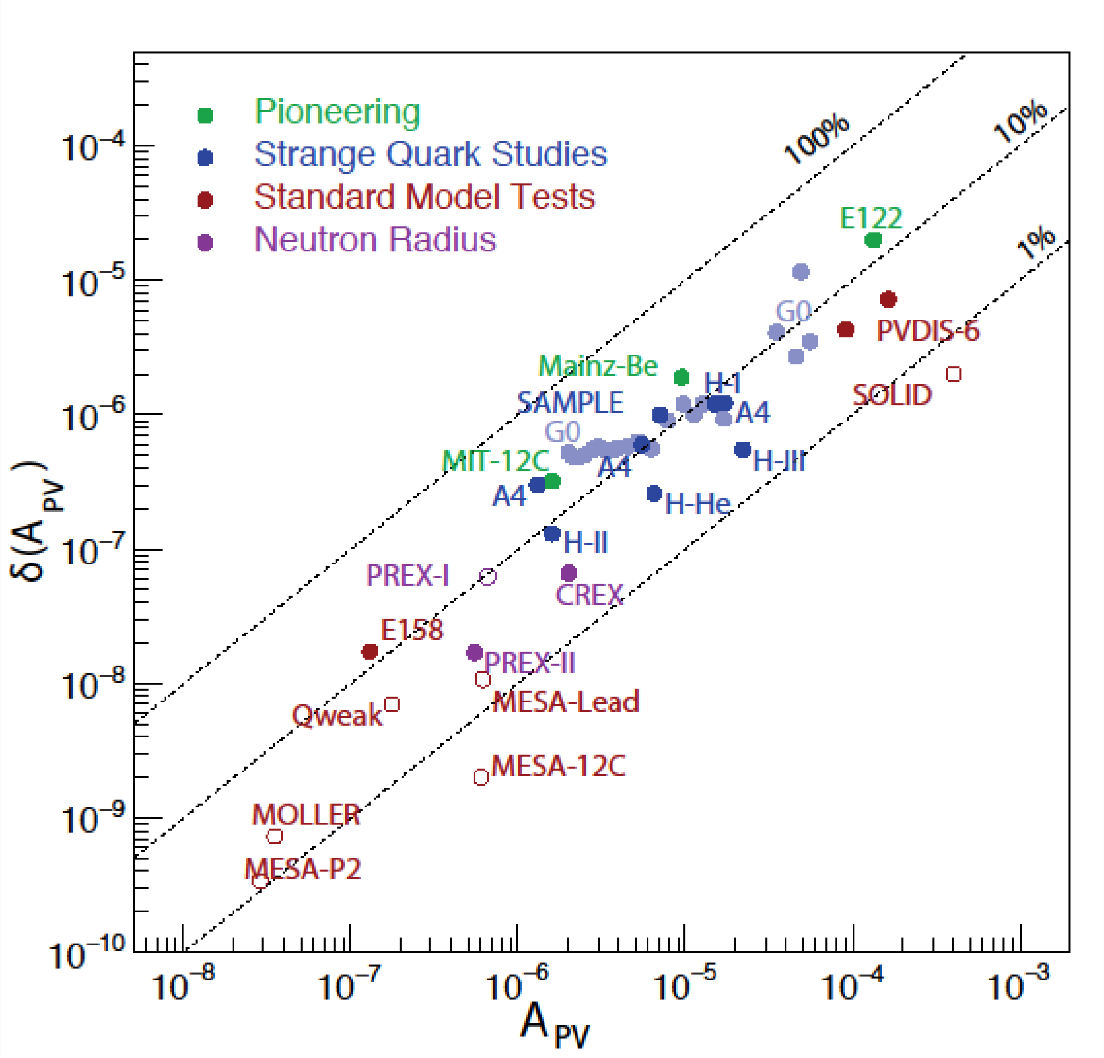
PV ASYMMETRY ON SPIN-0 NUCLEI



$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{F_{\rm wk}(Q^2)}{F_{\rm ch}(Q^2)} \frac{Q_W}{Z} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{Q_W}{Z}(1+\Delta)$$

Weak charge of the nucleus at tree level: $Q_W = Z\left(1 - 4\sin^2\theta_W\right) - N$

SUMMARY OF PVES EXPERIMENTAL PROGRAM



[Becker et al., EPJA, 2018]

WEAK CHARGES AT MESA

- Nucleus at tree level:

$$Q_W = Z\left(1-4\sin^2\theta_W\right)-N$$

- Proton: The weak charge is highly sensitive to the weak mixing angle.

$$Q_W^p \approx 1-4\sin^2\theta_W \approx 0.08 \quad \Rightarrow \quad \Delta\sin^2\theta_W/\sin^2\theta_W \approx 0.09\,\Delta Q_W^p/Q_W^p$$

- Carbon-12: Reduced beam time, theoretically easy to handle.

$$Q_W^{^{12}C} \approx -24\sin^2\theta_W \approx \text{-}5.52; \quad \Delta\sin^2\theta_W/\sin^2\theta_W = \Delta Q_W^{^{12}C}/Q_W^{^{12}C}$$

- Neutron: Weak interactions probe neutrons inside the nucleus.

$$Q_W^n \approx -1$$

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WEAK (NEUTRON) SKINS AT MESA

PV asymmetry:

$$A_{PV} = A_0 \frac{Q_W}{Z} \frac{F_{\mathrm{wk}}(Q^2)}{F_{\mathrm{ch}}(Q^2)} \approx A_0 \frac{Q_W}{Z} \left(1 - \frac{Q^2}{3} R_{\mathrm{wskin}} R_{\mathrm{ch}}\right)$$

