Capture Reactions in Effective Field Theories

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https://indico.mitp.uni-mainz.de/event/215/

Global Perspective

I look at several different reactions each with its own halo/cluster EFT.

Uncertainty quantification is system specific.

Still, what are the essential inputs for EFT?

Origin:

Bertulani, Hammer, van Kolck, NPA 712, 37 (2002)

Bedaque, Hammer, van Kolck, PLB 569, 159 (2003)

Review articles:

Hammer, Ji, Phillips, JPG 44, 103022 (2017)

Hammer, König, van Kolck, RMP 92, 025004 (2020)

One Slide on Effective Field Theories

Weinberg's 3rd law of progress in Theoretical Physics :

You may use any degree of freedom you like to describe a physical system, but if you use the wrong one, you will be sorry.

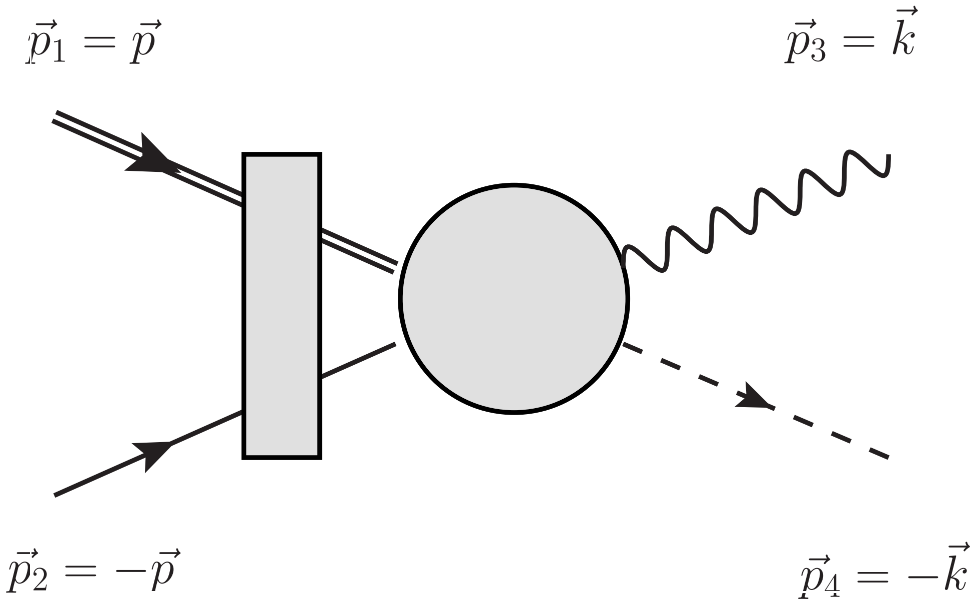
$$\mathcal{L}_{\text{interaction}} = c_0 \mathcal{O}^{(0)} + c_1 \mathcal{O}^{(1)} \dots$$

- $\mathcal{O}^{(i)}$: low-energy particle at momenta $p \sim Q$
- c_i hides short distance physics at momenta $\mathbf{\Lambda} \gg Q$
- Expansion in $\frac{Q}{\mathbf{\Lambda}}$ … which is system dependent

Platter and Phillips talks

Important: EFT is an expansion in energy/momentum not number of particles.

Anatomy of a Capture Reaction



$$\langle \psi_B | \, \mathrm{O}_{\mathrm{EM}} \, | \psi_i \rangle$$

Initial state: Phase shifts provide a model independent description

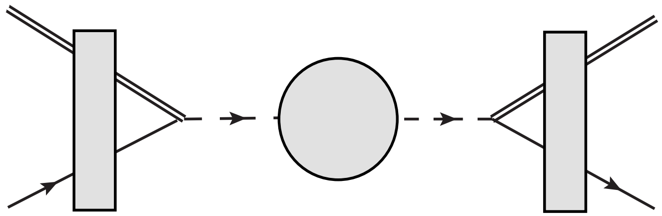
Final state: Again, phase shifts (affects overall normalization)

EM currents: One-body, two-body

These are the 3 sources of errors (in EFT).

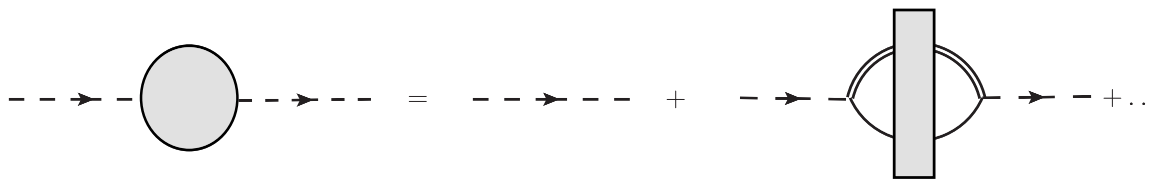
EFT and Phase Shift

$-iT_{SC} =$



$$T_{\rm SC}^{(l)} = -\frac{2\pi}{\mu}\frac{e^{2i\sigma_l}}{p\cot\delta_l - ip}$$

$iD(p_0, \boldsymbol{p}) =$



$$\left[\frac{\Gamma(2l+2)}{2^l\Gamma(l+1)}\right]^2 [C_l(\eta_p)]^2 p^{2l+1}(\cot\delta_l - i) = -\frac{1}{a_l} + \frac{1}{2}r_l p^2 - \frac{2k_C\, p^{2l}}{\Gamma(l+1)^2}\frac{|\Gamma(l+1+i\eta_p)|^2}{|\Gamma(1+i\eta_p)|^2}H(\eta_p),$$

$$H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \ln(i\eta),$$

Hamilton, Overbö, Tromborg, NPB 60, 443 (1973)
Higa, Rupak, Vaghani; EPJA 54, 89 (2018)

The numerical values of the scattering parameters a_l , r_l , etc. , affect the perturbation and so the uncertainty estimates.

