JET PHYSICS

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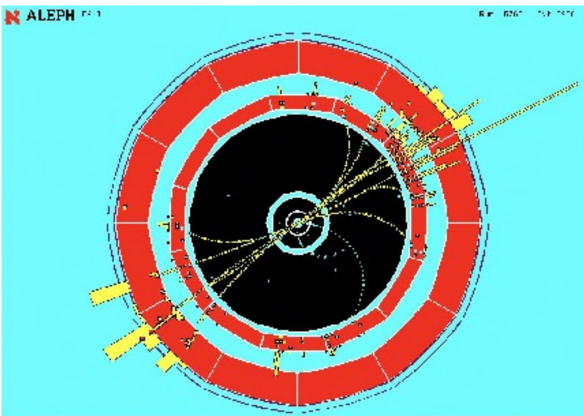
MITP School 2021
"The Amplitudes Games"

12 – 30 July 2021

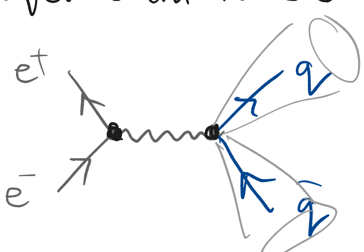
What is a Jet?

"collimated cluster / spray of particles (tracks, calorimeter deposits) or flow of energy"

EXPERIMENT:

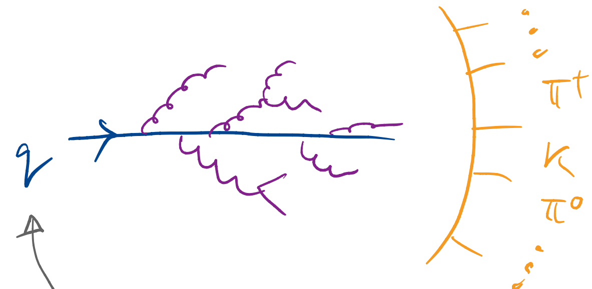


2 jet event in $e^+e^- \to Z/\gamma^* \to q\bar{q}$



interface

THEORY:



JETS – proxy for partons (particles of hadrons)

well-defined

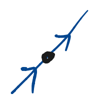
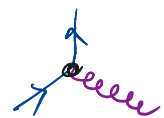
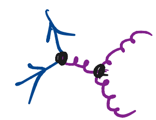
quarks / gluons
ill-defined

Why do we care?

* Jets are everywhere in QCD (partons $\leftrightarrow$ jets)

* Study QFTs (gauge theories)

  $\Rightarrow$ $e^+e^- \to$ 2 jets , 3 jets , 4 jets

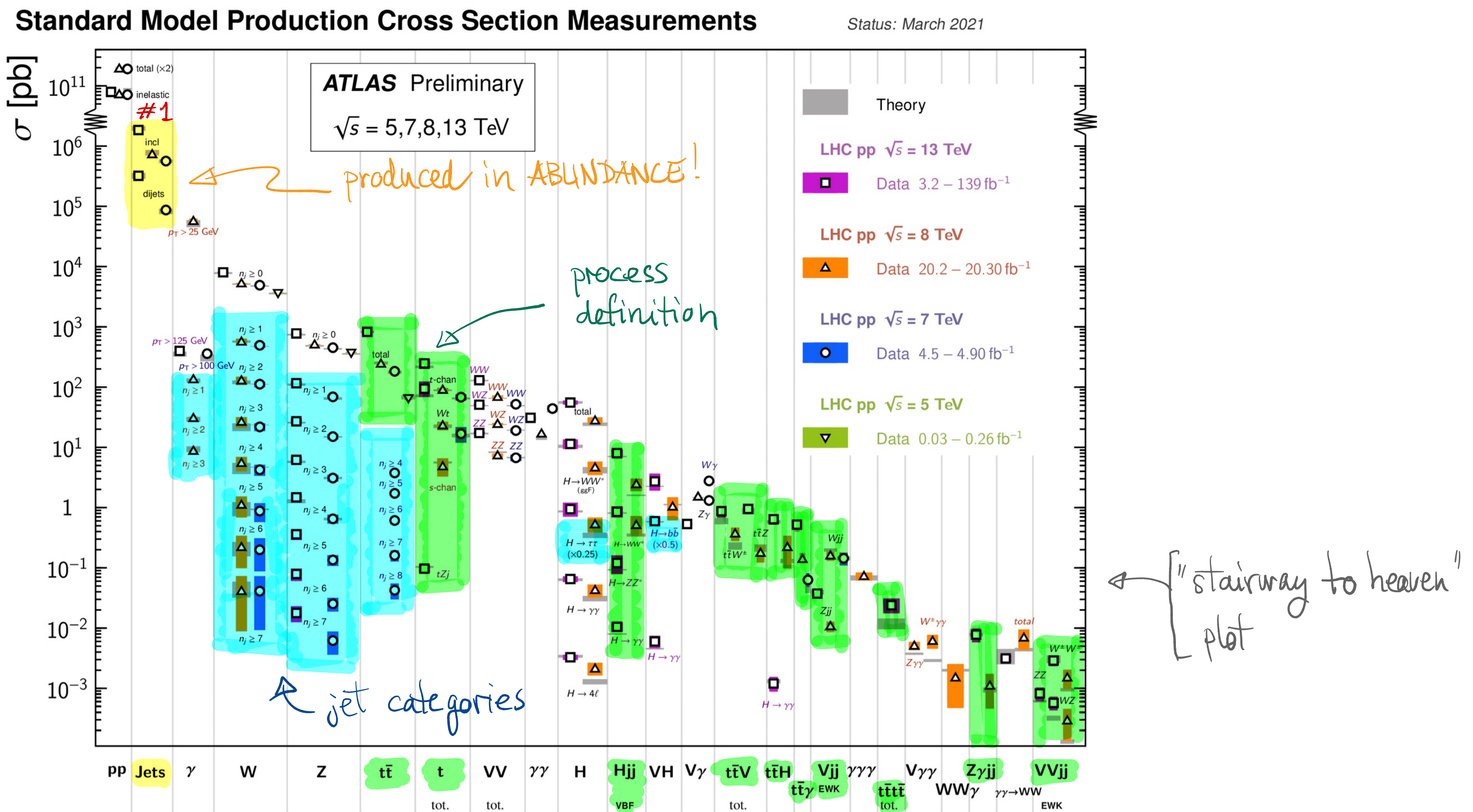
$\exists$ gluon!   non-Abelian

* New Physics Searches

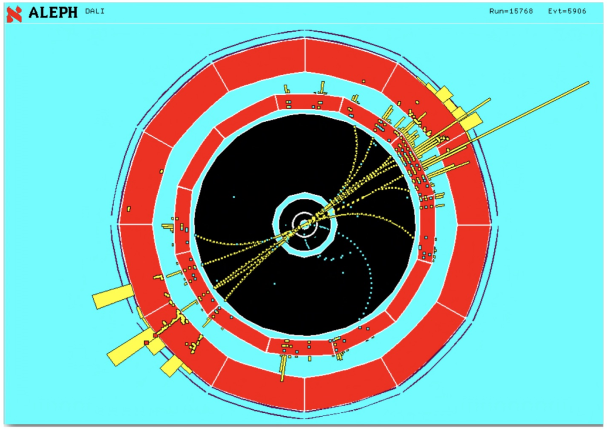
  $\Rightarrow$ study Higgs sector / Hierarchy Problem ; Dark Matter ?

  $\Rightarrow$ boosted objects

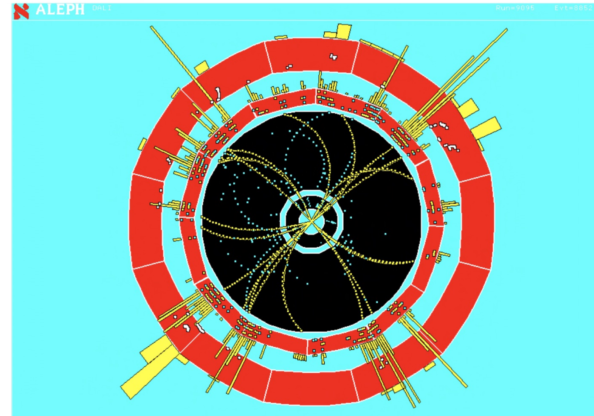
We cannot avoid them!



Jets are not unique ...



2 jets !



jets = ?

FREEDOM:

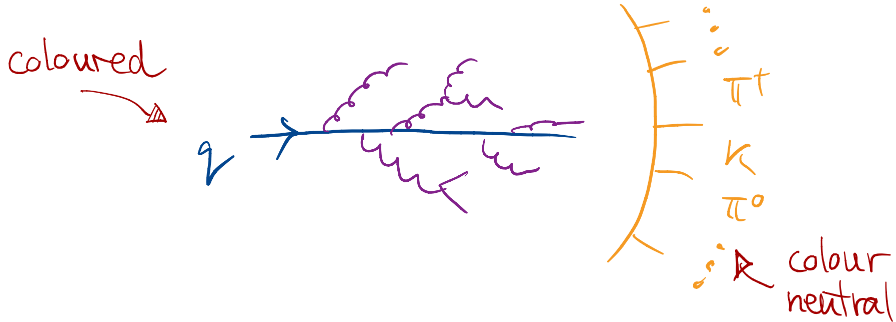
(1) which particles to put together ?

(2) how to combine them (momentum $p_{(ij)} = p_i + p_j$?)

→ JET DEFINITION (better respect infrared safety!)

... and fundamentally ambiguous

"proxy for a high-energetic parton"

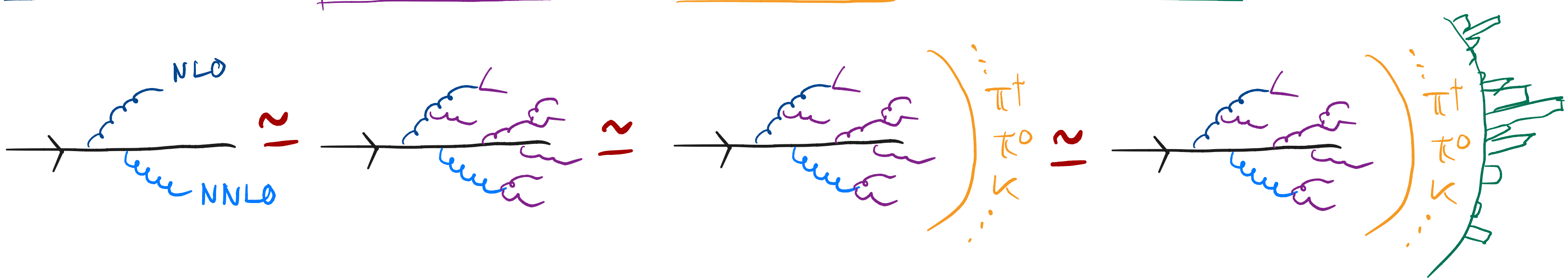


best we can hope for:

clusters of partons $\simeq$ clusters of hadrons

* ideally a robust definition:

fixed order   parton shower   hadronisation   detector



Outline

I. Introduction to Jets

II. Soft & Collinear Singularities and IR Safety

III. Jet Algorithms

IV. Jet Substructure

I. Introduction to Jets

* How do jets emerge from $\mathcal{L}_{QCD}$?

