Bootstrapping holographic defect correlators

Pedro Liendo

September 29 2021

"Quantum Field Theory at the Boundary"
MITP, JGU

arXiv:2108.13432 with Julien Barrat and Aleix Gimenez-Grau

Motivation

- **CFT**. Conformal field theories are of fundamental importance: they have applications in critical phenomena, string theory, mathematics, etc.

Motivation

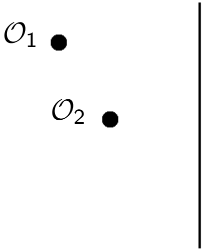
- **CFT**. Conformal field theories are of fundamental importance: they have applications in critical phenomena, string theory, mathematics, etc.
- **Defects**. With the advent of the bootstrap, huge progress has been made on the study of local operators. Less work has been done on extended objects.

Motivation

- **CFT**. Conformal field theories are of fundamental importance: they have applications in critical phenomena, string theory, mathematics, etc.
- **Defects**. With the advent of the bootstrap, huge progress has been made on the study of local operators. Less work has been done on extended objects.
- **Supersymmetry**. The addition of supersymmetry gives us strong analytic control: non-perturbative results, localization, integrability.

Half-BPS observables

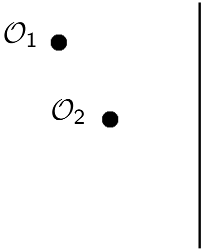
Half-BPS observables in $\mathcal{N} = 4$ SYM



Half-BPS observables in $\mathcal{N} = 4$ SYM

Half-BPS operators in $\mathcal{N} = 4$ SYM

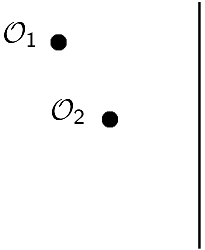
$$SO(4,2) \to SO(2,1) \times SO(3)$$
$$SO(6) \to SO(5)$$
$$PSU(2,2|4) \to OSP(4|4)$$



Half-BPS observables in $\mathcal{N} = 4$ SYM

Half-BPS operators in $\mathcal{N} = 4$ SYM

$$SO(4,2) \rightarrow SO(2,1) \times SO(3)$$
$$SO(6) \rightarrow SO(5)$$
$$PSU(2,2|4) \rightarrow OSP(4|4)$$



Analogous to four-point functions of local operators

The main characters

The main characters

Wilson loop

$$\mathcal{W} = Pe^{\int d\tau (iA_\tau + \phi^6)}$$

$$\langle \mathcal{W} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

Chiral primaries

$$\mathcal{O}_p = \operatorname{Tr} \phi^{\{i_1} \dots \phi^{i_p\}}$$

$$\langle \mathcal{O}_{p_1} \dots \mathcal{O}_{p_n} \rangle$$

The main characters

Wilson loop

$$\mathcal{W} = Pe^{\int d\tau (iA_\tau + \phi^6)}$$

$$\langle \mathcal{W} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

Chiral primaries

$$\mathcal{O}_p = \mathrm{Tr}\, \phi^{\{i_1} \dots \phi^{i_p\}}$$

$$\langle \mathcal{O}_{p_1} \dots \mathcal{O}_{p_n} \rangle$$

- Wilson loops are fundamental in localization.
- Chiral primaries play a key role in holography.

The main characters

Wilson loop

$$\mathcal{W} = Pe^{\int d\tau (iA_\tau + \phi^6)}$$

$$\langle \mathcal{W} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

Chiral primaries

$$\mathcal{O}_p = \operatorname{Tr} \phi^{\{i_1} \ldots \phi^{i_p\}}$$

$$\langle \mathcal{O}_{p_1} \ldots \mathcal{O}_{p_n} \rangle$$

- Wilson loops are fundamental in localization.
- Chiral primaries play a key role in holography.

Less work has been done on mixed correlators

$$\langle \mathcal{W} \mathcal{O}_p \rangle \,, \quad \langle \mathcal{W} \mathcal{O}_{p_1} \mathcal{O}_{p_2} \rangle \,, \quad \ldots$$

Preliminaries

Preliminaries

Wilson line

$$\mathcal{W} = \frac{1}{N}\,\mathcal{P}\exp\int_{-\infty}^{\infty} d\tau\,\left(i\dot{x}^{\mu}A_{\mu} + |\dot{x}|\theta_i\phi^i\right)$$

The extra scalar ensures this object is half-BPS.