Bound State Normalization

$$\frac{1}{\mathcal{Z}^{(\zeta)}} = \frac{\partial}{\partial p_0}[D^{(\zeta)}(p_0;\boldsymbol{p})]^{-1}\Big|_{p_0=p^2/(2\mu)-B}$$

p-wave bound states are a little subtle : $\mathcal{Z}^{(\zeta)} \propto \frac{1}{\rho_1^{(\zeta)} - f(k_C,\gamma)}$

Need both binding energy and effective momenta at LO. Small change in ρ_l can affect cross section by large amount

Rupak, Higa, PRL 106,222501 (2011)
Higa, Premarathna, Rupak, arXiv:2009.09324

$$f(k_C,\gamma) = 4k_C\, H\left(-i\frac{k_C}{\gamma}\right) + \frac{2k_C^2}{\gamma^3}(k_C^2-\gamma^2)\left[\psi'\left(\frac{k_C}{\gamma}\right) - \frac{\gamma^2}{2k_C^2} - \frac{\gamma}{k_C}\right]$$

$$\overset{k_C\to 0}{=} \quad 3\gamma$$

Connection to *ab initio* calculation Zhang, Nollett, Phillips, PRC 89, 024613 (2014)

Asymptotic Normalization Constant (ANC) $|C_b|^2 = \frac{\gamma^{2l}}{\pi\mu^{2l-2}}\left[\Gamma(l+1+\eta_b)\right]^2 \frac{2\pi}{\mu}\mathcal{Z}$

Higa, Premarathna, Rupak, arXiv:2010.13003

EM currents

1-body currents obtained from minimal substitution and magnetic moments

2-body currents are a source of uncertainty, usually subleading

Source of irreducible error, not constrained by Siegert/Ward-Takahashi theorem

$\nu + d$ Butler, Chen, NPA 675, 575 (2000)

$np \to d\gamma$ Rupak, NPA 678, 405 (2000)

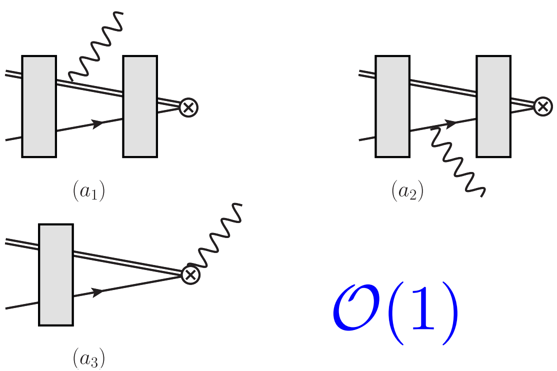
$^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$ in halo EFT

^{3}He and α as point particles

$\frac{3}{2}^{-}$ ground and $\frac{1}{2}^{-}$ excited state of ^{7}Be as p-wave bound state

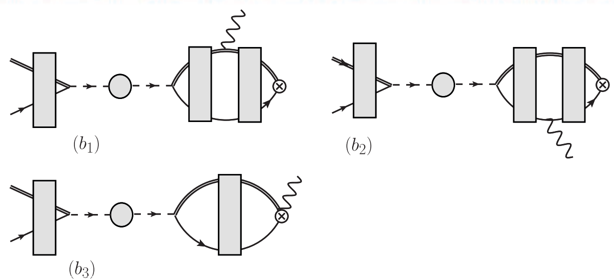
E1 capture from initial s- and d-wave state

