Bayesian analysis of ${}^{7}_{4}\mathrm{Be} + \mathrm{p} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma$ based on halo effective field theory

Xilin Zhang

The Ohio State University

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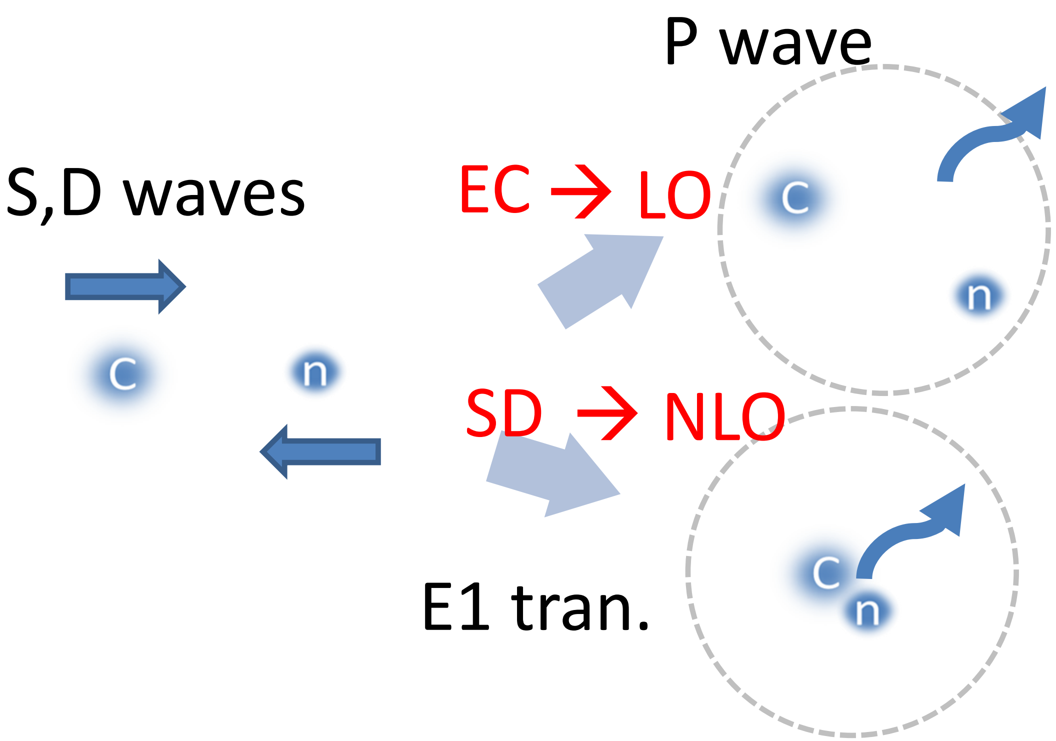
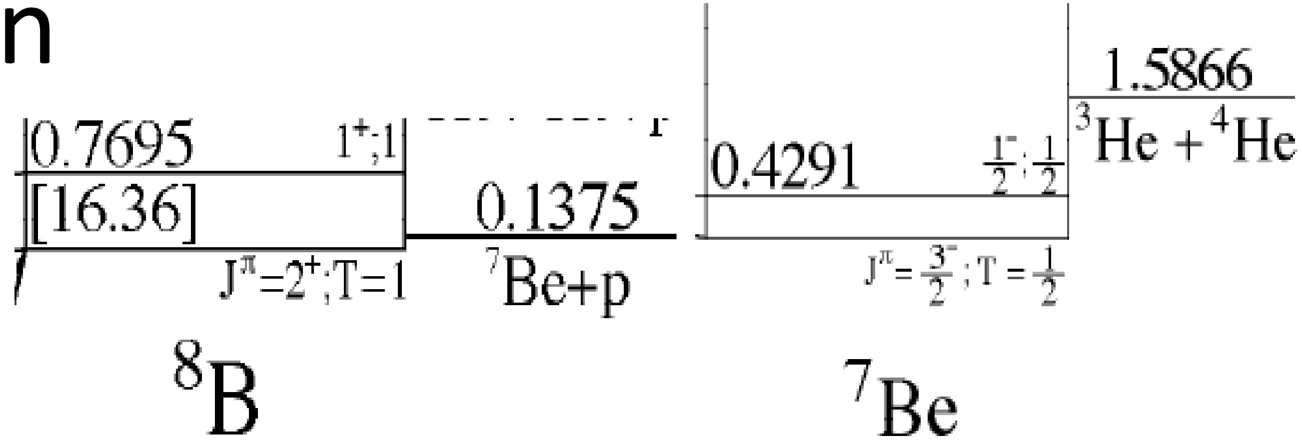
X.Z., K. Nollett and D. Phillips,
PRC 89, 051602 (2014), PLB 751, 535(2015); EPJ Web Conf. 113, 06001 (2016); PRC 98, 034616 (2018)

Outline

- Physics of cluster/halo EFT
- Bayesian analysis and results
- My recent development of ab initio calculation of nuclear scattering
- Summary

Physics: scale separation





Total spin S can be 1 or 2 → two reaction channels; low-E core excitation in S=1 channel

Momentum scale	Definition	Value
$k_C \sim \gamma$	$Q_c Q_n \alpha_{EM} M_R$	24.02 MeV
γ	$\sqrt{2M_R B_{^8\mathrm{B}}}$	15.04 MeV
Λ	$\sqrt{2M'_R B_{^7\mathrm{Be}}}$	70 MeV
$\gamma^* \sim \gamma$	$\sqrt{2M_R(B_{^8\mathrm{B}} + E^*)}$	30.53 MeV
$\gamma_\Delta \sim \gamma$	$\sqrt{2M_R E^*}$	26.57 MeV
$a_{^3S_1}, a_{^5S_2} \sim 1/\gamma$	scattering lengths	Varies
$r_0 \sim 1/\Lambda$	$l=0$ effective ranges	Varies
$a_1 \sim \gamma^{-2}\Lambda^{-1}$	scattering volume	1054.1 fm^3
$r_1 \sim \Lambda$	$l=1$ effective "range"	-0.34 fm^{-1}

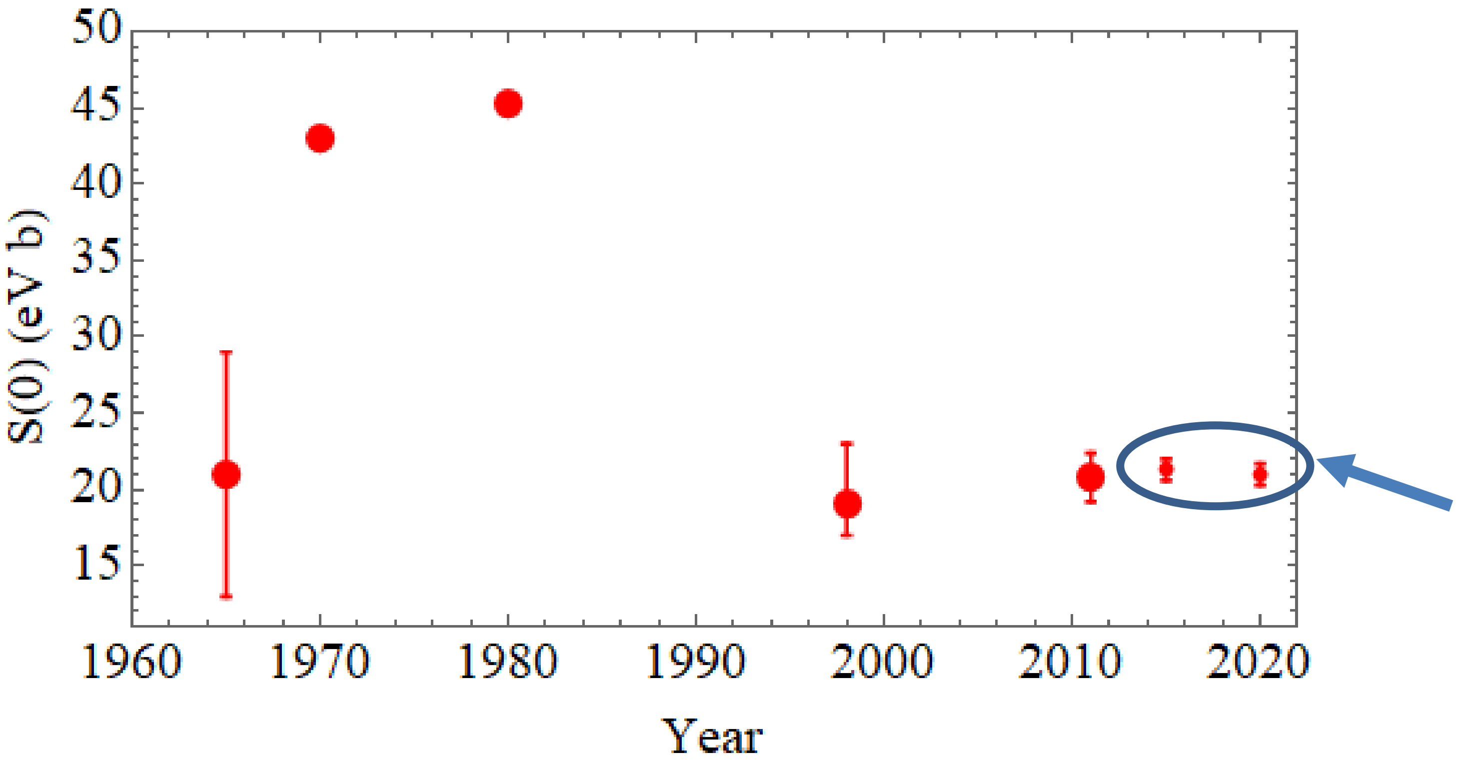
- EFT quantifies this picture, by expanding amplitudes in terms of $\frac{Q_{low}}{\Lambda} \sim 0.2$
- Not a Taylor expansion: non-analyticity due to strong initial scattering and Coulomb