  $\leftrightarrow$ properties of QCD

* Jets in a hadron collider environment.

Everything begins with $\mathcal{L}_{QCD}$

$\leftrightsquigarrow$ spin-½ quarks ⊕ local SU(N_c)

(ignored: $m_q = \emptyset$)

$$\mathcal{L}_{QCD} = \sum_q \bar{\psi}_q \left(i D_\mu \gamma^\mu \boxed{- m_q} \right) \psi_q$$

$$- \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$$

$$\psi_q = \begin{pmatrix} \psi_q^r \\ \psi_q^g \\ \psi_q^b \end{pmatrix} \quad (N_c = 3)$$

gluon field
$a = 1, \dots, N_c^2 - 1 = 8$

$q = u, d, s, c, b, t$
quark flavours

* covariant derivative $D_\mu = \partial_\mu + i g_s A^a_\mu t^a$

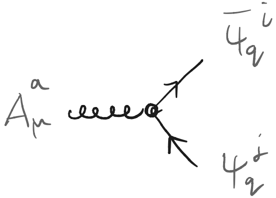
SU(N_c) generators
$t^a = \frac{1}{2} \lambda^a$
$\mathrm{Tr}\left[t^a t^b\right] = T_F \, \delta^{ab} \; ; \; T_F = \frac{1}{2}$
$\left[t^a, t^b\right] = i f^{abc} t^c$

* field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \underbrace{- g_s f^{abc} A^b_\mu A^c_\nu}_{\text{non-Abelian}}$$

$\rightsquigarrow$ self-interactions

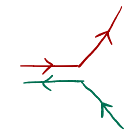
QCD Feynman rules



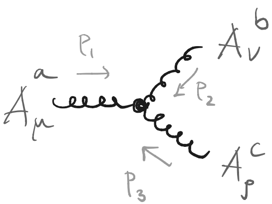
$$-i\, g_s \,(t^a)_{ij}\, \gamma_\mu$$

$$\bar\psi_q^r \quad t^1 \quad \psi_q^g$$

$$(1,0,0)\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

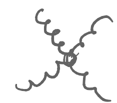


gluons carry colour & anti-colour



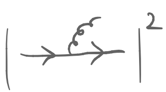
$$-g_s f^{abc}\left[g_{\mu\nu}\,(p_1-p_2)_\rho + \text{cyclic} \right]$$

;

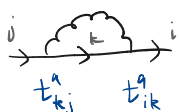


$\sim g_s^2 f^2$

* colour factors for emissions from a quark/gluon



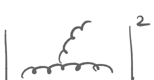
$\simeq$



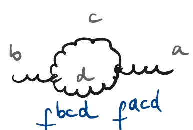
$= C_F\, \delta_{ij}$

$$C_F = T_F \frac{N_c^2-1}{N_c} = \frac{4}{3}$$

gluon ~ x2 "colour charge"



$\simeq$



$= C_A\, \delta^{ab}$

$$C_A = 2\, T_F\, N_c = 3$$

The running coupling

$$\alpha_s = \frac{g_s^2}{4\pi}$$

[5 loops [Chetyrkin et al. '16]]

$$\frac{1}{\alpha_s}\frac{\partial \alpha_s}{\partial \ln \mu^2} = \beta(\alpha_s) = -\left[\beta_0\left(\frac{\alpha_s}{2\pi}\right) + \beta_1\left(\frac{\alpha_s}{2\pi}\right)^2 \cdots + \beta_4\left(\frac{\alpha_s}{2\pi}\right)^5 + \ldots\right]$$

$$\beta_0 = \frac{11}{6}C_A - \frac{2T_F N_f}{3}$$

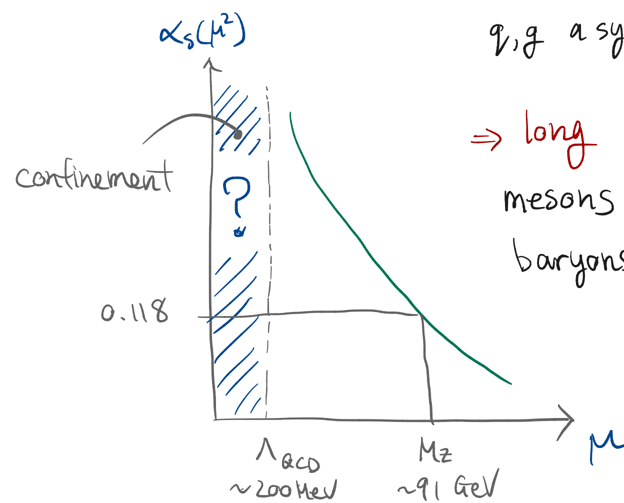
[# light quarks 3 (uds), ... 6

lowest order

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \beta_0 \frac{\alpha_s(\mu_0)}{2\pi}\ln\left(\mu^2/\mu_0^2\right)}$$

$$= \frac{1}{\frac{\beta_0}{2\pi}\ln\left(\mu^2/\Lambda_{QCD}^2\right)}$$

breakdown of pQCD

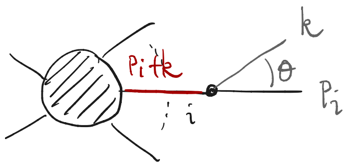


⇒ short distances: q, g asympt. free!

⇒ long distances: mesons ($q\bar{q}$) baryons (qqq)

Real emission in QCD

$2 \longrightarrow n+1$



propagator

$$(m_i^2 = k^2 = \emptyset) \qquad \frac{1}{(p_i+k)^2} = \frac{1}{2p_i \cdot k} = \frac{1}{2E_iE\,(1-\cos\theta)}$$

$\Rightarrow$ emissions are (potentially) enhanced in the

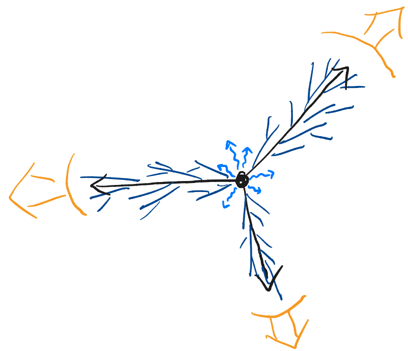
- $*$ soft limit: $E_i \to \emptyset$ or $E \to \emptyset$
- $*$ collinear limit: $\theta \to \emptyset$

in the soft & collinear limit (including LIPS)
we will show that the emission probability is simply:

$$d\,W^{s\&c}_{\phi\to\phi+g} = \frac{2\,\alpha_s}{\pi}\,C_\phi\,\frac{dE}{E}\,\frac{d\theta}{\theta}$$

two log-divergences

Emergent picture of QCD Jets



1. hard (high energetic) partons

2. asymptotic freedom & emission pattern

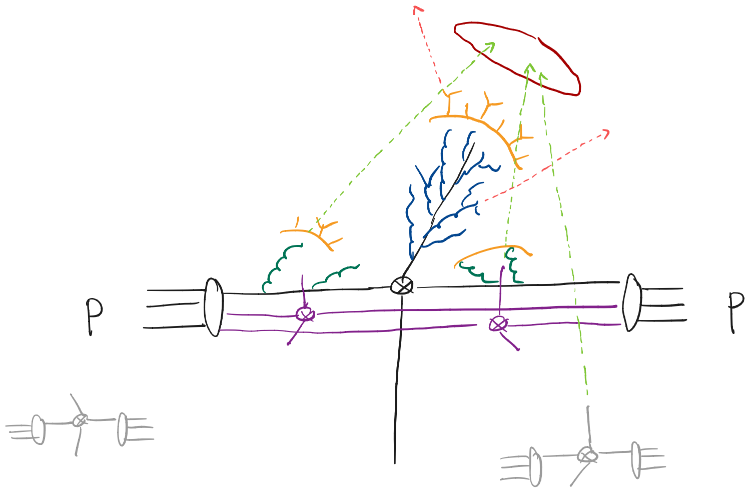
   $\hookrightarrow$ perturbative parton shower

3. long distance $\alpha_s \to 1 \rightsquigarrow$ hadronization

   $\hookrightarrow$ directions maintained ($m_{u,d} \ll \Lambda_{QCD}$)

   "cheap" to create $q\bar{q}$ pairs

Jets at hadron colliders



- final-state radiation (FSR) $\sim Q/2$
- initial-state radiation (ISR) $\sim Q/2$
- multiple parton interactions (MPI)
  aka underlying event (UE) $\sim$ GeV
- pile up (PU) $\sim n_{PU} \cdot 0.5$ GeV
- hadronisation $\sim \Lambda_{QCD}$

$$\text{Jet} = \underbrace{\left(\begin{array}{c}\text{hard parton} \\ + \\ \text{radiation}\end{array}\right)}_{\text{what we're after}} - \text{LOSS} + \text{CONTAMINATION}$$

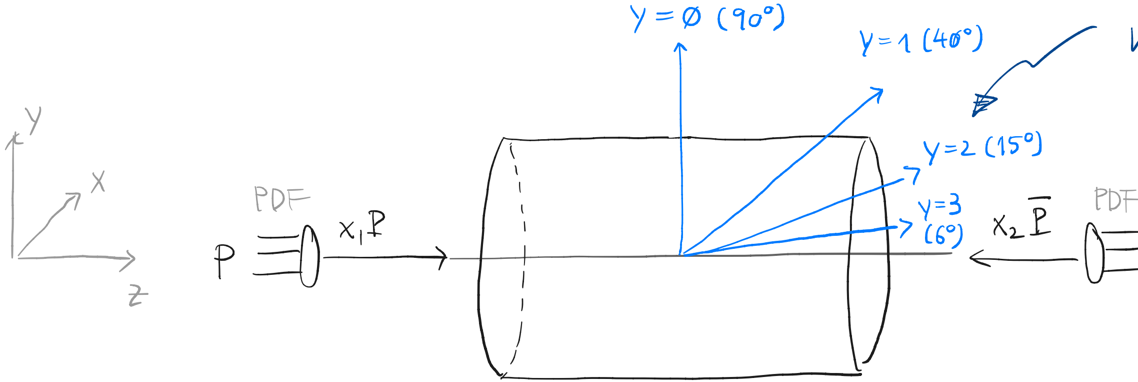
R bigger?