Preliminaries

Wilson line

$$\mathcal{W} = \frac{1}{N}\,\mathcal{P}\exp\int_{-\infty}^{\infty} d\tau\,\left(i\dot{x}^\mu A_\mu + |\dot{x}|\theta_i\phi^i\right)$$

The extra scalar ensures this object is half-BPS.

The "20 prime"

$$\mathcal{O}_2(x) = u_i u_j \mathrm{Tr}\phi^i(x)\phi^j(x) \qquad (\mathcal{B}_{[0,2,0]})$$

Protected by supersymmetry, also half-BPS

Preliminaries

Wilson line

$$\mathcal{W} = \frac{1}{N}\,\mathcal{P}\exp\int_{-\infty}^{\infty} d\tau\,\left(i\dot{x}^{\mu}A_{\mu} + |\dot{x}|\theta_i\phi^i\right)$$

The extra scalar ensures this object is half-BPS.

The "20 prime"

$$\mathcal{O}_2(x) = u_i u_j \mathrm{Tr}\phi^i(x)\phi^j(x) \qquad (\mathcal{B}_{[0,2,0]})$$

Protected by supersymmetry, also half-BPS

We will focus on

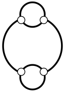
$$\langle \mathcal{W}\,\mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\rangle = \frac{(u_1\cdot\theta)^2(u_2\cdot\theta)^2}{x_1^2 x_2^2}\mathcal{F}(z,\bar{z},\omega)$$

R-symmetry channels

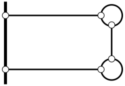
What is "ω"?

R-symmetry channels

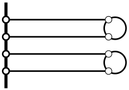
What is "ω"?



$(u_1 \cdot u_2)^2$



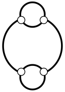
$(u_1 \cdot u_2)(u_1 \cdot \theta)(u_2 \cdot \theta)$



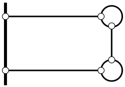
$(u_1 \cdot \theta)^2(u_2 \cdot \theta)^2$

R-symmetry channels

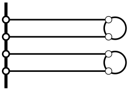
What is "ω"?



$(u_1 \cdot u_2)^2$



$(u_1 \cdot u_2)(u_1 \cdot \theta)(u_2 \cdot \theta)$



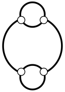
$(u_1 \cdot \theta)^2(u_2 \cdot \theta)^2$

Let's define

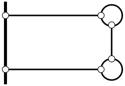
$$\frac{4\omega}{(1-\omega)^2} = -\frac{(u_1 \cdot \theta)(u_2 \cdot \theta)}{(u_1 \cdot u_2)}\,, \quad \sigma = \frac{(1-\omega)^2}{4\omega}$$

R-symmetry channels

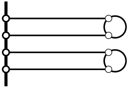
What is "ω"?



$(u_1 \cdot u_2)^2$



$(u_1 \cdot u_2)(u_1 \cdot \theta)(u_2 \cdot \theta)$



$(u_1 \cdot \theta)^2 (u_2 \cdot \theta)^2$

Let's define

$$\frac{4\omega}{(1-\omega)^2} = -\frac{(u_1 \cdot \theta)(u_2 \cdot \theta)}{(u_1 \cdot u_2)}\,, \quad \sigma = \frac{(1-\omega)^2}{4\omega}$$

Then,

$$\mathcal{F}(z,\bar{z},\omega) = \sigma^2 F_0(z,\bar{z}) + \sigma F_1(z,\bar{z}) + F_2(z,\bar{z})$$

Ward identities

Recall

$$\langle \mathcal{W}\, \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\rangle = \frac{(u_1\cdot\theta)^2(u_2\cdot\theta)^2}{x_1^2x_2^2}\mathcal{F}(z,\bar{z},\omega)$$

Ward identities

Recall

$$\langle \mathcal{W}\, \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\rangle = \frac{(u_1\cdot\theta)^2(u_2\cdot\theta)^2}{x_1^2 x_2^2}\mathcal{F}(z,\bar{z},\omega)$$

