Two-loop electroweak corrections to parity-violating electron scattering

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MITP Workshop on Parity Violation and Related Topics

1. **Motivation**

2. **Møller scattering: Calculational techniques**

3. **Møller scattering: Results**

4. **Some comments on ep scattering**

Motivation

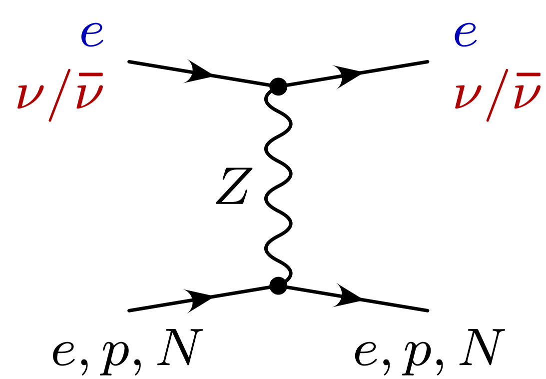
Low-energy parity violation

- Polarized ee, ep, ed scattering ($Q_W(e)$, $Q_W(p)$, eDIS)
  E158 '05; Qweak '17; JLab Hall A '13

- $\nu N/\bar{\nu} N$ scattering NuTeV '02

- Atomic parity violation ($Q_W(^{133}\mathrm{Cs})$) Wood et al. '97
  Guéna, Lintz, Bouchiat '05

$\longrightarrow$ Test of running $\overline{\mathrm{MS}}$ weak mixing angle $\sin^2\bar{\theta}(\mu)$



$$g_{AV}^{ef}\,[\bar{e}\gamma^\mu\gamma_5 e]\,[\bar{f}\gamma_\mu f]$$
$$g_{VA}^{ef}\,[\bar{e}\gamma^\mu e]\,[\bar{f}\gamma_\mu\gamma_5 f]$$

$$g_{AV}^{ef} = \frac{1}{2} - 2|Q_f|\sin^2\bar{\theta}(\mu)$$
$$g_{VA}^{ef} = \frac{1}{2} - 2\sin^2\bar{\theta}(\mu)$$

Low-energy parity violation

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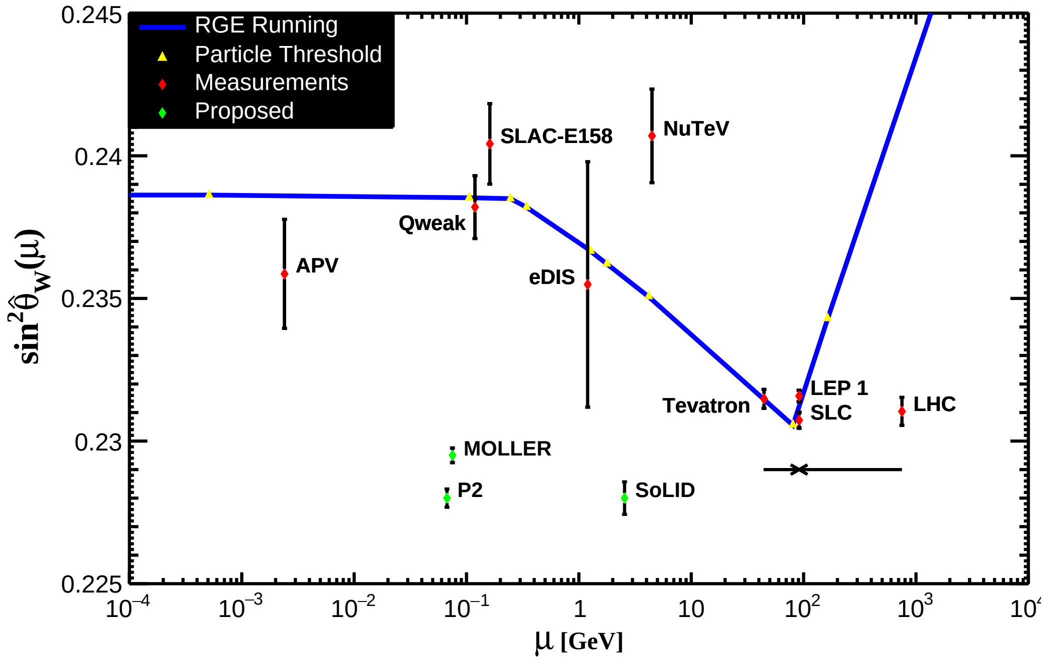
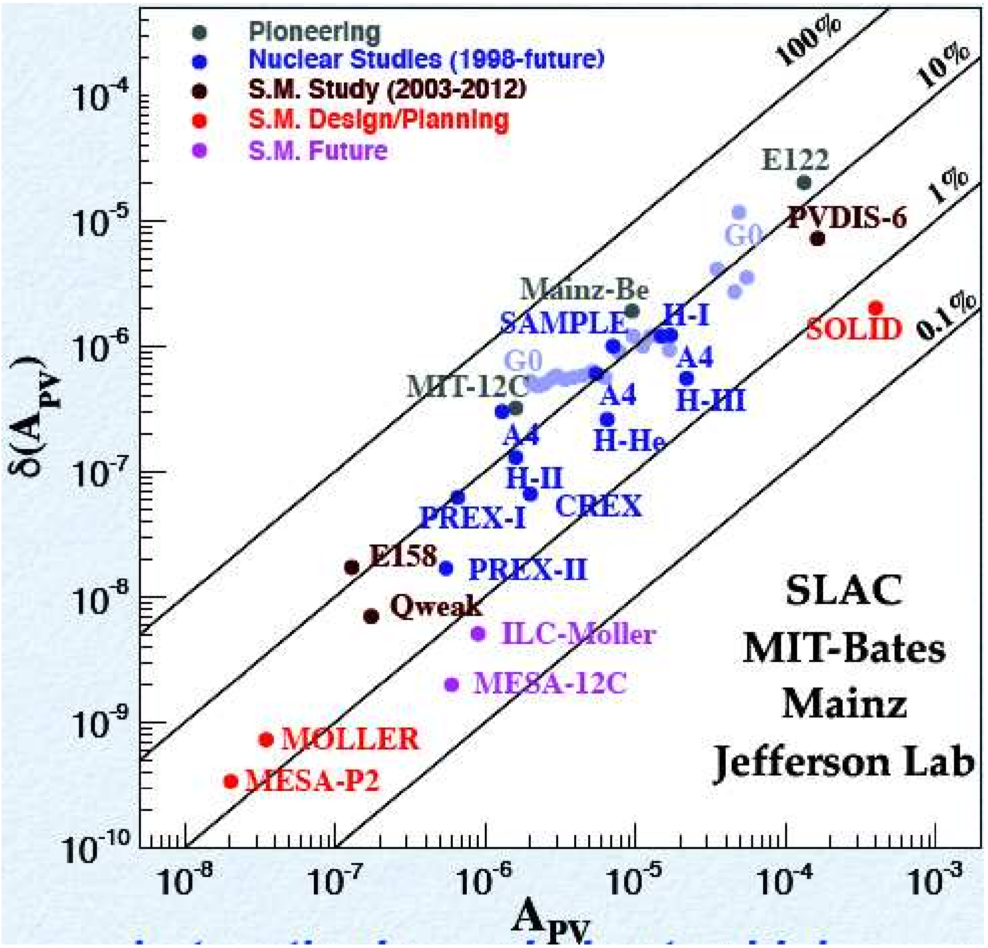
Erler, Ferro-Hernández ’17



