Defects and Anomalies in Quantum Field Theory

Yifan Wang
New York University

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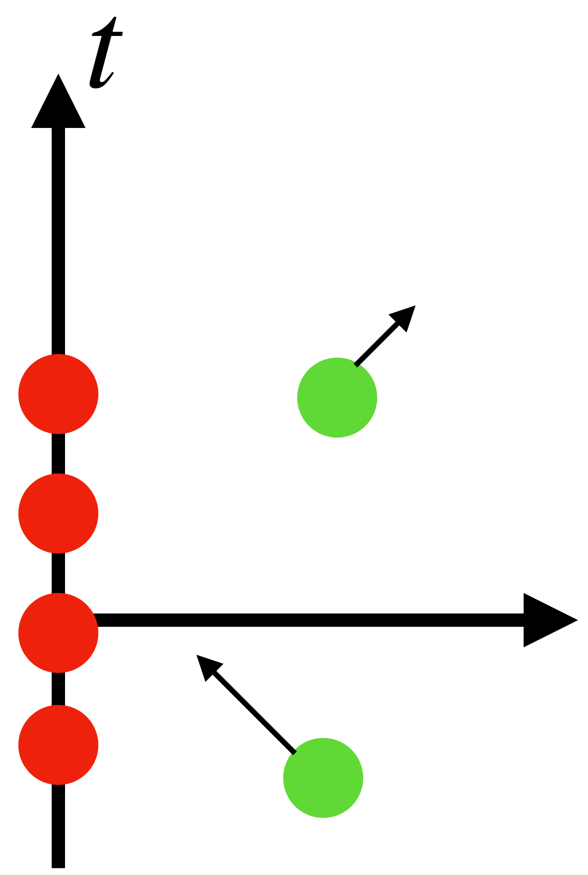
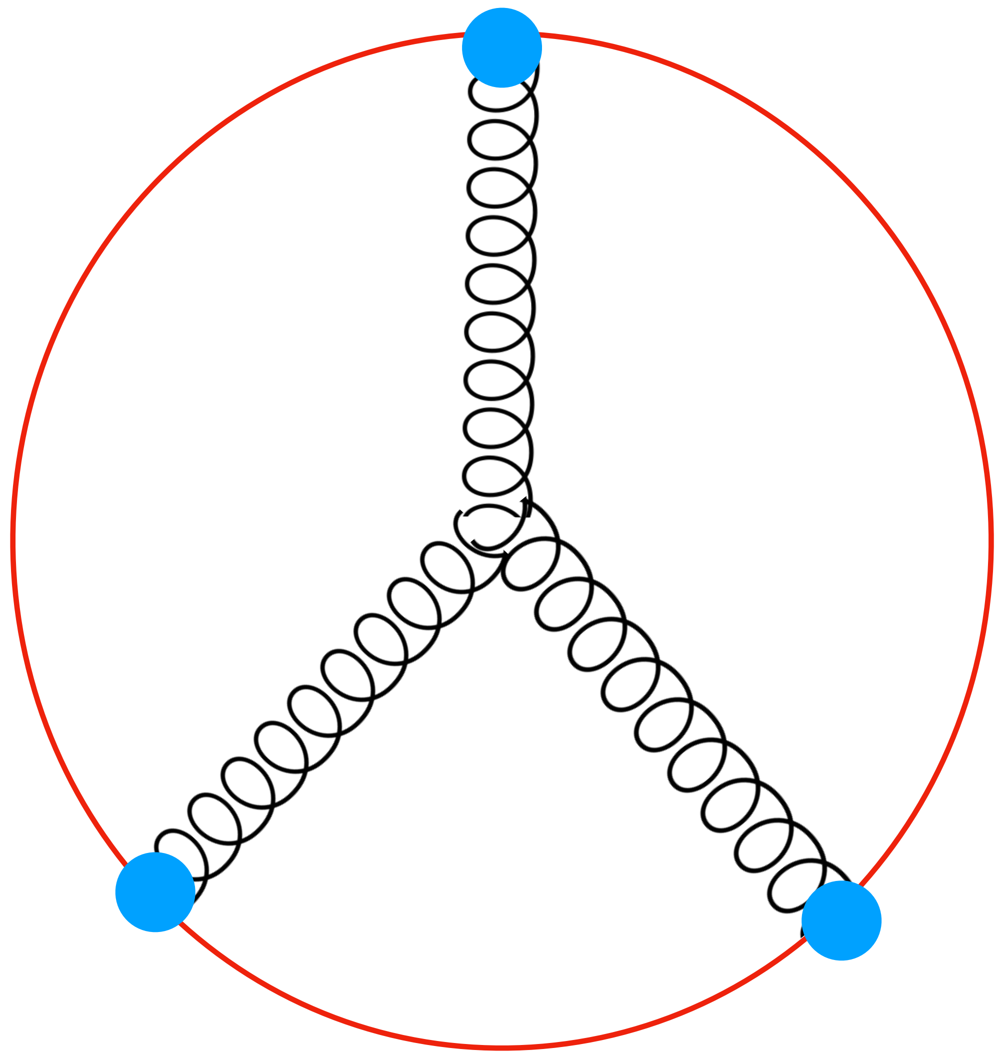
Defects are Ubiquitous

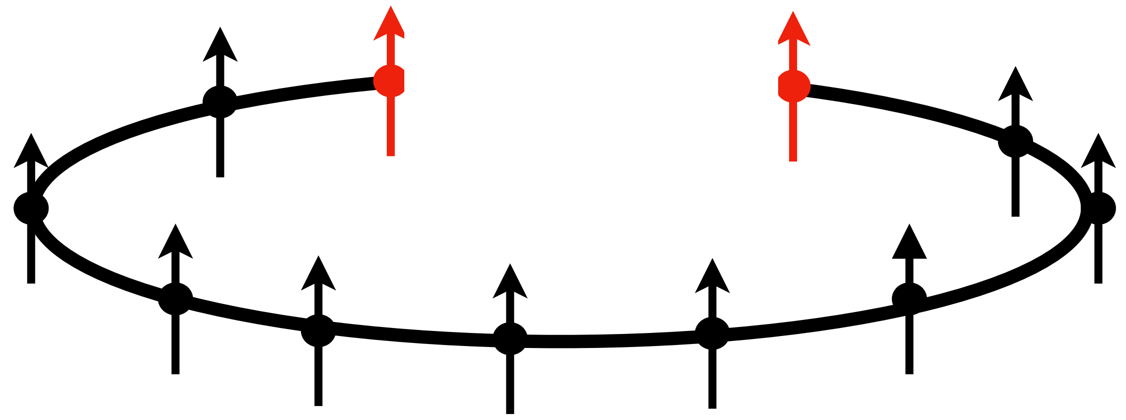
- Kondo effect: magnetic impurities in metal
- Worldlines of heavy charged particles
- Lattice systems (spin chain) with boundaries
- D-Branes in worldsheet string theory

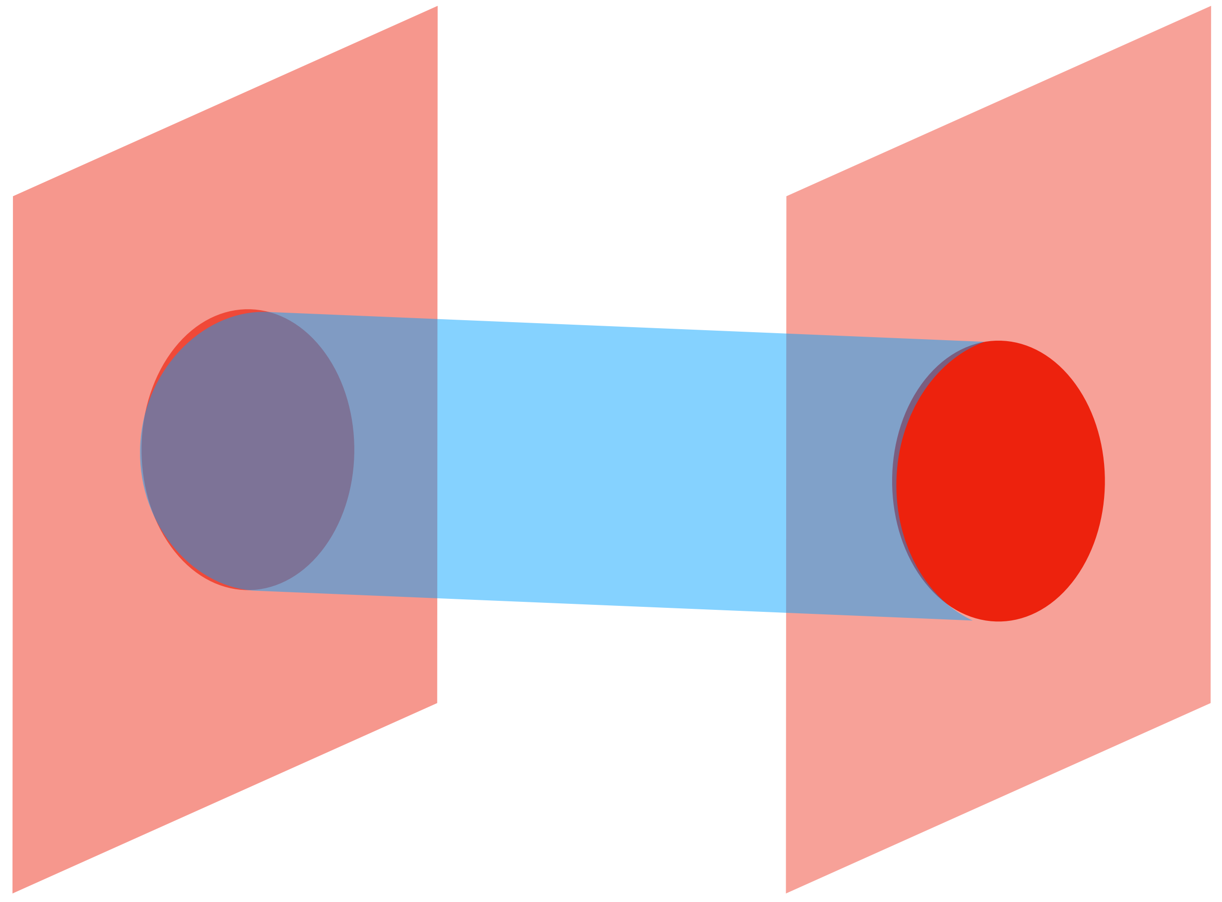
Precise Descriptions in QFT

- Defect lines in SU(2)k WZW [Affleck-Ludwig,…]
- Wilson/’t Hooft loops [Wilson, t’ Hooft, Kapustin,…]
- Conformal boundaries [Cardy, Diehl,…]
- 2d Cardy States [Cardy,…]