Bayesian analysis

S(0 keV) [S(20 keV)]

	S (eV b)	S'/S (MeV^{-1})	S''/S (MeV^{-2})
Median	21.33 [20.67]	−1.82 [−1.34]	31.96 [22.30]
$+\sigma$	0.66 [0.60]	0.12 [0.12]	0.33 [0.34]
$-\sigma$	0.69 [0.63]	0.12 [0.12]	0.37 [0.38]

E. G. Adelberger, et.al., Rev. Mod. Phys. 83, 195 (2011) recommend: $S(0) = 20.8 \pm 0.7\ (\mathrm{exp}) \pm 1.4\ (\mathrm{th})$ ev b



Tombrello(1965);
Aurdal(1970);
Rev.Mod.Phys.(1998);
Rev.Mod.Phys(2011);
XZ et.al., (2015);
R. Higa et.al., (2020)

Physics in r-space

- EC: external capture
- SD: short-distance contribution
- CX: core-excitation contribution

$$\mathcal{S}_{\rm EC} = \omega \int_0^\infty dr\ C_f\ W_{-\eta_B,\frac{3}{2}}(2\gamma r)\ r\ \frac{e^{i\delta_i}}{k}\left[\sin(\delta_i)\ G_0(k,r) + \cos(\delta_i)\ F_0(k,r)\right]$$

$$\mathcal{D}_{\rm EC} = \omega \int_0^\infty dr\ C_f\ W_{-\eta_B,\frac{3}{2}}(2\gamma r)\ r\ \frac{1}{k} F_2(k,r)$$

NLO

$$\mathcal{S}_{\rm SD} = \omega \frac{\sqrt{3}}{2} \frac{C_f \overline{L}_1}{\gamma\Gamma(2+\eta_B)} \frac{e^{i\delta_i}}{k} \frac{\sin(\delta_i)}{C_{\eta,0}}$$

$$C_{\eta_*,0}\left[G_0(k_*,r) + iF_0(k_*,r)\right] = \Gamma(1+i\eta_*) W_{-i\eta_*,\frac{1}{2}}(-2ik_* r)$$

$$\mathcal{S}_{\rm CX} = \omega \int_0^\infty dr\ C_{f*}\ W_{-\eta_{B*},\frac{3}{2}}(2\gamma^* r)\ r\ \varepsilon\ \frac{e^{i\delta_i}}{k} \sin(\delta_i)\ \frac{C_{\eta_*,0}}{C_{\eta,0}}\left[G_0(k_*,r) + iF_0(k_*,r)\right]$$

NLO (only exists in S=1)

$$U = \begin{pmatrix} 1 & -\varepsilon \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} e^{i2\delta_i} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \varepsilon \\ -\varepsilon & 1 \end{pmatrix} \approx \begin{pmatrix} e^{i2\delta_i} & (2i)\ \varepsilon\ e^{i\delta_i} \sin(\delta_i) \\ (2i)\ \varepsilon\ e^{i\delta_i} \sin(\delta_i) & 1 \end{pmatrix}$$

$$u_{ji}(r \gg r_s) = \begin{cases} (G_i - iF_i)\ \delta_{ji} - (G_j + iF_j)\ U_{ji} & \text{when } E \geq E_* \\ C_{ji}\ W_{-\eta_j, l_j+1/2}(2\gamma_j r) & \text{when } E < E_* \end{cases}$$

Initial state wave function in the EC region

Comments on Higa (2020)

Multiple scattering in the initial state

$$\mathcal{A}^{(11)} = \frac{2\pi}{\mu}[C_0(\eta_p)]^2 e^{i2\sigma_0}\Bigg\{ - a_{11}^{-1} - 2k_C H\left(\frac{k_C}{p}\right) + a_{12}^{-2}\left[\frac{1}{a_{22}} + 2k_C H\left(\frac{k_C}{p_\star}\right)\right]^{-1}\Bigg\}^{-1},$$

$$\mathcal{A}^{(12)} = \frac{2\pi}{\mu}[C_0(\eta_p)]^2 e^{i2\sigma_0}\Bigg\{ - a_{12}^{-1} + a_{12}\left[\frac{1}{a_{11}} + 2k_C H\left(\frac{k_C}{p}\right)\right]\left[\frac{1}{a_{22}} + 2k_C H\left(\frac{k_C}{p_\star}\right)\right]\Bigg\}^{-1}$$

$$a_{11} \gg a_{12} \gg a_{22}$$

$$\mathcal{A}^{11} \to [C_0(\eta)]^2\, e^{2i\sigma_0} \frac{1}{-\frac{1}{a_{11}} + \frac{a_{22}}{a_{12}^2} - 2k_c H(k_c/p)}$$

$$\mathcal{A}^{12} \to [C_0(\eta)]^2\, e^{2i\sigma_0} \frac{a_{22}}{a_{12}} \frac{1}{-\frac{1}{a_{11}} + \frac{a_{22}}{a_{12}^2} - 2k_c H(k_c/p)} = \frac{a_{22}}{a_{12}} \mathcal{A}^{11}$$

Final state

$$\langle j|T|i\rangle \overset{E\to E_B}{\approx} \langle j|V|B\rangle \frac{1}{E - E_B} \langle B|V|i\rangle$$

