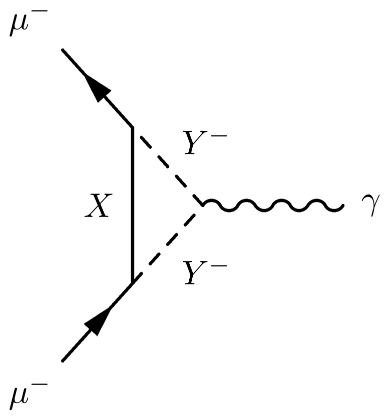
New Physics Contributions to g-2

Svjetlana Fajfer
Physics Department, University of Ljubljana and
Institute J. Stefan, Ljubljana, Slovenia



International Physics School
MUON DIPOLE MOMENTS AND HADRONIC EFFECTS
29 August to 3 Sept. 2021

Outline

Motivation

Possibility for New Physics

Leptoquaks

2HDM

Dark Matter

NP in g-2 using SMEFT

Constraints from processes at low and high energies

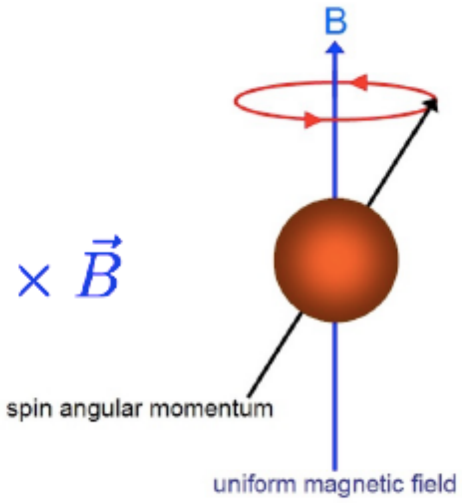
Motivation

$$\vec{\mu} = g_l \frac{e}{2m_l} \vec{S}$$

gyromagnetic ratio

Spin precession with frequency determined by g

$\vec{\mu} \times \vec{B}$



Interaction of magnetic moment (spin) with magnetic field B

a=(g-2)/2

g=2 (Dirac)

anomalous magnetic moment

$$a_\mu^{\mathrm{SM}} = (116\,591\,810 \pm 43) \cdot 10^{-11}$$

$$a_\mu^{\mathrm{average}} = (116\,592\,061 \pm 41) \cdot 10^{-11}$$

FERMILAB experiment
Brookhaven experiment , 2006

$$\Delta a_\mu = (251 \pm 59) \cdot 10^{-11}.$$

$$a_\mu^{\mathrm{SM}} = (116\,591\,810 \pm 43) \cdot 10^{-11}$$

$$a_\mu^{\mathrm{average}} = (116\,592\,061 \pm 41) \cdot 10^{-11}$$

$$a_\mu^{\mathrm{FNAL}} = (116\,592\,040 \pm 54) \times 10^{-11}$$
$$a_\mu^{\mathrm{BNL}} = (116\,592\,089 \pm 63) \times 10^{-11}$$
$$a_\mu^{2021} = (116\,592\,061 \pm 41) \times 10^{-11}$$

$$\Delta a_\mu = (251 \pm 59) \cdot 10^{-11}$$

$$a_e^{\mathrm{exp}} = (1\,159\,652\,180.73 \pm 0.28) \cdot 10^{-12}$$

$$\begin{aligned} \Delta a_e^{\mathrm{Cs}} &= a_e^{\mathrm{exp}} - a_e^{\mathrm{SM,\,Cs}} = -\,(0.88 \pm 0.36) \cdot 10^{-12} \\ \Delta a_e^{\mathrm{Rb}} &= a_e^{\mathrm{exp}} - a_e^{\mathrm{SM,\,Rb}} = (0.48 \pm 0.30) \cdot 10^{-12}\,. \end{aligned}$$

$$-2.4\sigma \;\; (+1.6\sigma)$$

$$\begin{aligned} -0.052 < \; & a_\tau < 0.013\,, && e^+e^- \to e^+e^-\tau^+\tau^- \\ -0.007 < \; & a_\tau < 0.005\,, && e^+e^- \to \tau^+\tau^- \end{aligned}$$

SM $a_\tau = (117\,721 \pm 5) \cdot 10^{-8}$

Eydelman et al., 2007

Why do we believe that there is New Physics(NP)?

Nomurapresented in his lecture (fist day of the school) reasons for physics beyond SM

- Neutrino masses & mixing matrix
- Why $\bar{\theta} \lesssim 2 \times 10^{-10}$? (strong CP problem)
- Why $m_{weak} \ll m_{GUT}$? (gauge hierarchy problem)
- Dark matter & dark energy
- Origin of the baryon number

  Gravity …

Flavour puzzle:
why Masses of fundamental fermions are so different?

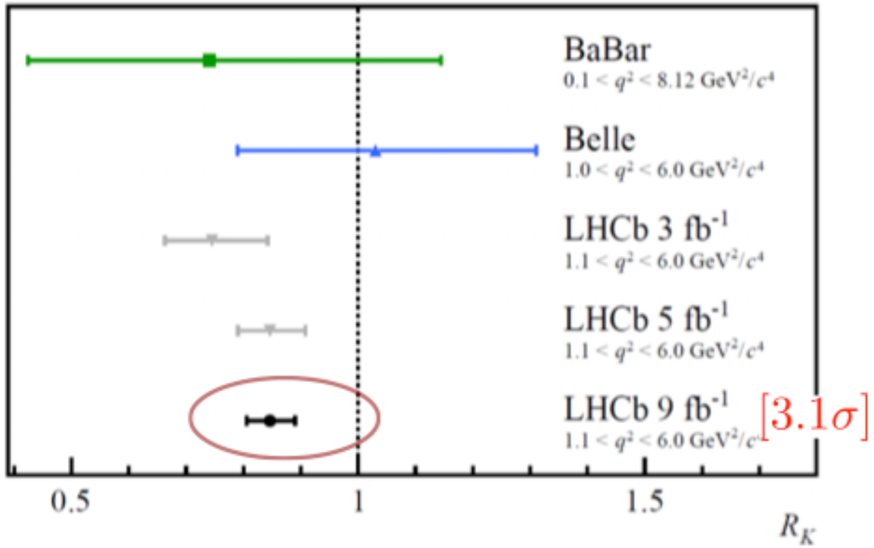
On experimental side LHC did not find any new particle.
Current experimental results are in agreement with the SM expectations.

However, in flavor physics, there are deviations of the measured from the predicted observables.

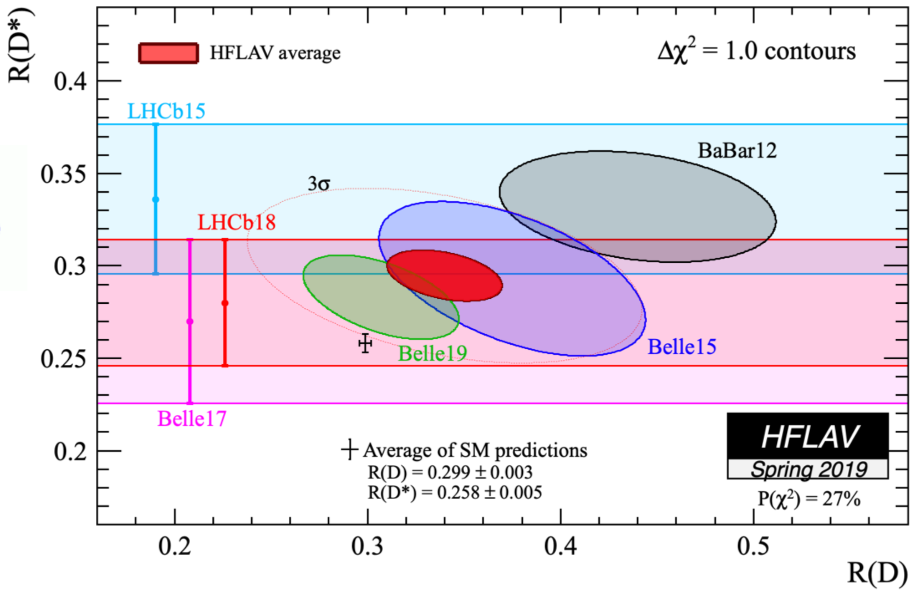
B anomalies

B meson anomalies

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)}{\mathcal{B}(B \to K^{(*)}ee)}\Bigg|_{q^2\in[q^2_{\min},q^2_{\max}]} \quad \& \quad R^{\rm exp}_{K^{(*)}} < R^{\rm SM}_{K^{(*)}}$$



$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})}_{\ell\in(e,\mu)} \quad \& \quad R^{\rm exp}_{D^{(*)}} > R^{\rm SM}_{D^{(*)}}$$



NP explaining both B anomalies

$$R^{exp}_{D^{(*)}} > R^{SM}_{D^{(*)}}$$

$$\mathcal{L}_{NP} = \frac{1}{(\Lambda^D)^2} 2\, \bar{c}_L \gamma_\mu b_L \bar{\tau} \gamma^\mu \nu_L$$

$$\Lambda^D \simeq 3\,\mathrm{TeV}$$

$$R^{exp}_{K^{(*)}} < R^{SM}_{K^{(*)}}$$

$$\mathcal{L}_{NP} = \frac{1}{(\Lambda^K)^2} \bar{s}_L \gamma_\mu b_L \bar{\mu}_L \gamma^\mu \mu_L$$

$$\Lambda^K \simeq 30\,\mathrm{TeV}$$

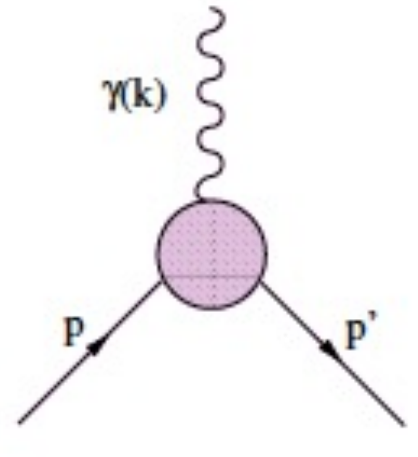
$$\Lambda^D \simeq \Lambda^K \equiv \Lambda$$

NP in FCNC $B \to K^{(*)}\mu^+\mu^-$
has to be suppressed

$$\frac{1}{(\Lambda^K)^2} = \frac{C_K}{\Lambda^2} \qquad C_K \simeq 0.01$$

suppression factor

Muon anomalous magnetic moment



=

$$= \quad (-ie)\bar{\mathrm{u}}(p')\left[\gamma^{\mu}\underbrace{F_1(k^2)}_{\text{Dirac}} + \frac{i\sigma^{\mu\nu}k_{\nu}}{2m}\underbrace{F_2(k^2)}_{\text{Pauli}}\right]\mathrm{u}(p)$$

$$F_1(0) = 1 \quad \text{and} \quad F_2(0) = a$$

(If P and CP symmetries hold)

- a_e - test of QED
- a_μ - important contributions from QED, QCD weak gauge sector

Muon anomalous magnetic moment in SM is finite

$$\mathcal{L}_{eff} = -\frac{e\,a_\mu}{4m_l}\bar{\mu}(x)\,\sigma^{\alpha\beta}\,\mu(x)\,F_{\alpha\beta}$$

$$a_\mu = F_2(0)$$

- can be calculated unambiguously in renormalizble QFT;
- no counter-term to absorb UV divegences;

New Physics explanation of $\Delta\, a_\mu$

The weak interaction contribution is $a_\mu^{weak} = 1.54 \times 10^{-9}$

$$\Delta a_\mu = (251 \pm 59) \cdot 10^{-11}$$

NP effects huge!

Assumption $\boxed{\Delta a_\mu \simeq a_\mu^{NP}}$

$$\mathcal{L}_{eff} = \frac{c_d}{\Lambda_{NP}^2} \bar{L}\, \sigma_{\mu\nu}\, l_R\, H\, F^{\mu\nu}$$

$$a_\mu^{NP} \simeq C \frac{m_\mu^2}{\Lambda_{NP}^2} \qquad\qquad a_\mu^{NP} \simeq C \frac{m_\mu\, m_t}{\Lambda_{NP}^2}$$

$$C \simeq 1$$

$$\boxed{\Lambda_{NP} \sim 1.9\,\mathrm{TeV}} \qquad\qquad \boxed{\Lambda_{NP} \sim 80\,\mathrm{TeV}}$$

Interesting

$\boxed{a_l \sim (m_l / M_{NP})^2}$

Muon a_μ 44000 more sensitive than electron a_e on M_{NP}

How to approach Physics Beyond SM?

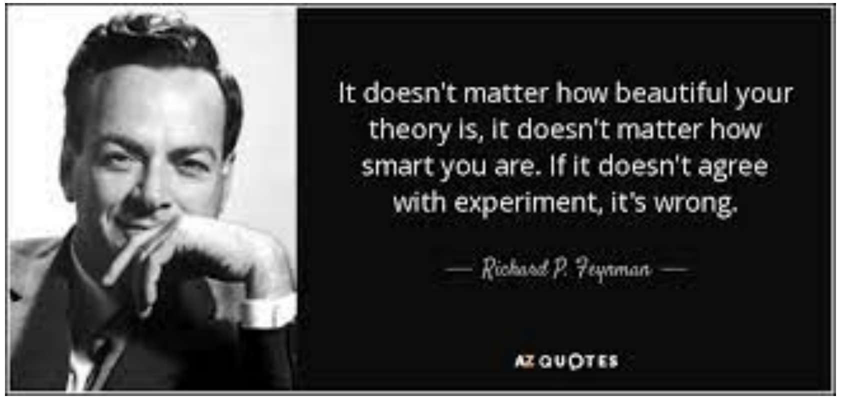
- find which new particles and their couplings to SM fermions can explain the difference, and then try to construct a new theory that can contain it!

- take your favorite model of NP and check which particles and couplings can explain the difference!

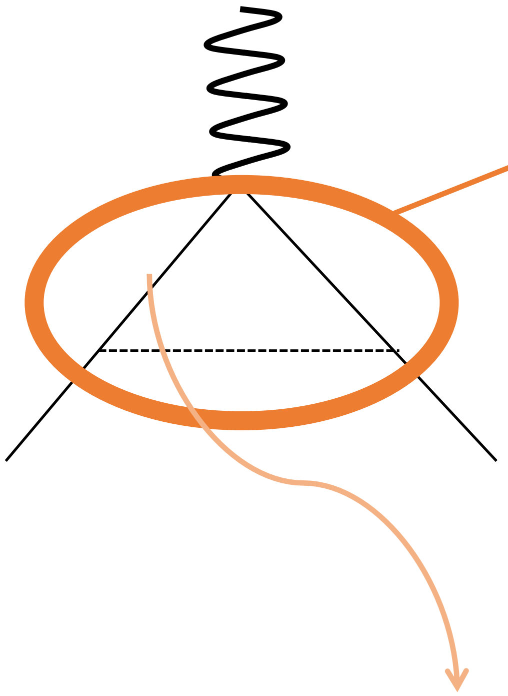
But

In both cases: SM results for all measured quantities should not be spoiled!

Check contributions to all existing measurements!



Possibilities for NP



NP particles

- SM fermions + new boson
- new fermions + SM bosons
- new fermions + new bosons

simplicity: only one new particle!

Increase significant for the large mass in the loop!

For a recent review on NP in g-2: Peter Athrona, Csaba Balazs, Douglas HJ Jacob Wojciech Kotlarski, Dominik Sẗockinger, Hyejung Sẗockinger-Kim 2104.03691

Particles that can explain Δa_μ

Model	Spin	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Result for $\Delta a_\mu^{\rm BNL}$, Δa_μ^{2021}
1	0	$(\mathbf{1}, \mathbf{1}, 1)$	Excluded: $\Delta a_\mu < 0$
2	0	$(\mathbf{1}, \mathbf{1}, 2)$	Excluded: $\Delta a_\mu < 0$
3	0	$(\mathbf{1}, \mathbf{2}, -1/2)$	
4	0	$(\mathbf{1}, \mathbf{3}, -1)$	Excluded: $\Delta a_\mu < 0$
5	0	$(\overline{\mathbf{3}}, \mathbf{1}, 1/3)$	
6	0	$(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$	Excluded: LHC searches
7	0	$(\overline{\mathbf{3}}, \mathbf{3}, 1/3)$	Excluded: LHC searches
8	0	$(\mathbf{3}, \mathbf{2}, 7/6)$	
9	0	$(\mathbf{3}, \mathbf{2}, 1/6)$	Excluded: LHC searches
10	1/2	$(\mathbf{1}, \mathbf{1}, 0)$	Excluded: $\Delta a_\mu < 0$
11	1/2	$(\mathbf{1}, \mathbf{1}, -1)$	Excluded: Δa_μ too small
12	1/2	$(\mathbf{1}, \mathbf{2}, -1/2)$	Excluded: LEP lepton mixing
13	1/2	$(\mathbf{1}, \mathbf{2}, -3/2)$	Excluded: $\Delta a_\mu < 0$
14	1/2	$(\mathbf{1}, \mathbf{3}, 0)$	Excluded: $\Delta a_\mu < 0$
15	1/2	$(\mathbf{1}, \mathbf{3}, -1)$	Excluded: $\Delta a_\mu < 0$
16	1	$(\mathbf{1}, \mathbf{1}, 0)$	
17	1	$(\mathbf{1}, \mathbf{2}, -3/2)$	UV completion problems
18	1	$(\mathbf{1}, \mathbf{3}, 0)$	Excluded: LHC searches
19	1	$(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$	UV completion problems
20	1	$(\overline{\mathbf{3}}, \mathbf{1}, -5/3)$	Excluded: LHC searches
21	1	$(\overline{\mathbf{3}}, \mathbf{2}, -5/6)$	UV completion problems
22	1	$(\overline{\mathbf{3}}, \mathbf{2}, 1/6)$	Excluded: $\Delta a_\mu < 0$
23	1	$(\overline{\mathbf{3}}, \mathbf{3}, -2/3)$	Excluded: proton decay

New weak scalar doublet (H^0, H^-)

Scalar leptoquarks

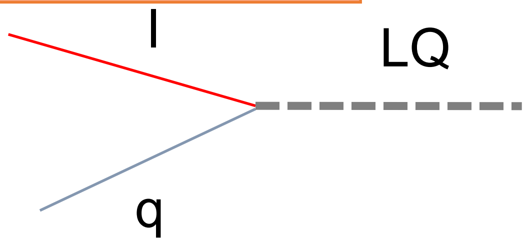
Single new fermion can not explain Δa_μ

New neutral gauge boson

Vector leptoquarks

See 2104.03691

Popular scenario: Leptoquarks as a resolution of B anomalies:



LQ=$(SU(3)_c, SU(2)_L)_Y$

or LQ=$(SU(3)_c, SU(2)_L, Y)$

$Q=I_3+Y$

no proton decay at tree level

Single scalar LQ (with coupling to μ) cannot solve simultaneously both anomalies!

