Data Input to HLbL

International Physics School on Muon Dipole Moments and Hadronic Effects

2021-09-02

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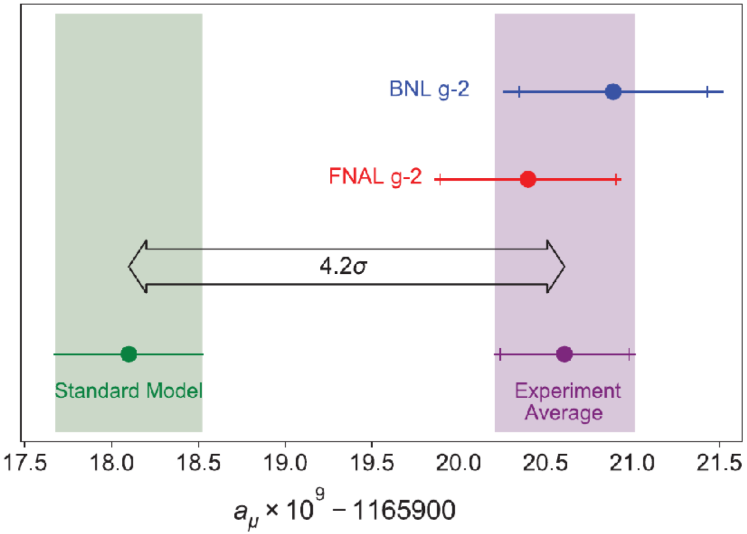
JG|U

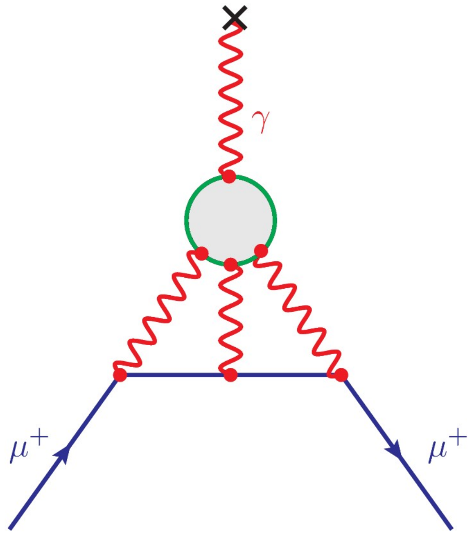
Anomalous Magnetic Moment of μ



Phys.Rev.Lett. 126 (2021) 141801

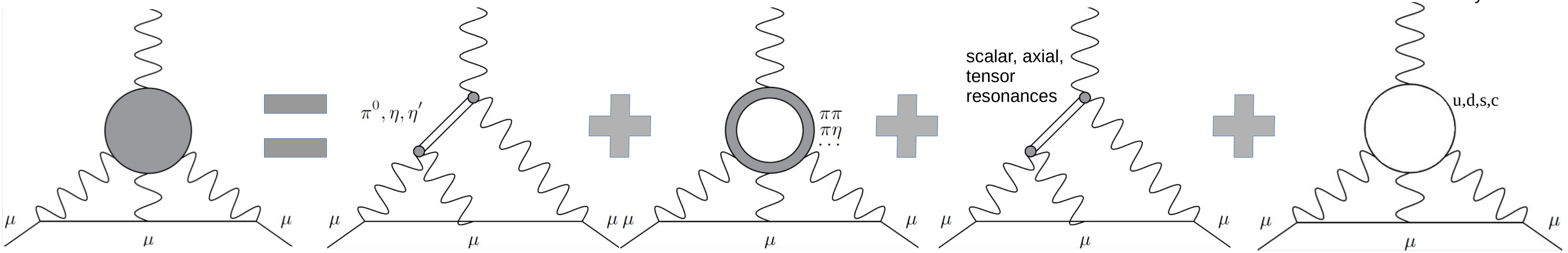
Contribution	in units of 10^{11}	
QED	116584718.931	± 0.104
Elektroweak	153.6	± 1.0
HVP	6845	± 40
HLbL	**92**	**± 18**

Phys.Rept. 887 (2020) 1



Hadronic Light-by-Light Scattering

Phys.Lett. B322 (1994) 239



Comparison of two frequently used compilations for HLbL in units of 10^{-11} from 2009 and a recent update with our estimate. Legend: PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N/JN = Nyffeler / Jegerlehner, Nyffeler; J = Jegerlehner.

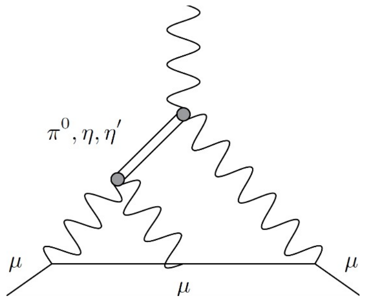
Contribution	PdRV(09) [475]	N/JN(09) [476,596]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Phys.Rept. 887 (2020) 1

Improvement using data driven approaches

What to measure?

- Largest contribution: Pseudoscalar poles
  - Lightest mesons contribute most
  - Transition Form Factor (TFF)
    - Coupling of mesons and two real/virtual photons
    - Function of photon virtualities $F(q_1^2, q_2^2)$
    - Contains structural information



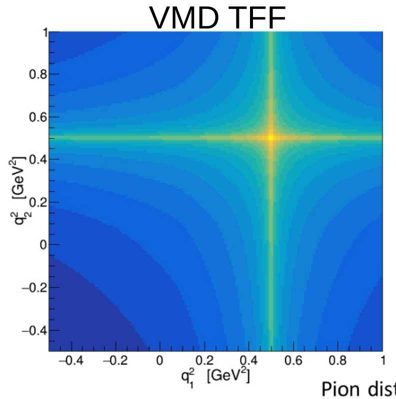
Transition Form Factors

- Real photons: Radiative width of meson

$$\Gamma_{P\to\gamma\gamma} = \frac{\pi\alpha^2 m_P^3}{4}|F_P(0,0)|^2$$

- Popular model: VMD

$$F(q_1^2, q_2^2) \sim \frac{M_V^2}{q_1^2 - M_V^2}\frac{M_V^2}{q_2^2 - M_V^2}$$



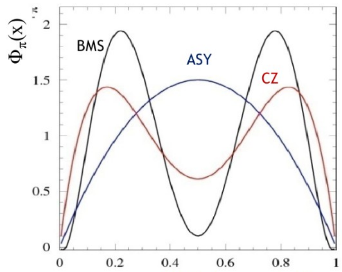
- Related to meson distribution amplitudes

$$F_{\pi^0}(-Q_1^2, -Q_2^2) = \frac{2f_\pi}{6}\int_0^1 \phi(x) T_H(x, Q_1^2, Q_2^2) dx$$

Distribution Amplitude (non-perturbative)

Function of quark's fraction of the meson momentum

Hard scattering kernel $\gamma\gamma \to q\bar{q}$



Brodsky-Lepage Limit

Phys.Rev D22 (1980) 2157
Phys.Rev D24 (1981) 1808

$$F_{\pi^0}(-Q_1^2,-Q_2^2) = \frac{2f_\pi}{6}\int_0^1 \phi(x) T_H(x,Q_1^2,Q_2^2)dx$$

$$T_H^{LO}(x,Q_1^2,Q_2^2) = \frac{2}{Q_1^2+Q_2^2}\left[\frac{1}{1-\omega(2x-1)}+\frac{1}{1+\omega(2x-1)}\right], \qquad \omega = \frac{Q_1^2-Q_2^2}{Q_1^2+Q_2^2}$$

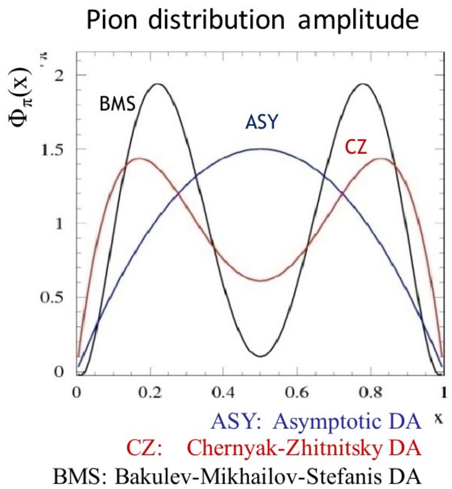
Here: Asymptotic DA $\phi(x) = 6x(1-x)$

Special case: Singly-virtual TFF $Q_2^2 \to 0$

$$\omega = 1 \longrightarrow T_H^{LO}(x,Q_1^2,0) = \frac{1}{Q_1^2}\left[\frac{1}{(1-x)}+\frac{1}{x}\right]$$

$$F_{\pi^0}(-Q_1^2,0) = \frac{2f_\pi}{6}\int_0^1 \phi_{ASY}(x) T_H(x,Q_1^2,0)dx = \frac{2f_\pi}{6}\int_0^1 6x(1-x)\frac{1}{Q_1^2}\left[\frac{1}{1-x}+\frac{1}{x}\right]dx$$

$$= \frac{2f_\pi}{Q_1^2}\int_0^1 x+1-x\,dx \qquad = \frac{2f_\pi}{Q_1^2}$$



pQCD predicts $\frac{1}{Q^2}$ behavior for singly virtual TFF

Where to measure?

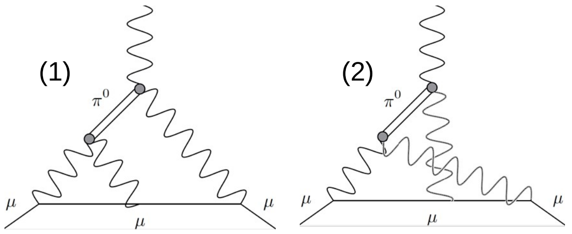
3D integral representation for PS-pole contribution:

$$a_\mu^{HLbL;\pi^0} = \left(\frac{\alpha}{\pi}\right)^3 \left[a_\mu^{HLbL;\pi^{0(1)}} + a_\mu^{HLbL;\pi^{0(2)}}\right]$$

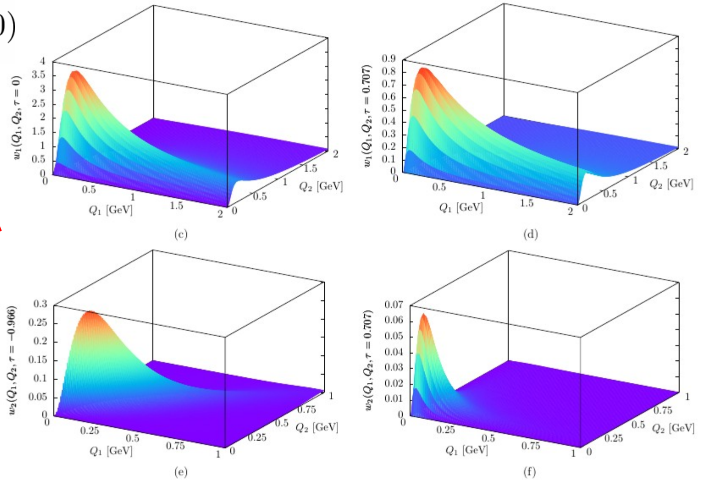
$$a_\mu^{HLbL;\pi^{0(1)}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\; w_1(Q_1,Q_2,\tau)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-(Q_1+Q_2)^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2,0)$$

$$a_\mu^{HLbL;\pi^{0(2)}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\; w_2(Q_1,Q_2,\tau)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-(Q_1+Q_2)^2,0)$$

$q_i^2 = -Q_i^2$	photon virtualities
$\tau = \cos\theta_{Q_1,Q_2}$	angle between virtualities
$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)$	transition form factor
$w_i(Q_1,Q_2,\tau)$	weighting functions



Phys.Rev.D94,053006, 2016



$$Q_i^2 \leq 1\,\mathrm{GeV}^2$$

How to measure?

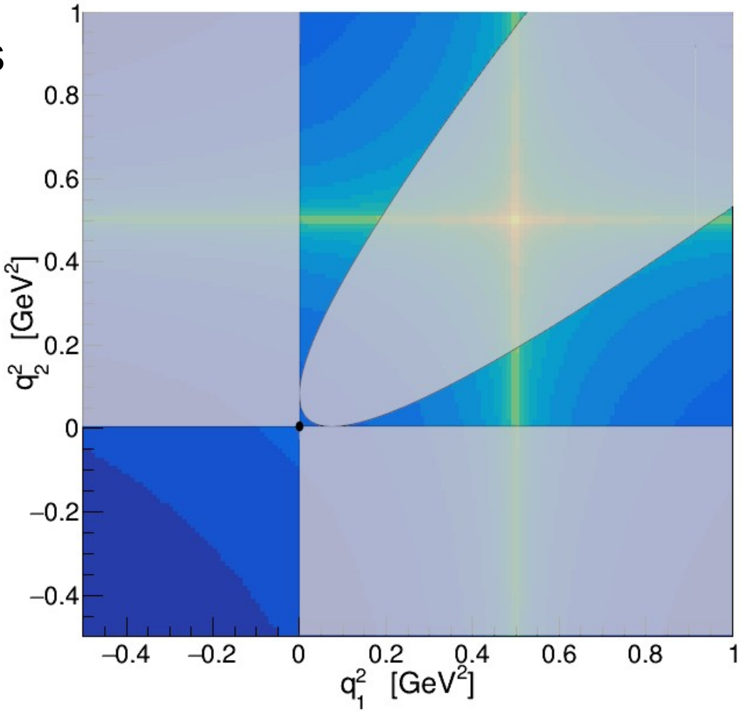
Obviously: Processes involving a meson and two photons

Real photons?

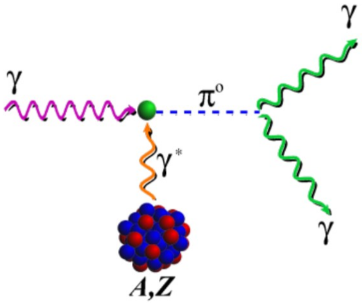
Virtual photons?

Spacelike?

Timelike?



