Energy Reflection and Transmission at 2D Holographic Interfaces

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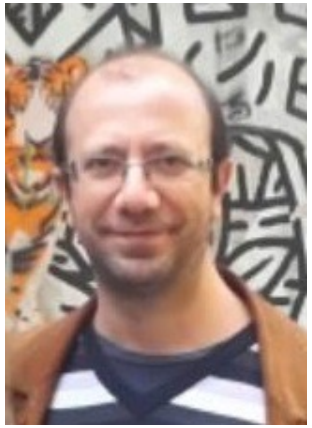


Quantum Field Theory at the Boundary - MITP

28 September 2021

Overview

Part I - 2d Conformal Interfaces

Part II - The Thin Brane Model

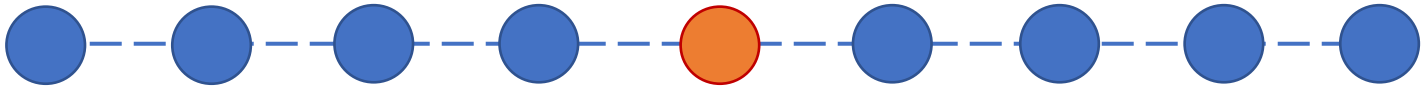
Part III - Energy Reflection and Transmission in Holography

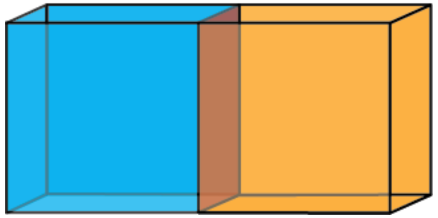
Summary and Outlook

Part 1 - 2d Conformal Interfaces

Conformal Interfaces in 2d

- **Interfaces** – codimension one extended objects which split the system into two.

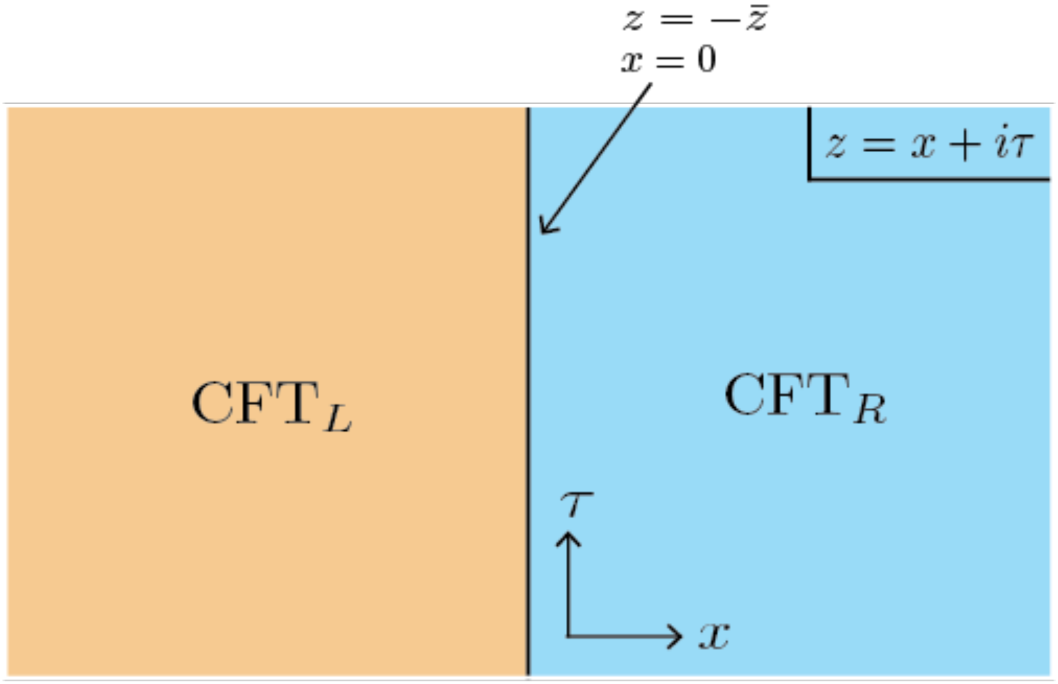




- **Conformal Interfaces** – separate two critical systems and preserve a large subgroup of the conformal symmetry.

- In 2d these are **impurities** which preserve one copy of the Virasoro algebra.