Form factors:

$$F_{\mathrm{ch}}(Q^2) = \int d^3r \rho_{\mathrm{ch}}(\vec{r}) e^{i\vec{q}\cdot\vec{r}} = 1 - \frac{Q^2}{3!} R_{\mathrm{ch}}^2 + \ldots$$

$$F_{\mathrm{wk}}(Q^2) = \int d^3r \rho_{\mathrm{wk}}(\vec{r}) e^{i\vec{q}\cdot\vec{r}} = 1 - \frac{Q^2}{3!} R_{\mathrm{wk}}^2 + \ldots$$

RMS radii:

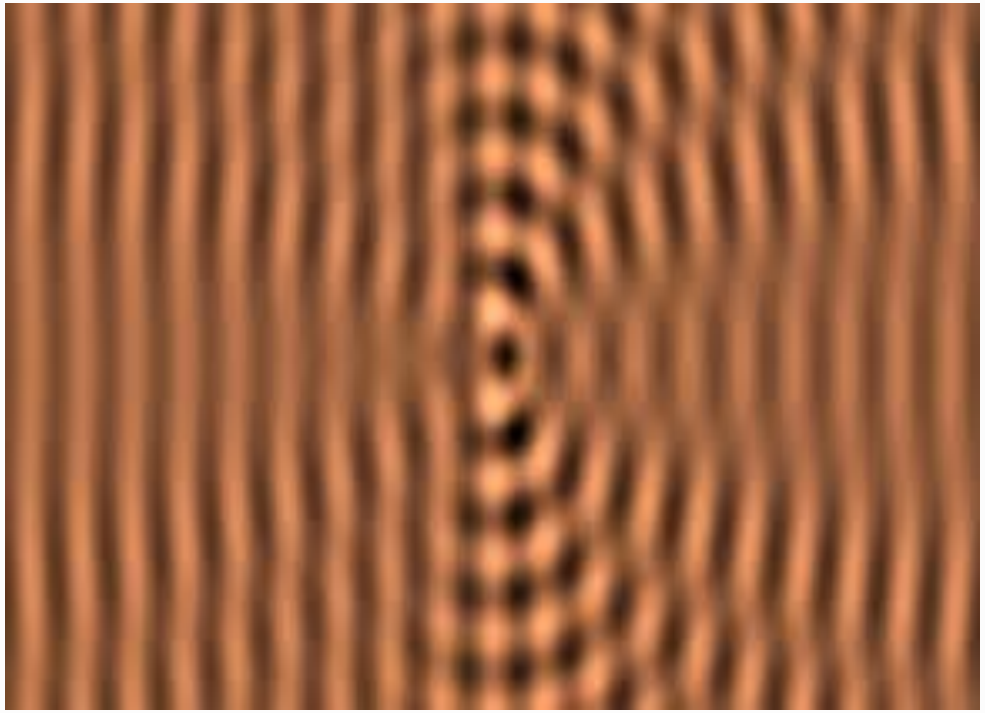
$$R_{\mathrm{ch}}^2 = \int r^2 \rho_{\mathrm{ch}}(r)\, d^3r \qquad R_{\mathrm{wk}}^2 = \int r^2 \rho_{\mathrm{wk}}(r)\, d^3r$$

Weak skin:

$$R_{\mathrm{wskin}} = R_{\mathrm{wk}} - R_{\mathrm{ch}} \qquad R_{\mathrm{nskin}} = R_n - R_p$$

The weak skin can be related to the neutron skin. Neutron skin of symmetric nuclei can help quantifying generic isospin symmetry-breaking effects, important in superallowed nuclear beta decays.

CALCULATION OF PV ASYMMETRY USING PARTIAL WAVES



[www.tcm.phy.cam.ac.uk]

The Dirac equation:

$$\left[-i\vec{\alpha}\cdot\vec{\nabla}+\beta m_e+V(\vec{r})\right]\psi=E\psi$$

Parametrize the potential realistically:

$$V_{\mathrm{ch}}(r)=-Z\alpha\int d^3r'\,\rho_{\mathrm{ch}}(r')/\left|\vec{r}-\vec{r}'\right|$$

Solve the Dirac equation:

$$\psi\sim\sum_{\kappa,m}\psi_{\kappa,m}\qquad\qquad \psi\underset{r\to\infty}{\sim}e^{i\vec{k}\cdot\vec{r}}+F(\theta,\varphi)\frac{e^{ikr}}{r}$$

Determine scattering amplitudes and CS.

COULOMB DISTORTION AND PV ASYMMETRY

The Dirac equation for
scattering of massless electron:

$$\left[-i\vec{\alpha}\cdot\vec{\nabla}+V_{R(L)}(r)\right]\psi_{R(L)}=E\psi_{R(L)}$$

$$V_{R(L)}(r)=V_{\mathrm{ch}}(r)\pm V_{\mathrm{wk}}(r)$$ [Horowitz, PRC, 1998]

Weak potential: $$V_{\mathrm{wk}}(r)=\frac{G_F Q_{\mathrm{wk}}}{2\sqrt{2}}\rho_{\mathrm{wk}}(r)$$

Modified the ELSEPA package to account for
Coulomb distortions. [Salvat, Jablonski, Powell, Comp. Phys. Com., 2004]

Studied nuclear structure uncertainty encoded in the weak
charge density distribution.

