Some basics on muon g-2

D. Nomura (IUHW/KEK)

talk at International Physics School
on Muon Dipole Moments and Hadronic Effects
in memoriam Simon Eidelman
hosted by JGU Mainz

30 August 2021

Refs (mainly reviews and books)

- T. Aoyama et al. (Muon g-2 Theory Initiative),
  "The anomalous magnetic moment of the muon in the Standard Model"
  ("Muon g-2 White Paper")
  Phys. Rept. **887** (2020) 1-166
  DOI:10.1016/j.physrep.2020.07.006
  `arXiv:2006.04822 [hep-ph].`
- F. Jegerlehner
  "The Anomalous Magnetic Moment of the Muon" (2nd edition)
  Springer Tracts Mod. Phys. **274** (2017) 1-693
  DOI:10.1007/978-3-319-63577-4
- B. Lee Roberts and W. J. Marciano (ed.)
  "Lepton Dipole Moments"
  Adv. Ser. Direct. High Energy Phys. 20 (2009) 1
  DOI:10.1142/7273

References

- F. Jegerlehner and A. Nyffeler
  "The Muon g-2"
  Phys. Rept. **477** (2009) 1-110
  `arXiv:0902.3360 [hep-ph]`
- J. P. Miller, E. de Rafael and B. Lee Roberts
  "Muon (g-2): Experiment and theory"
  Rept. Prog. Phys. **70** (2007) 795
  `hep-ph/0703049`
- K. Melnikov and A. Vainshtein
  "Theory of the muon anomalous magnetic moment"
  Springer Tracts Mod. Phys. **216** (2006) 1-176
  DOI:10.1007/3-540-32807-6

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Current status of the Standard Model (SM)

LHC data vs Standard Model

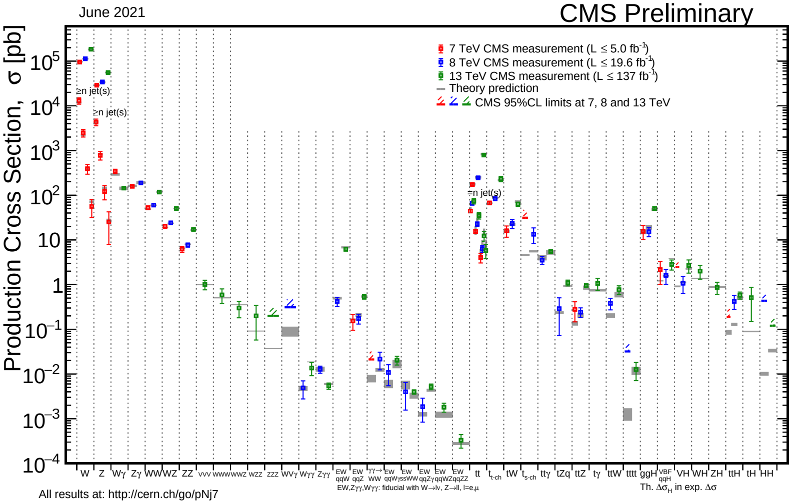
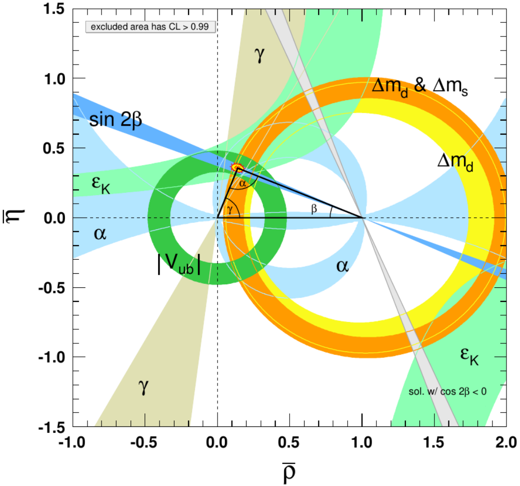


Fig. from CMS TWiki

LHC cross section data agree very well the SM

Unitarity of CKM matrix



$$\bar{\rho} \equiv -\mathrm{Re}\left[(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)\right]$$

$$\bar{\eta} \equiv -\mathrm{Im}\left[(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)\right]$$

Fig. from Particle Data Book, 2020

Data on CP violation support the SM, too.

Electroweak precision data vs SM

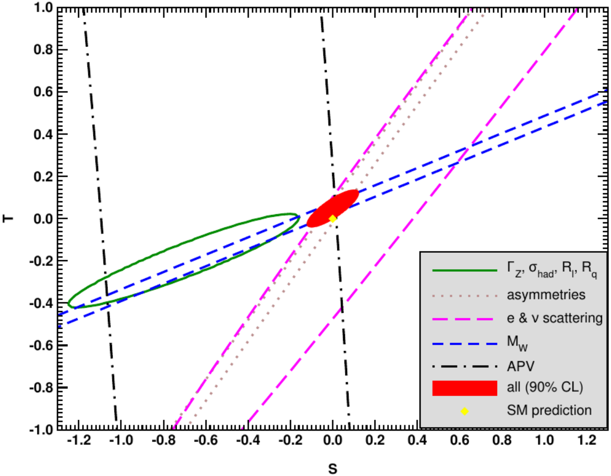


Fig. from Particle Data Book, 2020

Precision measurements at $\sim M_Z$ agree very well with the SM prediction.

Although the SM is such a successful theory, nobody believes that it is the ultimate theory.

Because...

What the SM cannot explain

- Why 3 generations? Why $SU(3)_C \times SU(2)_L \times U(1)_Y$?
- Many (19) free parameters

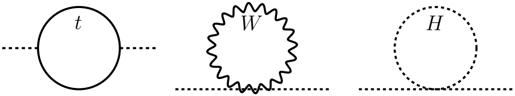
gauge couplings	g', g, g_s
vacuum expectation value (VEV)	v
Higgs boson mass	m_H
lepton masses	m_e, m_μ, m_τ
quark masses	$m_u, m_d, m_s, m_c, m_b, m_t$
quark mixing angles	ϕ_1, ϕ_2, ϕ_3
CKM phase	δ
(QCD θ-angle	$\bar{\theta}$)

- Neutrino masses & mixing matrix
- Why $\bar{\theta} \lesssim 2 \times 10^{-10}$? (strong CP problem)
- Why $m_{\text{weak}} \ll m_{\text{GUT}}$? (gauge hierarchy problem)
- Dark matter & dark energy
- Origin of the baryon number
- Gravity
- ⋮

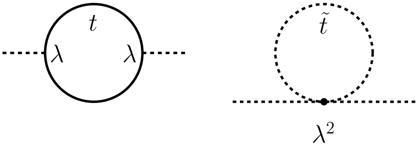
To solve these problems, new physics beyond the SM should exist.