The channels are not independent [PL, Meneghelli (2016)]

$$\left(\partial_z + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z,\bar{z},\omega)\Big|_{z=\omega} = 0\,,$$
$$\left(\partial_{\bar{z}} + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z,\bar{z},\omega)\Big|_{\bar{z}=\omega} = 0\,.$$

Ward identities

Recall

$$\langle \mathcal{W}\, \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\rangle = \frac{(u_1\cdot\theta)^2(u_2\cdot\theta)^2}{x_1^2 x_2^2}\mathcal{F}(z,\bar{z},\omega)$$

The channels are not independent [PL, Meneghelli (2016)]

$$\left(\partial_z + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z,\bar{z},\omega)\bigg|_{z=\omega} = 0\,,$$
$$\left(\partial_{\bar{z}} + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z,\bar{z},\omega)\bigg|_{\bar{z}=\omega} = 0\,.$$

See [Barrat, PL, Plefka (2020)] for a weak-coupling analysis.

Intro to Defect CFT

Defect CFT

Extended objects include Wilson and ’t Hoft lines, surface operators, boundaries, interfaces, . . .

Defect CFT

Extended objects include Wilson and 't Hoft lines, surface operators, boundaries, interfaces, ...

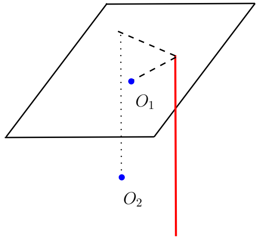


Figure: Local operatos in the presence of a defect.

Defect CFT

Extended objects include Wilson and 't Hoft lines, surface operators, boundaries, interfaces, ...

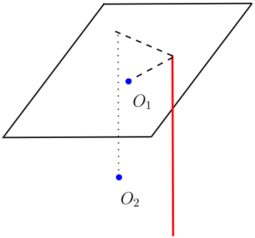


Figure: Local operatos in the presence of a defect.

We have $SO(4,2) \to SO(2,1) \times SO(3)$.

Defect CFT correlators

The $SO(4,2) \times SO(3)$ symmetry preserved by the defect implies that one-point functions are non-zero:

$$\langle \mathcal{O}(x) \rangle = \frac{a_{\mathcal{O}}}{(x^i)^{\Delta}} \, .$$

Defect CFT correlators

The $SO(4,2) \times SO(3)$ symmetry preserved by the defect implies that one-point functions are non-zero:

$$\langle \mathcal{O}(x) \rangle = \frac{a_{\mathcal{O}}}{(x^i)^{\Delta}}\,.$$

Two-point functions depend on two conformal invariants

$$\langle \phi(x_1)\phi(x_2) \rangle = \frac{1}{(z\bar{z})^{\Delta_{\phi/2}}} g(z,\bar{z})\,,$$

where $\bar{z} = z^*$ in Euclidean signature

Defect CFT correlators

The $SO(4,2) \times SO(3)$ symmetry preserved by the defect implies that one-point functions are non-zero:

$$\langle \mathcal{O}(x) \rangle = \frac{a_{\mathcal{O}}}{(x^i)^{\Delta}} \, .$$

Two-point functions depend on two conformal invariants

$$\langle \phi(x_1)\phi(x_2) \rangle = \frac{1}{(z\bar{z})^{\Delta_{\phi/2}}} g(z,\bar{z}) \, ,$$

where $\bar{z} = z^*$ in Euclidean signature

Remark. Compare with the four-point function in the bulk CFT.

Two-point function configuration

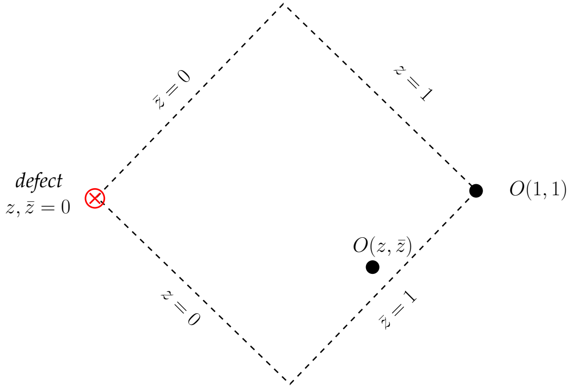


Figure: Configuration of the system in the plane orthogonal to the defect.