Conformal Interfaces in 2d



- Preserves Virasoro generators which do not displace the interface:

$$L_n + (-1)^n \bar{L}_n$$

- Energy conservation implies a gluing condition:

$$T_L - \bar{T}_L|_{x=0^-} = T_R - \bar{T}_R|_{x=0^+}$$

- Displacement operator $D(x) = T_L + \bar{T}_L - T_R - \bar{T}_R$

Conformal Interfaces in 2d

- Two-point functions of the stress tensor are completely fixed by the conformal symmetry

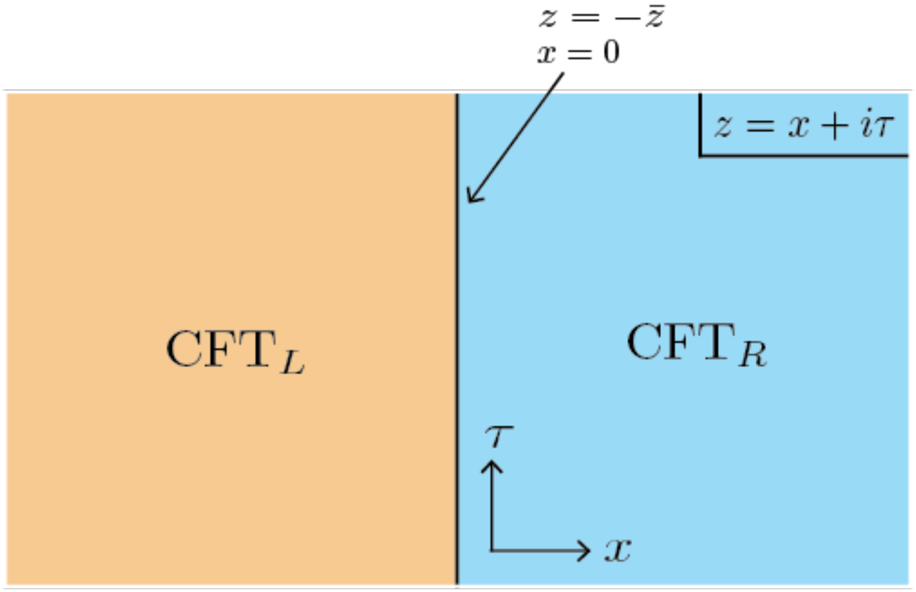
$$\langle T_L(z)T_L(w)\rangle_I = \frac{c_L/2}{(z-w)^4} \qquad \langle \bar{T}_L(\bar{z})\bar{T}_L(\bar{w})\rangle_I = \frac{c_L/2}{(\bar{z}-\bar{w})^4}$$

$$\langle T_R(z)T_R(w)\rangle_I = \frac{c_R/2}{(z-w)^4} \qquad \langle \bar{T}_R(\bar{z})\bar{T}_R(\bar{w})\rangle_I = \frac{c_R/2}{(\bar{z}-\bar{w})^4}$$

- New coefficient in left-right correlations

$$\langle T_L(z)T_R(w)\rangle_I = \frac{c_{LR}/2}{(z-w)^4} \qquad \langle \bar{T}_L(\bar{z})\bar{T}_R(\bar{w})\rangle_I = \frac{c_{LR}/2}{(\bar{z}-\bar{w})^4}$$

$$\langle T_L(z)\bar{T}_L(\bar{w})\rangle_I = \frac{(c_L-c_{LR})/2}{(z+\bar{w})^4} \qquad \langle T_R(z)\bar{T}_R(\bar{w})\rangle_I = \frac{(c_R-c_{LR})/2}{(z+\bar{w})^4}$$



Conformal Interfaces in 2d

- Stress-tensor three-point functions are fixed by the same coefficients ($z \leftrightarrow -\bar{z}$, $L \leftrightarrow R$)

$$\langle T_L(z_1)T_L(z_2)T_L(z_3)\rangle_I = \frac{c_L}{(z_1-z_2)^2(z_2-z_3)^2(z_3-z_1)^2}$$

$$\langle T_L(z_1)T_L(z_2)T_R(z_3)\rangle_I = \frac{c_{LR}}{(z_1-z_2)^2(z_2-z_3)^2(z_3-z_1)^2}$$

$$\langle T_L(z_1)T_L(z_2)\bar{T}_L(\bar{z}_3)\rangle_I = \frac{c_L-c_{LR}}{(z_1-z_2)^2(z_2+\bar{z}_3)^2(\bar{z}_3+z_1)^2}$$

- Four-point functions depend on the details of the theory.