It might exist at the TeV scale, because....

Hierarchy Problem in the Standard Model

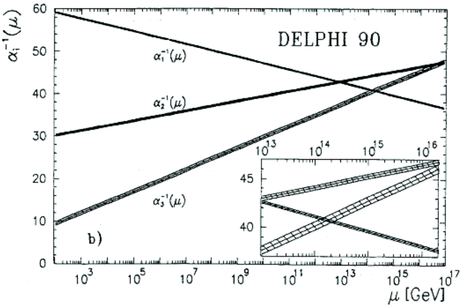


Radiative corrections to $m_{\mathbf{H}}^{\mathbf{2}}$ diverges as $\sim \mathbf{\Lambda^2}$. $\Leftrightarrow$ Physical Higgs mass $\sim m_{\mathbf{weak}}^{\mathbf{2}}$. (**Fine-tuning necessary** if $\mathbf{\Lambda} \gg m_{\mathbf{weak}}$)
In **SUSY Standard Models** this is automatically solved since **(softly broken) SUSY** ensures the cancellation of the quad. divergences. For example,

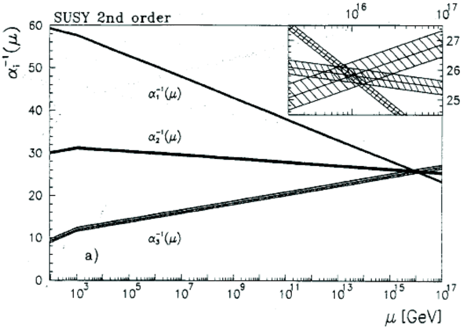


Gauge coupling unification:

SM case



MSSM case



Amaldi-de Boer-Fürstenau '91

SUSY particles change the 'running' of the gauge couplings above m_{SUSY}. Gauge unification also explains why the electric charges are quantized.

Muon g-2: Hint of new physics?

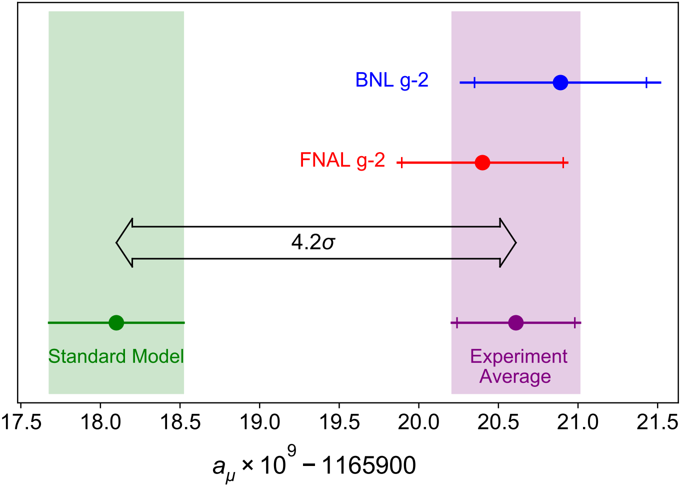


Fig. from Phys. Rev. Lett. 126 (2021) 141801 [arXiv:2104.03281]

4.2 σ discrepancy in $(g\text{-}2)_\mu$: new physics?

Many physicists thought that the LHC would discover new particles beyond the SM, once it started operation.

But the reality is ...

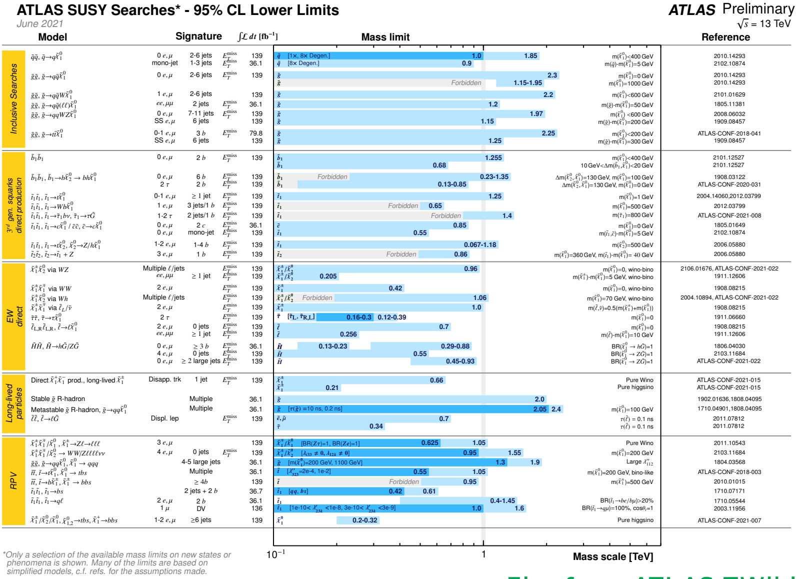


Fig. from ATLAS TWiki

No SUSY particles found so far

Current bound from LHC: $m_{\mathrm{SUSY}} \gtrsim 1$ TeV

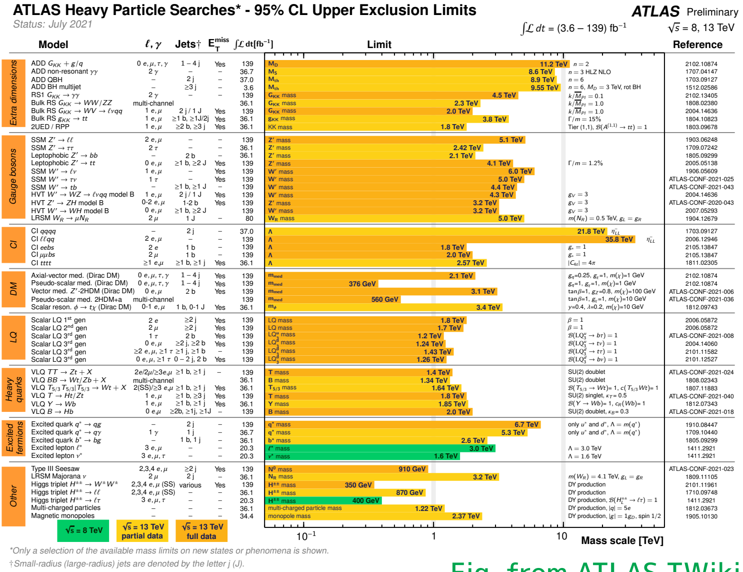


Fig. from ATLAS TWiki

Situation is similar for non-SUSY new particles

Current bound from LHC: $m_{\text{exotics}} \gtrsim 1$ TeV

But the LHC is not almighty.