Primakoff Effect



"production in the collision of a photon with an external, approximately static electric field; e.g. the Coulomb field of a nucleus"

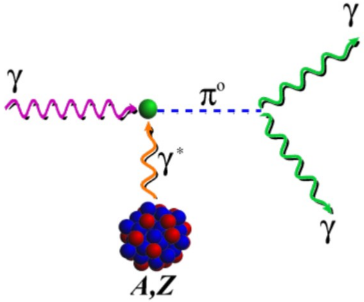
Phys. Rev. 81 (1951) 899

$$\frac{d\sigma_{Pr}}{d\Omega} = \Gamma_{\gamma\gamma}\frac{8\alpha Z^2}{m_\pi^3}\frac{\beta_\pi^2 E_\gamma^4}{Q^4}|F_{em}(Q)|^2 \sin^2\theta_\pi$$

Best observed with

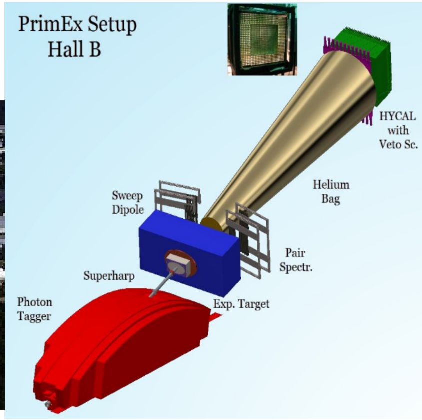
- High photon energies
- Heavy targets (large Z)
- small production angles

PrimEX



$$\frac{d\sigma_{Pr}}{d\Omega} = \Gamma_{\gamma\gamma}\frac{8^2}{m_\pi^3}\frac{\beta^2 E^4}{Q^4}|F_{em}(Q)|^2\sin\theta_\pi$$

Jefferson Lab

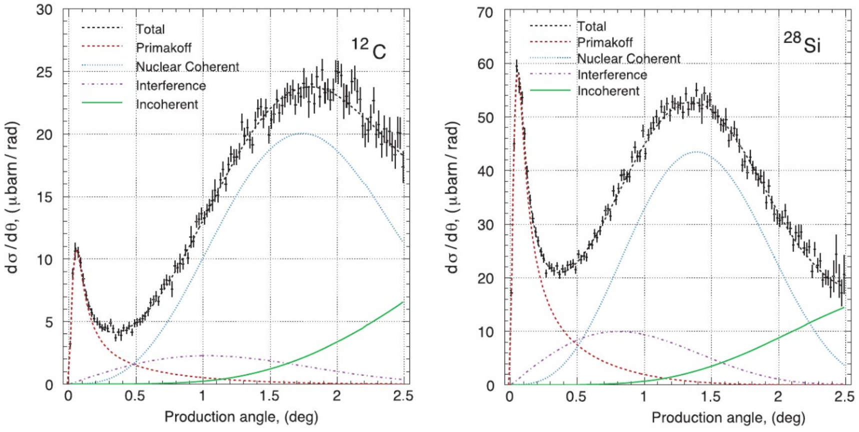


I. Larin, PhiPsi19 Workshop

- Most recent measurement by PrimEx at JLab
- Tagged photon beam in Hall B
- C and Si targets
- Disentangle Primakoff amplitude
- Combined result:

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.806 \pm 0.052_{\text{stat}} \pm 0.105_{\text{syst}}\,\text{eV}$$

Science 368 (2020) 506



Dalitz Decays

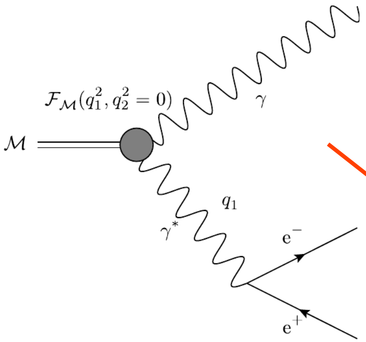
- Rare decay to photon and lepton pair
  - Best results from “meson factories“
- Mass of lepton pair defines virtuality
- Constraints: $m_{ll}^2 < q^2 < m_P^2$
- TFF from decay rate:

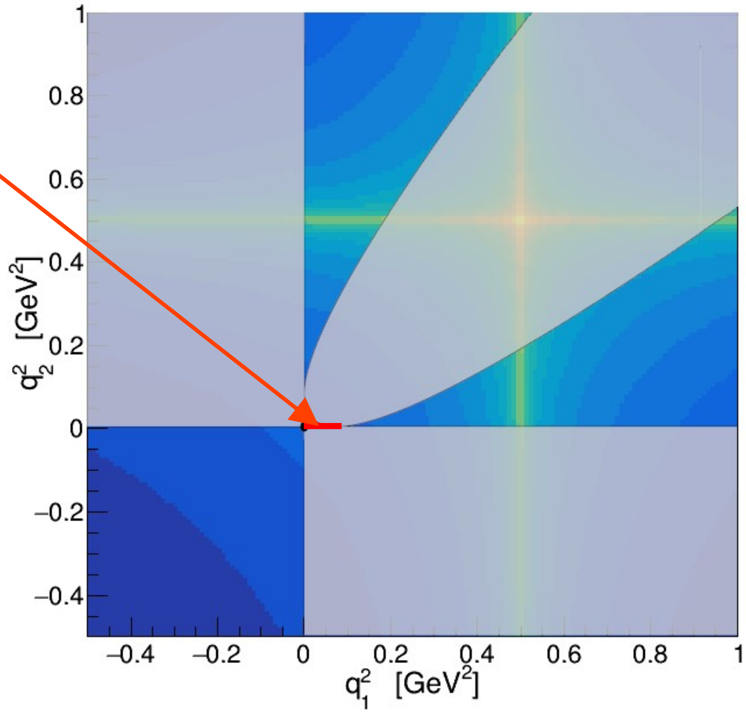
$$\frac{d\Gamma_{P\gamma\gamma^*}}{dq^2\Gamma_{\gamma\gamma}} = [QED]\left|\frac{F_{P\gamma\gamma^*}(q^2)}{F_{P\gamma\gamma}(0)}\right|^2$$

  - Slope parameter

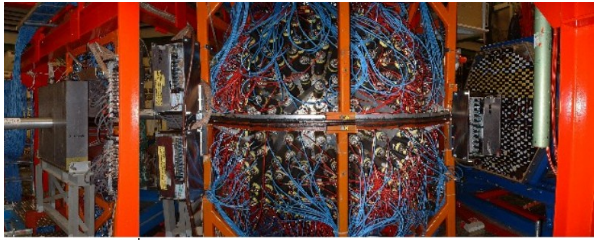
$$\Lambda^{-2} = \left.\frac{dF_{P\gamma\gamma^*}(q^2)}{dq^2}\right|_{q^2=0}$$

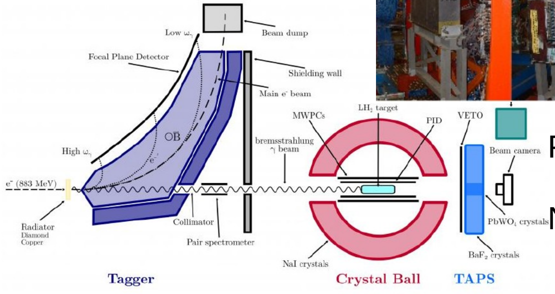
- Doubly-virtual process:
  - Decay to four leptons (Double Dalitz Decay)





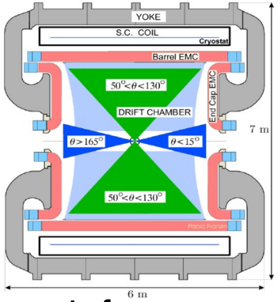
Meson Factories





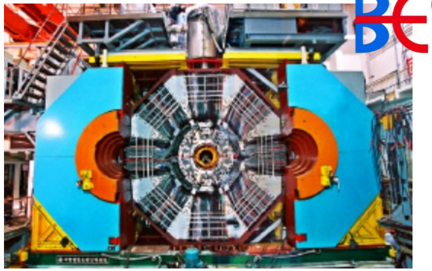
Photon beam facility at MAMI
$E_{max} \approx 1.6$ GeV
Non-magnetic spectrometer

KLOE-2 at DAΦNE



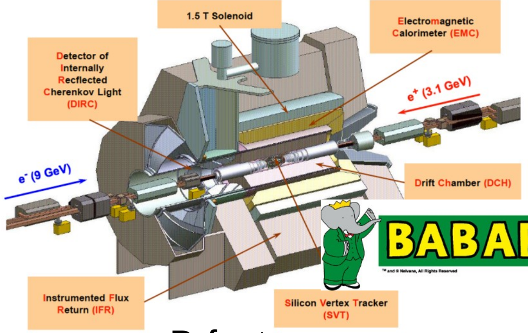
Φ-factory

BESIII at BEPCII

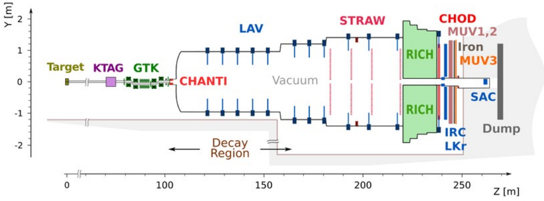


tau-charm-factory

BaBar at SLAC



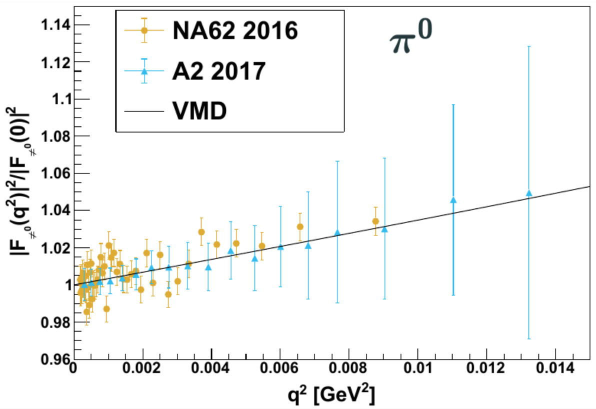
B-factory



Rare kaon decays at CERN SPS

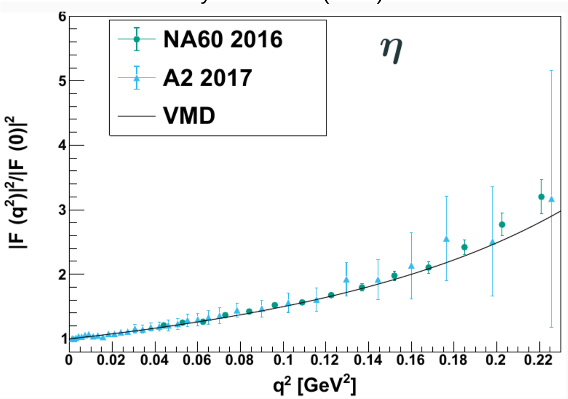
Dalitz Decays

Phys.Lett. B768 (2017) 38
Phys.Rev. C95 (2017) 025202



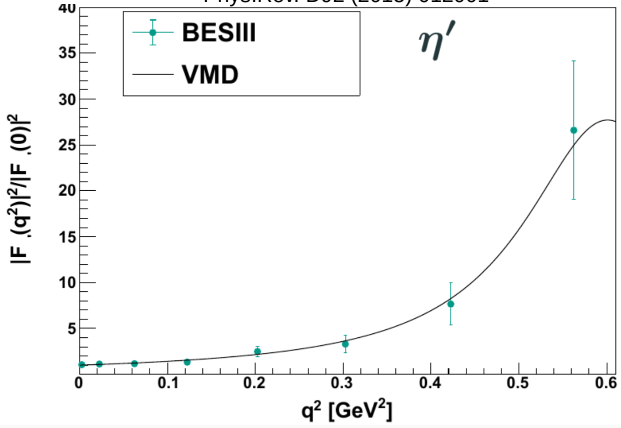
- A2/MAMI: $\gamma p \to p\pi^0$
  $\Lambda_{\pi^0} = (0.61 \pm 0.20)\,\mathrm{GeV}^2$
- NA62: $K^\pm \to \pi^\pm\pi^0$
  $\Lambda_{\pi^0} = (0.495 \pm 0.076)\,\mathrm{GeV}^2$
- Radiative corrections considered

Phys.Lett. B757 (2016) 437
Phys.Rev. C95 (2017) 035208



- A2/MAMI: $\gamma p \to p\eta$
  $\Lambda_\eta = (0.507 \pm 0.028)\,\mathrm{GeV}^2$
- NA60: Heavy ion experiment
  $\Lambda_\eta = (0.517 \pm 0.022)\,\mathrm{GeV}^2$

Phys.Rev. D92 (2015) 012001



- BESIII: $J/\psi \to \gamma\eta'$
  $\Lambda_{\eta'} = (0.625 \pm 0.073)\,\mathrm{GeV}^2$

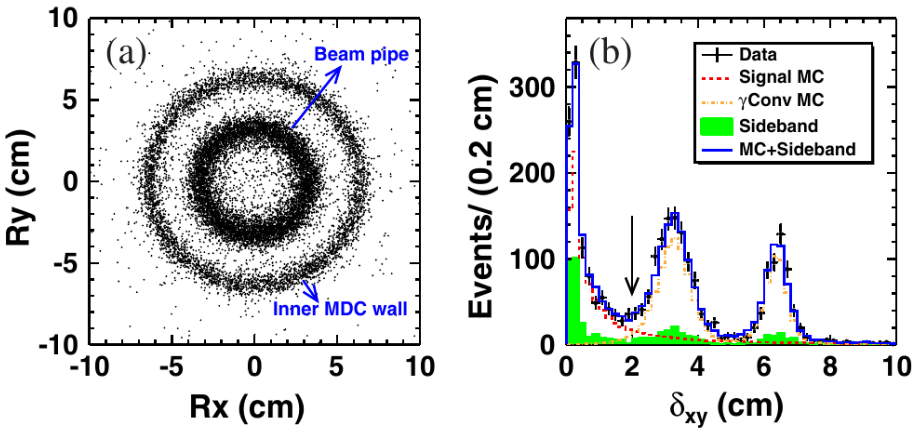
Good agreement between experiment and theory

Dalitz Decays

Photon Conversion

- Pair production in material
- Affects normalization channel
- Vertexing of lepton pairs

Example: BESIII Dalitz decay analysis

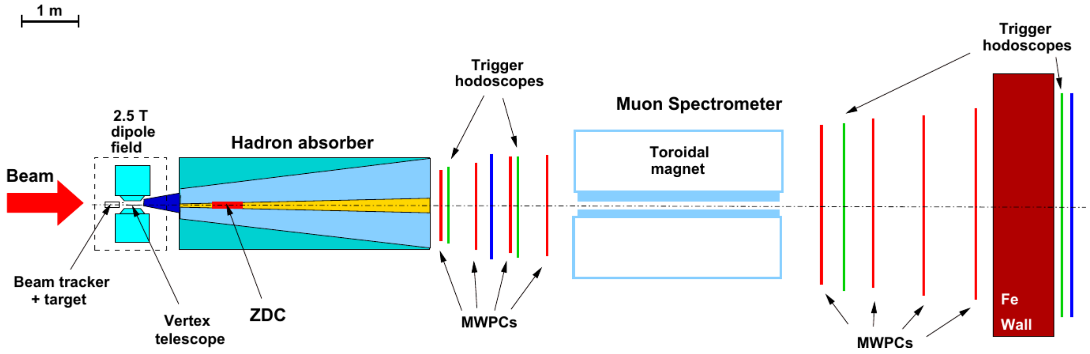


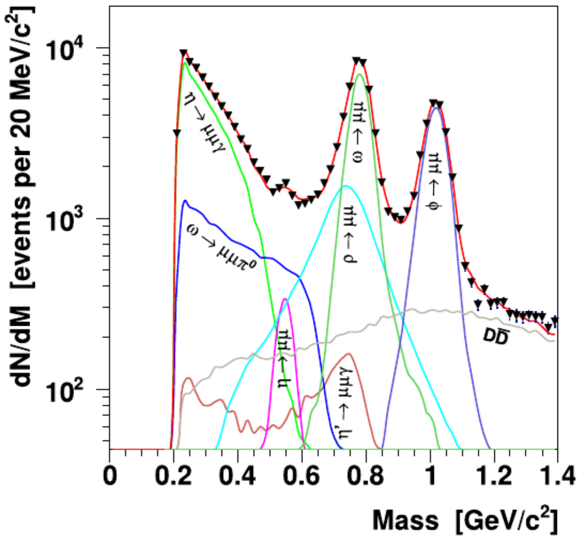
Phys.Rev. D92 (2015) 012001

Dalitz Decays

Dalitz decays at NA60

- Magnetic di-muon spectrometer
  - In-In collision — Phys. Lett. B677 (2009) 260
  - p-A collisions — Phys.Lett. B757 (2016) 437
- Measurement of inclusive di-muon mass spectrum
- Fit efficiency corrected spectrum for M < 0.65 GeV[2]
  - Normalizations (ρ,η,ω) and slope parameters floated
  - ρ contribution subtracted
  - η and ω contributions disentangled according to fit result

