Few-nucleon radiative captures in χEFT

Laura E. Marcucci

University of Pisa

INFN-Pisa

Outline

- Introduction
- State-of-the-art for theoretical studies: achievements and limits
- Why chiral effective field theory (χEFT)? Advantages and difficulties
- Chiral potentials and currents
- Preliminary results
- Outlook

Introduction

$p + d \rightarrow {}^3\mathrm{He} + \gamma$

- crucial reaction for BBN
- reaction of *pp*-chain
- reaction important for the evolution of protostars
- accurate experimental data in a wide energy-range $\rightarrow$ stringent test
  - $E_{cm} = 2$ MeV $\rightarrow \sigma(\theta), A_y, iT_{11}, T_{20}, T_{21}, T_{22}$
    (M.K. Smith and L.D. Knutson, Phys. Rev. Lett. **82**, 4591 (1999))
  - $E_{cm} = 32.4$ keV -262.9 keV $\rightarrow S-$factor
    (LUNA data - V. Mossa *et al.*, Nature **587**, 220 (2020))

$p + {}^3\mathrm{H} \rightarrow {}^4\mathrm{He} + \gamma$

- important for BBN
- experimental data for *S*-factor quite few and old

$n + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + \gamma$

- important for BBN
- thermal energies $\rightarrow$ total cross section

Both $p + {}^3\mathrm{H} \rightarrow {}^4\mathrm{He} + \gamma$ and $n + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + \gamma \longrightarrow$ search of the dark photon - $X17$

Ab-initio studies of nuclear reactions

1. Nucleus = system of A nucleons interacting among themselves and with external electro-weak probes
2. Realistic description of nuclear interactions and electro-weak currents
3. Exact* method to solve the quantum-mechanical problem (both bound and scattering states)
4. "True" predictions
5. Ideal case: estimate the theoretical error

* Exact ≡ no uncontrolled approximations

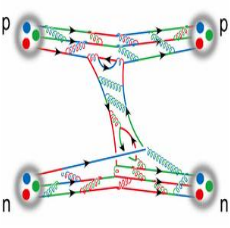
- controlled approximations are allowed (expansion on a certain basis) → converged results = *ab-initio* results
- comparison of *ab-initio* results with data → test of input ingredients

Method of choice: the Hyperspherical Harmonics (HH) method

- $A = 3, 4, 6$ bound states
- $A = 3, 4$ scattering states at low energies
- can be used with both local and non-local interactions
- no problem with Coulomb

The nuclear Hamiltonian and currents

The phenomenological approach

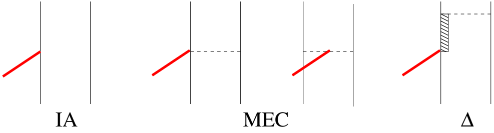


$H = T + V$

Nuclear interaction: $V = V_{NN} + V_{NNN}$
Until $\simeq$ 20–30 years ago: phenomenological potentials

- V_{NN} with $\simeq$ 40 parameters fitted to $A = 2$ data $\rightarrow \chi^2/\text{datum} \simeq 1$
- V_{NNN} with 2-3 parameters fitted to $B(A = 3, 4)$

Very common models: AV18+UIX, AV18+IL7

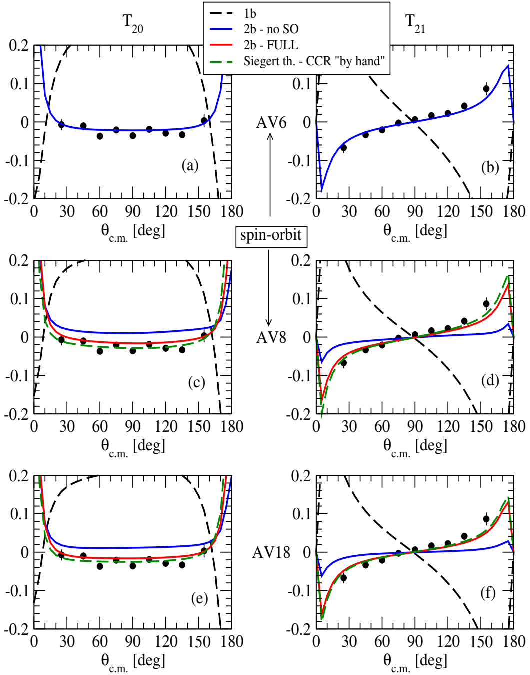


Nuclear EM current: $\mathbf{j}^\gamma$

- Realistic model: IA+MEC+$j(\Delta)$
- Current conservation relation (CCR) $\rightarrow \mathbf{q}\cdot\mathbf{j}^\gamma \propto [\rho^\gamma, H]$