Energy Reflection and Transmission

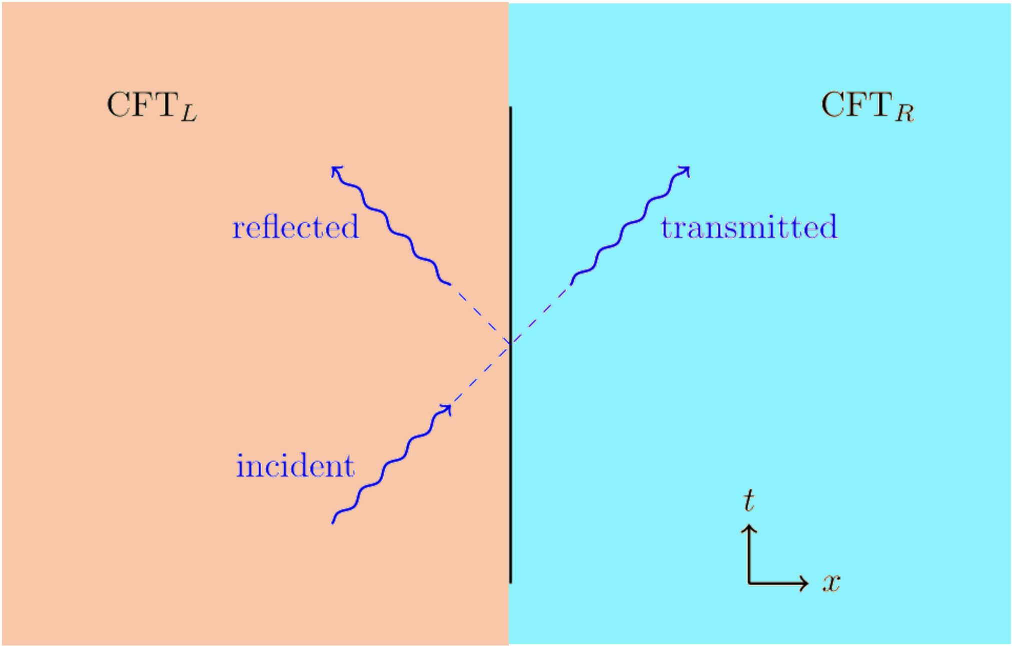
- Scattering experiment

$$\mathcal{T} = \frac{transmitted\ energy}{incident\ energy} \qquad \mathcal{R} = \frac{reflected\ energy}{incident\ energy}$$

- **Different transmission from left and right**

$$\mathcal{R}_{L,R}, \qquad \mathcal{T}_{L,R}$$

- **Universality** - scattered and reflected energy is completely independent of the details of the incoming excitation.



Quella, Runkel, Watts (2007)
Meineri, Penedones, Rousset (2019)

Energy Reflection and Transmission

$$\mathcal{T}_L = \frac{c_{LR}}{c_L} \qquad \mathcal{T}_R = \frac{\bar{c}_{LR}}{c_R} \qquad \mathcal{R}_{L,R} = 1 - \mathcal{T}_{L,R}$$

ANEC implies $\quad 0 \leq \mathcal{T}, \mathcal{R} \leq 1$

$$0 \leq c_{LR} \leq \min(c_L, c_R)$$

$$0 \leq \mathcal{T}_L \leq min\left(1, \frac{c_R}{c_L}\right)$$

$$0 \leq \mathcal{T}_R \leq min\left(1, \frac{c_L}{c_R}\right)$$

Quella, Runkel, Watts (2007)
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$$\mathcal{T}_L = \frac{c_{LR}}{c_L} \qquad \mathcal{T}_R = \frac{\bar{c}_{LR}}{c_R} \qquad \mathcal{R}_{L,R} = 1 - \mathcal{T}_{L,R}$$

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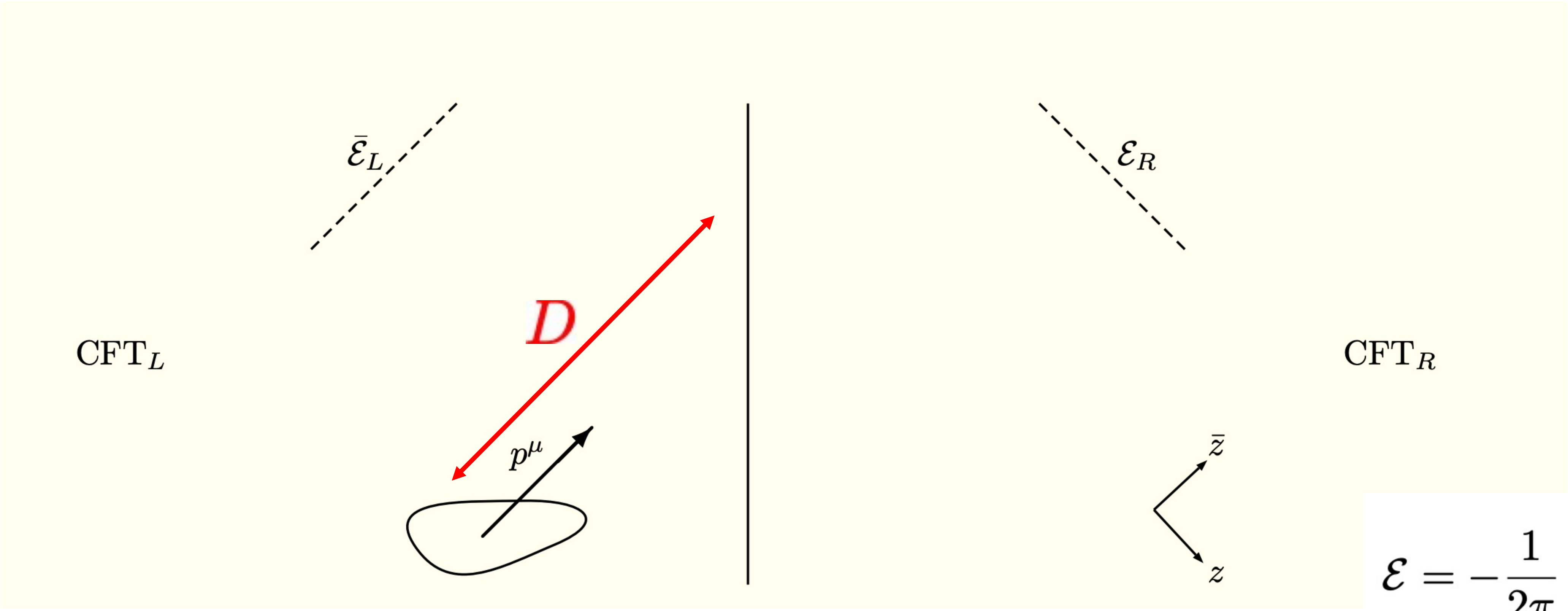
Meineri, Penedones, Rousset (2019)