COULOMB DISTORTION AND PV ASYMMETRY

The Dirac equation for scattering of massless electron:

$$\left[-i\vec{\alpha}\cdot\vec{\nabla}+V_{R(L)}(r)\right]\psi_{R(L)} = E\psi_{R(L)}$$

$$V_{R(L)}(r)=V_{\text{ch}}(r)\pm V_{\text{wk}}(r)$$

[Horowitz, PRC, 1998]

The Dirac equation for beam-normal single-spin asymmetry:

$$\left[-i\vec{\alpha}\cdot\vec{\nabla}+\beta m+V_{\text{ch}}(r)+i\beta V_{\text{abs}}(E,r)\right]\psi = E\psi$$

[Off topic, in progress, 2020]

Weak potential: $V_{\text{wk}}(r)=\dfrac{G_F Q_{\text{wk}}}{2\sqrt{2}}\rho_{\text{wk}}(r)$

Modified the ELSEPA package to account for Coulomb distortions.

[Salvat, Jablonski, Powell, Comp. Phys. Com., 2004]

Studied nuclear structure uncertainty encoded in the weak charge density distribution.

PV ASYMMETRY ON CARBON-12 AT MESA

Introduce weak skin parameter:

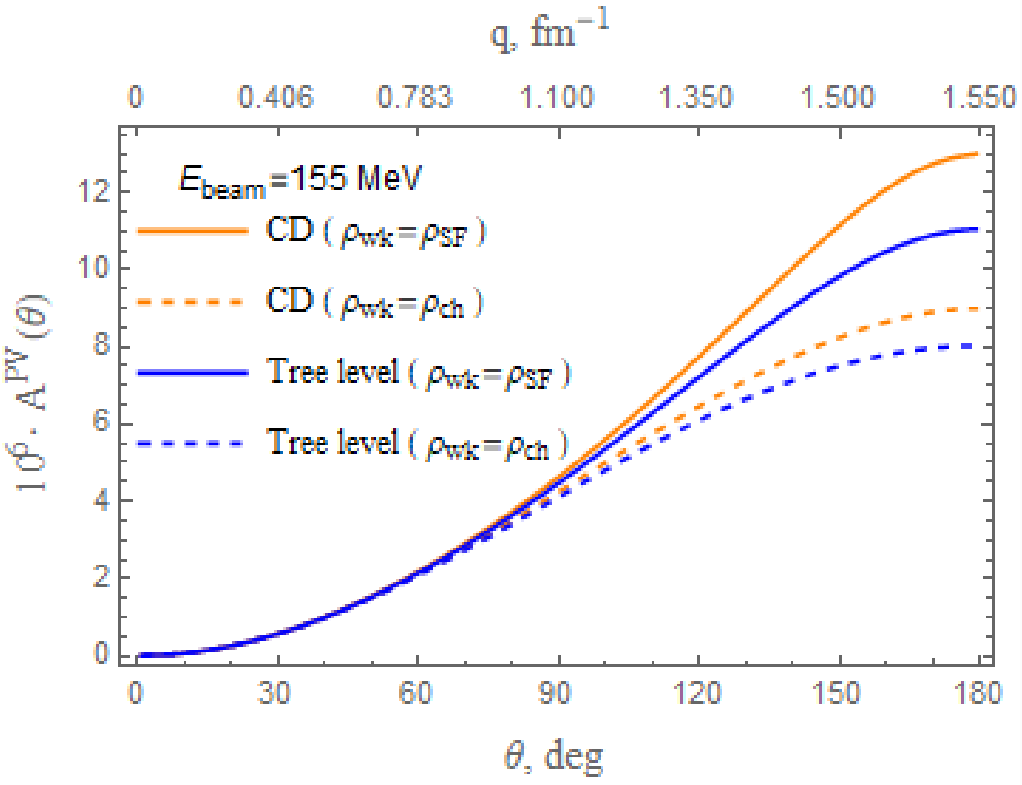
$$\lambda \equiv \left(R_{\text{wk}} - R_{\text{ch}}\right)/R_{\text{ch}} = R_{\text{wskin}}/R_{\text{ch}}$$

PV asymmetry using plane waves:

$$A_{PV} = A_0 \frac{Q_W}{Z} \frac{F_{\text{wk}}(Q^2)}{F_{\text{ch}}(Q^2)} = A_0 \frac{Q_W}{Z}\left(1 - \lambda \frac{Q^2 R_{\text{ch}}^2}{3} + \ldots.\right)$$

