# Some basics on muon g-2

### D. Nomura (IUHW/KEK)

talk at International Physics School on Muon Dipole Moments and Hadronic Effects in memoriam Simon Eidelman hosted by JGU Mainz

30 August 2021

# Refs (mainly reviews and books)

 T. Aoyama et al. (Muon g-2 Theory Initiative), "The anomalous magnetic moment of the muon in the Standard Model" ("Muon g-2 White Paper") Phys. Rept. 887 (2020) 1-166 DOI:10.1016/j.physrep.2020.07.006 arXiv:2006.04822 [hep-ph].

F. Jegerlehner

"The Anomalous Magnetic Moment of the Muon" (2nd edition) Springer Tracts Mod. Phys. **274** (2017) 1-693 DOI:10.1007/978-3-319-63577-4

 B. Lee Roberts and W. J. Marciano (ed.) "Lepton Dipole Moments" Adv. Ser. Direct. High Energy Phys. 20 (2009) 1 DOI:10.1142/7273

### References

- F. Jegerlehner and A. Nyffeler "The Muon g-2" Phys. Rept. 477 (2009) 1-110 arXiv:0902.3360 [hep-ph]
- J. P. Miller, E. de Rafael and B. Lee Roberts "Muon (g-2): Experiment and theory" Rept. Prog. Phys. 70 (2007) 795 hep-ph/0703049
- K. Melnikov and A. Vainshtein "Theory of the muon anomalous magnetic moment" Springer Tracts Mod. Phys. 216 (2006) 1-176 DOI:10.1007/3-540-32807-6

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# Current status of the Standard Model (SM)

# LHC data vs Standard Model



### LHC cross section data agree very well the SM

# **Unitarity of CKM matrix**



# **Electroweak precision data vs SM**



# Although the SM is such a successful theory, nobody believes that it is the ultimate theory.

Because...

# What the SM cannot explain

- Why 3 generations? Why  $SU(3)_C \times SU(2)_L \times U(1)_Y$ ?
- Many (19) free parameters

$g', g, g_s$	gauge couplings
$oldsymbol{v}$	vacuum expectation value (VEV)
$m_H$	Higgs boson mass
$m_e, m_\mu, m_ au$	lepton masses
$m_u, m_d, m_s, m_c, m_b, m_t$	quark masses
$\phi_1,\phi_2,\phi_3$	quark mixing angles
δ	CKM phase
$ar{ heta}$ )	(QCD $\theta$ -angle

- Neutrino masses & mixing matrix
- Why  $ar{ heta} \lesssim 2 imes 10^{-10}$ ? (strong CP problem)
- Why  $m_{\mathsf{weak}} \ll m_{\mathsf{GUT}}$ ? (gauge hierarchy problem)
- Dark matter & dark energy
- Origin of the baryon number
- Gravity

To solve these problems, new physics beyond the SM should exist.

# It might exist at the TeV scale, because....

### Hierarchy Problem in the Standard Model



Radiative corrections to  $m_{\rm H}^2$  diverges as  $\sim \Lambda^2$ .  $\Leftrightarrow$ Physical Higgs mass  $\sim m_{\rm weak}^2$ . (Fine-tuning necessary if  $\Lambda \gg m_{\rm weak}$ ) In SUSY Standard Models this is automatically solved since (softly broken) SUSY ensures the cancellation of the guad. divergences. For example,



### **Gauge coupling unification:**

SM case

MSSM case



SUSY particles change the 'running' of the gauge couplings above  $m_{\rm SUSY}$ . Gauge unification also explains why the electric charges are quantized.

# Muon g-2: Hint of new physics?



4.2  $\sigma$  discrepancy in (g-2)<sub> $\mu$ </sub>: new physics?

Many physicists thought that the LHC would discover new particles beyond the SM, once it started operation.

But the reality is ...