- Not suitable for very precise measurements due to various uncertainties (pdf, BG, ...)
- Limitation on $\sqrt{s}$ accessible in the near future.

$\Longrightarrow$ Important to combine with other methods

- e^+e^- colliders, various precision measurements (flavor physics, EDM searches, (g-2)$_{e,\mu}$, $0\nu\beta\beta$ decay searches...), dark matter searches, cosmology, ...

$\Longrightarrow$ Precision physics, in particular **the muon g-2**, is a good complement to the LHC.

Why Muon g-2?

- **4.2σ Anomaly Reported**
  Long standing anomaly ($\sim$ 20 yrs), in spite of careful studies on every aspect.
  (→ Major theoretical blunder unlikely.)
  Hint of New Physics beyond the Standard Model?
- No new physics at the LHC so far
  Intensity frontier: more and more important
- Long history of research
  1st $(g-2)_\mu$ exp.: Garwin, Lederman & Weinrich (1957)
  Well-established place to search for new physics
- Leptonic observable
  Experimentally and theoretically clean

Press Release from Fermilab (7 April 2021)

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First results from Fermilab's Muon g-2 experiment strengthen evidence of new physics

April 7, 2021

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Media contact

- Tracy Marc, Fermilab, media@fnal.gov, 224-290-7803

The long-awaited first results from the Muon g-2 experiment at the U.S. Department of Energy's Fermi National Accelerator Laboratory show fundamental particles called muons behaving in a way that is not predicted by scientists' best theory, the Standard Model of particle physics. This landmark result, made with unprecedented precision, confirms a discrepancy that has been gnawing at researchers for decades.

The strong evidence that muons deviate from t

hint at exciting new physics. Muons act as a window into the subatomic world and

from Fermilab webpage

Fermilab Muon g-2 exp 1st Results

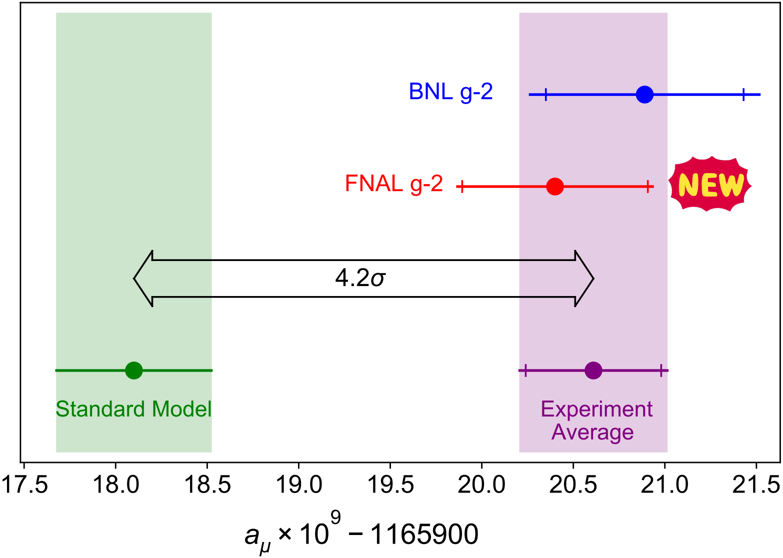

Fig. from Phys. Rev. Lett. 126 (2021) 141801 [arXiv:2104.03281]

Muon g-2 in the Media

The New York Times

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A Tiny Particle's Wobble Could Upend the Known Laws of

Experiments with particles known a
are forms of matter and energy vital
the cosmos that are not yet known to

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Science

Muons: 'Strong' evidence found for a new force of nature

By Pallab Ghosh
Science correspondent

Magnetic Moment: Definition

Suppose that there is a point particle f at rest in an external magnetic field $\vec{B}$. If the interaction Hamiltonian H_{mdm} between f and $\vec{B}$ is given by

$$H_{\text{mdm}} = -\vec{\mu} \cdot \vec{B} \ ,$$

then $\vec{\mu}$ is called the **magnetic dipole moment** of f.

- If f has a non-zero spin $\vec{s}$, then $\vec{\mu} \propto \vec{s}$
- H_{mdm} is P-even and T-even
- Its cousins:

  EDM $\vec{d}$: $H_{\text{EDM}} = -\vec{d} \cdot \vec{E}$ (P-odd, T-odd)
  (EDM: electric dipole moment)

  anapole $\vec{a}$: $H_{\text{ana}} = -\vec{a} \cdot (\nabla \times \vec{B})$ (P-odd, T-even)

Muon g-2: introduction

Lepton magnetic moment $\vec{\mu}$: $\vec{\mu} = g\frac{e}{2m}\vec{s}$ $\vec{s}$: spin

Anomalous magnetic moment a: $a \equiv (g-2)/2$

Historically,

- $g = 2$ at tree level (Dirac, 1928)
- $a = \alpha/(2\pi)$ at 1-loop (QED) (Schwinger, 1947)

Today, still important since...

- One of the **most precisely measured** quantities:

$$a_\mu(\text{exp}) = 11\,659\,206.1(4.1) \times 10^{-10} \quad (0.35\text{ppm})$$

(B. Abi et al., 2021)

- **Extremely useful** in probing/constraining new physics beyond the SM

Dipole moments of a spin-1/2 particle

For a spin-1/2 particle f,

$$\langle f(p')|J_\mu^{\text{em}}|f(p)\rangle = \bar{u}_f(p')\Gamma_\mu u_f(p) \,,$$

$$\Gamma_\mu = F_1(q^2)\gamma_\mu + \frac{i}{2m_f}F_2(q^2)\sigma_{\mu\nu}q^\nu$$
$$- F_3(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 - F_4(q^2)(\gamma_\mu q^2 - 2m_f q_\mu)\gamma_5$$

There are no other independent form factors of a spin-1/2 particle other than $F_1(q^2), \ldots, F_4(q^2)$ (See e.g., Nowakowski, Paschos, & Rodriguez, physics/0402058)

$F_1(0) = -eQ_f$	(electric charge)
$F_2(0) = -eQ_f a_f$	(a_f : anomalous magnetic moment)
$F_3(0) = d_f$	(EDM)
$F_4(0) = \tilde{a}_f$	(anapole moment)

If f is a Majorana particle, then $F_1(q^2) = F_2(q^2) = F_3(q^2) = 0$.