- Very successful approach
- CCR crucial

$\vec{d} + p \rightarrow {}^3\mathrm{He} + \gamma$
@ 2 MeV

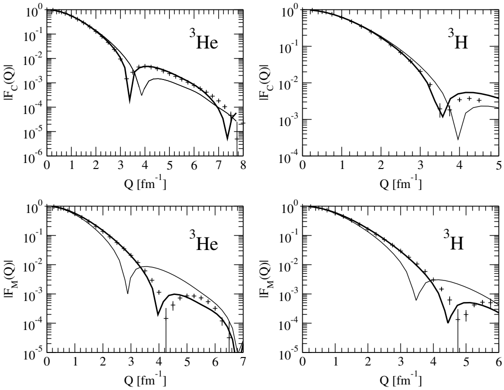


L.E. Marcucci *et al.* Phys. Rev. C **72**, 014001 (2005)

Discrepancies and limits

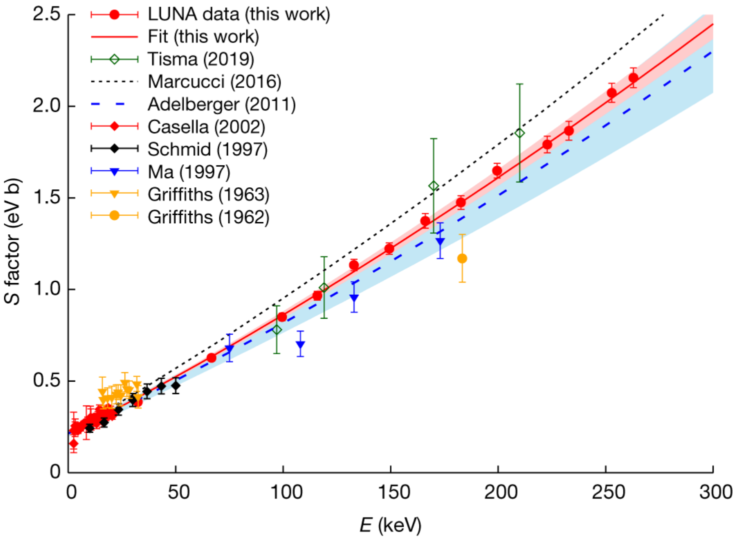
Some discrepancies

- The $A = 3$ magnetic form factors in the 1st diffraction region
- The *pd* *S*-factor in the BBN energy range



L.E. Marcucci *et al.*, J. Phys. G **43**, 023002 (2016)

The LUNA data for BBN



V. Mossa *et al.* (LUNA Collab.), Nature **587**, 210 (2020)

But ...

What is the theoretical accuracy?

- scattering w.fs. tested with $\langle\Psi|H|\Psi\rangle_{box} = E_{cm} + E_d$

$\Rightarrow$ theoretical accuracy from the scattering wfs. ~ 1 %

- Error associated with the nuclear interaction & currents?

Furthermore

- many parameters
- no connection with QCD

$\Rightarrow$ **Chiral Effective Field Theory**

Chiral Effective Field Theory (χEFT): a very short summary

- QCD $\rightarrow$ quarks and gluons ("high-energy" d.o.f.)
- Nuclear physics $\rightarrow$ nucleons and pions ("low-energy" d.o.f.)
- EFT $\rightarrow$ processes with $E \simeq p \simeq m_\pi \ll \Lambda_{\rm QCD} \sim 1$ GeV
  - ⋆ keep the "l-e" d.o.f.: π and N
  - ⋆ Lagrangians describing the interactions of π and N are expanded in powers of $O(p/\Lambda_{\rm QCD})^\nu$ $\rightarrow$ perturbative theory
  - ⋆ "h-e" d.o.f. integrated out $\rightarrow$ contact interactions with "l-e" d.o.f. and low-energy constants (LECs) obtained from experiment
- χEFT $\rightarrow$ EFT with spontaneous breaking of QCD's χ-symmetry
- Regularization of short-range terms with cutoff function $\rightarrow \Lambda \simeq 400 - 600$ MeV