How is this proven?



$$\mathcal{E} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} dz\, T(z)$$

$$|O_L, D\rangle_{\rm I} = \int d^2x\, f(z) f(\bar{z}+D) O_L(z,\bar{z})\, |0\rangle_{\rm I}$$

$$\bar{\mathcal{E}} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\bar{z}\, \overline{T}(\bar{z})$$

Meineri, Penedones, Rousset (2019)

How is this proven?

Proven using CFT techniques on

$$\mathcal{T}_L = \lim_{D\to\infty} \frac{\langle O_L, D|\mathcal{E}_R|O_L, D\rangle_{\mathrm{I}}}{\langle O_L, D|\mathcal{E}_L|O_L, D\rangle}$$

$$\mathcal{R}_L = \lim_{D\to\infty} \frac{\langle O_L, D|\bar{\mathcal{E}}_L|O_L, D\rangle_{\mathrm{I}} - \langle O_L, D|\bar{\mathcal{E}}_L|O_L, D\rangle}{\langle O_L, D|\mathcal{E}_L|O_L, D\rangle}$$

Limitations

- in the presence of an extended symmetry - different charges scatter differently.
- Multiple holomorphic quasi-primaries of spin two - reflection becomes state dependent.

Part II - The Thin Brane Model

Holographic Interfaces

A bottom up approach

Two different cosmological constants encode different central charges on the two sides via $c_{L,R} = \frac{3\ell_{L,R}}{2G_N}$

- Thin brane in AdS_3

$$S = \frac{1}{16\pi G_N}\int d^3x_L\sqrt{-g}\left(R+\frac{2}{\ell_L^2}\right) + \frac{1}{16\pi G_N}\int d^3x_R\sqrt{-g}\left(R+\frac{2}{\ell_R^2}\right)$$

Brane tension

$$-\sigma\int d^2x\sqrt{-\gamma}$$

Induced metric

- Solve Einstein equations in the left/right

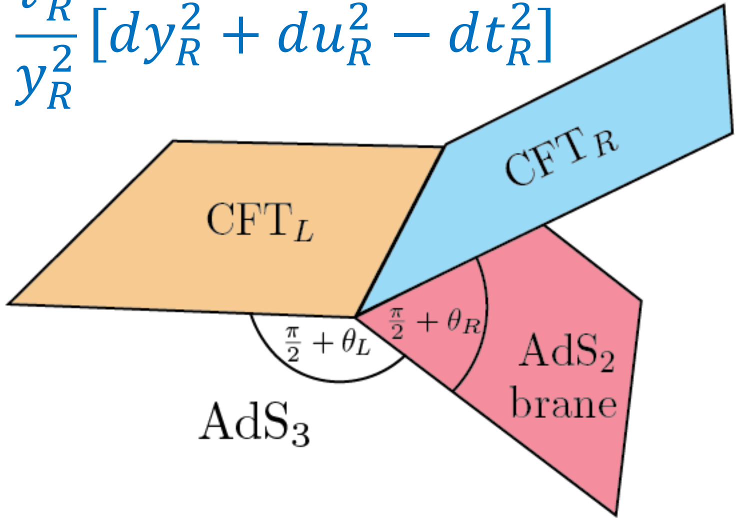
$$ds_L^2 = \frac{\ell_L^2}{y_L^2}[dy_L^2 + du_L^2 - dt_L^2]$$