### ATLAS Preliminary

#### ATLAS SUSY Searches\* - 95% CL Lower Limits

June 2021

	Model	S	Signatur	e ∫£	dt [fb-	9		Mass limit					Reference
5	$\tilde{q}\tilde{q}, \tilde{q} \rightarrow q \tilde{t}_1^0$	0 r. μ mono-jet	2-6 jets 1-3 jets	$E_T^{miss}$ $E_T^{miss}$	139 36.1	4 [1x, 8x D 3 [8x Deger	egen.]		1.0 0.9		1.85	m( <sup>2</sup> 1)>400 GeV m(j)-m(21)=5 GeV	2010.14293 2102.10574
arche	$\hat{g}\hat{g}, \hat{g} \rightarrow g\bar{g}\hat{\chi}_1^0$	0 e. µ	2-6 jets	$E_T^{\rm miss}$	139	2			Forbidden		1.15-1.95	m( $\tilde{k}_1^0$ )=0 GeV m( $\tilde{k}_1^0$ )=1000 GeV	2010.14293 2010.14293
S	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow q\bar{q}W\tilde{\chi}_{1}^{0}$	1 e, µ	2-6 jets		139	8					2.2	m(k <sup>0</sup> )<600 GeV	2101.01629
8	$\hat{g}\hat{g}, \hat{g} \rightarrow q\hat{q}(\ell\ell)\hat{\chi}_{1}^{0}$	ee, pp	2 jets	$E_T^{miss}$	36.1	8				1.2		m(į)-m(į <sup>0</sup> <sub>1</sub> )=50 GeV	1805.11381
clusi	$gg, g \rightarrow qqWZ \tilde{\chi}_1^0$	0 ε.μ SS ε.μ	7-11 jets 6 jets	$E_T^{max}$	139 139	8 8			1	.15	1.97	m(t <sup>2</sup> ) <600 GeV m(2)-m(t <sup>2</sup> )=200 GeV	2006.06032 1909.08457
3	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow t \tilde{\ell} \tilde{\chi}_1^0$	0-1 e.μ SS e.μ	3 <i>b</i> 6 jets	$E_T^{miss}$	79.8 139	R R				1.25	2.25	m(t <sup>0</sup> <sub>1</sub> )<200 GeV m(g)-m(t <sup>0</sup> <sub>1</sub> )=300 GeV	ATLAS-CONF-2018-041 1909.08457
	$\tilde{b}_1 \tilde{b}_1$	0 e, µ	2 b	$E_T^{miss}$	139	$\hat{b}_1 \\ \hat{b}_1$			0.68	1.255		m( $\hat{r}_1^0$ )<400 GeV 10 GeV<( $\hat{r}_1, \hat{r}_1^0$ )<20 GeV	2101.12527 2101.12527
arks tion	$\tilde{b}_1 \tilde{b}_1, \tilde{b}_1 \rightarrow b \tilde{\ell}_2^0 \rightarrow b h \tilde{\ell}_1^0$	0 ε, μ 2 τ	6 b 2 b	$E_T^{miss}$ $E_T^{miss}$	139 139	$\frac{\delta_1}{\delta_1}$	Forbidden		0.13-0.85	.23-1.35		$\Delta m(\hat{t}_{2}^{0}, \hat{t}_{1}^{0}) = 130 \text{ GeV}, m(\hat{t}_{1}^{0}) = 100 \text{ GeV}$ $\Delta m(\hat{t}_{2}^{0}, \hat{t}_{1}^{0}) = 130 \text{ GeV}, m(\hat{t}_{1}^{0}) = 0 \text{ GeV}$	1906.03122 ATLAS-CONF-2020-031
and a	$\bar{t}_1\bar{t}_1, \bar{t}_1 \rightarrow \hat{\alpha}_1^0$	0-1 e. µ	≥ 1 jet	$E_T^{mix}$	139	i <sub>1</sub>				1.25		m(x <sup>0</sup> )=1 GeV	2004.14050,2012.03799
1.5	$\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow Wh \tilde{t}_1^0$	1 e. µ	3 jets/1 b	$E_T^{miss}$	139	i <sub>1</sub>		Forbidden	0.65			m(R <sup>b</sup> <sub>1</sub> )+500 GeV	2012.03799
SCI 0	$\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow \tilde{\tau}_1 bv, \tilde{\tau}_1 \rightarrow \tau G$	1-2 7	2 jets/1 b	Erass. T	139	i <sub>1</sub>			Forbidden	1.4	4	m(t <sub>1</sub> )=800 GeV	ATLAS-CONF-2021-008
2.5	$\bar{t}_1\bar{t}_1, \bar{t}_1 \rightarrow c\mathcal{K}_1 / \bar{c}\bar{c}, \bar{c} \rightarrow c\mathcal{K}_1$	0 e, µ	2 c mono-jet	Etiin	36.1	2		0.55	0.85			m(2)=0 GeV m(2,2)-m(2)=5 GeV	1805.01649 2102.10874
	11. 1	1-2 6.4	1.4.6	Eniss	139	7.			0.057	1 18		m(2)-500 GeV	2006.05880
	$h_1h_1, h_1 \rightarrow h_2, h_2 \rightarrow h_1 + Z$	3 e. µ	1.6	ET	139	i.		Forbidden	0.86		,	n(x1)=360 GeV, m(x1)-m(x1)= 40 GeV	2006.05880
	$\tilde{\chi}_1^* \tilde{\chi}_2^0$ via $W\!Z$	Multiple ℓ/je er, μμ	ts ≥ljet	ET ET ET	139 139	$\hat{x}_{1}^{*}/\hat{x}_{1}^{*}$ $\hat{x}_{1}^{*}/\hat{x}_{1}^{*}$	0.205		0.96			$m(\hat{t}_1^0)=0$ , wino-bino $m(\hat{t}_1^0)=0$ (ino-bino)	2105.01676, ATLAS-CONF-2021-022 1911.12506
	$\tilde{x}_{1}^{*}\tilde{x}_{1}^{*}$ via WW	2 e. µ		ET	139	£1		0.42				m(f <sup>0</sup> <sub>1</sub> )=0, wino-bino	1908.08215
	$\tilde{\chi}_{1}^{*}\tilde{\chi}_{2}^{0}$ via $Wh$	Multiple //je	ts	ET	139	\$1/8 Fort	nabbid		1.0	6		m(t <sup>2</sup> )=70 GeV, wino-bino	2004.10894, ATLAS-CONF-2021-022
5 2	$\tilde{\chi}_1^* \tilde{\chi}_1^*$ via $\tilde{\ell}_L/\tilde{\nu}$	2 e, µ		Eniss T	139	$\hat{x}_{1}^{t}$			1.0			$m(\tilde{\ell}, \tilde{\nu}) = 0.5(m(\tilde{\ell}_1^n) + m(\tilde{\ell}_1^n))$	1908.08215
신음	$\hat{\tau}\hat{\tau}, \hat{\tau} \rightarrow \tau \hat{\chi}_1^0$	2 7		ET	139	* (TL, TR.L)	0.16-0	0.12-0.39				m( <sup>2</sup> )=0	1911.06560
v	$\ell_{L,R}\ell_{L,R}, \ell \rightarrow \ell \lambda_1''$	2 e, µ er, µµ	> 1 iet	ET.	139	1	0.255		0.7			m(t) ====================================	1908.08215
	44 A	0.0.0	> 3.6	Fairs	36.1		0 13-0 23		0.29-0.88			89.50	1806 04030
	111, 11-00,20	4 e. µ	0 jets	Ethins	139	B		0.55	5			$BP(\tilde{r}_1^0 \rightarrow ZG)=1$	2103.11684
		0 e. µ	≥ 2 large jet	S E <sub>T</sub>	139	B			0.45-0.93			$BP(\mathbb{F}_1^- \rightarrow ZG)=1$	ATLAS-CONF-2021-022
70	$\operatorname{Direct} \check{\mathcal{X}}_1^* \check{\mathcal{X}}_1^-$ prod., long-lived $\check{\mathcal{X}}_1^*$	Disapp. trk	c 1 jet	$E_T^{\rm miss}$	139	£.			0.66			Pure Wino	ATLAS-CONF-2021-015
60	Stable 5 B. badron		Multiple		~ .	~1 >	0.21						1003 01076 1808 04005
194 Dig	Motostable i B borken i		Multiple		36.1	8 (+(2) -10	os 02 cal			_	2.0	4 m(F <sup>2</sup> )-100 (m)/	1710 04901 1808 04095
pa	$\mathcal{U}, \mathcal{I} \rightarrow tG$	Displ. lep		$E_{\tau}^{mix}$	139	2, p			0.7	_		r(0 = 9.1 m	2011.07812
						÷		0.34				$\tau(l) = 0.1 \text{ ms}$	2011.07812
	$\tilde{y}_{1}^{*}\tilde{y}_{1}^{*}/\tilde{y}_{1}^{0}$ $\tilde{y}_{1}^{*}\rightarrow Z_{1}/U_{1}$	3 e. u			139	£7.78" (BR);	$(\tau)=1, BP(Z_{\ell})=1$	0.	625 1.05	5		Pure Wing	2011.10543
	$\tilde{\chi}_{1}^{+}\tilde{\chi}_{1}^{+}/\tilde{\chi}_{2}^{0} \rightarrow WW/Z\ell\ell\ell\ell_{YY}$	4 e. µ	0 jets	Eniss	139	$\hat{x}_{1}^{2}/\hat{x}_{2}^{0} = [\lambda_{cm}$	± 0, J <sub>115</sub> ± 0)		0.95	1	1.55	m(2):200 GeV	2103.11684
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow qq\tilde{\chi}_{1}^{0}, \tilde{\chi}_{1}^{0} \rightarrow qqq$		4-5 large jet	s	36.1	≹ [m(ξ <sup>0</sup> <sub>1</sub> )=20	0 GeV; 1100 GeV]			1.3	1.9	Large 3 <sub>112</sub>	1804.03568
>	$\tilde{H}, \tilde{t} \rightarrow t \tilde{\chi}_{1}^{0}, \tilde{\chi}_{1}^{0} \rightarrow thx$		Multiple		36.1	i (X <sub>22</sub> =20-4	. 1e-2]	0.55	1.05	5		m(R1)=200 GeV, bino-like	ATLAS-CONF-2018-003
5	$H, I \rightarrow b\chi_1^-, \chi_1^- \rightarrow bbs$		2.46		139	ĩ		Forbidden	0.95			m(t)=500 GeV	2010.01015
	$r_1r_1, r_1 \rightarrow m$	2	2 juci + 2 b		36.7	71 [199, 84] 7		0.42 0	1.61	0414	10	PD/2 - L- II MW	1710.07171
	stat's standa	2 ε,μ 1 μ	DV		136	i [1e-10< 2	<1e-8, 3e-10<	J' <3e-9]	1.0	0.4-1.4	1.6	BR(rsp)=100%, cost,=1	2003.11956
	$\tilde{\chi}_1^*/\tilde{\chi}_2^0/\tilde{\chi}_1^0, \tilde{\chi}_{1,2}^0 {\rightarrow} tbs, \tilde{\chi}_1^+ {\rightarrow} bbs$	1-2 e, µ	≥6 jets		139	£	0.2-	0.32				Pure higgsino	ATLAS-CONF-2021-007
										<u> </u>			i .
*Only i	a selection of the available ma	iss limits on	new state.	s or	10	D-1			1	1		Mass scale [TeV]	