General Defects in QFT

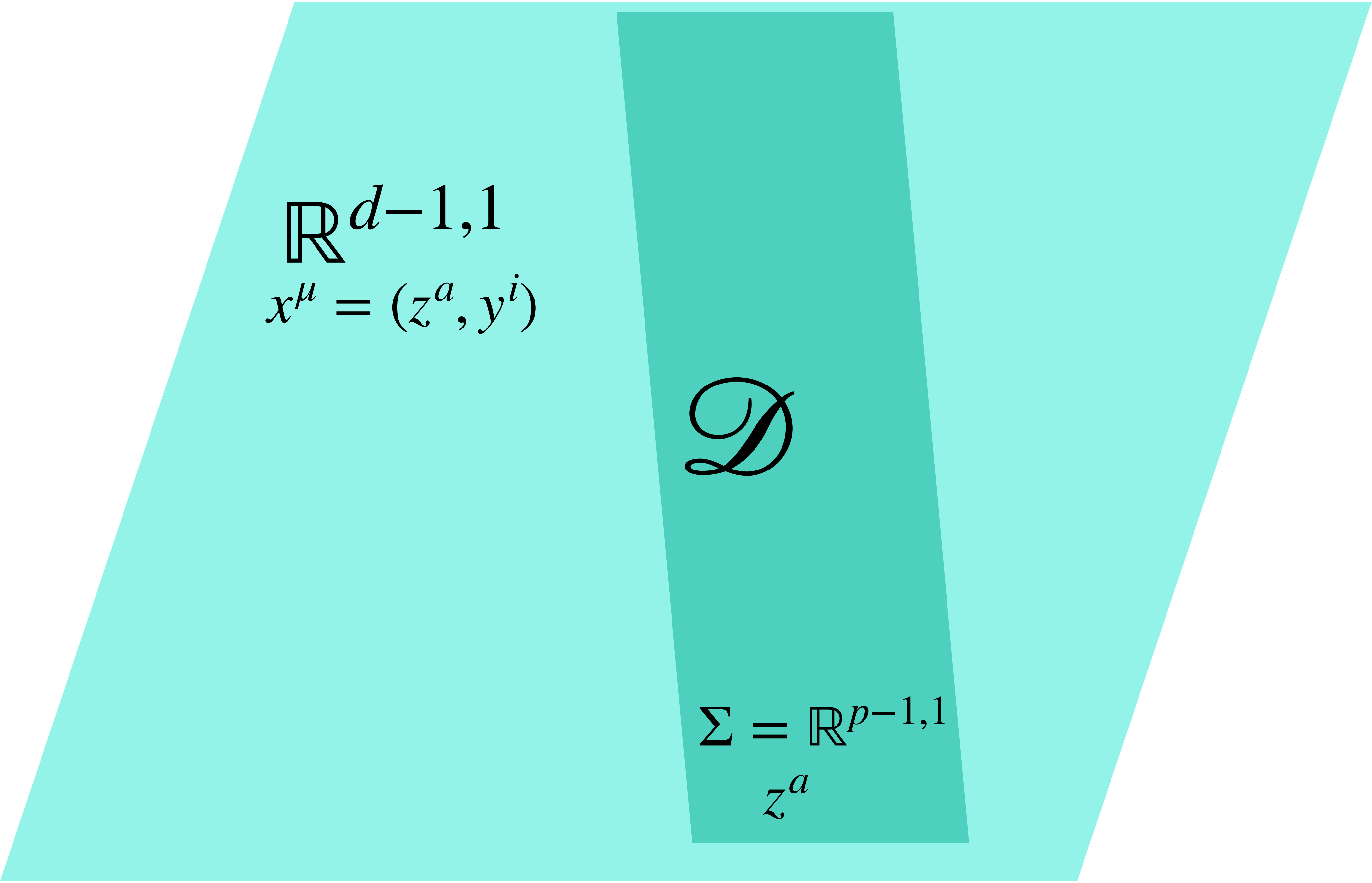
Defects are **extended** operators defined on a spacetime sub-manifold Σ

Order-type defects: e.g. Wilson lines …

$$\mathscr{D} = e^{i\int_\Sigma \mathscr{L}(\phi)}$$

Disorder-type defects: e.g. ’t Hooft lines,
vortex/twist operators …

$$\langle \mathscr{D} \ldots \rangle = \int D\phi \,|_{\phi(\Sigma)=\phi_0} (e^{iS(\phi)} \ldots)$$



Preserve Poincare symmetry on Σ but break transverse translation

More generally: extended operators characterized by the symmetries they **break/preserve**, correlation functions with both local and other defect operators

Extended Quantum Field Theory

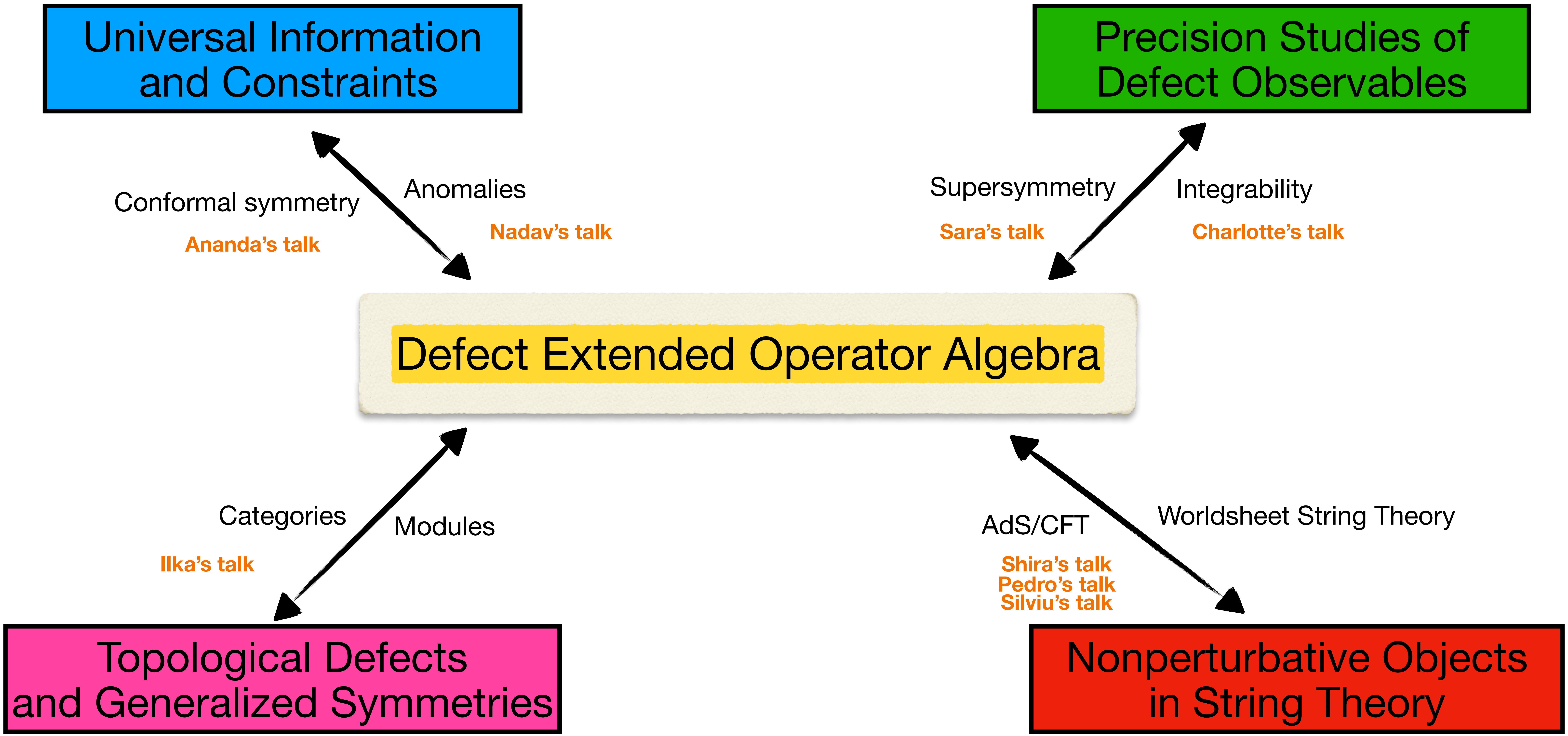
Defects are an indispensable part of QFT —> Extended QFT

- Topological defects as natural generalizations of symmetries [Gukov-Kapustin 2013, Gaiotto-Kapustin-Seiberg-Willett 2014, Bhardwaj-Tachikawa 2017, Chang-Lin-Shao-YW-Yin 2018] (also [Fuchs-Runkel-Schweigert-(Frohlich-Fjelstad) 2000s])