Polarized electron scattering: experiments



K. Kumar 'PVES 2018

MOLLER experiment at JLab (ee):

$\delta_{\rm exp} A_{\rm LR} = 0.73 \times 10^{-9}$ (2.4%)

$\delta_{\rm exp} \sin^2\theta_W \sim 0.1\%$

Current best ee (SLAC E158):

$\delta_{\rm exp} A_{\rm LR} = 14\%$

P2 experiment at MESA (ep):

$\delta_{\rm exp} A_{\rm LR} = 0.56 \times 10^{-9}$ (1.4%)

$\delta_{\rm exp} \sin^2\theta_W \sim 0.1\%$

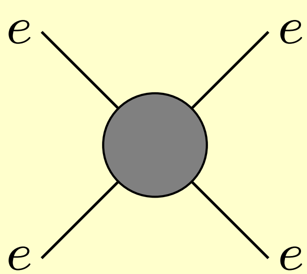
Current best ep (Qweak):

$\delta_{\rm exp} A_{\rm LR} = 8\%$

Sensitivity to new physics scales

4-lepton operator

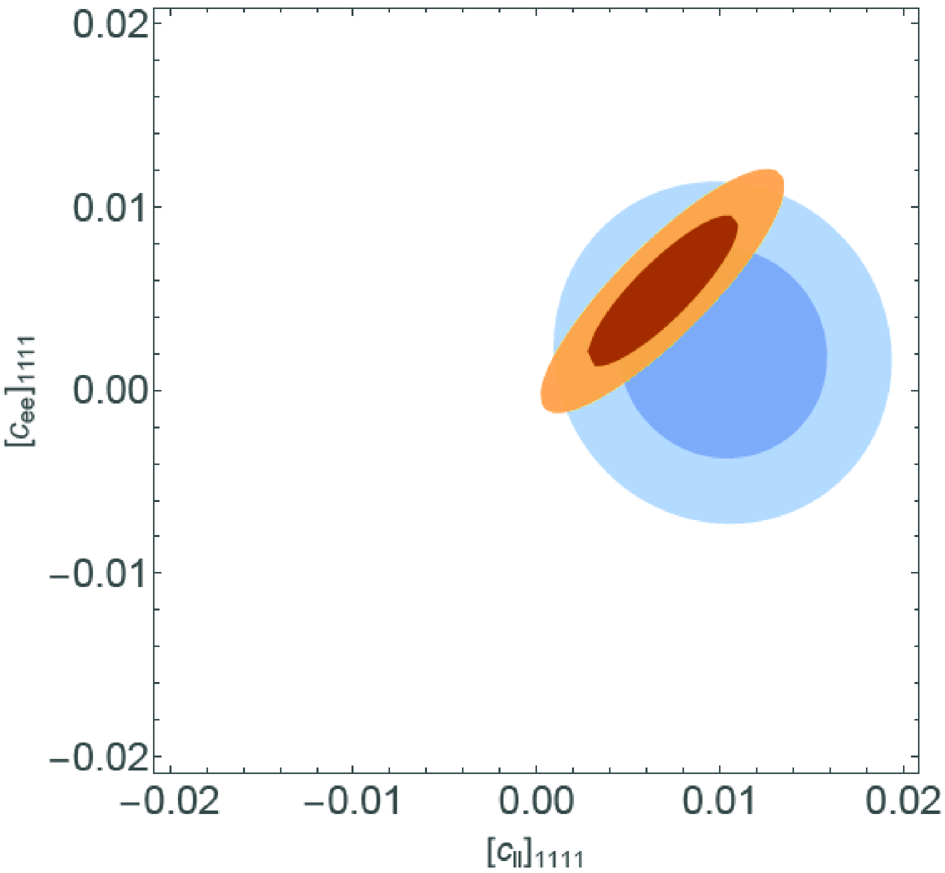
$$\frac{4\pi}{\Lambda^2}\,[\bar{e}\gamma^\mu\gamma_5 e]\,[\bar{e}\gamma_\mu e]$$