Breakdown of SM prediction for muon g-2

From Table 1 of the White Paper

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

HVP: Hadronic Vacuum Polarization contribution
HLbL: Hadronic Light-by-Light contribution

$$a_\mu(\text{exp, BNL}) - a_\mu(\text{SM}) = 27.9(7.6) \times 10^{-10}\ (3.7\ \sigma)$$

$$a_\mu(\text{exp, 2021}) - a_\mu(\text{SM}) = 25.1(5.9) \times 10^{-10}\ (4.2\ \sigma)$$

NEW

QED contribution (1)

QED contribution:

$$a_\mu(\text{QED}) = \frac{\alpha}{2\pi} + 0.765857425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32)\left(\frac{\alpha}{\pi}\right)^3$$
$$+ 130.8796(63)\left(\frac{\alpha}{\pi}\right)^4 + 752.2(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$
$$= 11658471.892(0.003) \times 10^{-10}\,,$$ (numbers from PDG 2020)

where the uncertainty is dominated by that of α.

- 5-loop calculation! (Aoyama, Hayakawa, Kinoshita & Nio)
- The 4-loop corrections $\simeq 38 \times 10^{-10} \simeq \mathcal{O}(a_\mu(\text{exp}) - a_\mu(\text{SM}))$.
- The 4-loop contribution now fully cross-checked by another group. Mass-independent part by S. Laporta (Phys.Lett. **B772** (2017) 232), and mass-dependent part by A. Kurz et al (Nucl. Phys. **B879** (2014) 1; Phys. Rev. **D92** (2015) 073019; ibid. **D93** (2016) 053017)
- The 5-loop contribution very small ($\simeq 0.5 \times 10^{-10} \ll a_\mu(\text{exp}) - a_\mu(\text{SM})$)

QED contribution (2)

QED contribution to the **electron** $g-2$:

$$a_e(\text{QED}) = \frac{\alpha}{2\pi} - (0.32847844400\ldots)\left(\frac{\alpha}{\pi}\right)^2 + (1.181234017\ldots)\left(\frac{\alpha}{\pi}\right)^3 - 1.91206(84)\left(\frac{\alpha}{\pi}\right)^4 + 7.79(34)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

(coefficients from CODATA 2014)

QED contributions to the **muon** $g-2$:

$$a_\mu(\text{QED}) = \frac{\alpha}{2\pi} + 0.765857425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.8796(63)\left(\frac{\alpha}{\pi}\right)^4 + 752.2(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

(coefficients from PDG 2020)

At higher orders, the coefficients of a_μ(QED) are much larger than those of a_e(QED). This happens because ....

logarithmic enhancement in muon g-2

the logarithmic enhancement $\ln(m_\mu/m_e) \approx 5.3$

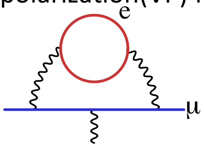
note: It does not exist for the lightest lepton, electron.

Two sources of the logarithm

1. Charge renormalization of the vacuum-polarization(VP) function

   2nd-order VP arises

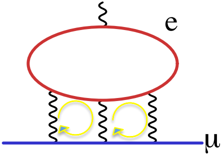
$$\frac{2}{3}\ln(m_\mu/m_e) - \frac{5}{9} \sim 3$$



   "Renormalization Group" estimate

2. Light-by-light scattering diagram

$$\frac{2}{3}\pi^2 \ln(m_\mu/m_e) \sim 35$$



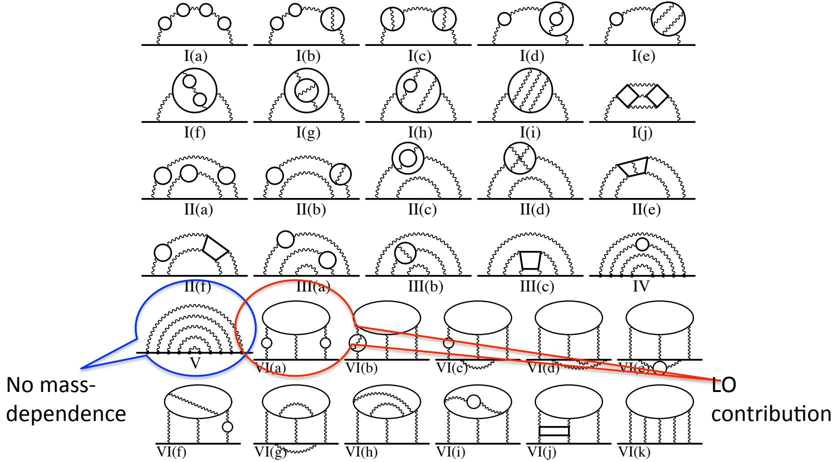
Coulomb photon loops provide the factor π^2

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

10^{th}-order contribution

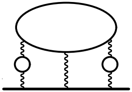
12,672 Feynman vertex diagrams contribute to the 10^{th} order .

They are classified into 32 gauge-invariant subsets over 6 sets.



Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

10^{th}-order leading term of $A_2^{(10)}$



VI(a)

The Leading Order(LO) contribution:

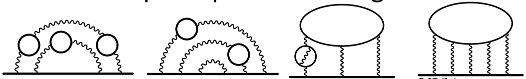
6^{th}-order light-by-light x two 2^{nd}-order vp's

estimate 20 x 3^2 x 6 ways ~ 1080

l-by-l 2 vp

Actually, its contribution is 542.760 ± 0.099 > $(\alpha/\pi)^{-1}$ ~430

Can also pick up the NLO diagrams:



II(a) III(a) VI(b) VI(k) ....