Conformal blocks and crossing symmetry

We have two operator expansions

Conformal blocks and crossing symmetry

We have two operator expansions

$$\phi(x)\phi(0) \sim \sum_{O} C_{\phi\phi O} d(x,\partial) O(0)$$

Conformal blocks and crossing symmetry

We have two operator expansions

$$\phi(x)\phi(0) \sim \sum_{O} C_{\phi\phi O} d(x, \partial) O(0)$$
$$\phi(x) \sim \sum_{\hat{O}} b_{\phi\hat{O}} D(x^i, \partial_{\vec{x}}) \hat{O}(\vec{x})$$

Conformal blocks and crossing symmetry

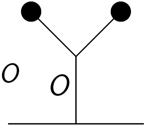
We have two operator expansions

$$\phi(x)\phi(0) \sim \sum_{O} C_{\phi\phi O}\, d(x,\partial)\, O(0)$$

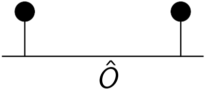
$$\phi(x) \sim \sum_{\hat{O}} b_{\phi\hat{O}}\, D(x^i, \partial_{\vec{x}})\, \hat{O}(\vec{x})$$

Equality of both expansions gives crossing symmetry

$$\sum_{\Delta,\ell} C_{\phi\phi O}\, a_O$$



$$= \sum_{\widehat{\Delta},s} b^2_{\phi\widehat{O}}$$



Superconformal blocks

The two-point function has two decompositions

$$\mathcal{F}(z,\bar{z},\omega) \sim \sum_{\chi} a_{\chi}\, C_{\mathcal{O}\mathcal{O}\chi}\, \mathcal{G}_{\chi}(z,\bar{z},\omega)$$

$$\mathcal{F}(z,\bar{z},\omega) \sim \sum_{\hat{\chi}} b^2_{\mathcal{O}\hat{\chi}}\, \hat{\mathcal{G}}_{\hat{\chi}}(z,\bar{z},\omega)$$

Superconformal blocks

The two-point function has two decompositions

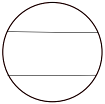
$$\mathcal{F}(z,\bar{z},\omega) \sim \sum_{\chi} a_\chi \, C_{\mathcal{O}\mathcal{O}\chi} \, \mathcal{G}_\chi(z,\bar{z},\omega)$$
$$\mathcal{F}(z,\bar{z},\omega) \sim \sum_{\hat{\chi}} b^2_{\mathcal{O}\hat{\chi}} \, \hat{\mathcal{G}}_{\hat{\chi}}(z,\bar{z},\omega)$$

Here $\mathcal{G}$s are superblocks:

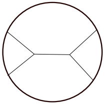
$$\mathcal{G}_\chi(z,\bar{z},\omega) \sim \sum h_K(\omega) f_{\Delta,\ell}(z,\bar{z}) \qquad (h_K : SO(6)_R)$$
$$\hat{\mathcal{G}}_\chi(z,\bar{z},\omega) \sim \sum \hat{h}_{\hat{K}}(\omega) \hat{f}_{\hat{\Delta},s}(z,\bar{z}) \qquad (\hat{h}_{\hat{K}} : SO(5)_R)$$

Strong coupling

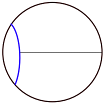
Strong coupling



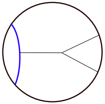
$\sim N^0$



$\sim \frac{1}{N^2}$

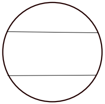


$\sim \frac{\lambda^{1/2}}{N}$

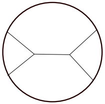


$\sim \frac{\lambda^{1/2}}{N^2}$

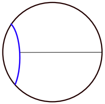
Strong coupling



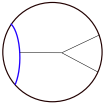
$\sim N^0$



$\sim \frac{1}{N^2}$



$\sim \frac{\lambda^{1/2}}{N}$

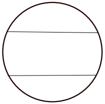


$\sim \frac{\lambda^{1/2}}{N^2}$

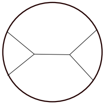
The natural parameters are λ/N^2 and $1/\sqrt{\lambda}$:

$$\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle = \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left(\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}}\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(2)}\right)\ldots$$