E158: $\Lambda \lesssim 17$ TeV

MOLLER: $\Lambda \lesssim 39$ TeV

Erler, Horowitz, Mantry, Souder '14



Falkowski, Gonzalez-Alonso, Mimouni '17
Falkowski et al. '18

$$\frac{c_{ee}}{v^2}\,[\bar{e}\gamma^\mu P_R e]\,[\bar{e}\gamma_\mu P_R e]$$

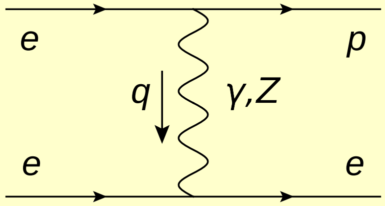
$$\frac{c_{\ell\ell}}{v^2}\,[\bar{e}\gamma^\mu P_L e]\,[\bar{e}\gamma_\mu P_L e]$$

Møller scattering

- Polarized e^- beam on e^- target
- LR asymmetry:

$$A_{\mathrm{LR}}^{ee} = \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \frac{G_\mu(-q^2)}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} Q_{\mathrm{W}}(e)$$

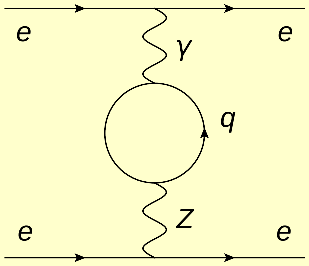
$$y \approx \tfrac{1}{2}(1-\cos\theta_{\mathrm{CM}})$$

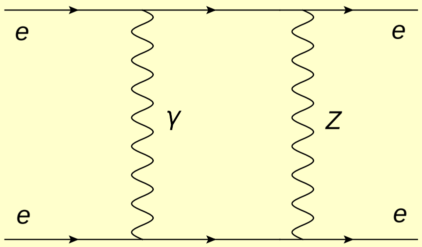


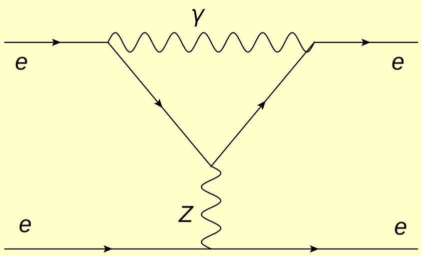
- $Q_{\mathrm{W}}(e) = 1 - 4\sin^2\theta_{\mathrm{W}} + \Delta Q_{\mathrm{W}}(e)$

- One-loop correction $\delta_{1\mathrm{l}} A_{\mathrm{LR}} \sim 40\%$
  $\delta_{1\mathrm{l}} \sin^2\theta_{\mathrm{W}} \sim 3\%$

  Czarnecki, Marciano '96







- IR radiation cancels in A_{LR}
  $\longrightarrow$ Real emission treated separately
- MOLLER exp. target:
  $\delta_{\mathrm{exp}} \sin^2\theta_{\mathrm{W}} \sim 0.1\%$
  $\longrightarrow$ 2-loop corrections needed

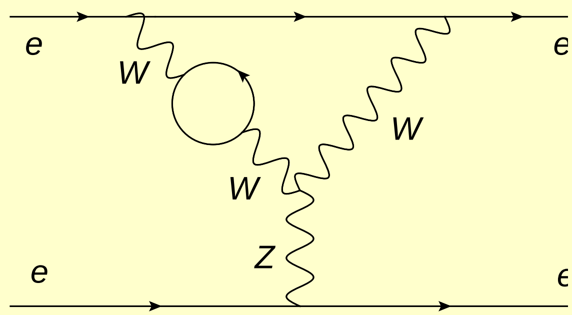
Electroweak corrections beyond one-loop

- As first step: EW 2-loop corrections with closed fermion loops

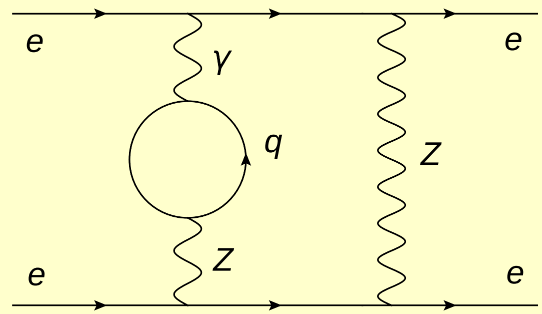
  Du, Freitas, Patel, Ramsey-Musolf '19

  $\longrightarrow$ Enhanced by N_f

  $\longrightarrow$ Experience from Z-pole EWPO:
  $\mathcal{O}(\alpha^2_{\text{ferm}}) \,/\, \mathcal{O}(\alpha^2_{\text{bos}}) \gtrsim 5$



- Computer tools to handle many diagrams and large expressions
  - Diagram generation (FEYNARTS)
  - Lorentz and Dirac algebra (PACKAGE-X, private code)
  - Integral simplication and expansion (in-house, two implementations)
  - Numerical integration of final set of loop functions (in-house, two impls.)