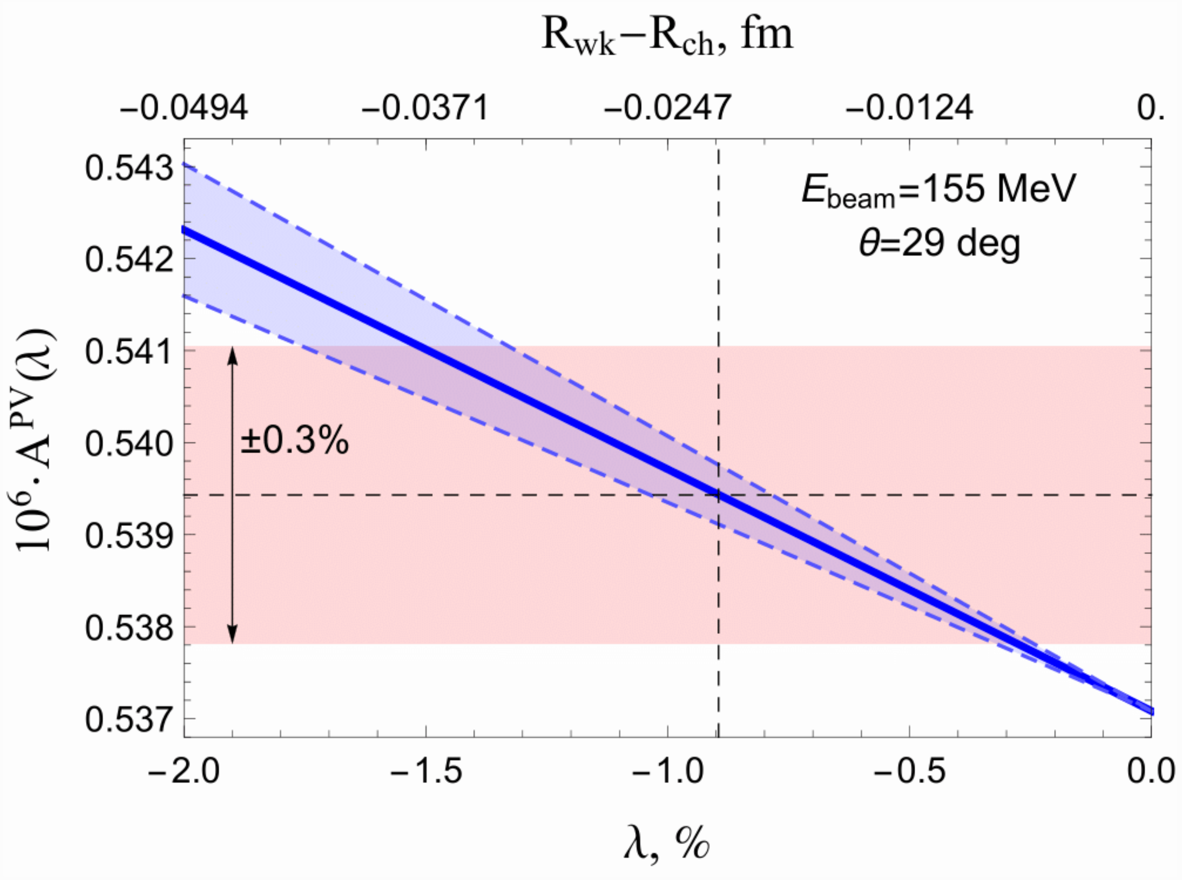
PV asymmetry using partial wave expansion:

$$A_{PV} = A_0 \frac{Q_W}{Z}\left(p_0 - \lambda p_1 + \ldots\right)\ ; \quad A_0 = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}$$



Effects due to Coulomb distortion and non-zero weak skin are clearly significant for a sub-percent asymmetry measurement on ^{12}C.