R smaller?

⇒ there is no single "best" jet definition (trade-offs; depends on application)

Hadron collider kinematics

partonic CoM $\neq$ hadronic CoM — z-boost



* choose variables that are invariant or transform simply w.r.t. longitudinal (z) boosts

$$p_T = \sqrt{p_x^2 + p_y^2} \qquad \text{transverse momentum}$$

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \qquad \text{rapidity} \quad \left(\overset{m=\emptyset}{\longleftrightarrow} \quad \eta = -\ln\tan\frac{\theta}{2}\right)$$

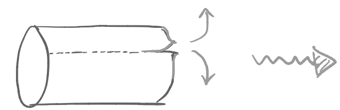
$$\varphi = \arctan\left(p_y/p_x\right) \qquad \text{azimuth}$$

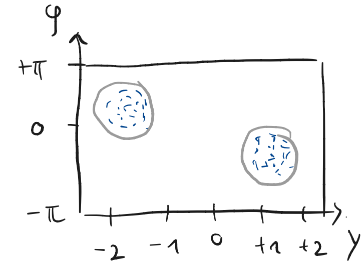
$$\left.\begin{matrix} p_T \\ y \\ \varphi \end{matrix}\right\} \xrightarrow{\text{z - boost}} \begin{matrix} p_T \\ y + \xi \\ \varphi \end{matrix}$$

$$\begin{pmatrix} \cosh\xi & \vec{0} & \sinh\xi \\ \vec{0} & \mathbb{1} & \vec{0} \\ \sinh\xi & \vec{0} & \cosh\xi \end{pmatrix}$$

Boost invariant distance measure

$$\boxed{\Delta R^2 = \Delta y^2 + \Delta \varphi^2}$$





* Comparison to standard opening angle $\Delta\Omega^2 = \Delta\theta^2 + \sin^2\theta\, \Delta\varphi^2$

$\Delta R^2 = \cosh^2 y\ \Delta\Omega^2$ for $y \sim \emptyset$ $(90°)$: $\Delta R^2 \sim \Delta\Omega^2$

in forward region: rescaled by $\cosh y$

$p_T \Delta R \simeq E\, \Delta\Omega$

* ISR (useful reparametrisation)

$$d\omega^{\mathrm{soc}}_{\phi\to\phi+g} \propto \frac{dE}{E}\,\frac{d\theta}{\theta} \longrightarrow \frac{dp_T}{p_T}\, dy$$

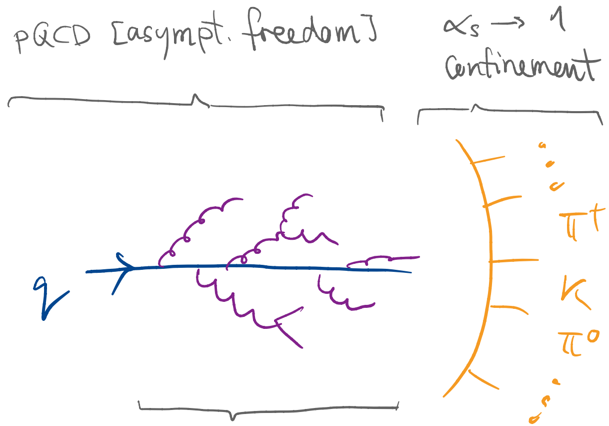
uniform emission probability in y!

$\Rightarrow$ choose $\Delta\Omega^2$ cone smaller for $\theta \to \emptyset$ $\rightsquigarrow$ uniform contamination

II. Soft & Collinear Singularities and IR Safety

Last Lecture

∗ Jets as an emergent feature of QCD:



direction partons $\approx$ direction hadrons [$m_{had} \ll \Lambda_{QCD}$]

emission pattern [S&C]

$$\frac{2\alpha_s}{\pi} C_F \boxed{\frac{dE}{E} \frac{d\theta}{\theta}}$$

[Looks divergent! ⇒ investigate this further

⤳ collimated spray of partons

Soft & collinear singularities in QCD

* SOFT LIMIT [factorization @ amplitude]

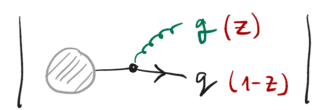
$$\mathcal{M}_{n+1}(p_1,\dots p_n, g(k)) \xrightarrow{k\to\emptyset} g_s\, \hat{J}^{a,\mu} \otimes \mathcal{M}_n(p_1,\dots,p_n)\ \varepsilon^a_\mu(k)^*$$

eikonal current: $\hat{J}^{a,\mu} = \sum_{i=1}^{n} \underbrace{\frac{p_i^\mu}{(p_i k)}}\, \hat{T}^a_i$ $\qquad \hat{T}^a_i = \begin{cases} t^a & \text{in } q, \text{ out } \bar{q} \\ -t^a & \text{out } q, \text{ in } q \\ T^a_{adj} & g \end{cases}$

[only colour charge & direction]

↳ $|\mathcal{M}|^2 \Rightarrow$ colour correlation between all part.

* COLLINEAR LIMIT [factorization @ $|\text{ampl.}|^2$]



$$\mathcal{M}_{n+1}(p_1,\dots,p_n, g(k)) \xrightarrow{p_i \parallel k} g_s^2 \frac{1}{(p_i\cdot k)} \boxed{C_F \frac{1+(1-z)^2}{z}} |\mathcal{M}_n(p_1,\dots (p_i+k)\dots p_n)|^2$$

↳ splitting function $P_{gq}(z)$

↳ local to leg i

↳ soft singularity $\sim \frac{1}{z}$

Phase space factorization

LIPS $\quad d\Phi_n = \prod_{i=1}^{n} [dp_i]\,(2\pi)^4\,\delta^{(4)}\left(Q - \sum_{i=1}^{n} p_i\right)\;;\quad [dp_i] \equiv \frac{d^4 p_i}{(2\pi)^4}(2\pi)\,\delta_+(p_i^2 - m_i^2) = \frac{d^3\vec{p}_i}{(2\pi)^3\,2E_i}$

* SOFT LIMIT

$$d\Phi_{n+1}(p_1,\dots,p_n,k) \overset{k\to 0}{\sim} d\Phi_n(p_1,\dots,p_n)\,[dk]$$

* COLLINER LIMIT

$$d\Phi_{n+1}(p_1,\dots,p_n,k) = \left(\prod_{j\neq i}[dp_j]\right)[dp_i]\,[dk]\,(2\pi)^4\,\delta^{(4)}\left(Q - \sum_{j\neq i} p_j - p_i - k\right)$$

$\Leftrightarrow$ parametrize $[dk]$ w.r.t. p_i direction : $\quad [dk] = \frac{1}{8\pi^2} E\,dE\,\overbrace{d\cos\theta}^{\sin\theta d\theta \simeq \theta d\theta}\,\frac{d\varphi}{2\pi}$

$$z = \frac{E}{E+E_i}\;;\quad E = \frac{z}{1-z}E_i\;;\quad dE = \frac{E_i}{(1-z)^2}dz$$

$$\frac{1}{8\pi^2}\left(\frac{z}{1-z}E_i\right)\frac{E_i}{(1-z)^2}\,dz\,\theta d\theta\,\frac{d\varphi}{2\pi}$$

$$= \left(\prod_{j\neq i}[dp_j]\right)\underbrace{(1-z)^2[d\tilde{p}_i]}_{[dp_i]}\,(2\pi)^4\,\delta^{(4)}\left(Q - \sum_{j\neq i} p_j - \tilde{p}_i\right)\frac{1}{8\pi^2}\frac{z\,\tilde{E}_i^2}{(1-z)}\,dz\,\theta d\theta\,\frac{d\varphi}{2\pi}$$

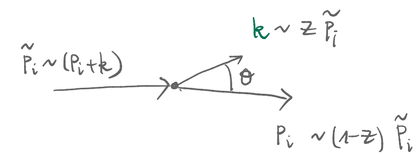
Phase space factorization

LIPS $\quad d\Phi_n = \prod_{i=1}^{n} [dp_i] \, (2\pi)^4 \, \delta^{(4)}\left(Q - \sum_{i=1}^{n} p_i\right) \; ; \quad [dp_i] \equiv \frac{d^4 p_i}{(2\pi)^4} (2\pi) \, \delta_+(p_i^2 - m_i^2) = \frac{d^3 \vec{p}_i}{(2\pi)^3 2E_i}$

* SOFT LIMIT

$$d\Phi_{n+1}(p_1, \dots, p_n, k) \overset{k \sim 0}{\sim} d\Phi_n(p_1, \dots, p_n) \, [dk]$$

* COLLINER LIMIT



$$d\Phi_{n+1}(p_1, \dots, p_n, k) = \left(\prod_{j \neq i} [dp_j] \right) [dp_i] \, [dk] \, (2\pi)^4 \, \delta^{(4)}\left(Q - \sum_{j \neq i} p_j - p_i - k\right)$$

$$\overset{p_i \parallel k}{\sim} d\Phi_n(p_1 \dots \tilde{p}_i \dots p_n) \, \frac{\tilde{E}_i^2}{8\pi^2} \, z(1-z) \, dz \; \theta \, d\theta \; \frac{d\varphi}{2\pi}$$

Emission probability in the collinear limit

$$d\sigma_{n+1} \propto |\mathcal{M}_{n+1}(p_1,\dots,p_n,k)|^2 \, d\Phi_n(p_1,\dots,p_n,k)$$

$$\xrightarrow{p_i \parallel k} g_s^2 \frac{1}{(p_i\cdot k)} P_{qq}(z) \underbrace{|\mathcal{M}_n(p_1,\dots(p_i+k)\dots p_n)|^2 \, d\Phi_n(p_1 \dots \tilde{p}_i \dots p_n)}_{\to\, d\sigma_n} \frac{\tilde{E}_i^2}{8\pi^2} z(1-z)\, dz\, \theta\, d\theta \frac{d\varphi}{2\pi}$$

⇒ emission probability for $q \to qg$ (coll.)

$$\boxed{dw^{coll}_{q\to q+g}} = \frac{\alpha_s}{2\pi} \frac{\tilde{E}_i^2}{\underbrace{E_i E}_{\tilde{E}_i^2 z(1-z)} \underbrace{(1-\cos\theta)}_{\frac{1}{2}\theta^2}} P_{qq}(z)\, z(1-z)\, dz\, \theta\, d\theta \frac{d\varphi}{2\pi} = \boxed{\frac{\alpha_s}{\pi} P_{qq}(z)\, dz \frac{d\theta}{\theta} \frac{d\varphi}{2\pi}}$$

* soft & collinear limit $\quad \left[P_{qq}(z) \to \frac{2}{z} \;\&\; \frac{dz}{z} = \frac{dE}{E} \right]$

$$\boxed{\begin{aligned} dw^{s\&c}_{q\to q+g} &= \frac{2\alpha_s}{\pi} C_F \frac{dE}{E} \frac{d\theta}{\theta} \frac{d\varphi}{2\pi} \\ dw^{s\&c}_{g\to g+g} &= \frac{2\alpha_s}{\pi} C_A \frac{dE}{E} \frac{d\theta}{\theta} \frac{d\varphi}{2\pi} \end{aligned}}$$

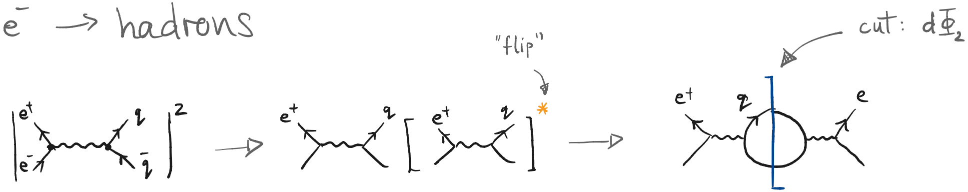
$C_F \leftrightarrow C_A$

⇒ Probability to emit a gluon is infinite!