Scales and integral expansion

- At Z pole: $s = M_Z^2 \sim M_W^2 \sim M_H^2 \sim m_t^2 \gg m_f^2 \quad (f \neq t)$
  $\longrightarrow$ Neglect all light fermion masses (except in $\Delta\alpha$)

- Low-energy ee scattering: $|q^2| \sim m_f^2 \ll M_{\text{weak}}^2$
  $\longrightarrow$ Expansion in large M_{weak}^2

- Techical realization: expansion by regions — Beneke, Smirnov '97
  $\longrightarrow$ Expansion in integrand, different categories for loop momenta $k_{1,2}$
  $\longrightarrow$ Here only soft+hard regions needed:

  hard–hard: $|k_1| \sim |k_2| \sim M_{\text{weak}} \gg Q, m_f \qquad Q = \sqrt{|q^2|}$

  soft–soft: $|k_1| \sim |k_2| \sim Q, m_f \ll M_{\text{weak}}$

  soft–hard: $|k_1| \sim Q, m_f \ll |k_2| \sim M_{\text{weak}}$ (and permutations)

- Coefficients are simpler integrals:
  hard–hard: 2-loop vacuum
  soft–hard: (1-loop) $\times$ (1-loop)
  soft–soft: 2-loop with fewer masses

Renormalization

- Form of one-loop result: Czarnecki, Marciano '96

$$A_{\mathrm{LR}} = \frac{\rho G_\mu \, q^2}{\sqrt{\pi}\alpha} \, \frac{-1+y}{1+y^4+(1-y)^4} \Big[1 - 4\kappa(q^2;\mu^2)\, \widehat{s}^2(\mu^2) + \text{boxes, QED} \Big]$$

- G_μ absorbs dependence on $\Delta\alpha$

- Corrections to G_μ known at 2-loop — Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon '02; Onishchenko, Veretin '02

- $\widehat{s}^2(\mu^2) \equiv \sin^2\theta_{\mathrm{W}}^{\overline{\mathrm{MS}}}(\mu^2)$

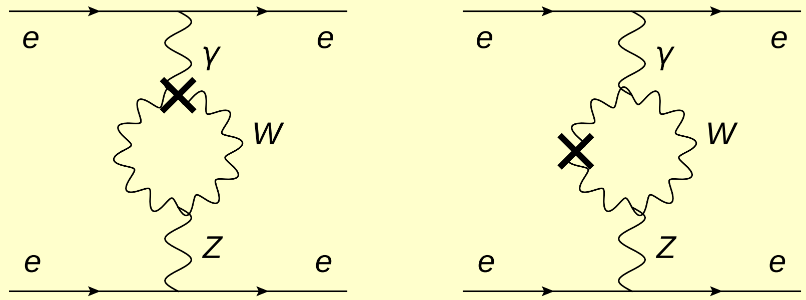
  Use $\mu = M_Z$ by default (full SM, no decoupled d.o.f.s)

- ρ, κ contain remaining loop corrections

EW corrections to Møller scattering: Renormalization

- At 2-loop level: need sub-loop renormalization
- Dependence on $\Delta\alpha$ re-enters
- Also need renormalization of M_W, but since $\hat{s}^2$ is used as input, δM_W is computed from $\hat{s}^2$ and M_Z:

$$\delta M_\mathrm{W}^2 = (1 - \hat{s}^2)\delta M_{\mathrm{Z,OS}}^2 - M_\mathrm{Z}^2 \delta \hat{s}^2$$

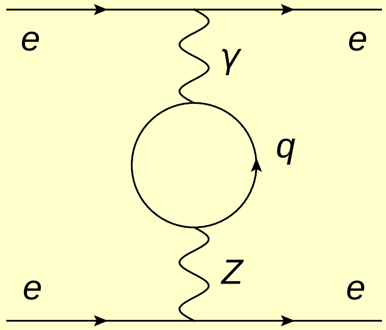


Hadronic effects

$$A_{\rm LR} = \frac{\rho G_\mu\, q^2}{\sqrt{\pi}\alpha}\,\frac{-1+y}{1+y^4+(1-y)^4}\Big[1 - 4\kappa(q^2;\mu^2)\,\hat{s}^2(\mu^2) + \text{boxes, QED}\Big]$$

- $\kappa(q^2) \approx \kappa(0)$ contains effect of γ–Z self-energy
- At 1-loop: $\kappa(0; M_Z^2) = 1 - \frac{\alpha}{6\pi s^2}\Delta_{\gamma Z} + \text{bosonic}$,

$$\Delta_{\gamma Z} = \sum_f (I_{3f} Q_f - 2s^2 Q_f^2) \ln \frac{m_f^2}{M_Z^2}$$



- Sensitivity to m_q: non-perturbative hadron physics
- $\Delta_{\gamma Z}$ relates to running of $\hat{s}^2(\mu^2)$ from $\mu = M_Z$ to 0,