Model	$R_{D^{(*)}}$	$R_{K^{(*)}}$	$R_{D^{(*)}}$ & $R_{K^{(*)}}$
$S_1 = (\bar{3}, 1)_{1/3}$	✓	✗	✗
$R_2 = (3, 2)_{7/6}$	✓	✗*	✗
$S_3 = (\bar{3}, 3)_{1/3}$	✗	✓	✗
$U_1 = (3, 1)_{2/3}$	✓	✓	✓
$V_2 = (3, 1)_{2/3}$	✗	✗	✗
$\widetilde{V}_2 = (\bar{3}, 2)_{-1/6}$	✗	✗	✗
$U_3 = (3, 3)_{2/3}$	✗	✓	✗

Spin 0

Spin 1

Doršner, SF, Greljo, Kamenik , Košnik, 1603.04993

Scalar LQ → simpler UV completion;

Only R_2 and S_1 might explain $(g\text{-}2)_\mu$ (both chiralities are required with the enhancement factor m_t/m_μ) Muller 1801.0338.

Vector LQ U_1 as (a gauge boson) can explein both B anomalies but cannot explain $(g\text{-}2)_\mu$ (Isidori's group)

Constraints from flavor observables

$$(g-2)_\mu$$

$$B_c \to \tau\nu \quad B \to \tau\nu$$

$$B \to K^{(*)}\nu\bar{\nu}$$

$$B_s^0 - \bar{B}_s^0$$

$$D^0 - \bar{D}^0$$

$$B \to D\mu\nu_\mu$$

$$K \to \mu\nu_\mu$$

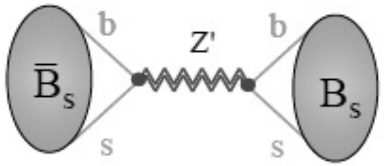
$$D_{d,s} \to \tau, \mu\nu$$

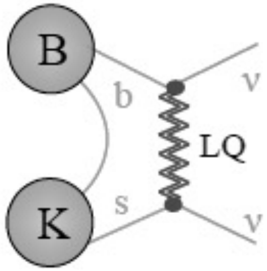
$$\tau \to K(\pi)\nu$$

$$K \to \pi\mu\nu_\mu$$

$$W \to \tau\bar{\nu},\ \tau \to \ell\bar{\nu}\nu$$

$$Z \to b\bar{b} \qquad Z \to l^+l^-$$





Constraints from LFV

$$\tau \to \mu\gamma$$

$$\mu \to e\gamma$$

$$\tau \to K(\pi)\mu(e)$$

$$K \to \mu e$$

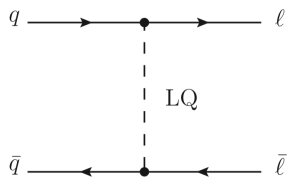
$$B \to K\mu e$$

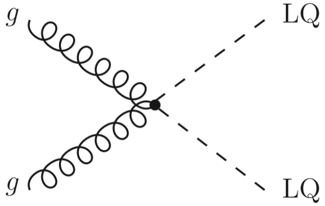
$$\tau \to \mu\mu\mu$$

$$\tau \to \phi\mu$$

$$t \to c\ell^+\ell'^-$$

Constraints from LHC (high p_T)





Becirevic et al., 1806.05689, 1608.07583, 1608.08501, Alonso et al., 1611.06676,…
Radiative constraints Feruglio et al.,1606.00524;

Scalar LQs

Symbol	$(SU(3)_c, SU(2)_L, U(1)_Y)$	Interactions	$F = 3B + L$
S_3	$(\overline{\mathbf{3}}, \mathbf{3}, 1/3)$	$\overline{Q}^C L$	-2
R_2	$(\mathbf{3}, \mathbf{2}, 7/6)$	$\overline{u}_R L$, $\overline{Q} e_R$	0
$\widetilde{R}_2$	$(\mathbf{3}, \mathbf{2}, 1/6)$	$\overline{d}_R L$	0
$\widetilde{S}_1$	$(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$	$\overline{d}_R^C e_R$	-2
S_1	$(\overline{\mathbf{3}}, \mathbf{1}, 1/3)$	$\overline{Q}^C L$, $\overline{u}_R^C e_R$	-2

Why LQs?
Goal: to explain B meson anomalies.

F≠0 proton destabilization

Is it possible to explain Δa_μ simultaneously?

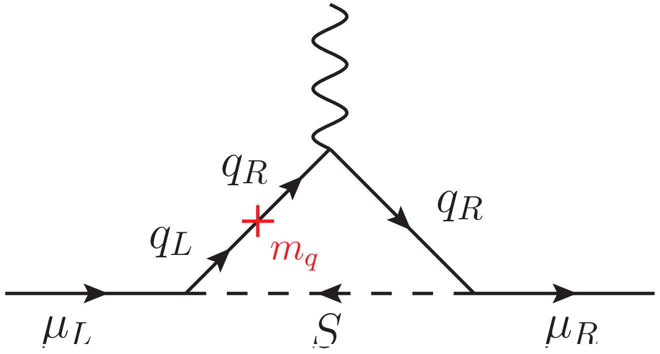
List of all scalar LQs, their SM quantum numbers, renormalizable interactions to the quark-lepton pairs, and associated fermion numbers. Interactions with right-handed neutrinos are not considered.

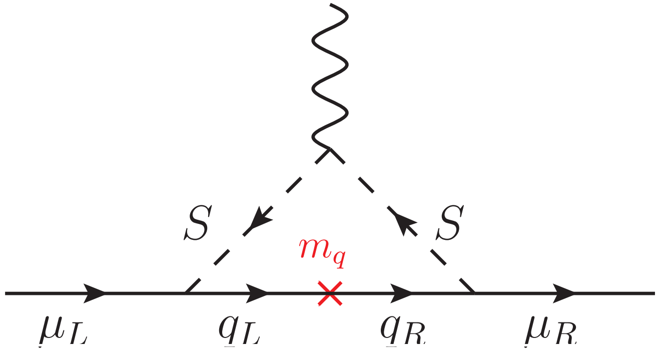
Doršner, SF, Greljo, Kamenik, Košnik, Phys. Rep. 641, 1 2016

Up-quark in weak doublet “talks” to down quarks via CKM!

$$Q_{iL} = \begin{bmatrix} V^*_{iJ}\, u_J \\ d_i \end{bmatrix}_L$$

Single LQ explanation





$$\bar{\mu}_L\, t_R\, S$$

$$\bar{\mu}_R\, t_L\, S$$

Both couplings are necessary – chirality flip

S no-chiral LQ 9can couple to both chiralities!

$$\mathcal{L}^{F=0} = \overline{q}_i \left(l^{ij} P_R + r^{ij} P_L\right) \ell_j \, S + \text{h.c.}$$

$$\mathcal{L}^{|F|=2} = \overline{q}_i^C \left(l^{ij} P_L + r^{ij} P_R\right) \ell_j \, S + \text{h.c.}$$

$$\boxed{\delta a_\mu = -\frac{N_c m_\mu}{8\pi^2 m_S^2} \sum_q \Big[\underbrace{m_\mu (|l^{q\mu}|^2 + |r^{q\mu}|^2)\, \mathcal{F}_{Q_S}(x_q)} + \underbrace{m_q \, \mathrm{Re}(r^{q\mu *}\, l^{q\mu})\, \mathcal{G}_{Q_S}(x_q)} \Big]}$$

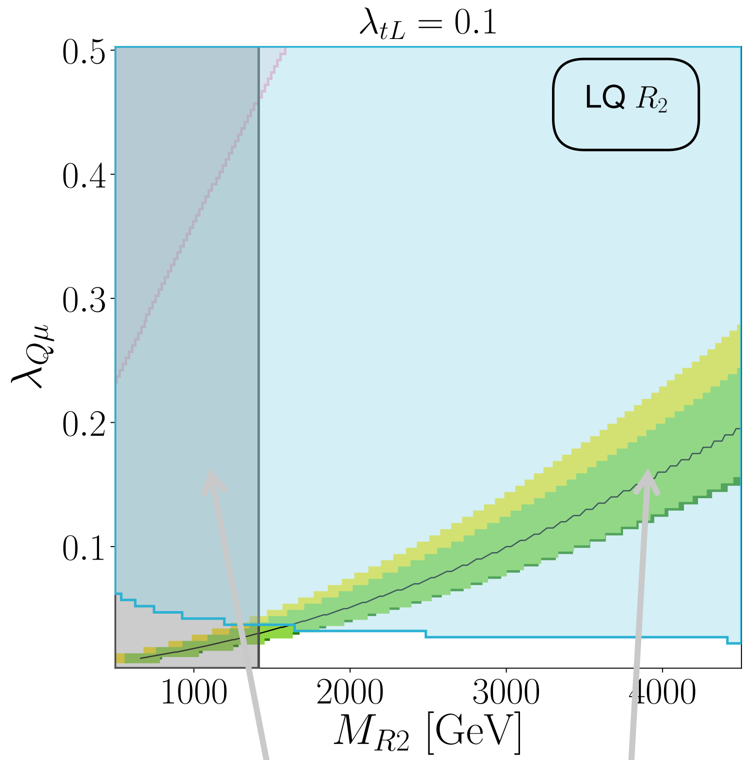
The same chirality

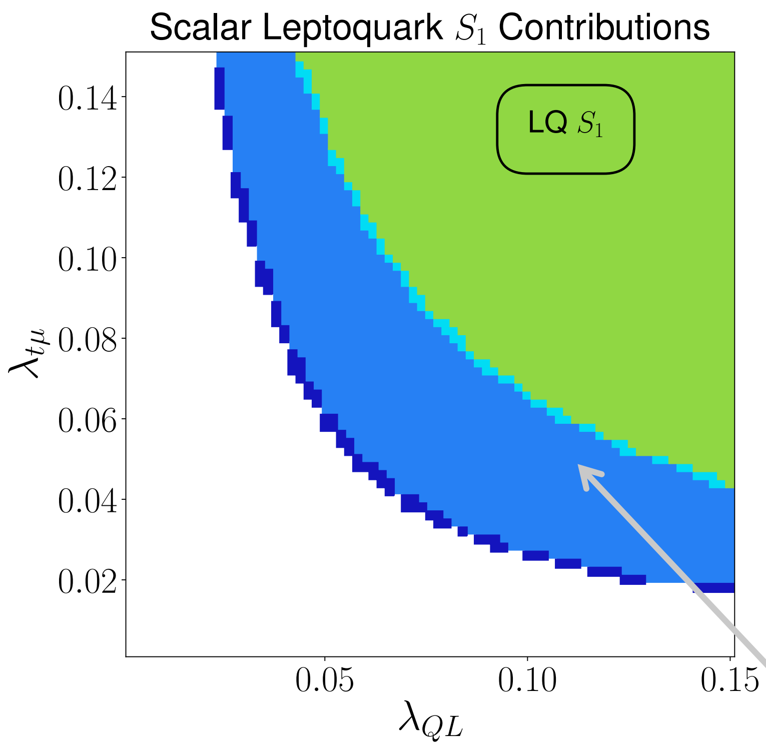
$$a_\mu^{NP} \simeq C \frac{m_\mu^2}{\Lambda_{NP}^2}$$

Both chiralities

$$a_\mu^{NP} \simeq C \frac{m_\mu \, m_t}{\Lambda_{NP}^2}$$

Only S_1 and R_2 can explain a_μ





satisfy direct LHC searches and $Z \to \mu\mu$ and $Z \to \nu\nu$ constraints and can explain Δa^{2021} (green) or both Δa_{BNL} and within 1σ

scalar leptoquark masses up to 4.5 TeV.

LHC excluded

Condition that radiative correction to μ mass

$$\frac{1}{2} < \frac{m_\mu^{\overline{\mathrm{MS}}}}{m_\mu} < 2$$

Δa_{BNL} and Δa^{2021} within 1σ are yellow and green

See 2104.03691

How to achieve LQ couplings with quarks and leptons having both chiralities?

LQs can mix when they have the same charge.

Mixing formalism

e.g.

$$\mathcal{M}^2 = \begin{pmatrix} m^2_{S_a} & \Omega \\ \Omega & m^2_{S_b} \end{pmatrix} \qquad \begin{pmatrix} S_-^{(Q)} \\ S_+^{(Q)} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} S_a^{(Q)} \\ S_b^{(Q)} \end{pmatrix}$$

$$\mathcal{L}_{\text{mix}}^{\widetilde{S}_1\,\&\,S_3} = \xi\, H^T i\tau_2(\vec{\tau}\cdot\vec{S}_3) H \widetilde{S}_1^* + \text{h.c.}$$

$$\mathcal{M}^2_{S^{(1/3)}} = \begin{pmatrix} m^2_{S_3} & -\frac{\xi v^2}{2} \\ -\frac{\xi v^2}{2} & m^2_{S_1} \end{pmatrix}$$

$$\theta \in [-\pi/2, \pi/2] \qquad \tan 2\theta = \frac{2\,\Omega}{m^2_{S_a} - m^2_{S_b}}.$$

$$m^2_{S_\pm^{(Q)}} = \frac{m^2_{S_a} + m^2_{S_b}}{2} \pm \frac{1}{2}\sqrt{(m^2_{S_a} - m^2_{S_b})^2 + 4\,\Omega^2}$$

Maximal mixing $\quad m_{S_a} = m_{S_b} \equiv m_S \qquad \theta = \pi/4$

$$\delta m_S^{(Q)} = m_{S_+^{(Q)}} - m_S \approx m_S - m_{S_-^{(Q)}}$$

Doršner, SF, Sumensari, 1910,03877
Doršner, SF, Saad, 2006.11624

$S_3 = (\bar{3},3,1/3)$ $S_1 = (\bar{3},1,1/3)$

Only left-handed quarks/leptons both chiralities of quarks/leptons

$$S_3 = \begin{cases} S_3^{4/3} \\ S_3^{1/3} \\ S_3^{-2/3} \end{cases}$$

can mix $S_1 = (\bar{3},1,1/3)$

LQ pairs	Mixing field(s)	$(g-2)_\mu$	ν-mass
S_1 & S_3	$H\ H$	u	–
$\widetilde{S}_1$ & S_3	$H\ H$	d	–
$\widetilde{R}_2$ & R_2	$H\ H$	d	–
$\widetilde{R}_2$ & S_1	H	–	d
$\widetilde{R}_2$ & S_3	H	–	d

example $S_1 = (\bar{3},\, 1,\, 1/3)\,\&\, S_3 = (\bar{3},\, 3,\, 1/3)$

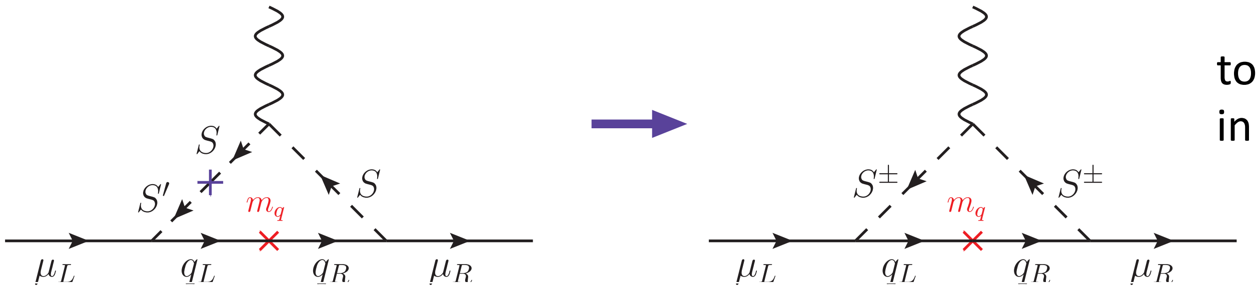
$$\left.\begin{array}{l} S_3^{1/3} \\ \\ S_1^{1/3} \end{array}\right. \longrightarrow \left.\begin{array}{l} S_+ \\ \\ S_- \end{array}\right\}$$ new mass eigen-states

$$\mathcal{L}_{S_1} = y_R^{ij}\, \overline{u}_{Ri}^C e_{Rj}\, S_1 + \text{h.c.}\,, \qquad \mathcal{L}_{S_3} = y_L^{ij}\, \overline{Q}_i^C i\tau_2(\vec{\tau}\cdot\vec{S}_3)L_j + \text{h.c.}$$

$$\boxed{\mathcal{L}_{\text{mix}}^{\widetilde{S}_1\,\&\,S_3} = \xi\, H^T i\tau_2(\vec{\tau}\cdot\vec{S}_3)H\widetilde{S}_1^* + \text{h.c.}}$$

$$\mathcal{M}^2_{S(1/3)} = \begin{pmatrix} m^2_{S_3} & -\frac{\xi v^2}{2} \\ -\frac{\xi v^2}{2} & m^2_{S_1} \end{pmatrix}$$

$$\begin{pmatrix} S_+^{(1/3)} \\ S_-^{(1/3)} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} S_3^{(1/3)} \\ S_1^{(1/3)} \end{pmatrix}$$



top quark
in the loop

$$\delta a_\mu \propto \frac{m_\mu^2}{m_S^2}(\dots) + m_\mu m_t \, y_L^{b\mu} \, y_R^{t\mu\,*} \left[\frac{\mathcal{G}_{1/3}(x_t^+)}{m_{S_+}^2} - \frac{\mathcal{G}_{1/3}(x_t^-)}{m_{S_-}^2} \right]$$