Just Coulomb, no unknown parameters up to ANC



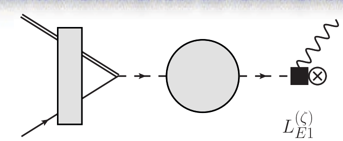
$\mathcal{O}(1)$

Initial strong interaction and ANC



$\sim a_0(B+J) \sim a_0 Q^3$

2-body current and ANC



$a_0 k_0 L_{\mathrm{E1}}$

$\sim a_0 Q^3 L_{\mathrm{E1}}$

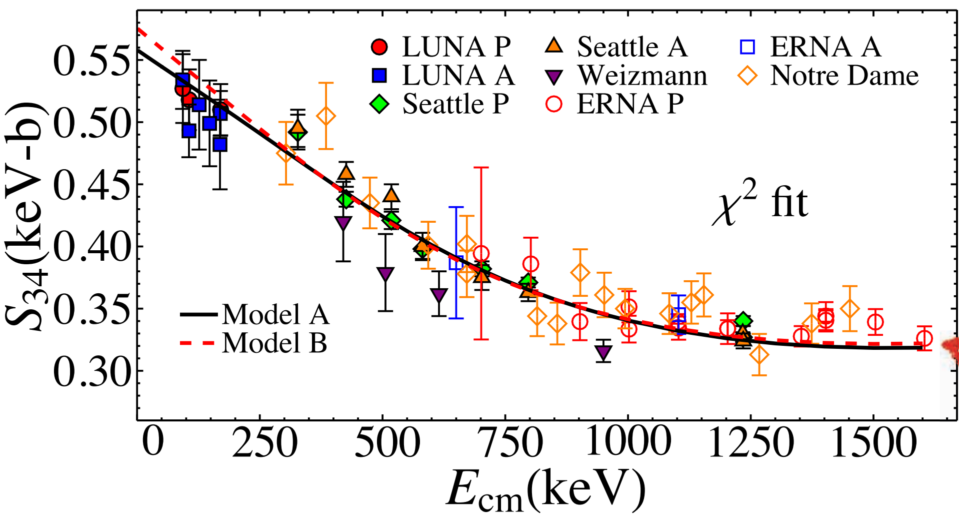
$Q \sim 60 - 70\ \mathrm{MeV}$

$\Lambda \sim 150 - 200\ \mathrm{MeV}$

Higa, Rupak, Vaghani; EPJA 54, 89 (2018)
Premarathna, Rupak; EPJA 56, 166 (2020)
Zhang, Nollett, Phillips; JPG 47, 054307 (2020)

Power Counting (Survey)

The size of a_0 determines the relative importance of initial state interaction and 2-body currents which can be as important as the LO "tree-level".



2-body current ($a_0 \sim 1/Q^3$)

No 2-body current, smaller a_0

Just a survey. Can't just drop two-body currents!

$$\chi^2_A \approx 0.9 \pm 0.2;\ \chi^2_B \approx 1.5 \pm 0.2$$

Higa, Rupak, Vaghani; EPJA 54, 89 (2018)

Knowledge of scattering phase shift helps in constructing the EFT and uncertainty estimates. How come potential models don't need 2-body currents?

$$\text{ANC} \propto \frac{1}{\rho_1^{(\zeta)} - f(k_C, \gamma)}$$

Sits near a pole in this system

Bayesian inferences for $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$

Fits	a_0 (fm)	r_0 (fm)	s_0 (fm^3)	$\rho_1^{(+)}$ (MeV)	$\sigma_1^{(+)}$ (fm)	$\rho_1^{(-)}$ (MeV)	$\sigma_1^{(-)}$ (fm)	$L_1^{(+)}$	$L_1^{(-)}$	K
χ^2	22 ± 3	1.2 ± 0.1	-0.9 ± 0.7	-55.4 ± 0.5	1.59 ± 0.03	-41.9 ± 0.7	1.74 ± 0.05	0.78 ± 0.06	0.83 ± 0.08	–
Model A I	48^{+2}_{-2}	$1^{+0.09}_{-0.1}$	$-1.8^{+1}_{-0.9}$	-72^{+5}_{-8}	$2.1^{+0.2}_{-0.2}$	-49^{+3}_{-6}	$2^{+0.2}_{-0.1}$	$1.4^{+0.2}_{-0.1}$	$1.2^{+0.2}_{-0.1}$	$0.3^{+0.4}_{-0.2}$
Model B I	38^{+3}_{-2}	$1.1^{+0.1}_{-0.1}$	-2^{+1}_{-1}	$-61.6^{+0.6}_{-0.6}$	$1.77^{+0.03}_{-0.04}$	-48^{+1}_{-7}	$2^{+0.2}_{-0.08}$	$1.13^{+0.03}_{-0.02}$	$1.2^{+0.3}_{-0.08}$	$0.3^{+0.3}_{-0.2}$
Model A* I	20^{+8}_{-5}	$-0.1^{+0.5}_{-0.7}$	-16^{+6}_{-8}	-89^{+9}_{-20}	–	-130^{+50}_{-70}	–	$3^{+1}_{-0.9}$	7^{+2}_{-3}	–
Model B* I	37^{+3}_{-10}	$1.1^{+0.1}_{-0.9}$	–	$-61.4^{+1}_{-0.8}$	–	-47^{+2}_{-6}	–	$1.14^{+0.09}_{-0.04}$	$1.2^{+0.2}_{-0.1}$	–
Model A II	40^{+5}_{-6}	$1.09^{+0.09}_{-0.1}$	$-2.2^{+0.8}_{-0.8}$	-59^{+1}_{-2}	$1.69^{+0.05}_{-0.06}$	-45^{+2}_{-2}	$1.84^{+0.08}_{-0.08}$	$1.02^{+0.06}_{-0.06}$	$1.07^{+0.08}_{-0.09}$	$0.3^{+0.3}_{-0.2}$
Model B II	$7.3^{+0.7}_{-0.7}$	$1.31^{+0.02}_{-0.02}$	6^{+1}_{-1}	$-53.5^{+0.1}_{-0.1}$	$1.53^{+0.05}_{-0.06}$	$-40.1^{+0.2}_{-0.2}$	$1.67^{+0.06}_{-0.06}$	$-0.04^{+0.08}_{-0.1}$	$-0.01^{+0.09}_{-0.1}$	$2.2^{+0.6}_{-0.5}$
Model A* II	46^{+10}_{-4}	$1^{+0.1}_{-0.3}$	-3^{+5}_{-2}	-62^{+5}_{-4}	–	-51^{+4}_{-70}	–	$1.1^{+0.1}_{-0.2}$	$1.3^{+2}_{-0.2}$	–
Model B* II	5^{+1}_{-2}	$1.24^{+0.04}_{-0.2}$	–	$-53.5^{+0.1}_{-0.2}$	–	$-40.2^{+0.2}_{-0.2}$	–	$-0.5^{+0.2}_{-1}$	$-0.4^{+0.2}_{-0.9}$	–