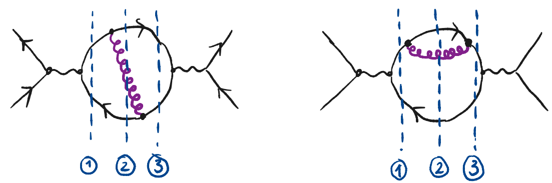
Cancellation of IR singularities

example : $e^+e^- \to$ hadrons

LO (α_s^0) :



NLO (α_s^1):



$$① + ③ \;=\; 2\,\mathrm{Re}\left[\left(\mathcal{M}^{\text{tree}}_{q\bar{q}}\right)^{*}\mathcal{M}^{\text{1-loop}}_{q\bar{q}}\right] d\Phi_2 \;=\; -\,\frac{\alpha_s}{2\pi}\,C_F\,\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-3\epsilon)}\left(\frac{4\pi\mu^2}{s}\right)^{\epsilon}\left[\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+8+\mathcal{O}(\epsilon)\right]$$

$$② \;=\; |\mathcal{M}_{q\bar{q}g}|^2\, d\Phi_3 \;=\; \frac{\alpha_s}{2\pi}\,C_F\,\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-3\epsilon)}\left(\frac{4\pi\mu^2}{s}\right)^{\epsilon}\left[\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{19}{2}\right]$$

[regularised in $D=4-2\epsilon$ $\quad \frac{dE}{E^{1+2\epsilon}} \quad \frac{d\theta}{\theta^{1+2\epsilon}}$]

$\Rightarrow \quad \sigma_{NLO} = \sigma_{LO}\left(1 + \frac{\alpha_s}{\pi}\right)$ singularities cancel between the REAL & VIRTUAL !

The measurement function

"differential"

In general, want to ask more detailed questions than σ_{tot}, e.g. JETS

$$\int |\mathcal{M}_n|^2 \, d\Phi_n \rightsquigarrow \int |\mathcal{M}_n|^2 \underbrace{\mathcal{F}^{(n)}(p_1, \dots, p_n)}_{\text{measurement function}} d\Phi_n$$

* fiducial cuts: $\sigma_{fid} \longleftrightarrow \mathcal{F}(\{p\}) = \Theta_{cut}(p_T^X > p_{T,min})$

* differential distributions: $\frac{d\sigma}{d\theta} \longleftrightarrow \mathcal{F}(\{p\}) = \delta(\theta - \hat{\theta}(\{p\}))$

* . . . , JETS ("projection")

What must $\mathcal{F}$ satisfy such that cancellation of IR singularities in tact?

↪ KLN: For sufficiently inclusive quantities!

Infrared safety

cancellation of IR singularities $\Longleftrightarrow$ $\mathcal{F}$ must be inclusive over degenerate states

* SOFT SAFETY   answer the same when particle w/ infinitesimal E added

$$\boxed{\mathcal{F}^{(n+1)}(p_1, \ldots, p_{n+1}) \xrightarrow{p_i \to \emptyset} \mathcal{F}^{(n)}(p_1, \ldots \cancel{p_i}, \ldots, p_{n+1})}$$

* COLLINEAR SAFETY   answer the same when particle splits exactly into two

$$\boxed{\mathcal{F}^{(n+1)}(p_1 \ldots, p_{n+1}) \xrightarrow{p_i \| p_j} \mathcal{F}^{(n)}(p_1, \ldots \cancel{p_i} \ldots \cancel{p_j} \ldots p_{n+1}, (p_i+p_j))}$$

$\hookrightarrow$ not safe? not calculable in pQCD!

(and likely sensitive to low-scale physics: hadronization ...)

Infrared subtraction

Achieving IR cancellation in differential predictions highly non-trivial

→ cancel singularity without integrating

* need to $\int [dk]$ to expose $1/\epsilon^n$ poles (IR)
* keep $[dk]$ in fact, since $\mathcal{F}$ depends on it (hard)

$\lightning$

@ NLO: conceptually solved [dipole, FKS]

@ NNLO: tremendous progress! $2 \to 2$ all done $2 \to 3$ new frontier

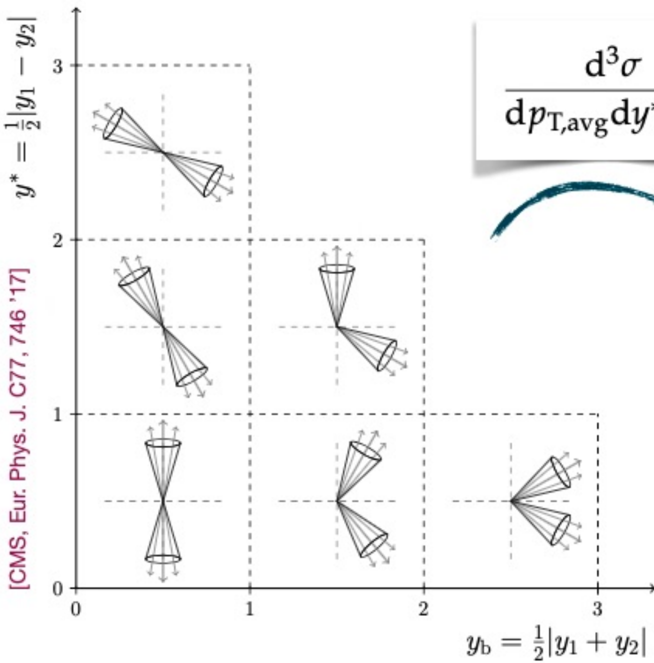
[antenna, CoLorFuL, q_T, Stripper, $\mathcal{T}_N$, nested SC, P2B, ...]

→ NNLO bottlenecks not just subtractions: availability of 2-loop amplitudes!

@ N^{3}LO: specific calculations targeted @ simple processes ($2 \to 1$)

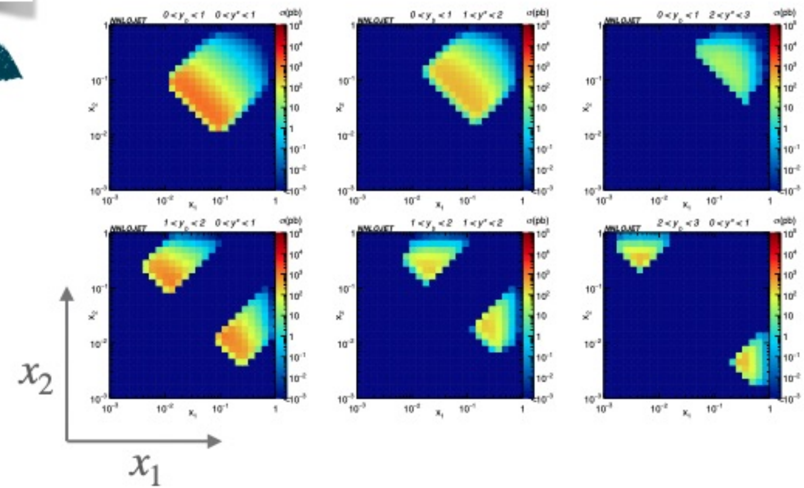
[q_T, P2B]

