Twisted Photons at the Gamma Factory

Valeriy G. SERBO
Novosibirsk State University, RUS–630090, Novosibirsk, Russia

Andrey SURZHYKOV
Physikalisch–Technische Bundesanstalt, D–38116 Braunschweig, Germany

Twisted whirl in water. Las Vegas

OUTLINE:

1. Introduction

We consider **the resonant elastic scattering of laser photons on ultra-relativistic partially striped ions** which is a basic process in the project of the Gamma Factory at the LHC. We find the matrix element of the process, including the case of heavy ions, and its non-relativistic limit.

We use these results to obtain the angular distribution and polarization of final photons in the case when **the laser photons are twisted**, i.e. when they have **non-zero orbital angular momentum**.

We find out that this case has **a close relation to the Compton backscattering of twisted photons** which has been discussed in a number of papers.

Then we argue that the twisted laser photons **transmit their twistedness to the final backscattered photons** with energies up to 400 MeV at a certain condition.

2. Parameters of the problem

Let us present some numbers for typical ions discussed in review

[1] D. Budker et al.,Ann. Phys. (Berlin) 2000204 (2020)

For definiteness, we consider a hydrogen-like ion in the ground state (nuclear charge Z, mass number A).

In the **ion rest frame (IRF)**, to excite the $1s \to 2p$ transition, a photon with the energy ω_1 close to the transition energy

$$\omega_{\nu i} = E_2 - E_1 = \frac{3}{8} m_e (Z\alpha)^2. \tag{1}$$

is required (here m_e is the electron mass, $\alpha = e^2 \approx 1/137$). Below we use relativistic units: $c = 1$, $\hbar = 1$

It is interesting to compare the corresponding wave length of such a photon $\lambda_{\nu i} = 2\pi/\omega_{\nu i}$ with the mean atom radius $\langle r \rangle = 3/(2 m_e Z\alpha)$.

In the discussed project, the energy of the laser photon in the **collider frame (CF)** ω_1' should be close to the value (here γ is Lorenz-factor of an ion)

$$\omega_0 = \frac{\omega_{\nu i}}{2\gamma}, \tag{2}$$

while the maximum energy of the backscattered photons in CF will be

$$(\omega_2')_{\mathrm{max}} = 2\gamma\omega_{\nu i} = (2\gamma)^2\omega_0. \tag{3}$$

Below we present some principal characteristic of the resonant photon scattering for typical ions discussed in Ref. [1]

	γ	$\omega_{\nu i}$ [keV]	ω_0 [eV]	$(\omega_2')_{\rm max}$ [MeV]	$\langle r\rangle/\lambda_{\nu i}$
Ar^{17+}	2940	3.3	0.56	19	0.012
Xe^{53+}	2800	30	5.3	170	0.035
Pb^{81+}	2690	69	13	370	0.054

From Table 1 one can see that the transition energy is **considerable less** than $m_e = 511$ keV.
For example, ratio $\omega_{\nu i}/m_e < 0.1$ for $Z < 71$.

Besides, the photon wave length in the IRF is **considerable larger** then the mean ion radius.

It means that in discussion about resonant scattering we can use the simple non-relativistic dipole approximation for a number of ions.
For symplicity, in this report
we concentrate just on this approximation.

However, the relativistic corrections may be essential for heavy ions. That is why it is quite important to have the expression for matrix element of the process with heavy ions **beyond the non-relativistic approximation**. Complicated details of these calculations can be found in our paper
V.G. Serbo and A. Surzhykov. Resonant scattering of twisted photons at the Gamma Factory (LHC). In preparation

Below we use the following notations:
$\boldsymbol{k}_{1,2}$ and $\boldsymbol{\epsilon}_{1,2}$ are the wave- and polarization-vectors of the incident and outgoing photons, respectively, in the IRF;
$\lambda_{1,2}$ are helicities of photons;
$\theta_{1,2}$, $\varphi_{1,2}$ and $\theta'_{1,2}$, $\varphi'_{1,2}$ are the spherical angles of photons in the IRF and CF, respectively.