  $\longrightarrow$ $\kappa(0;0)$ is free of $\ln m_f^2$ terms

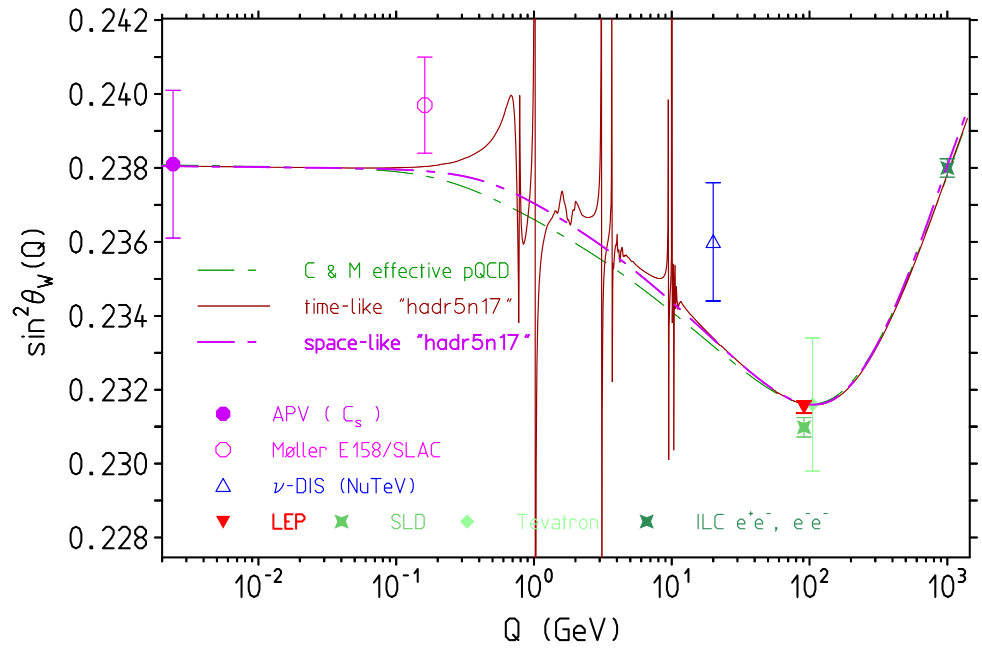
  $\longrightarrow$ RGE running of $\hat{s}^2(\mu^2)$ to resum $\ln m_f^2$ terms

Erler, Ramsey-Musolf '05

Determination of $\Delta_{\gamma Z} = \sum_f (I_{3f} Q_f - 2s^2 Q_f^2) \ln m_f^2/M_Z^2$:

a) Directly from e^+e^- data using dispersion relation and reweighting of different flavors
[SU(3)$_{u,d,s}$ symmetry,
pQCD for u, d, s at c, b thrsh.]

Wetzel '81; Marciano, Sirlin '84
Jegerlehner '86,17



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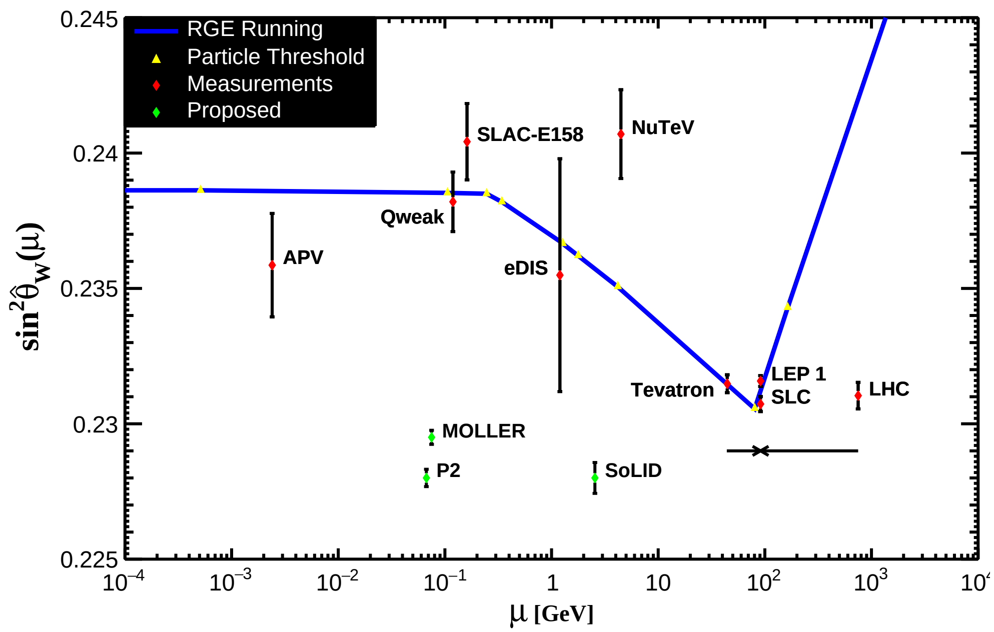
b) Determine "threshold masses" $\bar{m}_q$ from RGE analysis of $\Delta\alpha(q^2)$

Erler, Ramsey-Musolf '04
Erler, Ferro-Hernández '17

or tune effective masses to reproduce $\sin^2\theta(\mu)$ running from e^+e^-



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c) Lattice QCD

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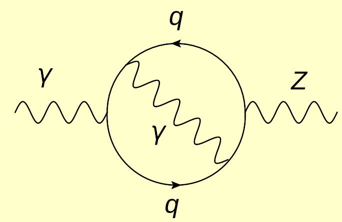
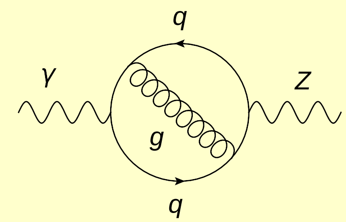
Hadronic effects

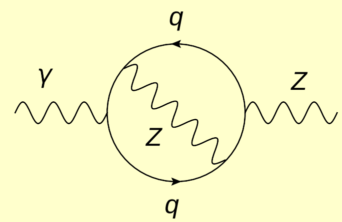
Two-loop contributions to γ–Z self-energy:

