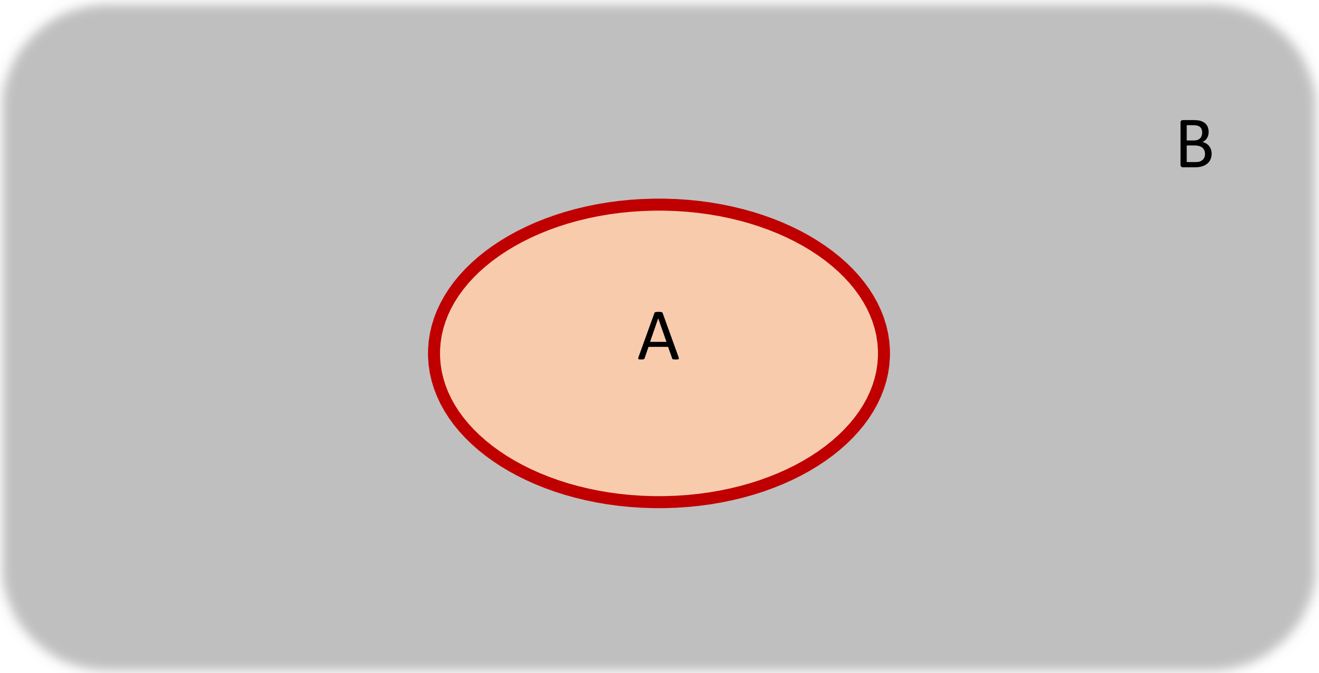
Entanglement in 1+1D conformal field theories with boundaries

Ananda Roy

Rutgers University

Quantum field theory at the boundary, Mainz Virtual Workshop
09/28/2021

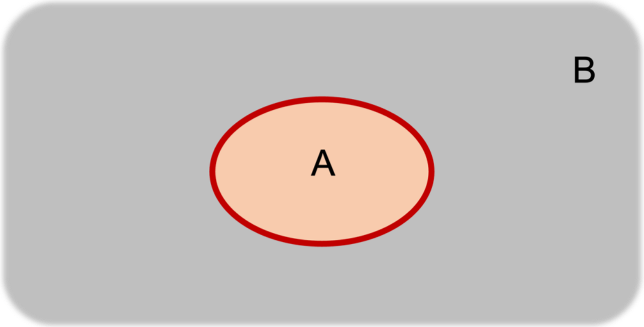
Entanglement in many body systems



A and B are entangled if $|\psi_{AUB}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$

Measures of entanglement

A and B are entangled if $|\psi_{A\cup B}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$



Measures of entanglement:

von Neumann entropy:
$$S_A = -\mathrm{Tr}\rho_A \ln \rho_A \,,$$
$$\rho_A = \mathrm{Tr}_\mathrm{B}\, \rho_{A\cup B}$$

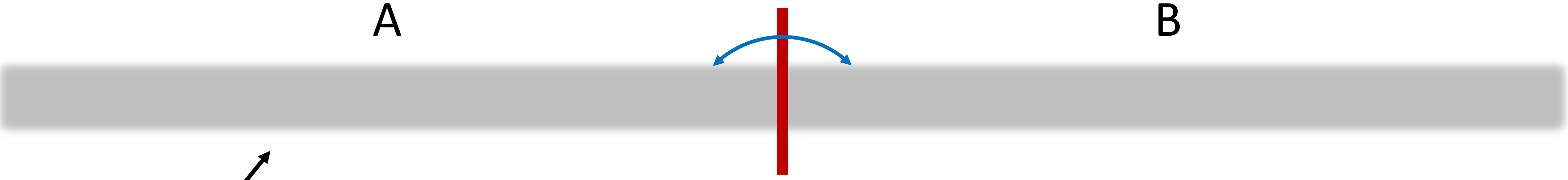
Entanglement Hamiltonian:
$$\mathcal{H}_A = -\frac{1}{2\pi} \ln \rho_A$$

log negativity:
$$\mathcal{E}_A = \ln\left|\rho_{\mathrm{A\cup B}}^{\mathrm{T_B}}\right|$$

Contain signatures of quantum critical phenomena, topological order, information scrambling, …

Eisert *et al* (2010), Haag (1992), Li and Haldane (2008), Vidal and Werner (2004)

Entanglement entropy in 1+1D quantum systems



1D quantum system at T = 0

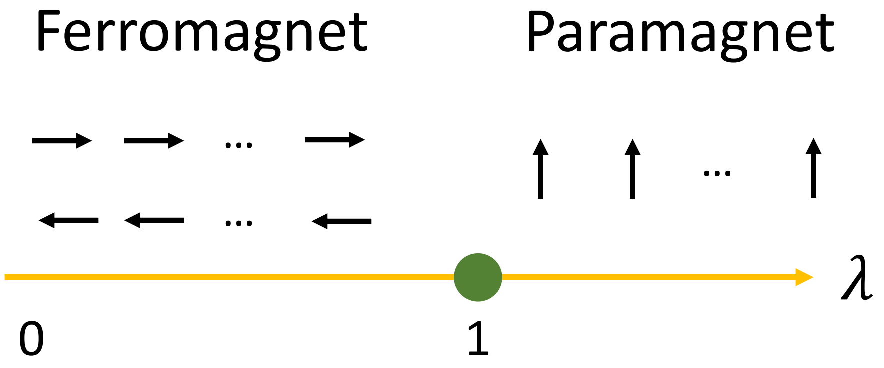
Generic behavior
for systems in ground state

gapped spectrum
$S_A \sim \ln \xi$,
ξ = correlation length

gapless spectrum
$S_A \sim \ln l_A$,
l_A = subsystem size

e.g., Ising chain:

$$H = -\sum_i \sigma_i^x \sigma_{i+1}^x - \lambda \sum_i \sigma_i^z$$



Holzhey *et al* (1994), Cardy and Calabrese (2004), Hastings (2007)

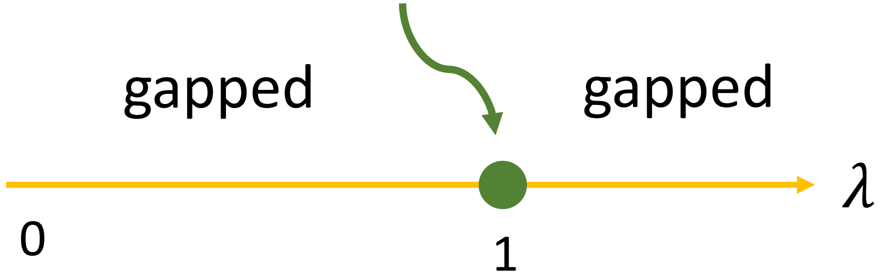
Entanglement entropy in 1+1D conformal-invariant quantum systems