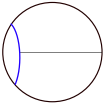
Strong coupling



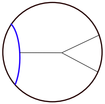
$\sim N^0$



$\sim \frac{1}{N^2}$



$\sim \frac{\lambda^{1/2}}{N}$



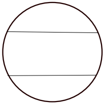
$\sim \frac{\lambda^{1/2}}{N^2}$

The natural parameters are λ/N^2 and $1/\sqrt{\lambda}$:

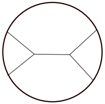
$$\langle\!\langle\, \mathcal{O}\mathcal{O}\,\rangle\!\rangle = \langle\!\langle\, \mathcal{O}\mathcal{O}\,\rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left(\langle\!\langle\, \mathcal{O}\mathcal{O}\,\rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}}\langle\!\langle\, \mathcal{O}\mathcal{O}\,\rangle\!\rangle^{(2)}\right)\dots$$

For the holographic setup see [Giombi, Pestun (2012)].

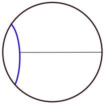
Strong coupling



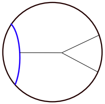
$\sim N^0$



$\sim \frac{1}{N^2}$



$\sim \frac{\lambda^{1/2}}{N}$



$\sim \frac{\lambda^{1/2}}{N^2}$

The natural parameters are λ/N^2 and $1/\sqrt{\lambda}$:

$$\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle = \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left(\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}}\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(2)}\right)\ldots$$

For the holographic setup see [Giombi, Pestun (2012)].

Today we are going to do bootstrap!

Inversion formula

Inversion formula

Following [Caron-Huot (2017)],

$$b(\widehat{\Delta}', s) \sim \int dz d\bar{z}\, J(z, \bar{z})\, \text{Disc}\, \mathcal{F}(z, \bar{z})$$

Inversion formula

Following [Caron-Huot (2017)],

$$b(\widehat{\Delta}', s) \sim \int dz d\bar{z}\, J(z, \bar{z})\, \text{Disc}\, \mathcal{F}(z, \bar{z})$$

where

$$b^2_{\mathcal{O}\hat{\mathcal{O}}} = -\operatorname*{Res}_{\widehat{\Delta}'=\widehat{\Delta}} b(\widehat{\Delta}', s) \qquad (\text{for } s > s^*)$$

Inversion formula

Following [Caron-Huot (2017)],

$$b(\widehat{\Delta}', s) \sim \int dz d\bar{z}\, J(z, \bar{z})\, \text{Disc}\, \mathcal{F}(z, \bar{z})$$

where

$$b^2_{\mathcal{O}\hat{\mathcal{O}}} = - \operatorname*{Res}_{\widehat{\Delta}'=\widehat{\Delta}} b(\widehat{\Delta}', s) \qquad \text{(for } s > s^*\text{)}$$

and

$$\text{Disc}\, \mathcal{F}(z, \bar{z}) = \mathcal{F}^{\circlearrowleft}(z, \bar{z}) - \mathcal{F}^{\circlearrowright}(z, \bar{z})$$

Inversion formula

Following [Caron-Huot (2017)],

$$b(\widehat{\Delta}', s) \sim \int dz d\bar{z}\, J(z,\bar{z})\, \text{Disc}\, \mathcal{F}(z,\bar{z})$$

where

$$b^2_{\mathcal{O}\hat{\mathcal{O}}} = -\underset{\widehat{\Delta}'=\widehat{\Delta}}{\mathrm{Res}}\, b(\widehat{\Delta}', s) \qquad \text{(for } s > s^*\text{)}$$

and

$$\text{Disc}\, \mathcal{F}(z,\bar{z}) = \mathcal{F}^{\circlearrowleft}(z,\bar{z}) - \mathcal{F}^{\circlearrowright}(z,\bar{z})$$

Around $\bar{z} = 1$ with z and ω fixed.