$x_t^{\pm} = m_t^2 / m_{S_\pm}^2$

Four mass eigenstates

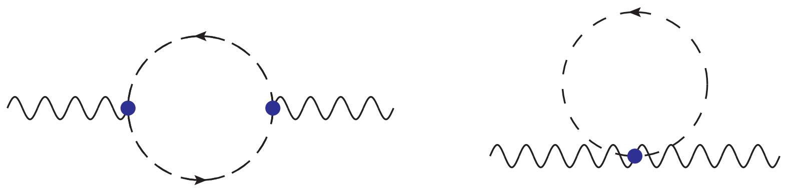
$$m_S = m_{S_3}^{(4/3)} = m_{S_3}^{(-2/3)}$$

$$m_{S_\pm} = m_{S_\pm}^{(1/3)}$$

$$\boxed{\delta a_\mu \propto \frac{m_\mu \, m_t}{m_S^2} \frac{\delta m_S}{m_S} y_R^{b\mu} \, y_L^{t\mu}}$$

Oblique corrections and mass splitting between $S_\pm$

T parameter



$$\Delta T = -\frac{N_c}{4\pi\, c_w^2\, s_w^2}\,\frac{1}{m_Z^2}\left[\cos^2\theta\, F(m_{S_3}, m_{S_-}) + \sin^2\theta\, F(m_{S_3}, m_{S_+})\right]$$

Expansion in δ m_S for maximal mixing ($\theta=\pi/4$)

$$\Delta T = \frac{N_c}{3\pi\, c_w^2\, s_w^2}\,\frac{\delta m_S^2}{m_Z^2} + \ldots$$

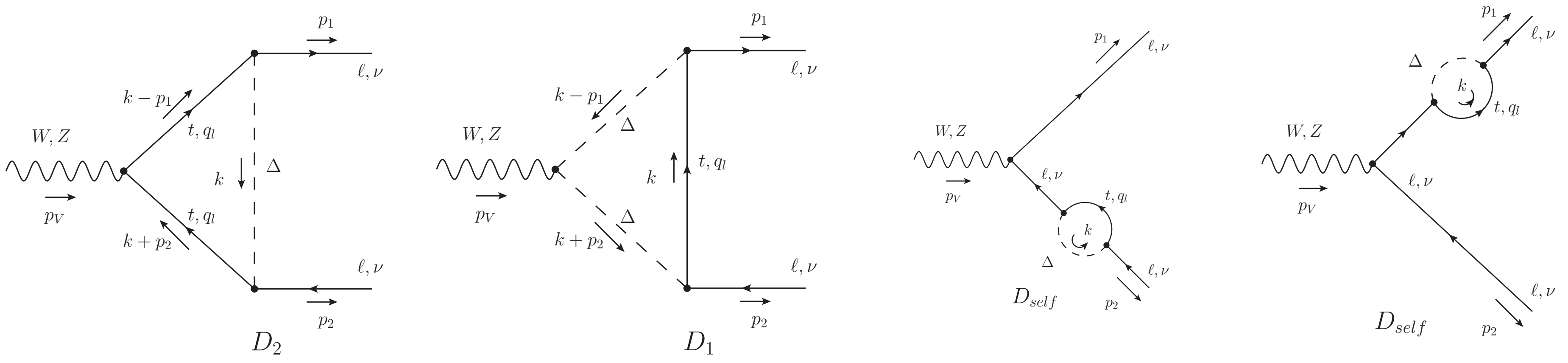
$\Delta T^{exp} = 0.05(12)$ implies $|\delta m_S| \leq 40\,\mathrm{GeV}$

Additional constraints

$$Z \to ll \;\&\; Z \to \nu\bar{\nu}$$

$$\delta\mathcal{L}_{\rm eff}^{Z} = \frac{g}{\cos\theta_W} \sum_{i,j} \bar{\ell}_i \gamma^\mu \left[g_{\ell_L}^{ij}\, P_L + g_{\ell_R}^{ij}\, P_R \right] \ell_j\, Z_\mu$$

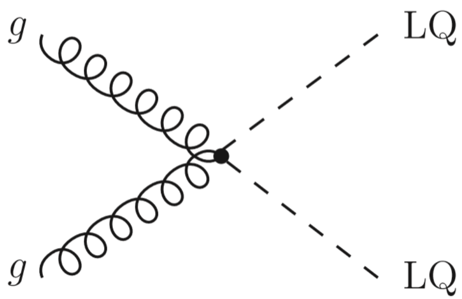
$$g_{\ell_{L(R)}}^{ij} = \delta_{ij}\, g_{\ell_{L(R)}}^{\rm SM} + \delta g_{\ell_{L(R)}}^{ij}$$



Arnan, Becirevic, Mescia, Sumensari, 1901.06315

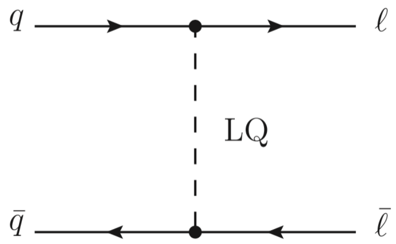
LHC constraints

Production at LHC



Angelescu, Becirevic, Faroughy and Sumensary, 2018

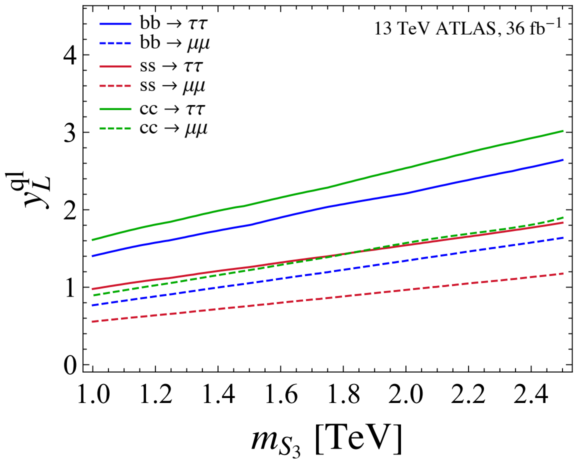
Faroughy, Greljo and Kamenik, 2015

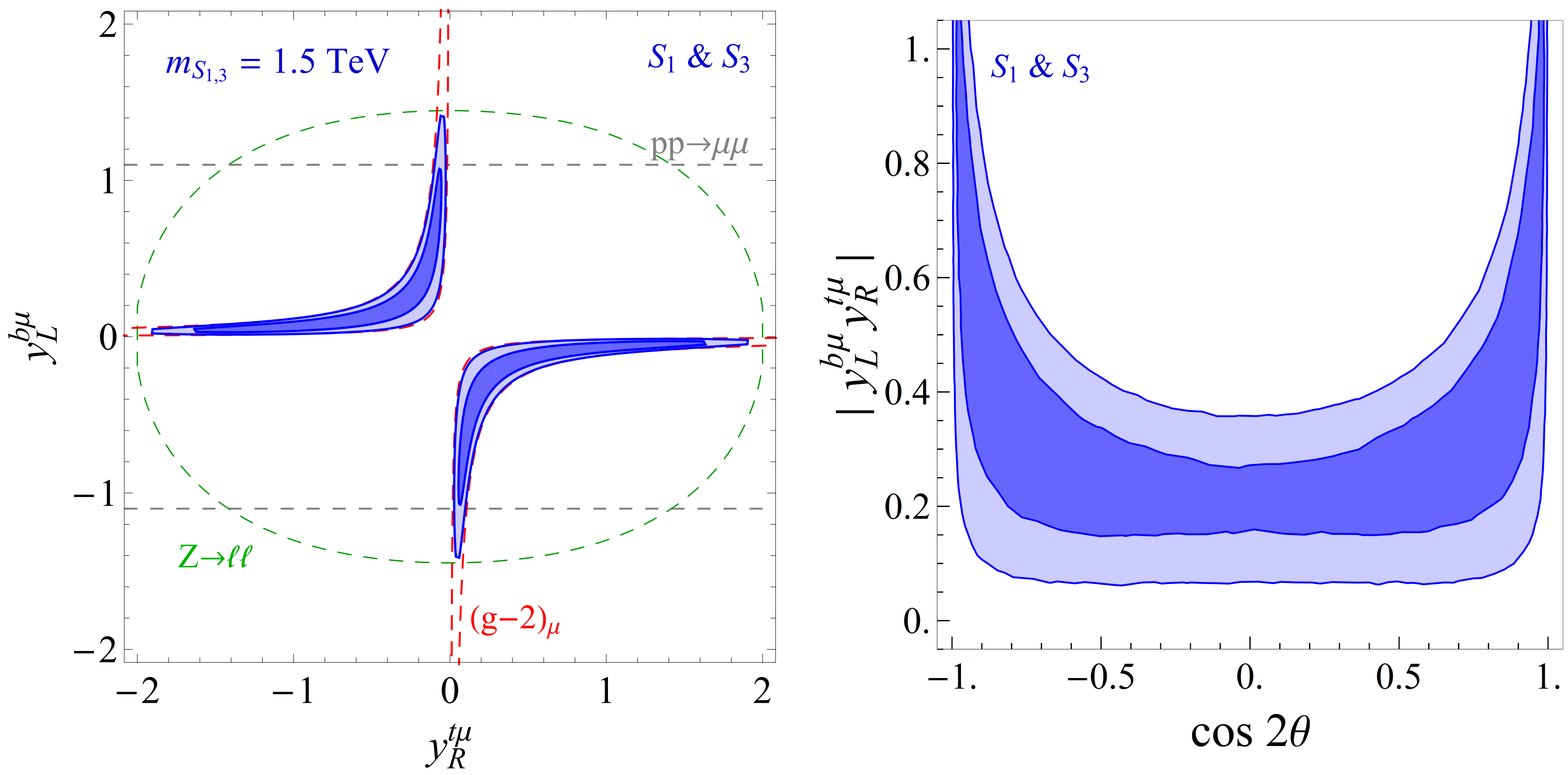


[CMS-PAS-EXO-17-003]

$$m_S \geq 1.6\,\text{TeV}$$

[ATLAS. 1707.02424,1709.07242]





Yukawa couplings should be perturbative

$(\leq \sqrt{4}\pi)$

$m_S < 15\,\mathrm{TeV}$

b quark in the loop

2/3 charge states mixes

$$R_2 = (3,2,7/6)\ \&\ \tilde{R}_2 = (3,2,1/6)$$

T parameter allows mass splitting ≤ 50 GeV
(Keith and Ma, 1997; Froggatt et al, 1992)

$$\mathcal{L}_{\widetilde{R}_2} = -y_L^{ij}\,\overline{d}_{Ri}\widetilde{R}_2 i\tau_2 L_j + \text{h.c.}\,,$$

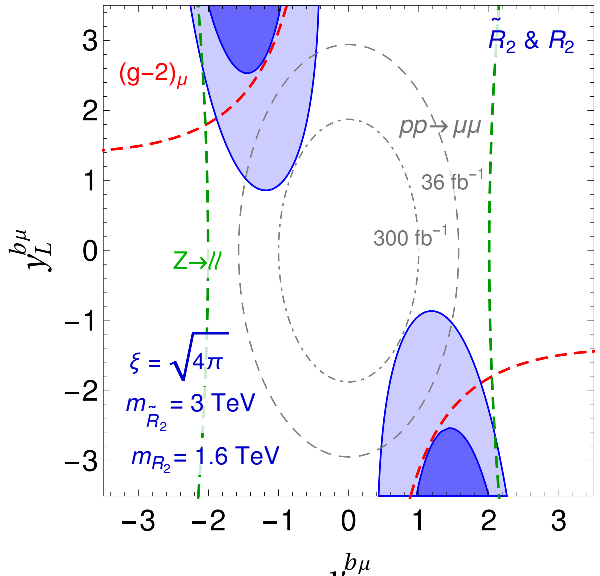
μ b coupling contribute only!

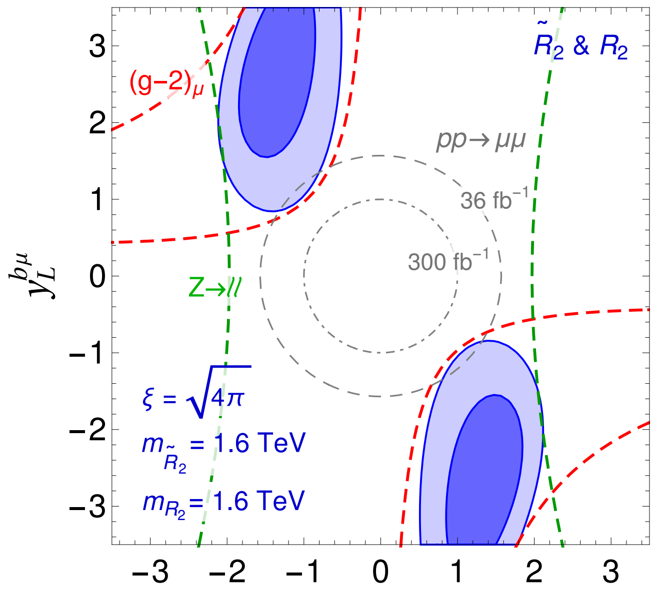
$$\mathcal{L}_{R_2} = y_R^{ij}\,\overline{Q}_i e_{Rj} R_2 + \text{h.c.}\,.$$

Mixing of
considered by Košnik,2012

$$R_2 \& \tilde{R}_2$$

$$\mathcal{L}_{\text{mix}}^{\widetilde{R}_2\,\&\,R_2} = -\xi\,(R_2^\dagger H)(\widetilde{R}_2^T i\tau_2 H) + \text{h.c.}$$

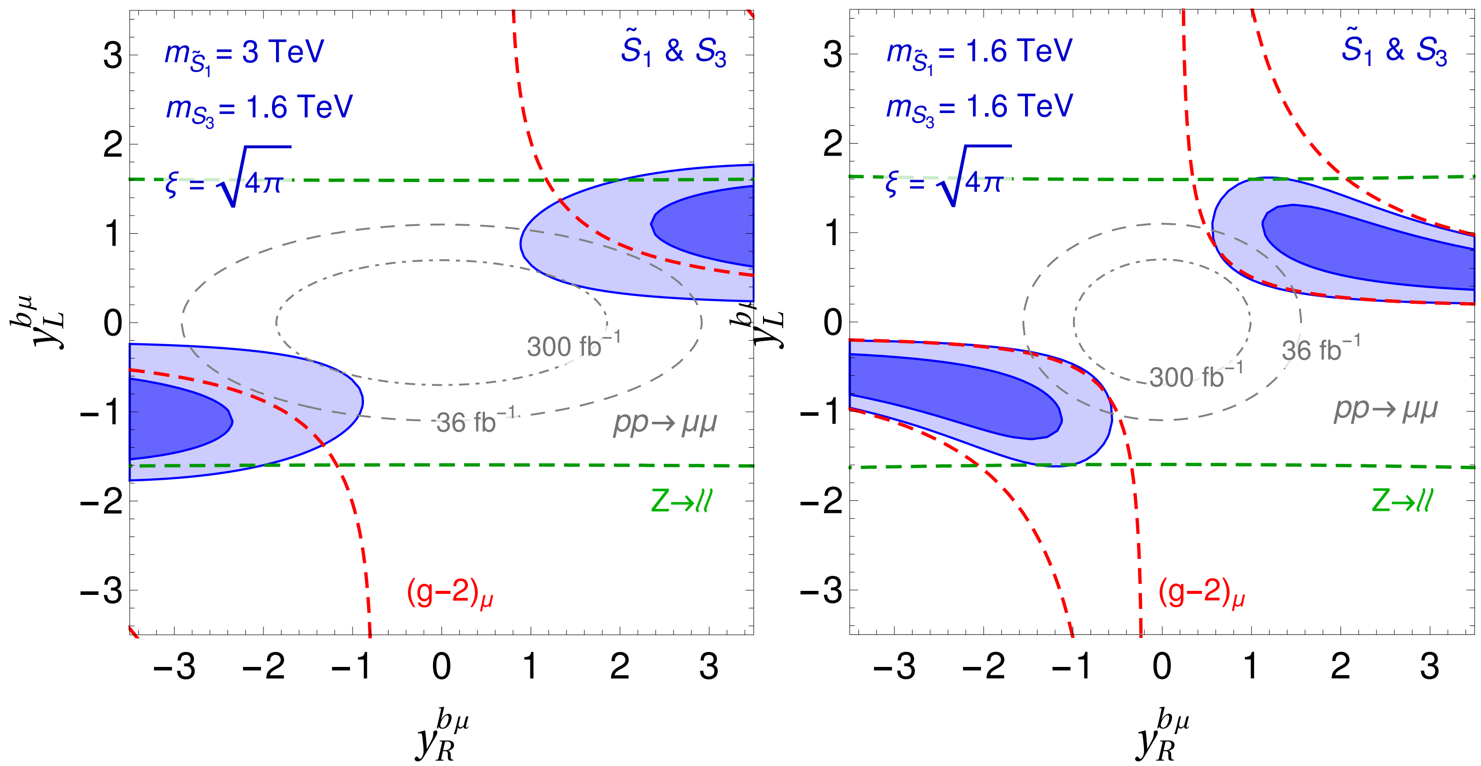




$$S_3 = (\bar{3}, 3, 4/3)\,\&\,\tilde{S}_1 = (3, 1, 4/3)$$

$$\mathcal{L}_{\widetilde{S}_1} = y_R^{ij}\,\bar{d}_{Ri}^{\,C}\,e_{Rj}\,\widetilde{S}_1 + \text{h.c.} \qquad \mathcal{L}_{S_3} = y_L^{ij}\,\bar{Q}_i^{\,C}\,i\tau_2(\vec{\tau}\cdot\vec{S}_3)L_j + \text{h.c.}$$

$$\mathcal{L}_{\text{mix}}^{\widetilde{S}_1\,\&\,S_3} = \xi\,H^T i\tau_2(\vec{\tau}\cdot\vec{S}_3)H\widetilde{S}_1^* + \text{h.c.}$$



How to build theory with two scalar LQs with masses in TeV region?