Disadvantage: limited to processes with $E \lesssim [2 \div 3]\, m_\pi$

Advantages

- nuclear force "hierarchy" $\rightarrow$ accurate $V_{NN} + V_{NNN}$
- consistent framework for interactions + currents (just add electro-weak field as d.o.f.)
- possibility to estimate the theoretical uncertainty (perturbative expansion+Λ-dependence)

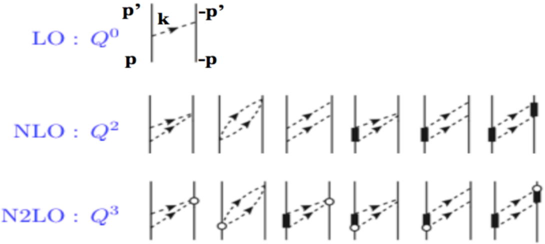
Until very recently, everything developed in momentum-space
$\rightarrow$ not user-friendly when charged particles

Local chiral V_{NN} with Δ's

M. Piarulli *et al.*, Phys. Rev. C **91**, 024003 (2015)

- $V_{NN} = v^{EM} + v^{LR} + v^{SR}$
- v^{EM}= electro-magnetic component including corrections up to α^2

$\rightarrow$ Chiral 1π and 2π exchange in v^{LR} with Δ's up to Q^3 (N2LO)



- v^{SR} contact terms up to Q^4 (N3LO) with 26 LECs

$\rightarrow$ v^{SR} regularized with $C_{R_S}(r) \propto \mathrm{e}^{-(r/R_S)^2}$ with $R_S = 0.8(0.7)$ fm [model *a* (*b*)]

$\rightarrow$ fit the 2013 Granada database

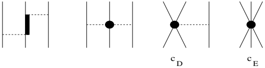
  - up to $E_{lab} = 125$ MeV ($\sim$ 2700 data) with χ^2/datum ≤ 1.1 (model I)
  - up to $E_{lab} = 200$ MeV ($\sim$ 3500 data) with χ^2/datum ≤ 1.4 (model II)

Local chiral V_{NNN} with Δ's (NV2+3/Ia* & NV2+3/Ib*)

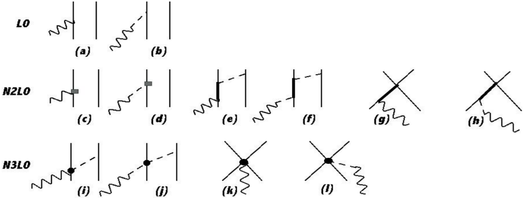
M. Piarulli *et al.*, Phys. Rev. Lett. **120**, 052503 (2018)

A. Baroni *et al.*, Phys. Rev. C **98**, 044003 (2018)

- Three-nucleon interaction V_{NNN} up to N2LO



- fit c_D & c_E $\rightarrow$ $B(^3\mathrm{H})$ and GT^{Exp} in ^{3}H β-decay $\Rightarrow j_A(q)$

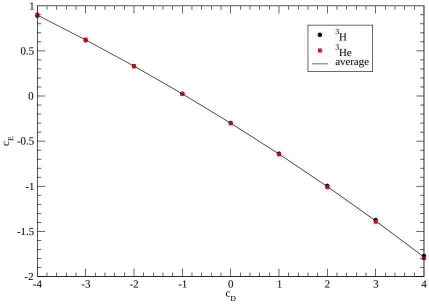


- CTs in (i) and (k)