\*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

### Fig. from ATLAS TWiki No SUSY particles found so far Current bound from LHC: $m_{SUSY} \gtrsim$ 1 TeV

D. Nomura (IUHW/KEK)

g-2 theory

30 August 2021

16/58

#### ATLAS Heavy Particle Searches\* - 95% CL Upper Exclusion Limits

Status: July 2021

 $\int f dt = (3.6 = 139) \text{ fb}^{-1}$ 

ATI AS Preliminary

 $\sqrt{c} = 9.12 \text{ To}/$ 

						J=== (ere : ee) :=	• • • • •
	Model	ί,γ	Jets†	E <sup>miss</sup> T	∫£ dt[ft	-1] Limit	Reference
Extra dimensions	$\begin{array}{l} \text{ADD } G_{KK} + g/q \\ \text{ADD non-resonant } \gamma\gamma \\ \text{ADD OBH} \\ \text{ADD BH multijet} \\ \text{RS1} G_{KK} \rightarrow \gamma\gamma \\ \text{Bulk RS} G_{KK} \rightarrow WW/ZZ \\ \text{Bulk RS} G_{KK} \rightarrow WV \rightarrow \ell \nu qq \\ \text{Bulk RS} g_{KK} \rightarrow tt \\ \text{ZUED} / \text{RPP} \end{array}$	0 e, μ, τ, γ 2 γ - 2 γ multi-channe 1 e, μ 1 e, μ 1 e, μ	1 - 4 j 2 j ≥3 j - 2 j / 1 J ≥1 b, ≥1 J ≥2 b, ≥3 j	Yes - - Yes 2) Yes Yes	139 36.7 37.0 3.6 139 36.1 139 36.1 36.1 36.1	Na 182000 - 3 2010 - 2010 - 2010 - 2010 Na 1920 - 2010 - 2010 - 2010 Na 1920 - 2010 - 2010 - 2010 Na 1920 - 2010 Na	2102.10874 1707.04147 1703.09127 1512.02586 2102.13405 1808.02380 2004.14658 1804.10823 1803.09678
Gauge bosons	$\begin{array}{llllllllllllllllllllllllllllllllllll$	$\begin{array}{c} 2  e, \mu \\ 2  \tau \\ - \\ 0  e, \mu \\ 1  e, \mu \\ 1  \tau \\ 1  e, \mu \\ 0.2  e, \mu \\ 0  e, \mu \\ 2  \mu \end{array}$	2b ≥1b,≥2J 2j/1J 2j/1J 1-2b ≥1b,≥2J 1J		139 36.1 139 139 139 139 139 139 139 139 139 80	2 mm         5.1700           2 max         2.42 TeV           2 max         2.1100           7 max         2.1100           7 max         2.1100           9 max         4.1500           9 max         2.160	1903.06248 1709.07242 1805.09299 2005.05138 1996.05620 ATLAS-CONF-2021-025 ATLAS-CONF-2021-045 2004.14638 ATLAS-CONF-2020-043 2007.05293 1904.12679
G	Cl qqqq Cl & dq Cl eebs Cl pybs Cl pybs Cl tttt	- 2 e, µ 2 e 2 µ ≥1 e,µ	2 j - 1 b ≥1 b, ≥1 j	- - Yes	37.0 139 139 139 36.1	A 21.8 TeV (°, A 28.8 TeV %, A 2.0 TeV 4.1 (°, 35.8 TEV %, A 2.0 TeV 4.1 (°, 55.8 TEV %, A 2.0 TeV 6.1 (°, 5.6 TEV 7.1 (°,	1703.09127 2006.12546 2105.13847 2105.13847 1811.02305
MQ	Axial-vector med. (Dirac DM) Pseudo-scalar med. (Dirac DM) Vector med. Z <sup>*</sup> -2HDM (Dirac DM) Pseudo-scalar med. 2HDM-a Scalar reson. $\phi \rightarrow t\chi$ (Dirac DM)	0 e, μ, τ, γ 0 e, μ, τ, γ 0 e, μ multi-channe 0-1 e, μ	1 - 4 j 1 - 4 j 2 b 1 b, 0-1 J	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	139 139 139 139 36.1	المست يولي 23 (26%) يولي (1/1) 10% يولي 23 (26%) يولي (1/1) 10% يولي 23 (26%) يولي (1/1) 10% يes (1/1) 10% يولي (1/1) 10% 20% abble (1/1) 10	2102.10874 2102.10874 ATLAS-CONF-2021-005 ATLAS-CONF-2021-035 1812.09743
10	Scalar LO 1 <sup>47</sup> gen Scalar LO 2 <sup>nd</sup> gen Scalar LO 3 <sup>rd</sup> gen Scalar LO 3 <sup>rd</sup> gen Scalar LO 3 <sup>rd</sup> gen Scalar LO 3 <sup>rd</sup> gen	2 e 2 µ 1 τ 0 e, µ ≥2 e, µ, ≥1 τ 0 e, µ, ≥1 τ	≥2 j ≥2 j ≥2 j, ≥2 b ≥1 j, ≥1 b 0 - 2 j, 2 t	145 145 145 145 145 145	139 139 139 139 139 139	LD mass         1.8 TeV (2) mass         p = 1           LD mass         1.2 TeV         R/LQ - bh = 1           LD mass         1.2 TeV         R/LQ - bh = 1           LD mass         1.2 TeV         R/LQ - bh = 1           LD mass         1.2 TeV         R/LQ - bh = 1           LD mass         1.4 TeV         R/LQ - bh = 1           LD mass         1.4 TeV         R/LQ - bh = 1           LD mass         1.4 TeV         R/LQ - bh = 1	2006.05872 2006.05872 ATLAS-CONF-2021-008 2004.14060 2101.11582 2101.12527
Heavy quarks	$ \begin{array}{l} VLQ\; TT \rightarrow Zt + X \\ VLQ\; BB \rightarrow Wt/Zb + X \\ VLQ\; BT_{5(2)}T_{5(2)}T_{5(2)} \rightarrow Wt + X \\ VLQ\; T \rightarrow Ht/Zt \\ VLQ\; T \rightarrow Wb \\ VLQ\; B \rightarrow Hb \end{array} $	2e/2µ/≥3e,µ multi-channe 2(SS)/≥3 e,µ 1 e,µ 1 e,µ 0 e,µ ≥	≥1 b, ≥1 j ≥1 b, ≥1 j ≥1 b, ≥3 j ≥1 b, ≥1 j 20, ≥1 j, ≥	- Yes Yes 1J -	139 36.1 36.1 139 36.1 139	Tasks         1.4 BW         SU3 codet           Branks         1.34 TeV         SU20 codet           Tra, mask         1.64 TeV         SU20 codet           Tra, mask         1.64 TeV         SU20 codet           Trans         1.8 TeV         SU20 codet           Y mask         1.8 TeV         SU20 codet           Y mask         1.8 TeV         SU20 codet, re 45           Rmask         1.8 TeV         SU20 codet, re 45           Rmask         2.0 TeV         SU20 codet, re 3.3	ATLAS-CONF-2021-024 1808.02343 1807.11883 ATLAS-CONF-2021-040 1812.07343 ATLAS-CONF-2021-018
Excited	Excited quark $q^* \rightarrow qg$ Excited quark $q^* \rightarrow q\gamma$ Excited quark $b^* \rightarrow bg$ Excited lepton $t^*$ Excited lepton $r^*$	1γ 	2 j 1 j 1 b, 1 j -	-	139 36.7 36.1 20.3 20.3	C mass         67.78V r mass         only ur and c', h = m(c') d' d' d	1910.08447 1709.10440 1805.09299 1411.2921 1411.2921
Other	Type III Seesaw LRSM Majorana v Higgs triplet $H^{*+} \rightarrow W^+W^+$ Higgs triplet $H^{*+} \rightarrow \ell\ell$ Higgs triplet $H^{*+} \rightarrow \ell\tau$ Multi-charged particles Magnetic monopoles	2,3,4 e, µ 2 µ 2,3,4 e, µ (SS 2,3,4 e, µ (SS 3 e, µ, τ - -	≥2j 2j ) various ) – – –	Yes - Yes 	139 36.1 139 36.1 20.3 36.1 34.4	Minima         915 GBV1         m(Ws) 24 1 first ga = ga           Minima         32 FeV1         D1 peddets           Minima         300 GBV1         D1 peddets           Minima         122 FeV1         D1 peddets           Minima         327 FeV1         D1 peddets	ATLAS-CONF-2021-023 1809.11105 2101.11961 1710.09748 1411.2921 1812.03673 1905.10130
	√s = 8 TeV vs par	= 13 TeV tial data	$\sqrt{s} = 13$ full d	3 TeV lata		10 <sup>-1</sup> 1 <sup>10</sup> Mass scale [TeV]	