$$ds_R^2 = \frac{\ell_R^2}{y_R^2}[dy_R^2 + du_R^2 - dt_R^2]$$

- Israel matching conditions determine the location of the brane

$$\gamma_{L,\alpha\beta} = \gamma_{R,\alpha\beta}$$

$$K^R_{\alpha\beta} - K^L_{\alpha\beta} = -8\pi G\sigma\gamma_{\alpha\beta}$$



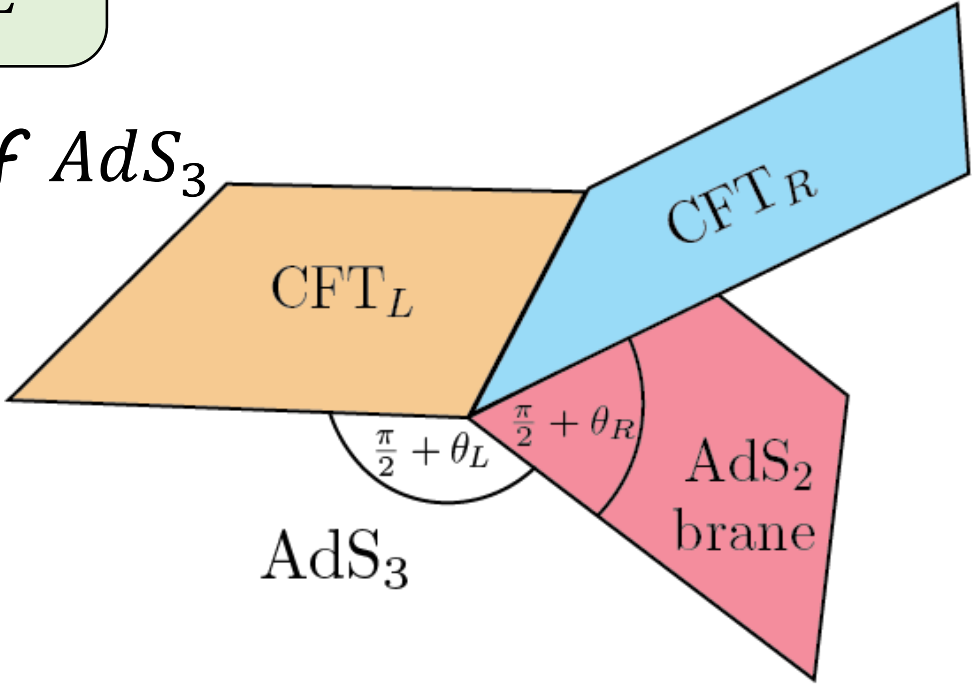
The Thin Brane Model

- Stable solutions with a thin AdS_2 brane exist as long as

$$\left|\frac{1}{\ell_R} - \frac{1}{\ell_L}\right| \leq 8\pi G\sigma \leq \frac{1}{\ell_R} + \frac{1}{\ell_L}$$

- The solution consists of two patches of AdS_3 connected along an AdS_2 brane with

$$\frac{\ell_L}{\cos\theta_L} = \frac{\ell_R}{\cos\theta_R} = \frac{\tan\theta_L + \tan\theta_R}{8\pi G\sigma}$$



- ~~Boundary~~ Interface entropy is fixed by the tension.

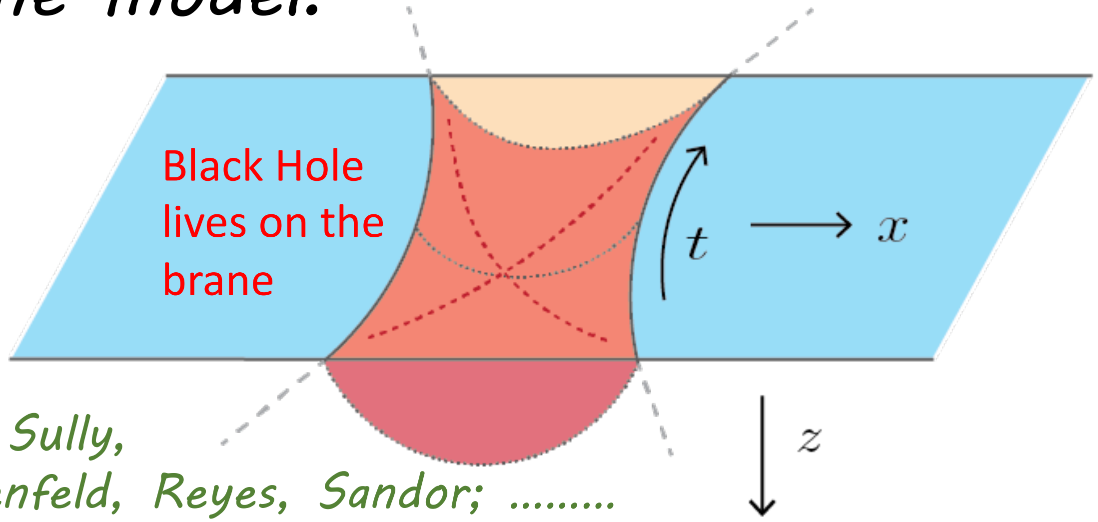
Bachas (2002)
Azeyanagi, Karch, Takayanagi, and Thompson (2007)

Energy Reflection and Transmission in the Thin Brane Model

Why?

- Better understanding of energy reflection and transmission in strongly coupled models with large central charges.
  - Can we improve the reflection and transmission bounds?

- Understand better the thin brane model: this model is being very much used recently in the studies of the Page curve.



Works by: Almheiri, Mahajan, Maldacena, Zhao; Rozali, Sully, Van Raamsdonk, Waddell, Wakeham; Chen, Myers, Neuenfeld, Reyes, Sandor; ………

Part III - Energy Reflection and Transmission in Holography

Final Result

- The transmission coefficients depends monotonically on the tension

$$\mathcal{T}_{L,R} = \frac{2}{\ell_{L,R}}\left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G\sigma\right]^{-1}$$

Higher tension = less transmission

- The transmission is fixed by the tension - just like the boundary entropy. But this is just because our model has a single parameter.
- Obtain bounds on the transport

$$\frac{c_R}{c_R+c_L} \leq \mathcal{T}_L \leq min\left(1, \frac{c_R}{c_L}\right) \qquad \frac{c_L}{c_R+c_L} \leq \mathcal{T}_R \leq min\left(1, \frac{c_L}{c_R}\right)$$

- Upper bound matches ANEC. Lower bound is stronger.