“Evidence” from data

Fit	$S_{34}(E_\star)$ (keV b)	$S'_{34}(E_\star)$ (10^{-4} b)
χ^2	$0.558 \pm 0.008 \pm 0.056$	$-2.71 \pm 0.20 \pm 0.27$
Model A I	$0.541^{+0.012}_{-0.014} \pm 0.054$	$-1.34^{+0.64}_{-0.59} \pm 0.13$
Model A II	$0.550^{+0.009}_{-0.010} \pm 0.055$	$-2.00^{+0.36}_{-0.35} \pm 0.20$
Model A* II	$0.551^{+0.021}_{-0.014} \pm 0.055$	$-1.86^{+0.72}_{-1.69} \pm 0.19$
Model B* II	$0.573^{+0.007}_{-0.007} \pm 0.017$	$-3.72^{+0.11}_{-0.10} \pm 0.11$

TABLE I. $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$: S_{34} and S'_{34} at threshold (defined as $E_\star = 60 \times 10^{-3}$ keV). The second set of errors are estimated from the EFT perturbation as detailed in the text.

We recommend A II if using shift information. Alternatively, A* II or B* II.

Higa, Rupak, Vaghani; EPJA 54, 89 (2018)
Premarathna, Rupak; EPJA 56, 166 (2020)

recommended value from the review in Ref. [1] is: $S_{34}(0) = [0.56 \pm 0.02(\text{expt.}) \pm 0.02(\text{theory})]$ keV b.

Adelberger et al., RMP 83, 195 (2011)

$^3\text{He} + \alpha$ Phase Shift from SONIK

Table 5.3: The *s*-wave scattering parameters for ^{3}He+^{4}He system for different choice of data sets and energy range. The scattering parameters are calculated at a=4.2 fm. The combined data set refers to the simultaneous fitting of the SONIK data and the Barnard *et al.* [BJP64].

"Elastic Scattering of ^{3}He+^{4}He with SONIK", S. N. Paneru, PhD thesis, 2020

Energy	Data Set	χ^2/N	a_0 (fm)	r_0 (fm)
$E[^3\text{He}]$<6 MeV	SONIK only	2.30	33.188	1.01
	Barnard *et al.* [BJP64] only	1.13	34.00	1.02
	Combined	1.86	33.57	1.01
$E[^3\text{He}]$<4 MeV	SONIK only	1.70	41.88	1.06
	Barnard *et al.* [BJP64] only	0.50	31.96	1.00
	Combined	1.37	38.43	1.04

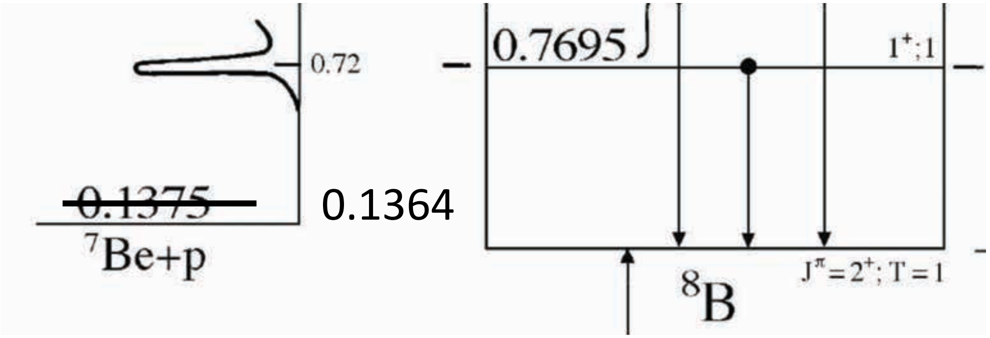
p-wave parameters would be good to know.

Table 5.4: *s*-wave scattering parameters for the ^{3}He+^{4}He system.

a_0 (fm)	r_0 (fm)	Method	Reference
7.7	-	NCSMC	J. Dohet-Eraly *et al.* [DENQ+16]
41.06	1.01	Microscopic Cluster Model	R. Kamouni and D. Baye [KB07]
40^{+5}_{-6}	$1.09^{+0.09}_{-0.1}$	EFT	P. Premarathna and R. Gautam [PR20]
50^{+7}_{-6}	0.97 ± 0.03	EFT	X. Zhang *et al.* [ZNP20]

$^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ in halo EFT

- ^{7}Be and p as point particles
- $\frac{3}{2}^-$ ground and $\frac{1}{2}^-$ excited state of ^{7}Be can contribute
- E1 capture from initial s- and d-wave state
- M1 capture from near the 1^+ ^{8}B resonance



TUNL database

^{7}Be excitation energy
$E_* \sim 0.429$ MeV

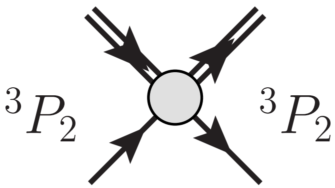
For $E > E_*$, an inelastic channel with excited ^{7}Be channel opens. Expect it to be important above about 500 keV in spin channel S=1.

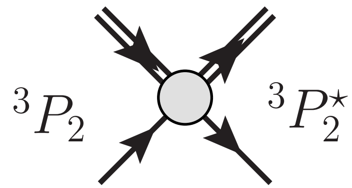
Spin channel S=2 is dominant.