3. Scattering of plane-wave photons

3.1. Resonant scattering

For definiteness, we consider the $1s \to 2p$ transition, when the photon energy ω_1 is close to $\omega_{\nu i}$. In this case the differential cross section in the IRF is

$$\frac{d\sigma}{d\Omega_2} = f(\omega_1) \left| \frac{3\lambda_{\nu i}}{4\pi} (\boldsymbol{\epsilon}_2^* \boldsymbol{\epsilon}_1) \right|^2 , \qquad (4)$$

where

$$f(\omega_1) = \frac{\Gamma_\nu^2/4}{(\omega_{\nu i} - \omega_1)^2 + \Gamma_\nu^2/4} \to 1, \text{ when } \omega_1 \to \omega_{\nu i}, \qquad (5)$$

and Γ_ν is the width of of the level $E_\nu = E_2$:

$$\Gamma_\nu = \left(\frac{2}{3}\right)^8 Z^4 \alpha^5 m_e. \tag{6}$$

In the standard case, when the initial plane-wave photon flies antiparallel to the z-axis, the expression which define **the polarization and angular distribution of the final photons** is:

$$\boldsymbol{\epsilon}_2^* \boldsymbol{\epsilon}_1 = e^{i\lambda_1(\varphi_1 - \varphi_2)} \frac{1}{2}(1 - \lambda_1 \lambda_2 \cos\theta_2) \tag{7}$$

Two conclusions: 1. For strictly backward scattering

$$\lambda_2 = -\lambda_1, \text{ when } \theta_2 = 0, \tag{8}$$

i.e. **helicity of the initial photon transmits to the final photon**.

2. For small deviations from the strictly backward scattering, the equality $\lambda_2 = -\lambda_1$ is violated and a nonzero probability of having $\lambda_2 = +\lambda_1$ appears:

$$\frac{d\sigma(\lambda_2 = +\lambda_1)}{d\sigma(\lambda_2 = -\lambda_1)} = \frac{(1 - \cos\theta_2)^2}{(1 + \cos\theta_2)^4} = \tan^4(\theta_2/2), \qquad (9)$$

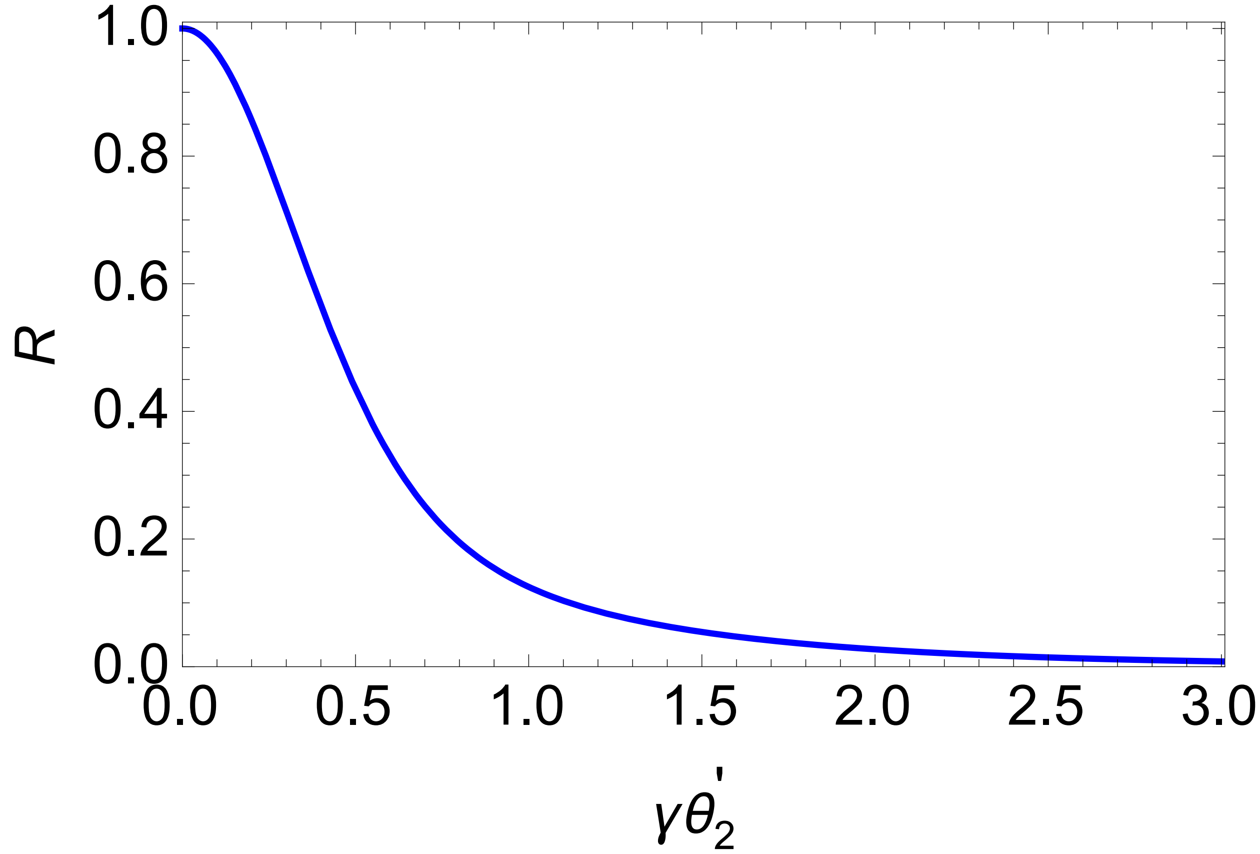
i.e. there will be some **distribution over values of λ_2.**