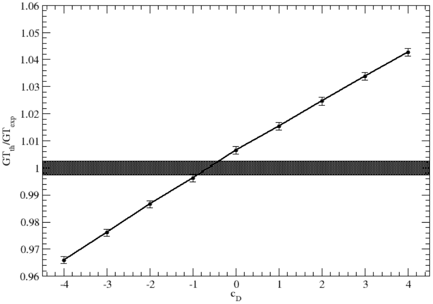
$$\mathbf{j}_{5,a}^{N3LO}(\mathbf{q};CT) = z_0\,\mathrm{e}^{i\mathbf{q}\cdot R_{ij}}\,\frac{\mathrm{e}^{-(r_{ij}/R_S)^2}}{\pi^{3/2}}(\tau_i\times\tau_j)_a(\sigma_i\times\sigma_j)$$

$$z_0 = \frac{g_A}{2}\frac{m_\pi^2}{f_\pi^2}\frac{1}{(m_\pi R_S)^3}\left[-\frac{m_\pi}{4g_A\Lambda_\chi}c_D+\frac{m_\pi}{3}(c_3+2c_4+c_3^\Delta+2c_4^\Delta)+\frac{m_\pi}{6m}\right]$$

$$\text{but}\quad c_3^\Delta+2c_4^\Delta=-\frac{h_A^2}{9m_{\Delta N}}+2\frac{h_A^2}{18m_{\Delta N}}=0\quad\text{with } h_A\equiv g_A^*$$



^{3}H
^{3}He
average
c_E
c_D



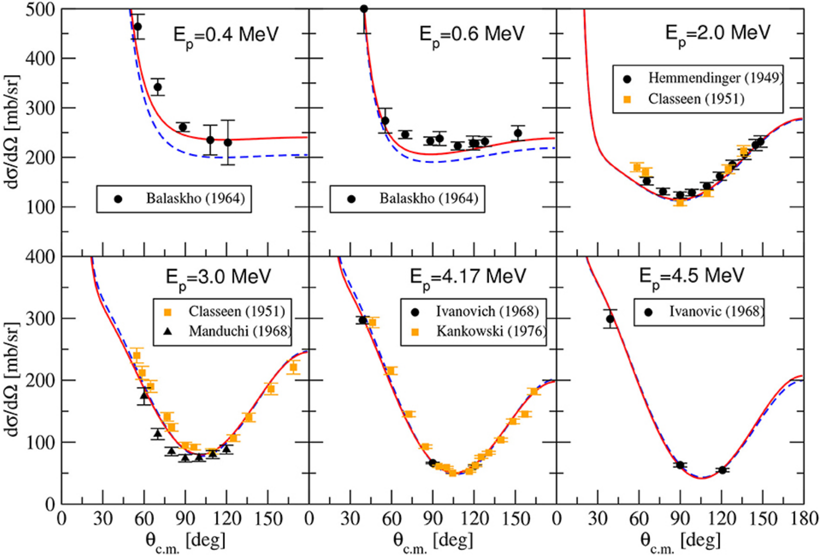
GT_{th}/GT_{exp}
c_D

$A = 3, 4$ binding energies and scattering lengths with NV potentials

Model	$B(^3H)$	$B(^3He)$	$B(^4He)$	a^2_{nd}	a^4_{nd}	a^2_{np}	a^4_{pd}
NVIa	8.718	7.090	25.15	1.119	6.326	0.959	13.596
NVIb	7.599	6.885	23.96	1.307	6.327	1.294	13.597
NV2+3/Ia*	<u>8.477</u>	<u>7.727</u>	28.30	0.638	6.326	0.070	13.596
NV2+3/Ib*	<u>8.469</u>	<u>7.724</u>	28.21	0.650	6.327	0.070	13.597
Exp.	8.475	7.725	28.30	0.645(10)	6.35(2)	–0.13(4)	14.7(2.3)

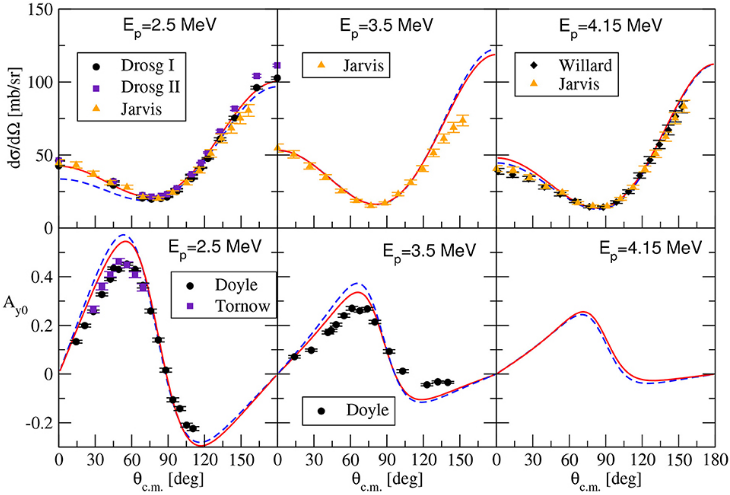
L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