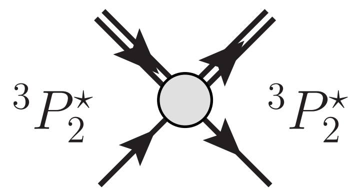
EFT for about 500 keV
$\mathrm{EFT}_\star$ above 500 keV

$$Q \sim \gamma \sim k_C \sim p$$
$$\Lambda \sim 70 \text{ MeV}$$

Coupled Channel Calculation







Low energy: 3 scattering volumes, 3 effective momenta

$$\mathcal{A}(p) = -9[C_1(\eta_p)]^2 e^{i2\sigma_1}\frac{2\pi}{\mu}\frac{p^2}{\mu^2}\mathcal{D}(E,0)\,,$$

$$\mathcal{D}^{-1}(E,0) = \mathcal{D}_0^{-1}(E,0) - \Sigma(E,0)$$

Free inverse Gorkov-like propagator, off-diagonal terms

Self-energy, diagonal matrix

Higa, Premarathna, Rupak, arXiv:2010.13003
Cohen, Gelman, van Kolck, PLB 588, 57 (2004)
Lensky, Birse, EPJA 47, 142 (2011)

$$\mathcal{Z}^{-1} = \frac{d}{dE}[\mathcal{D}(E,0)]^{-1}\Big|_{E=-B}$$
$$= -\frac{1}{\mu}\begin{pmatrix} \rho_{11} - 1.6\ \mathrm{MeV} & \rho_{12} \\ \rho_{12} & \rho_{22} + 5.1\ \mathrm{MeV} \end{pmatrix}$$

$^3P_2$, $^3P_2^\star$ ANCs not enough

2 fix the 3 effective momenta

Also include mixing in $^3S_1 - {}^3S_1^\star$

Zhang, Nollett, Phillips, PLB 751, 535 (2015)
PRC 98, 034616 (2018)

Power Counting

- Expect initial scattering at low-energy to be peripheral
- Capture should still proceed without strong interaction as ^{8}B is very shallow state

Strong interaction: $a_0(B+J) \sim a_0 Q^2$

$$a_0^{(2)} = -3.18^{+0.55}_{-0.50}\ \mathrm{fm} \sim 1/\Lambda\,, \qquad a_0^{(1)} = 17.34^{+1.11}_{-1.33}\ \mathrm{fm} \sim 1/Q$$

Paneru et al., PRC 99, 045807 (2019)

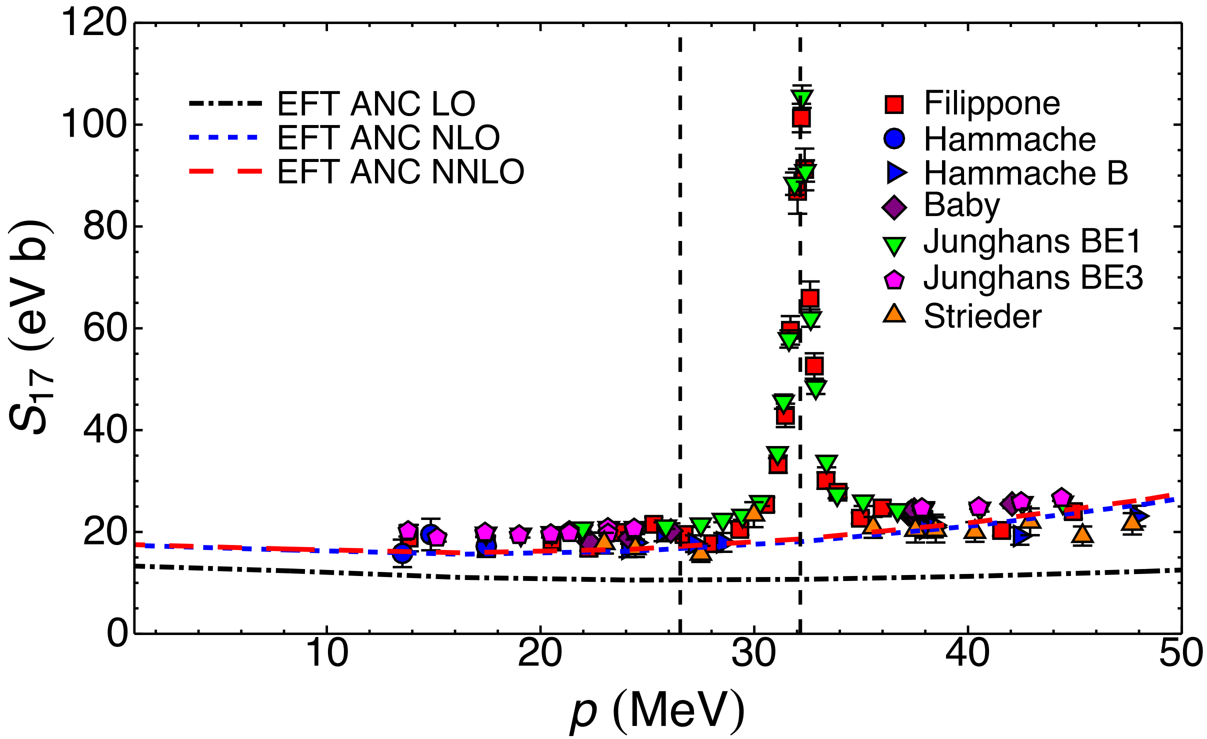
- LO: s-wave capture without strong interaction in spin S=2 channel
- NLO: d-wave capture in S=2, s-wave capture in S=1 without strong interaction
- NNLO: s-wave strong interaction in S=1,2 and d-wave in S=1 : excited core only relevant at NNLO

In 20/20 hindsight:

$$S_{17}/C^2_{1,\zeta} \approx 35.6(1 - a_0\, 0.00266\ \mathrm{fm}^{-1} + 0.0657 + \ldots)\,\mathrm{eV\ b\ fm}$$