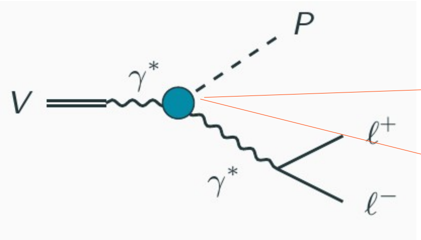


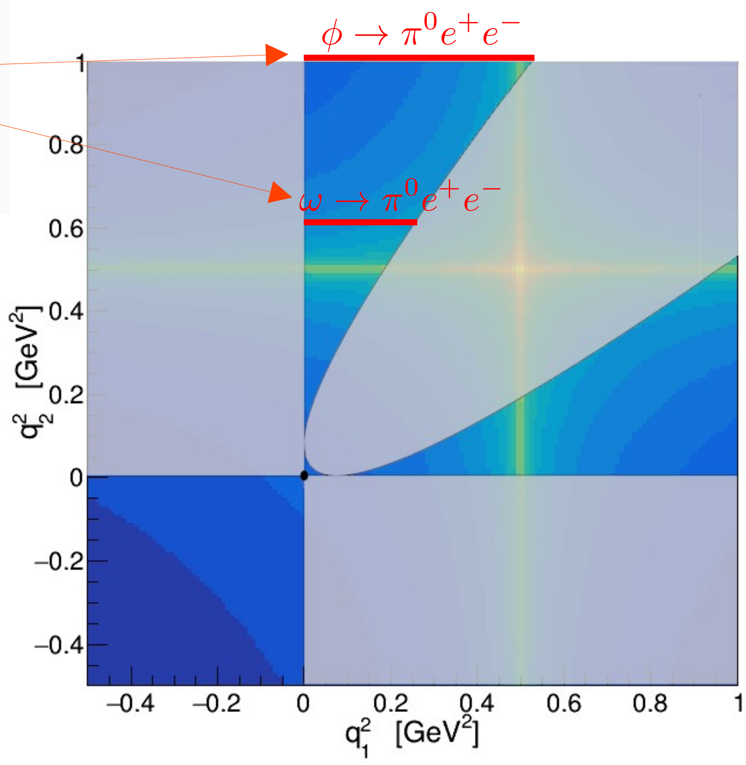


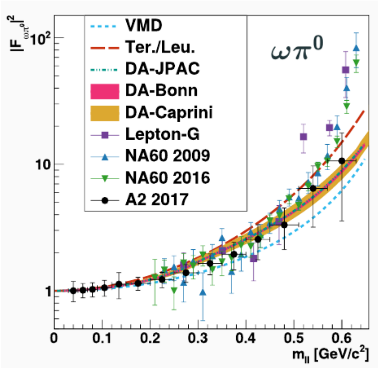
Vector Meson Dalitz Decays

Vector meson mass as 2nd virtuality

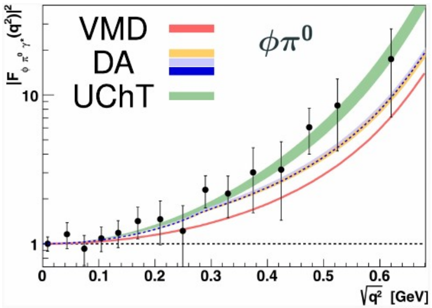
- $F_{PV}(q^2) \sim F_{P\gamma^*\gamma^*}(q^2, m_V^2)$



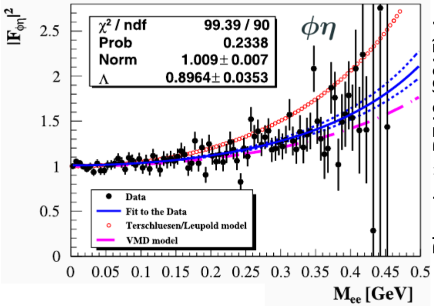




Phys.Rev. C95 (2017) 035208



Phys.Lett. B757 (2016) 362

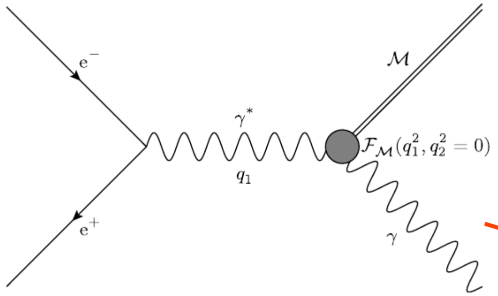


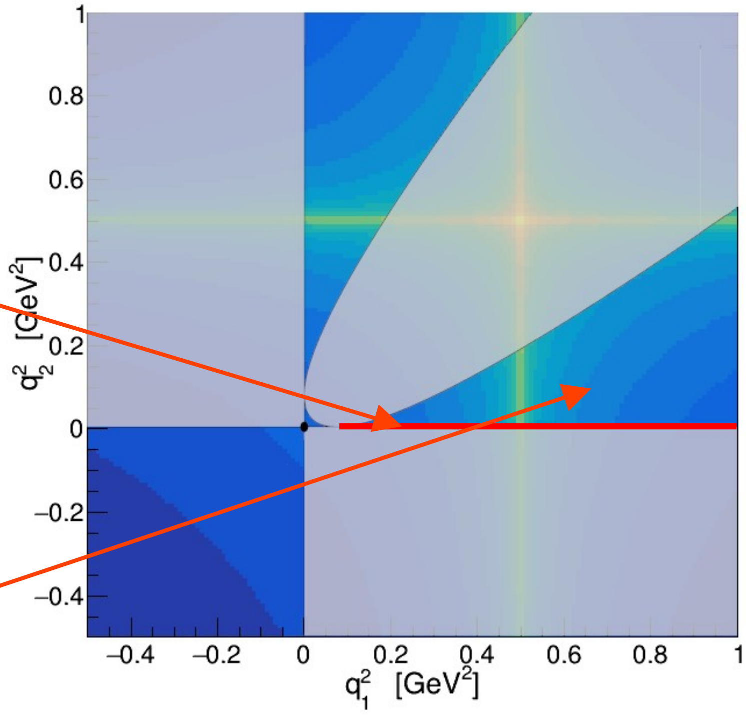
Phys.Lett. B742 (2015) 1

Intriguing tensions
between data and theory

Also: BESIII results on charmonia

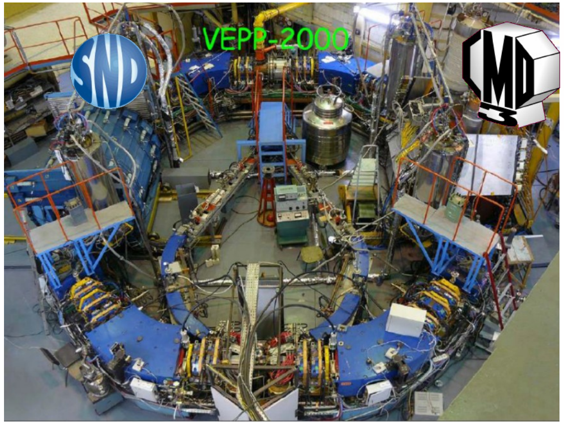
Radiative Production



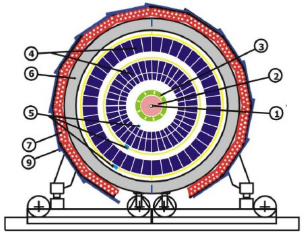


- Access to singly-virtual, timelike TFF
- Virtuality fixed to center-of-mass energy
  - $q^2 > m_P^2$
- $\sigma(e^+e^- \to P\gamma) = \frac{2\pi^2\alpha^3}{3}\frac{(q^2 - m_P^2)^3}{q^6}|F_{P\gamma^*\gamma}(q^2, 0)|^2$
- Doubly-virtual process: $e^+e^- \to Pl^+l^-$

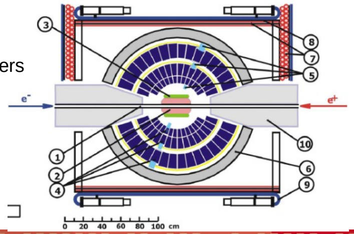
Radiative Production



VEPP-2000: 0.35 – 2.0 GeV, @$10^{32}cm^{-2}s^{-1}$

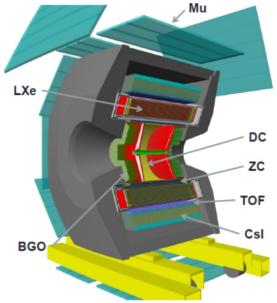


SND

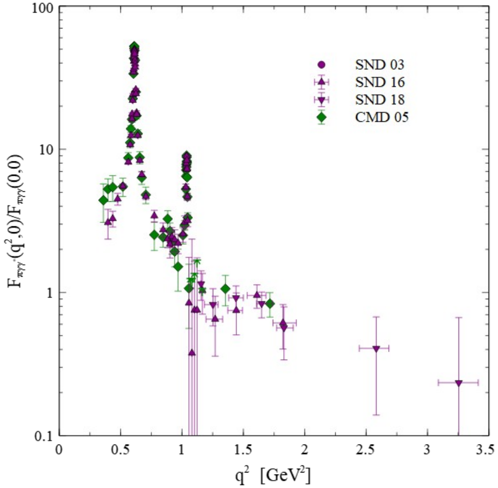


1 beam pipe
2 tracking system
3 aerogel Cherenkov counters
4 Na(Tl) crystals
5 phototriodes
6 iron muons absorber
7-9 muon detectors
10 focusing solenoids

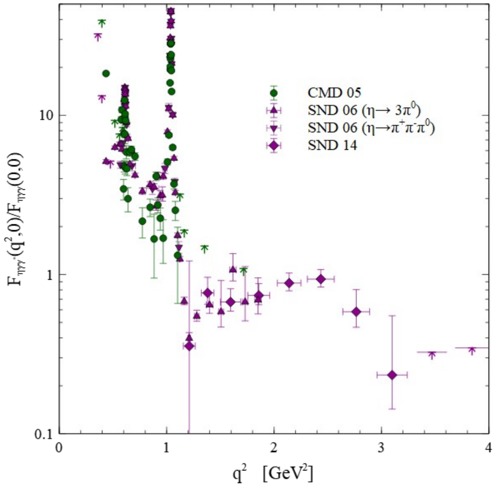
CMD-3



$e^+e^- \to \pi^0\gamma$



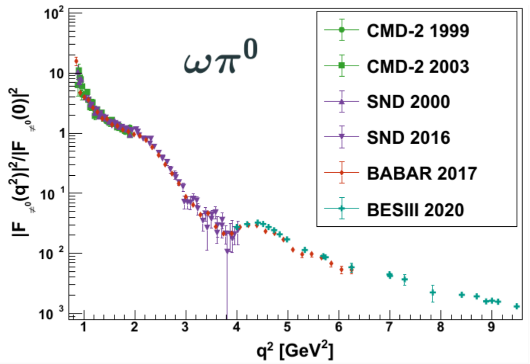
$e^+e^- \to \eta\gamma$

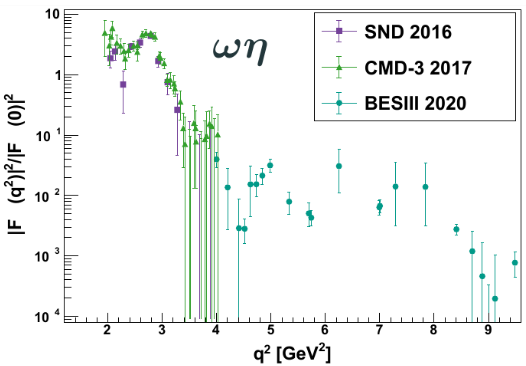


- Studies of light vector resonances
- At high q^2: Tests of pQCD
  - BaBar: q^2=112 GeV^2
  - CLEO: q^2=14 GeV^2

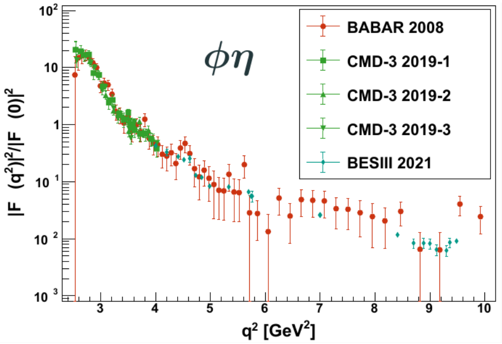
Vector Meson Production

- Access to excited vector meson states
- Also input to HVP
- Measurements in
  - Energy scan
  - ISR technique



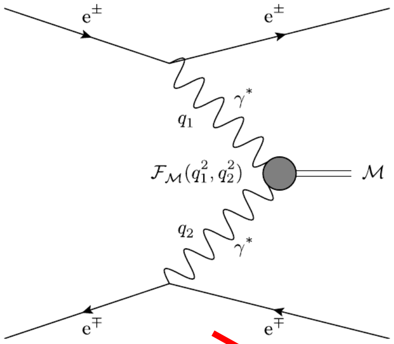


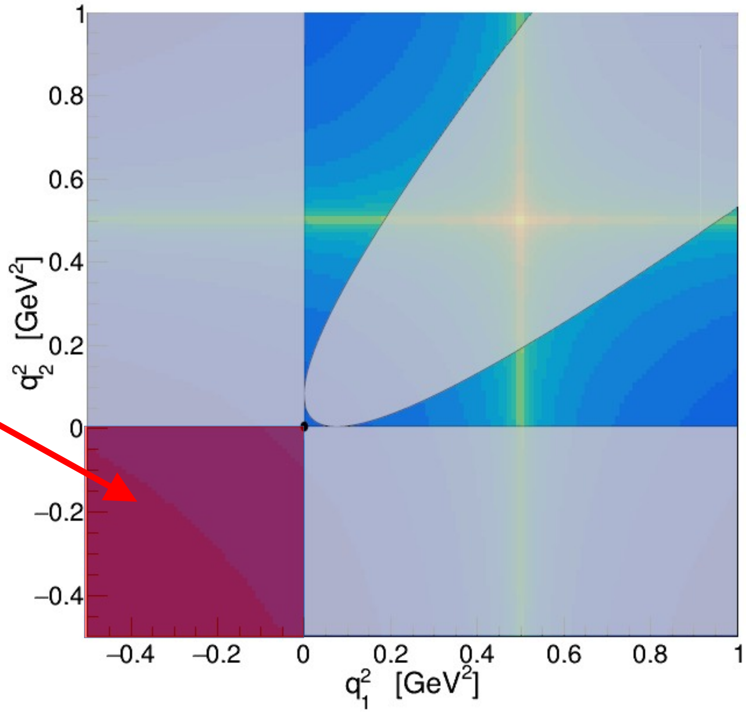
- $\sigma(e^+e^- \to VP) = \frac{4\pi^2\alpha^3}{3s^{3/2}}|F_{VP}(q_1^2)|^2 P_f(s)$
- Closely related to doubly-virtual timelike TFF
  - $F_{VP}(q_1^2) \sim F_{P\gamma^*\gamma^*}(q_1^2, m_V^2)$