After averaging over the polarizations of the initial and summing over the polarizations of the final photons, the angular distribution of photons in the IRF reads:

$$\frac{d\sigma}{d\Omega_2} = f(\omega_1)\left(\frac{3\lambda_{\nu i}}{4\pi}\right)^2 \frac{1}{2}\left(1 + \cos^2\theta_2\right). \tag{10}$$

In the CF we obtain (see Fig. 1)

$$\frac{d\sigma}{d\Omega_2'} = f(\omega_1)\left(\frac{3\lambda_{\nu i}}{4\pi}\right)^2 2\gamma^2 \frac{1 + (\gamma\theta_2')^4}{\left[1 + (\gamma\theta_2')^2\right]^4}, \tag{11}$$



The angular distribution (11), $R(\theta_2') = \frac{d\sigma(\theta_2')}{d\Omega_2'} \Big/ \frac{d\sigma(\theta_2' = 0)}{d\Omega_2'}$

The maximum values of the total cross section for the resonant scattering (when $\omega_1 = \omega_{\nu i}$) are

$$\sigma = \frac{3}{2\pi} \lambda_{\nu i}^2 = (1.5 \cdot 10^{-18} \div 6.7 \cdot 10^{-16})\ \mathrm{cm}^2 \qquad (12)$$

for ions from Table 1.

3.2. Comparison with the Compton scattering

The Compton backscattering of laser photons by ultra-relativistic electrons on storage rings is routinely used as **a source of the intensive high-energy beams of photons** – see, for example, reviews

V. G. Nedorezov, A. A. Turinge, and Yu. M. Shatunov, Phys.-Usp. **47**, 341 (2004);

H. R. Weller, M. W. Ahmed, H. Gao, W. Tornow, Y. K. Wu, M. Gai, and R. Miskimen. Progress in Particle and Nuclear Physics **62**, 257 (2009).

Therefore, it is useful to compare it with the discussed above resonant scattering of the same laser photons by partially stripped ions with the same Lorenz factor γ.

The differential cross section of the Compton scattering is

$$\frac{d\sigma_{\rm C}}{d\Omega_2} = \left|\frac{\alpha}{m_e}(\boldsymbol{\epsilon}_2^* \boldsymbol{\epsilon}_1)\right|^2 . \qquad (13)$$

From this expression we see that **the matrix element of the Compton scattering has the same structure as the matrix element of the resonant scattering**. Therefore, the angular distribution of the final photons is of the same form as above and the total Compton cross section in our energy range coincides with the Thompson cross section

$$\sigma_{\rm Th} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} = 6.7 \times 10^{-25}\ {\rm cm}^2 \qquad (14)$$

NOTE: the exact cross section differs from the Thompson cross section less then 20% for ions from Table 1 and turns out to be **nine to seven orders of magnitude less** than the resonance cross section.

4. Scattering of twisted photons

4.1. Twisted particles

For simplicity, we start with the non-relativistic electrons neglecting their spin states. There are **two** well-known types of waves: plane waves and spherical waves.