Final Result

$$\frac{c_R}{c_R+c_L} \le \mathcal{T}_L \le min\left(1, \frac{c_R}{c_L}\right) \qquad \frac{c_L}{c_R+c_L} \le \mathcal{T}_R \le min\left(1, \frac{c_L}{c_R}\right)$$

- Can't transmit fully from a higher central charge to a lower central charge.
- Complete transmission from both sides – equal central charges and a tensionless string (topological interface).
- Total reflection (zero transmission) from a given side – only for $c_R/c_L \to 0$ (depleting one of the CFTs of d.o.fs, relative to the other), BCFT limit. This is different from generic CFTs.

Derivation

- Bulk solution corresponding to a scattering experiment?
  want stress tensor with left and right moving waves

$$\langle T^L_{\alpha\beta}\rangle dx^\alpha_L dx^\beta_L = \epsilon\left[1\, e^{i\omega(t_L-u_L)} d(t_L-u_L)^2 + \mathcal{R}_L e^{i\omega(t_L+u_L)} d(t_L+u_L)^2\right] + c.c.$$

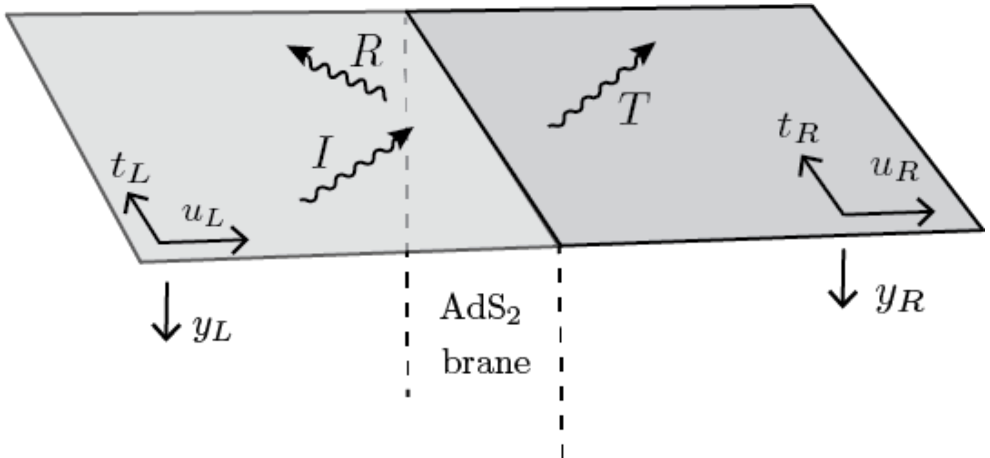
$$\langle T^R_{\alpha\beta}\rangle dx^\alpha_R dx^\beta_R = \epsilon\, \mathcal{T}_L e^{i\omega(t_R-u_R)} d(t_R-u_R)^2 + c.c$$

Characteristic frequency ω

- 2d Bulk solution is completely fixed

$$ds^2 = \frac{\ell^2 dy^2}{y^2} + \left[\frac{\ell^2 g^{(0)}_{\alpha\beta}}{y^2} + g^{(2)}_{\alpha\beta} + \frac{y^2}{4\ell^2} g^{(4)}_{\alpha\beta}\right] dw^\alpha dw^\beta$$

$$g^{(2)}_{\alpha\beta} = 4G\ell\langle T_{\alpha\beta}\rangle \qquad g^{(4)} = g^{(2)}\left(g^{(0)}\right)^{-1} g^{(2)}$$



Brane Ansatz

- Without the metric perturbation the two sides were matched along a brane with angles θ_L and θ_R.
- With the perturbation the brane gets perturbed and we want to find its shape.
- Location of the brane: $x_{L,R} = \tilde{\epsilon} e^{i\omega t} \delta_{L,R}(z) + c.c.$