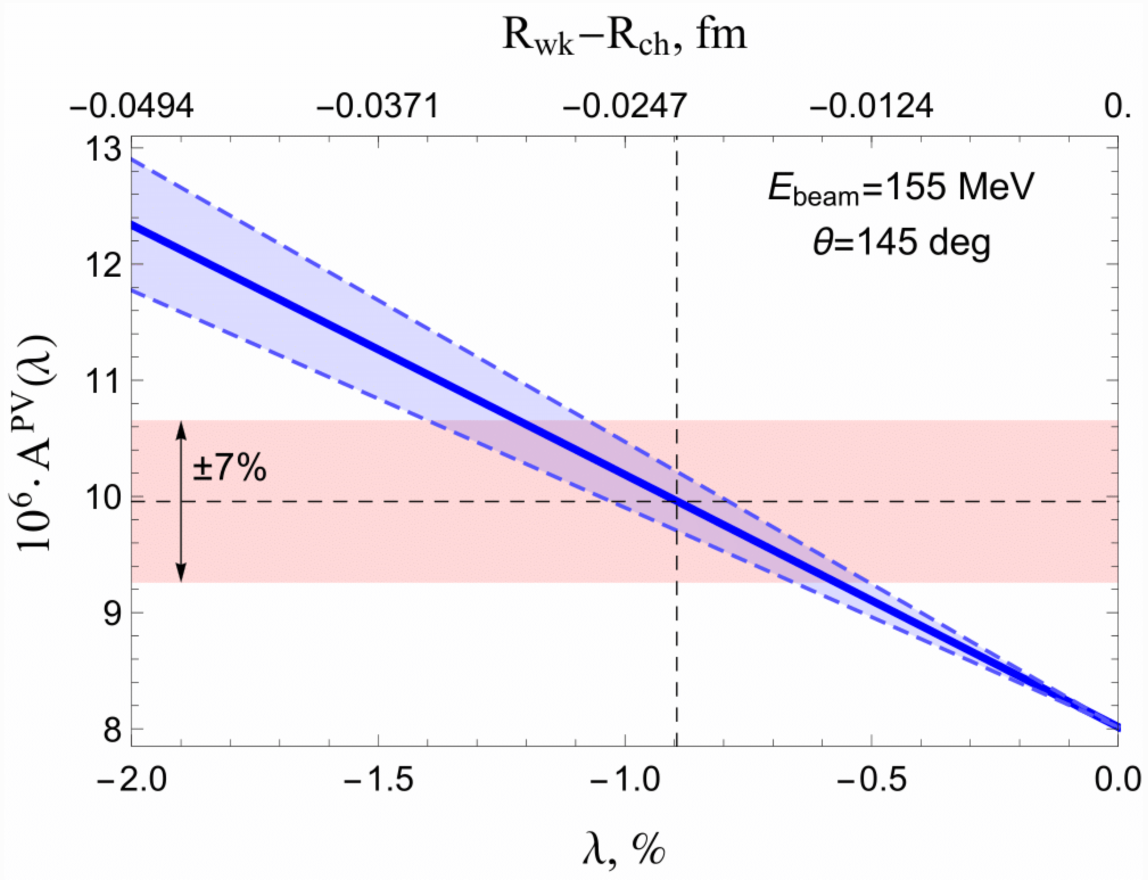
FORWARD ANGLE MEASUREMENT AT P2



[O. Koshchii et al., arXiv:2005.00479, 2020]

- In order to measure PV asymmetry to 0.3% we need to know the weak skin of Carbon-12 to 0.6% of the charge radius.
- Nuclear models suggest a larger uncertainty in the skin ⇒ with a single measurement and theory input alone 0.3% measurement is not feasible.
- Additional (backward) measurement can help achieving such precision.

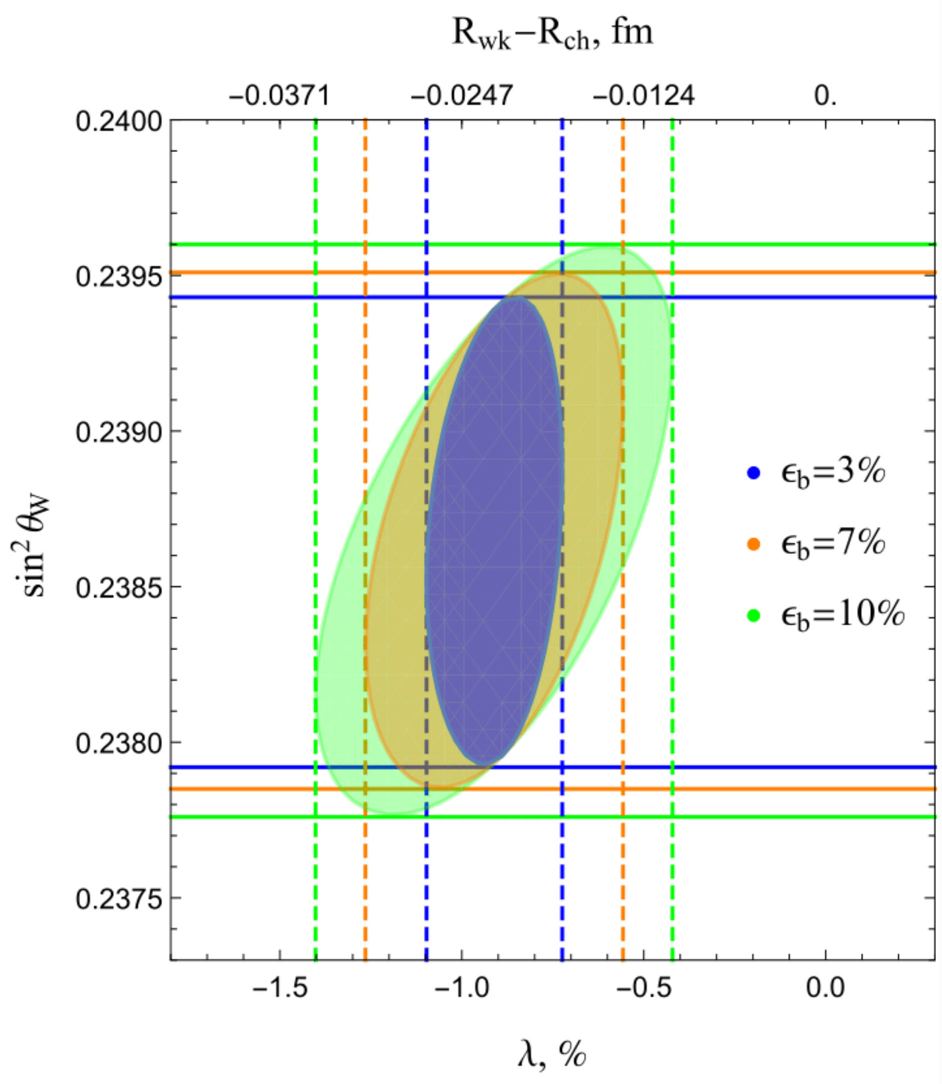
BACKWARD ANGLE MEASUREMENT AT P2



[O. Koshchii et al., arXiv:2005.00479, 2020]

- The backward measurement would be much more sensitive to the weak skin than the forward one. With 7% measurement we can determine the weak skin to 0.35% of the charge radius.
- Can be implemented in a relatively simple manner at P2 using the experimental setup designed for the measurement on the proton.

RESULTS



[O. Koshchii et al., arXiv:2005.00479, 2020]

- χ^2 fit of the combined forward and backward measurements of PV asymmetry with respect to free parameters of our parametrization.
- An additional 3-7% measurement at a backward angle of 145° will ensure a largely model-independent extraction of the weak mixing angle with a relative precision of 0.32-0.35% and the weak skin within 0.19-0.35% of the charge radius.