Inversion formula

Following [Caron-Huot (2017)],

$$b(\widehat{\Delta}', s) \sim \int dz d\bar{z}\, J(z,\bar{z})\, \mathrm{Disc}\, \mathcal{F}(z,\bar{z})$$

where

$$b^2_{\mathcal{O}\hat{\mathcal{O}}} = -\operatorname*{Res}_{\widehat{\Delta}'=\widehat{\Delta}} b(\widehat{\Delta}', s) \qquad \text{(for } s > s^*\text{)}$$

and

$$\mathrm{Disc}\, \mathcal{F}(z,\bar{z}) = \mathcal{F}^{\circlearrowleft}(z,\bar{z}) - \mathcal{F}^{\circlearrowright}(z,\bar{z})$$

Around $\bar{z} = 1$ with z and ω fixed.

[Lemos, PL, Meineri, Sarkar (2018)]

Strong coupling spectrum

The bulk expansion is

$$\mathcal{F}(z,\bar{z},\omega) \sim 1 + a_{\mathcal{O}_2}\, C_{\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2}\, \mathcal{G}_{\mathcal{O}_2}(z,\bar{z},\omega) + \sum_{\chi} a_{\chi}\, C_{\mathcal{O}_2\mathcal{O}_2\chi}\, \mathcal{G}_{\chi}(z,\bar{z},\omega)$$

Strong coupling spectrum

The bulk expansion is

$$\mathcal{F}(z,\bar{z},\omega) \sim 1 + a_{\mathcal{O}_2}\, C_{\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2}\, \mathcal{G}_{\mathcal{O}_2}(z,\bar{z},\omega) + \sum_{\chi} a_{\chi}\, C_{\mathcal{O}_2\mathcal{O}_2\chi}\, \mathcal{G}_{\chi}(z,\bar{z},\omega)$$

No anomalous dimensions at this order:

$$\Delta_{\chi} = 4 + 2n + \ell + \frac{1}{N^2}\left(a + \frac{b}{\lambda^{3/2}}\right) + \ldots$$

[Goncalves (2015)]

Strong coupling spectrum

The bulk expansion is

$$\mathcal{F}(z,\bar{z},\omega) \sim 1 + a_{\mathcal{O}_2}\, C_{\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2}\, \mathcal{G}_{\mathcal{O}_2}(z,\bar{z},\omega) + \sum_{\chi} a_{\chi}\, C_{\mathcal{O}_2\mathcal{O}_2\chi}\, \mathcal{G}_{\chi}(z,\bar{z},\omega)$$

No anomalous dimensions at this order:

$$\Delta_{\chi} = 4 + 2n + \ell + \frac{1}{N^2}\left(a + \frac{b}{\lambda^{3/2}}\right) + \ldots$$

[Goncalves (2015)]

Disc $\mathcal{G}_{\chi}(z,\bar{z},\omega) = 0$,

Strong coupling spectrum

The bulk expansion is

$$\mathcal{F}(z,\bar{z},\omega) \sim 1 + a_{\mathcal{O}_2}\, C_{\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2}\, \mathcal{G}_{\mathcal{O}_2}(z,\bar{z},\omega) + \sum_{\chi} a_\chi\, C_{\mathcal{O}_2\mathcal{O}_2\chi}\, \mathcal{G}_\chi(z,\bar{z},\omega)$$

No anomalous dimensions at this order:

$$\Delta_\chi = 4 + 2n + \ell + \frac{1}{N^2}\left(a + \frac{b}{\lambda^{3/2}}\right) + \dots$$

[Goncalves (2015)]

Disc $\mathcal{G}_\chi(z,\bar{z},\omega) = 0$, this implies

$$b(\widehat{\Delta}', s) \sim \int dz d\bar{z}\, J(z,\bar{z})\, \text{Disc}\; \mathcal{G}_{\mathcal{O}_2}(z,\bar{z},\omega)$$

Strong coupling spectrum

The bulk expansion is

$$\mathcal{F}(z,\bar{z},\omega) \sim 1 + a_{\mathcal{O}_2}\, C_{\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2}\, \mathcal{G}_{\mathcal{O}_2}(z,\bar{z},\omega) + \sum_{\chi} a_\chi\, C_{\mathcal{O}_2\mathcal{O}_2\chi}\, \mathcal{G}_\chi(z,\bar{z},\omega)$$

No anomalous dimensions at this order:

$$\Delta_\chi = 4 + 2n + \ell + \frac{1}{N^2}\left(a + \frac{b}{\lambda^{3/2}}\right) + \dots$$

[Goncalves (2015)]

Disc $\mathcal{G}_\chi(z,\bar{z},\omega) = 0$, this implies

$$b(\widehat{\Delta}', s) \sim \int dz d\bar{z}\, J(z,\bar{z})\, \text{Disc}\, \mathcal{G}_{\mathcal{O}_2}(z,\bar{z},\omega)$$

The CFT data can be reconstructed from a single block!