Triple differential Jet production



$$\frac{d^3\sigma}{dp_{T,avg}\,dy^*\,dy_b}$$

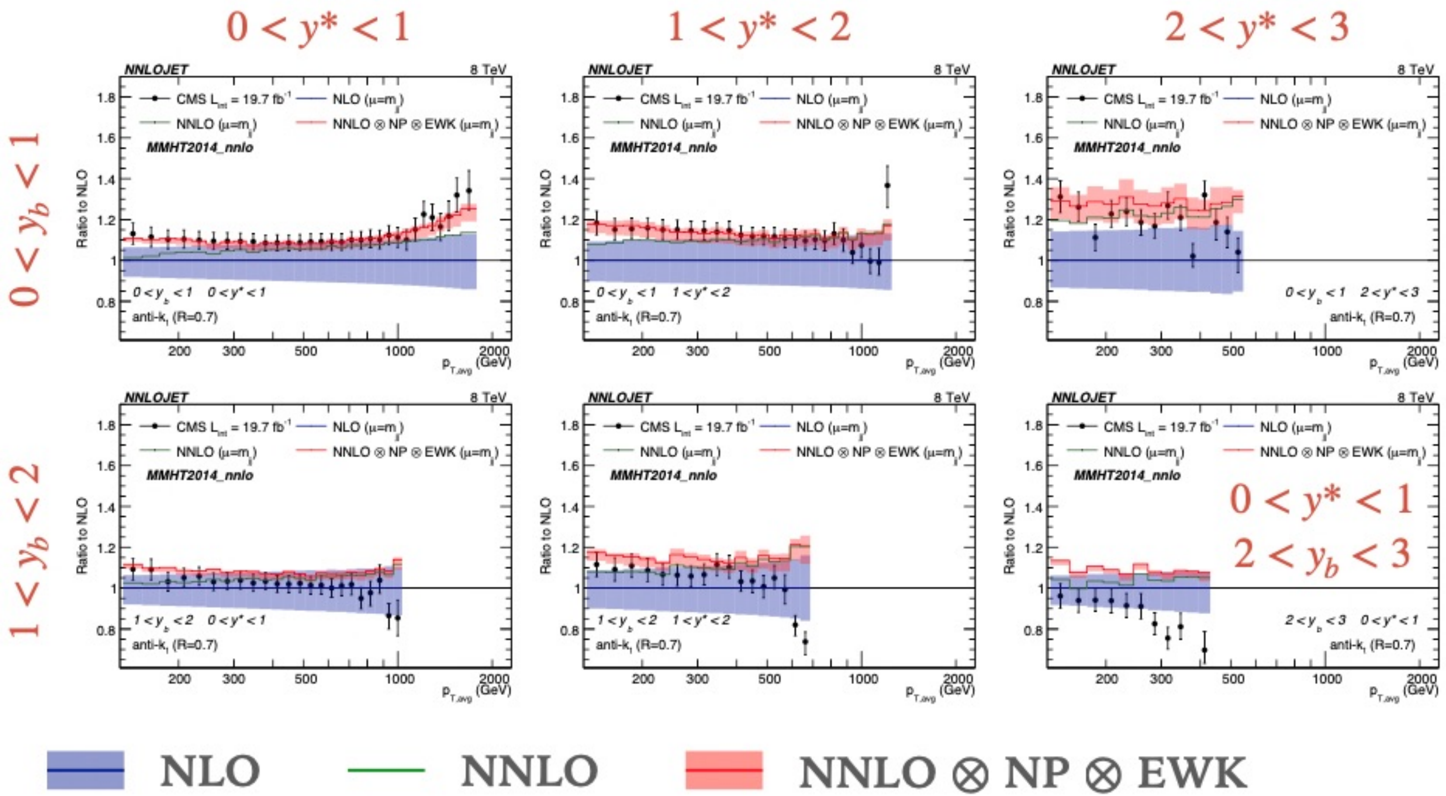
$$x_{1,2} = \frac{2p_{T,avg}}{\sqrt{s}} e^{\pm y_b} \cosh(y^*)$$



- study different **kinematic regimes**
- **disentangle** momentum fractions x_1 & x_2

Triple differential Jet production @ NNLO

[Gehrmann-De Ridder, Gehrmann, Glover, AH, Pires '19]



improved description of data & reduced uncertainties!

Calculations in the IR limits

Sometimes, we're mainly interested in the IR limits

$\hookrightarrow$ (double-) logs, all-order resummation, ...

* real emission simple: factorize $\overset{\text{S\&C}}{\propto} \frac{dE}{E}\frac{d\theta}{\theta}$ $\rightsquigarrow$ tackle analytically
* virtual corrections? $\rightsquigarrow$ trick to include them without calculating anything

---

SINGULARITIES CANCEL Think of the corrections as probabilities

	LO	NLO	...	N^kLO
$P_{\text{no-emit}}$	1	$-\int d\omega^{(1)}$		$-\int d\omega^{(k)}$
P_{emit}	$\emptyset$	$d\omega^{(1)}$		$d\omega^{(k)}$
Σ 1	1	$+\ \alpha_s \emptyset$	$+\ \ldots$	$+\ \alpha_s^k \emptyset$ $\ldots$

"UNITARITY"

(virtual corrections = "no-emission prob.")

III. Jet Algorithms

FERMILAB-Conf-90/249-E
[E-741/CDF]

Toward a Standardization of Jet Definitions *

* To be published in the proceedings of the 1990 Summer Study on High Energy Physics, *Research Directions for the Decade*, Snowmass, Colorado, June 25 - July 13, 1990.

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis; ← performance
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory; ← IR safety
5. Yields a cross section that is relatively insensitive to hadronization.

Jet Algorithms

$$\{p_i\} \longrightarrow \{j\}$$

[particles, momenta
calorimeter towers, ...]

A brief (incomplete) history of jets:

- Sterman-Weinberg jets '77
- k_T algorithm '93
- Cambridge / Aachen '97
- anti-k_T '08

TWO MAIN CLASSES:

① Cone [top down]

idea of directed energy flow

⇨ find coarse regions

(what we have been doing)

② Sequential recombination [bottom up]

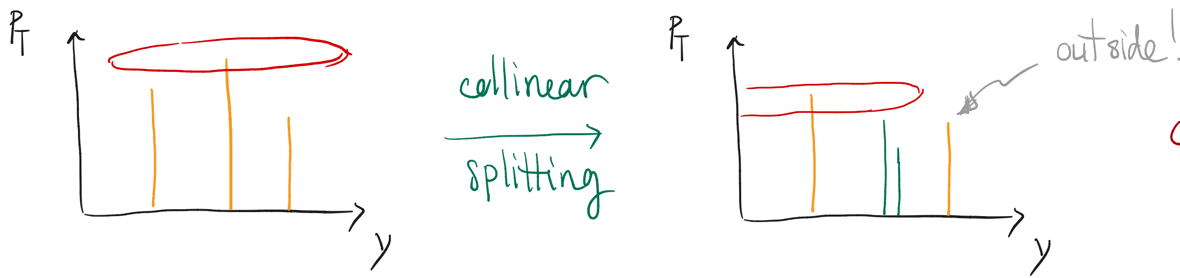
successively undo QCD branching

⇨ find "close" & aggregate

Potential IR issues using [seeded] cones

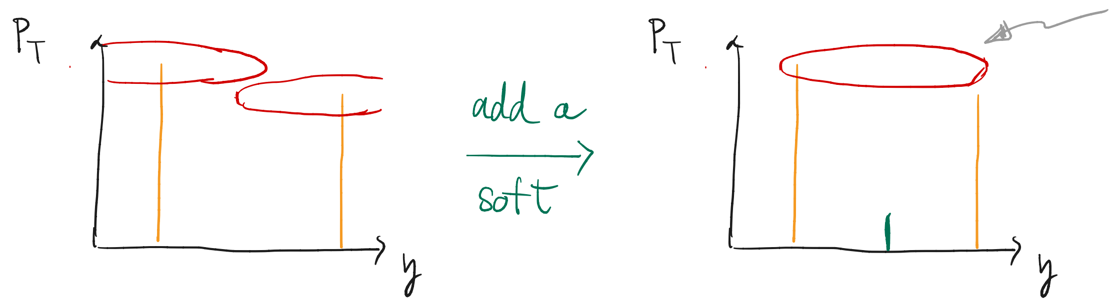
$y \equiv \phi$ in examples!

* start by placing cone around hardest particle



collinear unsafe!

* try placing cones around all particles & look for hardest



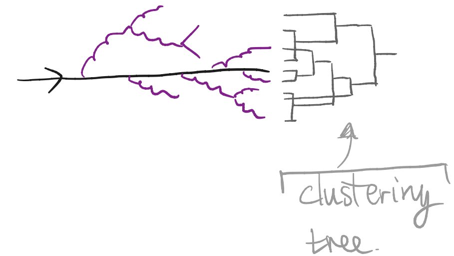
seeded around soft is new hardest

soft unsafe!

* one IR safe cone algorithm: SISCone

Sequential recombination algorithms

Try to work our way backwards through "branchings"



1) Compute distances between all particles

$$d_{ij} = ?$$

and to the beam [for hadron colliders]

$$d_{iB} = ?$$

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

$d_{ij} \Rightarrow$ merge i & j into a new "protojet"

E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

$d_{iB} \Rightarrow$ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

The k_T algorithm

Try to work our way backwards through "branchings"

1) Compute distances between all particles

$$d_{ij} = \min\left(p_{Ti}^2, p_{Tj}^2\right) \frac{\Delta R^2}{R^2}$$

[mimics the inverse of the S&C emission probability ~ relative k_T]

and to the beam [for hadron colliders]

$$d_{iB} = p_{Ti}^2$$

IR safe?

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

→ d_{ij} ⇒ merge i & j into a new "protojet"

E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

→ d_{iB} ⇒ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

The k_T algorithm

Try to work our way backwards through "branchings"

1) Compute distances between all particles

$$d_{ij} = \min\left(p_{T\,i}^2, p_{T\,j}^2\right) \frac{\Delta R^2}{R^2}$$

→ soft first
⇝ irregular shapes
↔ exp. challenges
↔ collects "junk"

and to the beam [for hadron colliders]

$$d_{iB} = p_{T\,i}^2$$

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

→ d_{ij} ⇒ merge i & j into a new "protojet"

E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

→ d_{iB} ⇒ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

4) Only retain jets above a minimum p_T threshold $p_T > p_{T,\mathrm{cut}}$

The generalised k_T algorithm

Try to work our way backwards through "branchings"

1) Compute distances between all particles

$$d_{ij} = \min\left(p_{Ti}^{2\alpha}, p_{Tj}^{2\alpha}\right) \frac{\Delta R^2}{R^2}$$

and to the beam [for hadron colliders]

$$d_{iB} = p_{Ti}^{2\alpha}$$

$$\alpha = \begin{cases} 1: & k_T \\ 0: & \text{Cambridge/Aachen [geometric]} \\ -1: & \text{anti-}k_T \end{cases}$$

anti-k_T: hard first

⟶ nearly perfect cones

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

⟶ d_{ij} ⇒ merge i & j into a new "protojet"