\*Only a selection of the available mass limits on new states or phenomena is shown. +Small-radius (large-radius) lets are denoted by the letter i (J).

### Fig. from ATLAS TWiki Situation is similar for non-SUSY new particles Current bound from LHC: $m_{\text{exotics}} \gtrsim 1 \text{ TeV}$

### But the LHC is not almighty.

- Not suitable for very precise measurements due to various uncertainties (pdf, BG, ...)
- Limitation on  $\sqrt{s}$  accessible in the near future.
- $\implies$  Important to combine with other methods
  - e<sup>+</sup>e<sup>-</sup> colliders, various precision measurements (flavor physics, EDM searches, (g-2)<sub>e,μ</sub>, 0νββ decay searches...), dark matter searches, cosmology, ...
- ⇒ Precision physics, in particular the muon g-2, is a good complement to the LHC.

# Why Muon g-2?

### 4.2σ Anomaly Reported

Long standing anomaly (~ 20 yrs), in spite of careful studies on every aspect. (→ Major theoretical blunder unlikely.) **Hint of New Physics beyond the Standard Model?** 

- No new physics at the LHC so far Intensity frontier: more and more important
- Long history of research
   1st (g 2)<sub>μ</sub> exp.: Garwin, Lederman & Weinrich (1957)
   Well-established place to search for new physics
- Leptonic observable Experimentally and theoretically clean

### Press Release from Fermilab (7 April 2021)



# Fermilab Muon g-2 exp 1st Results



Fig. from Phys. Rev. Lett. 126 (2021) 141801 [arXiv:2104.03281]

# Muon g-2 in the Media The New York Times

OUT THERE

### A Tiny Particle's Wobble Could Upend the Known Laws of 朝回新聞

DIGITAL Experiments with particles known a are forms of matter and energy vital トップ 社会 経済 政治 国際 スポーツ オピニオン IT・科学 文化・芸能 the cosmos that are not yet known tc

朝日新聞デジタル > 記事

### 素粒子物理学の根幹崩れた? 磁気の測定値に未知のずれ

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# Muons: 'Strong' evidence found for a new force of nature

By Pallab Ghosh Science correspondent

# **Magnetic Moment: Definition**

Suppose that there is a point particle f at rest in an external magnetic field  $\vec{B}$ . If the interaction Hamiltonian  $H_{mdm}$  between f and  $\vec{B}$  is given by

$$H_{
m mdm} = -ec{\mu} \cdot ec{B} \; ,$$

then  $\vec{\mu}$  is called the **magnetic dipole moment** of f.

• If f has a non-zero spin  $ec{s}$ , then  $ec{\mu}\proptoec{s}$ 

• Its cousins: EDM  $\vec{d}$ :  $H_{EDM} = -\vec{d} \cdot \vec{E}$  (P-odd, T-odd) (EDM: electric dipole moment) anapole  $\vec{a}$ :  $H_{ana} = -\vec{a} \cdot (\nabla \times \vec{B})$  (P-odd, T-even)

# **Muon g-2: introduction**

Lepton magnetic moment 
$$\vec{\mu}$$
:  $\vec{\mu} = g \frac{e}{2m} \vec{s}$   $\vec{s}$ : spin

Anomalous magnetic moment  $a \colon a \equiv (g-2)/2$ 

Historically,

• g = 2 at tree level (Dirac, 1928)

• 
$$a = \alpha/(2\pi)$$
 at 1-loop (QED) (Schwinger, 1947)

Today, still important since...

• One of the **most precisely measured** quantities:

$$a_{\mu}(\exp) = 11\ 659\ 206.1(4.1) \times 10^{-10}$$
 (0.35ppm)

(B. Abi et al., 2021)

 Extremely useful in probing/constraining new physics beyond the SM

# Dipole moments of a spin-1/2 particle

For a spin-1/2 particle f,

$$egin{aligned} &\langle f(p')|J^{\mathsf{em}}_{\mu}|f(p)
angle &= ar{u}_f(p')\Gamma_{\mu}u_f(p) \;, \ &\Gamma_{\mu} = F_1(q^2)\gamma_{\mu} + rac{i}{2m_f}F_2(q^2)\sigma_{\mu
u}q^
u \ &- F_3(q^2)\sigma_{\mu
u}q^
u\gamma_5 - F_4(q^2)(\gamma_{\mu}q^2 - 2m_fq_{\mu})\gamma_5 \end{aligned}$$

There are no other independent form factors of a spin-1/2 particle other than  $F_1(q^2), \ldots, F_4(q^2)$  (See e.g., Nowakowski, Paschos, & Rodriguez, physics/0402058)

 $\begin{array}{ll} F_1(0) = -eQ_f & (\text{electric charge}) \\ \hline F_2(0) = -eQ_f a_f & (a_f: \text{anomalous magnetic moment}) \\ F_3(0) = d_f & (\text{EDM}) \\ \hline F_4(0) = \tilde{a}_f & (\text{anapole moment}) \end{array}$ 

If f is a Majorana particle, then  $F_1(q^2) = F_2(q^2) = F_3(q^2) = 0$ .