Baye, PRC 62, 065803 (2000)
Zhang, Nollett, Phillips, PRC 98, 034616 (2018)
Higa, Premarathna, Rupak, arXiv:2010.13003

Just ANCs



ANCs-*ab initio* : Zhang, Nollett, Phillips, PRC 89, 051602 (2014)

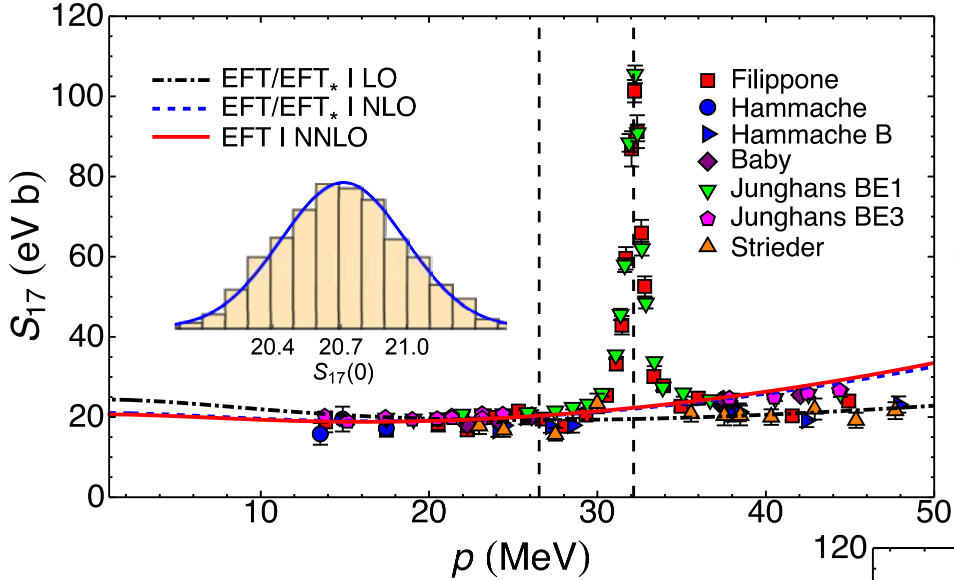
PRC 89, 024613 (2014)

Trache et al., PRC 67, 062801 (2003)

Tabacaru et al., PRC 73, 025808 (2006)

Nollett, Wiringa, PRC 83, 041001 (2011)

Bayesian fit below 500 keV



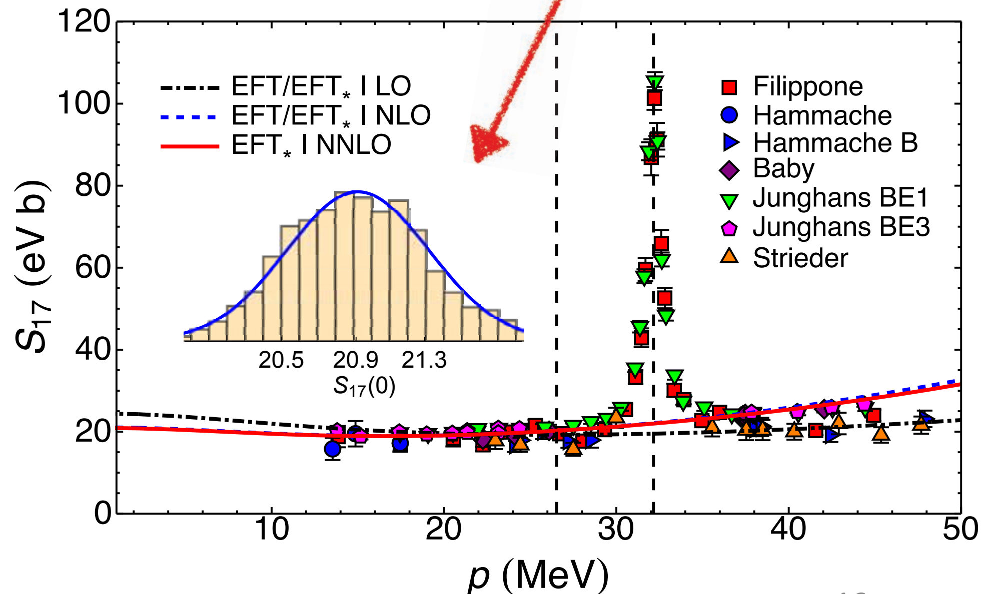
E ≤ 500 keV (p ≤ 28.6 MeV)