$A = 4$ scattering observables: $d\sigma/d\Omega$ for $p + {}^3$H



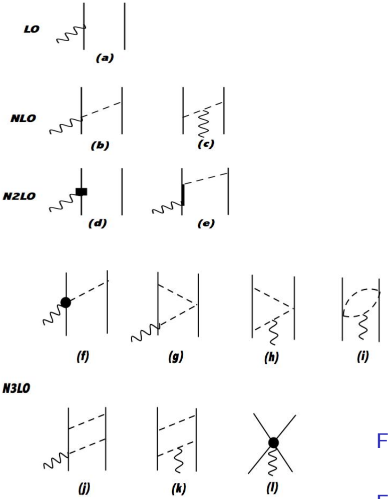
L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

$A = 4$ scattering observables for $p + {}^3\mathrm{H} \rightarrow n + {}^3\mathrm{He}$



L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

Electromagnetic current in χEFT



R. Schiavilla *et al.*, Phys. Rev. C **99**, 034005 (2019)

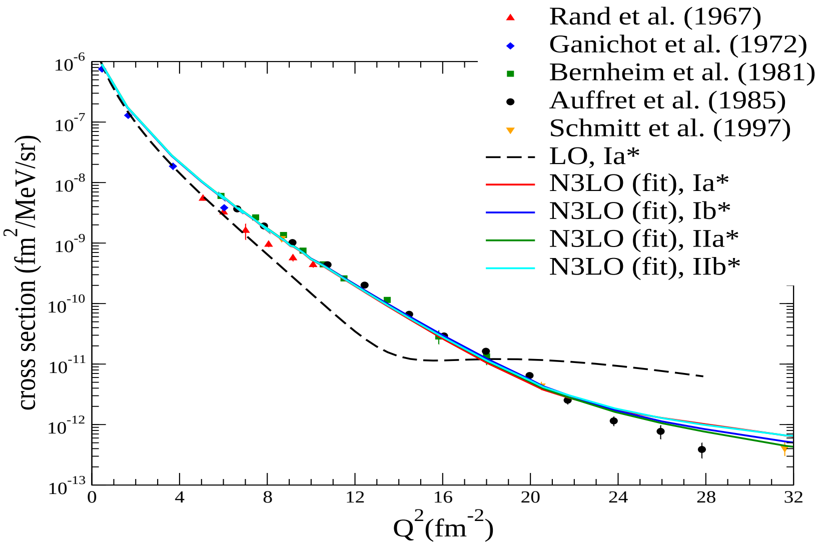
- $\mathbf{j}_{\Delta}^{\rm N2LO}(\mathbf{q})$ in panel (e) absent in $\not\Delta$-EFT
- not included the Δ intermediate states at N3LO
- $\mathbf{j}_{\rm OPE}^{\rm N3LO}(\mathbf{q}) \rightarrow d_2^S, d_2^V\, ; d_3^V$
- $\mathbf{j}_{\rm MIN}^{\rm N3LO}(\mathbf{q}) \rightarrow$ from πN scattering
- $\mathbf{j}_{\rm NM}^{\rm N3LO}(\mathbf{q}) \rightarrow d_1^S\, ; d_1^V$

To be noticed:

$$\mathbf{j}_{\rm OPE}^{\rm N3LO}(\mathbf{q}) \propto \frac{\sigma_j \cdot \mathbf{k}_j}{(m_\pi^2 + \mathbf{k}_j^2)}\, \mathbf{q} \times [(d_2^S \tau_i \cdot \tau_j + d_2^V \tau_j^z)\mathbf{k}_j + d_3^V (\tau_i \times \tau_j)^z \sigma_i \times \mathbf{k}_j]$$

FIT 1 $d_2^V\, ; d_3^V \rightarrow$ saturated with Δ-current of panel (e); d_1^S, d_2^S, d_1^V fitted to $A = 2, 3$ magnetic moments