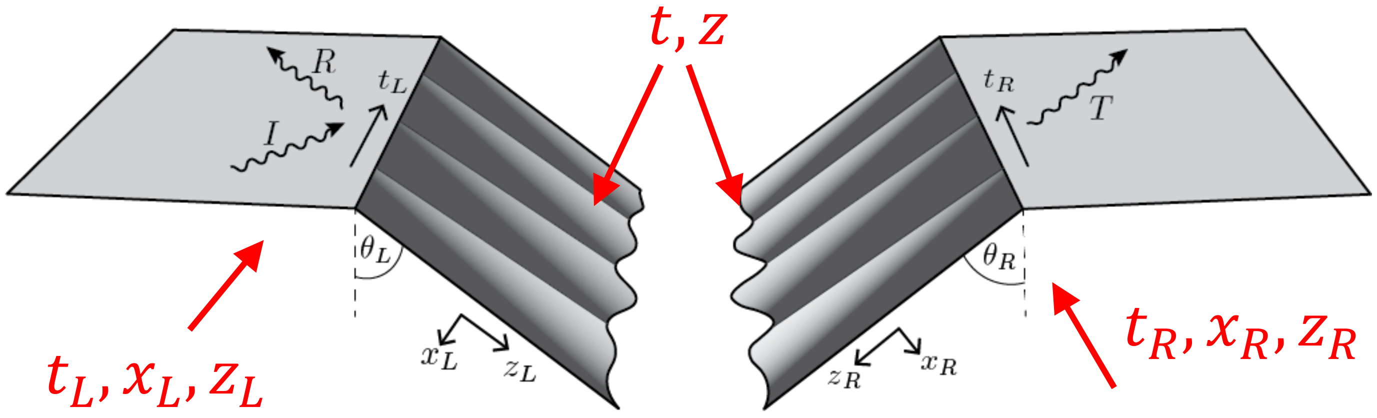
$$\tilde{\epsilon} \equiv \epsilon \frac{4\, G_N \cos\theta_L}{\ell_L} = \epsilon \frac{4\, G_N \cos\theta_R}{\ell_R}$$

- Matching the two sides:

$$z_{L,R} = z + \tilde{\epsilon}\, e^{i\omega t} \zeta_{L,R}(z)$$
$$t_{L,R} = t + \tilde{\epsilon} e^{i\omega t} \lambda_{L,R}(t)$$

Gauge invariance

$\zeta \equiv \zeta_L - \zeta_R$ and $\lambda \equiv \lambda_L - \lambda_R$



Brane Equations

Two of the equations are redundant due to momentum constraint: $D^{\alpha}K_{\alpha\beta} - D_{\beta}K = 0$

Israel matching

$$\gamma_{L,\alpha\beta} = \gamma_{R,\alpha\beta}$$
$$K^{R}_{\alpha\beta} - K^{L}_{\alpha\beta} = -8\pi G\sigma\gamma_{\alpha\beta}$$

$$\Delta + i\omega z\lambda = z^3\left[\frac{\cos\theta_L}{2}(\mathbf{I}+\mathbf{R}) - \frac{\cos\theta_R}{2}\mathbf{T}\right]$$

$$i\omega z\zeta - z\partial_z\lambda = z^3[\sin\theta_R\cos\theta_R\mathbf{T} + \sin\theta_L\cos\theta_L(\mathbf{I}-\mathbf{R})]$$

$$z\partial_z\zeta + \Delta = z^3\left[\frac{\sin^2\theta_R\cos\theta_R}{2}\mathbf{T} - \frac{\sin^2\theta_L\cos\theta_L}{2}(\mathbf{I}+\mathbf{R})\right]$$

$$z\partial_z D = z^3\left[\frac{1}{i\omega z}(\mathbf{I}-\mathbf{R}-\mathbf{T}) - \frac{\sin\theta_L\cos^2\theta_L}{2}(\mathbf{I}+\mathbf{R}) - \frac{\sin\theta_R\cos^2\theta_R}{2}\mathbf{T}\right]$$

$\mathbf{I} \equiv e^{-i\omega\sin\theta_L z}$, $\mathbf{R} = \mathcal{R}_L e^{i\omega\sin\theta_L z}$, $\mathbf{T} \equiv \mathcal{T}_L e^{i\omega\sin\theta_R z}$

Unknowns:

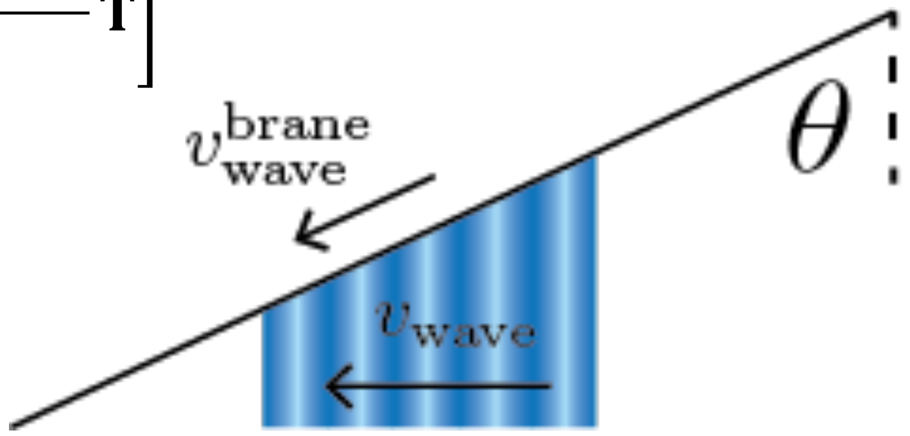
ζ - z matching

λ - t matching

$D \equiv \delta_L - \delta_R$,

$\Delta \equiv \tan\theta_L\delta_L + \tan\theta_R\delta_R - \zeta$

Superluminal waves: $e^{i\omega(t-\sin\theta\, z)}$
like sea waves hitting an oblique seashore