A. L. Kataev, PRD74(2006)073011

The numerical results are consistent

with the renormalization group estimate

The total of 31 subsets of the mass-dependent 10^{th}-order term

$$A_2^{(10)}(m_\mu/m_e) = 742.18(87)$$

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

Summary of 8th and 10th-order QED to muon g-2

$$A_2^{(8)}(m_\mu/m_e) = 132.6852\ (60)$$
$$A_2^{(8)}(m_\mu/m_\tau) = 0.042\ 34\ (12)$$
$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\ 72\ (4)$$
$$A_2^{(10)}(m_\mu/m_e) = 742.18\ (87)$$
$$A_2^{(10)}(m_\mu/m_\tau) = -0.068\ (5)$$
$$A_3^{(10)}(m_\mu/m_e, m_\mu/m_\tau) = 2.011\ (10)$$

QED contributions to the muon g-2 is now firmly established.

Rough estimate of the 12th-order contribution:

6th-order light-by-light x three 2nd-order vp x 10 ways

~ 20 x 3^3 x 10 $(\alpha/\pi)^6$ ~5,000 $(\alpha/\pi)^6$ ~ 0.08 x 10^{-11}

Recall the aimed goal of the on-going experiments ~ 12 x 10^{-11}

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

Electroweak Contribution

Electroweak (EW) contribution:

$$a_\mu(\text{EW}) = \underbrace{19.48 \times 10^{-10}}_{\text{1-loop}} + \underbrace{(-4.12(10) \times 10^{-10})}_{\text{2-loop}} + \underbrace{\mathcal{O}(10^{-12})}_{\text{3-loop leading log}}$$

$$= 15.36(10) \times 10^{-10} \,,$$ (Number taken from PDG 2020)

where the uncertainty mainly comes from quark loops.

- 1-loop result published by many groups (Bardeen-Gastmans-Lautrup, Altarelli-Cabibbo-Maiani, Jackiw-Weinberg, Bars-Yoshimura, Fujikawa-Lee-Sanda) in 1972, and now a textbook exercise (Peskin & Schroeder's textbook, Problems 6.3 (Higgs) and 21.1 (W, Z))
- 2-loop contribution ($\sim$ 1700 diagrams in the 't Hooft-Feynman gauge) enhanced by $\ln(m_Z/m_\mu)$ and also by a factor of $\mathcal{O}(10)$,

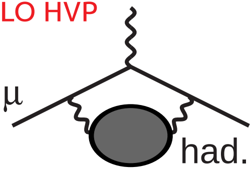
$$a_\mu(\text{EW, 2-loop}) \simeq -10 \left(\frac{\alpha}{\pi}\right) a_\mu(\text{EW, 1-loop}) \left(\ln \frac{m_Z}{m_\mu} + 1\right) \,,$$

where the factor of 10 appears since many "order one" diagrams accidentally add up. (Czarnecki-Krause-Marciano)

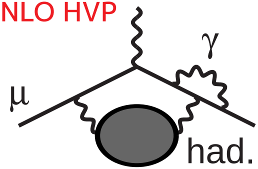
Hadronic Contributions

There are several hadronic contributions:

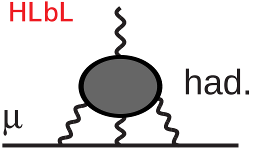
LO HVP



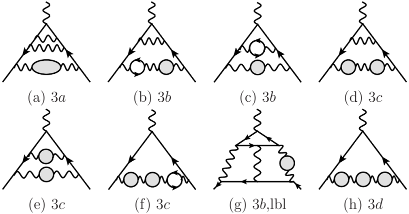
NLO HVP



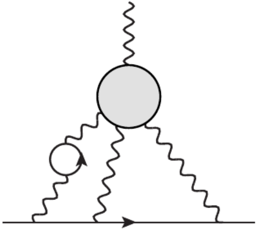
HLbL



LO HVP: Leading Order Hadronic Vacuum Polarization Contribution
NLO HVP: Next-to-Leading Order HVP Contribution
HLbL: Hadronic Light-by-Light Scattering Contribution



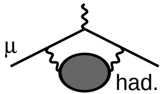
NNLO HVP Contributions



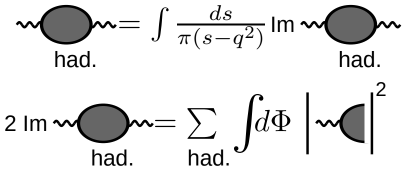
HLbL NLO Contrib.

LO Hadronic Vacuum Polarization Contribution

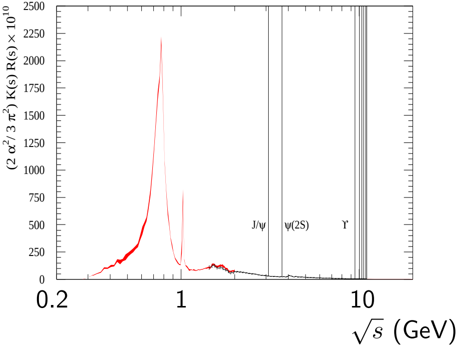
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\rm th}}^{\infty} ds\ \frac{1}{s} \hat{K}(s) \sigma_{\rm had}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

Channel	Energy range [GeV]	$a_\mu^{\rm had, LO\, VP} \times 10^{10}$	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) \times 10^4$	New data
		Chiral perturbation theory (ChPT) threshold contributions		
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	···
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	···
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	···
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	···
		Data based channels ($\sqrt{s} \le 1.937$ GeV)		
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	···
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\rm no\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	···
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\rm no\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	···
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	···
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	···
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to {\rm npp})K\bar{K}_{{\rm no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
		Estimated contributions ($\sqrt{s} \le 1.937$ GeV)		
$(\pi^+\pi^-3\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	···
$(\pi^+\pi^-4\pi^0)_{\rm no\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	···
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	···
$\omega(\to {\rm npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	···
$\omega(\to {\rm npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	···
$\omega(\to {\rm npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	···
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	···
		Other contributions ($\sqrt{s} > 1.937$ GeV)		
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	···	6.26 ± 0.19	7.07 ± 0.22	···
ψ'	···	1.58 ± 0.04	2.51 ± 0.06	···
$\Upsilon(1S-4S)$	···	0.09 ± 0.00	1.06 ± 0.02	···
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	···
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	···

Breakdown of contributions to a_μ(HVP) from various hadronic final states

KNT have included new data sets from $\sim$ 30 papers, in addition to those included in the HLMNT11 analysis