The plane wave is the stationary state with a defined momentum $\mathbf{k}$ and energy $E = \dfrac{k^2}{2m_e}$:

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}}.$$

The spherical wave is the stationary state with a defined energy $E = k^2/(2m_e)$,
angular momentum l
and projection of the angular momentum $l_z = m$:

$$\psi_{klm}(r, \theta_r, \varphi_r) = \sqrt{\frac{2\pi}{kr}}\, J_{l+1/2}(pr) Y_{lm}(\theta_r, \varphi_r),$$

where $J_n(x)$ is the Bessel function and $Y_{lm}(\theta_r, \varphi_r)$ the spherical function.

What is not so well-known is the fact that there is also **cylindrical waves also known as Bessel waves.**
The Bessel wave is the stationary state with a defined
(1) longitudinal momentum k_z,
(2) absolute value of the transverse momentum $\varkappa = |\mathbf{k}_\perp|$
(and therefore, $E = (\varkappa^2 + k_z^2)/(2m_e)$),
(3) projection of the angular momentum $l_z = m =$ $0; \pm 1, \pm 2, \ldots$:

$$\psi_{\varkappa m k_z}(r_\perp, \varphi_r, z) = J_m(\varkappa r_\perp)\, e^{i(m\varphi_r + k_z z)}$$

These states are briefly termed **twisted**. They differ from plane waves by the existence of a **nonzero projection** of the orbital angular momentum on the direction of motion, and from spherical waves by the existence of a **certain direction** of motion.

The function $\psi_{\varkappa m k_z}(\mathbf{r})$ can be expressed as **a superposition of plane waves** in the xy plane,

$$\psi_{\varkappa m k_z}(\mathbf{r}) = e^{ik_z z} \int a_{\varkappa m}(\mathbf{k}_\perp)\, e^{i\mathbf{k}_\perp \mathbf{r}_\perp} \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\,, \qquad (15)$$

where the Fourier amplitude $a_{\varkappa m}(\mathbf{k}_\perp)$ **is concentrated on the cone with $|\boldsymbol{k}_\perp| = \varkappa$:**
(the opening angle is $\theta_k = \arctan(\varkappa/k_z)$)

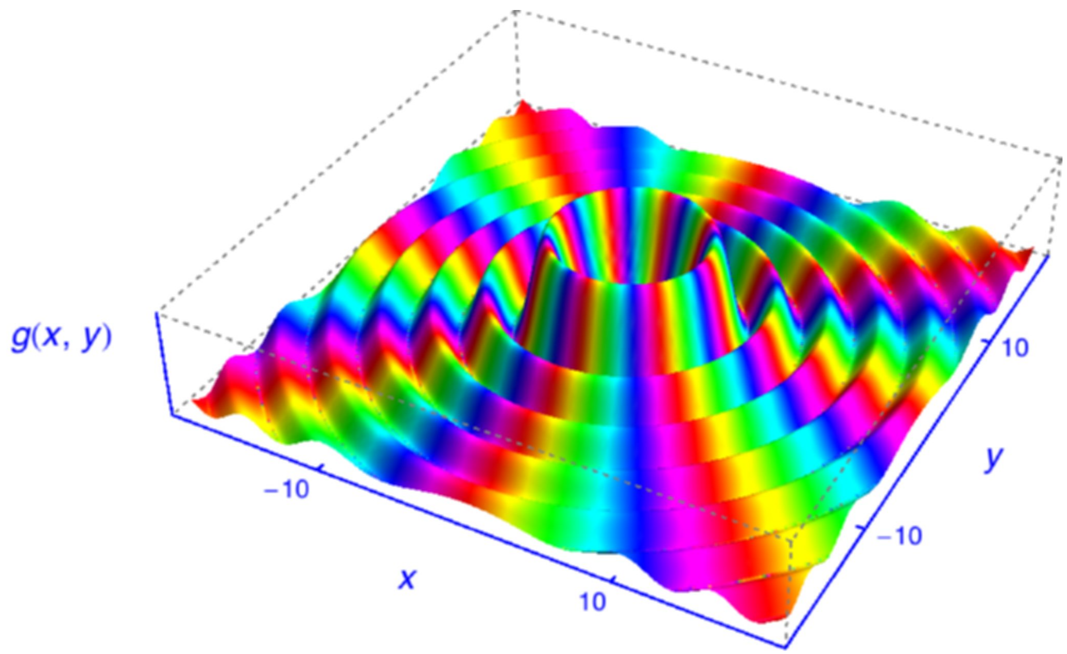
$$a_{\varkappa m}(\mathbf{k}_\perp) = (-i)^m\, e^{im\varphi_k} \sqrt{\frac{2\pi}{k_\perp}}\, \delta(k_\perp - \varkappa). \qquad (16)$$