2 parameter fit

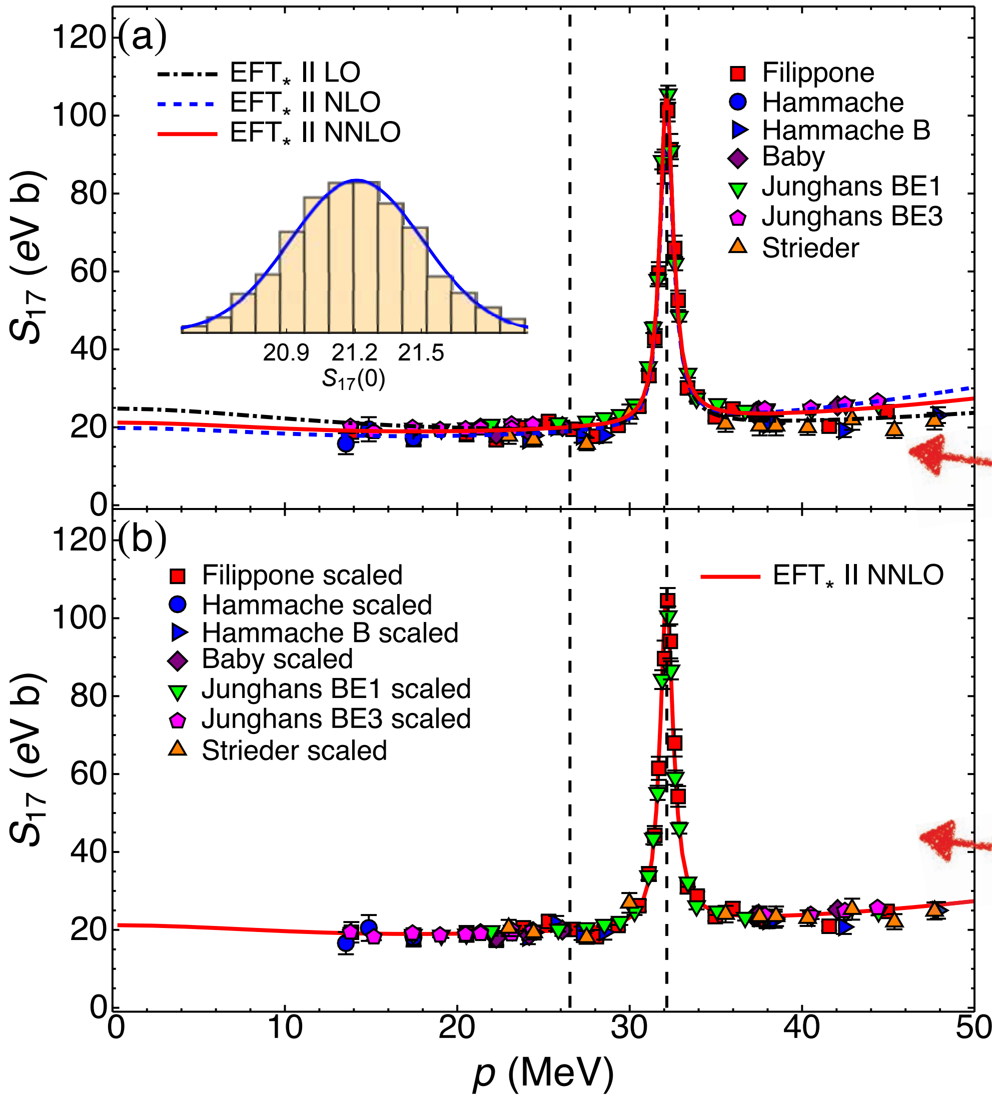
2+1+2 parameter fit

Small evidence for EFT over EFT$_\star$

$$\ln \frac{P(\mathrm{EFT}|D,H)}{P(\mathrm{EFT}_\star|D,H)} \approx 0.7 \pm 0.3$$



Bayesian fit up to 1000 keV



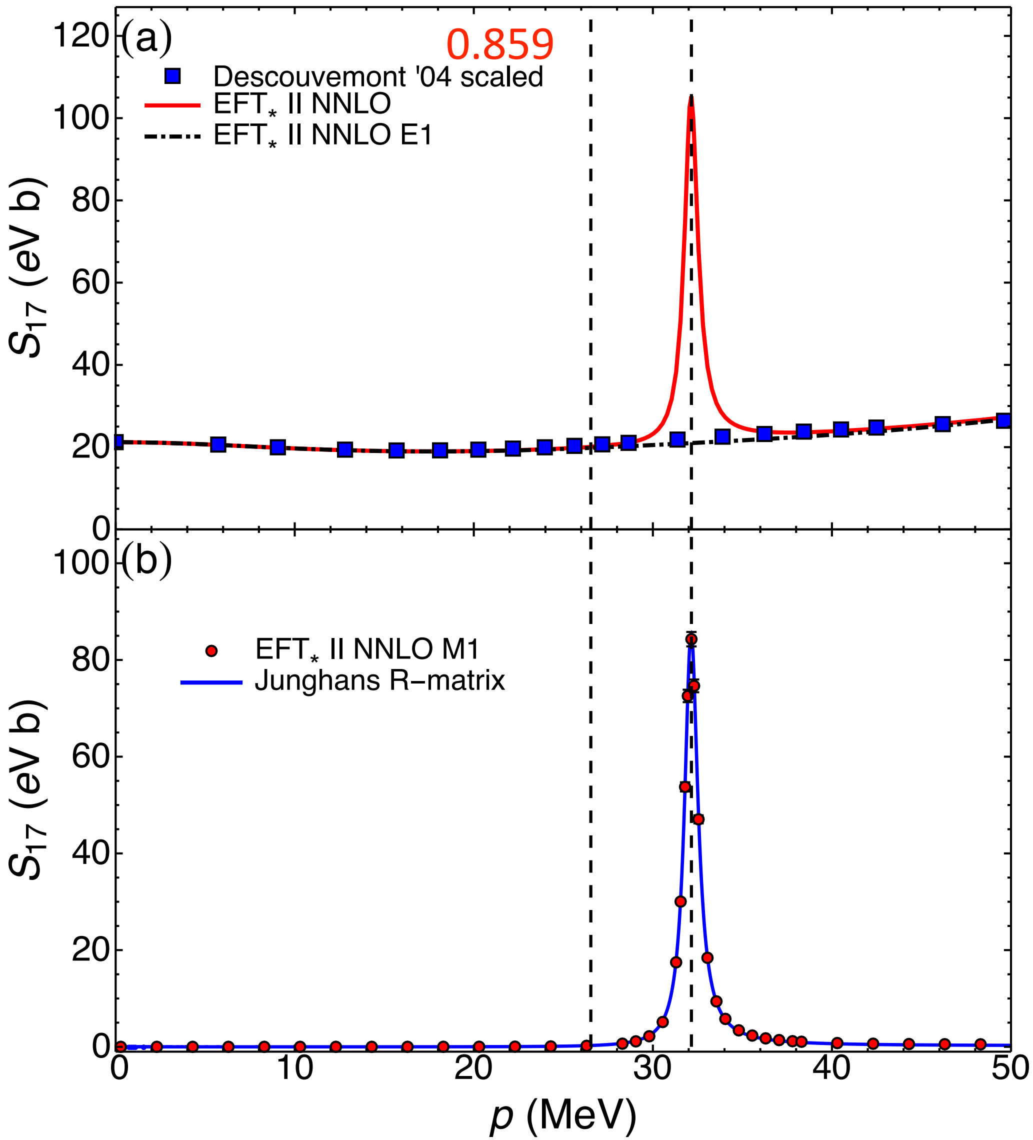
E ≤ 1 MeV (p ≤ 40.5 MeV)