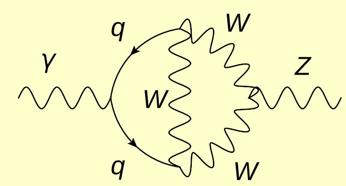
- Quark loops with photon or gluon
  $\longrightarrow$ Already contained in hadronic $\Delta_{\gamma Z}$



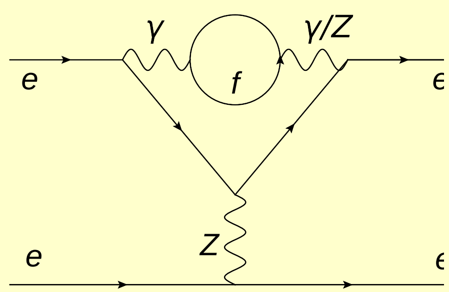


- Quark loops with W/Z boson
  $\longrightarrow$ Perform expansion by regions
  $\longrightarrow$ Result $\sim \alpha \sum_f c_f \ln m_f \times (W/Z\text{-loop})$





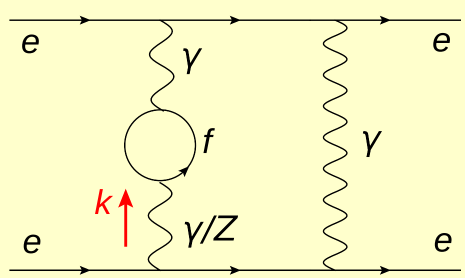
- Vertices/boxes with $\gamma\gamma$ and γZ self-energies:



$$\Sigma_{\mu\nu} = [g_{\mu\nu}k^2 - k_\mu k_\nu]\Pi_\mathrm{T}(k^2)$$

$$\Pi_\mathrm{T}(k^2) = c_\epsilon + \frac{k^2}{\pi}\int_0^\infty d\sigma \, \frac{\mathrm{Im}\{\Pi_\mathrm{T}(\sigma)\}}{\sigma(\sigma - k^2 - i0)}$$

$$c_\epsilon = \frac{N_c g_1 g_2}{12\pi}\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m_f^2}\Big)$$



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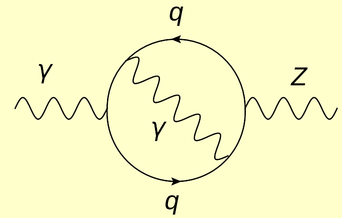

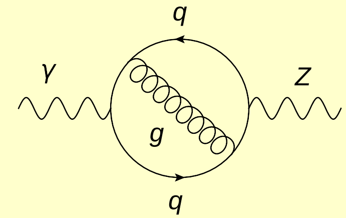

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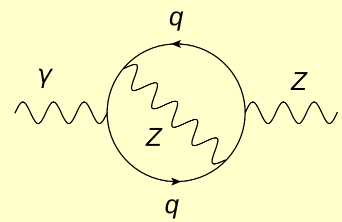
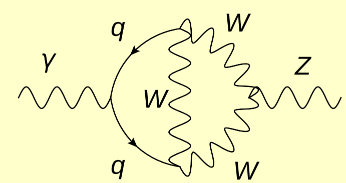




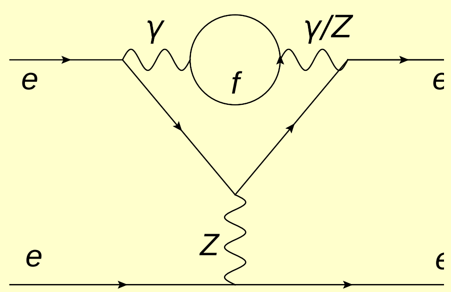
- Vertices/boxes with $\gamma\gamma$ and γZ self-energies:

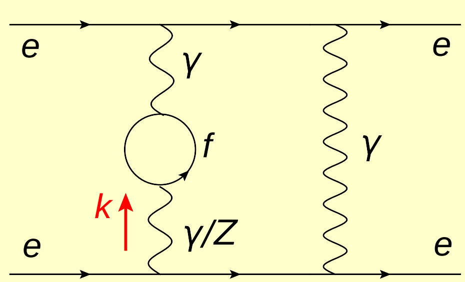
$$\Sigma_{\mu\nu} = [g_{\mu\nu}k^2 - k_\mu k_\nu]\Pi_T(k^2)$$

$$\Pi_T(k^2) = c_\epsilon + \frac{k^2}{\pi}\int_0^\infty d\sigma \, \frac{\text{Im}\{\Pi_T(\sigma)\}}{\sigma(\underbrace{\sigma - k^2 - i0}_{\text{extra prop. for } k \text{ loop}})}$$

$$c_\epsilon = \frac{N_c g_1 g_2}{12\pi}\left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_f^2}\right)$$

$\longrightarrow$ One-loop functions (from k-integral) in integrand of $\int d\sigma$ (numerical)