- 2 scalar LQ's can contribute to fermion masses (depends on the model);
- GUT models with two light LQs possible to construct;
- Neutrino masses can be generated radiatevely with the two scalar LQs in the loop;
- Both B-meson anomalies can be explained by S_3& R_2, S_3& S_1 .

Two-Higgs-doublet models and Δa_μ

Two $SU(2)_L$ doublets φ_1 and φ_2 , with two vevs v_1 and v_2

Global SU(2) $$\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \begin{pmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}, \qquad \tan\beta = \frac{v_2}{v_1}$$

In this basis, named the Higgs basis, Φ_2 has no vev while Φ_1 acquires a vev $v = \sqrt{v_1^2 + v_2^2}$

$$\boxed{\Phi_1 = \begin{pmatrix} G^+ \\ \frac{v+\rho_1+iG_0}{\sqrt{2}}\end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{\rho_2+i\eta}{\sqrt{2}}\end{pmatrix}}$$

This global transformation has the advantage of clearly isolating the Goldstone bosons $G^\pm$ and G^0 in the decomposition, which will be eaten to give mass to $W^\pm$ and Z^0.

The scalar fields $\rho_{1,2}$ can be additionally related with the physical Higgs field and a heavy neutral scalar by an orthogonal transformation

$$\begin{pmatrix}h\\ H^0\end{pmatrix} = \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\rho_1\\ \rho_2\end{pmatrix}$$

2HDM

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_Y + V(\Phi_1, \Phi_2)$$

$$\mathcal{L}_{kin} = D_\mu \Phi_1^\dagger D^\mu \Phi_1 + \partial_\mu \Phi_2^\dagger \partial^\mu \Phi_2$$

$$V(\Phi_1,\Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - \cancel{\left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right)} + \frac{\beta_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \cancel{\beta_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right)} + \cancel{\beta_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right)} + \cancel{\left[\frac{\beta_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right]} \, .$$

In the decoupling limit

$$m_{12} = \beta_3 = \beta_4 = \beta_5 = 0$$

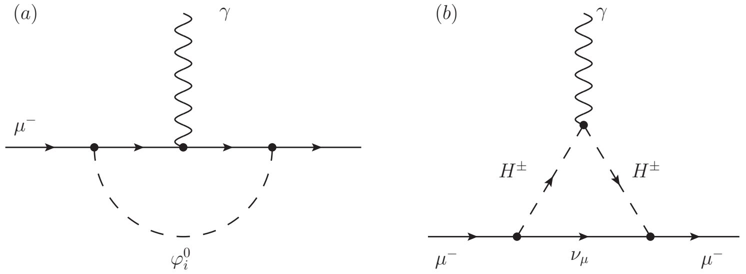
$$m_1 = m_h = 125 \text{ GeV} \qquad v^2 = -m_1^2/\beta_1 \qquad m_H^2 \Phi_2^\dagger \Phi_2 = m_H^2 \left(|H^+|^2 + \frac{1}{2}|\eta|^2 + \frac{1}{2}|H^0|^2\right)$$

$$m_2 \equiv m_H$$

$$\mathcal{L}_Y = Y^u_{st} \bar{q}_s \tilde{\Phi}_1 u_t + Y^d_{st} \bar{q}_s \Phi_1 d_t + Y^\ell_{pr} \bar{\ell}_p \Phi_1 e_r + Y'^{,\ell}_{pr} \bar{\ell}_p \Phi_2 e_r + Y'^{,u}_{st} \bar{q}_s \tilde{\Phi}_2 u_t + \text{h.c.}$$

$$\mathcal{L}_Y \supset (v+h) \sum_f \frac{m_f}{v} \bar{f} P_R f + \left(\frac{\eta + H^0}{\sqrt{2}}\right) \left[y'_t \bar{t} P_R t + y'_l \bar{l} P_R l\right]$$

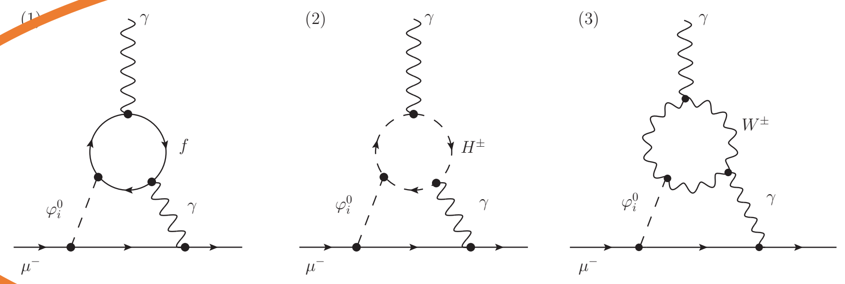
$$+ H^+ \left[y'_t V_{ti} \bar{t} P_L d_i + y'_l \bar{\nu}_l P_R l\right] + \text{h.c.} \, ,$$

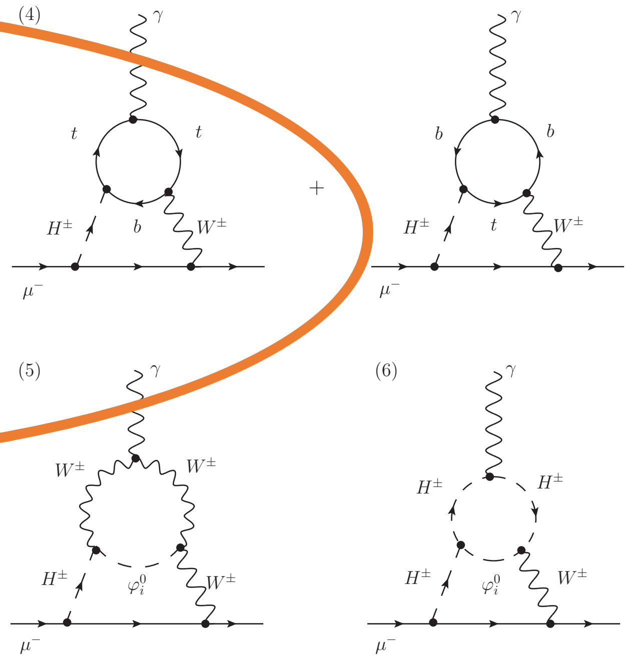


1502.04199 , Iliesie

One-loop contribution to Δa_μ in two-Higgs-doublet models.

Two-loop Barr-Zee type (with a charged Higgs and an internal W boson) contribution to Δa_μ in two-Higgs-doublet models.

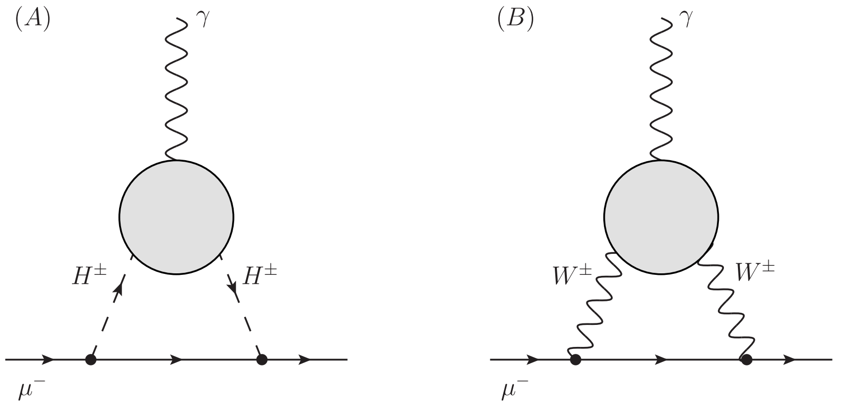




Dominant contributions!

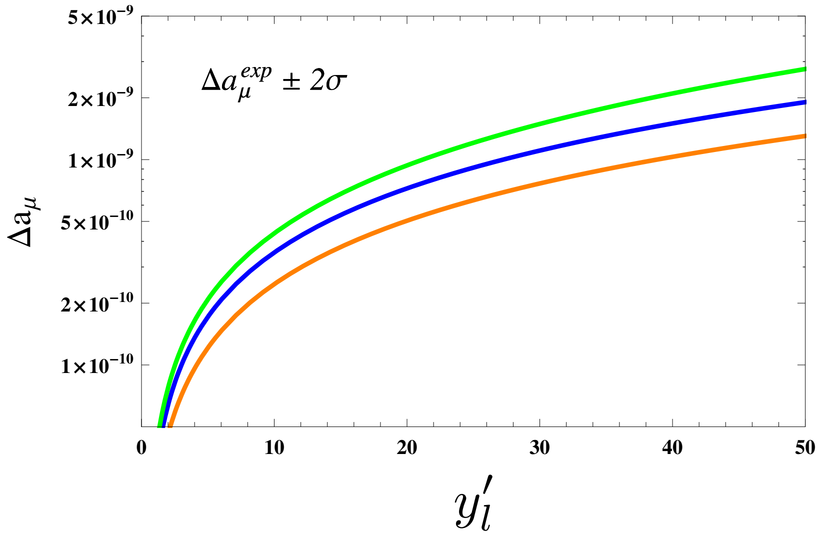
Two-loop Barr-Zee type (with an internal photon) contribution to Δa_μ in two-Higgs-doublet models .

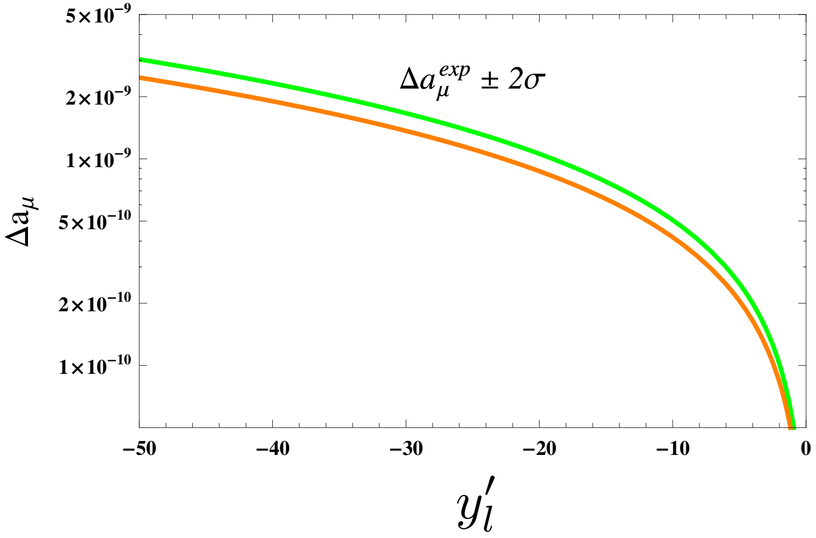
$m_\mu{}^2/M_W{}^2$
suppressed and can be safely discarded



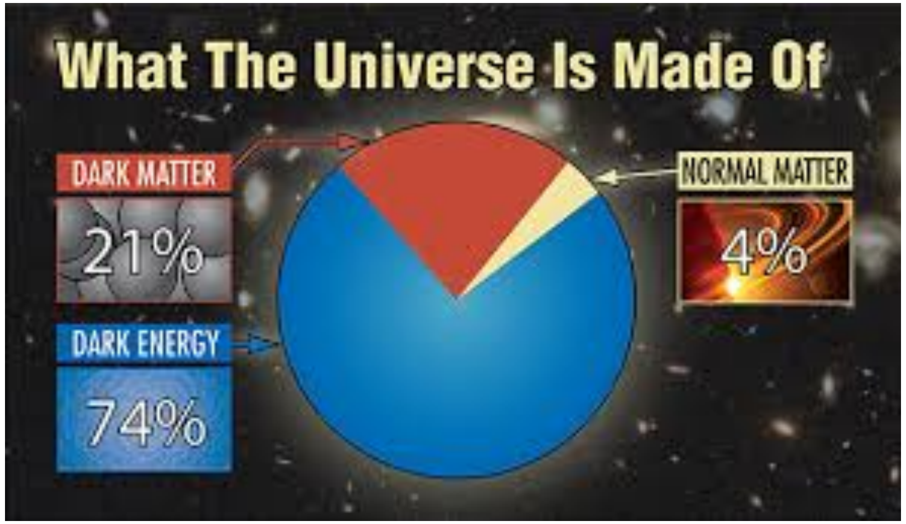
B can be sizeble

Generic two-loop Barr-Zee type contributions, with two internal charged Higges (left) and two internal W bosons (right), to Δa_μ in two-Higgs-doublet models.





Low-energy U(1) dark bosons



- Framework: Dark $U(1)_d$
- Model construction
- Dark gauge boson in $(g\text{-}2)_\mu$

Dark photon and dark Z explanations

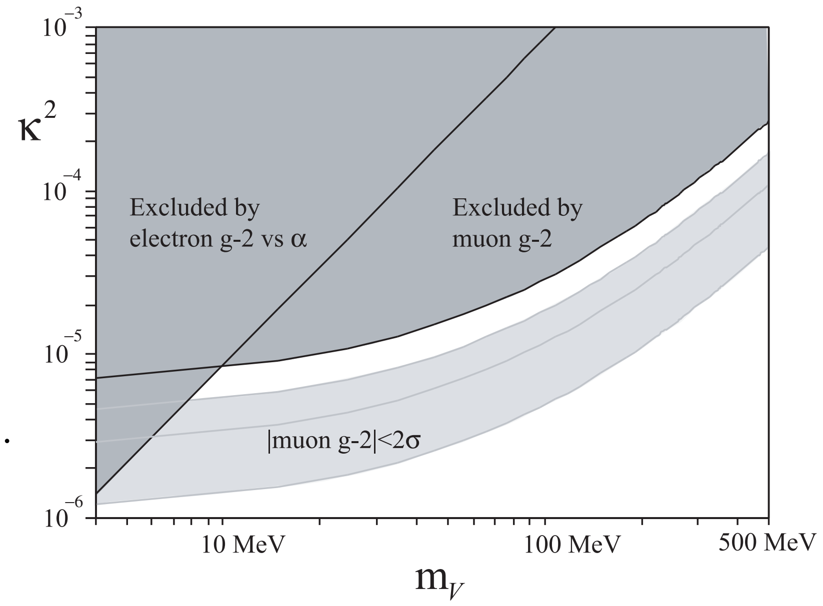
additional gauge field Z_d with (1,1,0) quantum numbers that arises from some additional $U(1)_d$ gauge symmetry

M. Pospelov, Secluded U(1) below the weak scale, Phys. Rev. D 80 (2009) 095002 [0811.1030]

V- Dark Photon

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}m_V^2 V_\mu^2 + \kappa V_\nu \partial_\mu F_{\mu\nu} + \mathcal{L}_{h'} + \dots$$
$$= -\frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}m_V^2 V_\mu^2 + \kappa e J_\mu V_\mu + \mathcal{L}_{h'} + \dots,$$

$$a_l^V = \frac{\alpha}{2\pi} \times \kappa^2 \int_0^1 dz \frac{2m_l^2 z(1-z)^2}{m_l^2(1-z)^2 + m_V^2 z} = \frac{\alpha\kappa^2}{2\pi} \times \begin{cases} 1 & \text{for } m_l \gg m_V, \\ 2m_l^2/(3m_V^2) & \text{for } m_l \ll m_V. \end{cases}$$



The dark photon scenario assumes that the known quarks and leptons have no $U(1)_d$ charge

Models with a general Higgs sector have both kinetic mixing of the SM B-field and Z_d and the mass mixing of the SM Z-field and Z_d.

$$\frac{\epsilon}{4} F_{\mu\nu}(A) F^{\mu\nu}(A_d)$$

$$\frac{m_{mix}^2}{2} A_\mu A_d^\mu$$

Mass mixing

Kinetic mixing induces an interaction between the SM fermions and the dark photon

$$10^{-6} < \epsilon^2 < 10^{-4}$$

- the leading contribution to a_μ is proportional to the kinetic mixing parameter ε,
- the region relevant for significant Δa_μ has first been found to be with dark photon masses in the range between 1 MeV … 500 MeV,
- the electron anomalous magnetic moment result reduces the mass range to 20 MeV $\cdots$ 500 MeV.