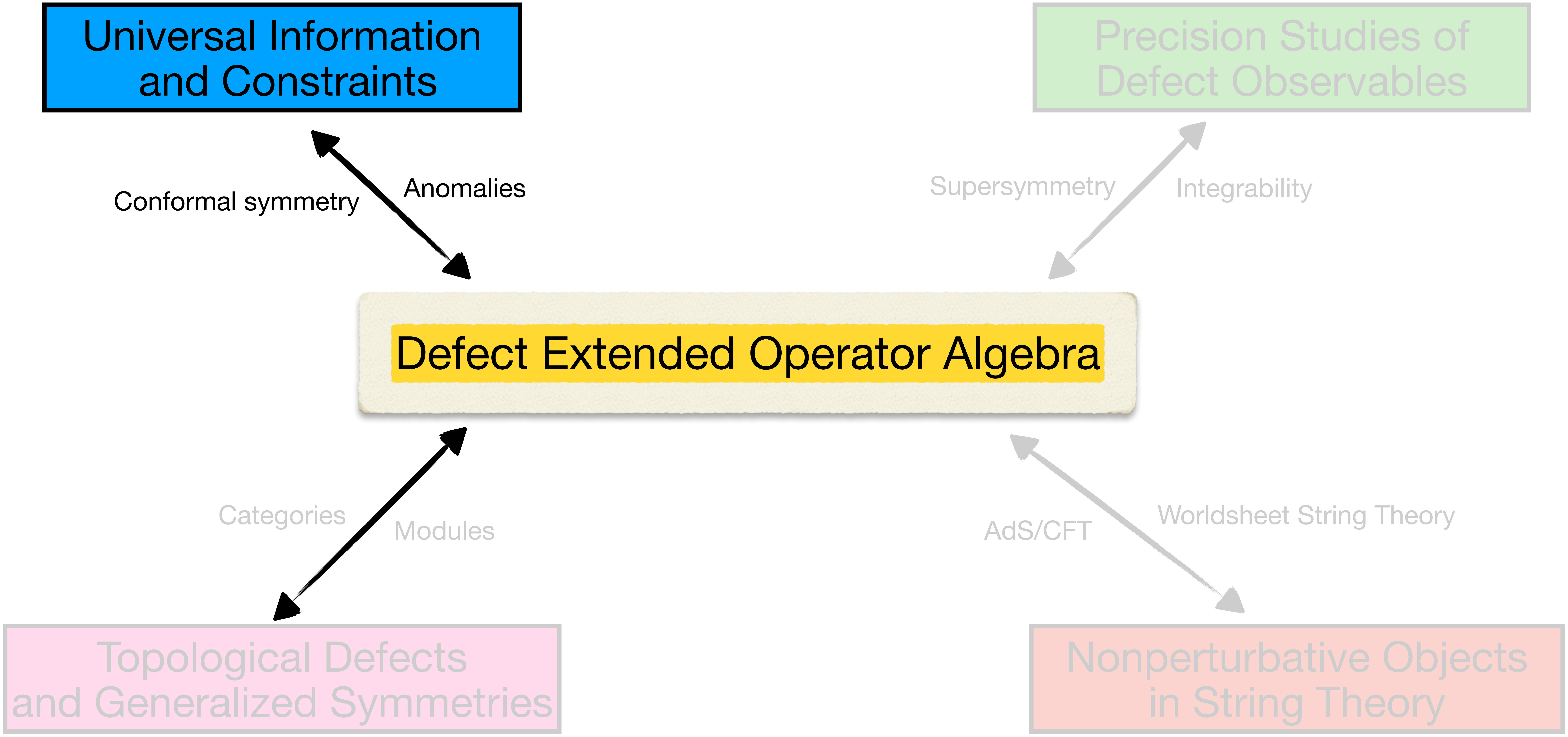
- Charged operators under generalized higher-form symmetries, order parameter for phase transitions (e.g. confinement-deconfinement transitions) [Gaiotto-Kapustin-Seiberg-Willett 2014]

- Sensitive to global structures of theory. There are finite classes of extended QFTs with identical local operators but distinct defects [Aharony-Seiberg-Tachikawa 2013]

- Laboratory for exciting and rich mathematics: e.g. AGT with defects [Alday-Gaiotto-Tachikawa 2009, Alday-Gaiotto-Gukov-Tachikawa-Verlinde 2009, Drukker-Gomis-Okuda-Teschner 2009, Drukker-Gaiotto-Gomis 2010,…]


Universal Information and Constraints
Precision Studies of Defect Observables
Conformal symmetry
Ananda's talk
Anomalies
Nadav's talk
Supersymmetry
Sara's talk
Integrability
Charlotte's talk
Defect Extended Operator Algebra
Categories
Ilka's talk
Modules
AdS/CFT
Shira's talk
Pedro's talk
Silviu's talk
Worldsheet String Theory
Topological Defects and Generalized Symmetries
Nonperturbative Objects in String Theory

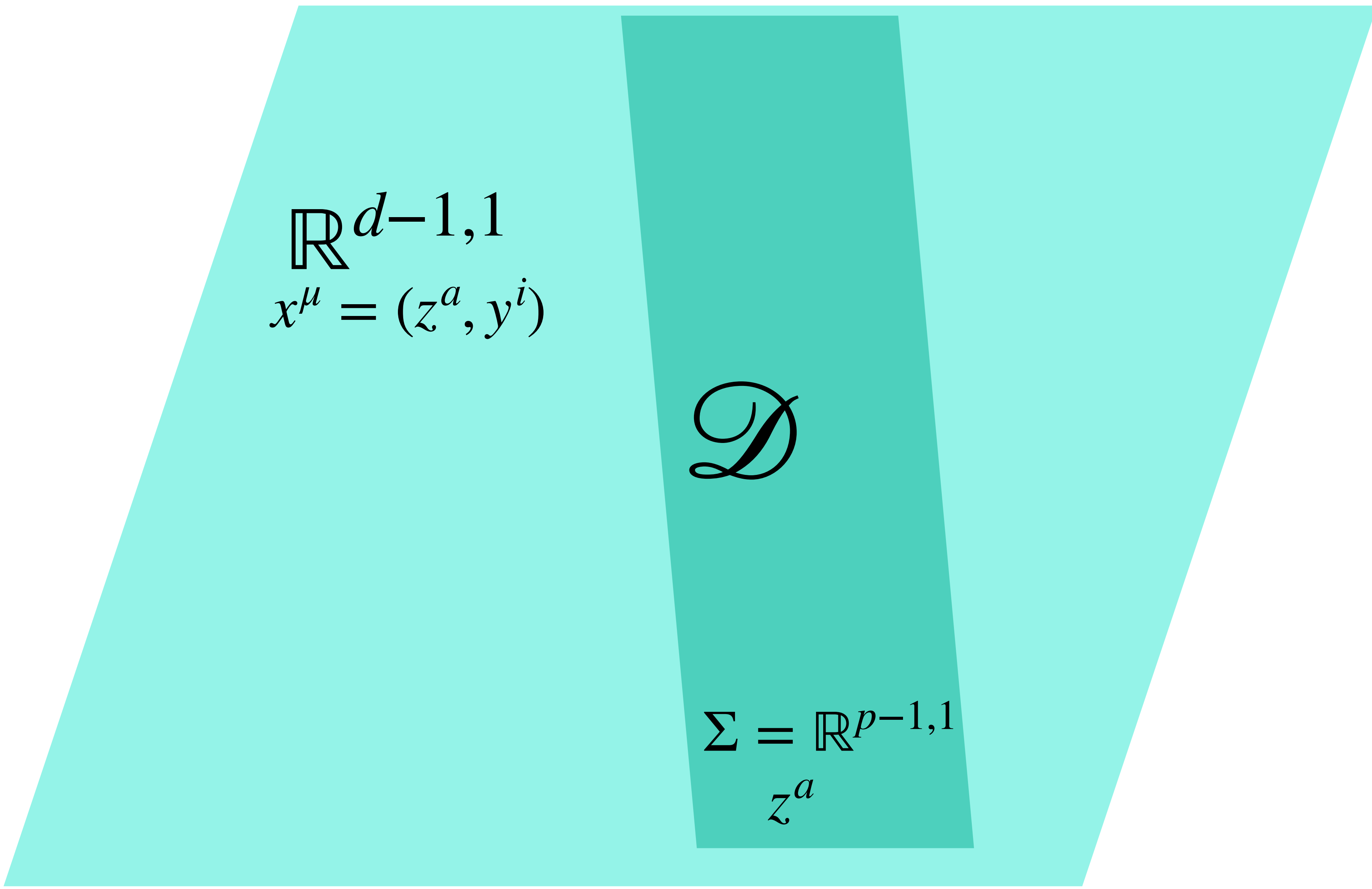
Today's Focus



Defect Symmetries

Two types of symmetries: Intrinsic vs Extrinsic

- **Instrinsic**: locally conserved symmetries on Σ:
  defect Noether current j^a
  $$\langle \partial_a J^a(z^a) \rangle_{\mathscr{D}} = 0$$
- **Extrinsic**: inherited from bulk:
  bulk Noether current J^μ
  $$\langle \partial_\mu J^\mu(z^a, y^i) \rangle_{\mathscr{D}} = 0$$
  $U_\alpha \equiv e^{\alpha \int_{\mathscr{M}_{d-1}} \star J}$ remains topological
  even if $\mathscr{M}_{d-1} \cap \Sigma \neq \varnothing$