Only one dimer for each bound state

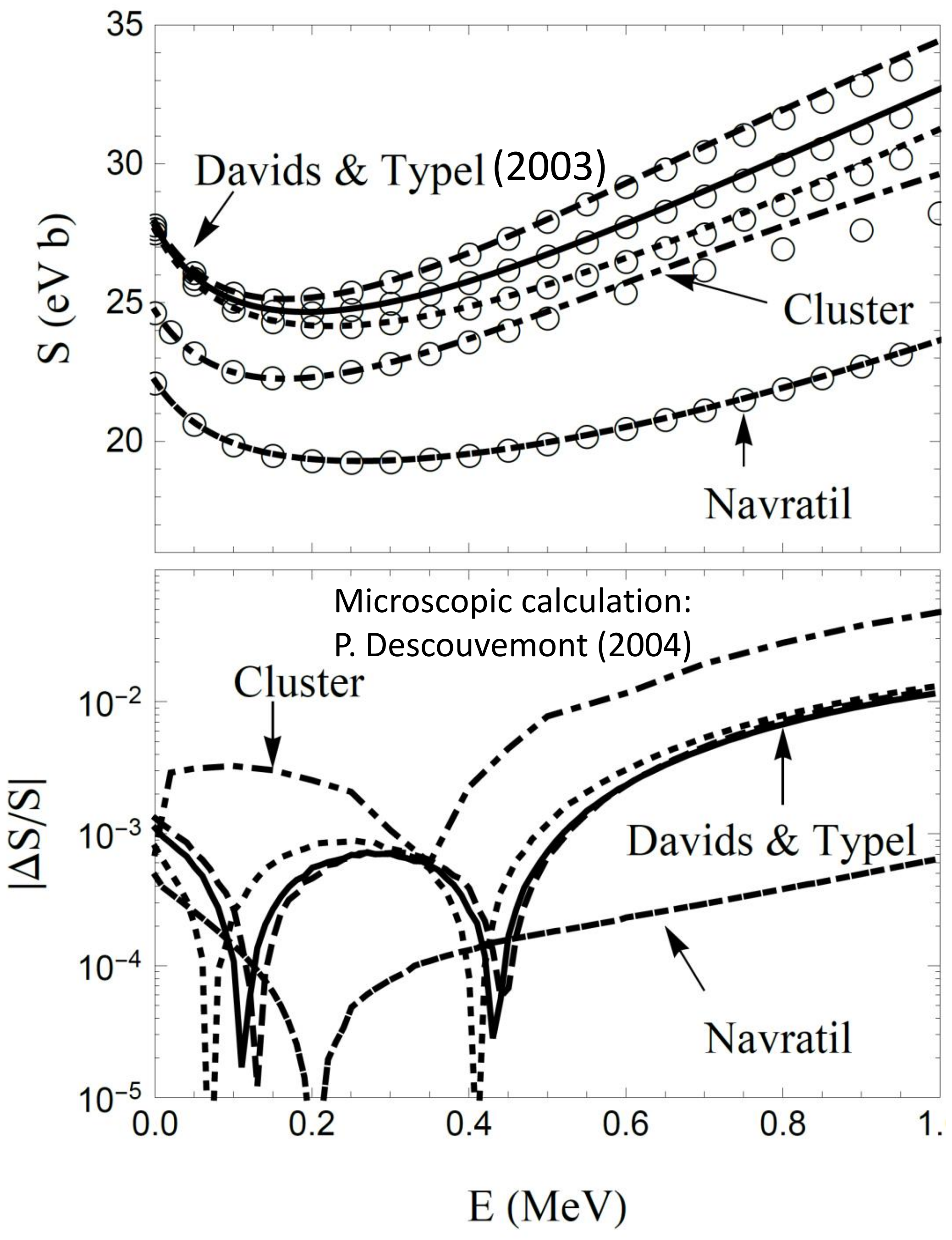
EFT: NLO

LO: 4 parameters $C_{(^3P_2)}, C_{(^5P_2)}, a_{(^3S_1)}, a_{(^5S_2)}$

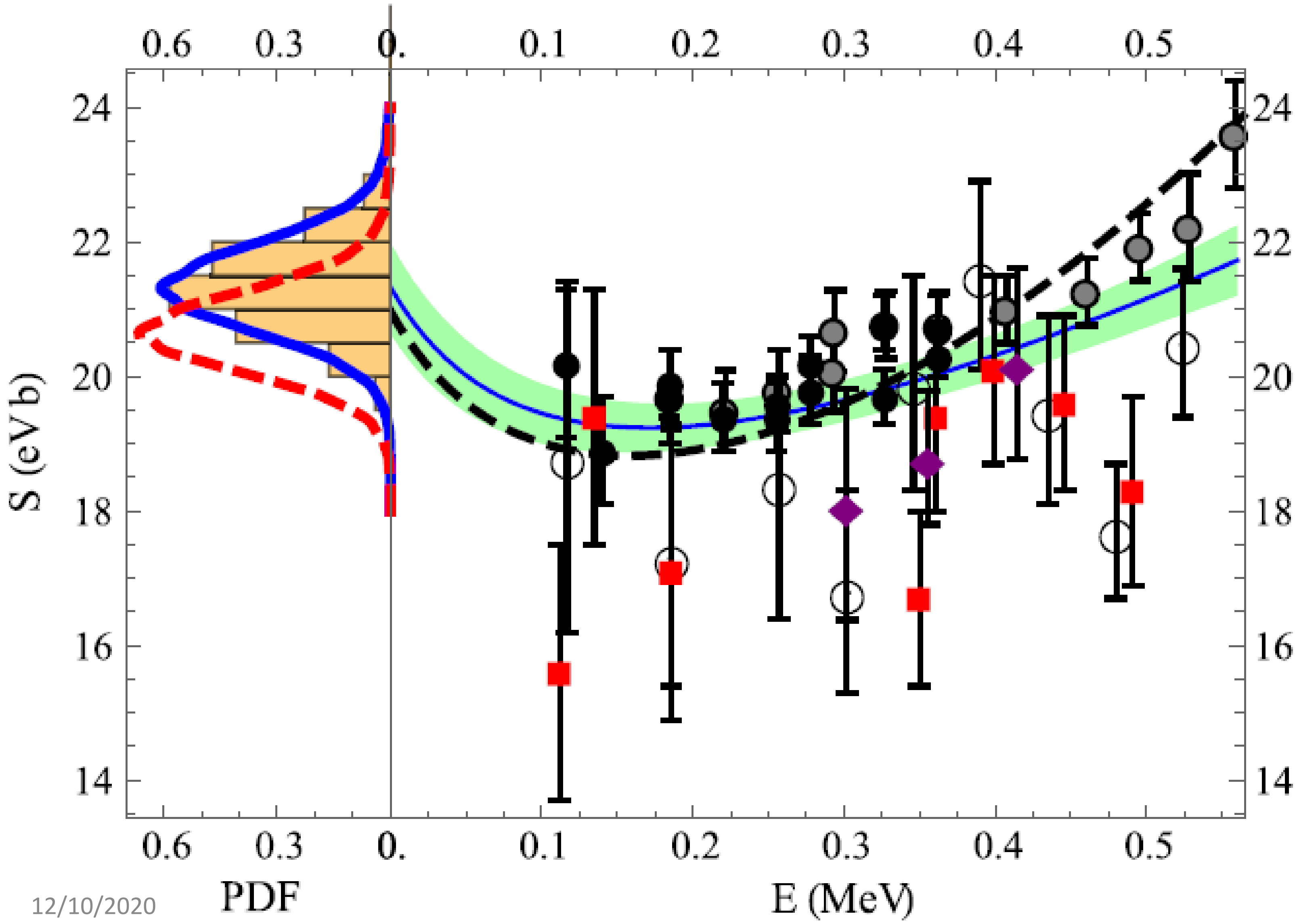
NLO: another 5 $r_{(^3S_1)}, r_{(^5S_2)}, \varepsilon_1, L_{E1}, L_{E2}$

$C^2_{(^3P_2)}$	$a_{(^3S_1)}$	$r_{(^3S_1)}$	ε_1	$\overline{L}_1$	$C^2_{(^5P_2)}$	$a_{(^5S_2)}$	$r_{(^5S_2)}$	$\overline{L}_2$
0.201	16.0	1.18	0	1.12	0.534	−10.0	3.93	2.69
0.201	25.0	1.36	0	1.27	0.533	−7.03	5.02	3.10
0.201	34.0	1.45	0	1.34	0.533	−4.03	8.56	4.19
0.109	−4.15	6.80	0	4.80	0.542	−6.91	3.57	3.73
0.108	7.19	0.785	0	0.725	0.480	7.19	0.785	0.725

EFT reproduces other models;
N2LO is about 1% below 1 MeV

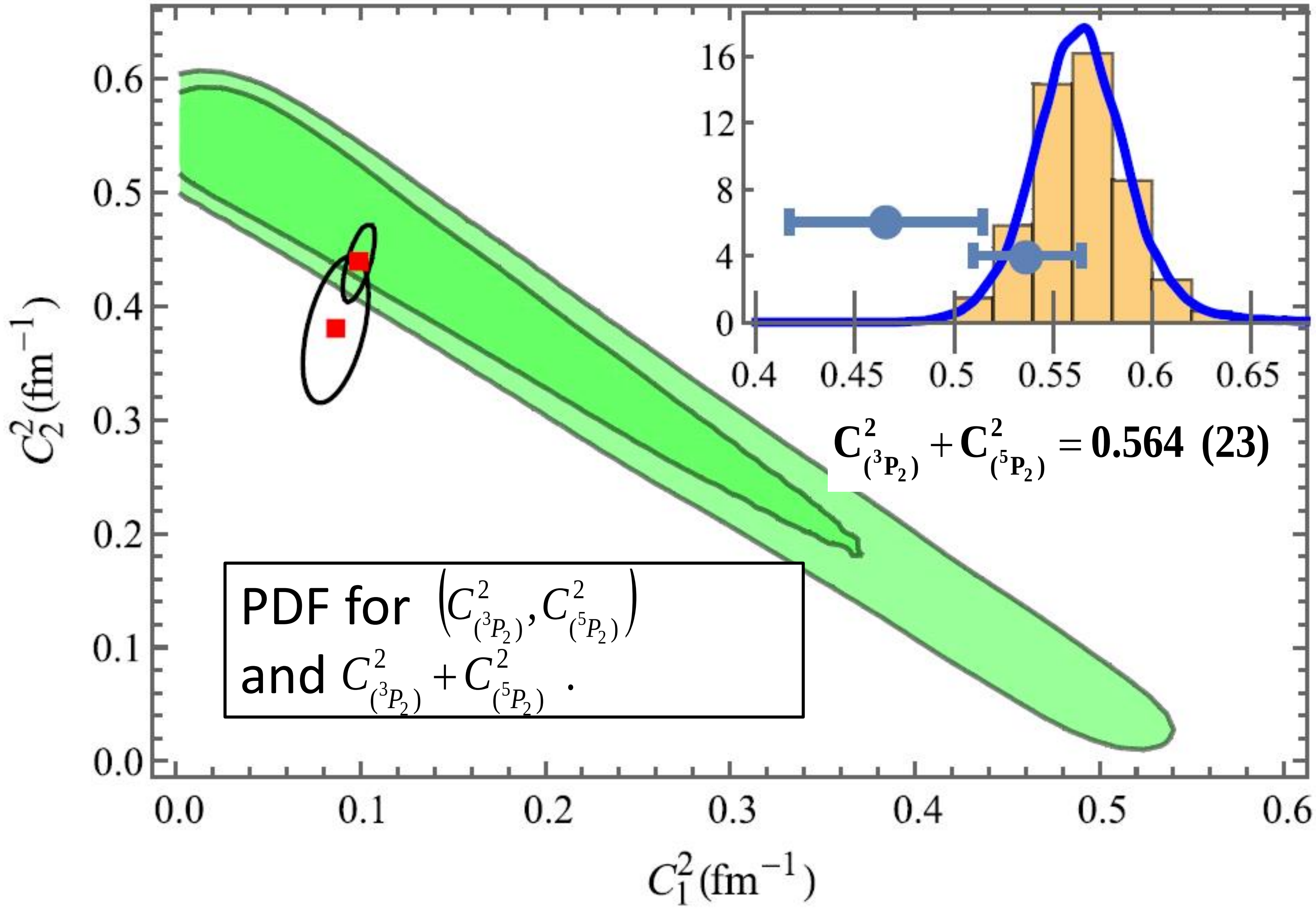


$$\mathrm{pr}\left(\boldsymbol{g},\{\xi_i\}|D;T;I\right)=\mathrm{pr}\left(D|\boldsymbol{g},\{\xi_i\};T;I\right)\mathrm{pr}\left(\boldsymbol{g},\{\xi_i\}|I\right)$$