First order correction

At leading order

$$\widehat{\Delta} = 2 + s\,, \qquad b_{2+s,s} = 1 + s$$

First order correction

At leading order

$$\widehat{\Delta} = 2 + s\,, \qquad b_{2+s,s} = 1 + s$$

The first correction

$$b_{2+s,s}\gamma_{\widehat{\Delta}} = \frac{1+2s}{16s}\,, \qquad \delta b_{2+s,s} = \frac{(2+s)(1+6s+2s^2)}{32(1+s)^2(5+2s)}$$

First order correction

At leading order

$$\widehat{\Delta} = 2 + s\,, \qquad b_{2+s,s} = 1 + s$$

The first correction

$$b_{2+s,s}\gamma_{\widehat{\Delta}} = \frac{1+2s}{16s}\,, \qquad \delta b_{2+s,s} = \frac{(2+s)(1+6s+2s^2)}{32(1+s)^2(5+2s)}$$

We can now do the sum

$$\mathcal{F}(z,\bar{z},\omega) = \sum_{\hat{\Delta},s} \delta b^2_{\mathcal{O}\hat{\Delta}}\, \hat{\mathcal{G}}_{\hat{\Delta}}(z,\bar{z},\omega) + \sum_{\hat{\Delta},s} b^2_{\mathcal{O}\hat{\chi}} \gamma_{\hat{\Delta}}\, \frac{\partial}{\partial \hat{\Delta}} \hat{\mathcal{G}}_{\hat{\Delta}}(z,\bar{z},\omega)$$

After resummation

After resummation

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

After resummation

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

$$F_1(z,\bar{z}) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar{z}}{(1-z)(1-\bar{z})(1-z\bar{z})}$$

After resummation

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

$$F_1(z,\bar{z}) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar{z}}{(1-z)(1-\bar{z})(1-z\bar{z})}$$

$$F_2(z,\bar{z}) = 0$$

After resummation

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

$$F_1(z,\bar{z}) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar{z}}{(1-z)(1-\bar{z})(1-z\bar{z})}$$

$$F_2(z,\bar{z}) = 0$$

This result is not supersymmetric!

After resummation

$$F_0(z,\bar z) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar z}{(1-z)(1-\bar z)}\left[\frac{1+z\bar z}{(1-z\bar z)^2}+\frac{2z\bar z\log z\bar z}{(1-z\bar z)^3}\right]$$

$$F_1(z,\bar z) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar z}{(1-z)(1-\bar z)(1-z\bar z)}$$

$$F_2(z,\bar z) = 0$$

This result is not supersymmetric!

Improved strategy:

- Add blocks with spin $s = 0, 1$

After resummation

$$F_0(z,\bar z) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar z}{(1-z)(1-\bar z)}\left[\frac{1+z\bar z}{(1-z\bar z)^2}+\frac{2z\bar z\log z\bar z}{(1-z\bar z)^3}\right]$$

$$F_1(z,\bar z) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar z}{(1-z)(1-\bar z)(1-z\bar z)} + \text{(spin 0)}$$

$$F_2(z,\bar z) = 0$$

This result is not supersymmetric!

Improved strategy:

- Add blocks with spin $s = 0, 1$

After resummation

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

$$F_1(z,\bar{z}) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar{z}}{(1-z)(1-\bar{z})(1-z\bar{z})}+\text{(spin 0)}$$

$$F_2(z,\bar{z}) = 0+\text{(spin 0 and 1)}$$

This result is not supersymmetric!

Improved strategy:

- Add blocks with spin $s=0,1$

After resumption

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

$$F_1(z,\bar{z}) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar{z}}{(1-z)(1-\bar{z})(1-z\bar{z})} + \text{(spin 0)}$$

$$F_2(z,\bar{z}) = 0 + \text{(spin 0 and 1)}$$

This result is not supersymmetric!