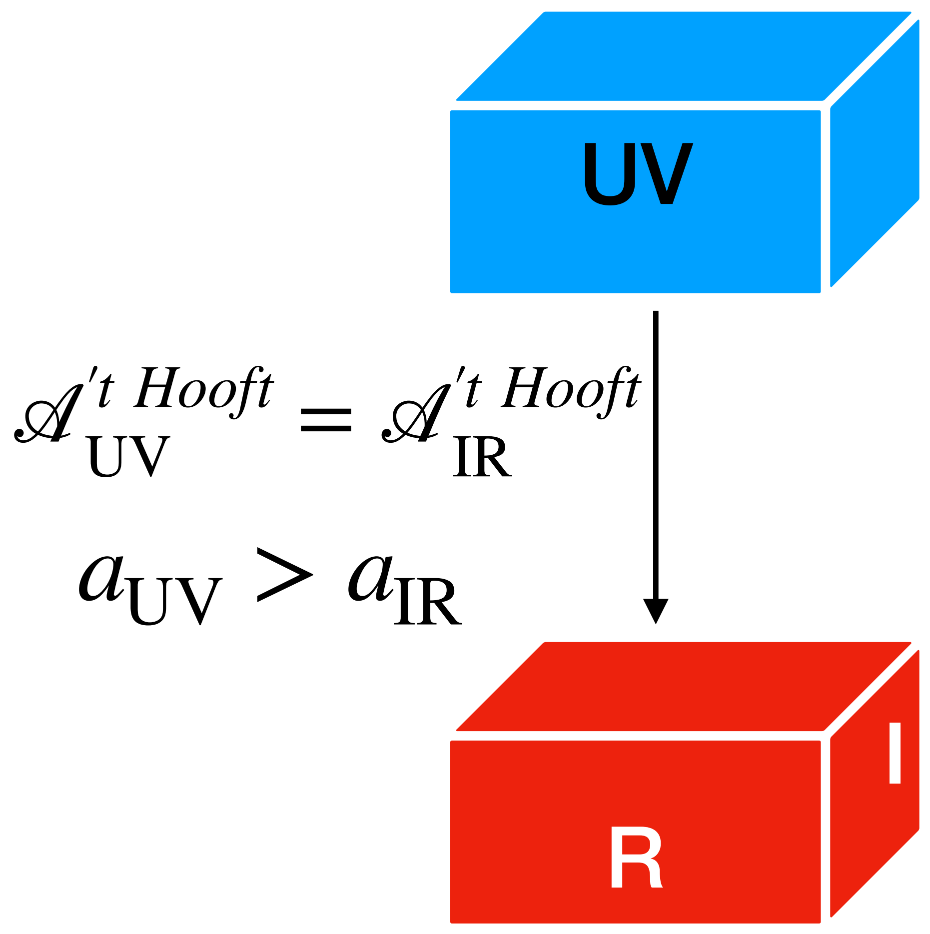
Generality of Anomalies

Features

- Modification of Ward identity by contact terms e.g. 2d $\langle \partial_\mu J^\mu(x) J^\nu(0) \rangle = \frac{k}{4\pi}\epsilon^{\mu\nu}\partial_\mu \delta(x)$
- Anomalous variation of the partition function $\delta_\lambda \log Z[A]$
- Encodes fine structures of symmetries (current algebra or fusion data) $\langle J^\mu(x) J^\nu(0) \rangle_{\text{odd}} = k \frac{\epsilon^{\mu\rho} I^\nu_\rho(x)}{|x|^2}$
- Solutions to Wess-Zumino Consistency Conditions $[\delta_\lambda, \delta_\rho] \log Z[A] = \delta_{[\lambda,\rho]} \log Z[A]$
- Anomaly inflow from anomaly field theory in one higher dimension

Powers

- Anomalies are invariant under symmetric deformations
- Universal and robust observables for symmetric theories
- 't Hooft anomalies are RG invariants
- Conformal a anomalies are RG monotonic (proven in d=2,4)
- Constraints on RG flows and phase structure



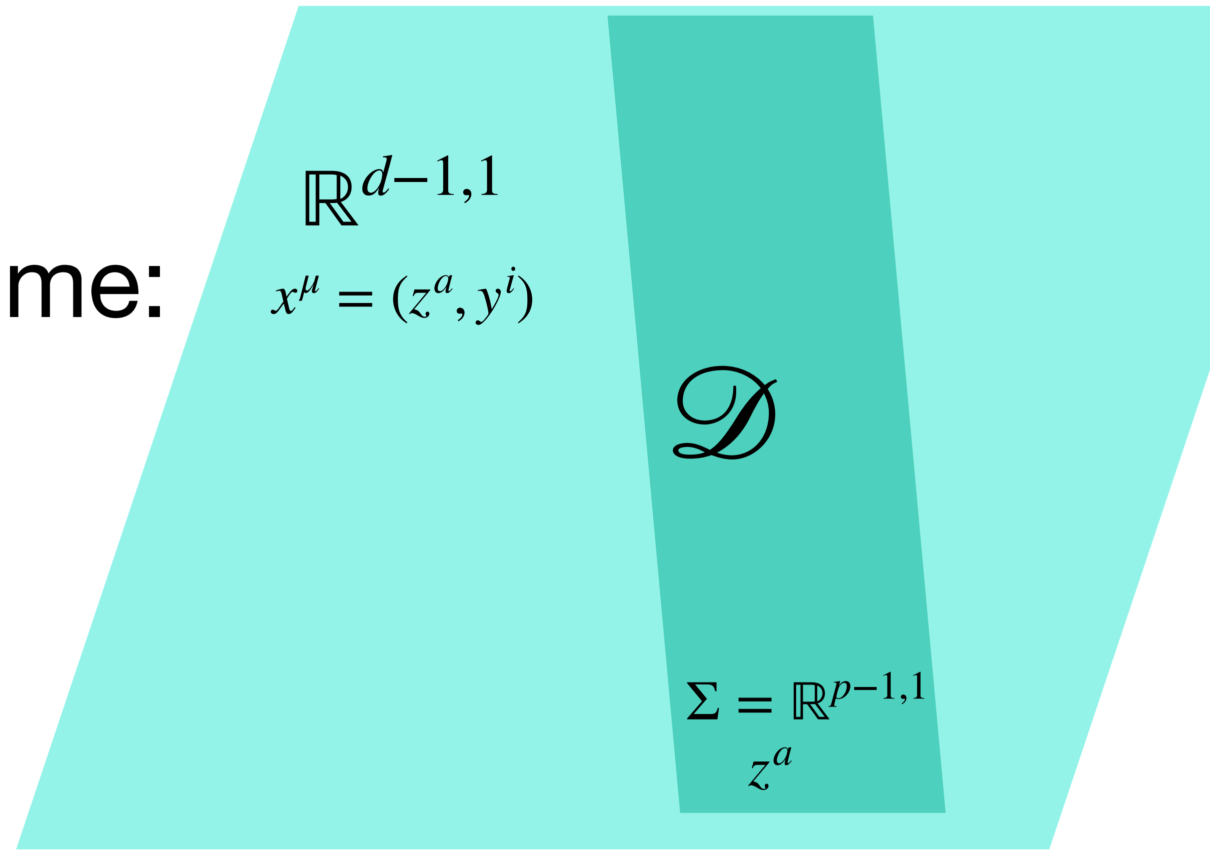
Structures of Anomalies w/ Defect Insertion?

Defect Anomalies

Features

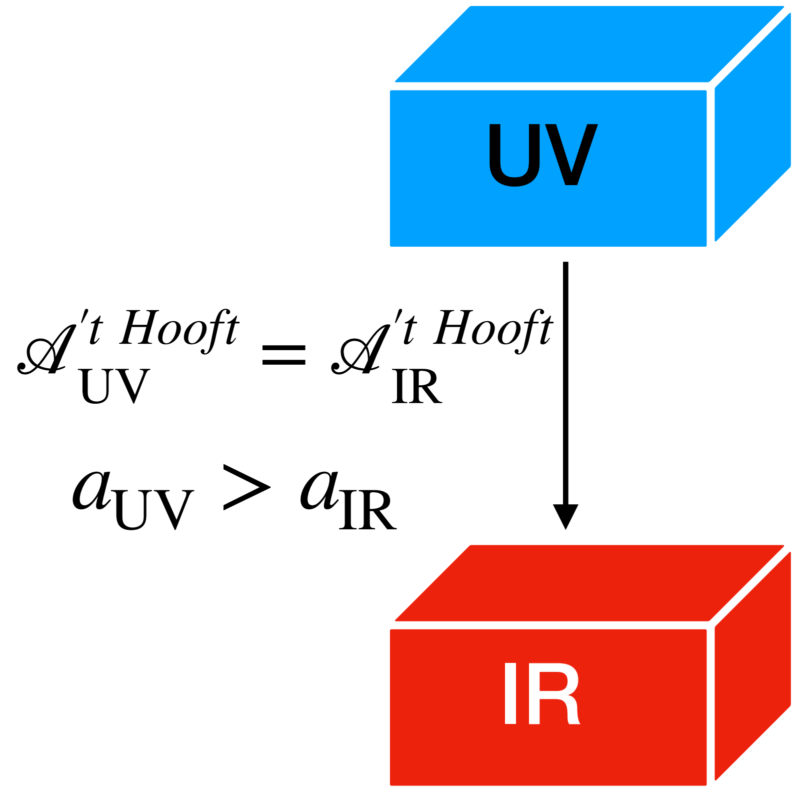
- Modification of Ward identity by contact terms on the **defect** worldvolume: $\partial_\mu J^\mu \sim k\delta(\Sigma) \times (\text{defect contact terms})$
- Anomalous variation of the **defect** partition function $\delta_{\lambda_\Sigma} \log Z_{\mathscr{D}}[A]$
- Solutions to WZ Consistency Conditions for the **residual** symmetries: more invariant structures, can be present even if the bulk is anomaly free
- Encodes current algebra with **defect** insertion
  e.g. surface defect in d=3: $\langle J^\mu(x) J^\nu(x') \rangle^{\mathscr{D}}_{\text{odd}} = k f(v) P^{\mu\nu}(x, x')$

v: conformal invariant cross ratio



Powers

- Anomalies are invariant under symmetric deformations on Σ
- Universal and robust observables for symmetric **defect** theories
- ’t Hooft anomalies are **defect** RG invariants
- **Defect** conformal “a”-type anomalies are RG monotonic
- Constraints on **defect** RG flows and **defect** phase structure



$$\mathscr{D}_{UV} \longrightarrow \mathscr{D}_{IR}$$

$$S_{\text{DFT}} = S_{\mathscr{D}_{\text{UV}}} + \int_\Sigma d^p z \sqrt{|h|} \sum_{\mathscr{O}_{\text{UV}} \in \mathscr{D}_{\text{UV}}} \lambda_{\mathscr{O}_{\text{UV}}} \mathscr{O}_{\text{UV}}(z)$$

Defect 't Hooft Anomalies: a free example

Boundary $U(1)$ anomaly of 3d Dirac fermion ψ

$$\partial_\mu J^\mu(x) = \frac{1}{4\pi}\delta(\Sigma)k\epsilon_{ab}F^{ab}$$

Projector in the transverse direction: $P_\pm \equiv \frac{1 \pm \gamma_y}{2}$

Standard U(1) symmetric boundaries: B_+:$P_+\psi|_\Sigma = 0$ or B_-:$P_-\psi|_\Sigma = 0$

Boundary $U(1)$ anomaly: $I_4(B_+) = \frac{1}{2}I_4(\chi_-^{2d}) = -\frac{1}{2}I_4(\chi_+^{2d}) = -I_4(B_-)$

2d chiral fermions

- Consider ψ on slab $I_{[0,L]} \times \Sigma$ with B_+ b.c. at two ends $\xrightarrow{L \to 0}$ $2I_4(B_+) = I_4(\chi_-^{2d})$

To see the 1/2

- The two b.c.s $B_\pm$ are related by $P_+\psi|_\Sigma = 0$ $\rightleftarrows$ $P_-\psi|_\Sigma = 0$

Parity flip

Boundary RG $\chi_-^{2d} + \int_\Sigma \bar\psi P_- \chi_-^{2d} + (c.c)$

$$I_4(B_+) = -I_4(B_-)$$
$$I_4(B_+) = I_4(B_-) + I_4(\chi_-^{2d})$$

What about more nontrivial defects?