Twisted photons:

Since 1992, the twisted photon energy is in the range from radio to X-ray, the angular momentum m up to 10^4

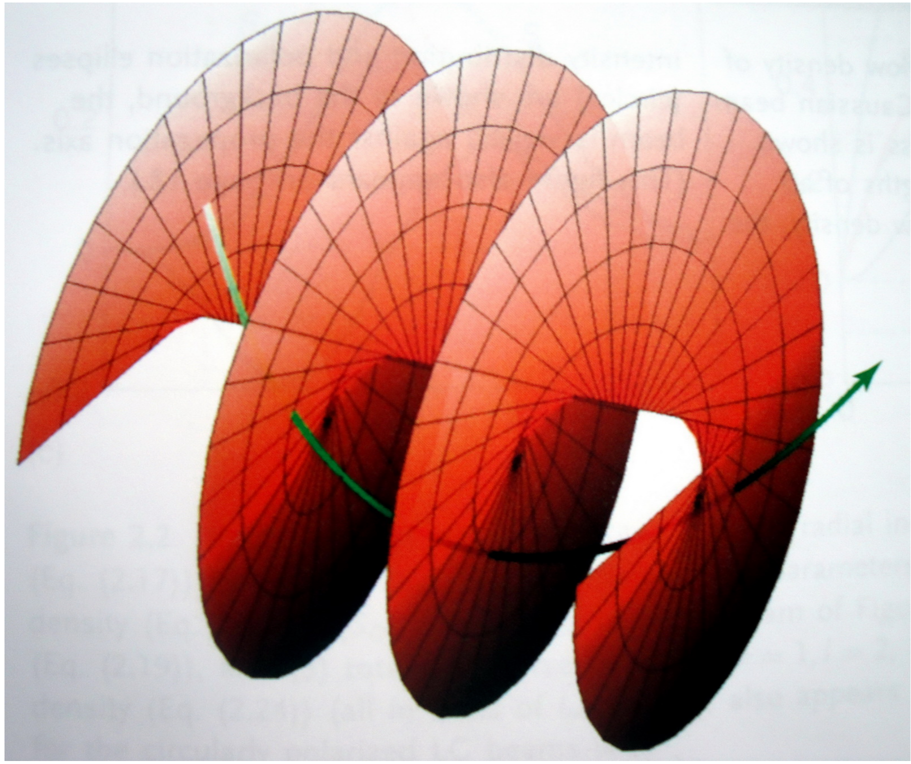
The vector potential of the twisted photon can be obtained as a natural generalization of the scalar wave function:

$$\mathbf{A}_{\varkappa m k_z \lambda}(\mathbf{r}) = \int \boldsymbol{\epsilon}_{\mathbf{k}\lambda}\, e^{i\mathbf{k}\mathbf{r}}\, a_{\varkappa m}(\mathbf{k}_\perp)\, \frac{d^2 k_\perp}{(2\pi)^2}$$



The vector potential $A^{\mu}_{\varkappa m k_z \lambda}(x, y, z)$ of the twisted photon is presented as $g(x, y) = |A^{\mu}_{\varkappa m k_z \lambda}(x, y, 0)|^2$, which is a function of x and y. The parameters are $\mu = 2$ (y component), $m = 5$, $\varkappa = 1$, $k_z = \sqrt{24}$ and $\lambda = 1$. From U.D. Jentschura, V.G. Serbo. EPJ C 71 (2011) 1571

The wavefront of such states **rotates around the propagation axis**, and their Poynting vector looks like **a corkscrew** (see green line):



Twisted photons have a number of application

Earlier researches — See book **"Twisted photons"** (Applications of light with orbital angular momentum) edd. by J. P. Torres and L. Torner (Wiley-VCH Weinheim, Germany 2011)

where the following problems have been discussed:

[1.] Micro-machines — it was demonstrated that micron-sized Teflon and calcite “particles” start to rotate after absorbing twisted photons

[2.] Astrophysics — the observation of orbital angular momentum of light scattered by rotating black holes could be very instructive