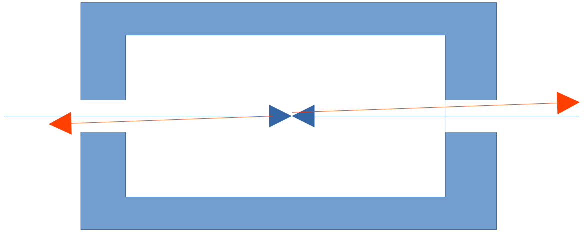
Two-Photon Collisions

- C-even states in e+e- collisions
- Direct access to $J^{PC} = 0,2^{\pm +}$
- Produced masses $m_X \ll \sqrt{s}$
- Energy dependence $\sigma \propto \alpha^2 \ln^2 E$
- Forward-peaked kinematics
- $\sigma = \sigma_{\text{point-like}} |F(Q_1^2, Q_2^2)|^2, \qquad Q^2 = -q^2$



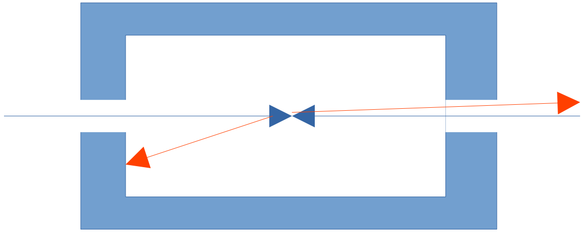


Experimental Constraints



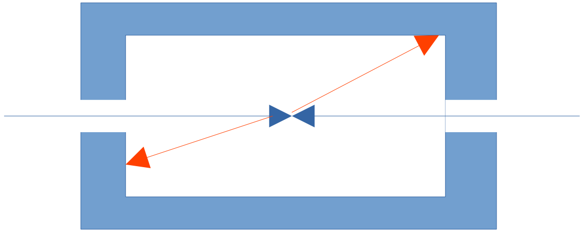
Untagged

- Both leptons unmeasured
- $Q_1^2 \approx Q_2^2 \approx 0$
- Largest cross section



Single-tag

- Only one lepton measured
- $Q_2^2 \approx 0$
- $F(Q_1^2, 0)$
- Reduced cross section



Double-tag

- All particles measured
- Full information
- $F(Q_1^2, Q_2^2)$
- Smallest cross section

Two-Photon Cross Section

Relation for unpolarized lepton beams:

$$d\sigma_{ee} = \frac{\alpha^2}{16\pi^4 q_1^2 q_2^2} \sqrt{\frac{(q_1 \cdot q_2)^2 - q_1^2 q_2^2}{(p_1 \cdot p_2)^2 - m_e^4}} \left[4\rho_1^{++}\rho_2^{++}\sigma_{TT} + 2|\rho_1^{+-}\rho_2^{+-}|\tau_{TT}\cos 2\tilde{\phi} + 2\rho_1^{++}\rho_2^{00}\sigma_{TL} + 2\rho_1^{00}\rho_2^{++}\sigma_{LT} + \rho_1^{00}\rho_2^{00}\sigma_{LL} - 8|\rho_1^{+0}\rho_2^{+0}|\tau_{TL}\cos\tilde{\phi} \right] \frac{d^3p_1' d^3p_1'}{E_1' E_2'}$$

$p_i, p_i', q_i = (p_i - p_i')$	Momenta of incoming and outgoing leptons and photons
$\sigma_{TT}, \sigma_{TL}, \sigma_{LT}, \sigma_{LL}$	Two-photon cross sections for Transversely and Longitudinally polarized photons
τ_{TT}, τ_{TL}	Two-photon cross sections correlation terms
ρ_i^{ab}	Elements of photon density matrix for helicities a,b=+,-,0; functions of q_i
$\tilde{\phi}$	Angle between planes of incoming and outgoing leptons in two-photon c.m.s

Two-Photon Cross Section

For pseudoscalar mesons:

Only transversely polarized photons contribute!

$$d\sigma_{ee} = \frac{\alpha^2}{16\pi^4 q_1^2 q_2^2}\sqrt{\frac{(q_1\cdot q_2)^2 - q_1^2 q_2^2}{(p_1\cdot p_2)^2 - m_e^4}}\left[4\rho_1^{++}\rho_2^{++}\sigma_{TT} + 2|\rho_1^{+-}\rho_2^{+-}|\tau_{TT}\cos 2\tilde{\phi} + 2\rho_1^{++}\rho_2^{00}\sigma_{TL} + 2\rho_1^{00}\rho_2^{++}\sigma_{LT} + \rho_1^{00}\rho_2^{00}\sigma_{LL} - 8|\rho_1^{+0}\rho_2^{+0}|\tau_{TL}\cos\tilde{\phi}\right]\frac{d^3p_1' d^3p_1'}{E_1' E_2'}$$

After integration over $\tilde{\phi}$ only one cross section left!

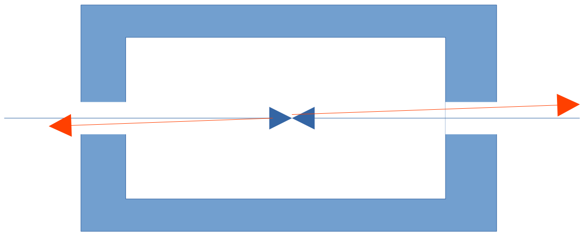
$$d\sigma_{ee} = \frac{\alpha^2}{16\pi^4 q_1^2 q_2^2}\sqrt{\frac{(q_1\cdot q_2)^2 - q_1^2 q_2^2}{(p_1\cdot p_2)^2 - m_e^4}}4\rho_1^{++}\rho_2^{++}\sigma_{TT}\frac{d^3p_1' d^3p_1'}{E_1' E_2'}$$

Two-photon luminosity function

Untagged

Both photons are quasi-real

- Luminosity function simplifies
- Equivalent Photon Approximation:
  - $\sigma_{ee} = \int dz \frac{\mathcal{L}_{\gamma\gamma}}{dz} \sigma_{\gamma\gamma\to P}(z)$
  - $z = \frac{W}{2E}, \quad W^2 = (q_1 + q_2)^2$
  - $\frac{d\mathcal{L}_{\gamma\gamma}}{dz} = \left(\frac{2\alpha^2}{\pi}\right)^2 \left(\ln \frac{E}{m_e}\right)^2 f(z)$
- Narrow spin-0 resonances: $\sigma_{\gamma\gamma\to P} = \frac{8\pi^2}{m_P} \Gamma_{P\to\gamma\gamma} \delta(W^2 - m_P^2)$
- Consider $q_i^2 \neq 0$ with TFF: $\sigma_{\gamma\gamma\to P} = \frac{8\pi^2}{m_P} \Gamma_{P\to\gamma\gamma} \delta((q_1 + q_2)^2 - m_P^2) |F(q_1^2, q_2^2)|^2$



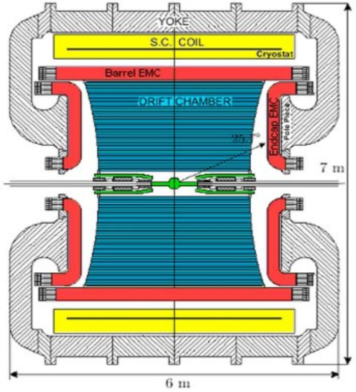
Phys. Rev. D4 (1971) 1532
Nucl. Phys. B54 (1973) 573
Phys. Rept. 15 (1975) 181

Access to radiative width $\Gamma_{X\to\gamma\gamma}$ in untagged measurements

Radiative Width

KLOE: η meson

- Off-peak data to reduce background
- $\eta \to \pi^0\pi^0\pi^0, \eta \to \pi^+\pi^-\pi^0$
- Combined fit to missing mass and momentum components



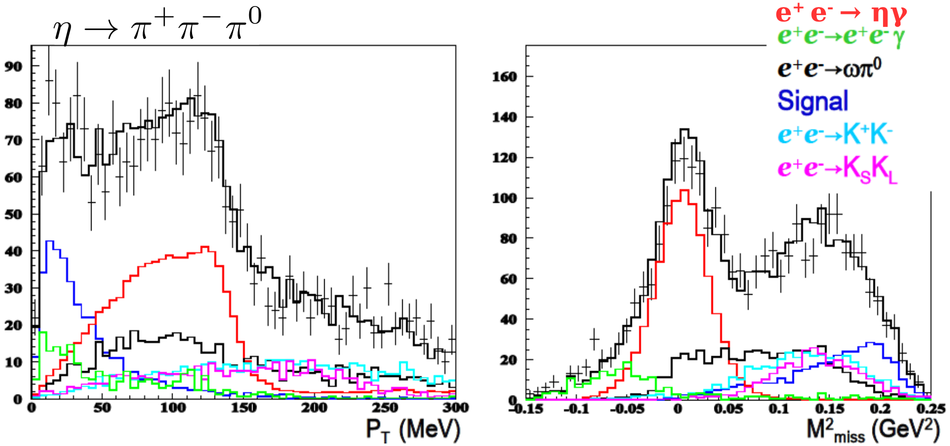
Combined (both decay modes) cross section

$$\sigma(e^+e^- \to e^+e^-\eta) = (32.7 \pm 1.3_{stat} \pm 0.7_{syst})\,\mathrm{pb}$$

Using VMD TFF:

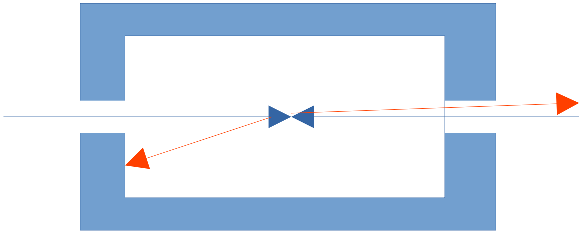
$$\Gamma_{\eta\to\gamma\gamma} = (520 \pm 20_{stat} \pm 13_{syst})\,\mathrm{eV}$$

JHEP 01 (2013) 119



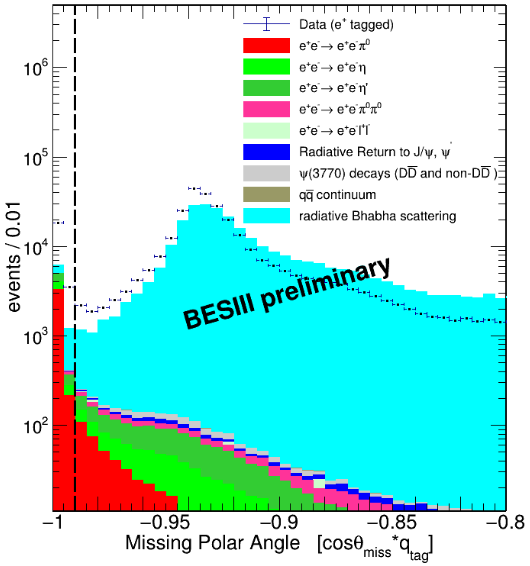
Single-Tagged

- Detect one lepton
- Detect produced meson(s)
- Unmeasured lepton from energy/momentum conservation



Example: π^0 TFF at BESIII

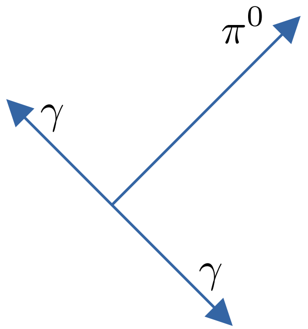
- $2.9 fb^{-1}$ at 3.773 GeV
- Require one lepton and two photons
- Require scattering angle of missing lepton to be small
  - Small virtuality of exchanged photon

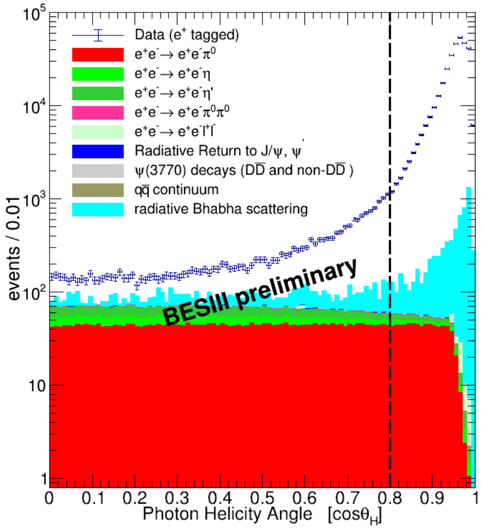


TFF measurement at BESIII

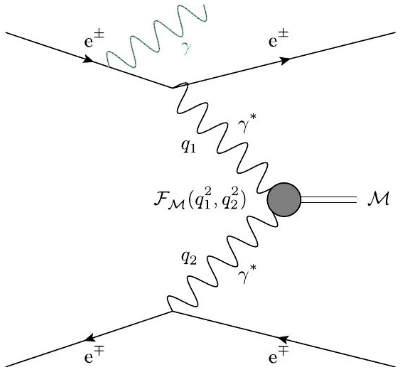
Helicity angle of photons

- Angle between photon in pion rest frame and pion momentum in lab frame
- Directly related to energy asymmetry of photons in lab frame
- Pion decays isotropically
- Useful to reject radiative Bhabha scattering





TFF measurement at BESIII



Radiative effects

- ISR leads to mistake in Q^2 calculation
  - Assumption: $-Q^2 = (p - p')^2$
  - ISR: $\tilde{p} = p - p_\gamma \quad \Rightarrow \quad -Q^2 = (\tilde{p} - p')^2$

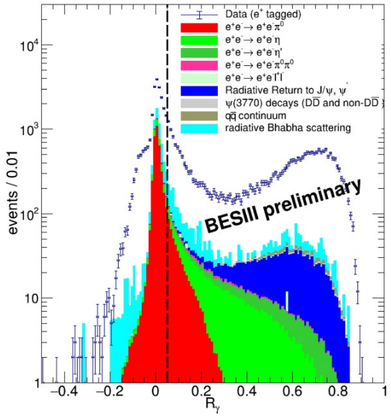
Belle

- Small deviation from expected energy of hadrons
- Add photons close to tagged electron

BaBar, BESIII

- Calculate energy of potential ISR photon
- Reject fraction too large

$$R_\gamma = \frac{\sqrt{s} - E^{CMS}_{e^\pm \pi^0} - p^{CMS}_{e^\pm \pi^0}}{\sqrt{s}}$$



Generators for TFF Measurements

TREPS

arXiv:1310.0157

- Using Equivalent Photon Approximation and taking into account virtual photons
- Calculates luminosity function
- Continuously adapted to Belle program
- Radiative corrections to tagged leptons