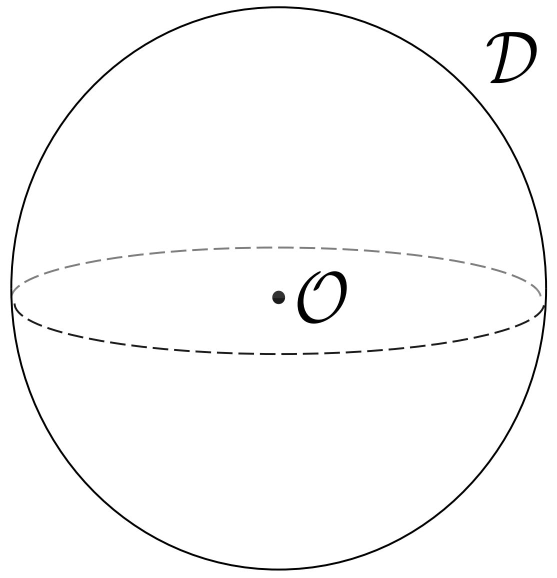
- Compute from deformed defects after symmetric deformations
- Anomaly inflow in string/M theory

Conformal Defects (DCFT)

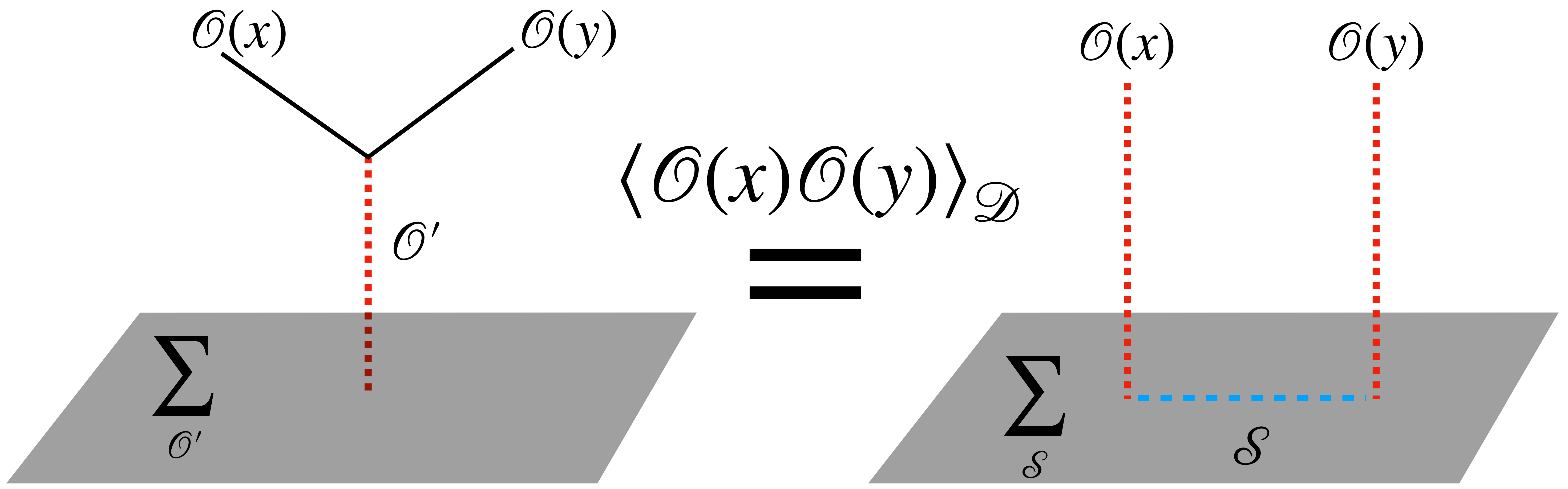
- Critical phase in the presence of boundaries/defects
- **Universality classes** of defect RG flows

$$SO(p,2) \times SO(d-p) \subset SO(d,2)$$

- No local p-dimensional stress tensor or currents (generically)
- **New critical exponents and OPE data**
  (e.g. defect local ops $\mathcal{S}$ and bulk local op. 1pf $\langle \mathcal{O} \rangle_{\mathcal{D}}$)



- Constrained by **defect bootstrap** equations
  (e.g. residual conf symmetry, crossing and unitarity)



General d [Liendo-Rastelli-van Rees, Gaiotto-Mazac-Paulos, Liendo-Meneghelli, Billo-Goncalves-Lauria-Meineri,…]

2d “Classifying Algebra” [Cardy-Lewellen, Lewellen, Fuchs-Schweigert-(Signer), Recknagel-Schomerus, …, Runkel-Watts]

Defect Conformal Anomalies

Ward identities w/ conformal defect $\mathscr{D}$

$$\partial_\mu T^{\mu a}(x) = 0$$
$$\partial_\mu T^{\mu i}(x) = \delta(\Sigma) D^i(z)$$
$$T^\mu_\mu(x) = 0$$

Displacement operator

Conformal Anomaly p=2: surface defect

Bulk stress tensor

Intrinsic curvature

Traceless extrinsic curvature

Ambient Weyl curvatuare

$$T^\mu_\mu(x) = \frac{1}{24\pi}\delta(\Sigma)(bR + d_1 \hat{K}_{ab}\hat{K}^{ab} + d_2 W_{ab}{}^{ab})$$

[Henningson-Skenderis, Schwimmer-Theisen,Graham-Witten,…]

Intrinsic defect anomalies: b-anomaly analogous to c anomaly of 2d CFT

Extrinsic defect anomalies: $d_{1,2}$ are new anomalies for surface defects

Features of Defect Conformal Anomalies

$$T^{\mu}_{\mu}(x) = \frac{1}{24\pi}\delta(\Sigma)(bR + d_1\hat{K}_{ab}\hat{K}^{ab} + d_2 W_{ab}{}^{ab})$$

	c	b	d_1	d_2
(D)CFT origin	$\langle T_{\mu\nu}(x)T_{\rho\sigma}(x')\rangle_{\mathrm{CFT}}$	$\langle T_{ab}(z)T_{cd}(z')\rangle_{\mathscr{D}}$ But not explicit	$d_1 \propto \delta_{ij}\langle D^i D^j\rangle_{\mathscr{D}}$	$d_2 \propto \delta_{ab}\langle T_{ab}\rangle_{\mathscr{D}}$
RG property	Bulk Monotonic	Defect Monotonic [Jensen-O'Bannon 16]	None	None
Positivity	Yes	No	Yes	Yes (ANEC) [Jensen-O'bannon-Robinson-Rodgers 19]
Dependence on defect marginal couplings		No	Yes	Yes
Dependence on bulk marginal couplings	No	Yes w/o SUSY (examples?)	Yes	Yes

[Herzog-Shamir 19, Bianchi 19]

Free examples: Defect Conformal Anomalies

Bulk stress tensor

Traceless extrinsic curvature

$$T^{\mu}_{\mu} = \frac{1}{24\pi}\delta(\Sigma)(bR + d_1 \hat{K}_{ab}\hat{K}^{ab})$$

Standard boundary conditions of 3d free fields

	Fields	cL	cR
2d	$\psi_-^{2d} = 0$	1/2	0
	$\psi_+^{2d} = 0$	0	1/2
	$\phi^{2d} = 0$	1	1
3d	$P_+\psi\|_\Sigma = 0$	-1/4	1/4
	$P_-\psi\|_\Sigma = 0$	1/4	-1/4
	$\phi\|_\Sigma = 0$	1/8	1/8
	$\partial_\perp\phi\|_\Sigma = 0$	-1/8	-1/8