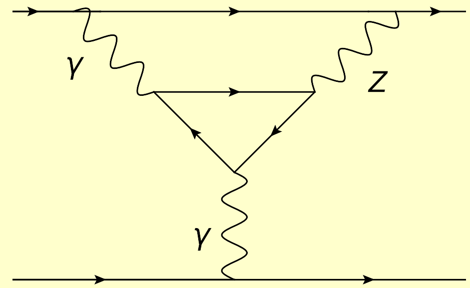
γ_5 and mass regulators

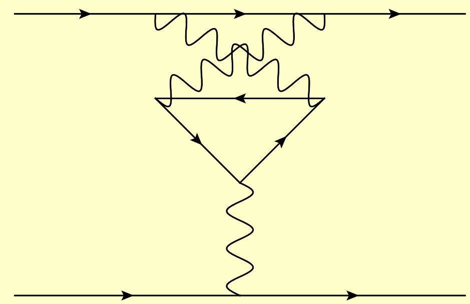
- Vertex diagrams with sub-loop triangles are sensitive to γ_5 problem

- DREG with naively anti-commuting γ_5:
  $\mathrm{Tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\} = 0$

- Contributions $\propto \mathrm{Tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\}$ are UV finite
  $\longrightarrow$ DREG not needed

- IR singularities from photon propagators
  (in individual diagrams, cancel in sum)
  $\longrightarrow$ Use mass regulator ($m_\gamma \neq 0$) in $D = 4$

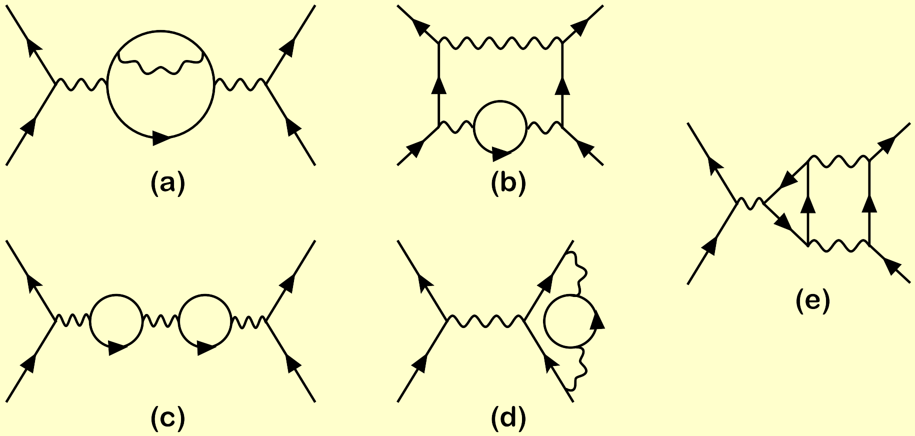




Combination and checks

- NNLO corrections to A_{LR}: genuine 2-loop, reducible, (1-loop)^2

Du, Freitas, Patel, Ramsey-Musolf '19



- Each building block computed in two independent setups

- Checks: UV finiteness (DREG), IR finiteness ($\log m_e^2$, $\log m_\gamma^2$)

- Real photon emission needed for eval. of exp. acceptance and particle ID, can be analyzed separately

Bucoveanu, Spiesberger '19

Banerjee, Engel, Signer, Ulrich '20

Møller scattering: Results

- Input parameters [GeV]: (light quark masses from Erler, Ferro-Hernández '17)

$$M_Z = 91.1876 \qquad m_\tau = 1.777 \qquad m_t = 173.0 \qquad m_b = 3.99$$

$$M_H = 125.1 \qquad m_\mu = 0.1057 \qquad m_c = 1.185$$

$$\hat{s}^2 = 0.2314 \qquad m_e = 5.11 \times 10^{-4} \qquad m_s = 0.342^{+0.048}_{-0.053}$$

$$\Delta\alpha = 0.02761_{\text{rad}} + 0.0314976_{\text{lep}} \qquad m_{u,d} = 0.246^{+0.054}_{-0.057}$$

- Correction to $\Delta Q^e_{W(L,N_f)}$:

  Du, Freitas, Patel, Ramsey-Musolf '19

Quantity	Contribution ($\times 10^{-3}$)
$1 - 4\sin^2\theta_W$	$+74.4$
$\Delta Q^e_{W(1,1)}$	-29.0
$\Delta Q^e_{W(1,0)}$	$+\ 3.1$
$\Delta Q^e_{W(2,2)}$	$-\ 2.12^{+0.014}_{-0.024}$
$\Delta Q^e_{W(2,1)}$	$+\ 1.65^{+0.010}_{-0.007}$
$\Delta Q^e_{W(2,0)}$	$\pm\ 0.18$ (estimate)

- Total fermion-loop NNLO: $\Delta Q^e_{W(2,2)} + \Delta Q^e_{W(2,1)} = -0.47^{+0.007}_{-0.014} \times 10^{-3}$

  MOLLER exp. target: $\delta_{\text{exp}} Q^e_W = 1.1 \times 10^{-3}$

Theory error estimate

Assumption: $\dfrac{\Delta Q^e_{\mathrm{W}(2,0)}}{\Delta Q^e_{\mathrm{W}(2,1)}} \sim \dfrac{\Delta Q^e_{\mathrm{W}(1,0)}}{\Delta Q^e_{\mathrm{W}(1,1)}}$

$\Rightarrow \quad \Delta Q^e_{\mathrm{W}(2,0)} \sim 0.18 \times 10^{-3}$

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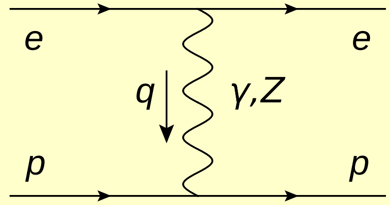
Some comments on ep scattering

- Polarized e^- beam on p target
- LR asymmetry:



$$A_{\mathrm{LR}}^{ep} = \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \frac{G_\mu(-q^2)}{4\sqrt{2}\pi\alpha} Q_{\mathrm{W}}(p)$$