### Breakdown of SM prediction for muon g-2

### From Table 1 of the White Paper

Contribution	Section	Equation	Value ×10 <sup>11</sup>	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO $(e^+e^-)$	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO $(e^+e^-)$	Sec. 2.3.8	Eq. (2.34)	-98.3(7)	Ref. [7]
HVP NNLO $(e^+e^-)$	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, udsc)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18-30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP $(e^+e^-, LO + NLO + NNLO)$	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2-8, 18-24, 31-36]
Difference: $\Delta a_{\mu} := a_{\mu}^{\exp} - a_{\mu}^{SM}$	Sec. 8	Eq. (8.14)	279(76)	

HVP: Hadronic Vacuum Polarization contribution HLbL: Hadronic Light-by-Light contribution

$$a_{\mu}(\mathsf{exp, BNL}) - a_{\mu}(\mathsf{SM}) ~= 27.9(7.6) imes 10^{-10}$$
 (3.7  $\sigma$  )

 $a_{\mu}({
m exp, 2021}) - a_{\mu}({
m SM}) = 25.1(5.9) imes 10^{-10}$  (4.2  $\sigma$ )



# QED contribution (1) QED contribution: $a_{\mu}(\text{QED}) = \frac{\alpha}{2\pi} + 0.765857425(17) \left(\frac{\alpha}{\pi}\right)^{2} + 24.05050996(32) \left(\frac{\alpha}{\pi}\right)^{3} + 130.8796(63) \left(\frac{\alpha}{\pi}\right)^{4} + 752.2(1.0) \left(\frac{\alpha}{\pi}\right)^{5} + \cdots$

 $= 11658471.892(0.003) \times 10^{-10}$ , (numbers from PDG 2020)

where the uncertainty is dominated by that of  $\alpha$ .

- 5-loop calculation! (Aoyama, Hayakawa, Kinoshita & Nio)
- The 4-loop corrections  $\simeq 38 \times 10^{-10} \simeq \mathcal{O}(a_{\mu}(\exp) a_{\mu}(\text{SM})).$
- The 4-loop contribution now fully cross-checked by another group. Mass-independent part by S. Laporta (Phys.Lett. B772 (2017) 232), and mass-dependent part by A. Kurz et al (Nucl. Phys. B879 (2014) 1; Phys. Rev. D92 (2015) 073019; ibid. D93 (2016) 053017)
- The 5-loop contribution very small  $(\simeq 0.5 \times 10^{-10} \ll a_{\mu}(\text{exp}) a_{\mu}(\text{SM}))$

### **QED contribution (2)** QED contribution to the electron g - 2:

$$a_e(\mathsf{QED}) = \frac{\alpha}{2\pi} - (0.32847844400\dots) \left(\frac{\alpha}{\pi}\right)^2 + (1.181234017\dots) \left(\frac{\alpha}{\pi}\right)^3 - 1.91206(84) \left(\frac{\alpha}{\pi}\right)^4 + 7.79(34) \left(\frac{\alpha}{\pi}\right)^5 + \dots$$

(coefficients from CODATA 2014)

QED contributions to the **muon** g - 2:

$$a_{\mu}(\mathsf{QED}) = \frac{\alpha}{2\pi} + 0.765857425(17) \left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32) \left(\frac{\alpha}{\pi}\right)^3 + 130.8796(63) \left(\frac{\alpha}{\pi}\right)^4 + 752.2(1.0) \left(\frac{\alpha}{\pi}\right)^5 + \cdots$$
(coefficients from PDG 2020)

At higher orders, the coefficients of  $a_{\mu}(\text{QED})$  are much larger than those of  $a_e(\text{QED})$ . This happens because ....

### logarithmic enhancement in muon g-2

the logarithmic enhancement  $\ln(m_{\mu}/m_e) \approx 5.3$ note: It does not exist for the lightest lepton, electron. Two sources of the logarithm

Charge renormalization of the vacuum-polarization(VP) function
 2<sup>nd</sup>-order VP arises

$$\frac{2}{3}\ln(m_{\mu}/m_e) - \frac{5}{9} \sim 3$$

"Renormalization Group" estimate

2. Light-by-light scattering diagram

$$\frac{2}{3}\pi^2 \ln(m_\mu/m_e) \sim 35$$



Coulomb photon loops provide the factor  $\pi^2$ Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

D. Nomura (IUHW/KEK)

g-2 theory

### 10<sup>th</sup>-order contribution

12,672 Feynman vertex diagrams contribute to the 10<sup>th</sup> order . They are classified into 32 gauge-invariant subsets over 6 sets.



Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

### $10^{\text{th}}$ -order leading term of $A_2^{(10)}$



The Leading Order(LO) contribution:

6<sup>th</sup>-order light-by-light x two 2<sup>nd</sup>-order vp's

estimate 20 x 3^2 x 6 ways ~ 1080 I-by-I 2 vp

Actually, its contribution is 542.760 ± 0.099 >  $(\alpha/\pi)^{-1}$  ~430

Can also pick up the NLO diagrams:



with the renormalization group estimate

The total of 31 subsets of the mass-dependent 10th-order term

$$A_2^{(10)}(m_\mu/m_e) = 742.18(87)$$

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

### Summary of 8<sup>th</sup> and 10<sup>th</sup>-order QED to muon g-2

$$A_2^{(8)}(m_{\mu}/m_e) = 132.6852 \ (60)$$

$$A_2^{(8)}(m_{\mu}/m_{\tau}) = 0.042 \ 34 \ (12)$$

$$A_3^{(8)}(m_{\mu}/m_e, m_{\mu}/m_{\tau}) = 0.062 \ 72 \ (4)$$

$$A_2^{(10)}(m_{\mu}/m_e) = 742.18 \ (87)$$

$$A_2^{(10)}(m_{\mu}/m_{\tau}) = -0.068 \ (5)$$

$$A_3^{(10)}(m_{\mu}/m_e, m_{\mu}/m_{\tau}) = 2.011 \ (10)$$

QED contributions to the muon g-2 is now firmly established.