Entanglement entropy: $S_A = \frac{c}{6}\ln l_A + \cdots,\ c$ = central charge

Ising chain:

$$H = -\sum_i \sigma_i^x \sigma_{i+1}^x - \lambda \sum_i \sigma_i^z$$

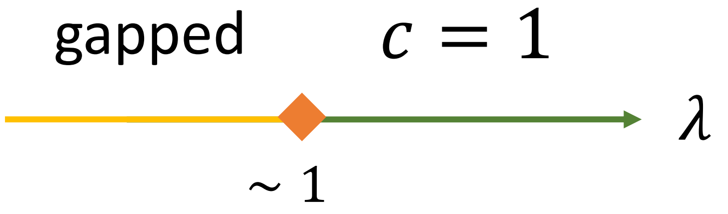
$$c = \frac{1}{2}$$



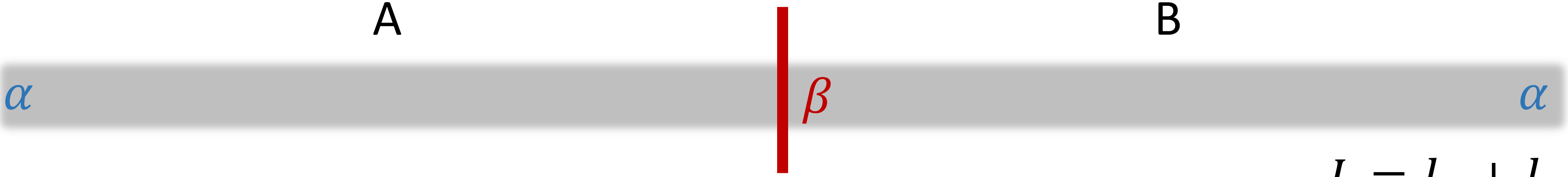
Quantum rotor chain:

$$H = \sum_i n_i^2 - \lambda \sum_i \cos(\phi_i - \phi_{i+1})$$

$$[n_i, e^{\pm i\phi_j}] = \pm e^{\pm i\phi_j}\,\delta_{ij}$$



Entanglement entropy in 1+1D conformal-invariant quantum systems with boundaries



$$L = l_A + l_B$$

S_0 = non-universal

Entanglement entropy:

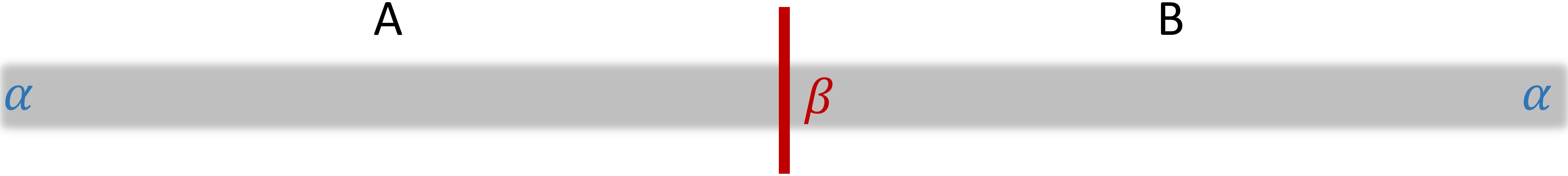
$$S_A(\alpha, \beta) = \frac{c}{6} \ln \left[\frac{2L}{\pi} \sin \frac{\pi l_A}{L} \right] + S_\alpha + S_\beta + S_0$$

$S_\alpha, S_\beta = \ln g_\alpha, \ln g_\beta$ = boundary entropies, α = inherited, β = free (from entanglement cut)

Easier to extract than conventional thermodynamic quantities like Casimir energy, impurity free energy, …

Cardy and Calabrese (2004, 2009), Cardy and Tonni (2016), Affleck and Ludwig (1991)

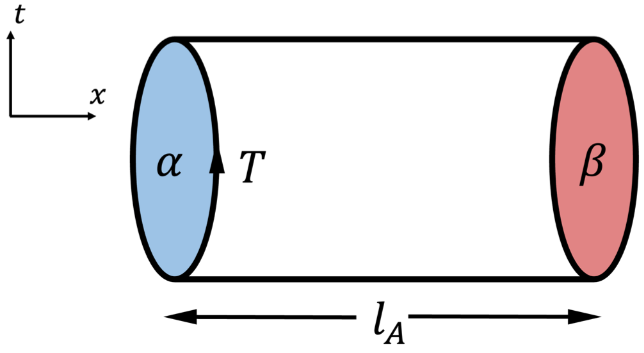
Entanglement Hamiltonian in 1+1D conformal-invariant quantum systems with boundaries



Entanglement Hamiltonian:

$$\mathcal{H}_A(\alpha,\beta) = -\frac{1}{2\pi}\ln \rho_A(\alpha,\beta) \simeq -\frac{1}{2\pi}\ln \frac{e^{-2\pi H_{\alpha\beta}}}{\mathrm{Tr}e^{-2\pi H_{\alpha\beta}}}$$

$H_{\alpha\beta}$ = boundary CFT Hamiltonian



Entanglement spectrum obtained from standard partition function computation

Cardy and Tonni (2016), AR *et al,* J. Stat. Mech (2020)

Entanglement entropy in the critical Ising chain with boundaries: Results

Hamiltonian:

$$H = -\sum_i \sigma_i^x \sigma_{i+1}^x - \sum_i \sigma_i^z + \lambda_0(\sigma_0^x + \sigma_{L-1}^x)$$

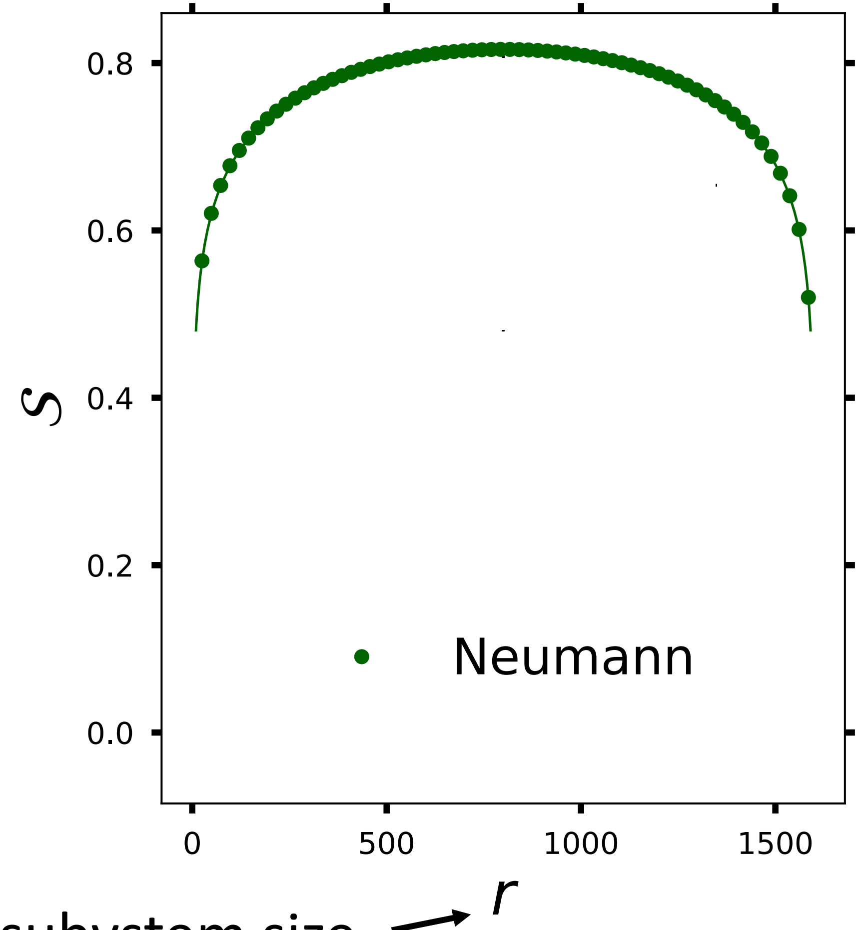
Neumann (free): $\lambda_0 = 0$, Dirichlet (fixed): $\lambda_0 \ll 1$

Central charge $= \ 1/2$

Change in boundary entropy:

$$\Delta S_{N\to D} = \frac{1}{2}\ln 2$$

DMRG ($\simeq$ matrix product state) computation



entanglement entropy

subsystem size

AR *et al,* J. Stat. Mech (2020), Affleck and Ludwig (1991)