GGResRC

Comput.Phys.Commun. 185 (2014) 236

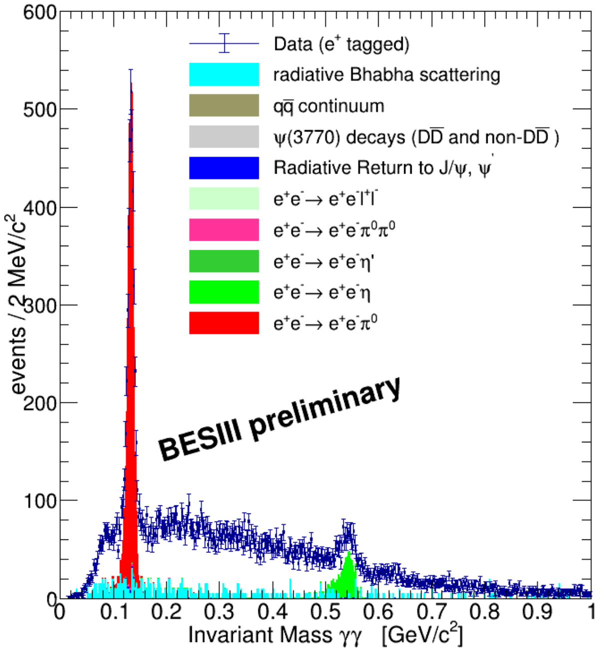
- Using exact equations for matrix elements
- Focused on tagged kinematics for pseudoscalars
- Radiative corrections to tagged leptons

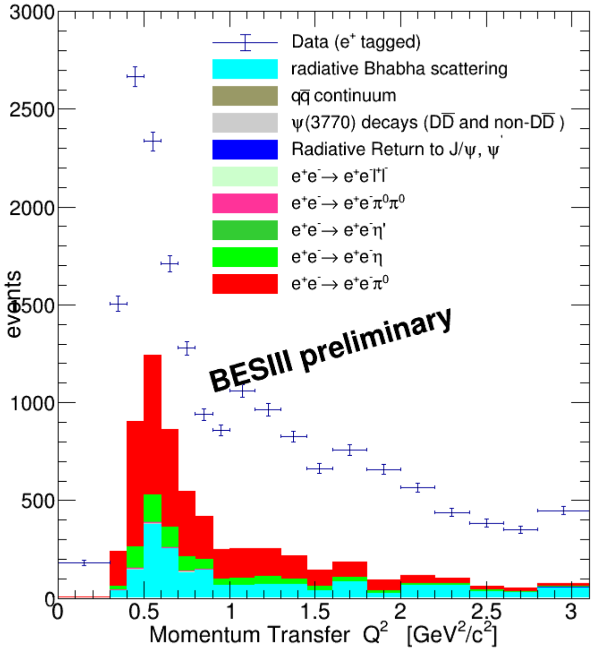
Ekhara 3

Comput.Phys.Commun. 234 (2019) 245

- Using exact equations for matrix elements
- Full phase space for pseudoscalars, pion pairs, χ_{cJ}
- Full QED terms for radiative corrections

TFF measurement at BESIII

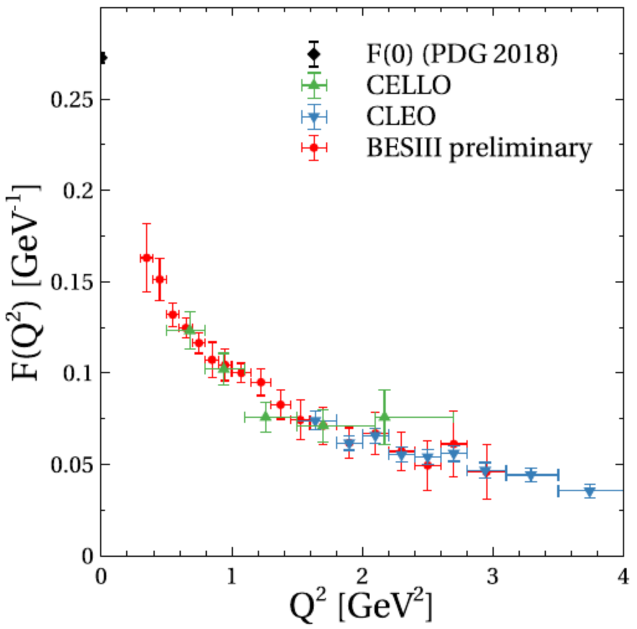


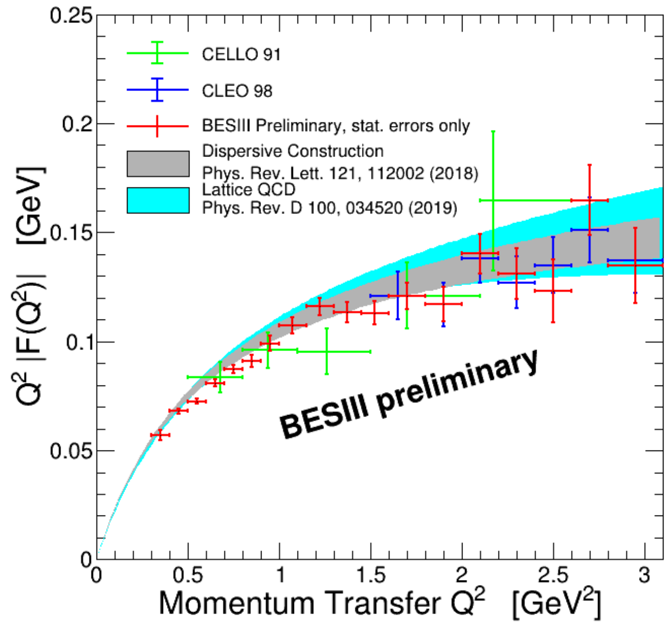


- Subtract background by „counting“ pions
- Convert event yield to differential cross section
- Calculate TFF using MC

$$F(Q^2)_{\exp} = \frac{\left.\frac{d\sigma}{dQ^2}\right|_{\exp}}{\left.\frac{d\sigma}{dQ^2}\right|_{\mathrm{MC}}} F(Q^2)_{\mathrm{MC}}$$

TFF measurement at BESIII

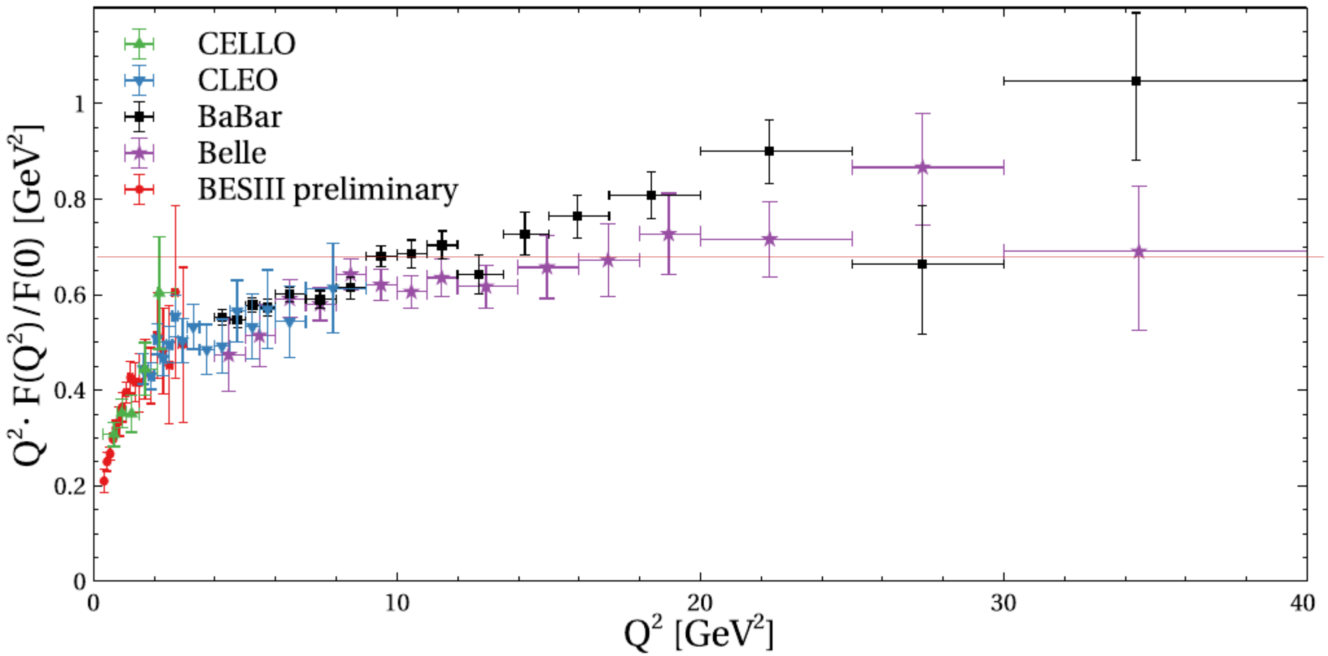




- $0.3 \leq Q^2[\mathrm{GeV}^2] \leq 3.1$
- Unprecedented accuracy for $Q^2 \leq 1.5\,\mathrm{GeV}^2$

- Measurement confirms
  - Dispersive construction of TFF
  - Lattice determination of TFF

Comparison to other measurements

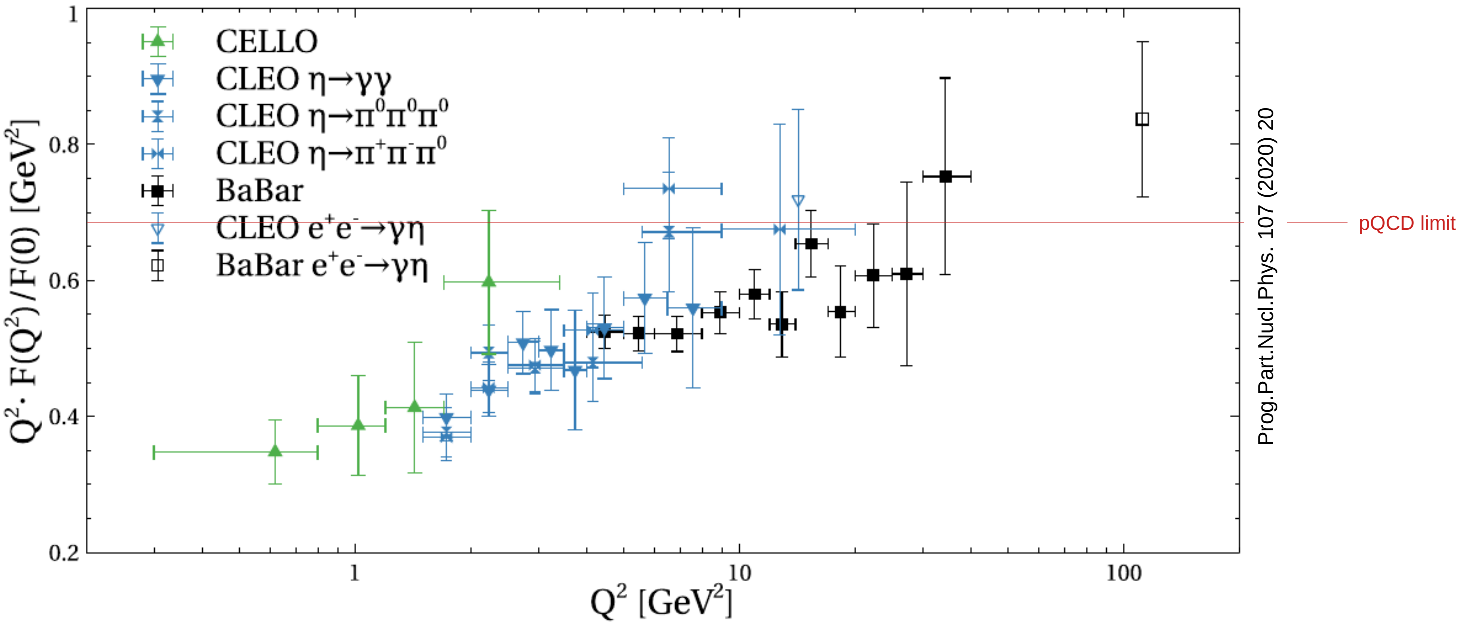


Prog.Part.Nucl.Phys. 107 (2020) 20

pQCD limit

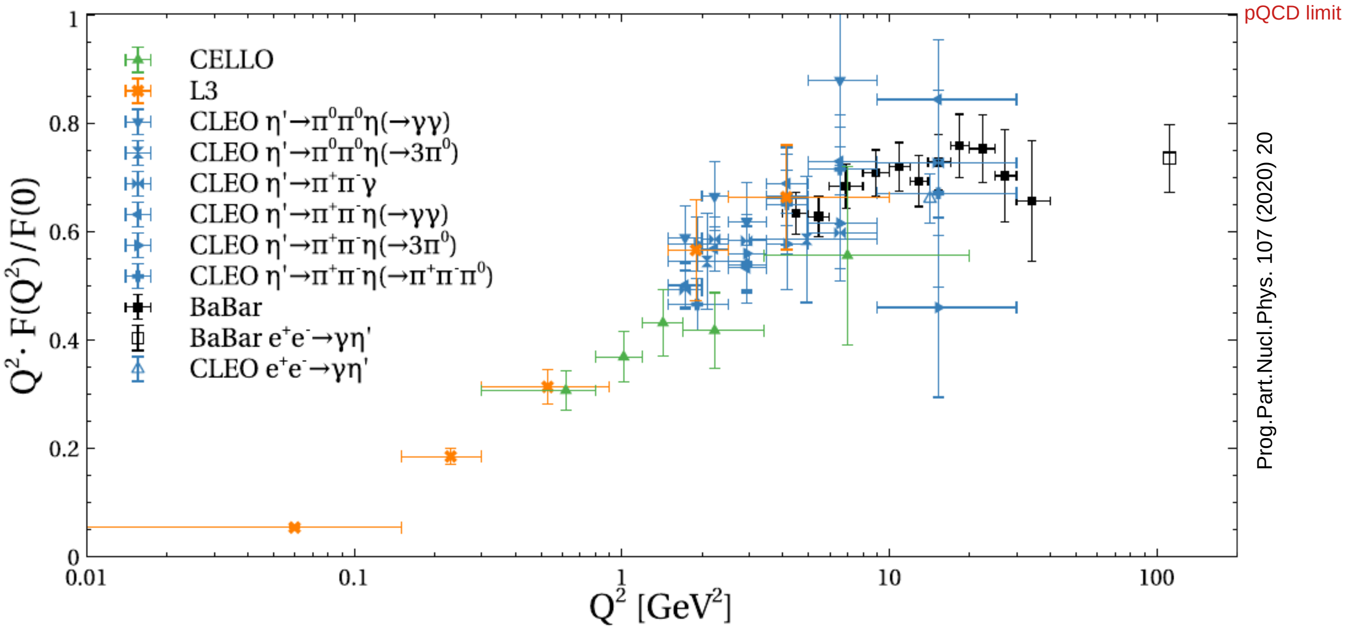
- World data dominated by B-factories
- Possible conflict with pQCD limit (BaBar-Belle puzzle)

η Transition Form Factor



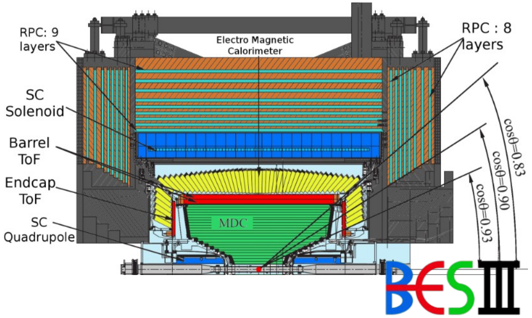
- World data dominated by B-factories
- Space- and timelike TFF in agreement with pQCD limit