Rough estimate of the 12<sup>th</sup>-order contribution:  $6^{th}$ -order light-by-light x three 2<sup>nd</sup>-order vp x 10 ways ~ 20 x 3^3 x 10 ( $\alpha/\pi$ )<sup>6</sup> ~ 5,000 ( $\alpha/\pi$ )<sup>6</sup> ~ 0.08 x 10<sup>-11</sup> Recall the aimed goal of the on-going experiments ~ 12 x 10<sup>-11</sup>

### Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

# **Electroweak Contribution**

Electroweak (EW) contribution:

$$\begin{split} a_{\mu}(\mathsf{EW}) &= \underbrace{19.48 \times 10^{-10}}_{\mbox{$1$-loop$}} + \underbrace{(-4.12(10) \times 10^{-10})}_{\mbox{$2$-loop$}} + \underbrace{\mathcal{O}(10^{-12})}_{\mbox{$3$-loop$ leading log}} \\ &= 15.36(10) \times 10^{-10} \ , \qquad (\mbox{Number taken from PDG 2020}) \end{split}$$

where the uncertainty mainly comes from quark loops.

- 1-loop result published by many groups (Bardeen-Gastmans-Lautrup, Altarelli-Cabibbo-Maiani, Jackiw-Weinberg, Bars-Yoshimura, Fujikawa-Lee-Sanda) in 1972, and now a textbook exercise (Peskin & Schroeder's textbook, Problems 6.3 (Higgs) and 21.1 (W, Z))
- 2-loop contribution ( $\sim$  1700 diagrams in the 't Hooft-Feynman gauge) enhanced by  $\ln(m_Z/m_\mu)$  and also by a factor of  $\mathcal{O}(10)$ ,

$$a_\mu({\sf EW}, \operatorname{2-loop}) \simeq -10 \left(rac{lpha}{\pi}
ight) a_\mu({\sf EW}, \operatorname{1-loop}) \left(\lnrac{m_Z}{m_\mu}+1
ight) \, ,$$

where the factor of 10 appears since many "order one" diagrams accidentally add up. (Czarnecki-Krause-Marciano)

# Hadronic Contributions There are several hadronic contributions:



LO HVP: Leading Order Hadronic Vacuum Polarization Contribution NLO HVP: Next-to-Leading Order HVP Contribution Hadronic Light-by-Light Scattering Contribution HLbL:





### LO Hadronic Vacuum Polarization Contribution

The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.





• Weight function  $\hat{K}(s)/s = \mathcal{O}(1)/s$   $\implies$  Lower energies more important  $\implies \pi^{+}\pi^{-}$  channel: 73% of total  $a_{\mu}^{\text{had,LO}}$ 

Channel	Energy range [GeV]	$a_{\mu}^{had,LO,VP} \times 10^{10}$	$\Delta \alpha_{had}^{(5)}(M_Z^2) \times 10^4$	New data	
	Chiral perturbation th	eory (ChPT) threshold conti	ributions		Breakdown of contributions
$\pi^0 \gamma$	$m_{\pi} \le \sqrt{s} \le 0.600$	$0.12 \pm 0.01$	$0.00 \pm 0.00$		$h_{0} = (H)/D$ from
π <sup>+</sup> π <sup>-</sup>	$2m_{\pi} \le \sqrt{s} \le 0.305$	$0.87 \pm 0.02$	$0.01 \pm 0.00$		to $a_{\mu}(\Pi VP)$ from
$\pi^{+}\pi^{-}\pi^{0}$	$3m_{\pi} \le \sqrt{s} \le 0.660$	$0.01 \pm 0.00$	$0.00 \pm 0.00$		various hadronis final states
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	$0.00 \pm 0.00$	$0.00 \pm 0.00$		various nauronic final states
	Data based c	channels ( $\sqrt{s} \le 1.937$ GeV)			
$\pi^0 \gamma$	$0.600 \le \sqrt{s} \le 1.350$	$4.46 \pm 0.10$	$0.36 \pm 0.01$	[65]	
<i>π</i> <sup>-</sup> <i>π</i> <sup>-</sup>	$0.305 \le \sqrt{s} \le 1.937$	$502.97 \pm 1.97$	$34.26 \pm 0.12$	[34,35]	
$\pi^{+}\pi^{-}\pi^{0}$	$0.660 \le \sqrt{s} \le 1.937$	$47.79 \pm 0.89$	$4.77 \pm 0.08$	[36]	
$\pi^{-}\pi^{-}\pi^{-}\pi^{-}$	$0.613 \le \sqrt{s} \le 1.937$	$14.87 \pm 0.20$	$4.02 \pm 0.05$	[40,42]	
$\pi^{-}\pi^{-}\pi^{'}\pi^{'}$	$0.850 \le \sqrt{s} \le 1.937$	$19.39 \pm 0.78$	$5.00 \pm 0.20$	[44]	
$(2\pi^{+}2\pi^{-}\pi^{0})_{nog}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03		KNT have included new data sets
$3\pi^{+}3\pi^{-}$	$1.313 \le \sqrt{s} \le 1.937$	$0.23 \pm 0.01$	$0.09 \pm 0.01$	[66]	
$(2\pi^{-}2\pi^{-}2\pi^{''})_{naqw}$	$1.322 \le \sqrt{s} \le 1.937$	$1.35 \pm 0.17$	$0.51 \pm 0.06$		from $\sim 30$ papers.
K <sup>+</sup> K <sup>-</sup>	$0.988 \le \sqrt{s} \le 1.937$	$23.03 \pm 0.22$	$3.37 \pm 0.03$	[45,46,49]	
$K_S^0 K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	$13.04 \pm 0.19$	$1.77 \pm 0.03$	[50,51]	in addition to those included
ККл	$1.260 \le \sqrt{s} \le 1.937$	$2.71 \pm 0.12$	$0.89 \pm 0.04$	[53,54]	
<i>KK2π</i>	$1.350 \le \sqrt{s} \le 1.937$	$1.93 \pm 0.08$	$0.75 \pm 0.03$	[50,53,55]	in the HLMNT11 analysis
ηγ +	$0.660 \le \sqrt{s} \le 1.760$	$0.70 \pm 0.02$	$0.09 \pm 0.00$ 0.20 $\pm 0.02$	[67]	
$\eta \pi \cdot \pi$ (t= -0)	1.091 ≤ √8 ≤ 1.937	$1.29 \pm 0.06$	0.39 ± 0.02	[08,09]	
$(\eta \pi \cdot \pi \pi^{-})_{now}$	$1.335 \le \sqrt{3} \le 1.937$	$0.00 \pm 0.13$ $0.08 \pm 0.01$	$0.21 \pm 0.03$ $0.03 \pm 0.00$	[70]	
η2η 2η	$1.338 \le \sqrt{3} \le 1.937$	0.05 ± 0.01	0.05 ± 0.00	[70,71]	KNT have included $\sim 30$ hadronic
$\eta \omega \rightarrow \pi^0 \gamma \pi^0$	$0.920 \le \sqrt{s} \le 1.937$	$0.88 \pm 0.02$	$0.10 \pm 0.01$ $0.19 \pm 0.00$	[72 73]	<i>a</i>
$ud \rightarrow \pi \gamma \pi$	$1569 \le \sqrt{s} \le 1937$	$0.42 \pm 0.03$	$0.15 \pm 0.01$	1.200	final states
$\phi \rightarrow unaccounted$	$0.988 \le \sqrt{s} \le 1.029$	$0.04 \pm 0.04$	$0.01 \pm 0.01$		
non <sup>0</sup>	$1.550 \le \sqrt{s} \le 1.937$	$0.35 \pm 0.09$	$0.14 \pm 0.04$	[74]	
$n(\rightarrow npp)K\bar{K}_{mb} = \kappa\bar{\kappa}$	$1.569 \le \sqrt{s} \le 1.937$	$0.01 \pm 0.02$	$0.00 \pm 0.01$	[53,75]	
DD 117 100-566	$1.890 \le \sqrt{s} \le 1.937$	$0.03 \pm 0.00$	$0.01 \pm 0.00$	[76]	At $2 \leq \sqrt{s} \leq 11$ GeV,
nñ	$1.912 \le \sqrt{s} \le 1.937$	$0.03 \pm 0.01$	$0.01 \pm 0.00$	[77]	
	Estimated con	tributions ( $\sqrt{s} < 1.937$ GeV	)		we use inclusively measured data
$(\pi^{+}\pi^{-}3\pi^{0})$	$1.013 < \sqrt{s} < 1.937$	$0.50 \pm 0.04$	$0.16 \pm 0.01$		
$(\pi^{+}\pi^{-}4\pi^{0})$	$1.313 \le \sqrt{s} \le 1.937$	$0.21 \pm 0.21$	$0.08 \pm 0.08$		
KK3#	$1569 \le \sqrt{s} \le 1.937$	$0.03 \pm 0.02$	$0.02 \pm 0.01$		At higher energies $> 11 \text{ GeV}$
$\omega(\rightarrow nnn)2\pi$	$1.285 \le \sqrt{s} \le 1.937$	$0.00 \pm 0.02$ $0.10 \pm 0.02$	$0.02 \pm 0.01$ $0.03 \pm 0.01$		At higher energies $\gtrsim 11$ GeV,
$\omega(\rightarrow npp)3\pi$	$1.322 \le \sqrt{s} \le 1.937$	$0.17 \pm 0.03$	$0.06 \pm 0.01$		
$\omega(\rightarrow npp)KK$	$1.569 \le \sqrt{s} \le 1.937$	$0.00 \pm 0.00$	$0.00 \pm 0.00$		we use pQCD
$\eta \pi^{+} \pi^{-} 2 \pi^{0}$	$1.338 \le \sqrt{s} \le 1.937$	$0.08 \pm 0.04$	$0.03 \pm 0.02$		
	Other contri	butions ( $\sqrt{s} > 1.937$ GeV)			
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	$43.67 \pm 0.67$	$82.82 \pm 1.05$	[56,62,63]	
$J/\psi$		$6.26 \pm 0.19$	$7.07 \pm 0.22$		
$\psi'$		$1.58 \pm 0.04$	$2.51 \pm 0.06$		
$\Upsilon(1S - 4S)$		$0.09 \pm 0.00$	$1.06 \pm 0.02$		
pQCD	$11.199 \le \sqrt{s} \le \infty$	$2.07 \pm 0.00$	$124.79 \pm 0.10$		
Total	$m_{\pi} \leq \sqrt{s} \leq \infty$	$693.26\pm2.46$	$276.11 \pm 1.11$		