[3.] Rotating atoms with light — rotating Bose-Einstein condensates

[4.] Spiral phase contrast microscopy

[5.] Optical torques in liquid crystals

[6.] Quantum information — quantum features in high-dimensional Hilbert spaces

Recent years have seen an increasing emphasis on **the basic interactions of twisted photons with electrons, atoms and nuclei** such as the Compton scattering, photoexitation and photoionization of atoms and so on −

See recent review B.A. Knyazev and V.G. Serbo **Beams of photons with nonzero orbital angular momentum projection: new results** Phys. Usp. 61 (5) (2018) DOI: 10.3367/UFNe.2018.02.038306

Twisted electrons:

Since 2010, twisted electrons carried the energy as high as 300 keV and the orbital quantum number up to $m \sim 10^3$.

See recent review K.Y. Bliokh, I.P. Ivanov et al. *Phys. Rep.* **690** 1 (2017) arXiv:1703.06879

Such electron vortex beams can be manipulated and focused just as the usual electron beams.

Twisted neutrons

Recently,the production of **twisted cold neutrons** with the energy of $E = 11$ meV and the wavelength of 0.27 nm was first examined experimentally at NIST
Clark C W et al *Nature* **525** 504 (2015)
and methods for their generation were further developed in D. Sarenac et al., New J. Phys. **20**, 103012 (2018).

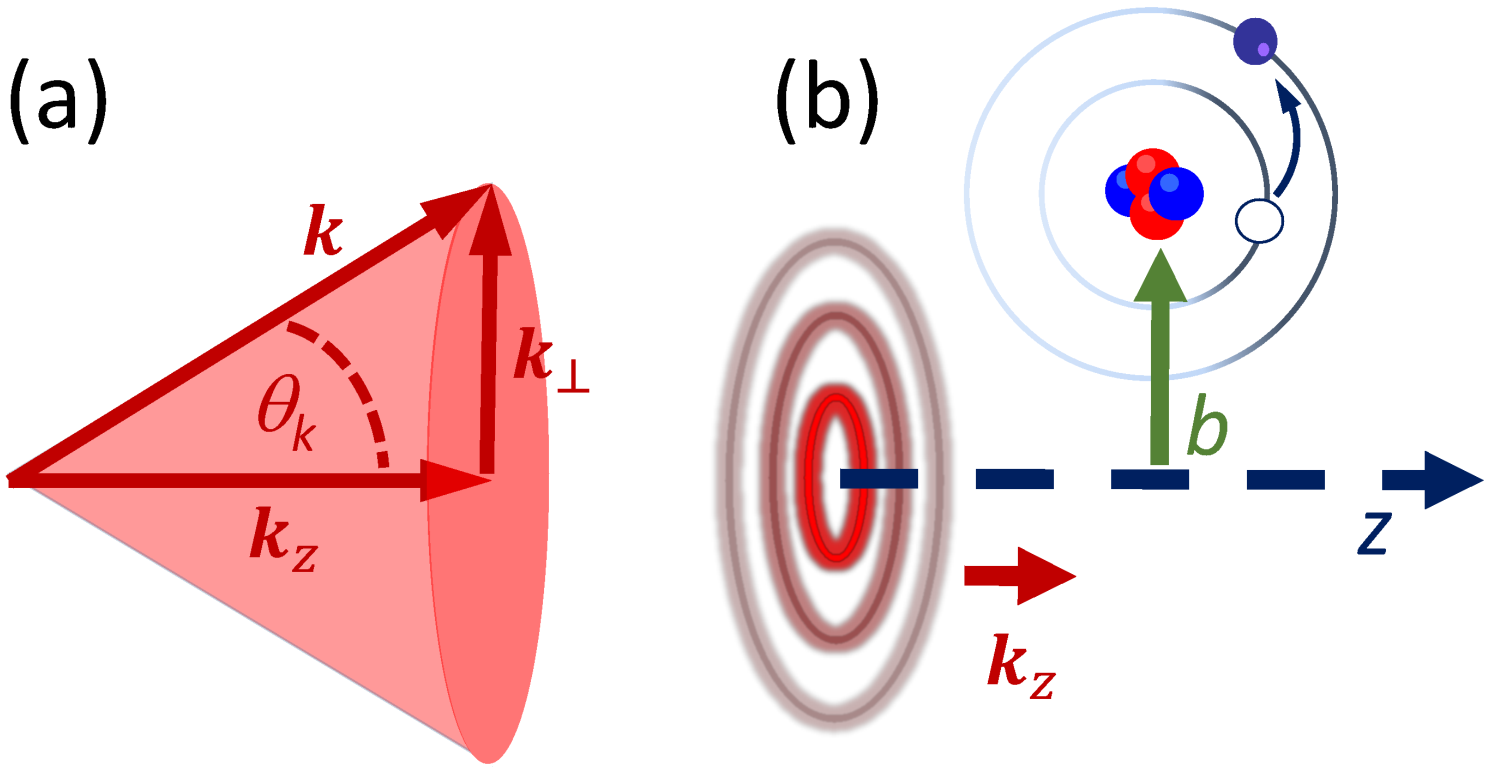
The first calculation was made of the processes with such neutrons A.Afanasev, V.G.Serbo, M. Solyanik,
J. of Phys. G 45 № 5 (2018) 055102
A.V. Afanasev, D.V. Karlovets, V.G. Serbo. Phys. Rev. C 100 (2019) 051601(R)