η’ Transition Form Factor

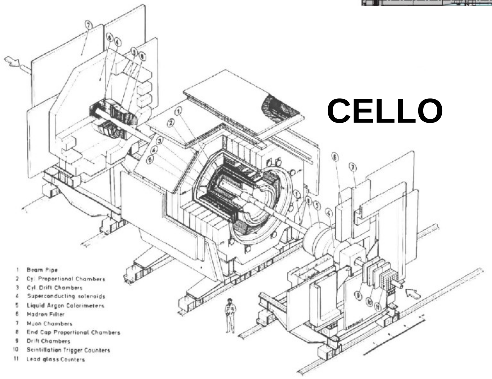


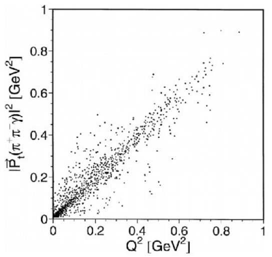
- World data dominated by B-factories
- Space- and timelike TFF in agreement with pQCD limit
- Smallest Q^2 range measured a LEP

Different Q^2 Ranges

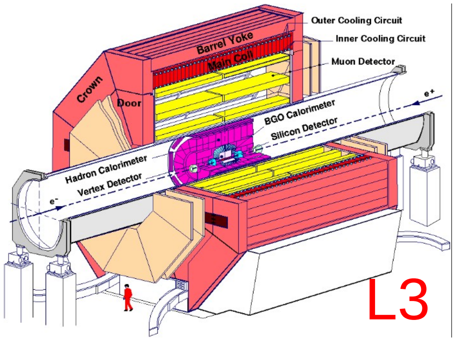


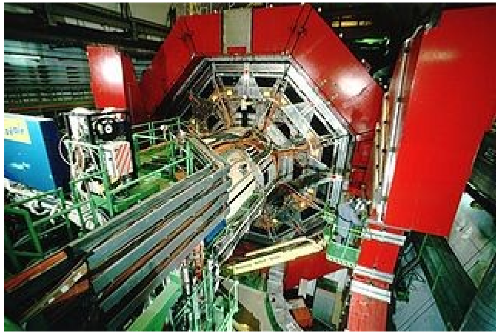
- BESIII, CLEO, BaBar, Belle only use main detector
  - Acceptance limited by kinematics
- CELLO at PETRA $(\sqrt{s} = 35\,\mathrm{GeV})$
  - small angle calorimeters (40-400 mrad)
- L3 at LEP $(\sqrt{s} = 91\,\mathrm{GeV})$
  - Small angle calorimeters (30-60 mrad)
  - Relation between p_T and largest Q^2 from simulation





Phys.Lett. B418 (1998) 399





Tagging Detectors

Install extra detectors to enhance acceptance

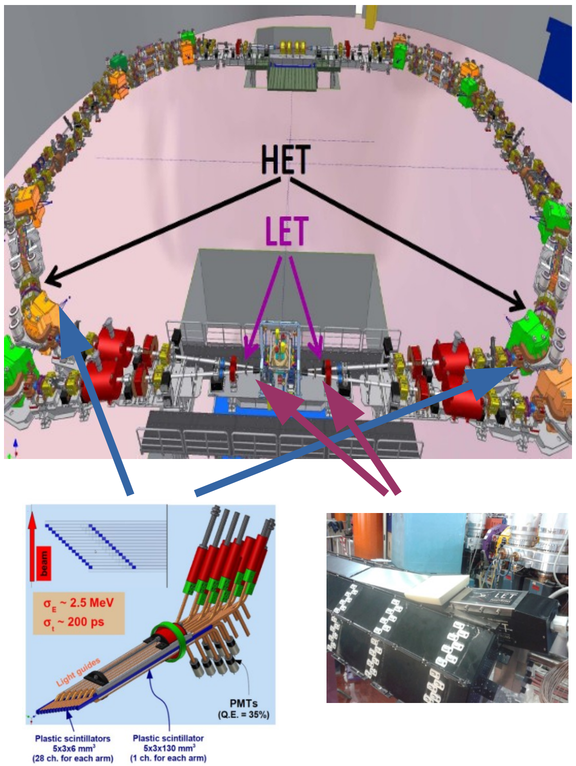
Example: KLOE-2

Low Energy Tagger

- 2 x 40 LYSO Crystals
- 1.5 m from the IP
- $150 < E_{e^\pm}[MeV] < 350$
- σ(E)/E < 10%

High Energy Tagger

- Scintillator hodoscope
- 11m from the IP
- DAΦNE dipoles as spectrometer
- $420 < E_{e^\pm}[MeV] < 495$



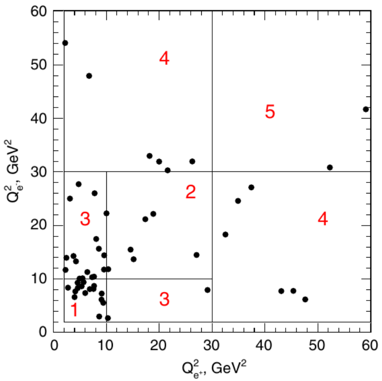
- Both leptons in HET → untagged configuration
  - $Q^2 < 10^{-3}$ GeV
- Leptons in KLOE and HET → single-tag configuration
  - $Q^2 < 0.1$ GeV²

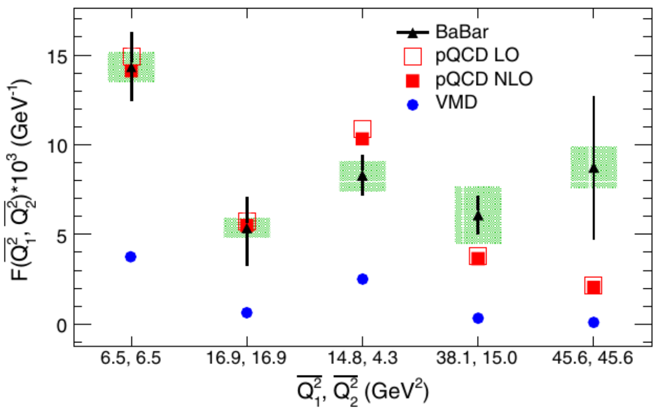
KLOE-2 data taking ended recently first results expected soon

Double-Tagged

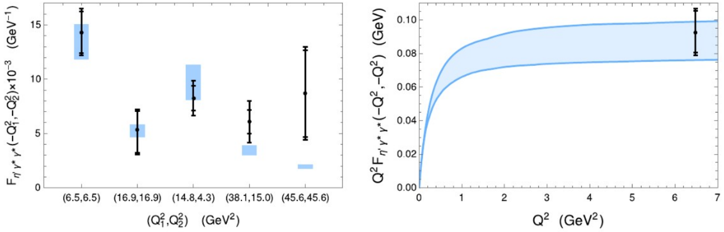
- First double-tagged measurement by Babar: $e^+e^- \to e^+e^-\eta'$
- 468.6 fb^{-1} at Y(4S)
- $\eta' \to \eta\pi^+\pi^-,\ \eta \to \gamma\gamma$
- Conditions on
  - total momentum and energy
  - Correlation of lepton energies
- Approx. 46 signal events
- five bins in (Q_1^2, Q_2^2)

Phys.Rev. D98 (2018) 112002



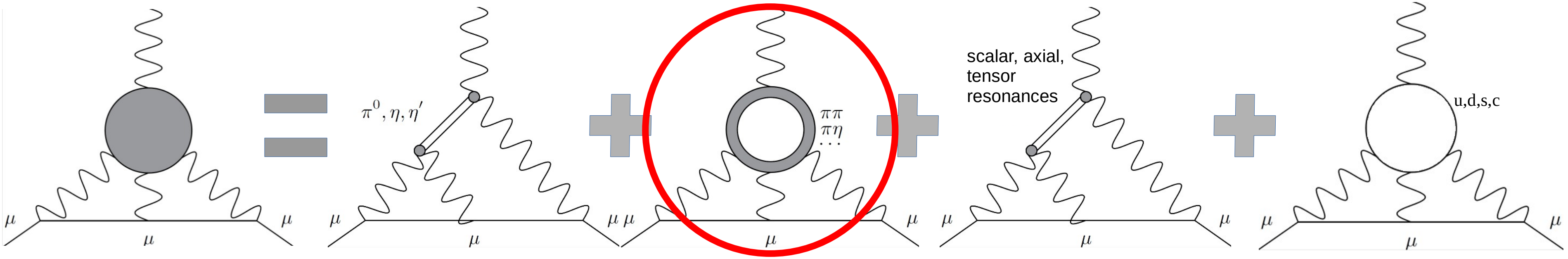


Comparison to Canterbury Approximants



Phys.Rept. 887 (2020) 1

Beyond Pseudoscalar Mesons



Required:

Partial waves of $\gamma^*\gamma^* \to \pi\pi, \gamma^*\gamma^* \to KK, \gamma^*\gamma^* \to \pi\eta, \ldots$ at arbitrary virtualities

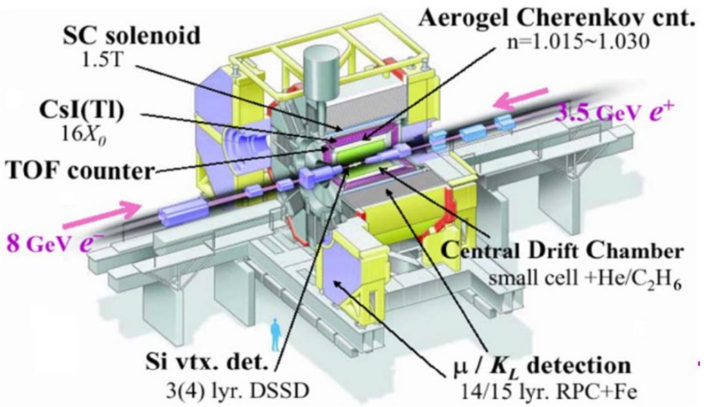
Two Pions

- Access to scalar and tensor resonance
  - Spectroscopy
  - Resonance parameters
- Polarizabilities
  - “stiffness” of pions
  - $\gamma\gamma \to \pi\pi$ related to $\gamma\pi \to \gamma\pi$
  - Information from mass threshold needed
- Rescattering effects

Untagged

Nakazawa, PhiPsi17

	GeV	|cosθ*| <	fb^{-1}	reference	year
γ J/ψ	3.2 - 3.8		32.6	PLB540, 33	2002
$\pi^+\pi^-$	2.4 - 4.1	0.6	88	PLB15, 39	2005
	0.8 -1.5	0.6	86	PRD75, 051101	2007
				JPhySocJpn76, 074102	2007
K^+K^-	1.4 - 2.4	0.6	67	EPJC32, 323	2003
	2.4 - 4.1	0.6	88	PLB15, 39	2005
ppbar	2.0 - 4.0	0.6	89	PLB621, 41	2005
4 mesons	2.75 - 3.75		395	EPJC53, 1	2006
KsKs	2.4 - 4.0	0.6	398	PLB651, 15	2007
	1.05 - 4.0	0.8	972	PTEP2013, 123C01	2013
$\pi^0\pi^0$	0.6 - 4.0	0.8	95	PRD78, 052004	2008
	0.6 - 4.1	0.8	223	PRD79, 052009	2009
$\eta\pi^0$	0.84 - 4.0	0.8	223	PRD80, 032001	2009
$\eta\eta$	1.096 - 3.8	1.0	393	PRD82, 114031	2010
ω J/ψ	3.9 - 4.2		694	PRL104, 092001	2010
ϕ J/ψ	4.2 - 5.0		825	PRL104, 112004	2010
$\omega\omega,\omega\phi,\phi\phi$	thr - 4.0		870	PRL108, 232001	2012
$\eta'\pi^+\pi^-$	1.4 - 3.4		673	PRD86, 052002	2012
π^0	$Q^2\in[4,40]GeV^2$		759	PRD86, 092007	2012
$\pi^0\pi^0$	$Q^2<30GeV^2$		759	PRD93, 032003	2016
ppbarK^+K^-	3.2 - 5.6		980	PRD93, 112017	2016



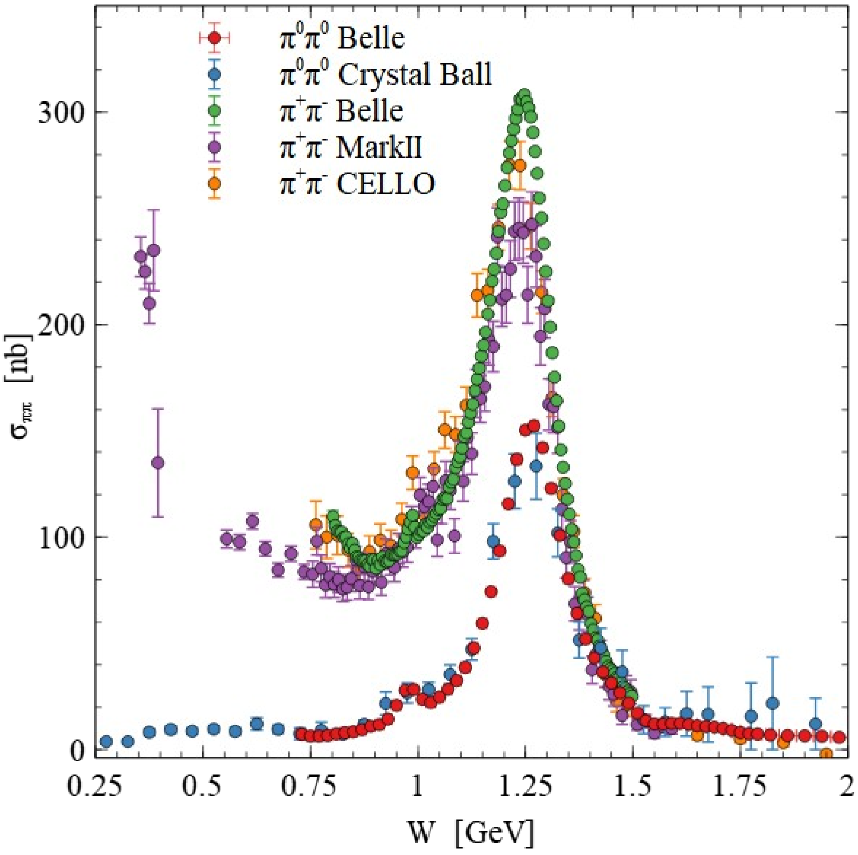
Untagged

- Detect two pions
- Require balanced p_T of pions
- Sum of measured energies/momenta $\lessapprox \frac{\sqrt{s}}{2}$

$$\frac{d^2}{dW\,d\cos\theta^*}(\gamma\gamma\to\pi\pi)\begin{cases}\pi^0\pi^0: & |\cos\theta^*|<0.8\\ \pi^+\pi^-: & |\cos\theta^*|<0.6\end{cases}$$