Entanglement spectrum in the critical Ising chain with boundaries: Results

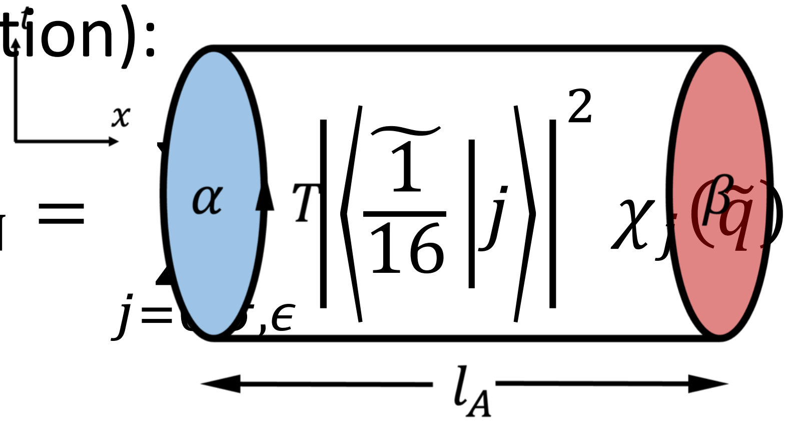
Boundary states:

$$\left|\tilde{0}\right\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|\epsilon\rangle + \frac{1}{2^{1/4}}|\sigma\rangle,$$

$$\left|\widetilde{\frac{1}{2}}\right\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|\epsilon\rangle - \frac{1}{2^{\frac{1}{4}}}|\sigma\rangle,$$

$$\left|\widetilde{\frac{1}{16}}\right\rangle = |0\rangle - |\epsilon\rangle$$

Partition function (Neumann boundary condition):



$$Z_{\mathrm{NN}} = \sum_{j=0,\epsilon} \left|\left\langle \widetilde{\frac{1}{16}} \middle| j \right\rangle\right|^2 \chi_j(\tilde{q}), \quad \tilde{q} = e^{-2L_{\mathrm{eff}}}$$

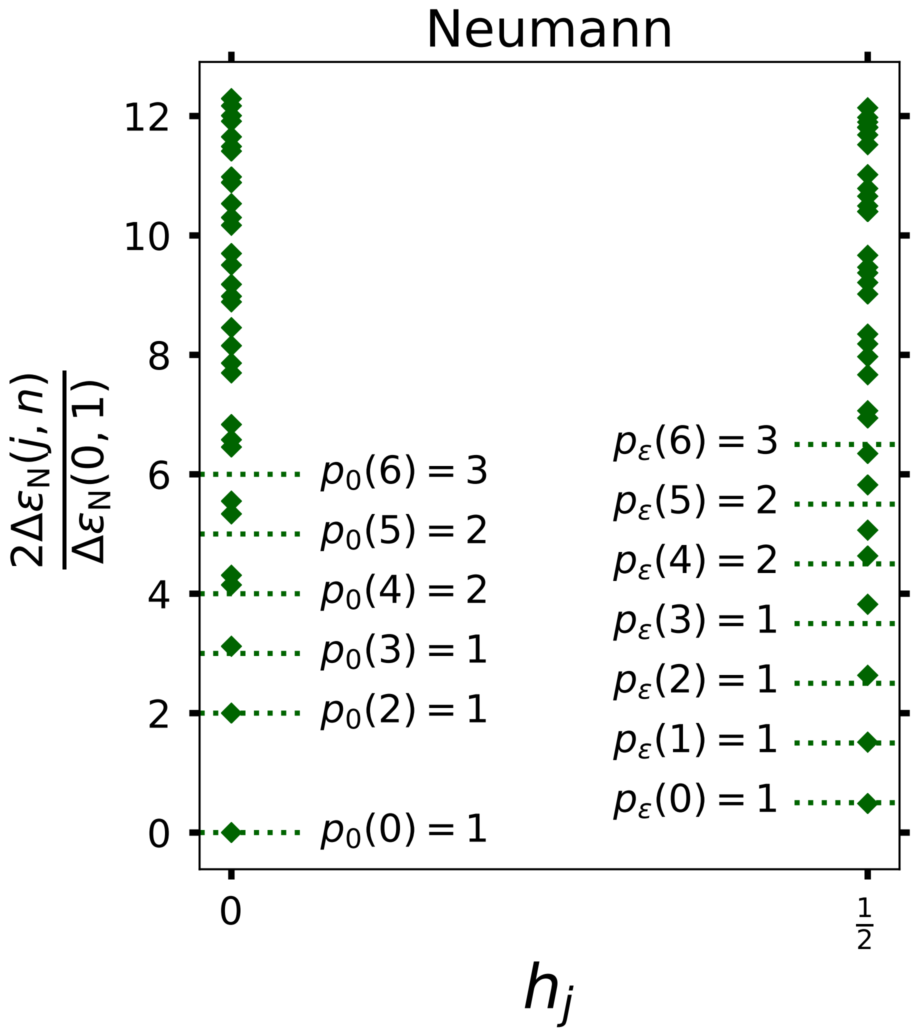
Entanglement spectrum (Neumann):

$$e(j,n) = \frac{L_{\mathrm{eff}}}{48\pi} + \frac{\pi}{L_{\mathrm{eff}}}\left(-\frac{1}{48} + h_j + n\right) + \frac{1}{2\pi}\ln \sum_{k=0,\epsilon}\sum_{m\geq 0} p_k(m) e^{-2L(h_k+m)}$$

degeneracy of $e(j,n) = p_j(n)$

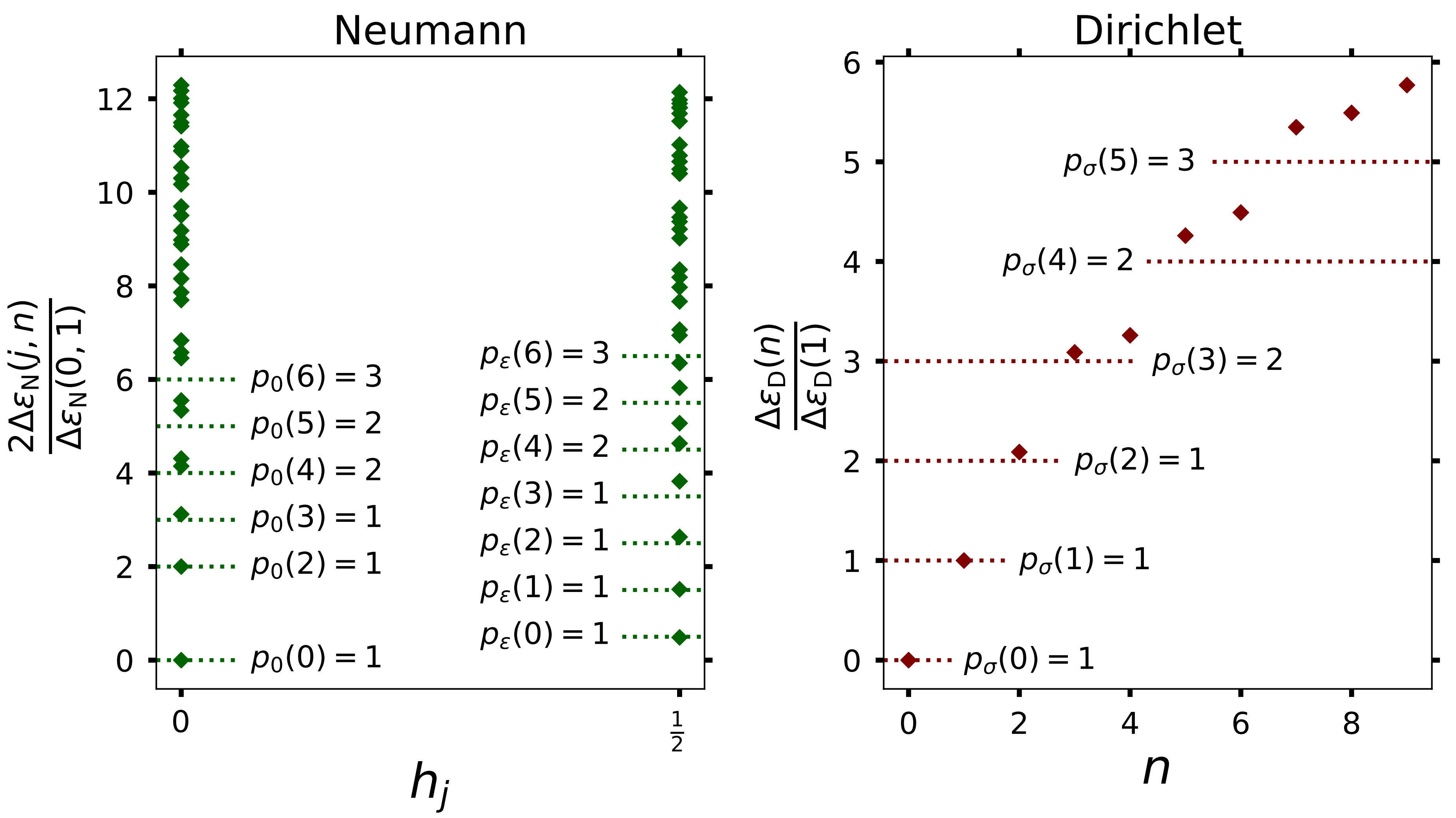
Ishibashi (1988), Cardy (1989), di Francesco *et al* (1997), AR *et al,* J. Stat. Mech (2020)