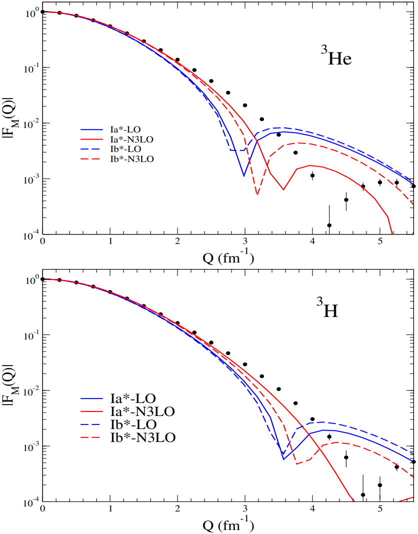
FIT 2 all 5 LECs fitted to $A = 2, 3$ magnetic moments and $d(e, e')pn$ at threshold



R. Schiavilla, private communication

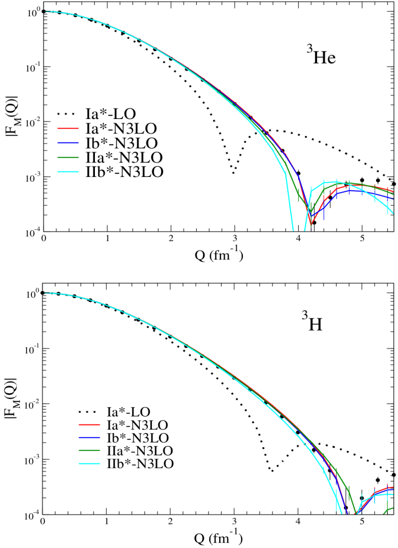
The $A = 3$ magnetic form factors

with FIT 1



R. Schiavilla *et al.*, Phys. Rev. C **99**, 034005 (2019)

with FIT 2



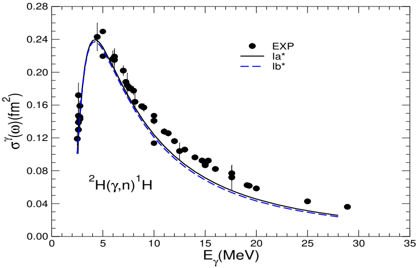
R. Schiavilla, private communication

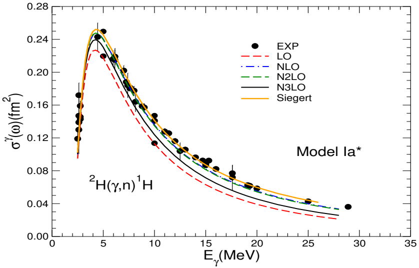
But CCR is not satisfied

CCR with $v_{ij}^{\mathrm{N3LO}} \rightarrow \mathbf{j}^{\mathrm{N5LO}}(\mathbf{q})$!!