Table from A. Keshavarzi, DN, & T. Teubner (KNT), Phys. Rev. D97 (2018) 114025

D. Nomura (IUHW/KEK)

q-2 theory

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Vacuum Polarization Corrections to  $\sigma(e^+e^- \rightarrow hadrons)$ Optical Theorem:



Experimentally observed cross section:



To evaluate  $a_{\mu}^{\text{LO, had}}$ , we need to subtract the vacuum polarization (VP) contribution.

It is straightforward to subtract the leptonic part of the VP, but the hadronic part is non-trivial: we need to do this recursively by using hadronic data, which introduces uncertainty.

Final State Radiation Corrections to  $\sigma(e^+e^- \rightarrow hadrons)$ 

**Optical Theorem:** 



To evaluate  $a_{\mu}^{\text{LO, had}}$ , by definition, we use the hadronic cross sections which include all the Final State Radiations (FSR).



In real experiments, people often impose cuts on the final state photons and/or miss photons in the final states. So we have to add back those missed photons, which introduces uncertainties.

# **Data Combination**

To evaluate the vacuum polarization contribution, we have to combine lots of experimental data.

To do so, we usually construct a  $\chi^2$  function and find the value of R(s) at each bin which minimizes  $\chi^2$ .

Naively, the  $\chi^2$  function defined as

$$\chi^2(\{\overline{R}_i\}) \equiv \sum_{n=1}^{N_{ ext{exp}}} \sum_{i=1}^{N_{ ext{bin}}} \sum_{j=1}^{N_{ ext{bin}}} (R_i^{(n)} - \overline{R}_i) (V_n^{-1})_{ij} (R_j^{(n)} - \overline{R}_j) \ ,$$

where  $V_n$  is the cov. matrix of the *n*-th exp.,

$$V_{n,ij} = \begin{cases} (\delta R_{i,\text{stat}}^{(n)})^2 + (\delta R_{i,\text{sys}}^{(n)})^2 & (\text{for } i = j) \\ (\delta R_{i,\text{sys}}^{(n)})(\delta R_{j,\text{sys}}^{(n)}) & (\text{for } i \neq j) \end{cases}$$

may seem OK, but when there are non-negligible normalization uncertainties in the data, we have to be more careful.  $\chi^2$  vs normalization error: d'Agostini bias G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306 We first consider an observable x whose true value is 1. Suppose that there is an experiment which measures xand whose normalization uncertainty is 10%. Now, assume that this experiment measured x twice:

$$\begin{array}{ll} \mbox{1st result:} & 0.9\pm 0.1_{\rm stat}\pm 10\%_{\rm syst} \;, \\ \mbox{2nd result:} & 1.1\pm 0.1_{\rm stat}\pm 10\%_{\rm syst} \;. \end{array}$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the  $\chi^2$  function are

$$egin{aligned} \mathsf{(cov.)} &= egin{pmatrix} 0.1^2 + 0.09^2 & 0.09 \cdot 0.11 \ 0.09 \cdot 0.11 & 0.1^2 + 0.11^2 \end{pmatrix} \ , \ \chi^2 &= egin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} \mathsf{(cov.)}^{-1} egin{pmatrix} x - 0.9 \ x - 1.1 \end{pmatrix} \ . \end{aligned}$$

 $\chi^2$  takes its minimum at x=0.98: Biased downwards!

d'Agostini bias (2): improvement by iterations What was wrong? In the previous page,

$$\begin{array}{ll} \mbox{1st result:} & 0.9\pm 0.1_{\rm stat}\pm 10\%_{\rm syst} \;, \\ \mbox{2nd result:} & 1.1\pm 0.1_{\rm stat}\pm 10\%_{\rm syst} \;. \end{array}$$

we took the syst. errors 0.09 and 0.11, respectively, which made the downward bias. Instead, we should take 10% of some estimator  $\bar{x}$  as the syst. errors. Then,

$$({ t cov.}) = egin{pmatrix} 0.1^2 + (0.1ar{x})^2 & (0.1ar{x})^2 \ (0.1ar{x})^2 & 0.1^2 + (0.1ar{x})^2 \end{pmatrix} \,, \ \chi^2 = egin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} ({ t cov.})^{-1} egin{pmatrix} x - 0.9 \ x - 1.1 \end{pmatrix} \,.$$

 $\chi^2$  takes its minimum at x = 1.00: Unbiased! In more general cases, we use iterations: we find an estimator for the next round of iteration by  $\chi^2$ -minimization. R.D.Ball et al, JHEP 1005 (2010) 075.