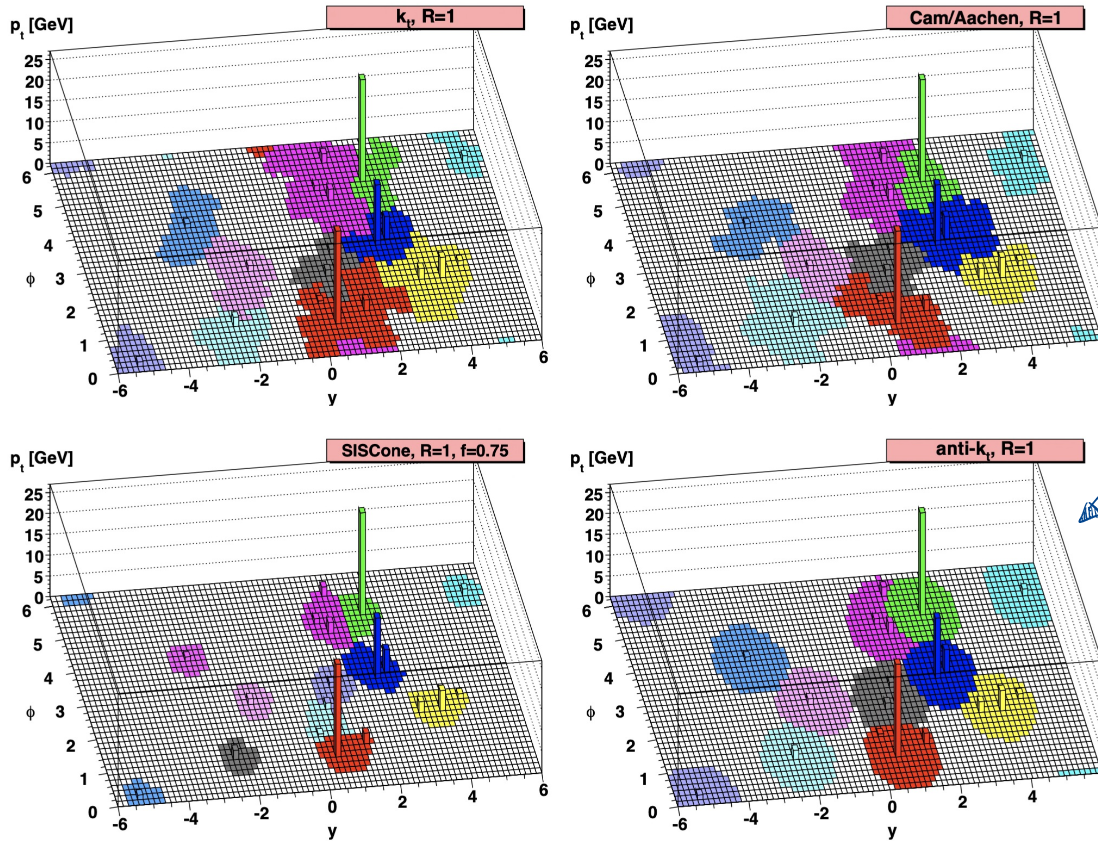
E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

⟶ d_{iB} ⇒ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

4) Discard jets with $p_T < p_{T,\text{cut}}$

Comparison of the algorithms



irregularities from <u>non-linear</u> behaviour w.r.t. soft emissions

solved

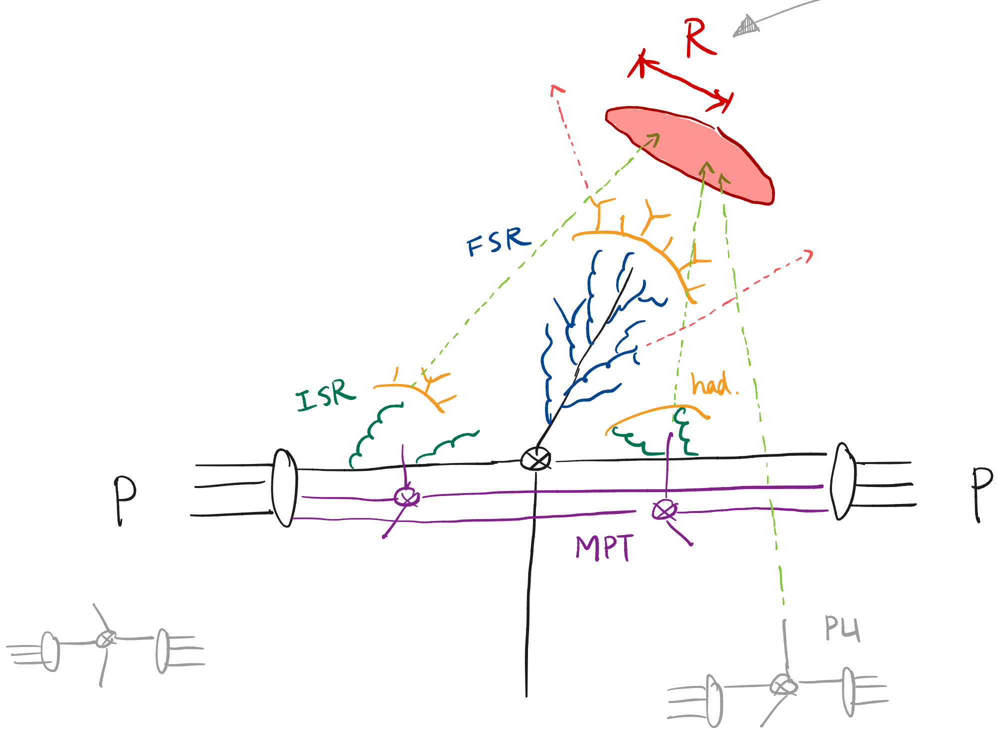
nearly perfect cones

(exp. happy)

* overlap ⇝ hard one

["Towards Jetography" – G. Salam '09]

Choices and how to fix them



Central parameter in all jet definitions

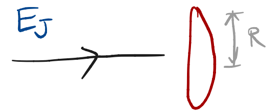
→ strongly impacts

* RADIATION LOSS
* CONTAMINATION

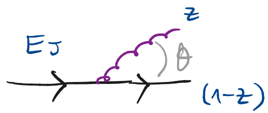
Emission v.s. R cone

Let's consider the energy at a jet

@ LO: $E_{jet} = E_J$



@ NLO:



$$d\omega^{coll}_{q\to qg} = \frac{\alpha_s}{\pi}\frac{d\theta}{\theta}P_{gq}(z)\,dz$$

via unitarity $\rightsquigarrow$ $-\int d\omega^{coll.}_{q\to qg}$

@ NLO

$$\frac{\alpha_s}{\pi}\int\frac{d\theta}{\theta}\,P_{gq}(z)\,dz \;\times\Big\{\cancel{E_J\,\Theta(\theta<R)}$$

real (inside cone)

$$+(1-z)\,E_J\,\Theta(\theta>R)\,\Theta(z<1/2)$$

$$+\;z\;E_J\,\Theta(\theta>R)\,\Theta(z>1/2)$$

outside cone

$$-E_J\Big\}$$

virtual

$$1=\cancel{\Theta(\theta<R)}+\Theta(\theta>R)$$

$$=\frac{\alpha_s}{\pi}\int_R\frac{d\theta}{\theta}\int_0^1 dz\,P_{gq}(z)\;E_J\Big\{-z\,\Theta(z<1/2)-(1-z)\,\Theta(z>1/2)\Big\}$$

$$=-E_J\,\frac{\alpha_s}{\pi}\ln(1/R)\Big\{\int_0^{1/2}dz\;z\,P_{gq}(z)+\int_{1/2}^{1}dz\,(1-z)\,P_{gq}(z)\Big\}$$

regulates $z\to 0$!

Emission v.s. R cone

Let's consider the energy of a jet

@ NLO: $E_{jet} = E_J \left(1 - \frac{\alpha_s}{\pi} \ln(1/R)\, L_x\right)$

$\hookrightarrow L_q = C_F \times 1.01129\ldots$ $\quad \Delta E/E \sim -5\%$ (R=0.4)

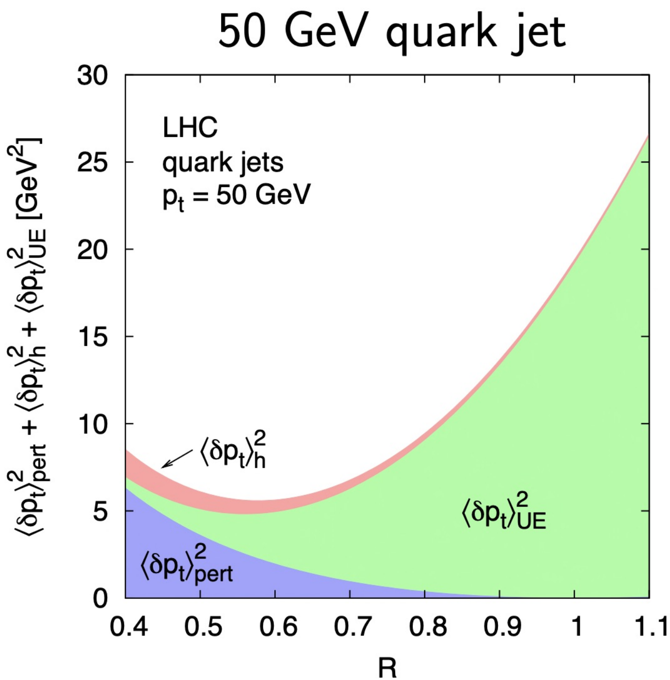
$\hookrightarrow L_g = C_A \times 0.94 + N_f \times 0.07$ $\quad \Delta E/E \sim -10\%$

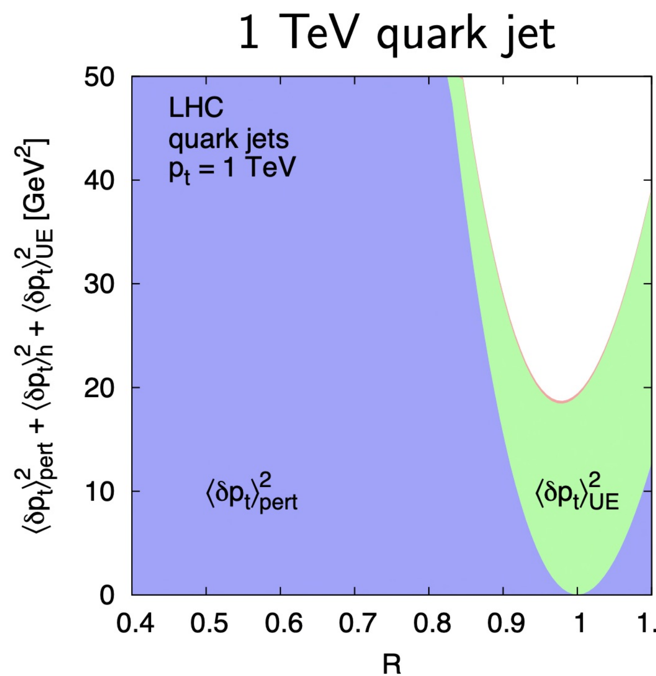
- final-state radiation (FSR) $\sim -\frac{\alpha_s}{\pi} C_i\, p_T \ln(1/R)$
- initial-state radiation (ISR) $\sim \frac{\alpha_s}{\pi} C_i\, p_T\, \pi R^2$
- multiple parton interactions (MPI) aka underlying event (UE) $\sim \rho^{MPI}\, \pi R^2$ $\quad [\rho^{MPI} \sim \mathcal{O}(1\,\text{GeV})$ @ LHC$]$
- pile up (PU) $\sim \rho^{PU}\, \pi R^2$ $\quad [\rho^{PU} \sim n_{PU} \times 0.5\,\text{GeV}]$ (100 – 1000)
- hadronisation $\sim -\Lambda_{QCD} \frac{1}{R}$

The "best" R cone ?