Solution

ζ - z matching
λ - t matching
$D \equiv \delta_L - \delta_R$
$\Delta \equiv \tan\theta_L \delta_L + \tan\theta_R \delta_R - \zeta$

$\mathbf{I} \equiv e^{-i\omega \sin\theta_L z}$
$\mathbf{R} \equiv \mathcal{R}_L e^{i\omega \sin\theta_L z}$
$\mathbf{T} \equiv \mathcal{T}_L e^{i\omega \sin\theta_R z}$

Two integration constants

$$\frac{\Delta(z)}{z} = a_+ e^{i\omega z} + a_- e^{-i\omega z} + \frac{1}{\omega^2 \cos\theta_L}(\mathbf{I}+\mathbf{R}) - \frac{1}{\omega^2\cos\theta_R}\mathbf{T}$$

$$\zeta(z) = \frac{i}{\omega}\left(a_+ e^{i\omega z} - a_- e^{-i\omega z}\right)$$
$$-\frac{\cos\theta_L z}{\omega^2}(\mathbf{I}+\mathbf{R}) - \frac{i}{\omega^3}(\mathbf{I}-\mathbf{R})\left(\tan\theta_L + \frac{\sin\theta_L\cos\theta_L}{2}\omega^2 z^2\right) - \frac{i}{\omega^3}\mathbf{T}\left(\tan\theta_R + i\cos\theta_R \omega z + \frac{\sin\theta_R\cos\theta_R}{2}\omega^2 z^2\right)$$

$$\lambda(z) = \frac{i}{\omega}\left(a_+ e^{i\omega z} + a_- e^{-i\omega z}\right)$$
$$+\frac{i}{\cos\theta_L \omega^3}(\mathbf{I}+\mathbf{R})\left(1 - \frac{\cos^2\theta_L}{2}\omega^2 z^2\right) - \frac{i}{\cos\theta_R \omega^3}\mathbf{T}\left(1 - \frac{\cos^2\theta_R}{2}\omega^2 z^2\right)$$

Source for the displacement operator

$$D(z) = d_0 - \frac{i}{\omega^3}(\mathbf{I}-\mathbf{R})\left(1 + \frac{\cos^2\theta_L}{2}\omega^2 z^2\right) + \frac{\sin\theta_L z}{\omega^2}(\mathbf{I}+\mathbf{R}) + \frac{i}{\omega^3}\mathbf{T}\left(1 - i\sin\theta_R \omega z + \frac{\cos^2\theta_R}{2}\omega^2 z^2\right)$$

Homogenous solution - $\zeta(z=0) = \lambda(z=0) = D(z=0) = 0$ all the integration constants vanish.

Solution

- Israel matching conditions – 4 differential equations for the unknown functions.
- Just 3 integration constants – to be fixed using boundary conditions.
- Imposing boundary conditions

$$\delta_R\,(0) = \delta_L(0) = \zeta(0) = \lambda(0) = 0$$

  fixes the three integration constants and $\Longrightarrow \mathcal{R}_L + \mathcal{T}_L = 1$

$$d_0 = 0; \qquad a_\pm = \frac{1}{2\omega^2}\left[\mathcal{T}_L\left(\frac{1 \pm \sin\theta_R}{\cos\theta_R} \pm \tan\theta_L\right) - \frac{(1+\mathcal{R}_L)}{\cos\theta_L}\right];$$

- All consistent for whatever reflection and transmission!
- What is missing?

Boundary Condition in the IR

- We need to impose a no-outgoing wave condition.

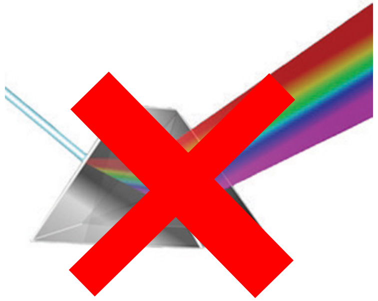
- For example, consider the traceless part of the extrinsic curvature

$$\hat{K}_{\pm\pm} = \frac{a_\pm \omega^2 \tilde{\epsilon}}{8\pi G_N \sigma} e^{i\omega x^\pm} + \mathcal{O}(\epsilon^2)$$

outgoing

$$a_\pm = \frac{1}{2\omega^2}\left[\mathcal{T}_L\left(\frac{1\pm\sin\theta_R}{\cos\theta_R} \pm \tan\theta_L\right) - \frac{(1+\mathcal{R}_L)}{\cos\theta_L}\right] \qquad x^\pm = t \pm z$$

ingoing



$$a_+ = 0 \Longrightarrow$$

$$\mathcal{T}_L = \frac{2}{\ell_L}\left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G\sigma\right]^{-1}$$

Universality! - result is independent of the frequency

Summary and Outlook

- Energy reflection and transmission are universal in holographic interfaces.
- In the thin brane model the transmission is fixed by the tension.
- In general boundary entropy and energy transmission will differ.
- Universality – try to shoot other things at the interface.
- Higher dimensional scattering?
- Under which conditions would there be a relation between energy and information transfer? Especially interesting in the context of the Page curve of black hole evaporation.

Lots to explore!

Thank you for listening!