$$b = c_L + c_R$$

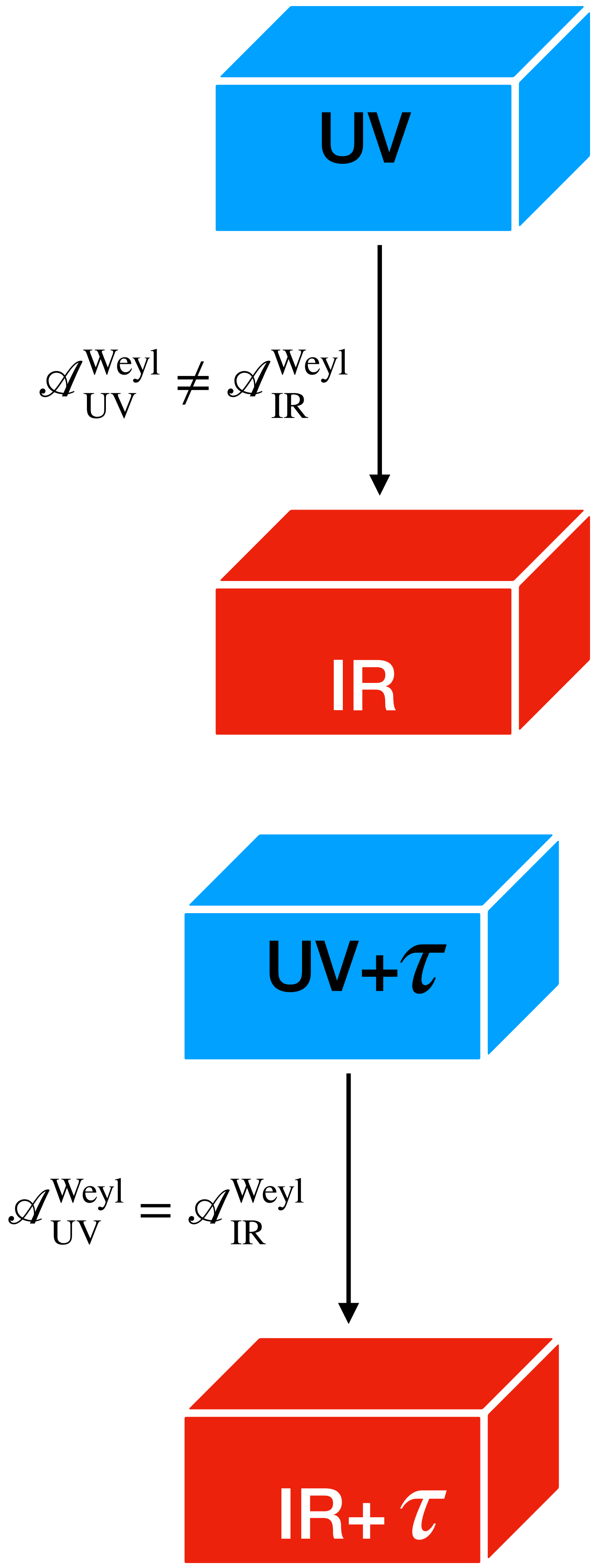
$$k_g = c_L - c_R$$

Defect gravitational anomaly

$P_\pm = \frac{1}{2}(1 \pm \gamma_\perp)$: boundary chiral projector

RG flow and a-theorem in QFT

General Strategy: [Komargodski-Schwimmer 2011]



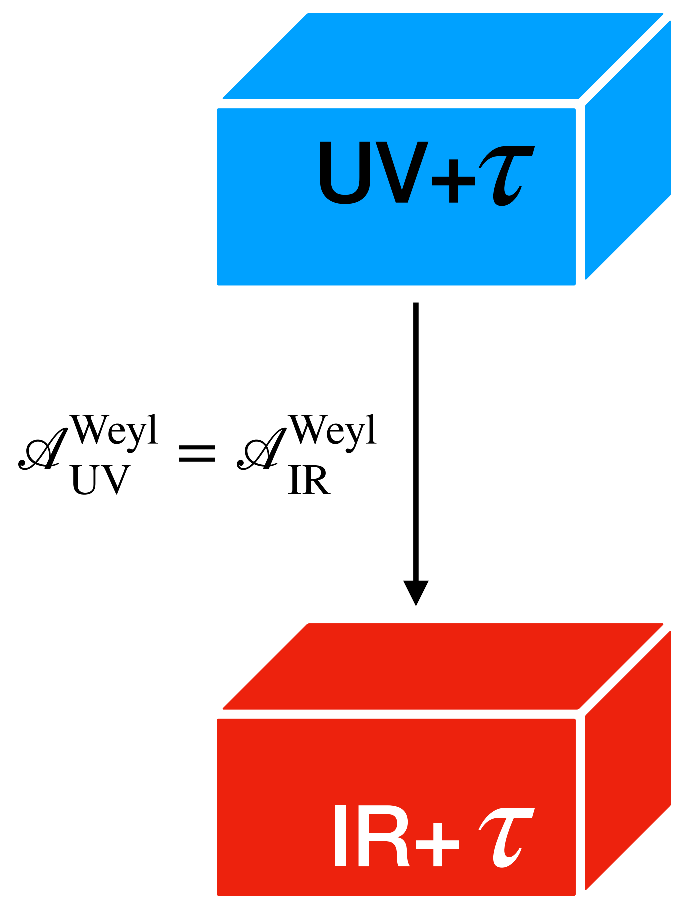
- Couple the QFT along the RG to spurious dilaton $\tau(z)$
- Restore Weyl symmetry by $\tau \to \tau + \sigma$ as $g_{\mu\nu} \to g_{\mu\nu} e^{2\sigma}$
- IR effective action $S_{\text{eff}} = S_{\text{IR}} + S_{\text{dilaton}}[\tau] + \ldots$
- Weyl anomaly matched between UV and IR

$$\delta_\sigma \log Z_{UV} = \delta_\sigma \log Z_{IR} + i\delta_\sigma S_{\text{dilaton}}[\tau]$$

- $S_{\text{dilaton}} = S_{\text{WZ}} + S_{\text{inv}}$ $\quad S^{2d}_{\text{dilaton}}[\tau] \sim -\Delta c_{2d} \int d^2 z (\partial\tau)^2 \quad S^{4d}_{\text{dilaton}}[\tau] \sim -\Delta a_{4d} \int d^4 z (\partial\tau)^4$
- a-theorem from unitarity constraints on S_{dilaton} (reflection positivity in 2d and optical theorem in 4d.

Defect RG flow and Defect a-theorem $\mathscr{D}_{UV} \xrightarrow[S_{\rm DFT} = S_{\mathscr{D}_{\rm UV}} + \int_\Sigma d^p z \sqrt{|h|} \sum_{\mathcal{O}_{\rm UV} \in \mathscr{D}_{\rm UV}} \lambda_{\mathcal{O}_{\rm UV}} \mathcal{O}_{\rm UV}(z)]{} \mathscr{D}_{IR}$

General Strategy: [Komargodski-Schwimmer 2011] extended to defects



- Couple the **DFT** along the RG to spurious **defect** dilaton $\tau(z)$
- Restore **defect** Weyl symmetry by $\tau \to \tau + \sigma$ as $g_{\mu\nu} \to g_{\mu\nu} e^{2\sigma}$
- IR effective action $S_{\rm eff} = S_{\mathscr{D}_{\rm IR}} + S_{\rm dilaton}[\tau] + \ldots$
- Weyl anomaly matched between UV and IR

$$\delta_\sigma \log Z_{\mathscr{D}_{UV}} = \delta_\sigma \log Z_{\mathscr{D}_{IR}} + i\delta_\sigma S_{\rm dilaton}[\tau]$$

Isolate potential complications in the defect case due to the extrinsic anomalies

- $S_{\rm dilaton} = S_{\rm WZ} + S_{\rm inv}$ $\quad S^{2d}_{\rm dilaton}[\tau] \sim -\Delta b_{2d} \int d^2 z (\partial\tau)^2 \quad S^{4d}_{\rm dilaton}[\tau] \sim -\Delta a_{4d} \int d^4 z (\partial\tau)^4$
- a-theorem from unitarity constraints on $S_{\rm dilaton}$ [Jensen-O'Bannon 16, YW 21] (reflection positivity of defect correlators in 2d, defect optical theorem in 4d)

Defect Anomalies beyond Free Theories?

Superconformal Defects

- Ubiquitous in SCFTs (constructions from intersecting branes and branes wrapping geometries in string/M/F theory)

- Breaks bulk superconformal algebra to a smaller superconformal subalgebra

- Displacement operator (and other broken current operators) sits in unitary superconformal multiplets -> Constraints on symmetry breaking pattern -> classification of superconformal defects according to symmetries

[Bianchi-Preti-Vescovi 18, Bianchi-Lemos-Meineri 18, Liendo-Meneghell-Mitaev 18, Agmon-YW 20]

For superconformal lines (preserving transverse rotations)

- No superconformal lines in 4d N=1 and 6d N=(1,0) or (2,0)

- half-BPS lines in 5d N=1, 4d N=2, 2/3-BPS lines in 4d N=3

- Half and quarter-BPS lines in 4d N=4

- Superconformal lines in d>3 are all rigid

- Superconformal lines in d=3 can have (and sometimes must have) conformal manifolds

[Agmon-YW 20]

Superconformal surface defect anomalies

$\mathcal{N} = (0,2)$ superconformal surface defect: $\mathfrak{so}(1,2) \times \mathfrak{osp}(2|2,\mathbb{R})$

Anomaly multiplet relation

Global $U(1)_r$ symmetry J_μ

Conformal symmetry $T_{\mu\nu}$

't Hooft anomalies

Conformal anomalies

SUSY

$$\partial_\mu J^\mu(x) = \frac{1}{4\pi}\delta(\Sigma)k\epsilon_{ab}F^{ab}$$

$$T^\mu_\mu = \delta(\Sigma)(bR + d\hat{K}_{ab}\hat{K}^{ab})$$

RG invariant, easy

RG variant, hard

$U(1)_r$ current
Identify by b-maximization
(Generalizing [Benini-Bobev 2012])

- Anomaly multiplet relations $\boxed{c_L = 3k - k_g,\ c_R = 3k, \text{ or } b = 3k - \frac{k_g}{2}}$ [YW 20]

Defect gravitational anomaly

Example 1: Super Ising SCFT

N=2 SQFT
with superpotential

$$W = \Phi^3$$

$$\Phi \to (\phi, \psi)$$

$$\longrightarrow \quad \mathscr{L} = |\partial\phi|^2 + \lambda|\phi|^4 + \text{fermions}$$

- Flow to strongly coupled 3d CFT
- appear in various CM setups: e.g. optical lattice of cold atoms and boundaries of topological superconductors
- What can we say about its boundary universality classes?