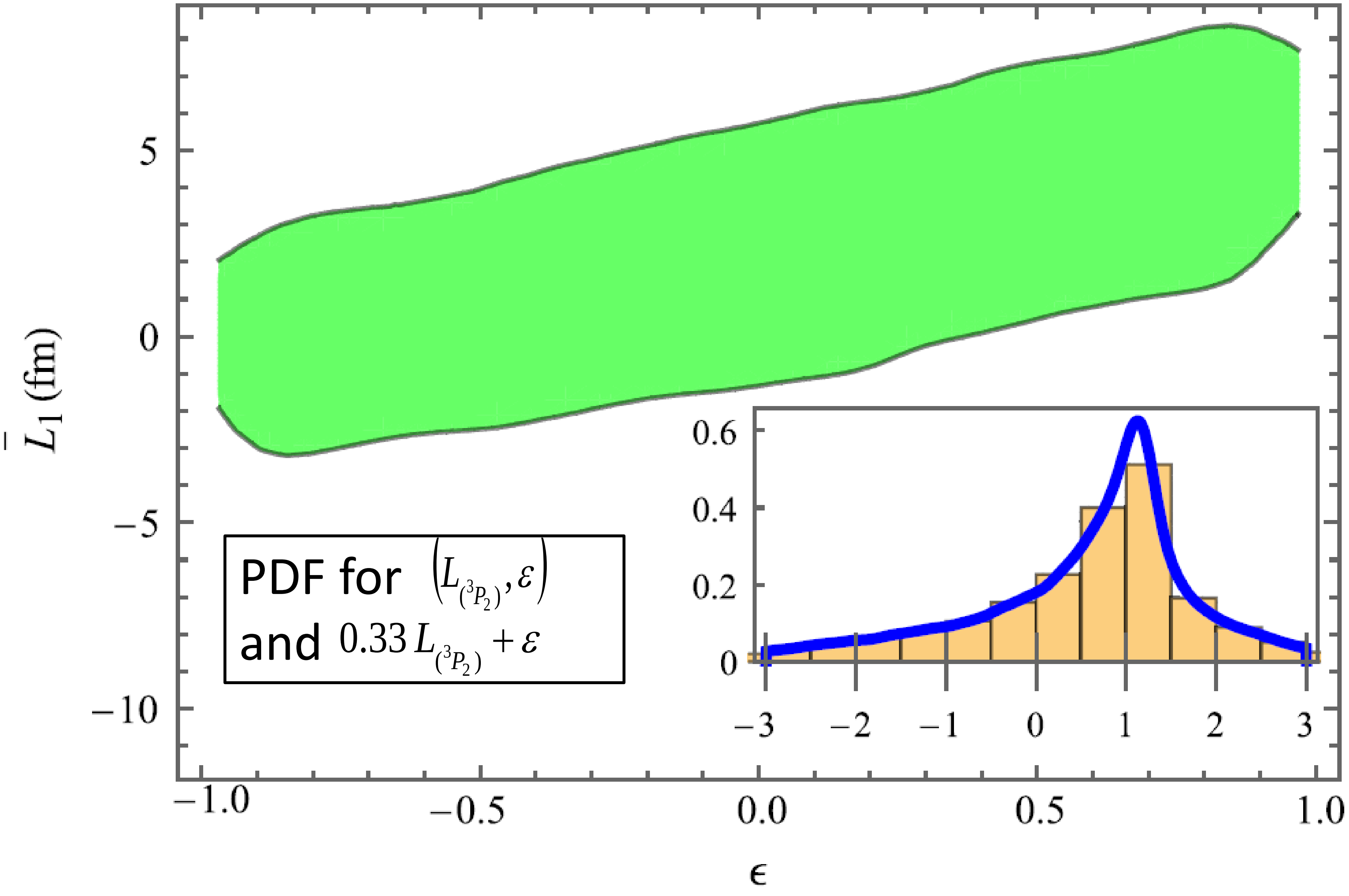
- Junghans BE1 and BE3 (filled circle), Filippone (open circle), Baby (filled diamond), Hammache (filled box)
- ξ_i: data rescaling factor
- $\boldsymbol{g}$: theory parameters
- ***I:*** prior information on para.

Green band is our 1-standard deviation error band: 3% error



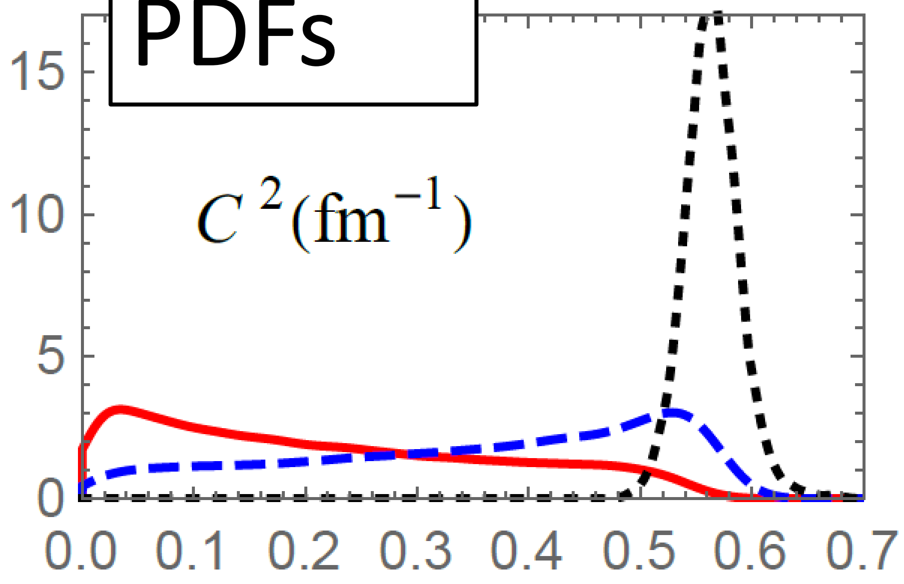
Direct capture reaction constrains total squared ANCs!

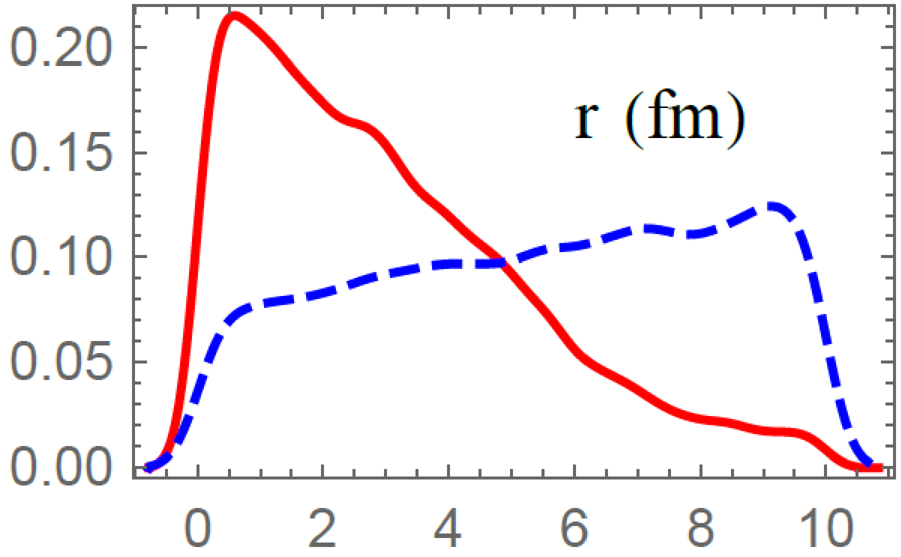
Tabacaru *et.al.*, measurements by transfer reaction (large eclipse)
Nollett *et.al.*, *ab initio* calculation (small eclipse)