Find clear evidence for EFT★ at NNLO

5+1 parameter fit

"Artist's" illustration

Cross Checks



Descouvemont, PRC 70, 065802 (2004)

Junghans et al., PRC 68, 065803 (2003)

S-factor Extrapolations

TABLE III. S_{17} and its first two energy derivatives at $E_0 = 50 \times 10^{-3}$ keV. The first set of errors are from the fits. The second set is the estimated LO 30%, NLO 10% and NNLO 3% EFT errors, respectively, from higher order corrections.

Theory	S_{17} (eV b)	S'_{17}/S_{17} (MeV^{-1})	S''_{17}/S_{17} (MeV^{-2})
EFT/EFT$_\star$ I LO	24.4(0.3)(7.3)	−2.44(0.05)(0.73)	35.8(0.7)(10.8)
EFT/EFT$_\star$ I NLO	21.1(0.3)(2.1)	−1.87(0.04)(0.19)	32.4(0.6)(3.2)
EFT I NNLO	20.7(0.3)(0.6)	−1.79(0.04)(0.05)	31.9(0.6)(1)
EFT$_\star$ I NNLO	20.9(0.4)(0.6)	−1.82(0.08)(0.05)	31.9(0.8)(1)
EFT$_\star$ II LO	24.8(0.3)(7.4)	−2.44(0.04)(0.73)	35.8(0.6)(10.8)
EFT$_\star$ II NLO	19.8(0.2)(2)	−1.91(0.03)(0.19)	32.7(0.5)(3.3)
EFT$_\star$ II NNLO	21.2(0.3)(0.6)	−1.89(0.04)(0.06)	31.9(0.6)(1)

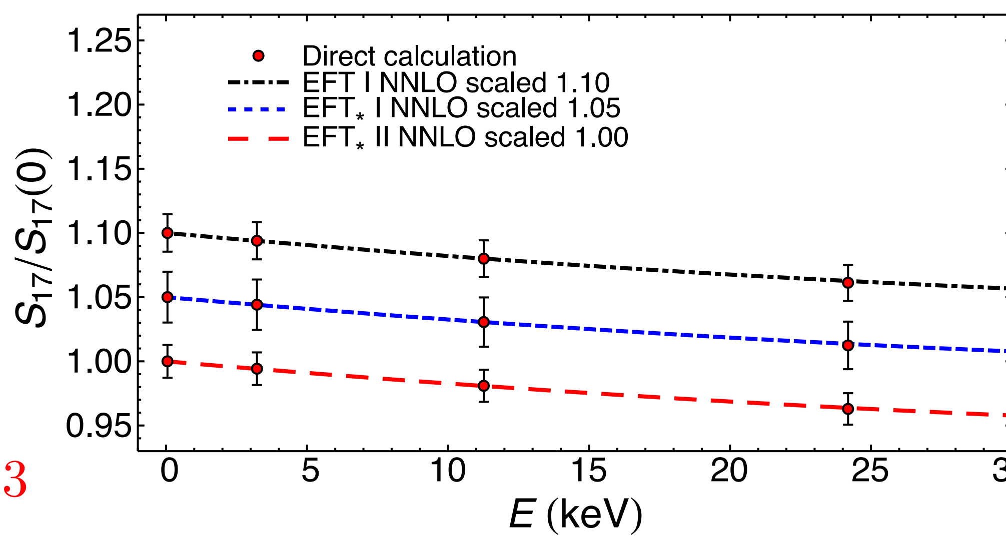
$S_{17}(0) = 21.0(7)$ eV b

Higa, Premarathna, Rupak, arXiv:2010.13003

Fitting + theory error

Solar II [3] is $S_{17}(0) = 20.8(16)$ eV b.

Adelberger et al., RMP 83, 195 (2011)



EFT S''_{17}/S_{17} larger by a factor of 3

Conclusions

- Initial state is constrained by phase shift parameters. Affects EFT power counting, and so error estimates. Higher orders kinematically suppressed. $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B},$ and much more

- Final state also related to phase shift. Large effect but exact at NLO: zed-parameterization. Interaction should describe binding energy and ANC. For example, $^{7}\mathrm{Li}(n,\gamma)^{8}\mathrm{Li}$

- 2-body current usually higher order but not kinematically suppressed. Not constrained by Siegert theorem.

- Bayesian estimate of higher order EFT error?