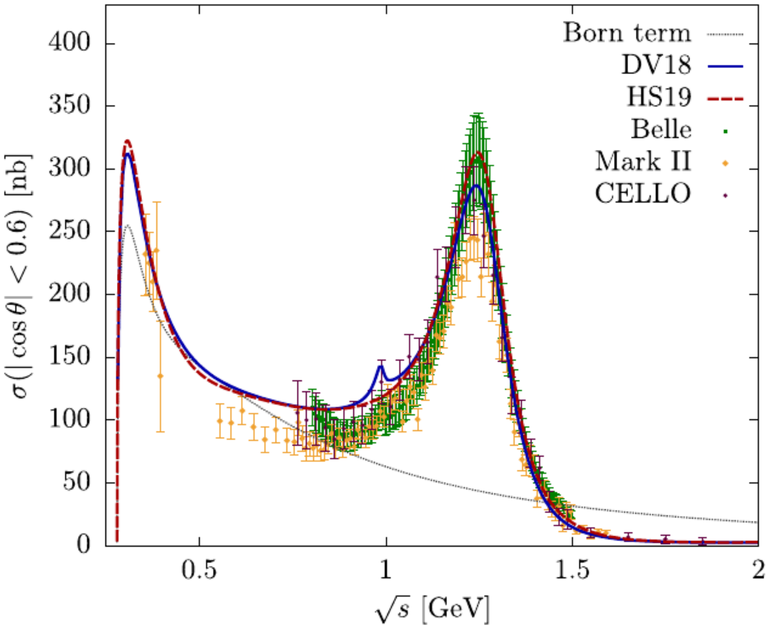
- Partial wave analysis to obtain resonance parameters

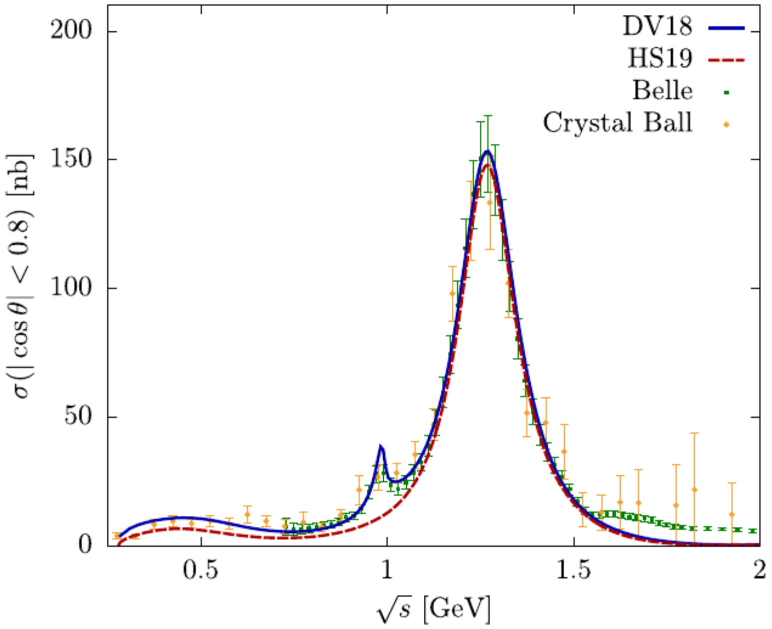
$$\frac{d\sigma}{d\Omega}(\gamma\gamma\to\pi^+\pi^-)=|S_0Y_0^0+D_0Y_0^2|^2+|D_2Y_2^2|^2$$



Untagged

Comparison to dispersive analyses





Phys.Rept. 887 (2020) 1

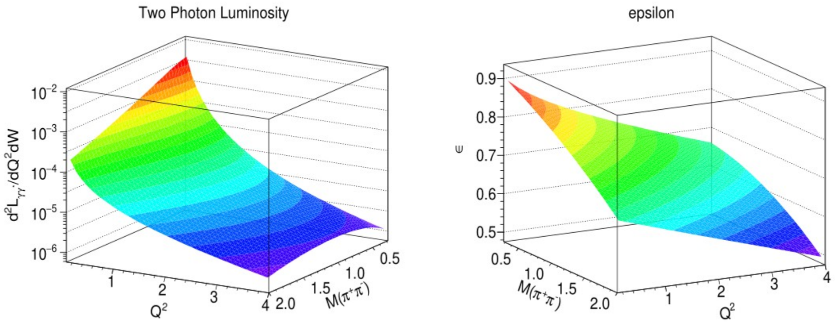
Information including virtual photons needed!

Two-photon cross Section

Longitudinal polarization contributes to cross section in a single-tagged measurement!

$$d\sigma_{ee} = \frac{\alpha^2}{16\pi^4 q_1^2 q_2^2}\sqrt{\frac{(q_1\cdot q_2)^2 - q_1^2 q_2^2}{(p_1\cdot p_2)^2 - m_e^4}}\left[4\rho_1^{++}\rho_2^{++}\sigma_{TT} + 2\rho_1^{++}\rho_2^{00}\sigma_{TL} + 2\rho_1^{00}\rho_2^{++}\sigma_{LT}\right]\frac{d^3p_1' d^3p_1'}{E_1' E_2'}$$

$$\frac{d^2\sigma_{ee}}{dQ^2 dW} = \frac{d\mathcal{L}_{\gamma\gamma}}{dQ^2 dW}\left(\sigma_{TT}(Q^2,0,W) + \varepsilon\sigma_{TL}(Q^2,0,W)\right)$$



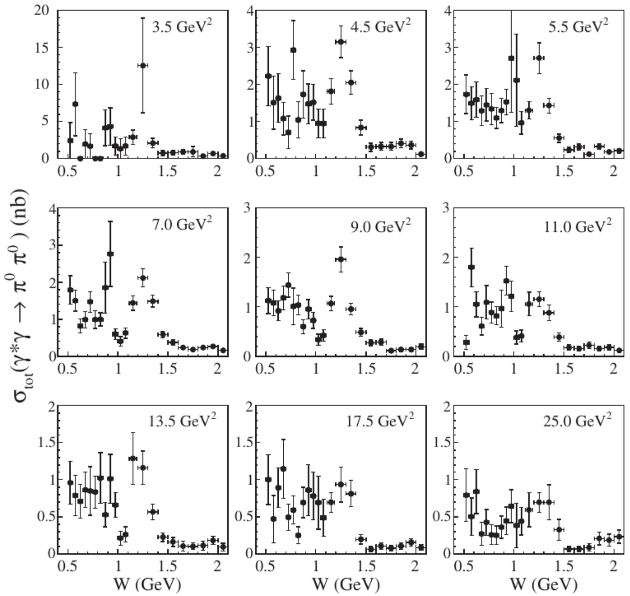
Calculated for BESIII measurement

Using:
Phys. Rept. 15 (1975) 181
Nucl.Phys. B54 (1973) 573

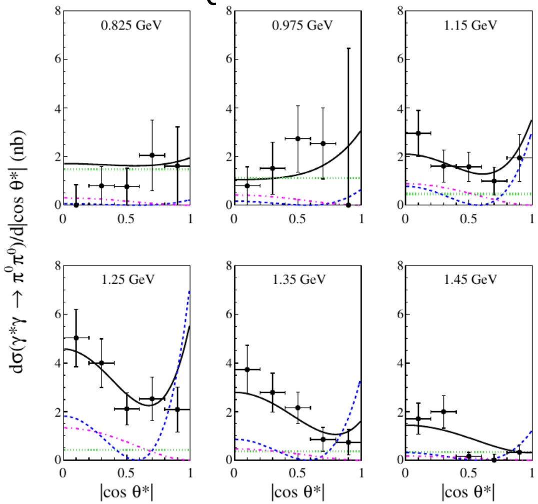
Single-Tagged

Phys.Rev. D93 (2016) 032003
Phys.Rev. D97 (2018) 052003

- First measurement performed at Belle $\pi^0\pi^0(K_sK_s)$
- Cross section studied depending on $W, Q^2, \cos\theta^*, \phi^*$
  - $3 \leq Q^2[\text{GeV}^2] \leq 30$
    - e^- -tag from 3.5 GeV
    - e^+ -tag from 5.5 GeV
  - $0.5 \leq W[\text{GeV}] \leq 2$
  - $|\cos\theta^*| \leq 1$



Q² = 4.5 GeV²

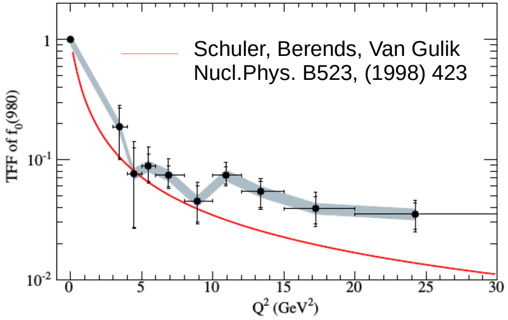


- Partial wave analysis to obtain TFFs

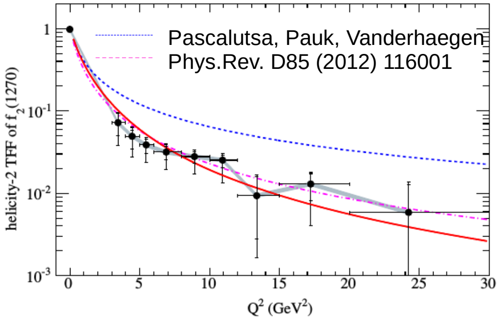
$$\frac{d\sigma(\gamma\gamma\to\pi^0\pi^0)}{4\pi d|\cos\theta^*|} = |S_0Y_0^0 + D_0Y_0^2|^2 + 2\varepsilon|D_1Y_1|^2 + |D_2Y_2^2|^2$$

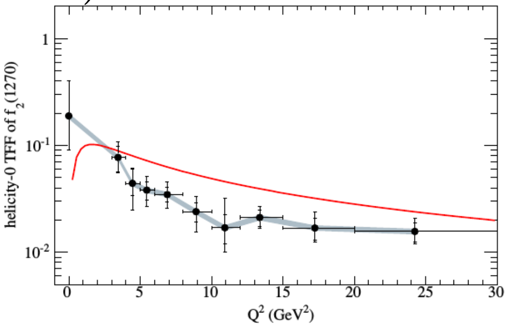
Single-Tagged

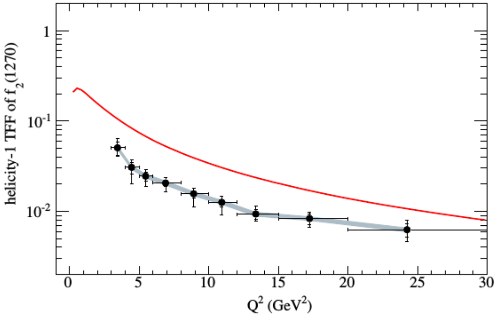
$f_0(980)$



$f_2(1270)$





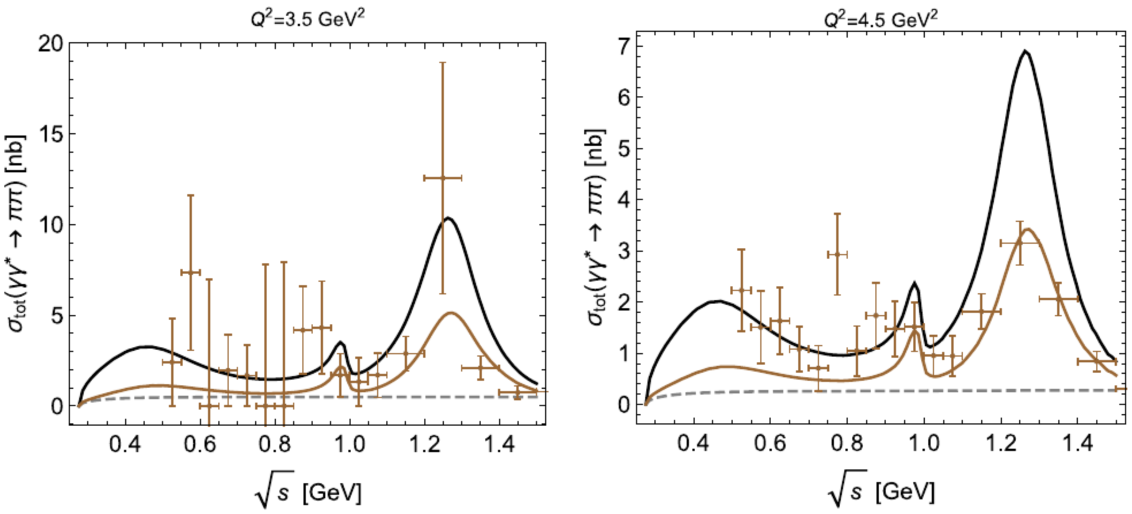


First measurement of scalar and tensor TFFs

- All helicities measured
  - Significant helicity-0 contribution
  - Non-zero helicity-1 contribution
  - Good agreement with theories for helicity-2

Single-Tagged

Comparison to dispersive analysis



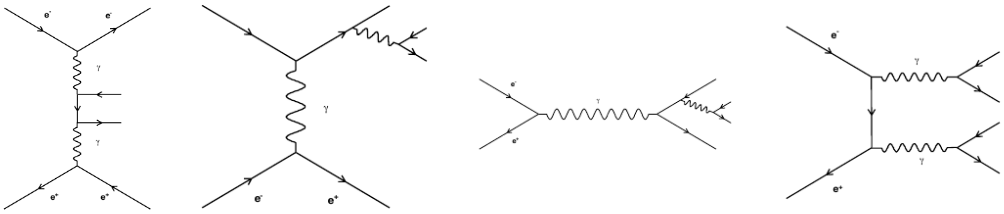
Prog.Part.Nucl.Phys. 107 (2020) 20

Agreement within uncertainties

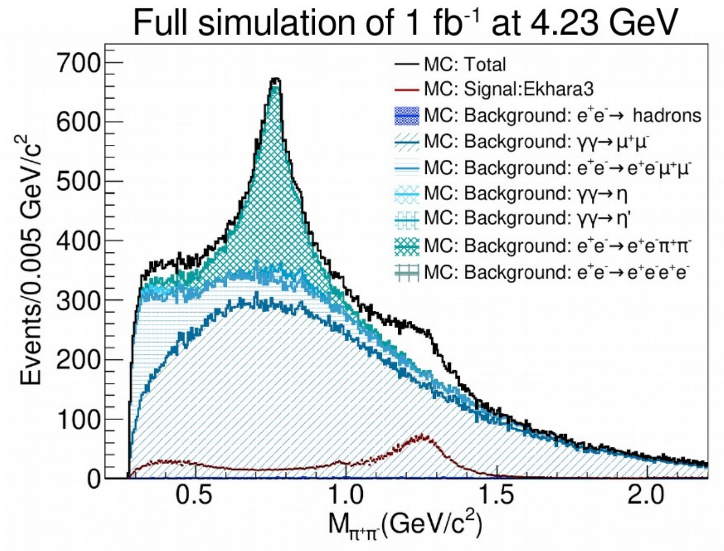
Tagged charged

Ongoing analysis at BESIII

- Event selection analoguosly to π TFF
- Reducible background $e^+e^- \to e^+e^-\mu^+\mu^-$
  - Dominating process



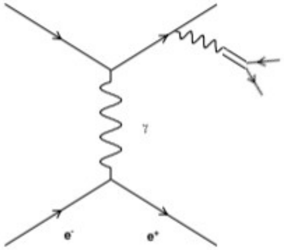
  - Reliable simulations of all amplitudes exist
  - Use machine learning for improved pion-muon separation
  - Subtract remaining contributions