Super Ising boundary

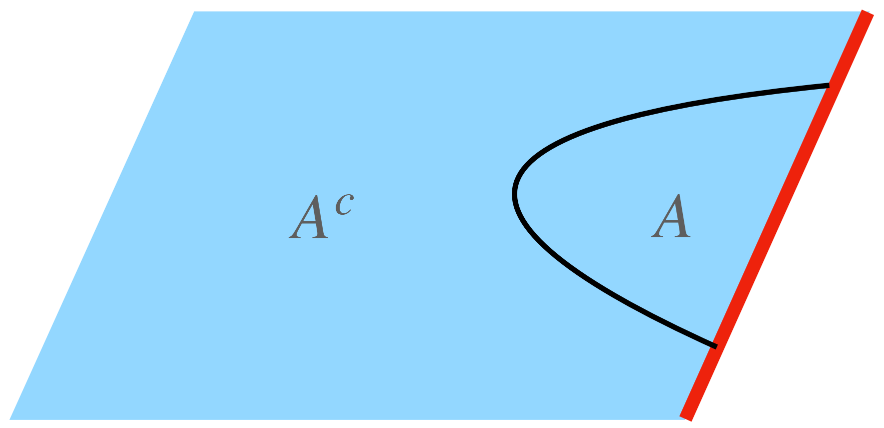
UV field content | complex boson ϕ and fernion ψ

Simplest boundary (supersymmetric Dirichlet) $$\phi|_\Sigma = P_+\psi|_\Sigma = 0$$

$U(1)_r$ charge

't Hooft anomalies from the fermion with $r = -\frac{1}{3}$ $$k = -\frac{1}{2}\left(\frac{1}{3}\right)^2, \quad k_g = -\frac{1}{2}$$

Defect conformal anomalies from SUSY anomaly multiplet relations $c_L = \frac{1}{3}, \quad c_R = -\frac{1}{6}$

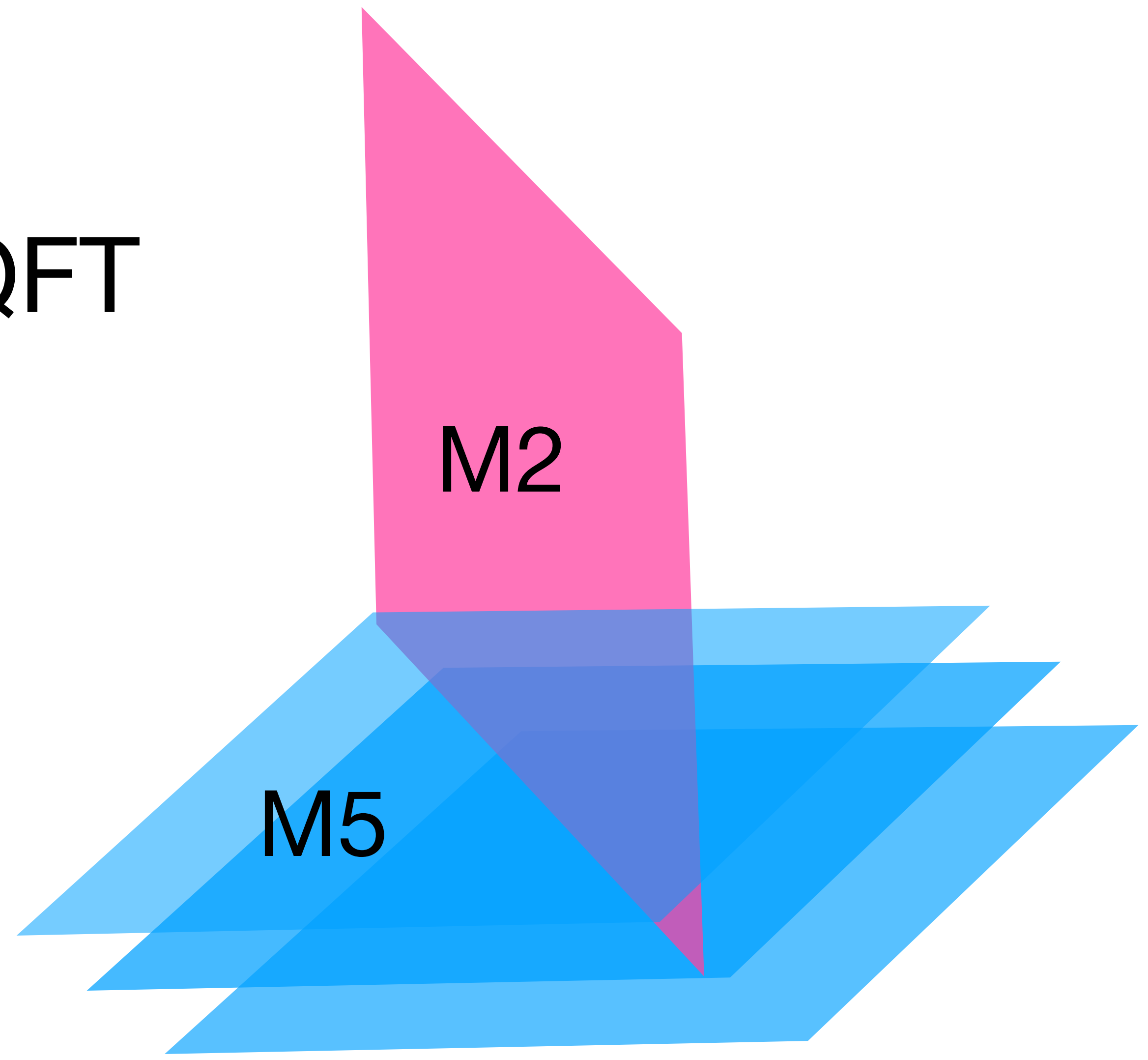


Boundary Casimir energy,
Boundary entanglement entropy,
….

Example 2: Surface defects in 6d SCFTs

- Mysterious 6d (2,0) SCFTs with no Lagrangian, no parameter, labelled by ADE Lie algebra $\mathfrak{g}$
- Existence suggested by string/M theory constructions: M5 branes, IIB on ADE singularities
- Mother of many QFTs: compactification -> unifying classification of lower dimensional QFTs, geometric pictures of strong-weak dualities …
- Contain rich spectrum of defects -> defects and local ops in the lower dim QFT
- M theory predicts surface defects from M2 branes $\mathscr{D}_\lambda[\mathfrak{g}]$

Weight vector λ describes how M2 intersects the M5s (before taking the conformal/singular limit)



- Defect conformal anomaly b?
- SUSY says $b = 3k$
  k is the 't Hooft anomaly for R-symmetry in the $\mathfrak{osp}(2\,|\,2,\mathbb{R}) \times \mathfrak{so}(2,1)$ subalgebra

Example: Surface defects in 6d SCFTs

Defect symmetry

Residual 6d R-symmetry — Transverse rotation

$$SU(2)_L \times SU(2)_R \times SU(2)_F \times SU(2)_I$$

$$\mathscr{D}_\lambda[\mathfrak{g}]$$

Defect U(1)R symmetry

$$r = 2(R - 2I)$$

[Kim-Kim-Park 16, Shimizu-Tachikawa 16]

Defect 't Hooft anomalies from inflow on the tensor branch

$$I_4 = \frac{1}{2}(\lambda, \lambda)(c_2(F_L) - c_2(F_R)) + (\lambda, \rho)(c_2(F_I) - c_2(F_F))$$

Weyl vector for $\mathfrak{g}$

Defect U(1)R 't Hooft anomaly

$$k = (\lambda, \lambda) + 8(\lambda, \rho)$$

Defect conformal anomalies

$$b = c_L = c_R = 3k = 3(\lambda, \lambda) + 24(\lambda, \rho)$$

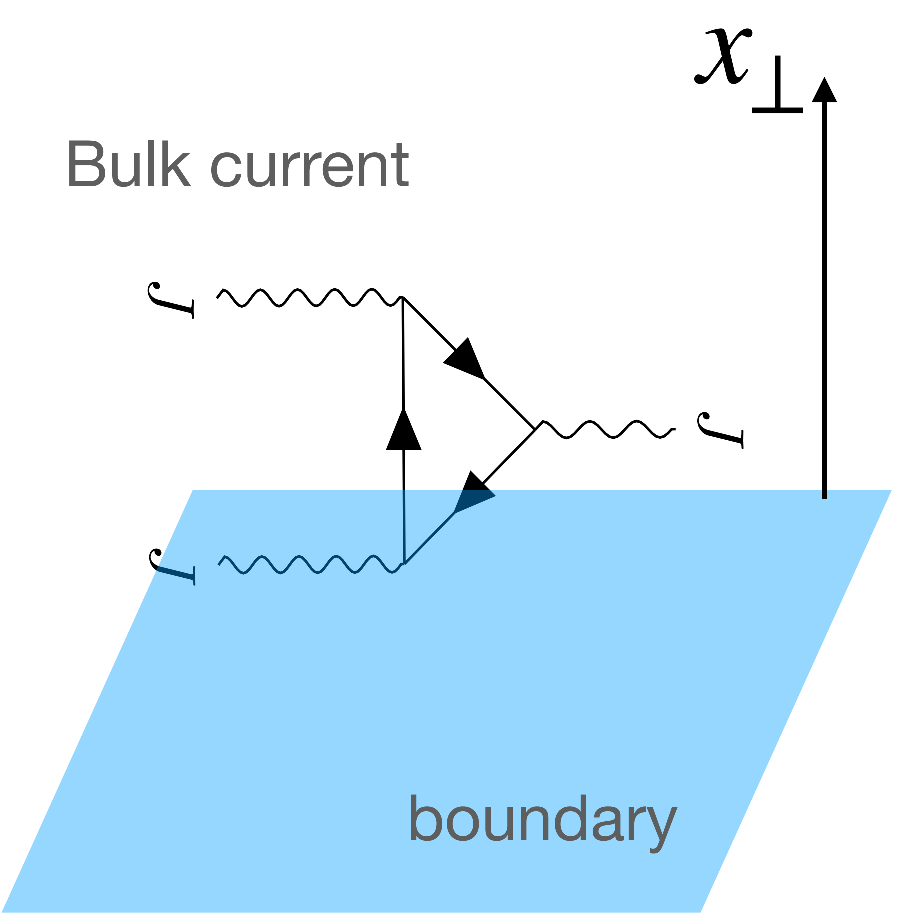
[YW 20]

Agrees with holographic computations for SU(N) case in large N limit [Estes-Krym-O'Bannon-Robinson-Rodgers 18]

also [Drukker-Probst-Trepanier 20] and see Nadav's talk

Constraints from 't Hooft anomalies on defects?