Entanglement spectrum in the critical Ising chain with boundaries: DMRG Results



AR *et al,* J. Stat. Mech (2020)

Entanglement spectrum in the critical Ising chain with boundaries: DMRG Results



Entire spectrum from the ground state!

AR *et al*, J. Stat. Mech (2020)

Entanglement in the 1+1D free, compact boson CFT

Euclidean action:

$$A = \frac{1}{2\pi K}\int \left(\partial_\mu \phi\right)^2 dx dt\,,$$

$K =$ Luttinger parameter, $\left\langle e^{i\phi(0)} e^{-i\phi(r)} \right\rangle \sim \frac{1}{r^{K/2}}$

$R =$ compactification radius

Boundary states:

$$|D(\phi_0)\rangle = \frac{1}{\sqrt{2R\sqrt{\pi}}} \sum_n e^{-\frac{in\phi_0}{R\sqrt{\pi}}} \prod_{k>0} e^{-\tilde{a}^\dagger_{-k}\tilde{a}^\dagger_k} |n,0\rangle \qquad \text{Dirichlet}$$

$$\left|N\left(\tilde{\phi}_0\right)\right\rangle = \sqrt{R\sqrt{\pi}} \sum_n e^{-\frac{im\tilde{\phi}_0 R}{2\sqrt{\pi}}} \prod_{k>0} e^{+\tilde{a}^\dagger_{-k}\tilde{a}^\dagger_k} |0,m\rangle \qquad \text{Neumann}$$

Callan *et al* (1987), Affleck and Oshikawa (1997)

Entanglement in the free, compact boson CFT: Results

Euclidean action:

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K = Luttinger parameter, $\left\langle e^{i\phi(0)} e^{-i\phi(r)} \right\rangle \sim \frac{1}{r^{K/2}}$

Change in boundary entropy from Neumann to Dirichlet:

$$\Delta S_{N \to D} = \frac{1}{2} \ln \frac{2}{K}$$

degeneracy $p(l)$ = # of integer partitioning of l

Entanglement spectrum (Neumann):

$$\varepsilon_N(k, l) = \varepsilon_N(0,0) + \frac{\pi}{L_{\text{eff}}}\left(\frac{K}{2} k^2 + l\right)$$

exact form available

dim. of primary fields

descendant level

AR *et al,* J Stat. Mech. (2020), see also: Saleur (1998)

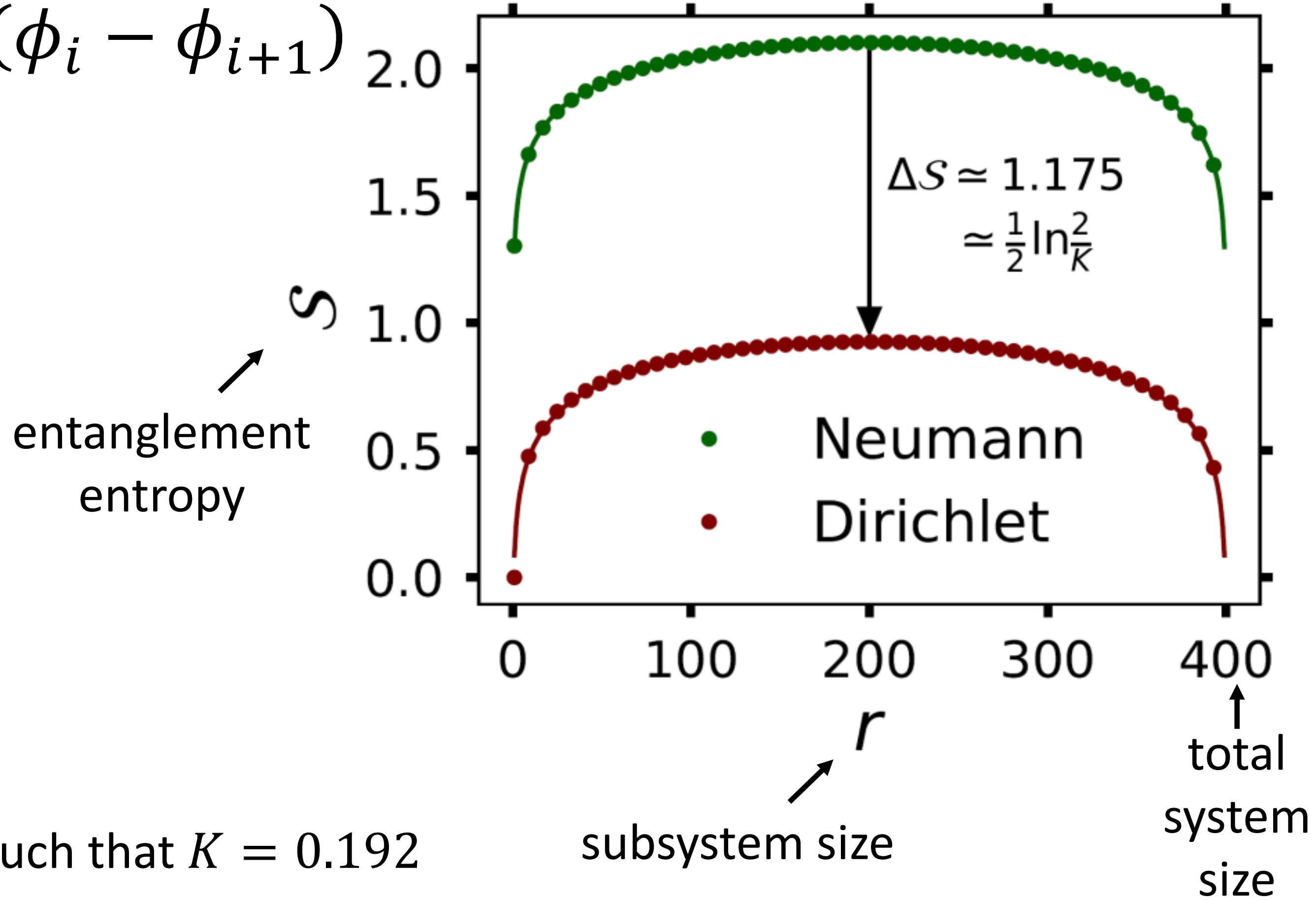
Entanglement in the free, compact boson CFT: DMRG results for the quantum rotor chain

Hamiltonian with nearest-neighbor interaction:

$$[n_i, e^{\pm i\phi_j}] = \pm e^{\pm i\phi_j}\,\delta_{ij}$$

$$H = \sum_i n_i^2 + \epsilon \sum_i n_i n_{i+1} - \lambda \sum_i \cos(\phi_i - \phi_{i+1})$$

Free, compact boson for $\lambda > 1, \epsilon \ll 1$



Chose λ, ϵ such that $K = 0.192$

AR *et al,* J Stat. Mech. (2020)