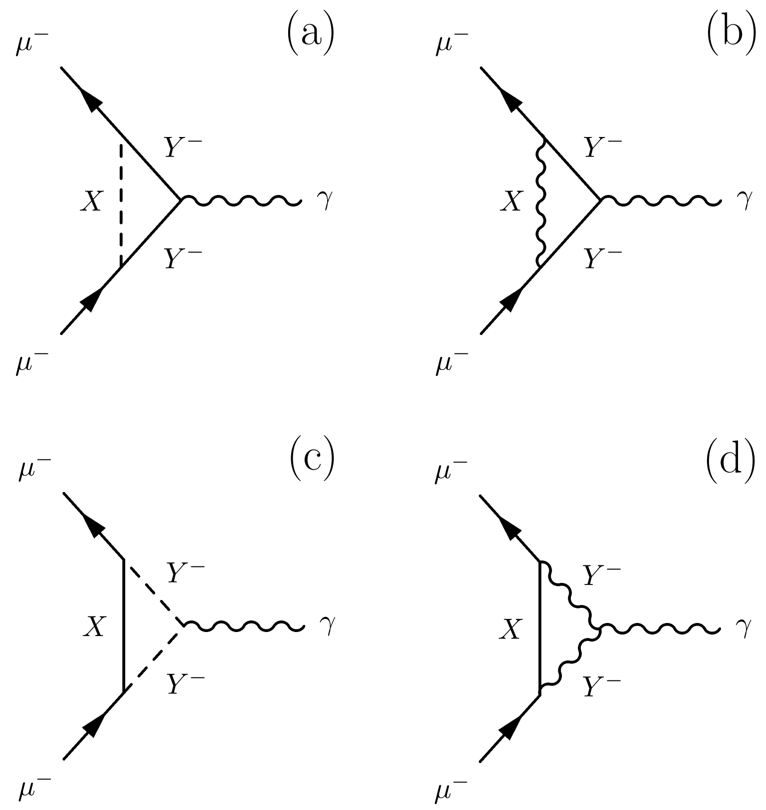
The remaining range is excluded

- by the various experimental results from A1 in Mainz (radiative dark photon production in fixed-target electron scattering with decays into e^+e^- pairs),
- BaBar (pair production in e^+e^- collision with subsequent decay into e^+e^- or $\mu^+\mu^-$ pairs),
- NA48/2 at CERN (π^0 decay modes via dark photon and subsequent decay into e^+e^- -pair),
- from dark matter production via dark photon from NA46 at the CERN.

A pure dark photon models cannot accommodate significant contributions to a_μ.

Contributions to the Muon's Anomalous Magnetic Moment from a Hidden Sector

David McKeen 0912.1076

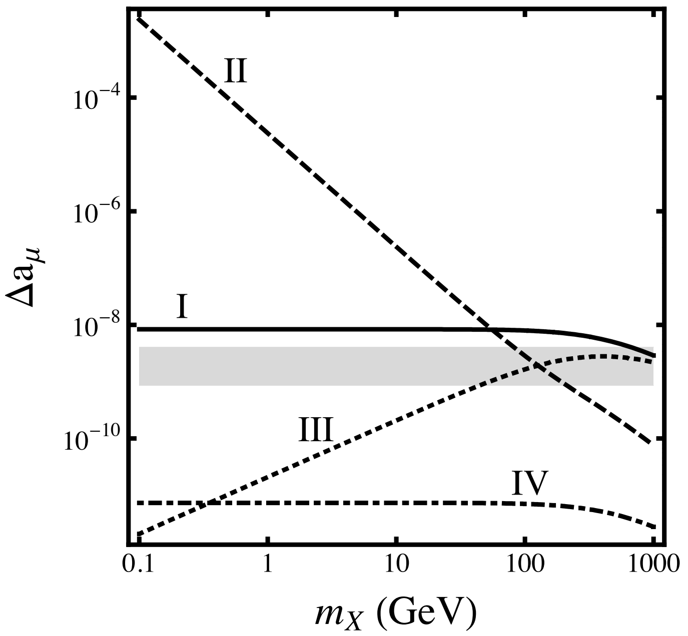


I $\mathcal{L}_{\text{int}} = \lambda_L X \bar{Y}_R \mu_L + \lambda_R X \bar{Y}_L \mu_R + \text{H.c.}$

II $\mathcal{L}_{\text{int}} = \lambda_L X^\mu \bar{Y}_L \gamma_\mu \mu_L + \lambda_R X^\mu \bar{Y}_R \gamma_\mu \mu_R + \text{H.c.}$

III $\mathcal{L}_{\text{int}} = \lambda_L Y_L \bar{X} \mu_L + \lambda_R Y_R \bar{X} \mu_R + \text{H.c.}$

IV $\mathcal{L}_{\text{int}} = \lambda_L Y_L^\nu \bar{X} \gamma_\nu \mu_L + \lambda_R Y_R^\nu \bar{X} \gamma_\nu \mu_R + \text{H.c.}$



Contributions to a_μ as functions of m_X for $\lambda_L = 0.1$, $\lambda_R = 0$, and $m_Y = 400$ GeV in
Cases I (solid), II (dashed), III (dotted) and IV (dot-dashed).
Note that we have plotted $-(\Delta a_\mu)^2$ in Case II (dashed) since it is negative for these choices of $\lambda_{L,R}$.
Full one loop expressions are used,
The light gray band shows values of Δa_μ for which the discrepancy between the theoretical and experimental values of a_μ is reduced to 1σ.

How to build theory with hidden sector?

SM× $U(1)_X$

Particle content

- SM gauge bosons + new B^X_μ;
- Two-Higgs doublet model;
- Three gauge couplings: g, g_Y and g_X;
- Fermions are doublets of $SU(2)_L$; and there are $SU(2)_L$ singlets $\chi_R, l_{iR}, u_{iR}, d_{iR}$
- Y hypercharge

$$Y_L = -\frac{1}{2}; \quad Y_Q = \frac{1}{6}; \quad Y_l = -1; \quad Y_\chi = 0; \quad Y_u = \frac{2}{3}; \quad Y_d = -\frac{1}{3};$$

- X hypercharge

$$X_L = 0; \quad X_Q = 0; \quad X_{e2} = 1; \quad X_{\chi_R} = -1; \quad X_{u2} = -1; \quad X_{d2} = 1;$$

- Higgs doublets Φ^0, Φ^X and singlet s

$$Y_0 = Y_X = \frac{1}{2}; \quad X_0 = 0; \quad X_X = -1; \quad Y_s = 0 \quad X_s = 1$$

Basic properties of the model

- **Minimality**: Introducing a minimal set of new degrees of freedom;
- **Non-Universality**: Selected puzzles as a signal of favored flavors;
- **Standard Model features** :

a) Fermions are accommodated within the same representations as in SM ;

b) Cancellation of anomalies per generation;

- **Low-Energy Phenomenology** (1103.0721):

a) Interactions νe or νN not stronger than G_F ;
b) Absence of fundamental electrically charged particles with mass < 100 GeV;
c) The model must have the possibility of a UV completion at or above the weak scale;
d) The model must be consistent with a variety of tests from QED and particle physics in the MeV energy range.

Anomaly cancellations

- $U(1)_X^3$
- $U(1)_Y U(1)_X^2$
- $U(1)_Y^2 U(1)_X$
- $SU(2)^2 U(1)_X$
- $SU(3)^2 U(1)_X$
- $grav^2 U(1)_X$

SM fermions + single DM fermion (singlet of the SM gauge group),
SM quarks and leptons non-zero $U(1)_X$ charges
the DM fermion must have a vector-like $U(1)_X$ coupling.
e.g. Ellis et al, 1704.03850, 1705.03447
Babu et al., 1705.01822,...

Our approach:

- anomalies are solved per generations as in SM
- dark fermion is stable!

Electroweak Lagrangian

$$
\begin{aligned}
\mathcal{L} =\; & -\frac{1}{4}\mathbf{W}^{\mu\nu}\cdot\mathbf{W}_{\mu\nu} - \frac{1}{4}B^{Y\mu\nu}B^{Y}_{\mu\nu} - \frac{1}{4}B^{X\mu\nu}B^{X}_{\mu\nu} + \frac{\epsilon}{2}B^{Y\mu\nu}B^{X}_{\mu\nu} + \\
& +(D_\mu\phi^0)^\dagger(D^\mu\phi^0) + (D_\mu\phi^X)^\dagger(D^\mu\phi^X) + (D_\mu s)^\dagger(D^\mu s) - V(\phi^0,\phi^X,s) \\
& - \sum_{\alpha=1,2,3}\left(\sum_{\beta=1,3}\overline{L}_{\alpha L}\phi^0 Y^{l}_{\alpha\beta}e_{\beta R} + \overline{L}_{\alpha L}\phi^X Y^{l}_{\alpha 2}e_{2R} + h.c.\right) \\
& - \sum_{\alpha=1,2,3}\sum_{\beta=1,3}\left(\overline{Q}_{\alpha L}\phi^0 Y^{D}_{\alpha\beta}d_{\beta R} + \overline{Q}_{\alpha L}\tilde{\phi}^0 Y^{U}_{\alpha\beta}u_{\beta R} + h.c.\right) \\
& - \sum_{\alpha=1,2,3}\left(\overline{Q}_{\alpha L}\phi^X Y^{D}_{\alpha 2}d_{2R} + \overline{Q}_{\alpha L}\tilde{\phi}^X Y^{U}_{\alpha 2}u_{2R} + h.c.\right) \\
& -Y_s\,\overline{\chi_L}\chi_R s - Y_s^*\,\overline{\chi_R}\chi_L s^* + \\
& +i\sum_{\alpha=1,2,3}\Big[\overline{L}_{\alpha L}\not{D}L_{\alpha L} + \overline{Q}_{\alpha L}\not{D}Q_{\alpha L} + \\
& +\bar{l}_{\alpha R}\not{D}l_{\alpha R} + \overline{d}_{\alpha R}\not{D}d_{\alpha R} + \overline{u}_{\alpha R}\not{D}u_{\alpha R}\Big] + i\overline{\chi}_R\not{D}\chi_R\,.
\end{aligned}
$$

Gauge bosons mixing

$$\mathcal{L}_{\text{g.b.}} = -\frac{1}{4}\mathbf{W}^{\mu\nu}\cdot\mathbf{W}_{\mu\nu} - \frac{1}{4}B^{Y\mu\nu}B^{Y}_{\mu\nu} - \frac{1}{4}B^{X\mu\nu}B^{X}_{\mu\nu} + \frac{\epsilon}{2}B^{Y\mu\nu}B^{X}_{\mu\nu}$$

Field redefinition $B^Y_\mu \to B^Y_\mu + \epsilon B^X_\mu$

$$\hat{\mathcal{O}}^{\mu\nu} = \partial^\mu\partial^\nu - \partial^2 g^{\mu\nu}$$

$$\mathcal{L}_{k.m.} \supset -\frac{1}{2}(B^Y_\mu + \epsilon B^X_\mu)\hat{\mathcal{O}}^{\mu\nu}(B^Y_\nu + \epsilon B^X_\nu) - \frac{1}{2}B^X_\mu\hat{\mathcal{O}}^{\mu\nu}B^X_\nu + \epsilon\, B^X_\mu\hat{\mathcal{O}}^{\mu\nu}(B^Y_\nu + \epsilon B^X_\nu)$$

$$\mathcal{L}_{k.m.} \supset -\frac{1}{2}B^Y_\mu\hat{\mathcal{O}}^{\mu\nu}B^Y_\nu - \frac{1}{2}B^X_\mu\hat{\mathcal{O}}^{\mu\nu}B^X_\nu + \mathcal{O}(\epsilon^2)$$

the crossed terms vanishes and the mixing effect is converted into the covariant derivative

$$D_\mu \to D_\mu = \partial_\mu - ig\mathbf{W}_\mu\cdot\tau - ig_Y B^Y_\mu Y^p - i(\kappa Y^p + g_X X^p)B^X_\mu$$

$$\boxed{\epsilon g_Y \equiv \kappa}$$

Masses of gauge boson

$$D_\mu \phi^p = \left[\partial_\mu - i\frac{g}{\sqrt{2}}(W^+\mathbb{I}_+ + W^-\mathbb{I}_-) - ig\tau_3 W^3_\mu - ig_Y Y^p B^Y_\mu - i(\kappa Y^p + g_X X^p)B^X_\mu\right]\phi^p\,,$$

$$D_\mu s = (\partial_\mu - ig_X X^s B^X_\mu)s\,.$$

Hypercharge assignment

$$Y_0 = Y_X = \frac{1}{2};\quad X_0 = 0;\quad X_X = -1;\quad Y_s = 0\quad X_s = 1$$

Scalars

$$\phi_0 = \begin{pmatrix} \varphi_0^+ \\ \frac{v_0 + H_0 + i\chi_0}{\sqrt{2}} \end{pmatrix},\qquad \phi_X = \begin{pmatrix} \varphi_X^+ \\ \frac{v_X + H_X + i\chi_X}{\sqrt{2}} \end{pmatrix},\qquad s = \frac{v_s + H_s + i\chi_s}{\sqrt{2}}$$

SM

$$\mathbb{M}^0 = \frac{v^2}{8}\begin{pmatrix} g^2 & -gg_Y & g(2g_X c_\beta^2 - \kappa) \\ -gg_Y & g_Y^2 & -g_Y(2g_X c_\beta^2 - \kappa) \\ g(2g_X c_\beta^2 - \kappa) & -g_Y(2g_X c_\beta^2 - \kappa) & 4[g_X^2\frac{\bar{v}^2}{v^2} - g_X\kappa c_\beta^2] + \kappa^2 \end{pmatrix}$$

$$v^2 \equiv (v_0^2 + v_X^2),\qquad \bar{v}^2 \equiv (v_s^2 + v_X^2),\qquad c_\beta^2 = \frac{v_X^2}{v^2}$$

Parameter space:

Based on F. Correia and SF , 1609.00860
and F. Correia and SF, 1905.03867, 1905.03872S

Reproducing SM W and Z masses,
parameter space reduced to

SM $P := [g, g_Y, v]$

SM⊗U(1)

$P := [\kappa, g, g_Y, g_X, v_X, v_0, v_s, \mathbb{F}]$

Constraints

- ρ parameter
- $(g\text{-}2)_\mu$, $(g\text{-}2)_e$
- K leptonic decays
- parity non-conserving processes

(Proton puzzle in the $U(1)_X$?)

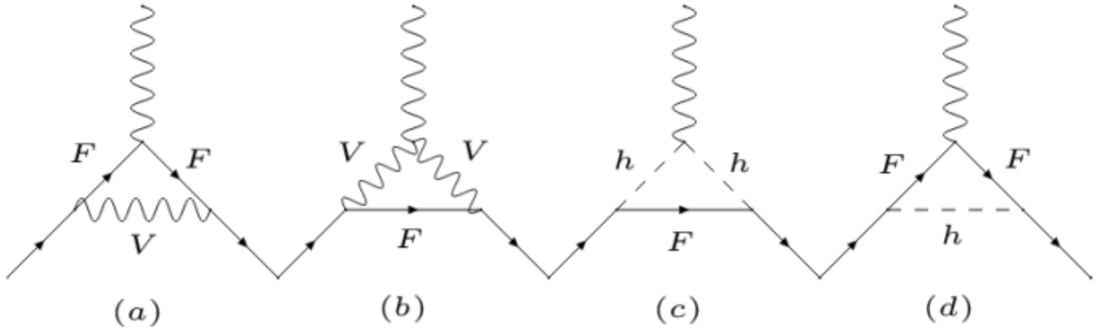
$(g\text{-}2)_\mu$

$$\mathcal{L} = \frac{1}{2}\sum_F \bar{\mu}[x_V\gamma^\rho + x_A\gamma^\rho\gamma^5]F\ X_\rho$$