$\Rightarrow$ Observables sensitive to E_1 are not reproduced

The deuteron photodisintegration

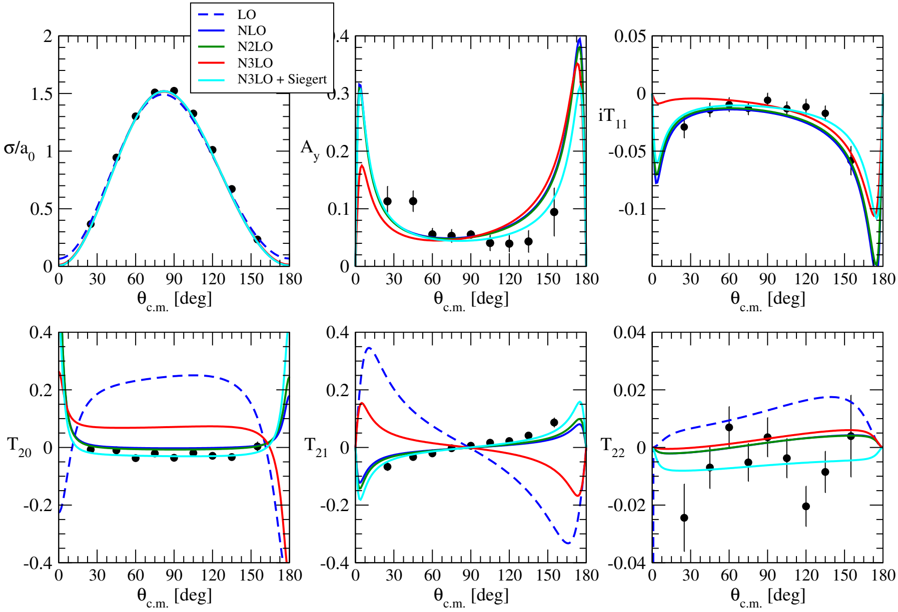




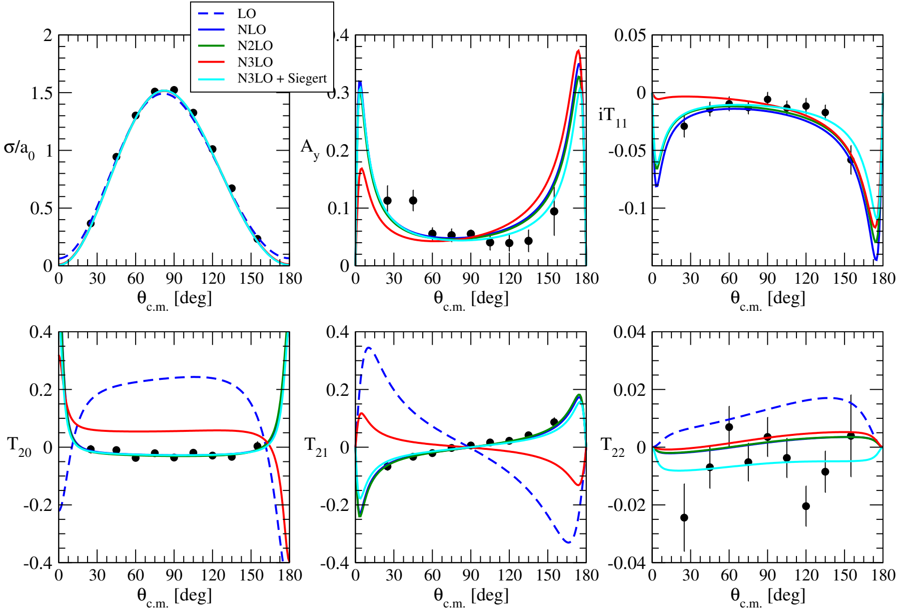
$\Rightarrow$ Use Siegert theorem (CCR imposed by hand)

R. Schiavilla *et al.*, Phys. Rev. C **99**, 034005 (2019)

The $p + d \to {}^3\mathrm{He} + \gamma$ reaction at $E_{cm} = 2$ MeV: PRELIMINARY (NV2+3/Ia*)



The $p + d \to {}^3\mathrm{He} + \gamma$ reaction at $E_{cm} = 2$ MeV: PRELIMINARY (NV2+3/Ib*)



The *pd* astrophysical *S*-factor [eV b]: PRELIMINARY

E_{cm} [keV]	Exp.[1]	AV18/UIX[2]	NV2+3/Ia*	NV2+3/Ib*
2000	–	25.86	25.45	25.57
262.9	$2.156 \pm 0.020 \pm 0.054$	2.34	2.35	2.29
252.9	$2.073 \pm 0.012 \pm 0.052$	2.27	2.25	2.21
232.9	$1.866 \pm 0.012 \pm 0.051$	2.08	2.07	2.04
222.8	$1.791 \pm 0.006 \pm 0.045$	1.99	1.98	1.94

[1] V. Mossa, Nature **587**, 210 (2020)

[2] L.E. Marcucci *et al.*, Phys. Rev. Lett. **116**, 102501 (2016)