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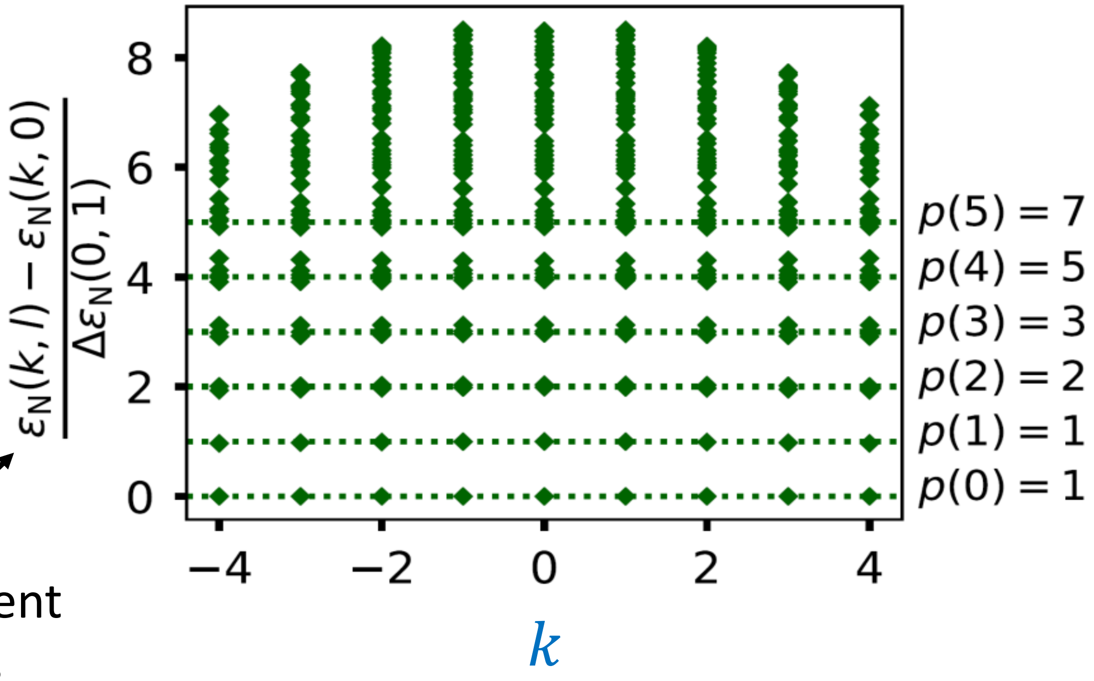
Entanglement spectrum (Neumann):

$$\varepsilon_N(k,l) = \varepsilon_N(0,0) + \frac{\pi}{L_{\text{eff}}}\left(\frac{K}{2}k^2 + l\right)$$

rescaled entanglement energies



Non-generic behavior, characteristic of an integrable model

AR *et al*, J Stat. Mech. (2020)

Chose λ, ϵ such that $K = 0.192$

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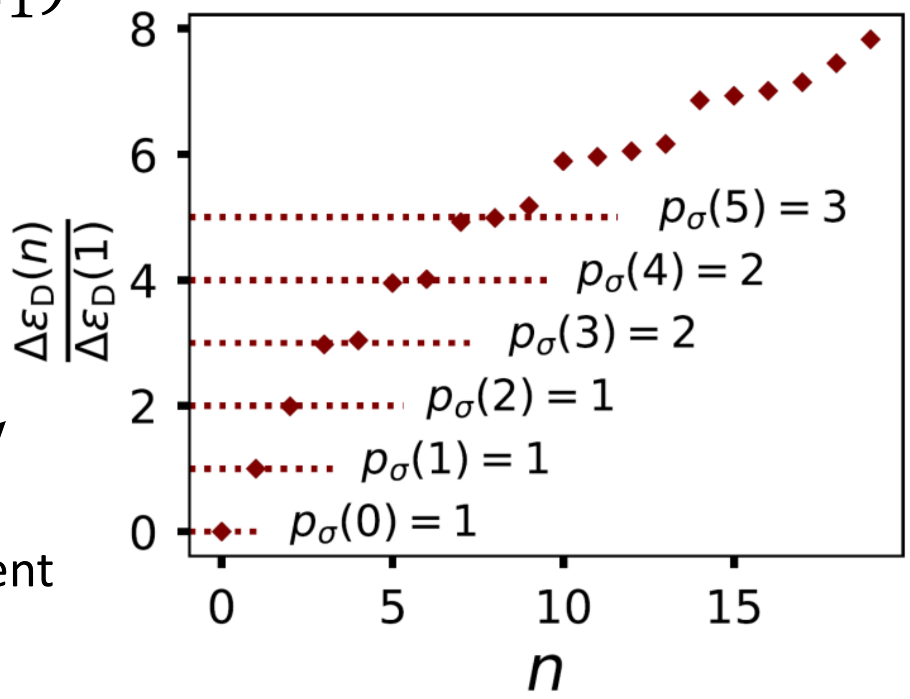
Free, compact boson for $\lambda > 1, \epsilon \ll 1$

Entanglement spectrum (Dirichlet):

$$\varepsilon_D(n) = \varepsilon_D(0) + \frac{\pi}{2L_{\mathrm{eff}}} n$$

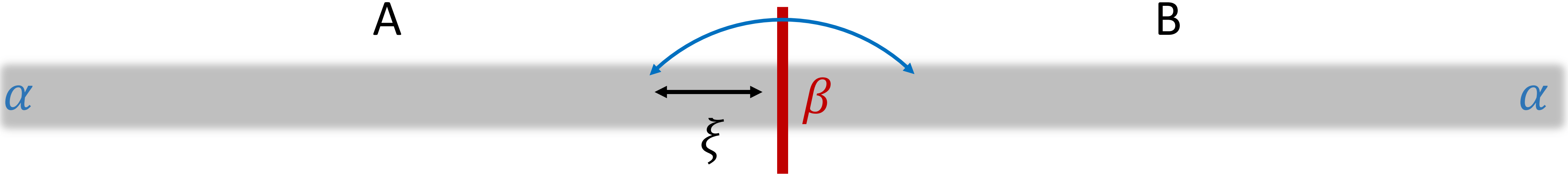
rescaled entanglement energies



AR *et al,* J Stat. Mech. (2020)

Chose λ, ϵ such that $K = 0.192$

Entanglement in 1+1D CFTs perturbed by a primary field



Hamiltonian:

$$H = H_{\mathrm{CFT}} + \lambda' \int dx\, \Phi$$

e.g., Ising chain with $\lambda \neq 1$:

$$H = -\sum_i \sigma_i^x \sigma_{i+1}^x - \lambda \sum_i \sigma_i^z$$

Entanglement entropy:

$$S_A = \frac{c}{6} \ln \xi + \cdots$$

c = central charge, ξ = correlation length

Entanglement Hamiltonian:

$$\mathcal{H}_A(\alpha, \beta) \sim -\frac{1}{2\pi} \ln \frac{e^{-2\pi H_{\Phi\beta}}}{\mathrm{Tr} e^{-2\pi H_{\Phi\beta}}}$$

Entanglement gap: $\varepsilon \sim 1/\ln \xi$

Peschel (1999, 2001), Cho *et al* (2017)

does not require lattice integrability!

Entanglement spectrum of the quantum sine-Gordon model : DMRG results

Hamiltonian with nearest-neighbor interaction:

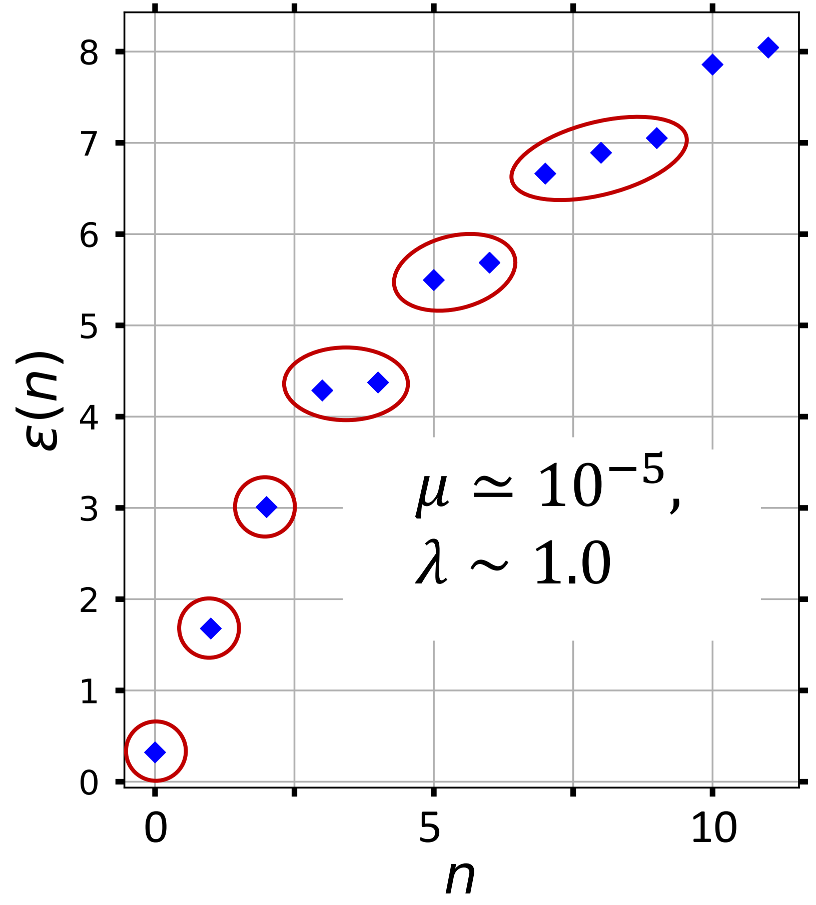
$$H = H_0 + H'$$

$$H_0 = \sum_i n_i^2 + \epsilon \sum_i n_i n_{i+1} - \lambda \sum_i \cos(\phi_i - \phi_{i+1})$$