Contribution of axial coupling sign analysis leads to $[a_\mu]_a < 0$

To explain observed difference

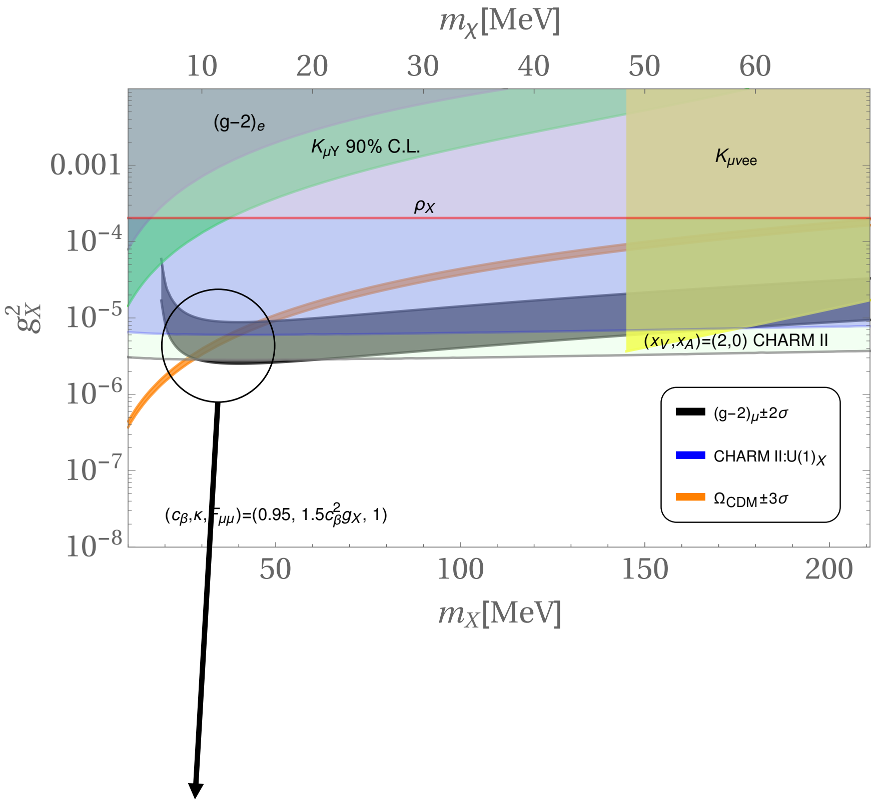
$$[a_\mu]_a > 0$$

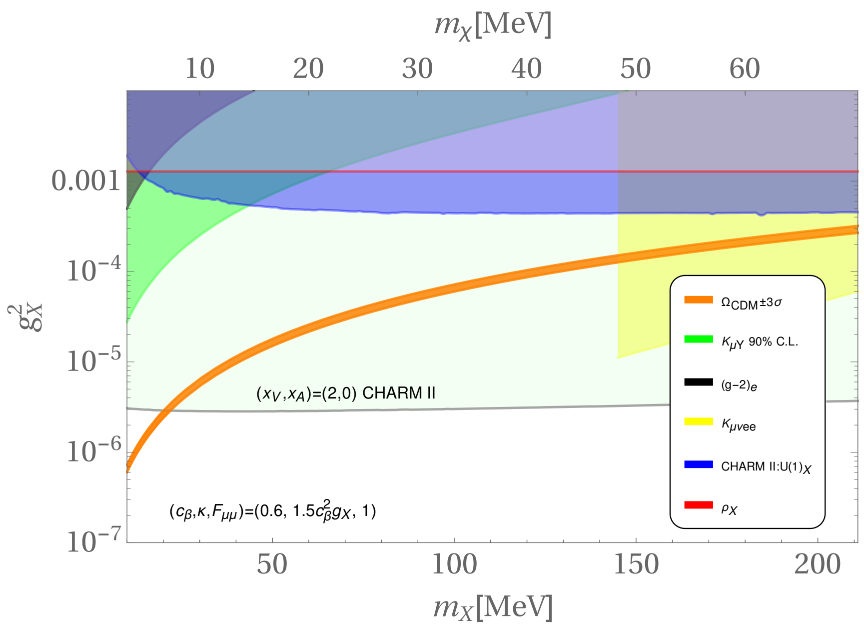


$$I_V(m_X^2) = \int_0^1 dz \frac{z^2(1-z)}{[m_l^2 z^2 + m_X^2(1-z)]} \overset{m_X \gg m_l}{\longrightarrow} \frac{1}{3m_X^2},$$

$$I_A(m_X^2) = \int_0^1 dz \frac{z(1-z)(z-4) - \left(2\frac{m_l^2}{m_X^2}\right) z^3}{[m_l^2 z^2 + m_X^2(1-z)]} \overset{m_X \gg m_l}{\longrightarrow} -\frac{5}{3m_X^2}$$

V≡ X

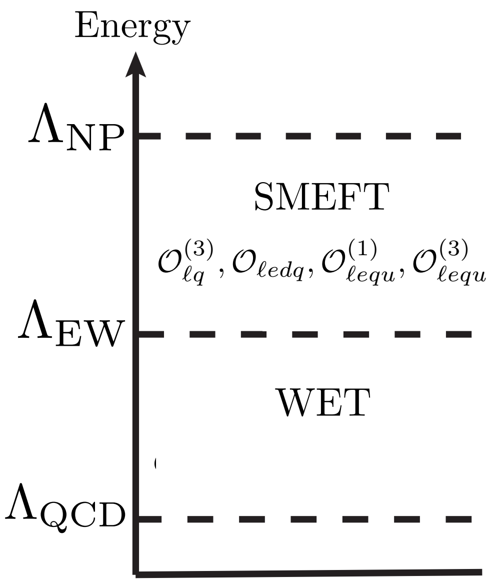




In this region only muon anomalous magnetic moments and relic abundance can be resolved

Standard Model Effective Field Theory (SMEFT)

LHC: no observation of particles beyond the Standard Model (SM)



Indication

New Physics (NP) is either very weakly coupled

or there are new heavy particles with masses well above the electroweak scale

The effects of NP on experiments at energies below the threshold of the new heavy particles can be described by an effective field theory (EFT) that contains the SM particles only -SMEFT

SMEFT enables parametrization of some high scale physics beyond the Standard Model using SM fields.

"Warsaw" operator basis

$$X^3 : f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu} \qquad H^6 : (H^\dagger H)^3$$

$$H^4 D^2 : \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right) \qquad \psi^2 H^3 : (H^\dagger H)(\bar{l}_p e_r H)$$

$$X^2 H^2 : H^\dagger H \, G_{\mu\nu}^A G^{A\mu\nu} \qquad \psi^2 X H : (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$$

$$\psi^2 H^2 D : (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r) \qquad \psi^4 : (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$

B. Grzadkowski, M. Iskrzyński,
M. Misiak and J. Rosiek , 1008.4884

$$\mathcal{L} = \mathcal{L}_{D\leq 4} + c_i \mathcal{O}_i + \ldots \qquad [c_i] = \frac{1}{\text{mass}^2} \qquad c_i \to \frac{c_i}{\Lambda^2}$$

Λ is determined from experiment after you see deviations from the SM

- 53 *CP*-even and 23 *CP*-odd operators for one generation (total 76)
- 1350 *CP*-even and 1149 *CP*-odd operators for three generations (total 2499)
- 59 operators in the original table — some are real and some complex.

SMEFT gives a model independent way of testing for deviations from the SM.

Physics below EW scale

SM → LEFT : expansion in p/M_W
SMEFT → LEFT : expansion in p/Λ , v/Λ , p/M_W
HEFT → LEFT : expansion in p/Λ , p/M_W

SMEFT puts constraints on LEFT parameters from $SU(2) \times U(1)$ invariance.

With EFT $\longrightarrow$ scales are separated $\quad \boldsymbol{\Lambda} \longrightarrow v \longrightarrow m_{q,l}$

It simplifies loop calculations through renormalization-group equations (RGEs).
Systematic re-summation of large logarithms in a leading-log expansions known as RG-improved perturbation theory

The one-loop running in the SMEFT for the operators up to dimension six was calculated in

R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, JHEP 04, 159 (2014), [arXiv:1312.2014 [hep-ph]].
J. Aebischer, M. Fael, C. Greub, and J. Virto, JHEP 09, 158 (2017), [arXiv:1704.06639 [hep-ph]].
E. E. Jenkins, A. V. Manohar, and P. Stoffer, JHEP 03, 016 (2018), [arXiv:1709.04486 [hep-ph]].
E. E. Jenkins, A. V. Manohar, and P. Stoffer, JHEP 01, 084 (2018), [arXiv:1711.05270 [hep-ph]]

Dipole operators

Dipole operators in LEFT are dimension five
(Note that no dimension five $\Delta B = \Delta L = 0$ operators in SMEFT)

$$\mathcal{O}_{e\gamma} = \bar{e}_{Lp}\sigma^{\mu\nu} e_{Rr}\, F_{\mu\nu}$$

$$\bar{e}_L\sigma^{\mu\nu} e_R\, F_{\mu\nu}, \quad \bar{\mu}_L\sigma^{\mu\nu} \mu_R\, F_{\mu\nu}, \quad \bar{e}_L\sigma^{\mu\nu} \mu_R\, F_{\mu\nu}, \quad \bar{s}_L\sigma^{\mu\nu} b_R\, F_{\mu\nu}$$

Experimental studies

Electron $g - 2$
Muon $g - 2$
Electron EDM
$\mu \to e\gamma$
NEDM (from quark operators) $b \to s\gamma$

For $SU(2)_L \otimes U(1)_Y$ invariance Higgs field must enter effective operators

$$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}, \quad (\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu} \qquad \to \qquad v\, \bar{e}_L \sigma^{\mu\nu} e_R\, F_{\mu\nu}$$

$$\frac{1}{\Lambda} \to \frac{v}{\Lambda^2}$$

Dimension 6 operators

This means that SMEFT dipole effects are v/Λ suppressed in LEFT

Remember: The effect on low-energy observables depends on the initial conditions of the high-energy Wilson coefficients at the scale Λ.

In muon anomalous moment and Higgs decay to $\mu^+\mu^-$

$$\mathcal{O}_{1,pr} = \left(\varphi^\dagger\varphi\right)\left(\bar{\ell}_p e_r \varphi\right) \,,$$
$$\mathcal{O}_{2,pr} = \left(\bar{\ell}_p \sigma^{\mu\nu} e_r\right) \tau^a \varphi W^a_{\mu\nu} \,,$$
$$\mathcal{O}_{3,pr} = \left(\bar{\ell}_p \sigma^{\mu\nu} e_r\right) \varphi B_{\mu\nu} \,.$$

$$\mathcal{O}_{4,prst} = \left(\bar{\ell}^j_p e_r\right) \epsilon_{jk} \left(\bar{q}^k_s u_t\right) \,,$$
$$\mathcal{O}_{5,prst} = \left(\bar{\ell}^j_p \sigma_{\mu\nu} e_r\right) \epsilon_{jk} \left(\bar{q}^k_s \sigma^{\mu\nu} u_t\right) \,,$$

SF, J.F. Kamenik and M. Tammaro, 2103.10859

We consider only top quark contributions due to $m_q/m_t \longrightarrow 0$, q=u,d,s,c,b

$s = t$; $p = r = e, \mu, \tau$

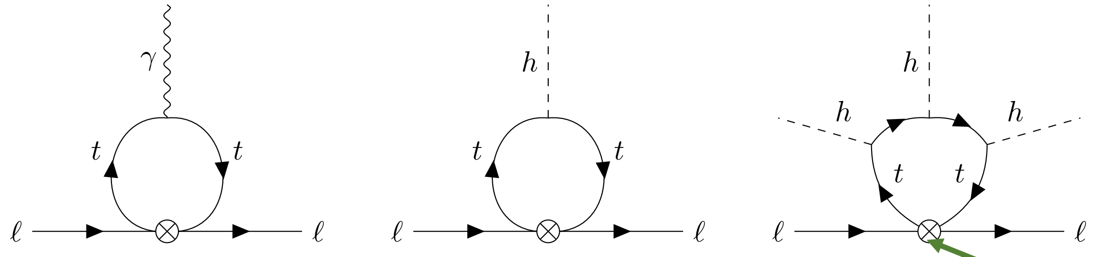
- the gauge couplings, the Higgs mass and the top quark Yukawa evolve according to the SM equations
- the Higgs quartic coupling λ and the muon Yukawa y_μ receive one-loop contributions from the operators O_1 and O_4
- The RG evolution of these operators can hint at the low-energy signatures. As an illustration of the interplay between them, we set the initial conditions at the high scale $\Lambda = 10^8$ GeV and solve the RG evolution down to the weak scale v.

$$\dot{\mathcal{C}}_{1,\ell} = \left[-\frac{27}{4}g_2^2 - 3\left(3Y_\ell^2 + 3Y_e^2 - 4Y_\ell Y_e\right)g_1^2 + 3N_c y_t^2 + 24\lambda\right]\mathcal{C}_{1,\ell} + 4N_c y_t\left(y_t^2 - \lambda\right)\mathcal{C}_{4,\ell}$$
$$- 3\left(4g_1^2 g_2 Y_h\left(Y_e + Y_\ell\right) + 3g_2^3\right)\mathcal{C}_{2,\ell} - 6\left(4g_1^3 Y_h^2\left(Y_e + Y_\ell\right) + g_2^2 g_1 Y_h\right)\mathcal{C}_{3,\ell}\,,$$
$$\dot{\mathcal{C}}_{2,\ell} = \left[\left(3c_{F,2} - b_{0,2}\right)g_2^2 + \left(-3Y_e^2 + 8Y_e Y_\ell - 3Y_\ell^2\right)g_1^2 + N_c y_t^2\right]\mathcal{C}_{2,\ell}$$
$$+ g_1 g_2\left(3Y_\ell - Y_e\right)\mathcal{C}_{3,\ell} - 2g_2 N_c y_t \mathcal{C}_{5,\ell}\,,$$
$$\dot{\mathcal{C}}_{3,\ell} = \left[-3c_{F,2}g_2^2 + \left(3Y_e^2 + 4Y_e Y_\ell + 3Y_\ell^2 - b_{0,1}\right)g_1^2 + N_c y_t^2\right]\mathcal{C}_{3,\ell}$$
$$+ 4c_{F,2}g_1 g_2\left(3Y_\ell - Y_e\right)\mathcal{C}_{2,\ell} + 4g_1 N_c y_t\left(Y_u + Y_q\right)\mathcal{C}_{5,\ell}\,,$$
$$\dot{\mathcal{C}}_{4,\ell} = -\left[6\left(Y_e^2 + Y_e\left(Y_u - Y_q\right) + Y_q Y_u\right)g_1^2 + 3\left(N_c - \frac{1}{N_c}\right)g_3^2 + y_t^2\left(2N_c + 1\right)\right]\mathcal{C}_{4,\ell}$$
$$- \left[24\left(Y_q + Y_u\right)\left(2Y_e - Y_q + Y_u\right)g_1^2 - 18g_2^2\right]\mathcal{C}_{5,\ell}\,,$$
$$\dot{\mathcal{C}}_{5,\ell} = g_1\left(Y_q + Y_u\right)y_t\mathcal{C}_{3,\ell} - \frac{3}{2}g_2 y_t \mathcal{C}_{2,\ell} + \left[2\left(Y_e^2 - Y_e Y_q + Y_e Y_u - 2Y_q^2 + 5Y_q Y_u - 2Y_u^2\right)g_1^2\right.$$
$$\left. - 3g_2^2 + \left(N_c - \frac{1}{N_c}\right)g_3^2 + y_t^2\right]\mathcal{C}_{5,\ell} + \frac{1}{8}\left[-4\left(Y_q + Y_u\right)\left(2Y_e - Y_q + Y_u\right)g_1^2 + 3g_2^2\right]\mathcal{C}_{4,\ell}\,.$$

RGE running from
E. E. Jenkins, A. V. Manohar, and M. Trott,
JHEP 04, 159 (2014).

$$\dot{y}_\ell = y_\ell\left(\frac{3}{4}y_\ell^2 + \frac{3}{2}\left(y_t^2 + y_\ell^2\right) - \frac{9}{8}\left(g_1^2 + g_2^2\right)\right) + m_h^2\left(3\hat{\mathcal{C}}_{1,\ell} - N_c y_t \hat{\mathcal{C}}_{4,\ell}\right)\,,$$
$$\dot{\lambda} = 2\left(12\lambda^2 + 2N_c y_t^2\lambda - N_c y_t^4 + m_h^2 y_\ell \hat{\mathcal{C}}_{1,\ell}\right)\,,$$
$$\dot{m}_h = m_h\left(6\lambda + N_c y_t^2 - \frac{9}{4}g_2^2 - \frac{3}{4}g_1^2\right)\,.$$

One-Loop Matching of O$_{4,l}$ to Q$_{1,l}$



$$\mathcal{M} = \left(-i\frac{y_t}{\sqrt{2}}\right)(iN_c\hat{\mathcal{C}}_{4,\ell})\bar{v}_2\int\frac{d^dl}{(2\pi)^d}\mu^{2\varepsilon}\left[\mathrm{Tr}\left(\frac{i(\not{l}+m_t)i(\not{l}'+m_t)P_R}{(l^2-m_t^2)(l'^2-m_t^2)}\right)\right]u_1$$

$$= \sqrt{2}y_tN_c\hat{\mathcal{C}}_{4,\ell}\bar{v}_2\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\mu^{2\varepsilon}\left[\frac{k^2+m_t^2}{(k^2-\Delta)^2}\right]u_1\,.$$

$$\mathcal{M} = \sqrt{2}y_tN_c\hat{\mathcal{C}}_{4,\ell}\frac{im_t^2}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\left(\frac{1}{m_t^2}\right)^{2-d/2}\left[1-\frac{d/2}{1-d/2}\right]\mathcal{Q}_{1,\ell}$$

$$\mathcal{M} = \sqrt{2}y_tN_c\hat{\mathcal{C}}_{4,\ell}\frac{im_t^2}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\left(\frac{1}{m_t^2}\right)^{2-d/2}\left[1-\frac{d/2}{1-d/2}\right]\mathcal{Q}_{1,\ell}$$

The Feynamn rule for the effective vertex is

$$FR_4 = i\hat{\mathcal{C}}_{4,\ell}\delta_{c_3,c_4}P_{1,2}P_{3,4}$$

$$P_{i,j} = P_R = (1+\gamma_5)/2$$

$$\boxed{\delta c_{1,\ell}(\mu_w)|_{1-\mathrm{loop}} = y_tN_c\hat{\mathcal{C}}_{4,\ell}\frac{3m_t^2}{8\sqrt{2}\pi^2}\left[\frac{1}{3}+\ln\left(\frac{m_t^2}{\mu_w^2}\right)\right]}$$

One-Loop Matching of $O_{5,l}$ to $Q_{2,l}$

$$\mathcal{M} = iN_c Q_t e \hat{\mathcal{C}}_{5,\ell} \bar{u}_2 \int \frac{d^d l}{(2\pi)^d} \mu^{2\varepsilon} \frac{i}{2} \gamma_\mu \gamma_\nu \left[\mathrm{Tr}\left(\frac{i(\not{l} + m_t)\not{\epsilon}(q) i(\not{l}' + m_t)\gamma^\nu \gamma^\mu P_R}{(l^2 - m_t^2)(l'^2 - m_t^2)} \right) - \mathrm{Tr}(\mu \leftrightarrow \nu) \right] u_1$$

$$= -N_c Q_t e \hat{\mathcal{C}}_{5,\ell} \bar{u}_2 \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \mu^{2\varepsilon} \left[\frac{4 m_t \not{q} \not{\epsilon}(q)}{(k^2 - \Delta)^2} \right] u_1 \,,$$

$$\mathcal{M} = \frac{-4iN_c m_t Q_t e \hat{\mathcal{C}}_{5,\ell}}{(4\pi)^2} \bar{u}_2 \not{q} \not{\epsilon} u_1 \left(\frac{1}{\hat{\varepsilon}} - \ln\left(\frac{m_t^2}{\mu^2} \right) \right)$$

$$\boxed{\delta c_{2,\ell}(\mu_w)|_{1-\mathrm{loop}} = \frac{N_c Q_t \hat{\mathcal{C}}_{5,\ell}(\mu_w) m_t m_\ell}{4\pi^2} \ln\left(\frac{m_t^2}{\mu_w^2} \right)}$$