CONCLUSIONS

- Obtained prediction for the PV asymmetry including Coulomb distortions.
- Studied sensitivity of the forward measurement to nuclear structure. Observed considerable effect of the weak skin on extraction of the weak mixing angle in forward kinematics.
- Showed an enhancement of the contribution due to the weak skin in backward kinematics. This enhancement can be used to determine experimentally the weak skin of Carbon-12 at MESA.
- Simultaneous sub-percent extraction of the weak charge and the weak radius of the Carbon-12 nucleus is possible with two measurements of the asymmetry.

BACK UP SLIDES

PARAMETRIZATIONS OF DENSITY DISTRIBUTIONS

- Parametrization providing the best description of experimental data – a sum of Gaussians (SG):

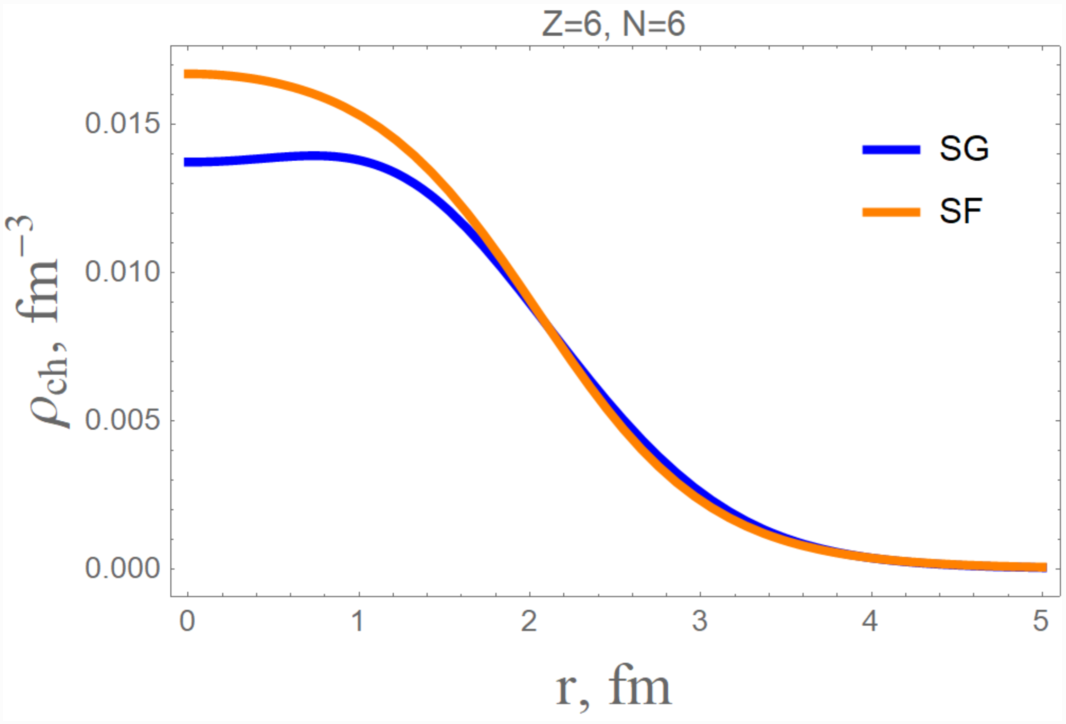
$$\rho_{\mathrm{ch}}(r) = \sum_{i=1}^{12} A_i \left[\exp\left(-\frac{(r-R_i)^2}{\gamma^2} \right) + \exp\left(-\frac{(r+R_i)^2}{\gamma^2} \right) \right]$$

[H. de Vries et al., ADNDT, 1987]

- 2 parameter symmetrized Fermi (SF):

$$\rho_{\mathrm{SF}}(r) = \frac{3}{4\pi c\left(c^2+\pi^2 a^2\right)} \frac{\sinh(c/a)}{\cosh(r/a)+\cosh(c/a)}$$

[J. Piekarewicz et al., PRC, 2016]



Can be used to parametrize the distribution of the weak charge.