Free, compact boson for $\lambda > 1, \epsilon \ll 1$

$$H' = -\mu \sum_i \cos\phi_i$$

Predicted degeneracies:
1,1,1,2,2,3,4,…



Lattice integrable regularization: XYZ spin-chain

AR *et al*, Nucl. Phys. B (2021)

The XYZ regularization of the quantum sine-Gordon model

Baxter's XYZ spin-chain

$$H_{\mathrm{XYZ}} = -\frac{1}{2}\sum_{i=1}^{L}\left[J_x\sigma_i^x\sigma_{i+1}^x + J_y\sigma_i^y\sigma_{i+1}^y + J_z\sigma_i^z\sigma_{i+1}^z\right], J_x > J_y \geq |J_z|$$

XYZ to sine-Gordon operator mapping: $\sigma^+ \sim e^{\frac{i\beta\phi}{2}}$

Relation to eight-vertex model parameters (in the principal regime):

$$\Gamma = \frac{J_y}{J_x}, \Delta = \frac{J_z}{J_x},$$

$$\frac{2\sqrt{k}}{1+k} = \sqrt{\frac{1-\Gamma^2}{\Delta^2-\Gamma^2}}, -i\,\mathrm{sn}(i\lambda, k) = \frac{1}{\sqrt{k}}\sqrt{\frac{1-\Gamma}{1+\Gamma}}$$

Baxter (1982), Luther (1975), Lukyanov (1997, 2003)

The XYZ regularization of the quantum sine-Gordon model

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$$\frac{2\sqrt{k}}{1+k} = \sqrt{\frac{1-\Gamma^2}{\Delta^2-\Gamma^2}}, -i\,\mathrm{sn}(i\lambda, k) = \frac{1}{\sqrt{k}}\sqrt{\frac{1-\Gamma}{1+\Gamma}}$$

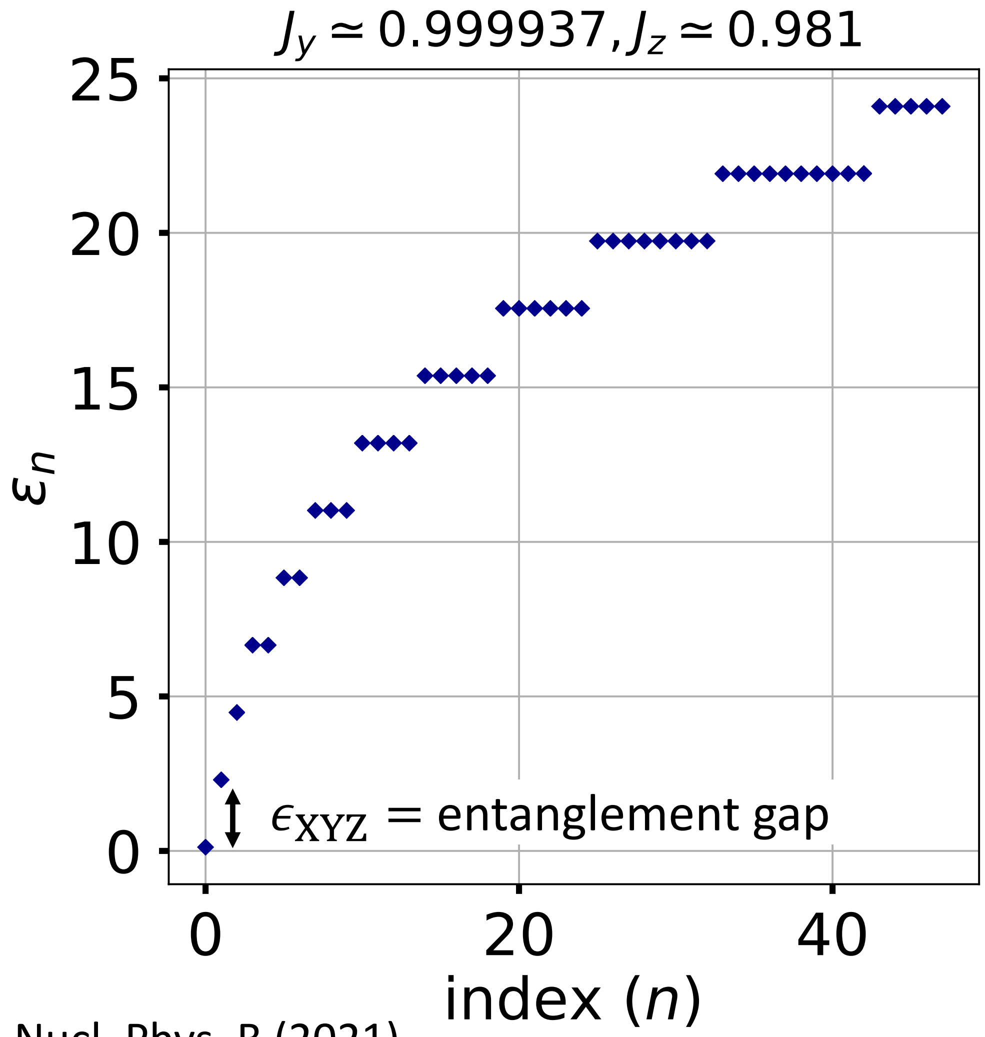
Entanglement spectrum obtained from corner transfer matrix spectrum, level spacing:

$$\varepsilon_{\mathrm{XYZ}} = \frac{\pi\lambda}{I(k)}$$

Baxter (1982)

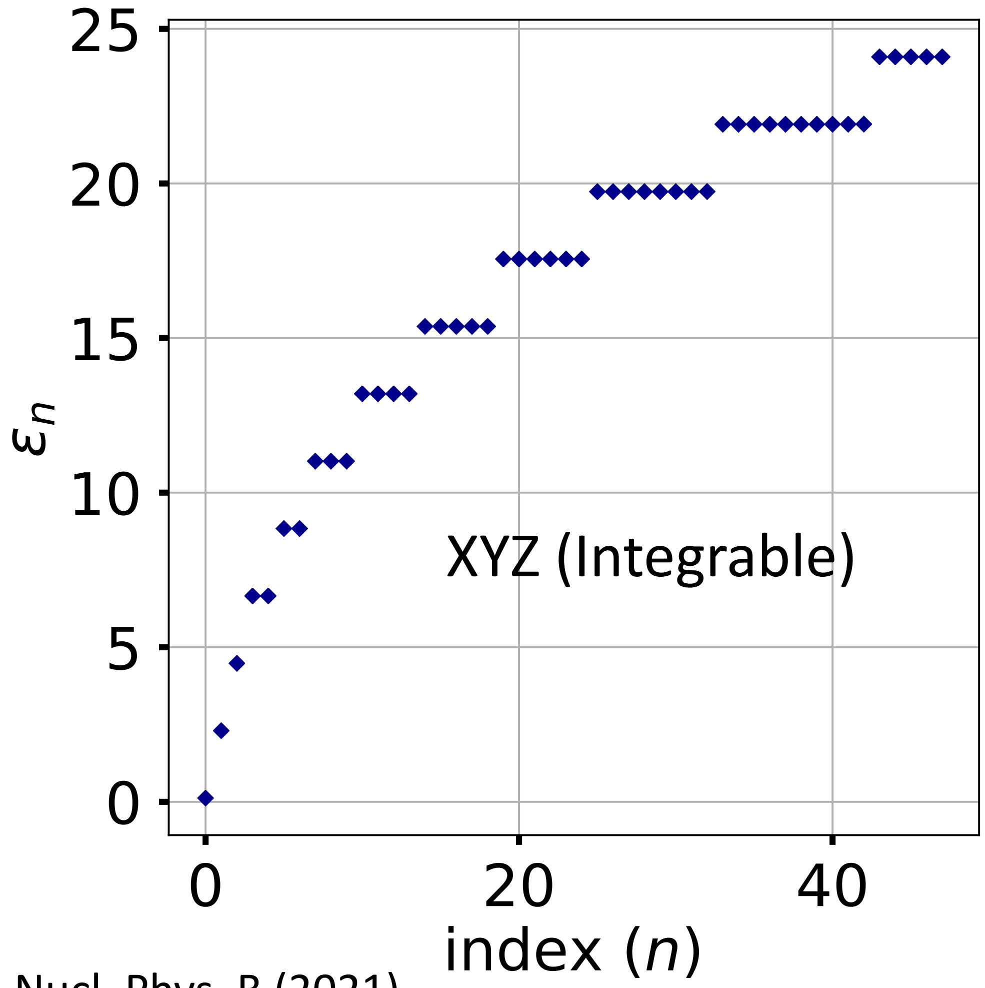
AR *et al,* Nucl. Phys. B (2021)