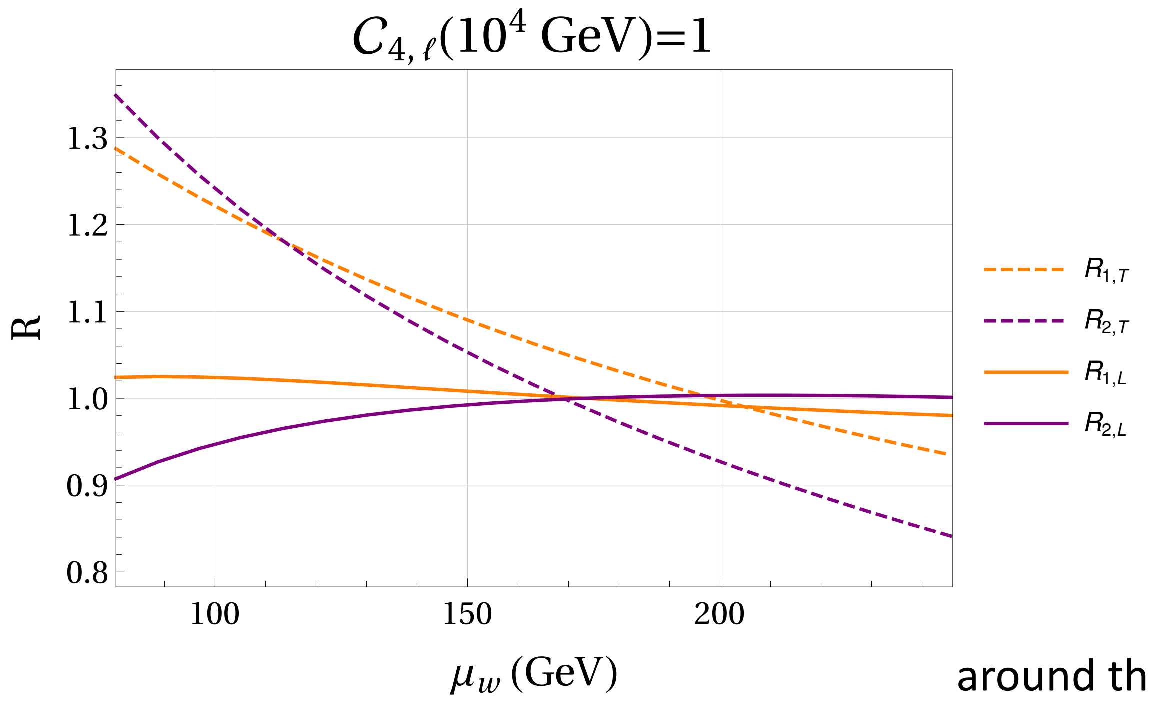
trick

$$\not{q}\not{\epsilon} = \gamma_\mu \gamma_\nu q^\mu \epsilon^\nu = \frac{2i\sigma_{\mu\nu} + 2\eta_{\mu\nu}}{2} \frac{q^\mu \epsilon^\nu - q^\nu \epsilon^\mu}{2} = -\frac{1}{2} \sigma_{\mu\nu} F^{\mu\nu} \to -\frac{m_\ell}{2e} \mathcal{Q}_{2,\ell}$$

Note that $C_{1,l}$ does not appear in the RG equations for $C_{2,3,4,5,l}$
The UV models that only generate $O_{1,l}$ can be probed exclusively via Higgs decays
On the other hand, $C_{2,l}$ and $C_{3,l}$ can induce a sizeble $C_{1,l}$ through their RG evolution .

To parametrize the relative importance of matching scale and loop corrections, we define the ratio

$$R_{i,j}(\mu_w) = \frac{c_{i,\ell}(\mu_w)|_j}{c_{i,\ell}(m_t)|_L}\,, \quad i = 1, 2\,,$$



around the top mass

The tree-level matched result has a sizable μ_w dependence: starting with a fixed value of C4,\`
at the high scale of 10^4 GeV.
Including the one-loop correction, the scale dependence is greatly reduced to around 5% .

Higgs decay to two leptons

$$\mathcal{L} \supset y_\ell^{SM} \bar{\ell}_\ell \varphi e_\ell + \text{h.c.} = y_\ell^{SM} \left(\frac{h+v}{\sqrt{2}} \right) \bar{l}_\ell l_\ell$$

- $\mathcal{Q}_{1,\ell}$ contributes

$$\mathcal{L} \supset \left(\frac{m_\ell}{v} + \delta c_{1,\ell} \right) h \bar{l}_\ell l_\ell \qquad \delta c_{1,\ell}(\mu_w)|_{tree} = \hat{\mathcal{C}}_{1,\ell}(\mu_w) \frac{v^2}{\sqrt{2}}$$

$$\delta c_{1,\ell}(\mu_w)|_{loop} = \frac{N_c m_t^3}{8\pi^2 v} \hat{\mathcal{C}}_{4,\ell}(\mu_w) \left[\frac{1}{3} + \ln\left(\frac{m_t}{\mu_w} \right) \right]$$

$$\kappa_\ell^2 = \frac{\Gamma(h \to \ell^+ \ell^-)}{\Gamma_{SM}(h \to \ell^+ \ell^-)}$$

$$\kappa_\ell = \left(1 + \frac{\delta c_{1,\ell} v}{m_\ell} \right)$$

where we evaluate $\delta c_{1,l}$ at the Higgs mass ($\mu_w = m_h$)

Anomalous magnetic moment

$$\delta c_{2,\ell}(\mu_w)|_{tree} = \frac{v\, m_\ell}{\sqrt{2}e}\left(c_w \hat{C}_{3,\ell}(\mu_w) - s_w \hat{C}_{2,\ell}(\mu_w)\right)$$

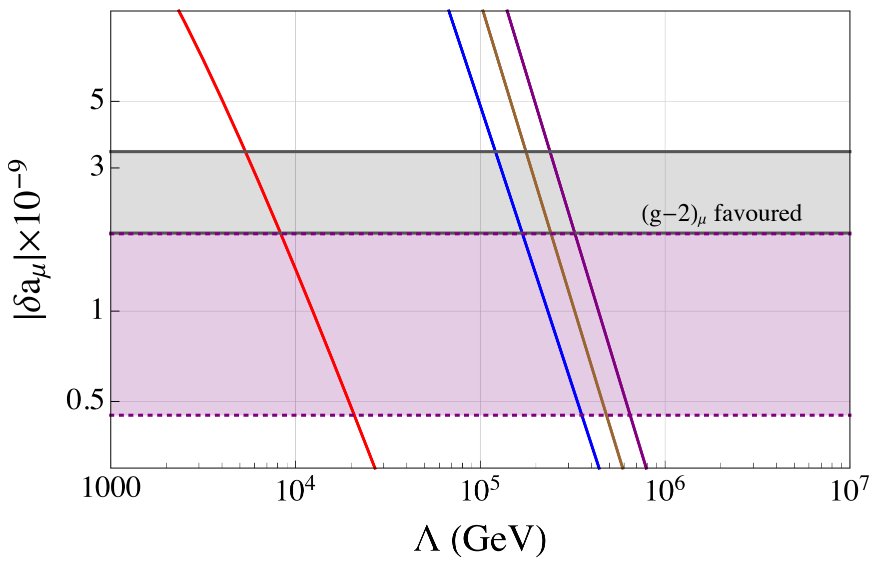
$$\delta c_{2,\ell}(\mu_w)|_{1-loop} = -\frac{N_c Q_t \hat{C}_{5,\ell}(\mu_w) m_t m_\ell}{2\pi^2} \ln\left(\frac{\mu_w}{m_t}\right) + \mathcal{O}(\alpha)$$

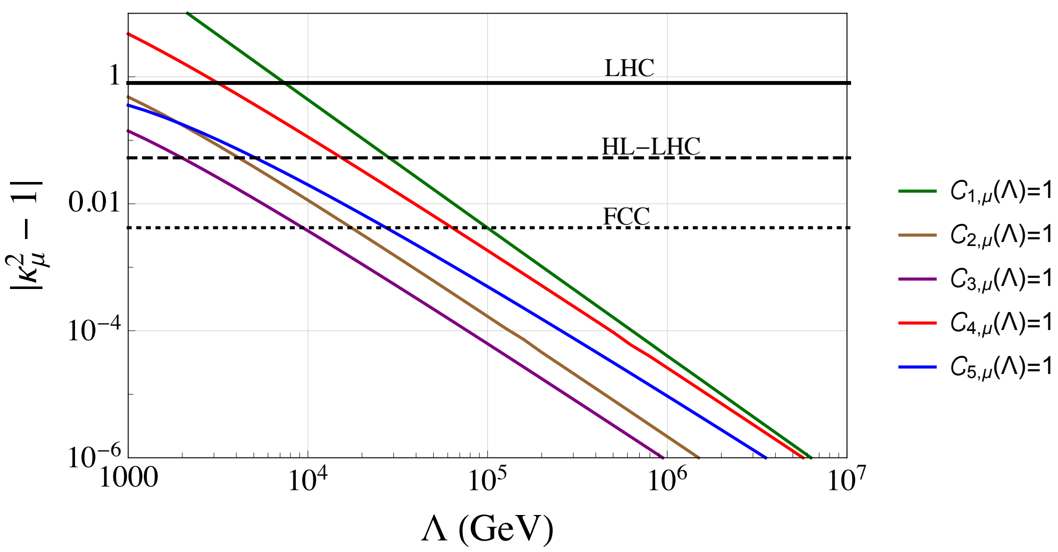
This term stands for all the one-loop contributions from $O_{2,\ell}$ and $O_{3,\ell}$, but are numerically sub-dominant (they include both finite threshold effects and logarithmic scale dependent terms)

The subsequent running from the weak scale to the lepton mass scale is driven by QED interactions only. They induce a shift in c_{2l} of at most a few percent, and so in light of the residual matching scale variance can be safely neglected. In our phenomenological analysis

$$\delta c_{2,\ell}(m_t) \simeq \delta c_{2,\ell}(m_\ell)$$

$$\boxed{\delta a_\ell = 4\delta c_{2,\ell}(m_\ell)}$$





The horizontal gray and purple bands show the (g − 2)μ favored region assuming δa_μ as in i) or in ii) respectively.

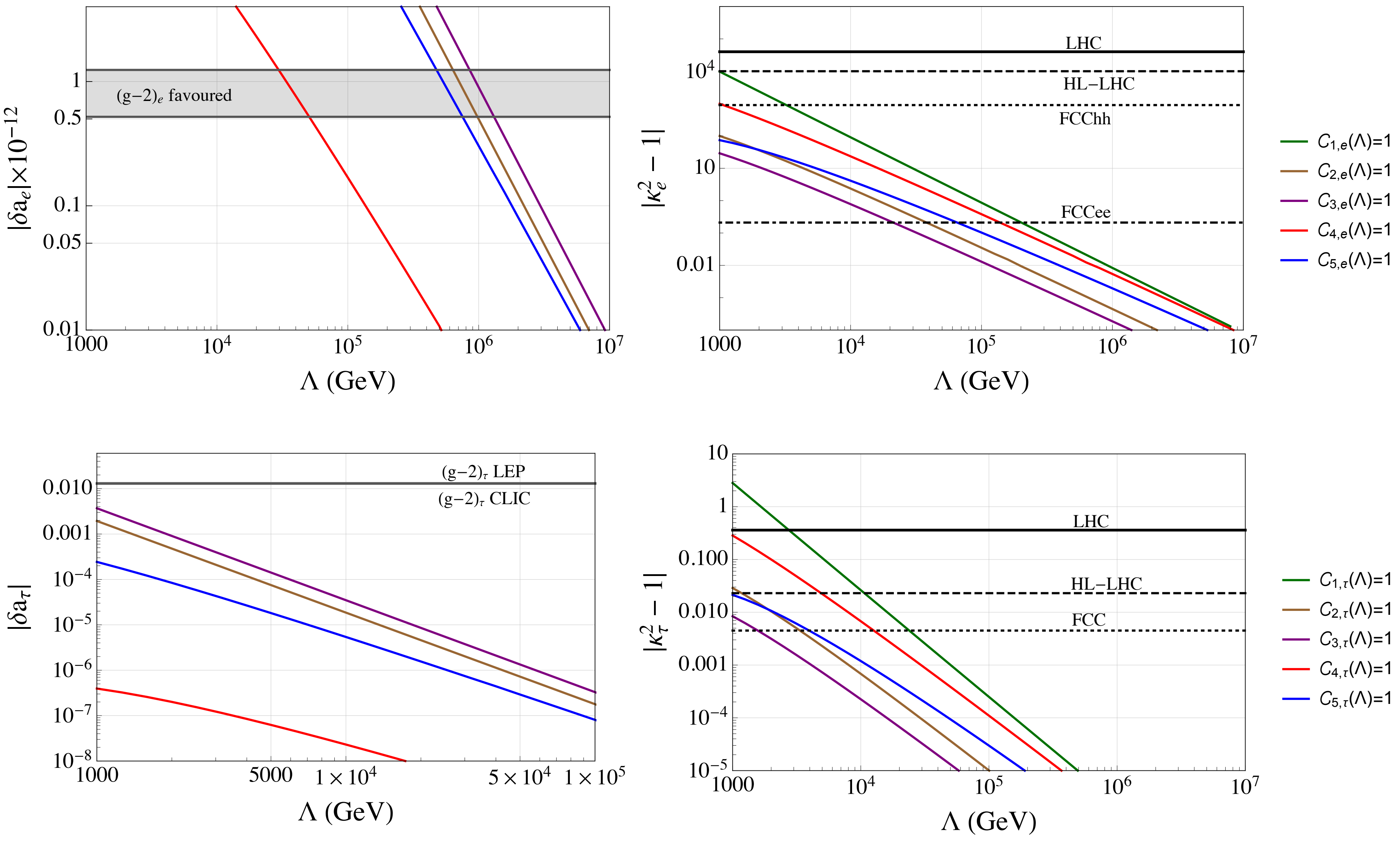
i) $$\delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (261 \pm 79) \times 10^{-11} \quad \sim 3.3\sigma$$

ii) $$\delta a_\mu = (113 \pm 68) \times 10^{-11}$$

Note that the recent BMW Lattice result (S. Borsanyi et al., (2020), 2002.12347) updates the value of the leading order hadron vacuum polarization (LO-HVP) contribution.

Induced shift in Higgs to two muons decay from $\delta c_{1,\mu}$ as function of the NP scale Λ. The horizontal solid line shows present bounds from measurements at LHC, while dashed and dotted lines show projections from HL-LHC and FCC respectively.