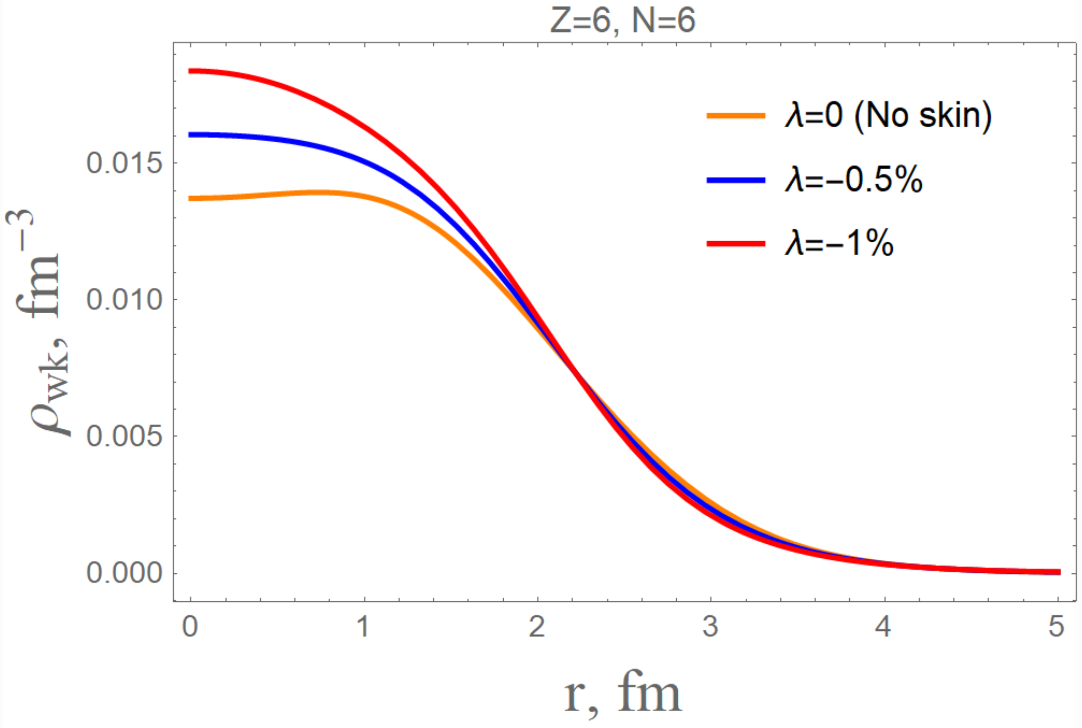
WEAK SKIN PARAMETRIZATION

The choice of a particular form for the weak charge distribution introduces model dependence that may be difficult to quantify. To avoid this, use:

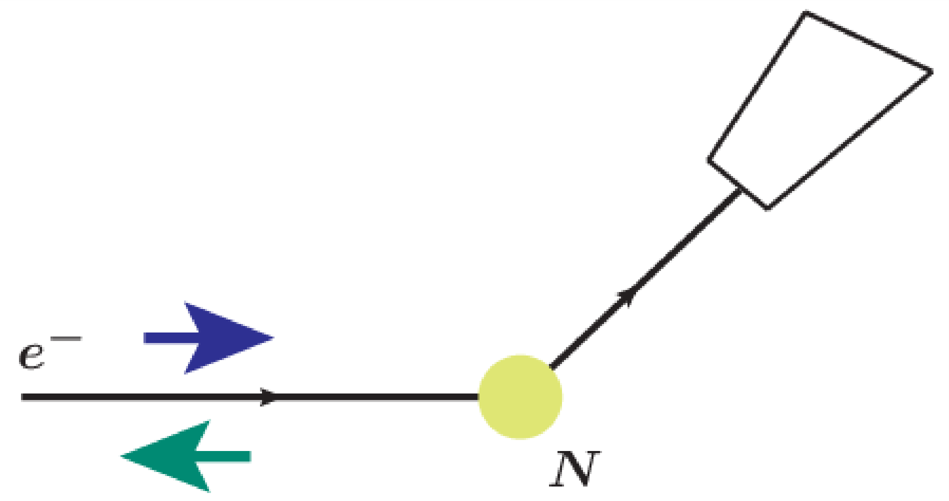
$$\rho_{\mathrm{wk}}(r) = \rho_{\mathrm{ch}}(r) + \rho_{\mathrm{wskin}}(r)$$

$$\rho_{\mathrm{wskin}}(r) = \lambda/\lambda_{\mathrm{SF}}\left(\rho_{\mathrm{SF}}(r) - \rho_{\mathrm{ch}}(r)\right)$$

$$\rho_{\mathrm{SF}}(r) = \frac{3}{4\pi c\left(c^2+\pi^2 a^2\right)} \frac{\sinh(c/a)}{\cosh(r/a)+\cosh(c/a)}$$



PV ASYMMETRY ON PROTON AND CARBON-12 AT MESA



$$A_{PV} = \frac{N^+ - N^-}{N^+ + N^-}$$

$$\Delta A_{PV} \sim \frac{1}{\sqrt{N^+ + N^-}} = \frac{1}{\sqrt{N}}$$

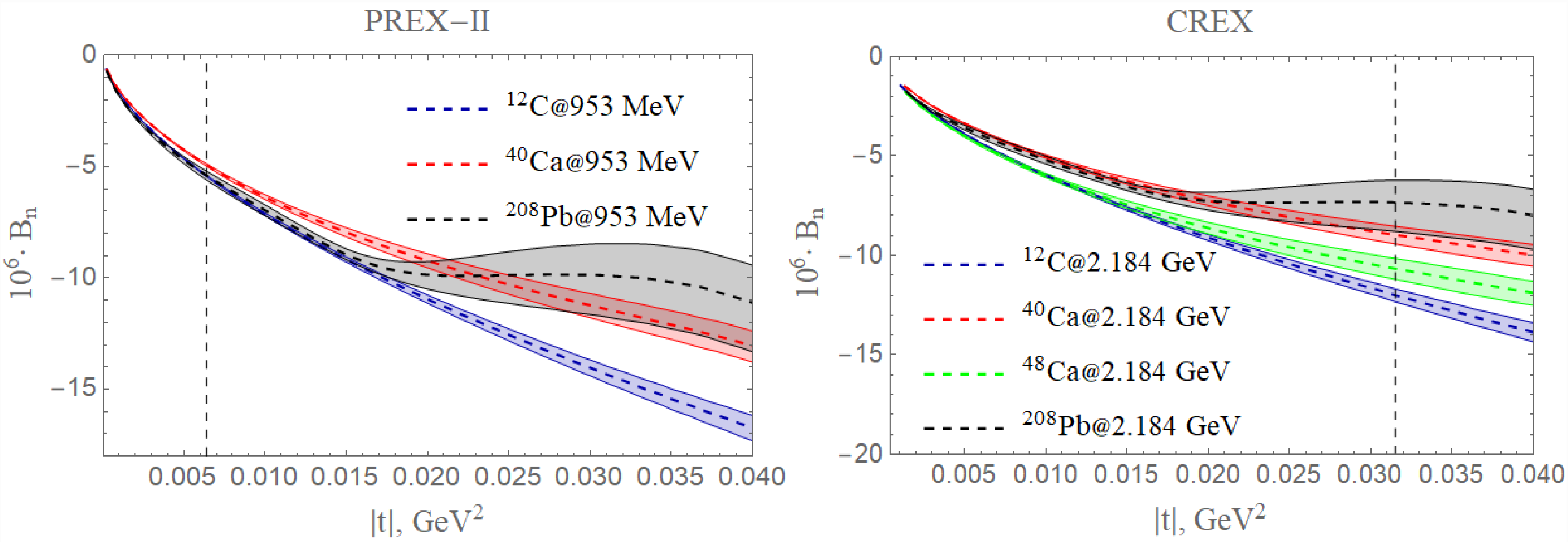
$$\dot{N} = \frac{d\sigma}{d\Omega} \times L \times d\Omega$$