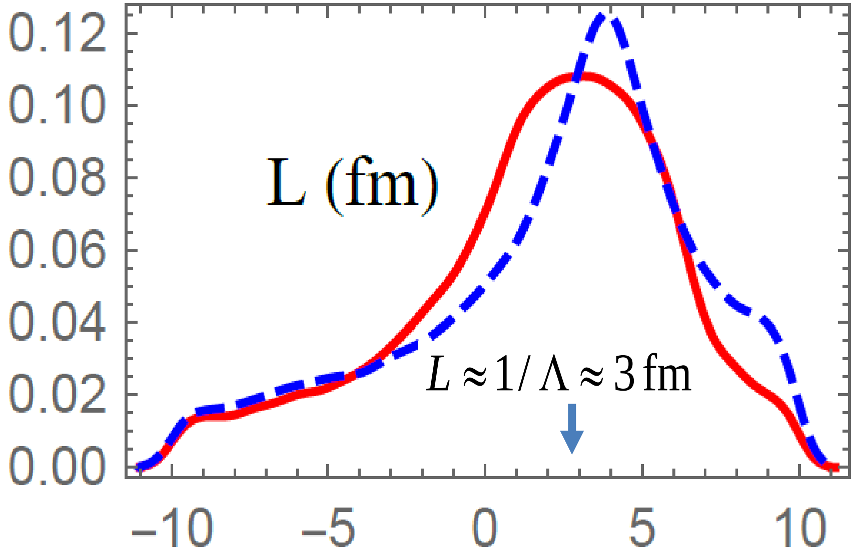
Core excitation and short-range term not distinguished by low energy data

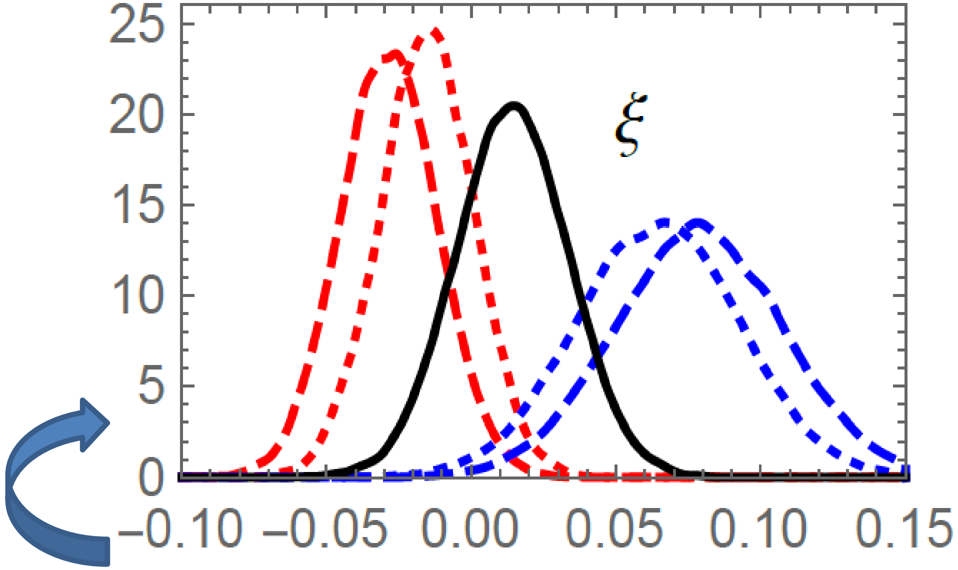
PDFs





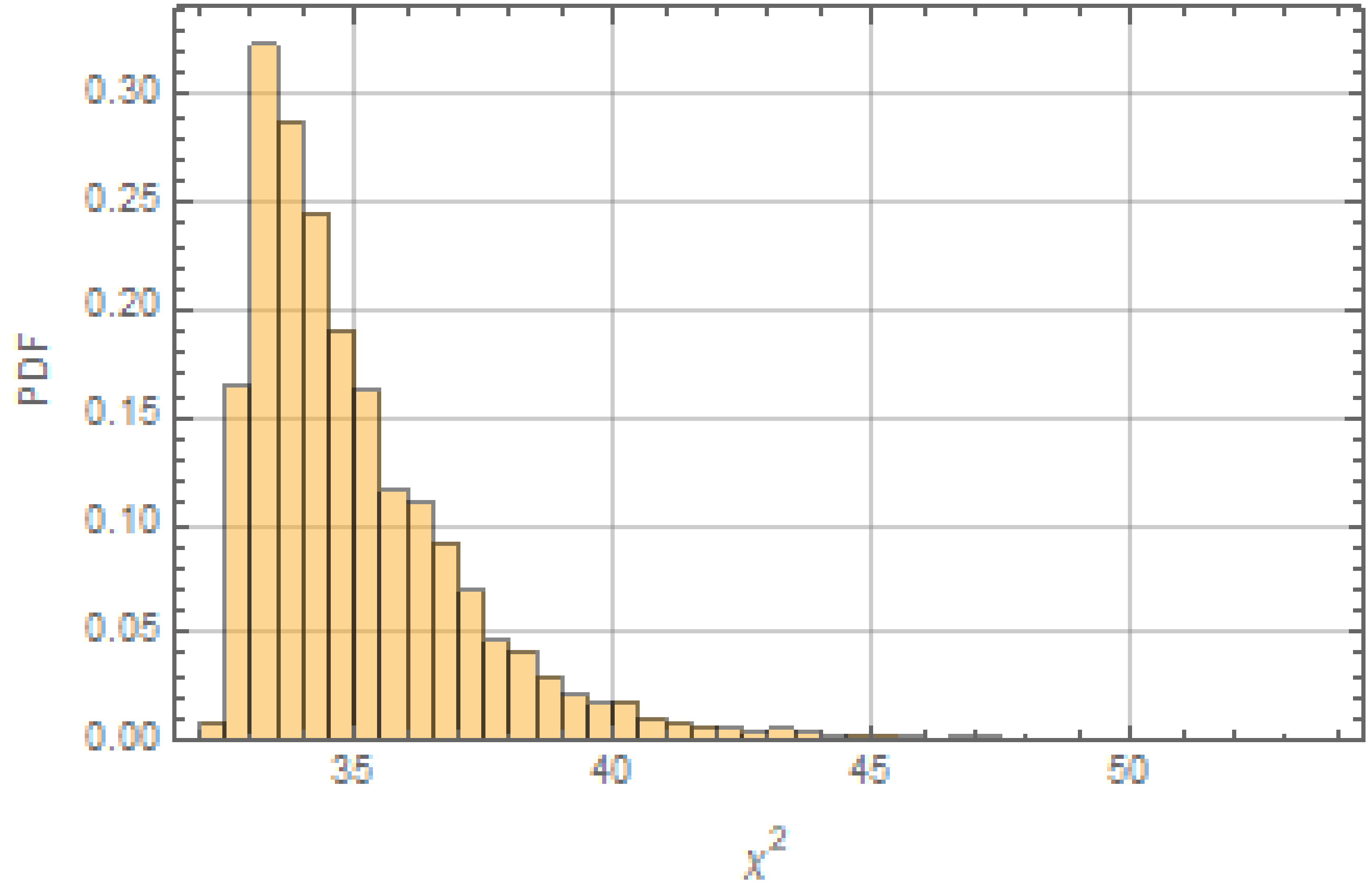
Red for S=1, Blue for S=2.





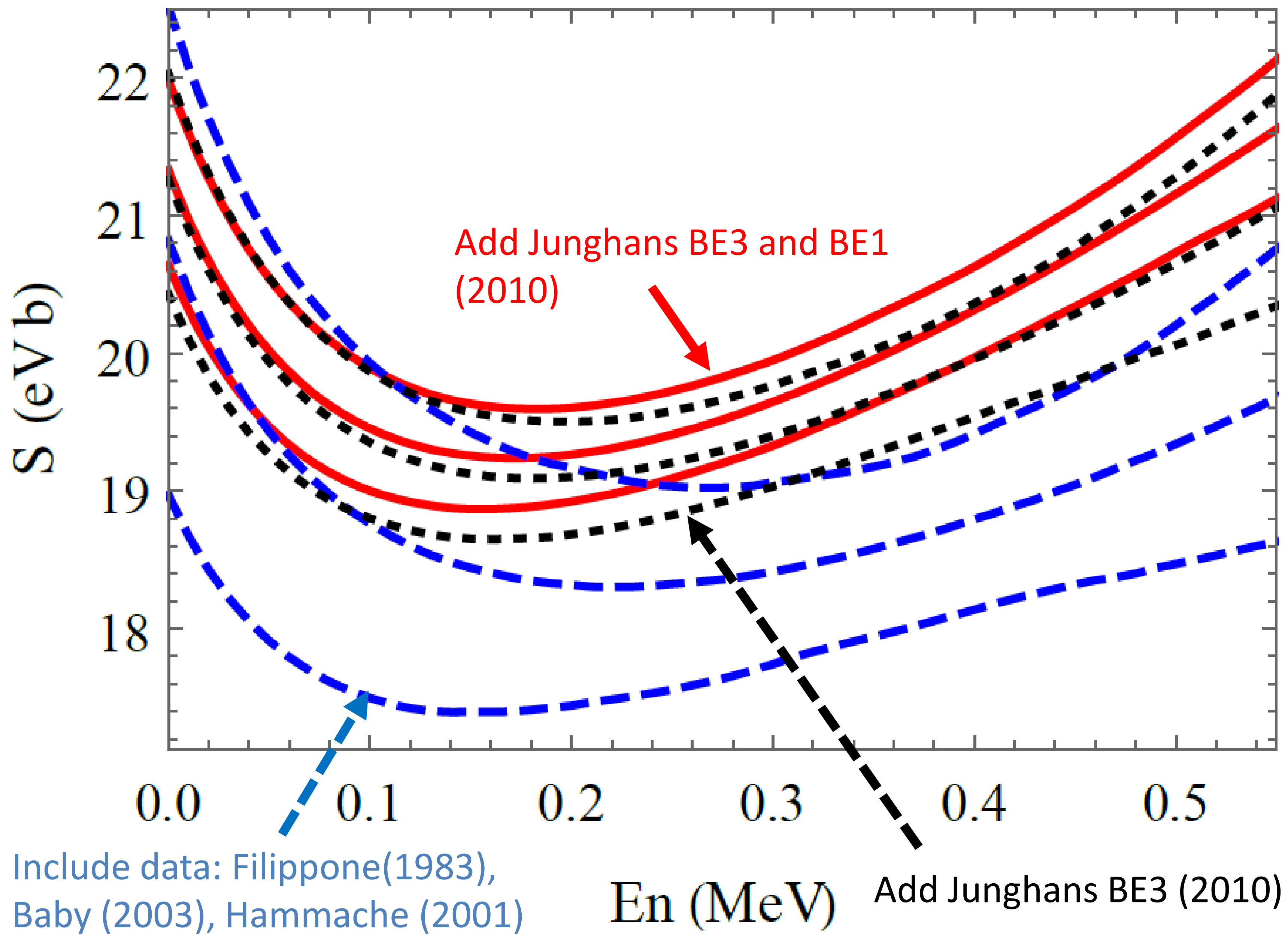
From left to right: Junghans (BE1and BE3) Baby, Hammache, Filipponne