Anomalies and Boundaries



Symmetry G	Consequence of anomalies
Usual 't Hooft anomalies w/o bdy	Obstruction for gauging G
t Hooft anomalies w/ bdy	Obstruction for G-sym boundary condition

2d continuous sym [Masataka 2016, Billo-Goncalves-Lauria-Meineiri 2016, Jensen-Shaverin-Yarom 2017, Hellerman-Orlando-Watanabe 2021]

General [Thorngren-YW 2020]

Anomalies and Boundaries

d>2

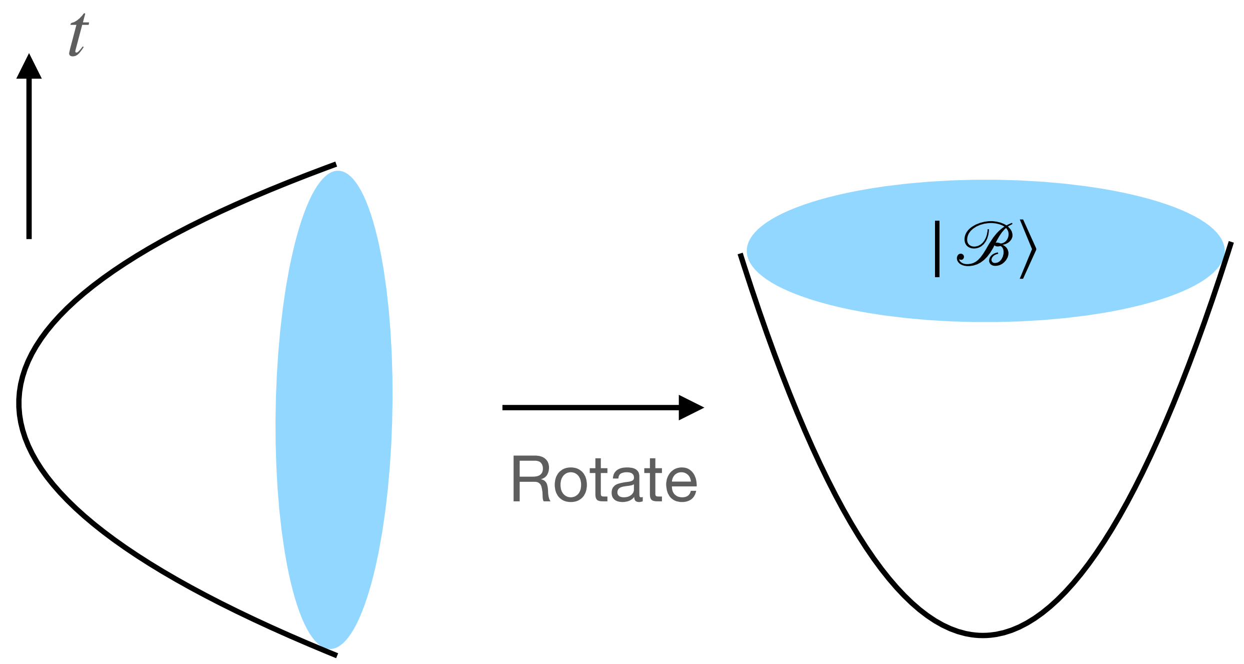
Assume a G-symmetric boundary $\mathscr{B}$ $\qquad \partial_\mu J^\mu = 0$ Up to c number terms (e.g anomalies)

Structure of anomalies constrained by **Wess-Zumino consistency condition** on manifold with boundary

$\downarrow$

The anomaly does not lead to a projective rep of G on Hilbert space ($|\mathscr{B}\rangle$ defines a G-symmetric state)

$\downarrow$



The anomaly must be abelian (from descent equations)

Also [Jensen-Shaverin-Yarom 2017]

Anomalies and Boundaries

d=2n>2

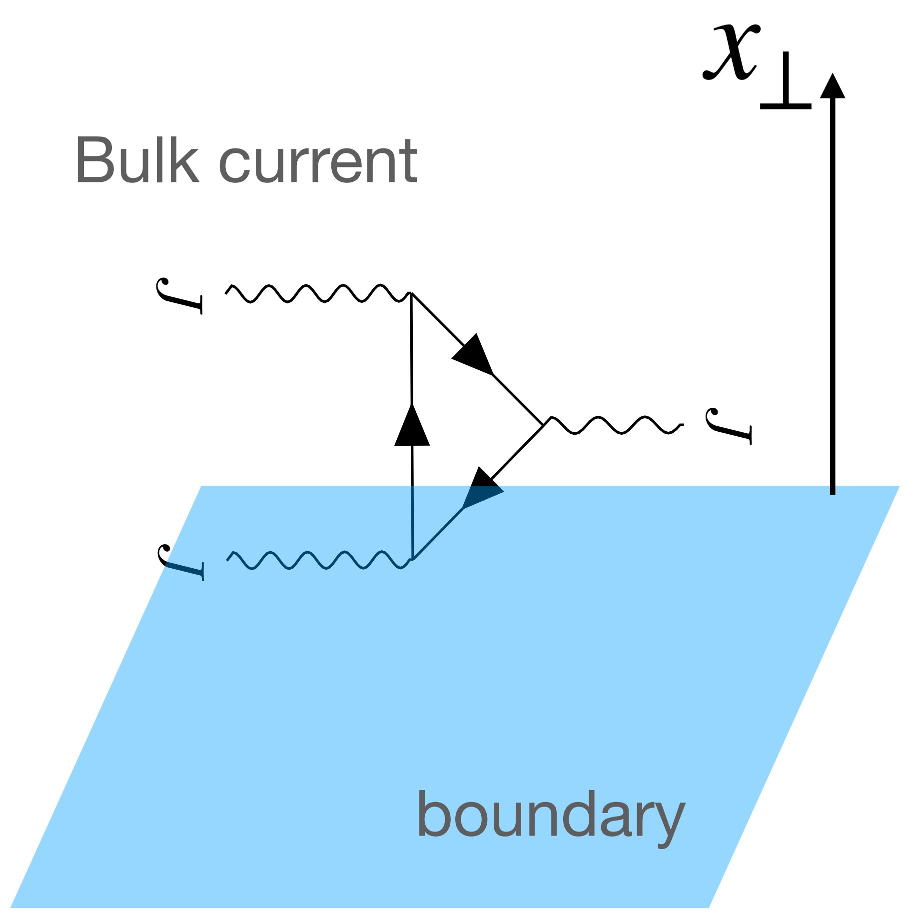
Assume a G-symmetric boundary $\mathscr{B}$ $\partial_\mu J^\mu = 0$ Up to c number terms (e.g) anomalies

Focus on the IR **conformal** boundary for G=U(1)

Current Ward identity (w/ boundary) $\langle \partial_\mu J^\mu(x) J^{\mu_1}(x_1)\dots J^{\mu_n}(x_n)\rangle = -\frac{k}{(n+1)!(2\pi)^n}\epsilon^{\mu_1\dots\mu_n\nu_1\dots\nu_n}\prod_{i=1}^{n}\frac{\partial}{\partial x_i^{\nu_i}}\delta^d(x-x_i)$

Unitarity requires as $x_\perp \to 0$ $J_\perp = 0$ As an operator equation

Together $k = 0$



Anomalous symmetries must be broken at the boundary

Application: Boundaries of N=4 Super-Yang-Mills

N=4 super-Yang-Mills

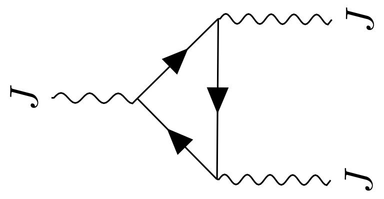
Symmetry

$SO(6)_R$ — Maximial subgroups → $SU(2)_F \times SU(2)_R \times U(1)_R$

$SO(6)_R$ — Maximial subgroups → $SO(3)_H \times SO(3)_C$

Anomalies

Pert. 't Hooft anomaly $c_3(SO(6)_R)$



Witten anomaly for $SU(2)_F$

$\pi_4(SU(2)) = \mathbb{Z}_2$

Implications

$SO(6)_R$ or $SU(2)_F$ symmetric boundary conditions not possible

- Known maximally symmetric boundary preserves $SO(3)_H \times SO(3)_C$ [Gaiotto-Witten]
- Both anomalies trivialize

Application: Boundary conditions for 2d fermions w/chiral symmetry

$\boxed{\text{Symmetry}}$ $\mathbb{Z}_2^L \times \mathbb{Z}_2^R$ fermion parity: $(-1)^{F_L}$ and $(-1)^{F_R}$

N_f 2d Majorana fermions

$\boxed{\text{Anomaly}}$ $\Omega^3_{Spin}(B\mathbb{Z}_2) = \mathbb{Z}_8$ [Fidkowski-Kitaev 2010]

$\boxed{\text{Implications}}$ $\mathbb{Z}_2^L \times \mathbb{Z}_2^R$ symmetric boundary condition possible only if $N_f \in 8\mathbb{Z}$

- Indeed, explicit constructions of symmetric boundary states only possible for these cases [Smith-Tong 2020]

More Open Questions

- Universal (“positivity”) bounds on defect conformal anomalies (and free energy) e.g. collider bounds from positive energy conditions [Hofman-Maldacena 08, Herzog-Schaub 21]
- Discrete (and “exotic”) symmetries and defect ’t Hooft anomalies
- Inversion (conformal) anomalies for odd-dimensional defects [Drukker-Gross]
- Holographic dual of the defects and constraints on world-volume couplings
- Bootstrap & Symmetry constraints on non-BPS branes/ boundaries in String/M-theory (via AdS/CFT) and swampland conjectures for quantum gravity [McNamara-Vafa 2019]

Thank you!