A few words about a theoretical approach:

Usually, for theory considerations of the processes with the twisted particles, we start with the basic formulae provide relations between S-matrix elements of these processes involving a **convolution of Fourier amplitude of the twisted state with corresponding standard (plane-wave) S-matrix elements**

H. M. Scholz-Marggraf, S. Fritzsche, V. G. Serbo, A. Afanasev, A. Surzhykov. Phys. Rev. A 90 (2014) 013425

Besides, we need to account for the fact that the twisted particles has a symmetry axis aligned with the z-direction, while a nucleus is located at a certain **impact parameter** $\mathbf{b} = (b_x, b_y, 0)$ from this axis:


(a)
k
k⊥
θk
kz
(b)
b
z
kz

$$\mathcal{M}_{fi}^{(\text{tw})}(m,\varkappa,k_z,\mathbf{b}) = \int \mathcal{M}_{fi}^{(\text{st})}(\mathbf{k}) e^{i\mathbf{k}_\perp \mathbf{b}} a_{\varkappa m}(\mathbf{k}_\perp) \frac{d^2 k_\perp}{(2\pi)^2},$$

As a result, the angular distributions of final particles and their polarization usually significantly differ from the standard, plane-wave, expressions.

It provides a dual use of the processes with twisted particles. **First**, they can be used to obtain **an additional information on the fundamental interaction** between involved particles. **Secondly**, these processes can be used for **the diagnostics of the twisted beams themselves**.

4.2. Angular distribution of final plane-wave photons for the case when the initial photons are twisted

Let us consider the case when an initial ion in the ground state performs a head-on collision with an initial twisted photon in the state $|\varkappa_1, m_1, k'_{1z}, \lambda_1\rangle$ propagating along the $(-z)$ axis with momentum $k'_{1z} < 0$ (since $\pi/2 < \theta'_1 < \pi$).

Here m_1 is projection of the total angular momentum of photon onto the axis of its motion
and $\varkappa_1$ is the module of the photon transverse momentum.
The energy of this state ω'_1 should be close to the value

$$\omega_0 \frac{2}{1 + |\cos\theta'_1|} > \omega_0. \tag{17}$$

The matrix element for such a scattering $\mathcal{M}_{fi}^{(\mathrm{tw})}$ can be expressed via the standard (plane-wave) matrix element $\mathcal{M}_{fi}^{(\mathrm{st})}$ as

$$\mathcal{M}_{fi}^{(\mathrm{tw})}(\boldsymbol{b}) = \int_0^{2\pi} \frac{d\varphi_1'}{2\pi} e^{-im_1\varphi_1' + i\varkappa_1 b\cos(\varphi_1' - \varphi_b)}\, \mathcal{M}_{fi}^{(\mathrm{st})}, \quad (18)$$

where we take into account that ion is located at a certain impact parameter $\boldsymbol{b} = b(\cos\varphi_b, \sin\varphi_b, 0)$ from the z-axis. Using this equation we obtain

$$\left|\mathcal{M}_{fi}^{(\mathrm{tw})}(\boldsymbol{b})\right| = \left|J_{m_1-\lambda_1}(\varkappa_1 b)\mathcal{M}_{fi}^{(\mathrm{st})}\right|. \quad (19)$$

Now we should **define the differential cross section.**

It is convenient to characterize the process by the cross section **averaged over the impact parameters of the ions of the bounded target**.