- $Q_{\mathrm{W}}(p) = 1 - 4\sin^2\theta_W + \Delta Q_{\mathrm{W}}(p)$
  $= 2(2C_{1\mathrm{u}} + C_{1\mathrm{d}}), \qquad C_{1\mathrm{u}} = 2g_A^e g_V^u, \quad C_{1\mathrm{d}} = 2g_A^e g_V^d$

Marciano, Sirlin '83

Higher-order corrections:

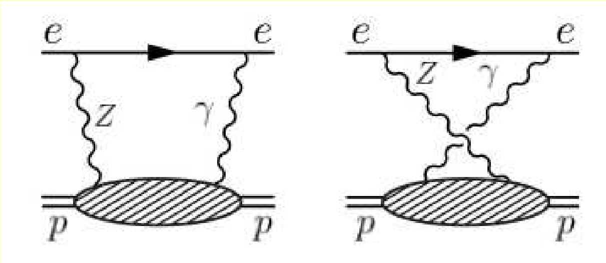
- Renormalization as for Møller scattering
- Large mass expansion: $|q^2| \sim m_f^2 \ll M_{\mathrm{weak}}^2$
- For most diagrams:
  - soft region: vanishes due to IR cancellations
  - hard region: gauge bosons couple to u, d quarks (as for $C_{1\mathrm{u,d}}$ above)

$\longrightarrow$ Same type of integrals as for ee scattering, except $m_e \ll m_p$

Box diagrams

γZ boxes: Sensitivity to physics at Λ_{QCD}

Gorchstein, Horowitz '08
Sibirtsev, Blunden, Melnitchouk, Thomas '10
Rislow, Carlson '11,13
Gorchstein, Horowitz, Ramsey-Musolf '11
Blunden, Melnitchouk, Thomas '12
Gorchstein, Zhang '15
Hall, Blunden, Melnitchouk, Thomas '13,15
Erler, Gorchtein, Koshchii, Seng, Spiesberger '19



(also for $\gamma\gamma$ box, but can be treated as external corr.)

$$\Box_{\gamma Z} = \Box^V_{\gamma Z} + \Box^A_{\gamma Z}$$

$$\Box^V_{\gamma Z} = \int_{W^2_{\text{min}}}^{\infty} dW^2 \int_0^\infty \frac{dQ^2}{Q^2(Q^2+M_Z^2)} \left[C_1(W^2,Q^2,E)\, F_1^{\gamma Z}(x,Q^2) + C_2(W^2,Q^2,E)\, F_2^{\gamma Z}(x,Q^2)\right]$$

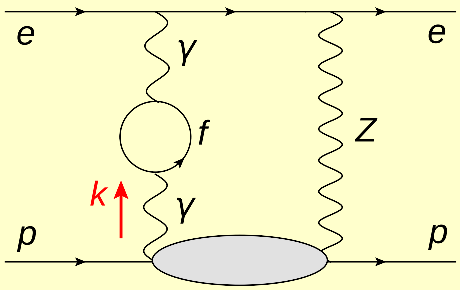
$$\Box^A_{\gamma Z} = \int_{W^2_{\text{min}}}^{\infty} dW^2 \int_0^\infty \frac{dQ^2}{Q^2(Q^2+M_Z^2)} C_3(W^2,Q^2,E)\, F_3^{\gamma Z}(x,Q^2)$$

$E = E^e_{\text{in}}$, $Q^2 = -k_\gamma^2$, $W^2 =$ had. inv. mass, $x = Q^2/(W^2 - m_p^2 + Q^2)$

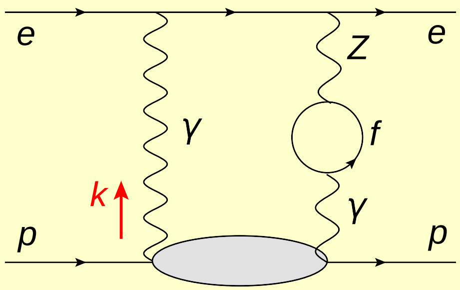
$F^{\gamma Z}_{1,2,3}$ extracted from data (with some model assumptions)

Box diagrams

Fermion-loop NNLO corrections:



$$\frac{1}{Q^2} \to \frac{1}{Q^2}\Pi_T^{\gamma\gamma}(Q^2) \quad \text{in} \quad \int dW^2 \int \frac{dQ^2}{Q^2(Q^2+M_Z^2)}$$



$$\frac{1}{Q^2+M_Z^2} \to \frac{1}{Q^2+M_Z^2}\Pi_T^{\gamma Z}(Q^2)$$

Summary

- Precision of future PVES measurenments requires 2-loop SM corrections

- No fully automated procedure for 2-loop calculations
  $\longrightarrow$ Each process requires new work
  $\longrightarrow$ Combination of analytic and numerical methods
  $\longrightarrow$ Careful treatment of hadronic effects

- Wish list of further refinements:
  - Match to running $\sin^2\theta_{\rm W}^{\overline{\rm MS}}(\mu^2)$ for $\mu \sim 0$
  - Exact q^2 dependence in γ–Z self-energy (rather than $q^2 \approx 0$)
  - More detailed estimate of missing "bosonic" $\Delta Q^e_{\rm W(2,0)}$ corrections.
    Is $\Delta Q^e_{\rm W(2,0)} \ll \delta_{\rm exp} Q^e_{\rm W}$ robust?