KNT have included $\sim$ 30 hadronic final states

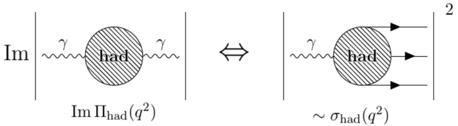
At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

At higher energies $\gtrsim 11$ GeV, we use pQCD

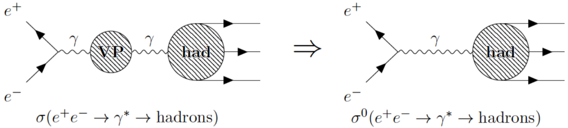
Table from A. Keshavarzi, DN, & T. Teubner (KNT), Phys. Rev. D97 (2018) 114025

Vacuum Polarization Corrections to $\sigma(e^+e^- \to \text{hadrons})$

Optical Theorem:



Experimentally observed cross section:

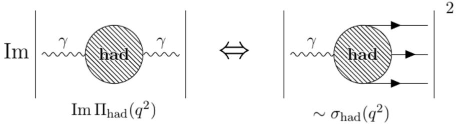


To evaluate $a_\mu^{\text{LO, had}}$, we need to subtract the vacuum polarization (VP) contribution.

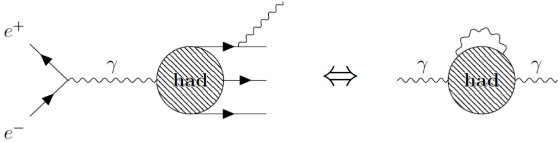
It is straightforward to subtract the leptonic part of the VP, but the hadronic part is non-trivial: we need to do this recursively by using hadronic data, which introduces uncertainty.

Final State Radiation Corrections to $\sigma(e^+e^- \to$ hadrons)

Optical Theorem:



To evaluate $a_\mu^{\text{LO, had}}$, by definition, we use the hadronic cross sections which include all the Final State Radiations (FSR).



In real experiments, people often impose cuts on the final state photons and/or miss photons in the final states. So we have to add back those missed photons, which introduces uncertainties.

Data Combination

To evaluate the vacuum polarization contribution, we have to combine lots of experimental data.

To do so, we usually construct a χ^2 function and find the value of $R(s)$ at each bin which minimizes χ^2.

Naively, the χ^2 function defined as

$$\chi^2(\{\overline{R}_i\}) \equiv \sum_{n=1}^{N_{\text{exp}}} \sum_{i=1}^{N_{\text{bin}}} \sum_{j=1}^{N_{\text{bin}}} (R_i^{(n)} - \overline{R}_i)(V_n^{-1})_{ij}(R_j^{(n)} - \overline{R}_j) \, ,$$

where V_n is the cov. matrix of the n-th exp.,

$$V_{n,ij} = \begin{cases} (\delta R_{i,\text{stat}}^{(n)})^2 + (\delta R_{i,\text{sys}}^{(n)})^2 & (\text{for } i = j) \\ (\delta R_{i,\text{sys}}^{(n)})(\delta R_{j,\text{sys}}^{(n)}) & (\text{for } i \neq j) \end{cases}$$

may seem OK, but when there are non-negligible normalization uncertainties in the data, we have to be more careful.

χ^2 vs normalization error: d'Agostini bias

G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306

We first consider an observable x whose true value is 1. Suppose that there is an experiment which measures x and whose normalization uncertainty is 10%. Now, assume that this experiment measured x twice:

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ ,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ .$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the χ^2 function are

$$(\text{cov.}) = \begin{pmatrix} 0.1^2 + 0.09^2 & 0.09 \cdot 0.11 \\ 0.09 \cdot 0.11 & 0.1^2 + 0.11^2 \end{pmatrix} ,$$
$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} (\text{cov.})^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} .$$

χ^2 takes its minimum at $x = 0.98$: Biased downwards!

d'Agostini bias (2): improvement by iterations

What was wrong? In the previous page,

1st result: $0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}}$,
2nd result: $1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}}$.

we took the syst. errors 0.09 and 0.11, respectively, which made the downward bias. Instead, we should take 10% of some estimator $\bar{x}$ as the syst. errors. Then,

$$\text{(cov.)} = \begin{pmatrix} 0.1^2 + (0.1\bar{x})^2 & (0.1\bar{x})^2 \\ (0.1\bar{x})^2 & 0.1^2 + (0.1\bar{x})^2 \end{pmatrix} ,$$

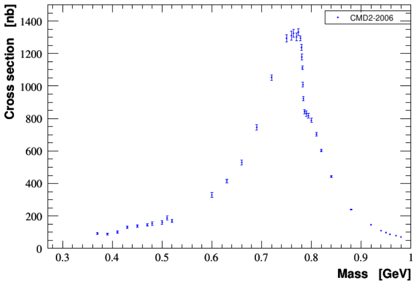
$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} \text{(cov.)}^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} .$$

χ^2 takes its minimum at $x = 1.00$: Unbiased!

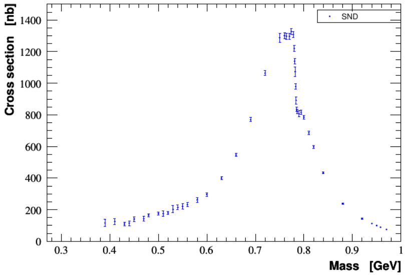
In more general cases, we use iterations: we find an estimator for the next round of iteration by χ^2-minimization. R.D.Ball et al, JHEP 1005 (2010) 075.