Let us present two limiting cases.
First, when $\theta_1' \to \pi$, we return to the plane-wave result

$$d\sigma^{(\mathrm{tw})} \to \delta_{m_1,\lambda_1}\, d\sigma^{(\mathrm{st})}(\theta_1' = \pi, \theta_2'), \qquad (20)$$

since in this limit $\varkappa_1 \to 0$ and $J_n(\varkappa_1 b) \to \delta_{n,0}$.

Second, we consider the most realistic experimental set-up, when **the target is large** and corresponds to ions quite uniformly distributed over the entire transverse plane. It is not difficult to show (see, for example, Sect. II.B in paper H. M. Scholz-Marggraf, S. Fritzsche, V. G. Serbo, A. Afanasev, A. Surzhykov. Absorption of twisted light by hydrogen like atoms. Phys. Rev. A **90**, 013425 (2014))
that in this case

$$d\sigma^{(\mathrm{tw})} = \frac{1}{|\cos\theta_1'|} \int_0^{2\pi} \frac{d\varphi_1'}{2\pi}\, d\sigma^{(\mathrm{st})}(\varphi_1'). \tag{21}$$

4.3. How the twistedness of the initial photon transmits to the final photon

Now we consider the case when **not only initial, but the final photon is twisted** as well. Let m_1 (m_2) be the projection of the total angular momentum of the initial (final) photon on its axis of motion.

We briefly point out the idea of consideration this problem. We have already mentioned in Sect. II that for a number of ions we can use the simple non-relativistic dipole approximation when calculating the resonant photon scattering.

It means that in a wide range of Z-values we can neglect the electron spin and assume that its z-projection does not change. For strictly backward scattering we can immediately conclude that the equality

$$m_2 = -m_1 \tag{22}$$

follows from a law of conversation of the z-projection of the total angular momentum of the system.

More detailed consideration can be found in our paper
V.G. Serbo and A. Surzhykov. Resonant scattering of twisted photons at the Gamma Factory (LHC). In preparation

Let us now compare this case with the Compton scattering

The Compton scattering of a twisted laser photon is considered in a number of papers

U.D. Jentschura, V.G. Serbo. Phys. Rev. Letters **106**, 013001 (2011);

U.D. Jentschura, V.G. Serbo. Eur. Phys. J. C **71**, 1571 (2011);

I.P. Ivanov, V.G. Serbo. Phys. Rev. A **84**, 033804 (2011); Erratum **84**, 069906(E) (2011); Addendum **84**, 065802 (2011)

I.P. Ivanov. Phys. Rev. A **85**, 033813 (2012)

and its main features are well known. When scattering **strictly backward, the scattered photon turns out to be twisted, and the twistedness of the laser photon is transmits to it**, i. e. $m_2 = -m_1$.

For small deviations from the scattering of a photon strictly backward, this equality is violated, a nonzero probability arises to have different values of m_2, i.e. there **appears some distribution over the values of** m_2.

Since the structure of the resonant scattering matrix element is the same as in the case of the Compton scattering, then all further calculations remain almost the same.

5. Conclusion

1. For the resonant scattering the photon energy in the IRF ω_1 must be close to the transition energy $\omega_{\nu i}$. **In the standard case**, to get such an energy the laser energy must be

$$\omega_1' = \frac{\omega_1}{2\gamma}, \tag{23}$$

while in the case of **the twisted initial photon** the laser energy must be larger

$$\omega_1' = \frac{\omega_1}{\gamma + |\cos\theta_1'|}, \tag{24}$$

where $\pi/2 < \theta_1' < \pi$.

2. In the CF there is the following angular distribution for **the initial twisted photons** and **the large ion target**

$$d\sigma^{(\mathrm{tw})} = \frac{1}{|\cos\theta_1'|} \int_0^{2\pi} \frac{d\varphi_1'}{2\pi}\, d\sigma^{(\mathrm{st})}(\varphi_1'). \qquad (25)$$

3. For **strictly backward scattering** the initial twisted photon **transmits its twistedness to the final photon**, $m_2 = -m_1$, while for **small deviations** from the strictly backward scattering, **some distribution** over the values of m_2 is appeared.

Thank you for attention!