Is it a “good fit”?



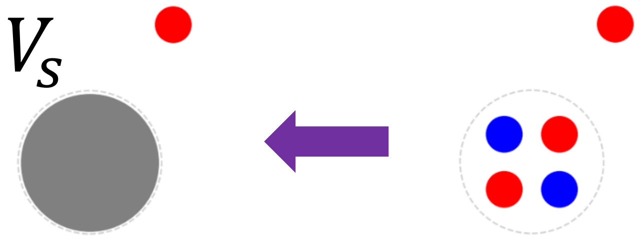
- 42 data points
- 9 EFT parameters
- 5 ξ_i parameters fixed to their mean

Choice of data sets

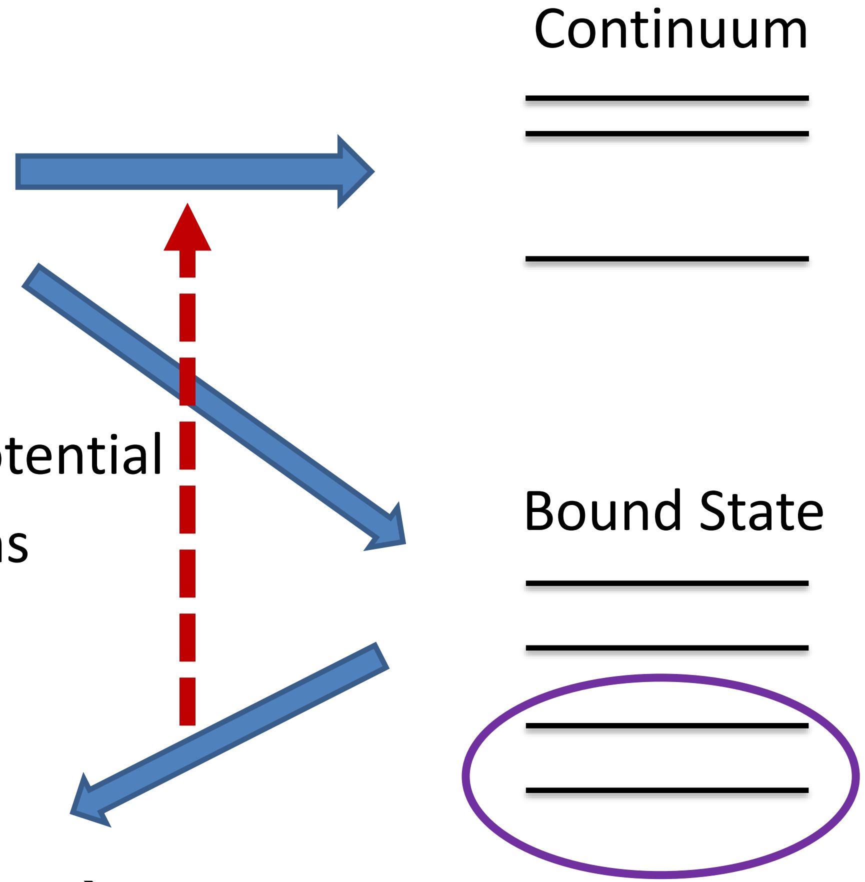


Ab initio scattering calculation

Computer experiment (CE)



Trap them in $\frac{1}{2} M_N \omega_T^2 r^2$ potential within ab initio calculations



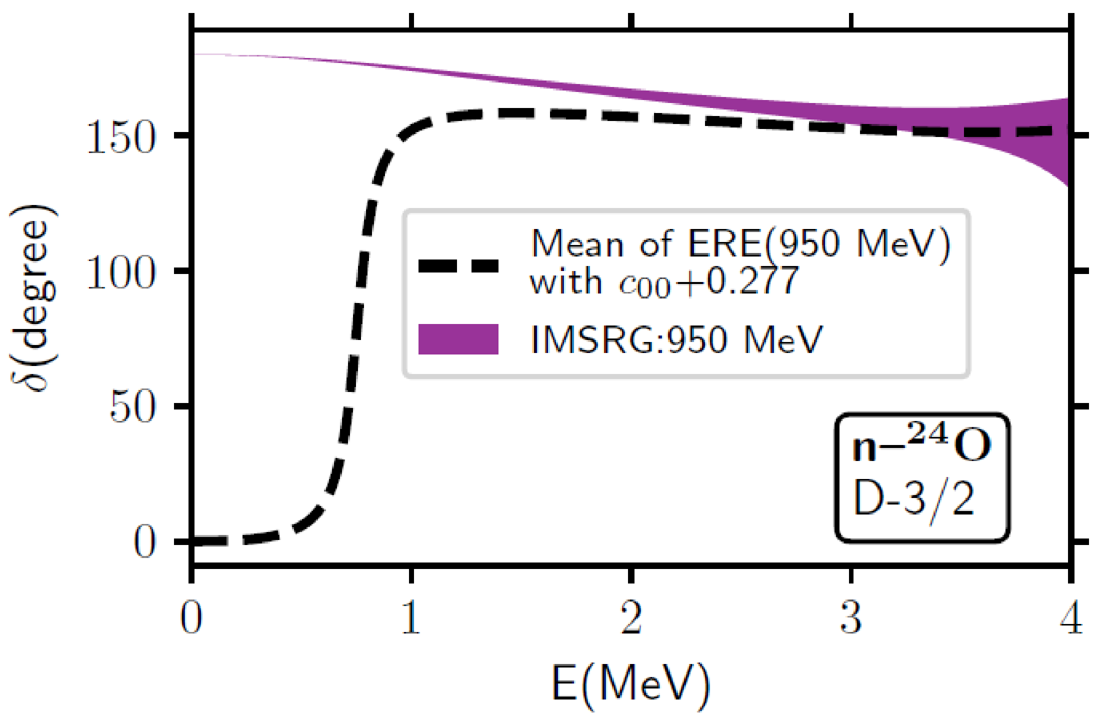
Constrain EFT (or model on V_s) → compute scattering and reaction