$$t_b = \frac{N_b}{\dot{N}_b} = \frac{d\sigma_f / d\Omega}{d\sigma_b / d\Omega} \left(\frac{\Delta A_{PV}^f}{\Delta A_{PV}^b} \right)^2 t_f$$

$$\frac{d\sigma_f / d\Omega}{d\sigma_b / d\Omega} = 3.9 \cdot 10^5$$

	P2	C-12: Forward	C-12: Backward
A_{PV}	$2 \cdot 10^{-8}$	$5.4 \cdot 10^{-7}$	$1 \cdot 10^{-5}$
ΔA_{PV}	2%	0.3%	7%
N	$6.25 \cdot 10^{18}$	$3.82 \cdot 10^{17}$	$2.04 \cdot 10^{12}$
Meas. Time	11000 h	2500 h	5000 h

BEAM-NORMAL SINGLE-SPIN ASYMMETRY



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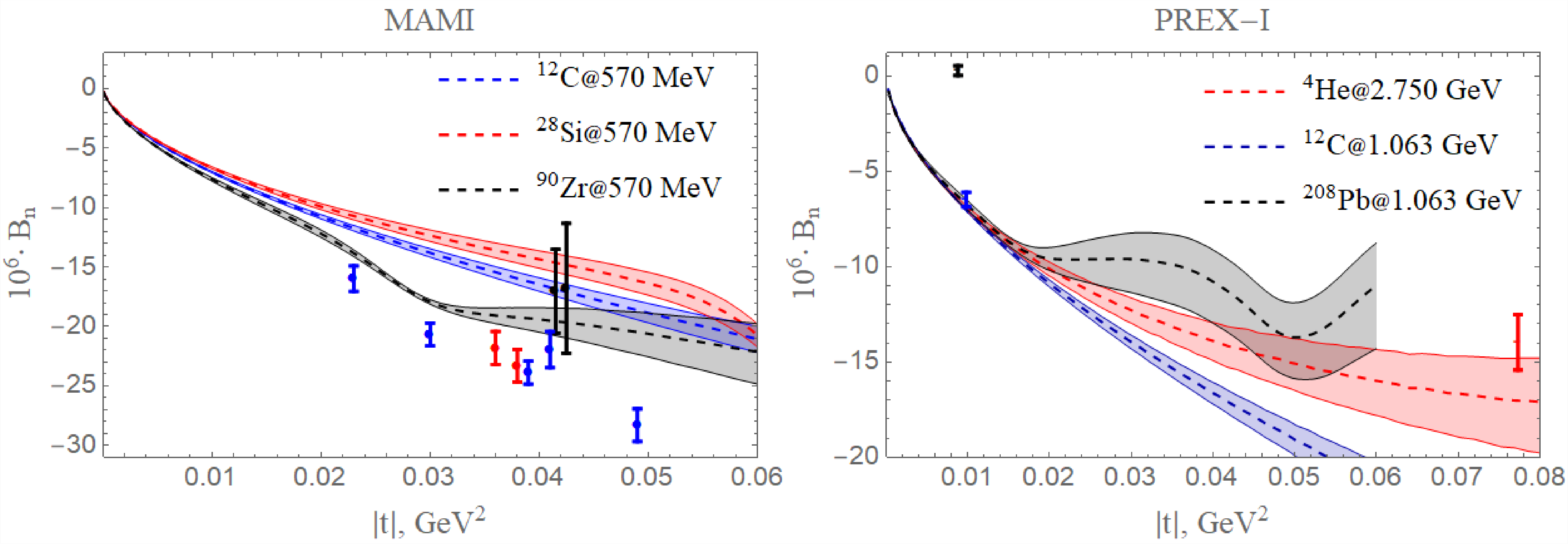

ENERGY DEPENDENCE OF B_n AT FIXED MOMENTUM TRANSFER

$$B_n(E_{beam},|t|) = B_0(A,|t|) \times \frac{1}{E_{beam}^2} \int_{\omega_\pi}^{E_{beam}} d\omega\ \omega\ \sigma_{\gamma p}(\omega) \ln\left[\frac{|t|}{m^2}\left(\frac{E_{beam}}{\omega} - 1\right)^2\right]$$

$$I_0 \equiv \frac{1}{E_{beam}^2} \int_{\omega_\pi}^{E_{beam}} d\omega\ \omega\ \sigma_{\gamma p}(\omega) \ln\left[\frac{|t|}{m^2}\left(\frac{E_{beam}}{\omega} - 1\right)^2\right]$$