* get the different $\langle \delta p_T^2 \rangle$ to balance

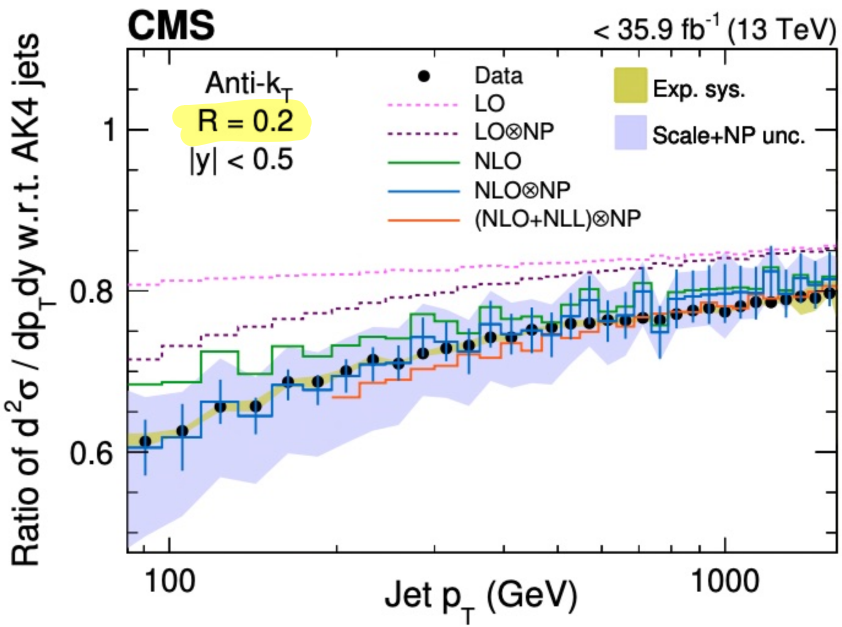
["Towards Jetography" – G. Salam '09]



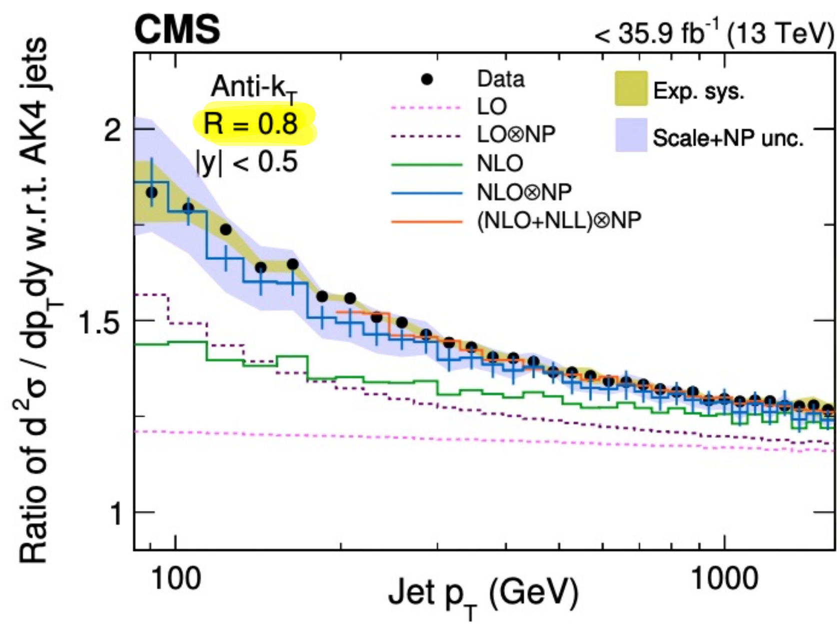


* R small to limit impact of UE

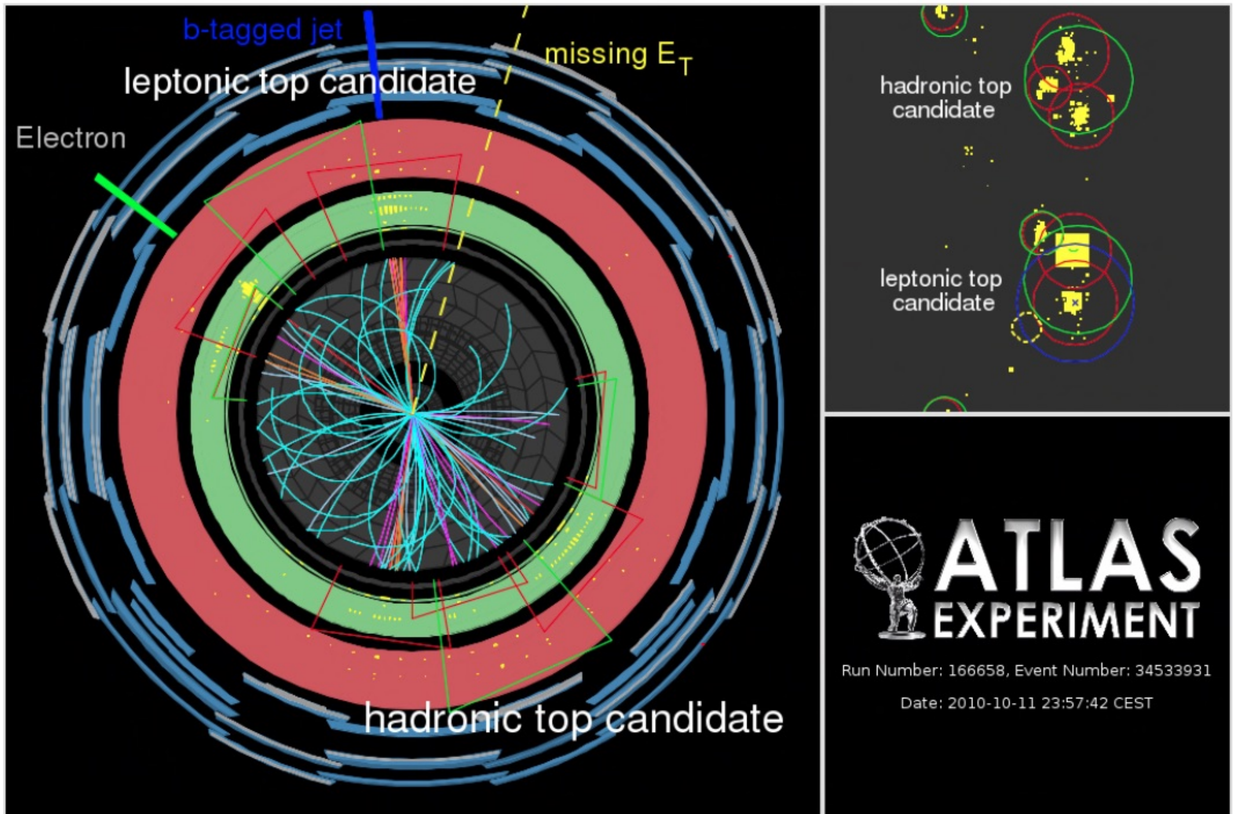
* R large to tame FSR



* "NP" $\hat{=}$ hadr. + MPI



IV. Jet Substructure

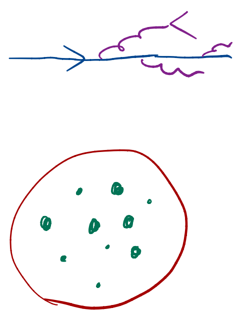
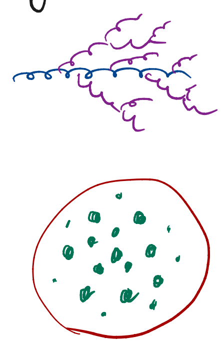
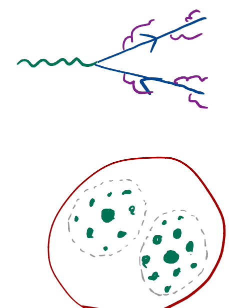


Looking inside Jets

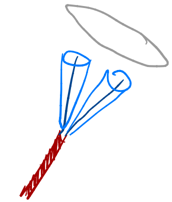
* At the end of jet finding ⇝ collection of constituents ⟷ p^{μ}_{jet}

  ↪ more information / physics than just the momentum

* what is the arrangement of the constituents inside the jets?

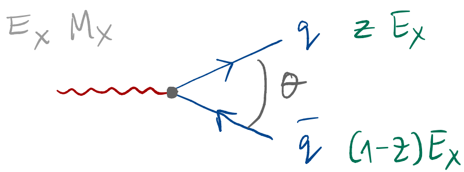
quark?	gluon?	boosted object?
		
$\frac{2\alpha_s}{\pi} C_F \frac{dE}{E} \frac{d\theta}{d\theta}$	$\frac{2\alpha_s}{\pi} C_A \frac{dE}{E} \frac{d\theta}{\theta}$	[H/W/Z ⟷ 2 prongs top quark ⟷ 3 prongs]

Boosted objects



* In extreme kinematic configurations,
  massive hadronically decaying object ⇝ fat jets

* what cone sizes are we talking about?



$$m_J^2 = M_X^2 = 2 E_X^2 z(1-z) \overbrace{(1-\cos\theta)}^{\frac{1}{2}\theta^2}$$

$$\Rightarrow \theta = \frac{M_X}{E_X} \frac{1}{\sqrt{z(1-z)}} \overset{z\sim 1/2}{\sim} \frac{2M_X}{E_X}$$

put in some numbers :

$$\left.\begin{array}{l} M_X = M_W \simeq 80 \text{ GeV} \\ E_X \sim 1 \text{ TeV} \end{array}\right\} \theta \sim 0.15 \rightsquigarrow \text{likely end up in 1 jet}$$

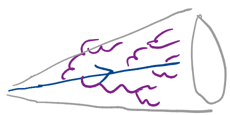
↪ how to distinguish this from a QCD jet?