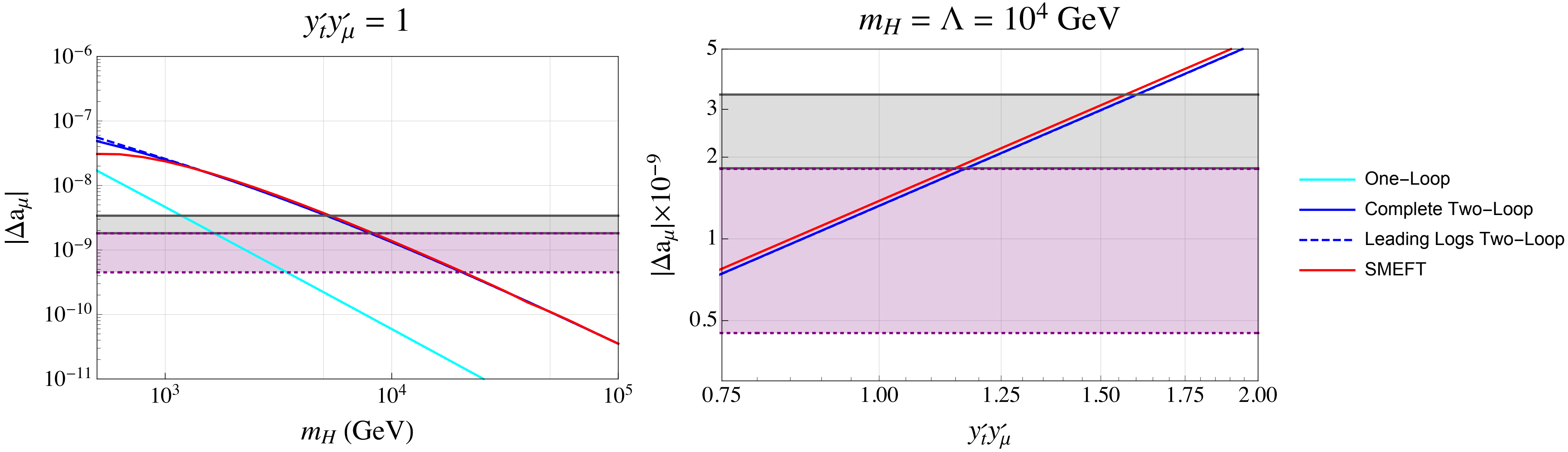
Comparison: electron and tau



electron

tau

2HDM and SMEFT



Obviously SMEFT works perfectly! No need to calculate full two-loop (Bar Zee diagrams)

Scalar Leptoquarks and SMEFT

$$\mathcal{G}_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{L}_{S_1} \supset y^{LL}_{1,ij}\bar{q}^{C,ia}S_1\epsilon^{ab}\ell^{j,b} + y^{RR}_{1,ij}\bar{u}^{C,i}S_1\epsilon^{ab}e^j + \text{h.c.}\,,$$

$$\mathcal{L}_{R_2} \supset -y^{RL}_{2,ij}\bar{u}^i R^a_2\epsilon^{ab}\ell^{j,b} + y^{LR}_{2,ij}\bar{e}^i R^a_2{}^{\,*} q^{j,a} + \text{h.c.}\,,$$

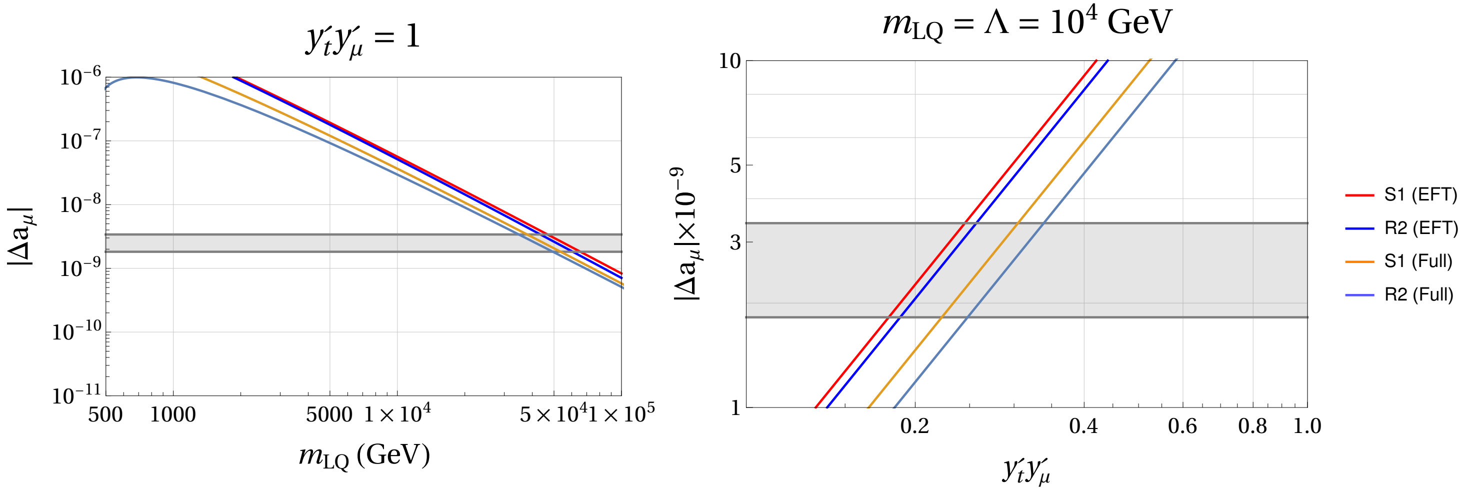
$$\mathcal{L} \supset -\frac{4G_F}{\sqrt{2}}\left[g^{LL}_{ij,ks}\left(\bar{q}^i_L q^j_R\right)\left(\bar{\ell}^k_L\ell^s_R\right) + h^{LL}_{ij,ks}\left(\bar{q}^i_L\sigma_{\mu\nu}q^j_R\right)\left(\bar{\ell}^k_L\sigma^{\mu\nu}\ell^s_R\right)\right]$$

$$S_1 : g^{LL}_{ij,ks} = -4h^{LL}_{ij,ks} = \frac{v^2}{4m^2_{LQ}}y^{RR}_{1,js}\left(y^{LL}_1\right)^*_{ik}$$

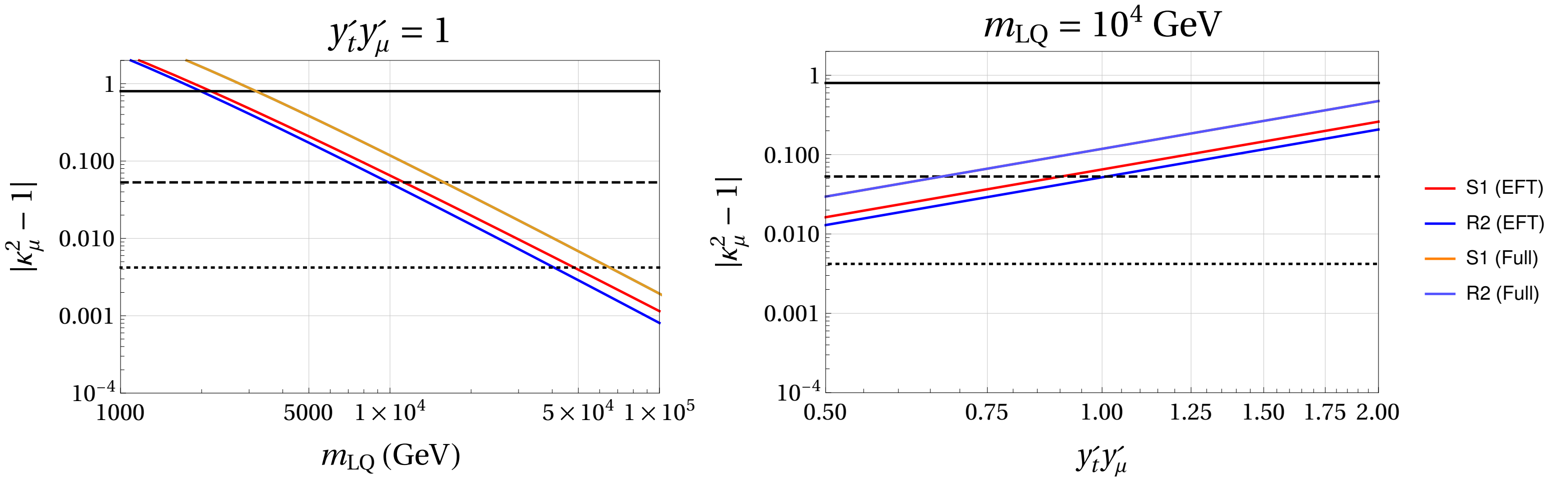
$$R_2 : g^{LL}_{ij,ks} = 4h^{LL}_{ij,ks} = -\frac{v^2}{4m^2_{LQ}}y^{RL}_{2,jk}\left(y^{LR}_2\right)^*_{si}$$

$$Y \equiv y^{RR}_{1,32}\left(y^{LL}_1\right)^*_{32} = y^{RL}_{2,32}\left(y^{LR}_2\right)^*_{32}.$$

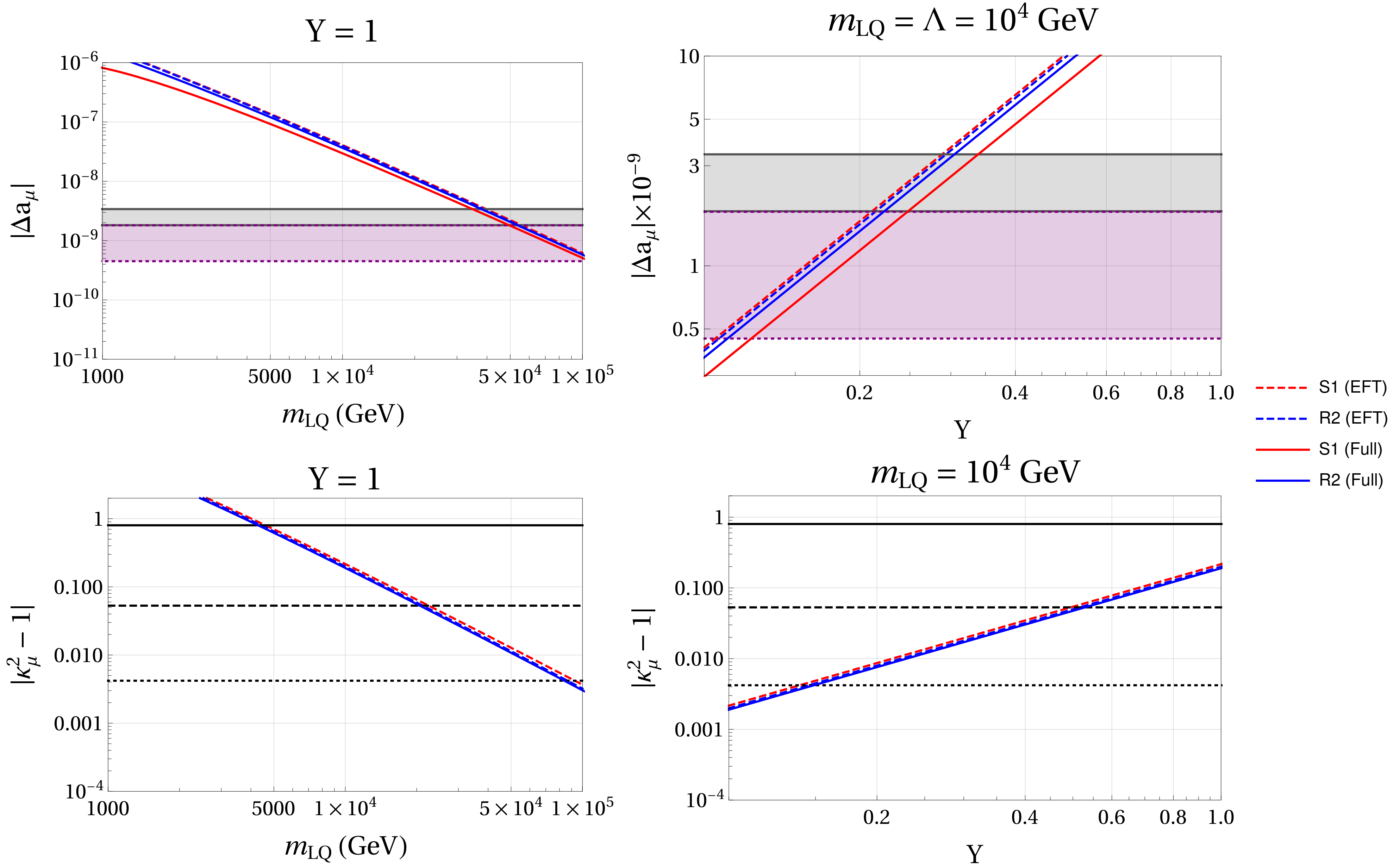
$$\hat{\mathcal{C}}_{4,\mu} = \mp 4\hat{\mathcal{C}}_{5,\mu} = -\frac{4G_F}{\sqrt{2}}g^{LL} = \frac{v^2 G_F}{\sqrt{2}m^2_{LQ}}Y$$



$y'_t y'_\mu = 1$
$|\Delta a_\mu|$
$m_{\rm LQ}$ (GeV)
$m_{\rm LQ} = \Lambda = 10^4$ GeV
$|\Delta a_\mu| \times 10^{-9}$
$y'_t y'_\mu$
S1 (EFT)
R2 (EFT)
S1 (Full)
R2 (Full)



$y'_t y'_\mu = 1$
$|\kappa_\mu^2 - 1|$
$m_{\rm LQ}$ (GeV)
$m_{\rm LQ} = 10^4$ GeV
$|\kappa_\mu^2 - 1|$
$y'_t y'_\mu$
S1 (EFT)
R2 (EFT)
S1 (Full)
R2 (Full)

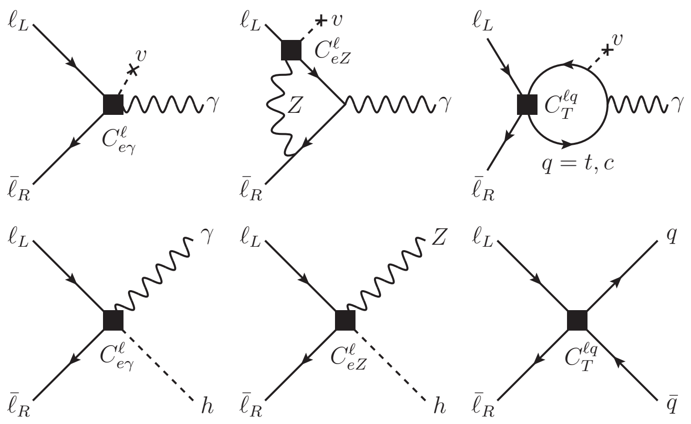


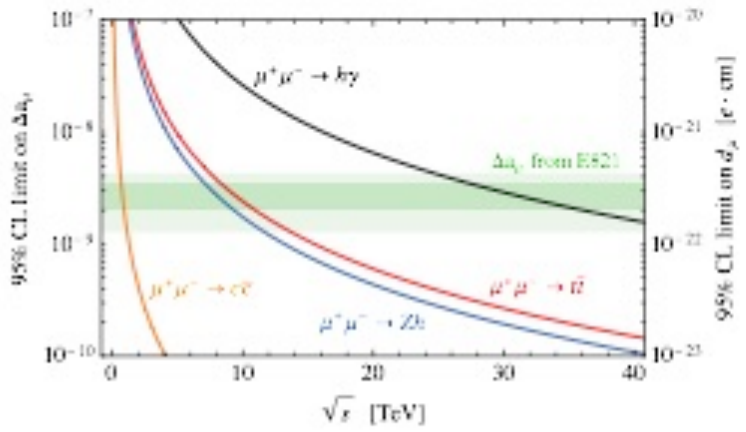
$Y = 1$
$|\Delta a_\mu|$
$m_{\rm LQ}$ (GeV)
$m_{\rm LQ} = \Lambda = 10^4$ GeV
$|\Delta a_\mu| \times 10^{-9}$
Y
$Y = 1$
$|\kappa_\mu^2 - 1|$
$m_{\rm LQ}$ (GeV)
$m_{\rm LQ} = 10^4$ GeV
$|\kappa_\mu^2 - 1|$
Y
S1 (EFT)
R2 (EFT)
S1 (Full)
R2 (Full)

$(g\text{-}2)_\mu$ and muon collider

Buttazzo& Patradizi 2012.02769.

“The capability of a foreseen Muon Collider to probe the muon g-2 is systematically investigated. We demonstrate that a Muon Collider, running at center-of-mass energies of several TeV, can provide the first model-independent high-energy test of new physics in the muon g-2, being sensitive to deviations of few × 10^{-9} .This achievement would be of the utmost importance to shed light on the long-standing muon g-2 anomaly”

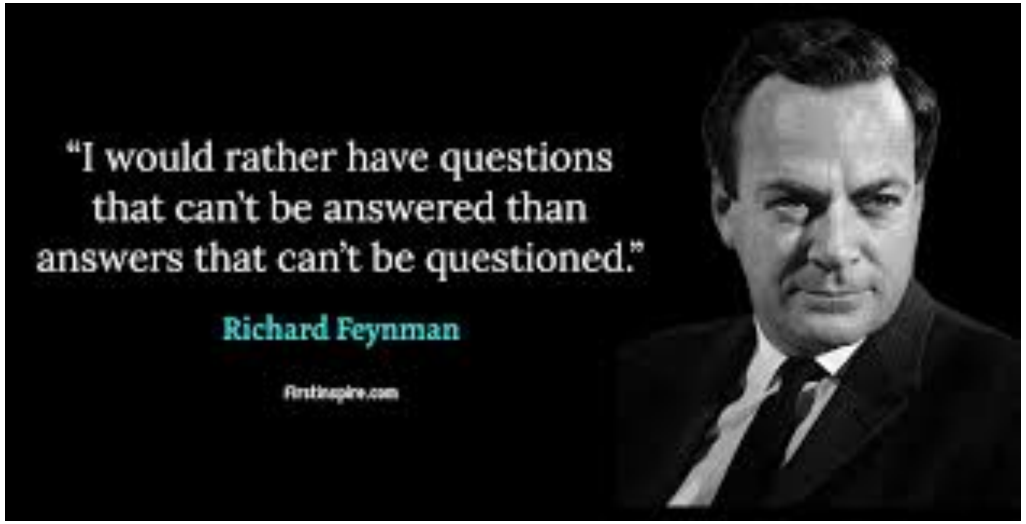




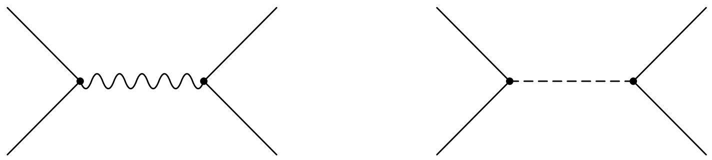
the collider center-of-mass energy

Summary

- Current deviation of the muon anomalous magnetic moment can be approached by new physics;
- Minimal solution for $(g\text{-}2)_\mu$ scalar leptoquark(s), new scalar weak doublet, additional Z’ gauge boson;
- All solutions are strongly constrained by the low-energy physics as well as LHC physics;
- Instead of particular model SMEFT can be used.


"I would rather have questions
that can't be answered than
answers that can't be questioned."
Richard Feynman

EFT can be used even for axions



$$-\frac{g^2}{2M_W^2} + \frac{1}{f_a^2}\frac{q^2}{q^2 - m_a^2} = -\frac{2}{v^2}\left[1 + \frac{v^2}{2f_a^2}\frac{q^2/m_a^2}{q^2/m_a^2 - 1}\right]$$

$$v = 246\,\text{GeV},\ f_a \sim 2\times 10^{12}\,\text{GeV},\ v/f_a \sim 10^{-10},\ m_a \sim 2\mu\text{eV}$$

$$\left|\frac{q^2}{m_a^2} - 1\right| \sim \frac{v^2}{f_a^2} \sim 10^{-20} \implies \Delta q \sim 10^{-25}\,\text{GeV}$$

EFT is valid unless one is doing very
Only in specialized experiments with high precision EFT should not be used..