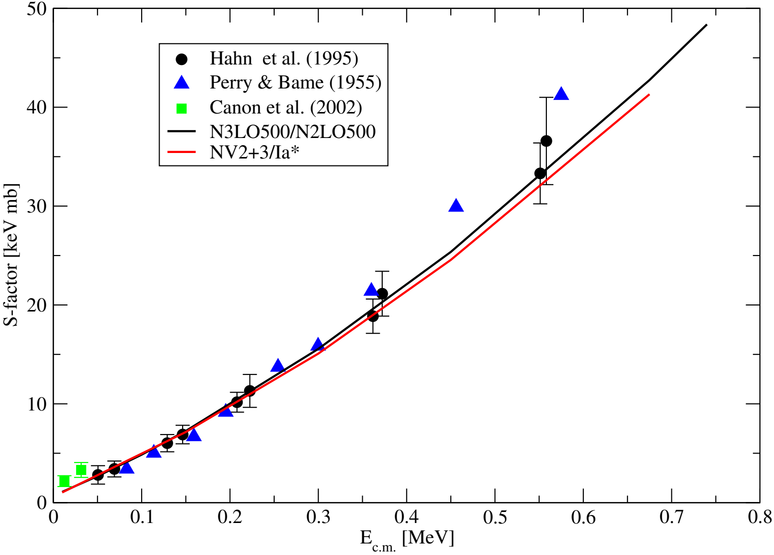
Current contrib. at $E_{cm} = 262.9$ keV

Chiral order	NV2+3/Ia*	NV2+3/Ib*
LO	2.04	2.03
NLO	2.21	2.23
N2LO	2.27	2.30
N3LO	2.32	2.34
N3LO-Siegert	2.35	2.29

J^π contrib. at $E_{cm} = 262.9$ keV (NV2+3/Ib*)

J^π	N3LO	N3LO-Siegert
$1/2^+$	0.056	0.056
$3/2^+$	0.030	0.030
$5/2^+$	0.001	0.001
$\mathbf{1/2^-}$	**0.753**	**0.731**
$\mathbf{3/2^-}$	**1.500**	**1.467**
$5/2^-$	0.001	0.001

The $p + {}^3\mathrm{H} \to {}^4\mathrm{He} + \gamma$ reaction: PRELIMINARY



Dominated by the E_1 transition $1^- \to 0^+ \Rightarrow$ Siegert theorem

The $n + {}^3\mathrm{He} \to {}^4\mathrm{He} + \gamma$ reaction: PRELIMINARY

- $\sigma_{tot}[\mu\mathrm{b}]$ at thermal energies
- Dominated by the M_1 transition $1^+ \to 0^+$

$\Rightarrow$ very sensitive to MEC

	AV18/UIX hybrid	N3LO/N2LO $\Lambda = 500$ MeV	N3LO/N2LO $\Lambda = 600$ MeV	NV2+3/Ia* [$\Lambda = 500$ MeV]	NV2+3/Ib* [$\Lambda = 570$ MeV]
LO	15.3	9.9	12.4	14.9	14.3
NLO	2.7	7.2	5.1	1.7	5.2
N2LO	0.7	2.8	1.7	5.9	19.0
N3LO	54.5	78.8	75.2	69.2	46.7
Exp.			55 ± 3		

- N2LO contribution with N3LO/N2LO and AV18/UIX potentials $\to$ relativistic correction while N2LO contribution with NV2+3/I potentials $\to$ Δ-current
- N3LO contribution large due to cancellations between LO and NLO
- order-by-order ... **non**-convergence
- AV18/UIX and NV2+3/I potentials $\to$ better agreement with experiment

Summary and outlook

Summary

- First studies in χEFT of $A = 3, 4$ radiative captures
- **PRELIMINARY** results
- Still a long what-to-do list: for example for $p + d$
  - Study of the convergence of the $\Psi(p + d)$
  - Study of model-dependence
    - local chiral potentials: NV2+3/IIa*, NV2+3/IIb*
    - non-local chiral potentials: N3LO/N2LO 500/600
  - Study of order-by-order convergence consistently in the interaction+current

  $\Rightarrow$ refit the LECs in j^γ

Outlook

- Study of other reactions
  - $n + d \to {}^3\mathrm{H} + \gamma$ at thermal energies
  - $d + d$ $(ddn, ddp, dd\gamma)$ at BBN energies
  - $A > 4$ nuclei $\to$ HH method for $A = 6$

    (A. Gnech *et al.*, Phys. Rev. C **102**, 014001 (2020))

In collaboration with:

- A. Kievsky and M. Viviani (INFN Pisa - Italy)
- L. Girlanda (INFN Lecce and Univ. of Salento - Italy)
- A. Gnech (JLab - USA)
- M. Piarulli (WUSTL - USA)
- R. Schiavilla (ODU & JLab - USA)

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THANK YOU!