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### $\pi^+\pi^-$ data from CMD-2 and SND

 $e^+e^- 
ightarrow \pi^+\pi^-$  data



Fig. 2 of White Paper

### $\pi^+\pi^-$ data from KLOE



### $\pi^+\pi^-$ data from BaBar



## $\pi^+\pi^-$ data from BESIII and CLEO-c



Fig. 5 of White Paper

### $\pi^+\pi^-$ data: comparison



 $\pi^+\pi^-$  data: comparison







### $\pi^+\pi^-$ channel: ACD vs CHS vs DHMZ vs KNT

 $\pi^+\pi^-$  Contribution to a (HVPIO)

~		on to $a_{\mu}$						
	Energy range	ACD18	CHS18	DHMZ19	DHMZ19'	KNT19		
	$\leq 0.6  \text{GeV}$		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)		
	$\leq 0.7  \mathrm{GeV}$		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)		
	$\leq 0.8  { m GeV}$		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)		
	$\leq 0.9  \text{GeV}$		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)		
	$\leq 1.0{ m GeV}$		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)		
-	[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)		
	[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)		
	[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)		
	[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)		
	$\leq 0.63  \text{GeV}$	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)		
	[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)		
	$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)		

ACD18: B. Ananthanarayan et al, PRD 98 (2018) 114015 CHS19: G. Colangelo et al, JHEP 02 (2019) 006 DHMZ19: M. Davier et al, EPJC 80 (2020) 241 KNT19: A. Keshavarzi et al, PRD 101 (2020) 014029

Table 6 of White Paper

# DHMZ vs KNT (table 5 of WP)

### Contributions from major channels to $a_{\mu}$ (HVP LO):

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^{+}\pi^{-}\pi^{0}\pi^{0}$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
$K^+K^-$	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without cc)	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7,∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_{\mu}^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_{\psi}(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Difference in the  $\pi^+\pi^-$  channel is mainly from the way to combine the data sets.

- KNT19: Global  $\chi^2$  minimization
- DHMZ19: Takes the average of "all but KLOE" and "all but BaBar" as the mean value, and counts the half of the diff of the two as an additional systematic uncertainty.

# **Comparison with Lattice Results**



Lattice 2021, 26-30 July 2021

### Talk by A. El-Khadra (U. of Illinois) at Lattice 2021

D. Nomura (IUHW/KEK)

a-2 theory

### Breakdown of SM prediction for muon g-2

### From Table 1 of the White Paper

Contribution	Section	Equation	Value ×10 <sup>11</sup>	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO $(e^+e^-)$	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO $(e^+e^-)$	Sec. 2.3.8	Eq. (2.34)	-98.3(7)	Ref. [7]
HVP NNLO $(e^+e^-)$	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, udsc)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18-30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP $(e^+e^-, LO + NLO + NNLO)$	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2-8, 18-24, 31-36]
Difference: $\Delta a_{\mu} := a_{\mu}^{\exp} - a_{\mu}^{SM}$	Sec. 8	Eq. (8.14)	279(76)	

HVP: Hadronic Vacuum Polarization contribution HLbL: Hadronic Light-by-Light contribution

$$a_{\mu}(\mathsf{exp, BNL}) - a_{\mu}(\mathsf{SM}) ~= 27.9(7.6) imes 10^{-10}$$
 (3.7  $\sigma$  )

 $a_{\mu}({
m exp, 2021}) - a_{\mu}({
m SM}) = 25.1(5.9) imes 10^{-10}$  (4.2  $\sigma$ )



### The muon g-2 $\iff \Delta \alpha$ connection

Massimo Passera INFN Padova

KEK-PH Lectures and Workshops May 11<sup>th</sup> 2021

Talk by M. Passera at KEK-PH-2021, May 11, 2021

D. Nomura (IUHW/KEK)

g-2 theory

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# **Precision Electroweak Fit**

- The electroweak sector of the Standard Model can be parametrized by 3 parameters,  $\{g, g', v\}$ .
- Instead of  $\{g, g', v\}$ , we usually choose 3 most precisely measured quantities  $\{M_Z, G_F, \alpha(M_Z)\}$  as input, where  $\alpha(M_Z)$  is the least accurately known.
- By using  $\{M_Z, G_F, \alpha(M_Z)\}$  as input, we can indirectly predict the Higgs boson mass by comparing observables (such as  $M_W$ ,  $Br(Z \to f\bar{f})$ , ...) with the SM predictions.
- This is possible since the Higgs boson gives a contribution to these observables through radiative corrections.

- Can Δa<sub>u</sub> be due to missing contributions in the hadronic σ(s)?
- An upward shift of  $\sigma(s)$  also induces an increase of  $\Delta \alpha_{had}^{(5)}(M_z)$ .
- Consider:

$$\begin{aligned} \mathbf{a}_{\mu}^{\text{HLO}} &\to \\ \mathbf{a} &= \int_{4m_{\pi}^{2}}^{s_{u}} ds \, f(s) \, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^{3}}, \, s_{u} < M_{Z}^{2}, \\ \mathbf{\Delta}\alpha_{\text{had}}^{(5)} &\to \end{aligned} \\ b &= \int_{4m_{\pi}^{2}}^{s_{u}} ds \, g(s) \, \sigma(s), \qquad g(s) = \frac{M_{Z}^{2}}{(M_{Z}^{2} - s)(4\alpha\pi^{2})}, \end{aligned}$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

 $\epsilon$ >0, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \quad \longrightarrow \quad$$

M Passera KEK May 11th 2021

### Talk by M. Passera at KEK-PH-2021, May 11, 2021

D. Nomura (IUHW/KEK)

g-2 theory

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#### Shifts $\Delta \sigma(s)$ to fix $\Delta a_{\mu}$ are possible, but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

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Δα

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D. Nomura (IUHW/KEK)

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#### How large are the required shifts $\Delta \sigma(s)$ ?



Shifts below ~1 GeV conflict with the quoted exp. precision of  $\sigma(s)$ 

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)

M Passera KEK May 11th 2021

Talk by M. Passera at KEK-PH-2021, May 11, 2021

D. Nomura (IUHW/KEK)

g-2 theory

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Δα

# **Summary**

- SM prediction for  $(g-2)_{\mu}$ : 4.2 $\sigma$  deviation from measured value  $\implies$  New Physics?
- Recent data-driven evaluations of HVP contributions seem convergent
- To better establish the  $(g-2)_{\mu}$  anomaly, better data for  $e^+e^- \rightarrow \pi^+\pi^-$  welcome (from BESIII, CMD-3, Belle II, ...)
- In general, lattice results still suffer from large uncertainties, but the BMW collaboration claim a smaller uncertainty and a better agreement with a<sub>µ</sub>(exp).
   (Which is correct, data-driven or BMW?)
- The EW precision data seem to favor the data-driven analysis (although I may be biased...)