$\pi^+\pi^-$ data from CMD-2 and SND

$e^+e^- \to \pi^+\pi^-$ data



CMD-2 data



SND data

Fig. 2 of White Paper

$\pi^+\pi^-$ data from KLOE

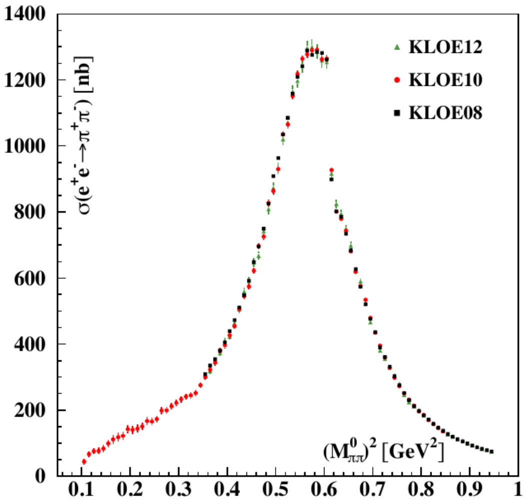


Fig. 3 of White Paper

$\pi^+\pi^-$ data from BaBar

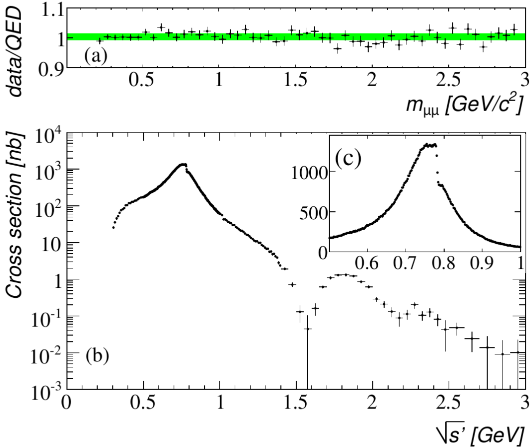
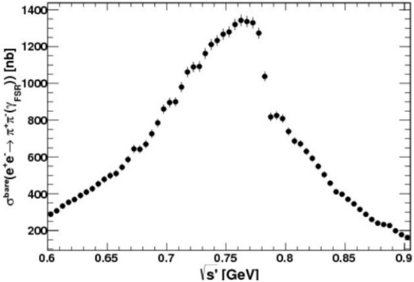
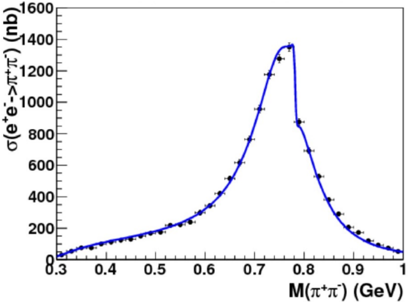


Fig. 4 of White Paper

$\pi^+\pi^-$ data from BESIII and CLEO-c



BESIII data



CLEO-c data

Fig. 5 of White Paper

$\pi^+\pi^-$ data: comparison

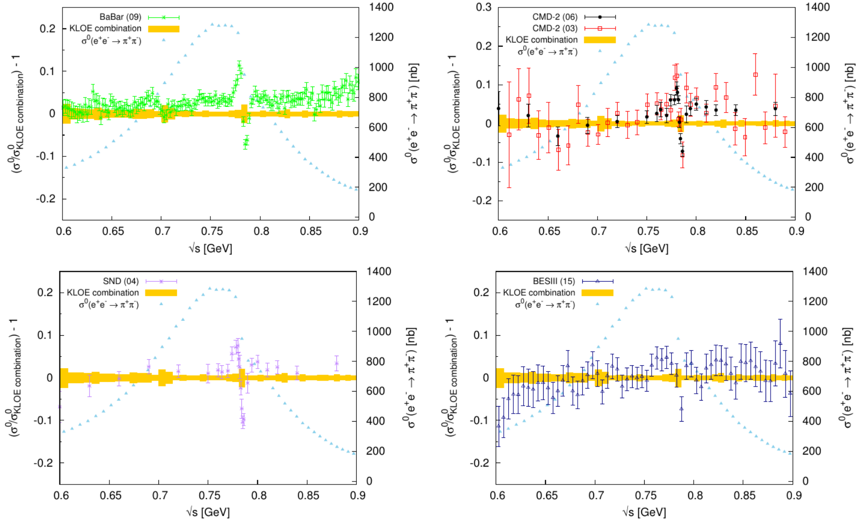


Fig. 13 of White Paper

$\pi^+\pi^-$ data: comparison

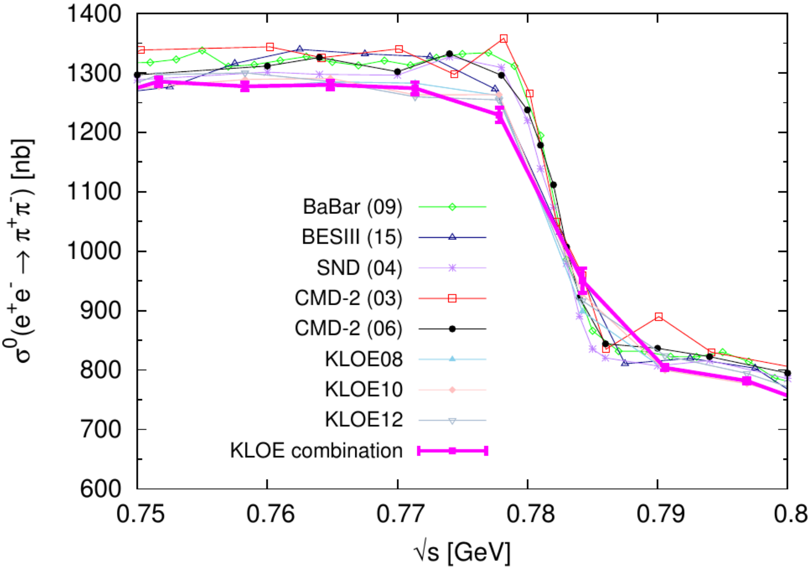


Fig. 14 of White Paper

a_μ from $\pi^+\pi^-$ data: comparison

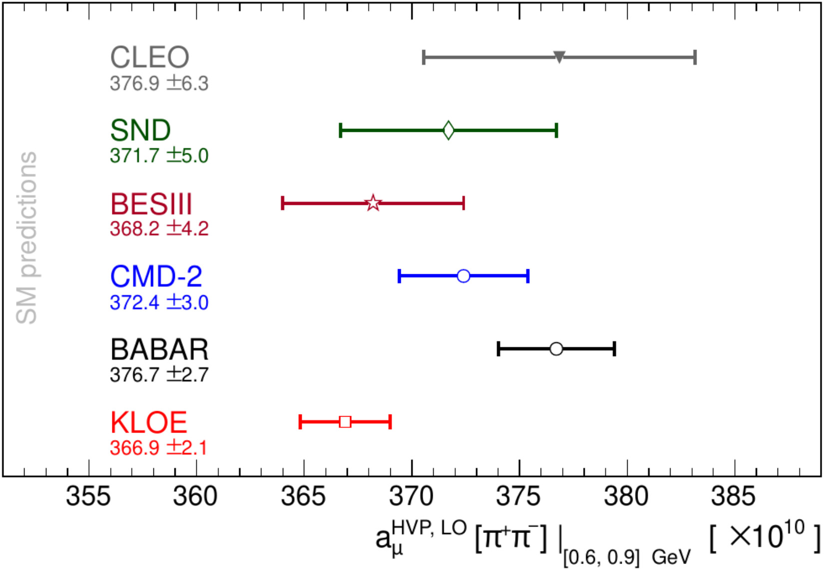


Fig. 15 of White Paper

$\pi^+\pi^-$ channel: ACD vs CHS vs DHMZ vs KNT

$\pi^+\pi^-$ Contribution to a_μ(HVP LO): Table 6 of White Paper

Energy range	ACD18	CHS18	DHMZ19	DHMZ19′	KNT19
≤ 0.6 GeV		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)
≤ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)
≤ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)
≤ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)
≤ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)