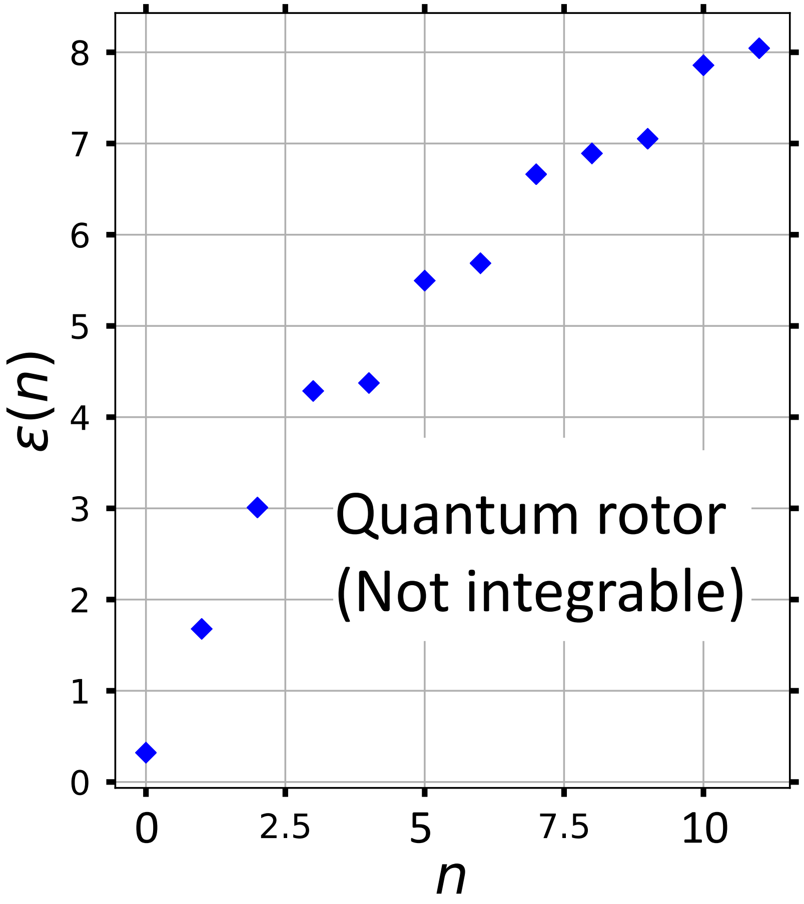
Entanglement spectrum of the quantum sine-Gordon model : DMRG vs corner transfer matrix results



AR *et al,* Nucl. Phys. B (2021)

Entanglement spectrum for integrable vs non-integrable regularizations

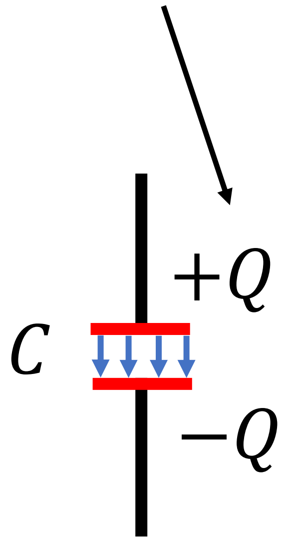




AR *et al*, Nucl. Phys. B (2021)

Outlook: experimental realization with superconducting quantum electronic circuits

electric charge

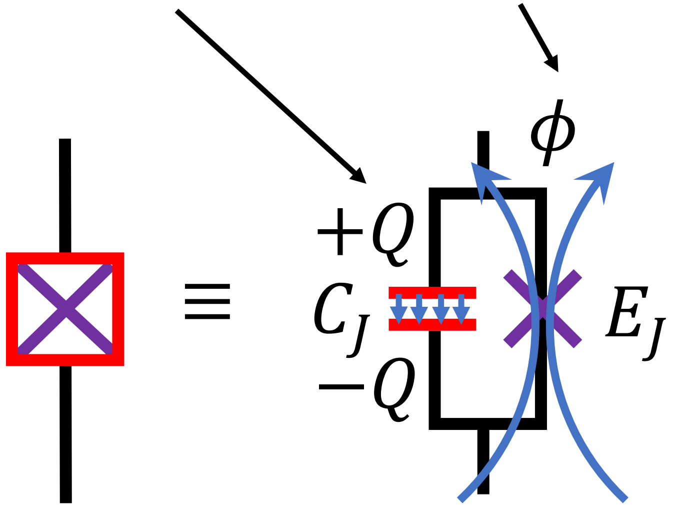


Capacitor

$$H = \frac{Q^2}{2C}$$

electric charge

magnetic flux



Josephson junction

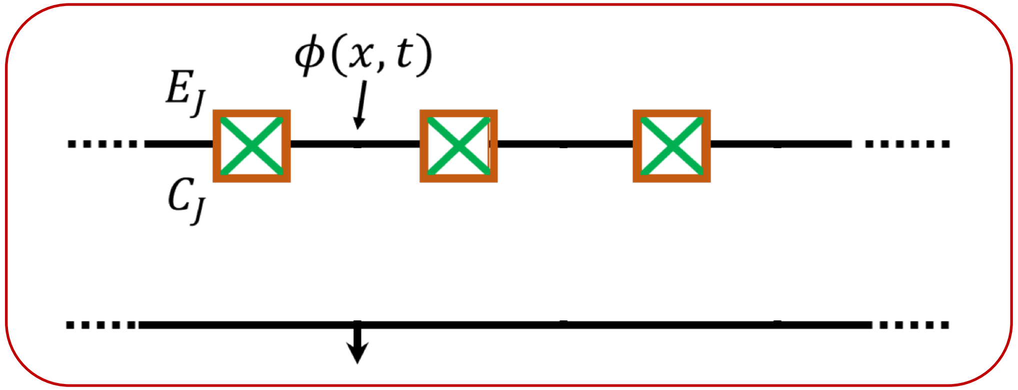
$$H = \frac{Q^2}{2C_J} - E_J \cos\frac{\phi}{\varphi_0}$$

$$[\phi, Q] = i\hbar$$

$\varphi_0 = \hbar/2e$

The free, compact boson CFT with superconducting quantum electronic circuits

Quantum circuit

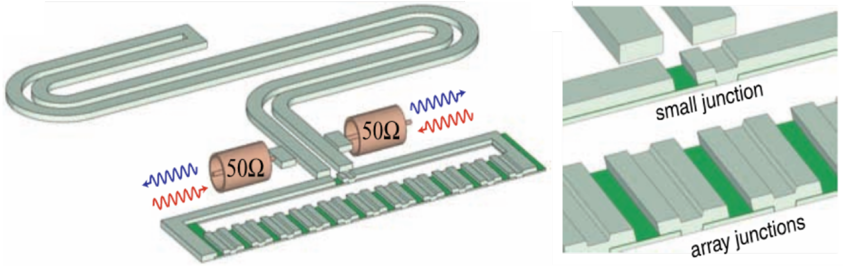


$$H = \epsilon \sum_i n_i n_{i+1} - \lambda \sum_i \cos(\phi_i - \phi_{i+1})$$

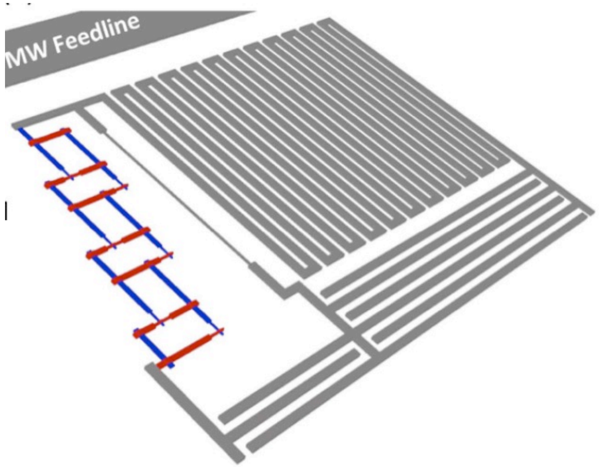
Free, compact boson for $\lambda > 1, \epsilon \ll 1$