The Jet mass

* naive expectation (common misconception)

  jet from "X" has mass M_X, whereas q/g jets are massless

* The jet mass of QCD partons



$$m^2 = \Big[\sum_{i \in jet} p_i\Big]^2$$

consider the cumulant:

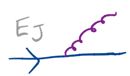
$\Sigma(m_J^2)$ = probability for the jet to have mass^2 < m_J^2

$$= \frac{1}{\sigma}\int dm'^2 \, \frac{d\sigma}{dm'^2}$$

@ LO:

$m^2 \equiv \phi \Rightarrow \Sigma(m_J^2) = 1$

@ NLO:



soft & collinear limit $\Rightarrow \; dw^{s\&c} = \frac{2\alpha_s}{\pi} C_F \frac{dE}{E} \frac{d\theta}{\theta} = \frac{\alpha_s}{\pi} C_F \frac{dz}{z} \frac{d\theta^2}{\theta^2}$

$$m^2 = 2 p_i p_j \overset{\text{coll.}}{\simeq} E_J^2 z(1-z)\theta^2 \overset{\text{soft}}{\simeq} E_J^2 z \theta^2$$

<u>@ NLO</u>

$$\alpha_s \Sigma^{(1)}(m^2) = \frac{\alpha_s}{\pi} C_F \int \frac{d\theta^2}{\theta^2} \int \frac{dz}{z}$$

$$\times \Big\{ \Theta(E_J^2 z \theta^2 < m^2)\, \Theta(\theta < R) \quad \leftarrow \text{real } \in \text{jet}$$

$$+ \Theta(\emptyset < m^2)\, \Theta(\theta > R) \quad \leftarrow \text{real } \notin \text{jet}$$

$$- \Theta(\emptyset < m^2) \Big\} \quad \leftarrow \text{virtual}$$

(last two terms together: $-\Theta(\theta < R)$)

$$= -\frac{\alpha_s}{\pi} C_F \int \frac{d\theta^2}{\theta^2} \int \frac{dz}{z}\, \Theta(E_J^2 z \theta^2 > m^2)$$

$*\ \theta^2 > \frac{m^2}{E_J^2 z}$; $R^2 > \theta^2$

$*\ z > \frac{m^2}{E_J^2 R^2}$

$$= -\frac{\alpha_s}{\pi} C_F \int_{\frac{m^2}{E_J^2 R^2}}^{1} \frac{dz}{z} \boxed{\int_{\frac{m^2}{E_J^2 z}}^{R^2} \frac{d\theta^2}{\theta^2}} = -\frac{\alpha_s}{\pi} C_F \frac{1}{2} \ln^2\left(\frac{E_J^2 R^2}{m^2}\right)$$

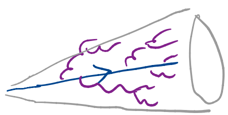
$$\boxed{\;\cdot\;} \to \ln\left(\frac{R^2 E_J^2 z}{m^2}\right)$$

The Jet mass

* naive expectation (common misconception)

  jet from "X" has mass M_X, whereas q/g jets are massless

* The jet mass of QCD partons



$$m^2 = \Big[\sum_{i \in jet} p_i\Big]^2$$

consider the cumulant:

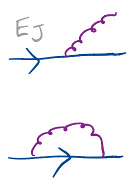
$\Sigma(m_J^2)$ = probability for the jet to have mass^2 < m_J^2

$$= \frac{1}{\sigma}\int dm'^2 \, \frac{d\sigma}{dm'^2}$$

@ LO:

$$m^2 \equiv \emptyset \Rightarrow \Sigma(m_J^2) = 1$$

@ NLO:



$$\Rightarrow \Sigma(m_J^2) = 1 - \frac{\alpha_s}{2\pi} C_F \ln^2\left(\frac{E_J^2 R^2}{m_J^2}\right)$$

not good ($m_J \to \emptyset$)!
higher orders won't help either
$\sim \alpha_s^n \ln^{2n}\left(\frac{E_J^2 R^2}{m_J^2}\right)$

The resummed Jet mass

* need to account for these logs to all orders!

$$\Sigma(m_J^2) = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left[\int_\delta \frac{d\theta_i^2}{\theta_i^2} \int_\epsilon \frac{dz_i}{z_i} \frac{\alpha_s C_F}{\pi} \Theta_{i\in jet} \right] \cdot \Theta\left(\left[\sum_{i=1}^{n} p_i\right]^2 < m_J^2 \right)$$

real emissions inside the jet

$$\sum_{m=0}^{\infty} \frac{1}{m!} \prod_{j=1}^{m} \left[\int_\delta \frac{d\tilde\theta_j^2}{\tilde\theta_j^2} \int_\epsilon \frac{d\tilde z_j}{\tilde z_j} \frac{\alpha_s C_F}{\pi} \left(\Theta_{j\notin jet} - 1 \right) \right]$$

$\leftrightarrow$ do not change m^2

real out of cone

virtual

* $\left[\sum_{i=1}^{n} p_i\right]^2 \simeq E_J^2 \sum_{i=1}^{n} z_i \theta_i^2$

* we're interested in the leading logs (LL) $\Rightarrow$ widely separate scales

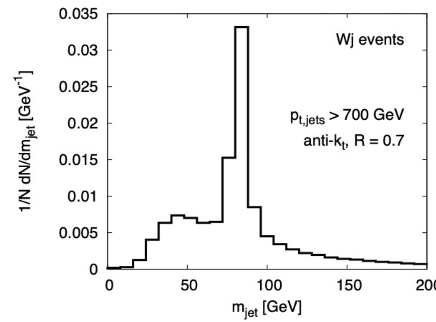
$\Rightarrow$ among all $z_i\theta_i^2$ one is dominant!

$$\Theta\left(E_J^2 \sum_i z_i\theta_i^2 < m_J^2\right) \simeq \Theta\left(E_J^2 \max\{z_i\theta_i^2\} < m_J^2\right) = \prod_{i=1}^{n} \Theta\left(E_J^2 z_i \theta_i^2 < m_J^2\right)$$

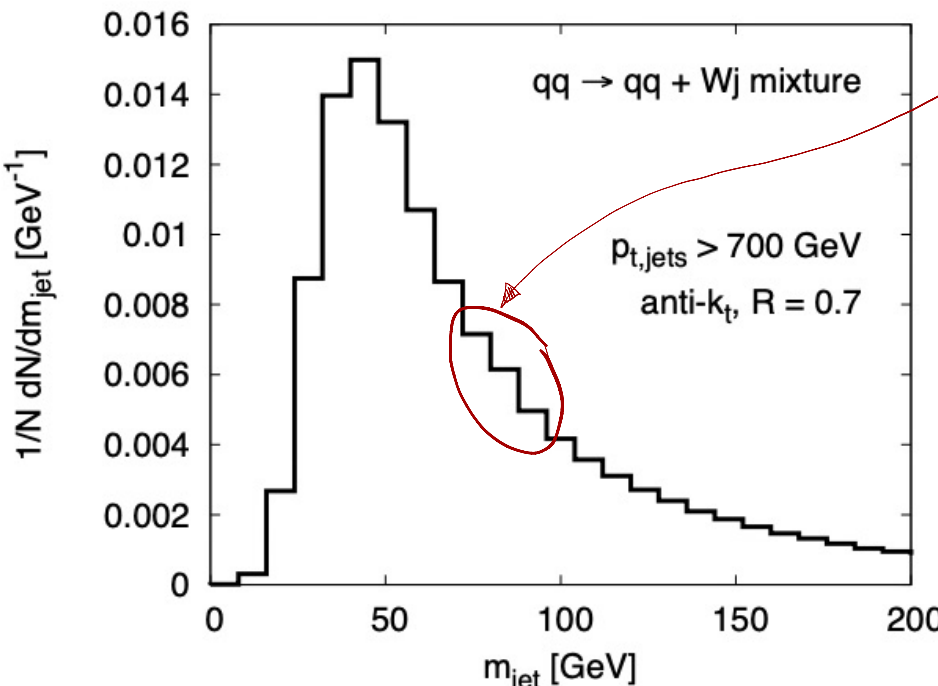
$$= \exp\left[-\int \frac{d\theta^2}{\theta^2} \int \frac{dz}{z} \frac{\alpha_s C_F}{\pi} \Theta(\theta < R)\, \Theta\left(E_J^2 z \theta^2 < m_J^2\right) \right] = \exp\left[-\frac{\alpha_s C_F}{2\pi} \ln^2\left(\frac{E_J^2 R^2}{m_J^2}\right)\right]$$

The Jet mass in real life

$$\frac{d\sigma}{dm_J^2} = \frac{d\Sigma}{dm_J^2} = \frac{1}{m_J^2}\,\frac{\alpha_s C_F}{\pi}\ln\left(\frac{E_J^2R^2}{m_J^2}\right)\underbrace{\exp\left[-\frac{\alpha_s C_F}{2\pi}\ln^2\left(\frac{E_J^2R^2}{m_J^2}\right)\right]}_{\text{“Sudakov”}}$$



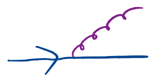
→ clear sign of W

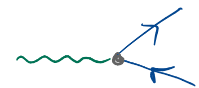


→ but QCD jets massive too

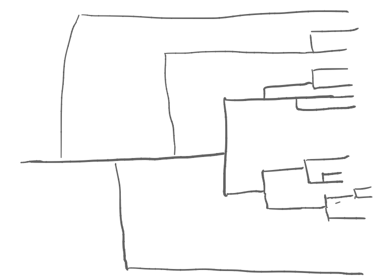
NEED TO REJECT QCD BACKGROUND & ENHANCE THE SIGNAL!

mMDT'13 / Soft Drop'14 ($\beta = 0$)   (BDRS '08)

 $\sim \frac{1}{z}$ [soft]

 ~ 1 [balanced momentum sharing]

1. Take the constituents of the jet and recluster using C/A
   ↪ angular-ordered tree

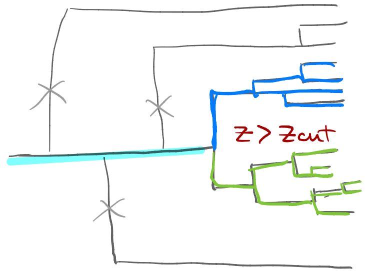


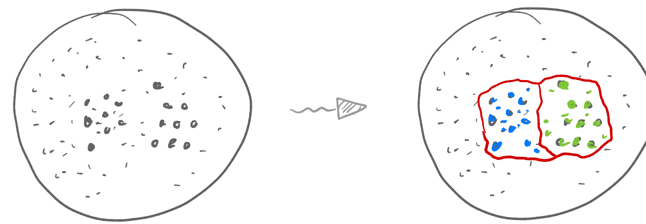
2. Decluster jet, disregard the softer branch until

$$z = \frac{\min(E_i, E_j)}{E_i + E_j} > z_{cut}$$

mMDT '13 / Soft Drop '14 ($\beta = 0$) (BDRS '08)

k_T would keep the soft junk





* Removes soft radiation from periphery of jet
  [because Cambridge-Aachen for declustering]
* Dynamically shrinks jet radius to match hard core

} Jet cleaning "grooming"

* Information on the 2-prong kinematics

} Jet discrimination

* $$\Sigma_1(m_J^2) = \exp\left[-\frac{\alpha_s}{\pi} C_F \ln\left(1/z_{cut}\right) \ln\left(\frac{E_J^2 R^2}{m_J^2}\right)\right] \longleftrightarrow$$ [much smoother (& smaller) background

soft singularity regulated ⇒ single log!