ACD18: B. Ananthanarayan et al, PRD 98 (2018) 114015
CHS19: G. Colangelo et al, JHEP 02 (2019) 006
DHMZ19: M. Davier et al, EPJC 80 (2020) 241
KNT19: A. Keshavarzi et al, PRD 101 (2020) 014029

DHMZ vs KNT (table 5 of WP)

Contributions from major channels to a_μ(HVP LO):

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Difference in the $\pi^+\pi^-$ channel is mainly from the way to combine the data sets.

KNT19: Global χ^2 minimization

DHMZ19: Takes the average of "all but KLOE" and "all but BaBar" as the mean value, and counts the half of the diff of the two as an additional systematic uncertainty.

Comparison with Lattice Results

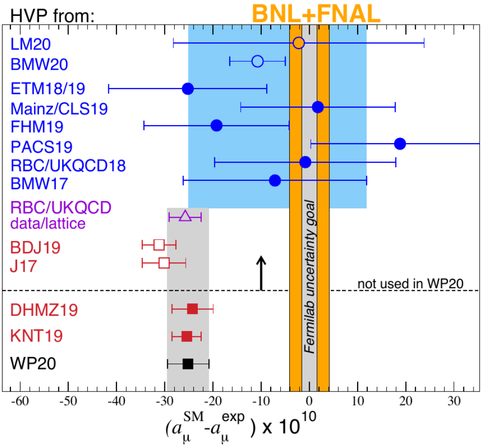
$$a_\mu^{\rm SM} < a_\mu^{\rm HVP} + [a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}]$$

Lattice QCD + QED

hybrid: combine data & lattice

data driven

+ unitarity/analyticity constraints



A. El-Khadra

Lattice 2021, 26-30 July 2021

Talk by A. El-Khadra (U. of Illinois) at Lattice 2021

Breakdown of SM prediction for muon g-2

From Table 1 of the White Paper

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

HVP: Hadronic Vacuum Polarization contribution
HLbL: Hadronic Light-by-Light contribution

$$a_\mu(\text{exp, BNL}) - a_\mu(\text{SM}) = 27.9(7.6) \times 10^{-10}\ (3.7\,\sigma)$$

$$a_\mu(\text{exp, 2021}) - a_\mu(\text{SM}) = 25.1(5.9) \times 10^{-10}\ (4.2\,\sigma)$$

NEW

The muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

Massimo Passera
INFN Padova

KEK-PH Lectures and Workshops
May 11th 2021

Talk by M. Passera at KEK-PH-2021, May 11, 2021

Precision Electroweak Fit

- The electroweak sector of the Standard Model can be parametrized by 3 parameters, $\{g, g', v\}$.
- Instead of $\{g, g', v\}$, we usually choose 3 most precisely measured quantities $\{M_Z, G_F, \alpha(M_Z)\}$ as input, where $\alpha(M_Z)$ is the least accurately known.
- By using $\{M_Z, G_F, \alpha(M_Z)\}$ as input, **we can indirectly predict the Higgs boson mass** by comparing observables (such as M_W, Br$(Z \to f\bar{f})$, $\cdots$) with the SM predictions.
- This is possible since the Higgs boson gives a contribution to these observables through radiative corrections.

- Can Δa_μ be due to missing contributions in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$
$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

$\varepsilon>0$, in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

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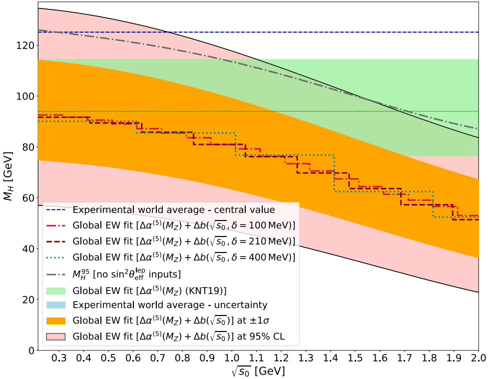
Talk by M. Passera at KEK-PH-2021, May 11, 2021

Muon g-2: connection with the SM Higgs mass (2020)

Δα



Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible,
but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

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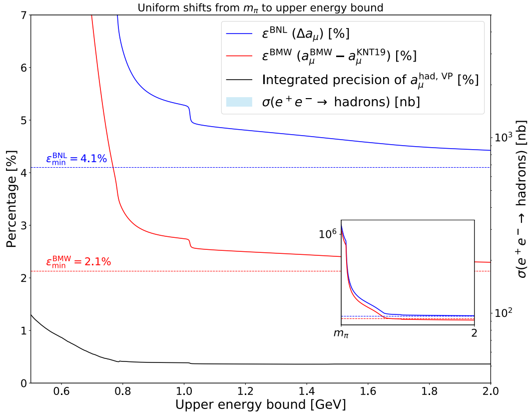
How large are the required shifts Δσ(s) ?

Δα



Shifts below ~1 GeV conflict with the quoted exp. precision of σ(s)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)

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Summary

- SM prediction for $(g-2)_\mu$: 4.2σ deviation from measured value $\Longrightarrow$ New Physics?
- Recent data-driven evaluations of HVP contributions seem convergent
- To better establish the $(g-2)_\mu$ anomaly, better data for $e^+e^- \to \pi^+\pi^-$ welcome (from BESIII, CMD-3, Belle II, . . .)
- In general, lattice results still suffer from large uncertainties, but the BMW collaboration claim a smaller uncertainty and a better agreement with a_μ(exp).
  (Which is correct, data-driven or BMW?)
- The EW precision data seem to favor the data-driven analysis (although I may be biased...)