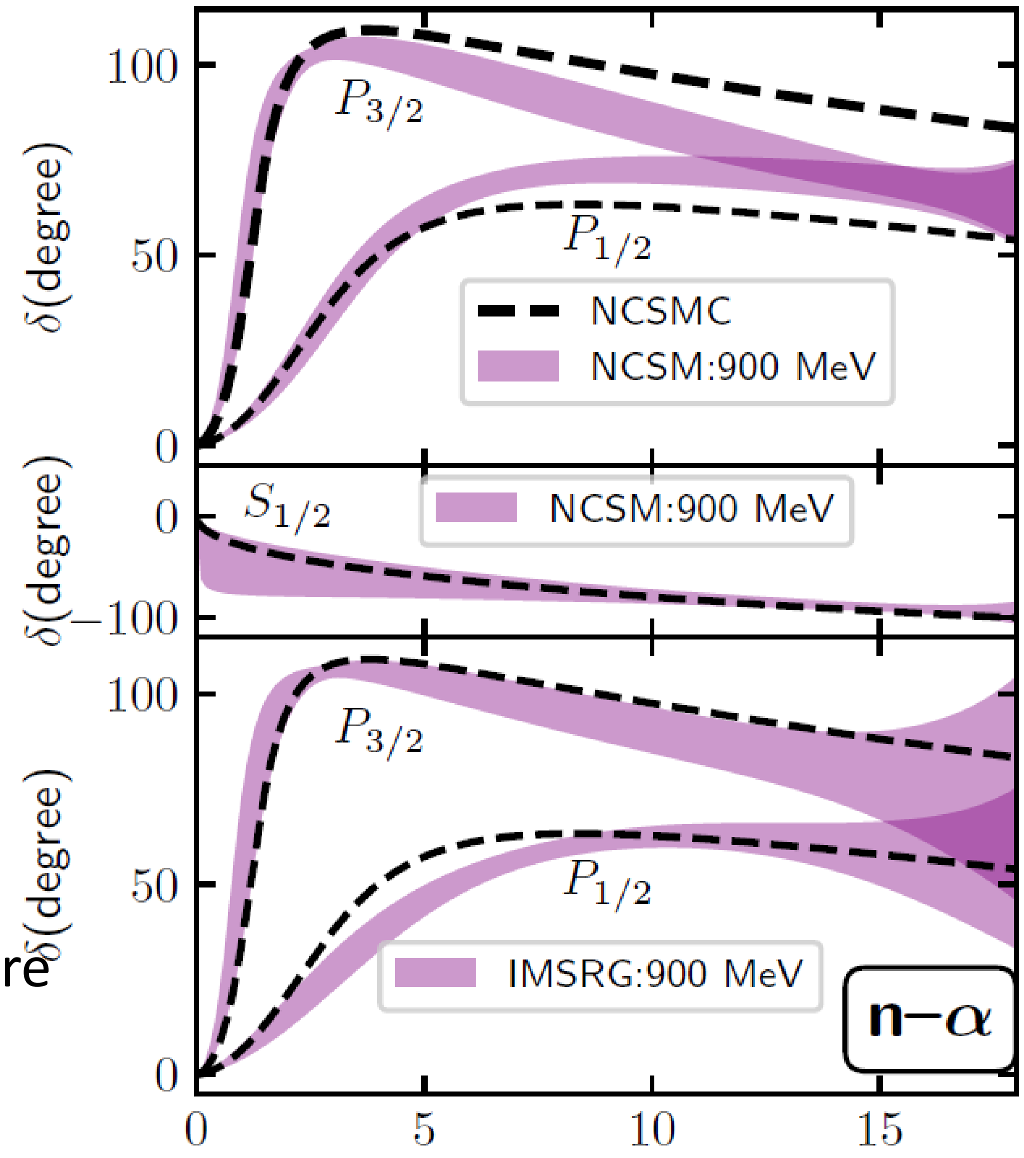
- Works for systems computable by structure methods (traps are important!)
- Matches nucleon-level and cluster-level theories through observables
- *Weinberg's Third Law of Progress in Theoretical Physics: use right DOFs*

Simplification:
$(\boldsymbol{E}, \boldsymbol{\omega_T}) \rightarrow \boldsymbol{\delta_l(E)}$

Promising results: n-α and n-O24 scatterings

XZ et.al., (2020)



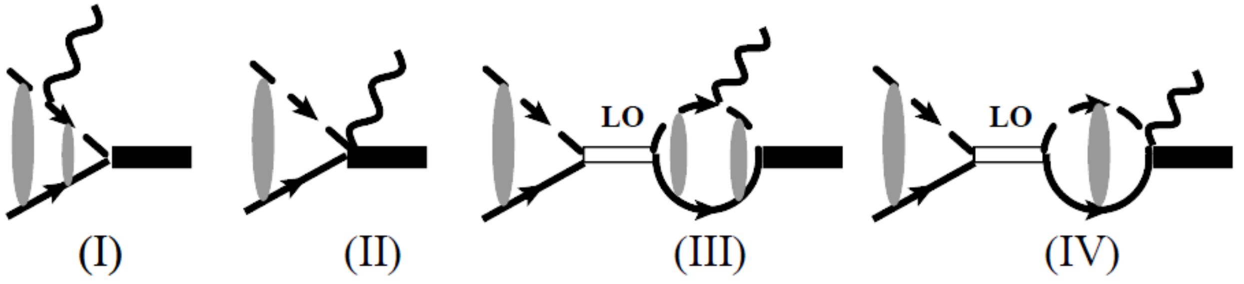


- No-core-shell-model (NCSM) and In-medium similarity renormalization group (IMSRG) structure methods→ phase shift extraction
- NCSM+C(continuum): direct scattering calcs.

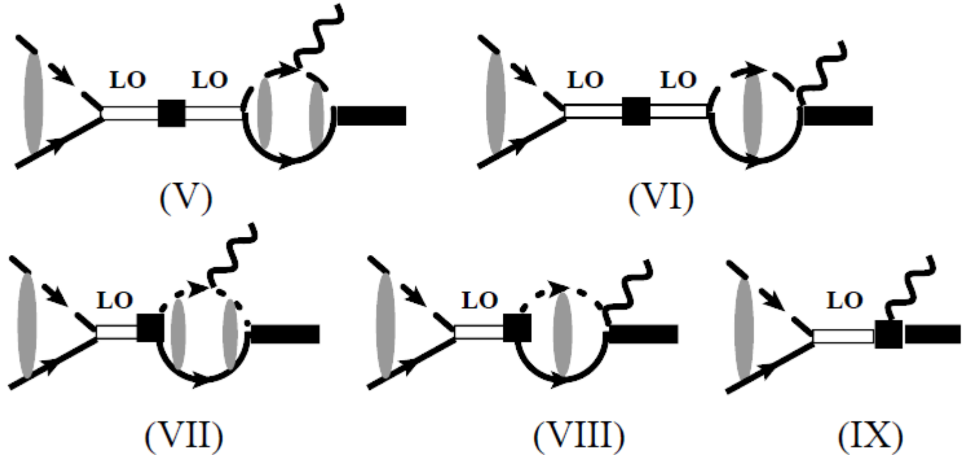
Summary

- Bayesian analysis + Halo-EFT is a useful tool for data analysis
- Computer experiment strategy enables ab initio calculations of scattering and reactions

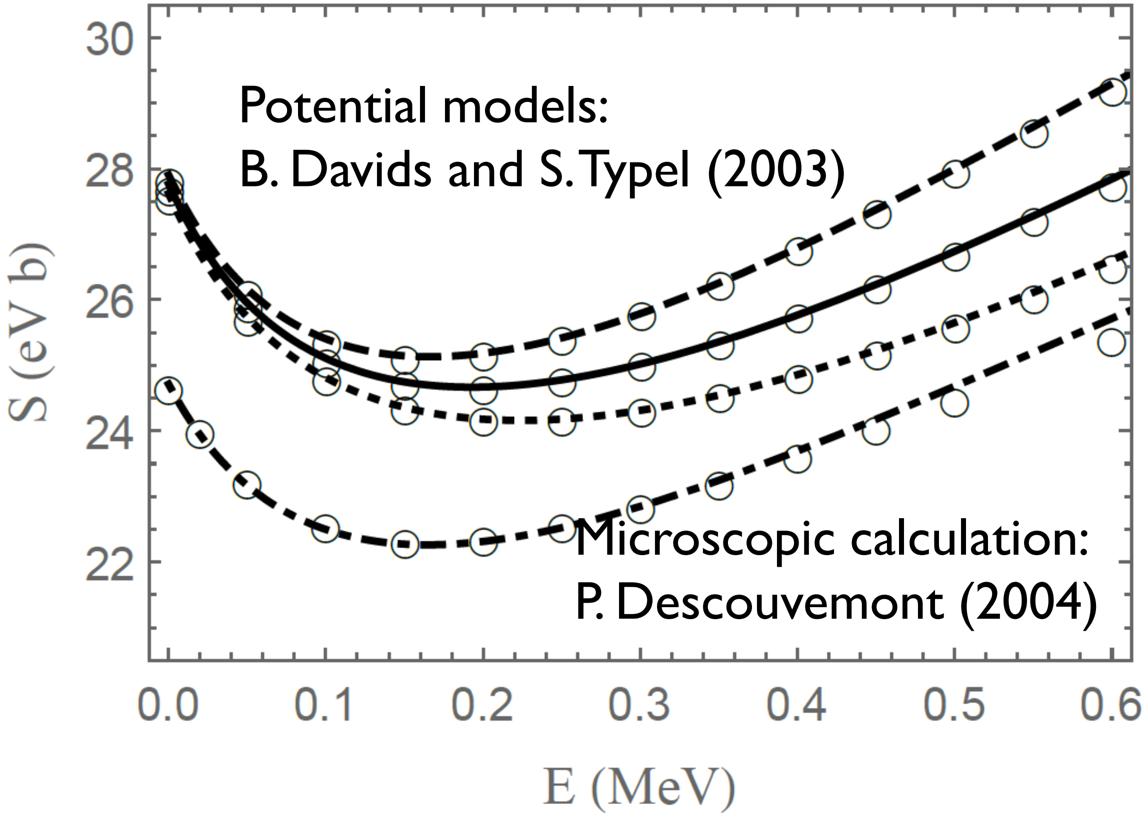
Halo-EFT: NLO



LO: 4 parameters $C_{(^3P_2)}, C_{(^5P_2)}, a_{(^3S_1)}, a_{(^5S_2)}$



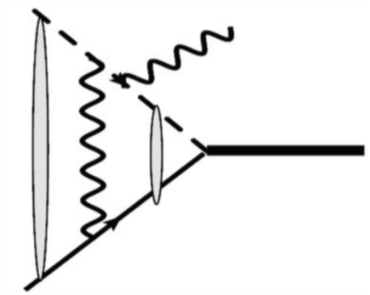
NLO: another 5 $r_{(^3S_1)}, r_{(^5S_2)}, \varepsilon_1, L_{E1}, L_{E2}$