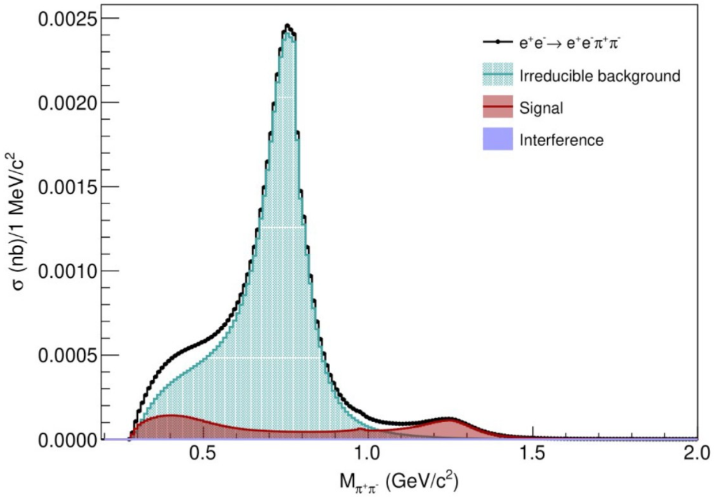


Tagged charged

Ongoing analysis at BESIII

- Event selection analoguosly to π TFF
- Reducible background $e^+e^- \to e^+e^-\mu^+\mu^-$
- Irreducible background $e^+e^- \to e^+e^-\pi^+\pi^-$
  - Radiative Bhabha scattering coupling to ρ mesons
  - Potential interferences
  - Study with optimized MC generators

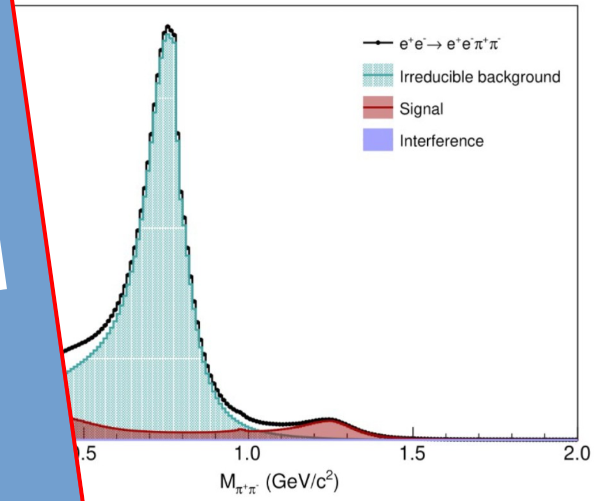




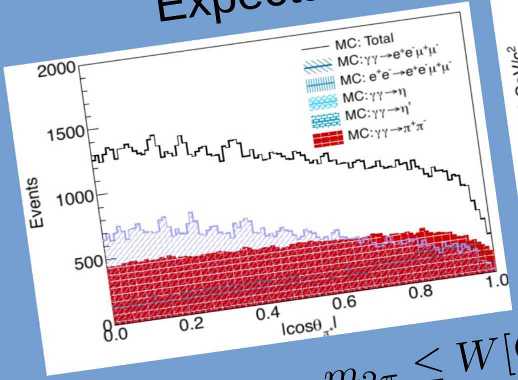
Tagged charged

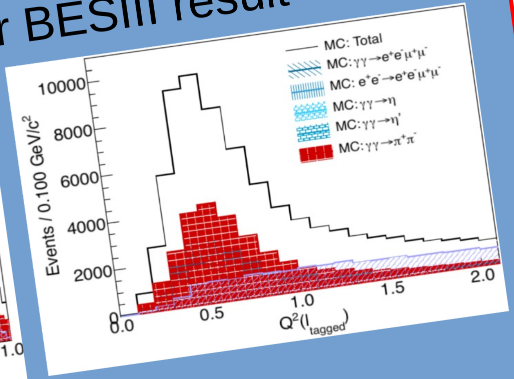
Ongoing analysis at BESIII

- Event selectio
- Reducible ba
- Irreducible ba
  - Radiative
  - Potential
  - Study with



Expectations for BESIII result





$$m_{2\pi} \leq W[\mathrm{GeV}] \leq 2$$

$$0.2 \leq Q^2[\mathrm{GeV}^2] \leq 2$$

$$|\cos\theta^*| \leq 1$$

Axial Mesons

Landau-Yang Theorem

Dokl. Akad. Nauk Ser. Fiz. 60 (1948) 207
Phys. Rev. 77 (1950) 242

- Two photons couple only to J=0,2
- Not valid for virtual photons!

Axial mesons accessible in single-tagged measurements!

- Coupling at small Q^2 defined by TL-polarized photons

$$\tilde{\Gamma} = \lim_{Q^2 \to 0} \frac{M^2}{Q^2} \Gamma^{TL}_{\gamma\gamma*}$$

- Contribution of TT-polarized photons at large Q^2

Cross Section

- In general three TFFs in cross section
- Single-tagged:

$$d\sigma_{ee} = \frac{\alpha^2}{16\pi^4 q_1^2 q_2^2}\sqrt{\frac{(q_1\cdot q_2)^2 - q_1^2 q_2^2}{(p_1\cdot p_2)^2 - m_e^4}}\left[4\rho_1^{++}\rho_2^{++}\sigma_{TT} + 2\rho_1^{++}\rho_2^{00}\sigma_{TL} + 2\rho_1^{00}\rho_2^{++}\sigma_{LT}\right]\frac{d^3p_1' d^3p_1'}{E_1' E_2'}$$

- Q^2 dependence unknown → models needed

- Use effective TFF:

Nucl. Phys. B 523 (1998) 423

$$|\tilde{F}|^2 = \lim_{Q_2^2\to 0}\kappa(f_{TT} + \varepsilon f_{TL})$$

$$\tilde{F} = \frac{M^2}{M^2+Q^2}\sqrt{2\varepsilon}\frac{Q}{M}$$ for small Q^2

$$\sigma(\gamma\gamma* \to R) = \frac{24\pi\Gamma_{\gamma\gamma}\Gamma}{(W^2-M^2)^2+\Gamma^2M^2}\left(1+\frac{Q^2}{M^2}\right)\left[\frac{Q^2}{M^2}\left(1+\frac{Q^2}{2M^2}\right)\frac{2}{(1+\frac{Q^2}{\Lambda^2})^4}\right]$$

Phys.Lett. B526 (2002) 269

More complete VMD based TFF descriptions developed recently:

Phys.Rev. D 96 (2017) 076004
Phys.Lett.B 800 (2020) 135117

JHEP 05 (2020) 159
JHEP 07 (2021) 106

$f_1(1285)$

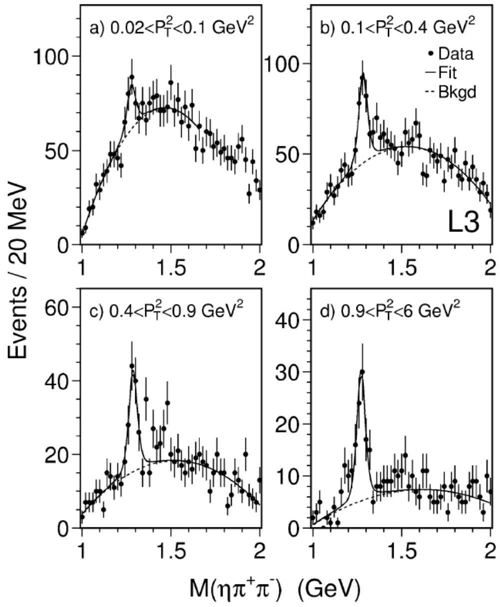
Lastest measurement from L3

$f_1(1285) \to \pi^+\pi^-\eta$

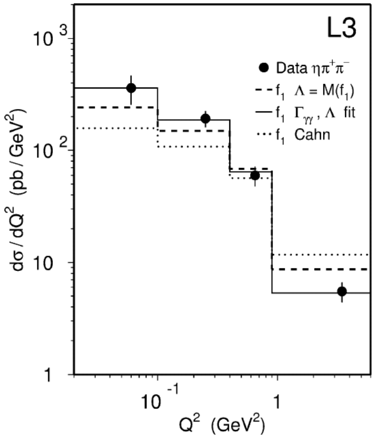
- Dominated by intermediate $a_0(980)$
- Q^2 dependence from p_T
- Lepton-based cross section
  - fitted with different models

$$\Lambda = 1.04 \pm 0.06 \pm 0.05\,\mathrm{GeV}$$

$$\tilde{\Gamma} = 3.5 \pm 0.6 \pm 0.5\,\mathrm{keV}$$



Phys.Lett. B526 (2002) 269



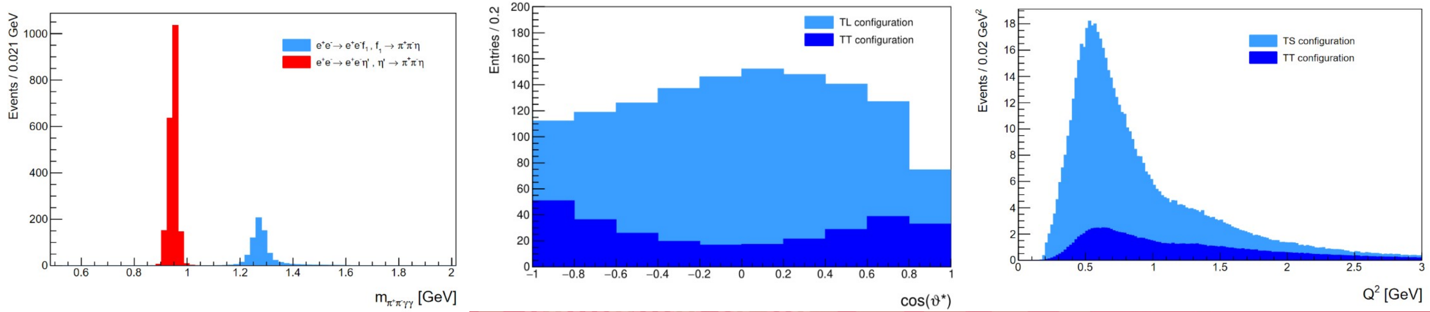
$f_1(1285)$

Studies in preparation at BaBar and BESIII

- Complementary Q^2 ranges
- $f_1(1285) \to \pi^+\pi^-\eta$
- Make use of intermediate state $a_0(980)$
- GGResRC generator to simulate individual helicity contributions
- Separate TT and TL contributions using helicity angle

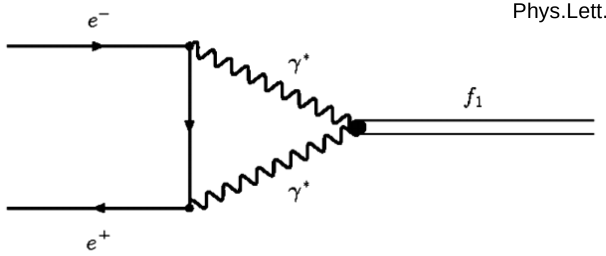
Many thanks to
E. Kozyrev et al.

Simulations from feasibility study at BESIII



A Different Approach to $f_1(1285)$

Phys.Lett. B800 (2020) 135074

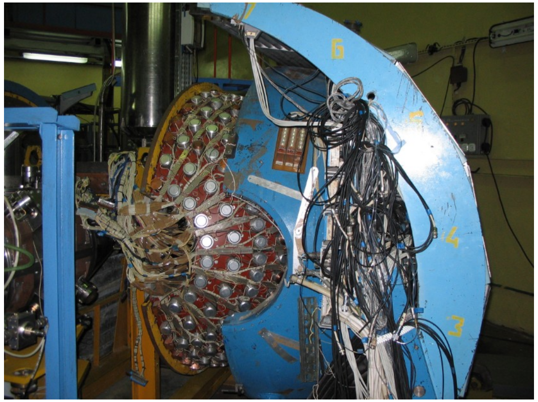


- Direct production in „two-photon annihilation“

  $$e^+e^- \to f_1(1285)$$

- Essential to constrain TFFs
- Suppressed compared to normal annihilation processes
- Choice of final state essential

- SND measurement of $f_1(1285) \to \eta\pi^0\pi^0$
  - 15 pb^{-1} in 12 scan points from 1200 – 1400 MeV
  - 3.5 pb^{-1} at 1282±0.63 MeV

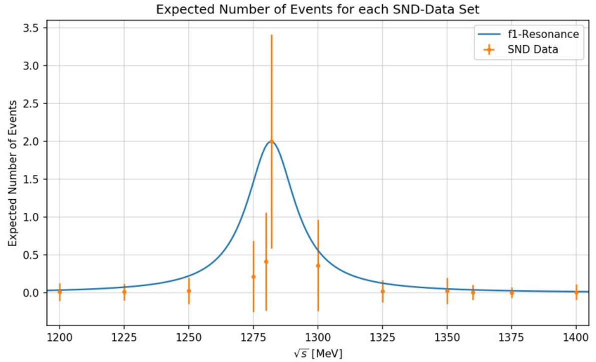


A Different Approach to $f_1(1285)$

Phys.Lett. B800 (2020) 135074

- Background from $e^+e^- \to \omega\pi^0, \omega\pi^0\pi^0$ with $\omega \to \pi^0\gamma$, and $e^+e^- \to \eta\gamma$
- Rejected by
  - General conditions on energy/momentum conservation
  - Kinematic fits to signal and background hyotheses
  - ω veto
- 2 event candidates at peak energy
  - $\epsilon = (0.79 \pm 0.08)\%$
  - $\sigma(e^+e^- \to f_1(1285)) = 45^{+33}_{-24}\,\mathrm{pb}$
  - $\mathcal{B}(f_1(1285) \to e^+e^-) = 5.1^{+3.7}_{-2.7} \cdot 10^{-9}$
  - Consistent with prediction $\sigma(e^+e^- \to f_1(1285)) = 31 \pm 16\,\mathrm{pb}$

Phys.Rev. D96 (2017) 076004
Phys.Lett. B800 (2020) 135117



A Theoretician's Whish List

Priorities for new experimental input and cross-checks.

issue	experimental input [I] or cross-checks [C]
axials, tensors, higher pseudoscalars	$\gamma^{(*)}\gamma^* \to 3\pi,\ 4\pi,\ K\bar{K}\pi,\ \eta\pi\pi,\ \eta'\pi\pi$ [I]
missing states	inclusive $\gamma^{(*)}\gamma^* \to$ hadrons at 1–3 GeV [I]
dispersive analysis of $\eta^{(\prime)}$ TFFs	$e^+e^- \to \eta\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-e^+e^-$ [I]
	$\gamma\pi^- \to \pi^-\eta$ [C]
dispersive analysis of π^0 TFF	$\gamma\pi \to \pi\pi$ [I]
	high accuracy Dalitz plot $\omega \to \pi^+\pi^-\pi^0$ [C]
	$e^+e^- \to \pi^+\pi^-\pi^0$ [C]
	$\omega, \phi \to \pi^0 l^+ l^-$ [C]
pseudoscalar TFF	$\gamma^{(*)}\gamma^* \to \pi^0, \eta, \eta'$ at arbitrary virtualities [I,C]
pion, kaon, $\pi\eta$ loops (including scalars and tensors)	$\gamma^{(*)}\gamma^* \to \pi\pi,\ K\bar{K},\ \pi\eta$ at arbitrary virtualities, partial waves [I,C]

Phys.Rept. 887 (2020) 1

Summary

- Data driven approaches require input
- World wide efforts to provide relevant data