AR *et al,* J. Stat. Mech. (2020)

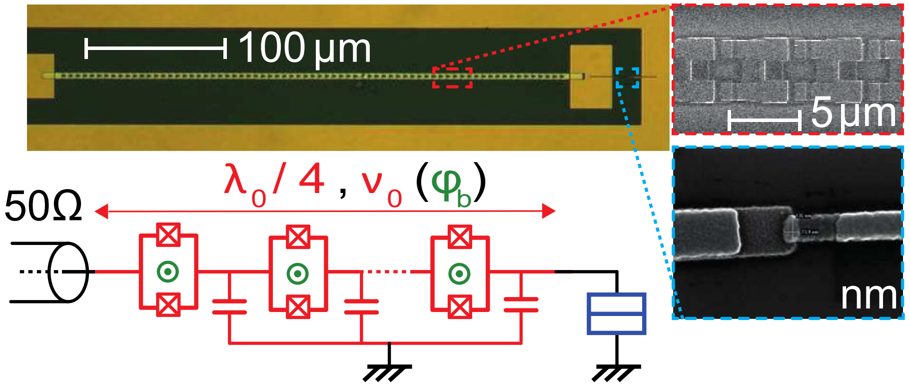
Where are the experiments?



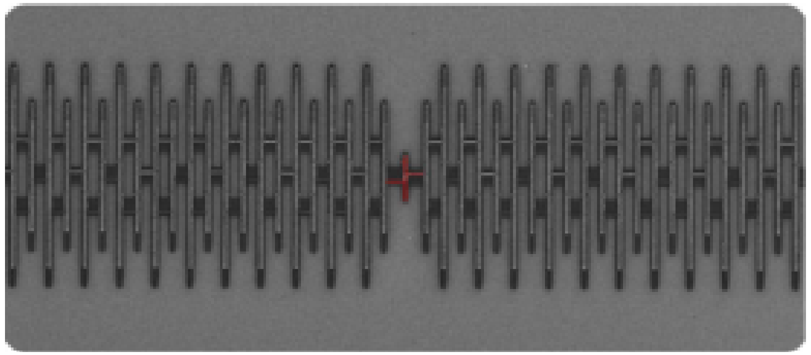
Devoret group (Yale, 2009):
$N = 43$



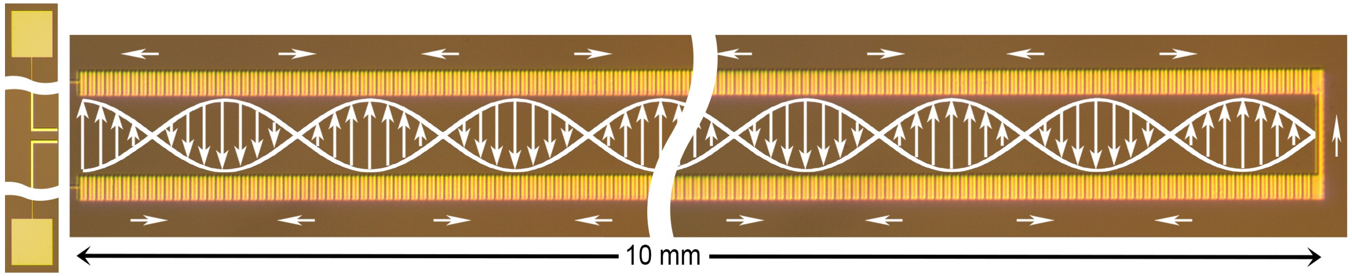
Gershenson group
(Rutgers, 2012): $N = 6,\ 24$



Esteve group (Saclay, 2013):
$N \sim 100$



Roch group (Grenoble , 2019): $N \sim 1500$

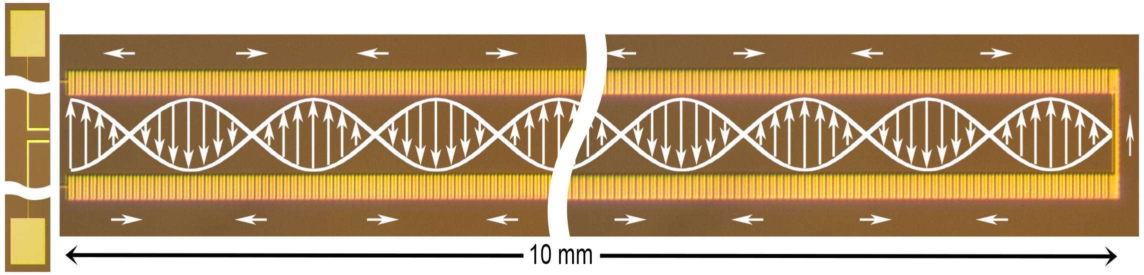


Manucharyan group (Maryland, 2018, 2019):
$N \sim 33000$

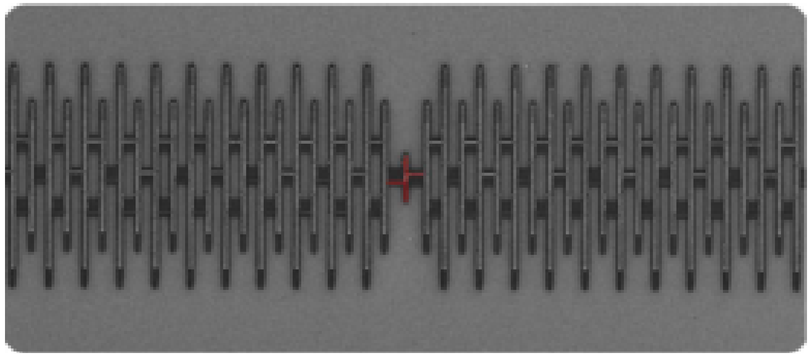
Probing strongly-interacting quantum field theories

Early experiments: Delft (1990-s)

Quantum circuits as analog free boson QFT simulators



Maryland group (2018, 2019): $N \sim 33000$

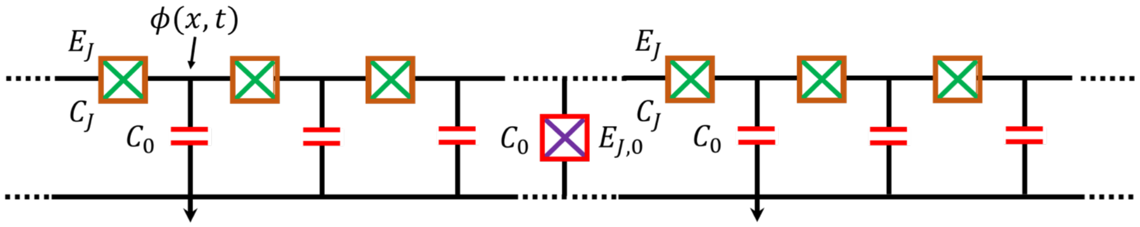


Grenoble group (2019): $N \sim 1500$

Quantum circuit

Maryland group (2018, 2019): $N \sim 33000$

Quantum circuit



More interesting field theories with quantum circuits:
AR and H. Saleur, Phys. Rev. B (2019), AR *et al,* Nucl. Phys. B (2021)

Thank You!

Johannes Hauschild (UC Berkeley)

Frank Pollmann (TU Munich)

Hubert Saleur (CEA Saclay)