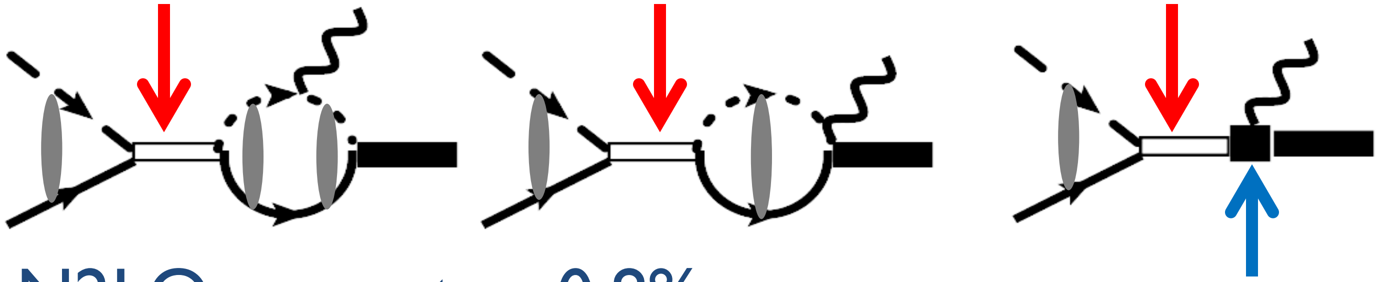


EFT reproduces other models; N2LO is about 1% below 1 MeV

EFT N2LO corrections

- E2, M1 contributions (S factor): < 0.01%
- Radiative corrections: ~0.1%
- EFT s-wave scattering: ~0.8%

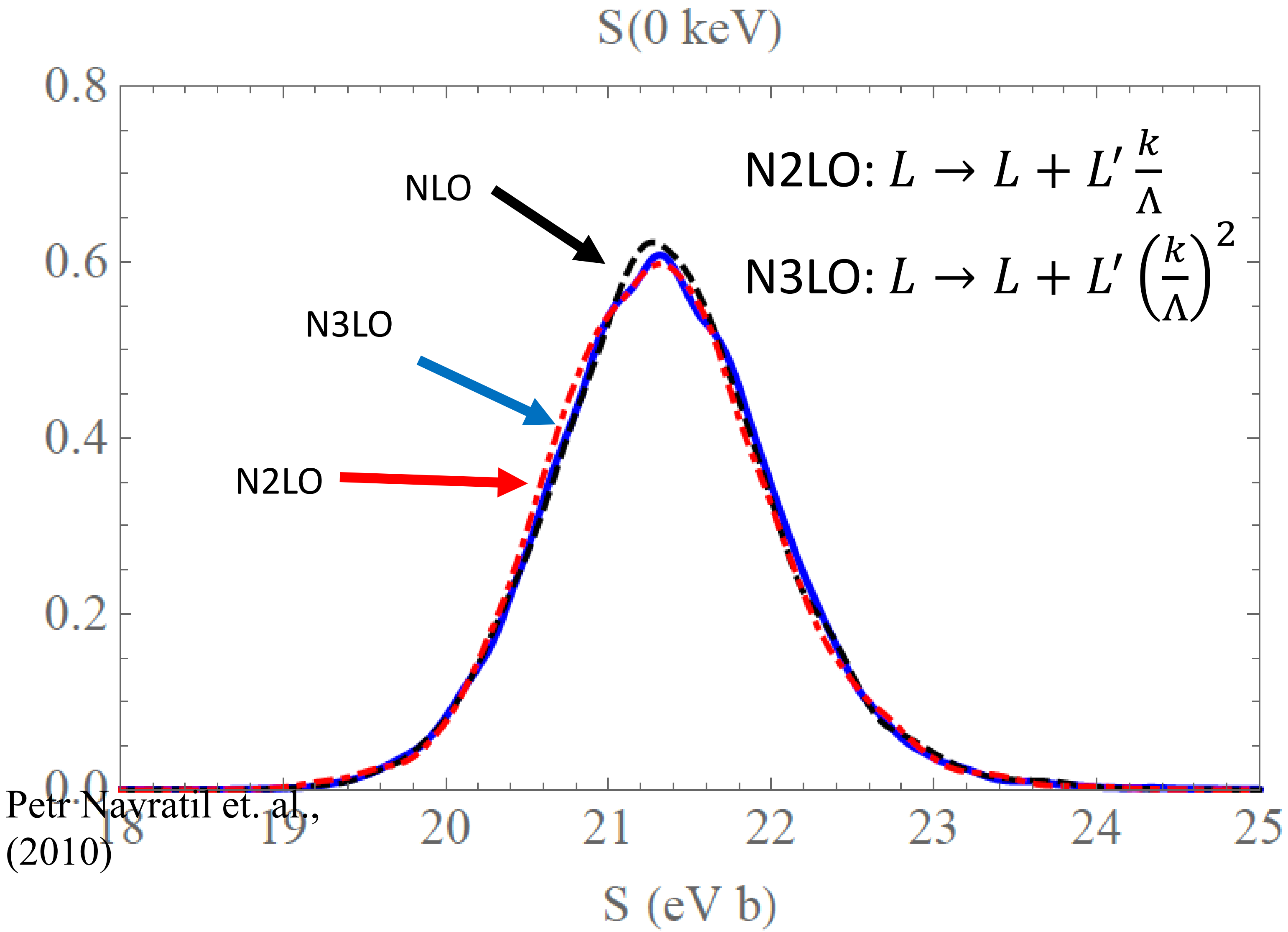




- EFT N2LO currents: ~0.8%
- Notice B8 BE=136.4(1.0) keV: ~ 0.8%

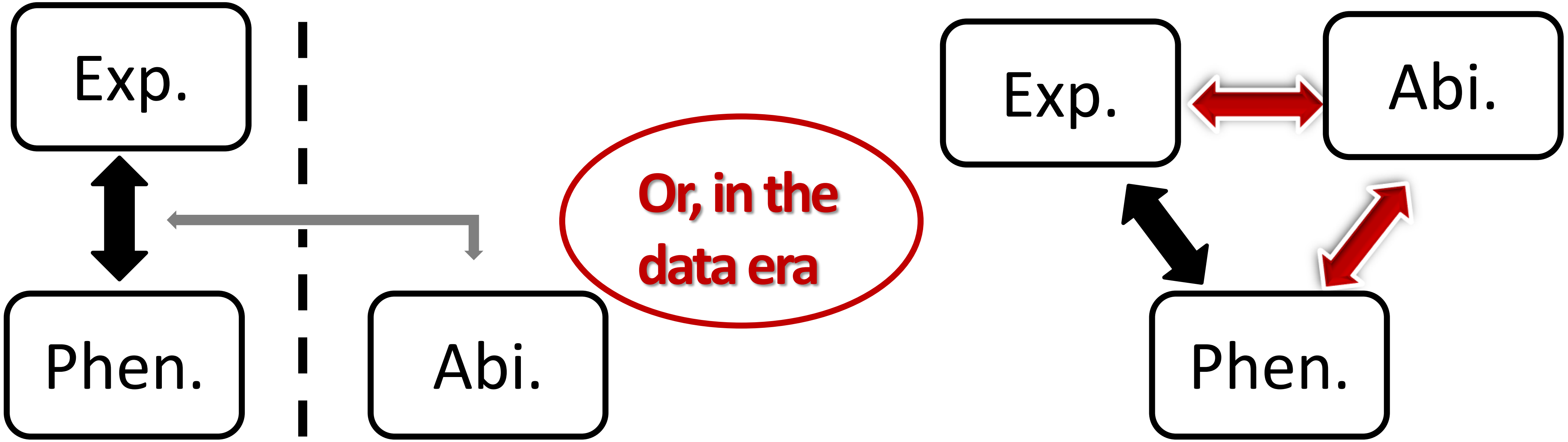
Recall EFT fitted to various potential model and RGM calculation results: deviation <~1% up to 1MeV (cm E).

N2LO impact on Bayesian analysis



Adding N2LO shifts S(0) by << 1%.

Synergy: relationships among **exp., ab initio calculations,** and **phen.** (cluster theory/optical potentials/R-matrix)



- Phen. Analysis (Cluster theory):
  - design CE, analyze Abi. results
  - require phen. errors under control
  - **efficient** platform for combining information (in contrast to tuning NN int. in Abi.)
  - **Exp.** Vs. **Abi**: complementary and/or competing
- Opens the door for data science tools