Improved strategy:

- Add blocks with spin $s = 0, 1$
- Impose $\left(\partial_z + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z,\bar{z},\omega)\big|_{z=\omega} = 0$

After resummation

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

$$F_1(z,\bar{z}) = 2\frac{\sqrt{\lambda}}{N^2}\frac{z\bar{z}}{(1-z)(1-\bar{z})(1-z\bar{z})} + \text{(spin 0)}$$

$$F_2(z,\bar{z}) = 0 + \text{(spin 0 and 1)}$$

This result is not supersymmetric!

Improved strategy:

- Add blocks with spin $s = 0, 1$
- Impose $\left(\partial_z + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z,\bar{z},\omega)\big|_{z=\omega} = 0$
- Consistency with the bulk expansion

The final result

The final result

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

The final result

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

$$F_1(z,\bar{z}) = \frac{\sqrt{\lambda}}{N^2}\left[-2\log(1+\sqrt{z\bar{z}})-\frac{2z\bar{z}}{(1-z\bar{z})^2}\right.$$
$$\left.-\frac{z\bar{z}\big(5z\bar{z}-2z^2\bar{z}^2+z^3\bar{z}^3-(z+\bar{z})(2-z\bar{z}+z^2\bar{z}^2)\big)\log z\bar{z}}{(1-z)(1-\bar{z})(1-z\bar{z})^3}\right]$$

The final result

$$F_0(z,\bar z) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar z}{(1-z)(1-\bar z)}\left[\frac{1+z\bar z}{(1-z\bar z)^2}+\frac{2z\bar z\log z\bar z}{(1-z\bar z)^3}\right]$$

$$F_1(z,\bar z) = \frac{\sqrt{\lambda}}{N^2}\left[-2\log(1+\sqrt{z\bar z}) - \frac{2z\bar z}{(1-z\bar z)^2}\right.$$
$$\left. - \frac{z\bar z\big(5z\bar z - 2z^2\bar z^2 + z^3\bar z^3 - (z+\bar z)(2 - z\bar z + z^2\bar z^2)\big)\log z\bar z}{(1-z)(1-\bar z)(1-z\bar z)^3}\right]$$

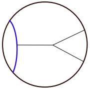
$$F_2(z,\bar z) = \frac{\sqrt{\lambda}}{8N^2}\left[-3 - \frac{2(z+\bar z)}{\sqrt{z\bar z}} + \frac{(z+\bar z)(1+z\bar z) - 4z\bar z}{(1-z\bar z)^2}\right.$$
$$+\frac{2\big((z+\bar z)(1+z\bar z) - 4z\bar z\big)\log(1+\sqrt{z\bar z})}{z\bar z}$$
$$\left.+\frac{z\bar z\big((z+\bar z)(3-2z\bar z+z^2\bar z^2) - 6 + 6z\bar z - 4z^2\bar z^2\big)\log z\bar z}{(1-z\bar z)^3}\right]$$

The final result

$$F_0(z,\bar z) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar z}{(1-z)(1-\bar z)}\left[\frac{1+z\bar z}{(1-z\bar z)^2}+\frac{2z\bar z\log z\bar z}{(1-z\bar z)^3}\right]$$

$$F_1(z,\bar z) = \frac{\sqrt{\lambda}}{N^2}\left[-2\log(1+\sqrt{z\bar z}) - \frac{2z\bar z}{(1-z\bar z)^2} - \frac{z\bar z\big(5z\bar z - 2z^2\bar z^2 + z^3\bar z^3 - (z+\bar z)(2 - z\bar z + z^2\bar z^2)\big)\log z\bar z}{(1-z)(1-\bar z)(1-z\bar z)^3}\right]$$

$$F_2(z,\bar z) = \frac{\sqrt{\lambda}}{8N^2}\left[-3 - \frac{2(z+\bar z)}{\sqrt{z\bar z}} + \frac{(z+\bar z)(1+z\bar z)-4z\bar z}{(1-z\bar z)^2} + \frac{2\big((z+\bar z)(1+z\bar z)-4z\bar z\big)\log(1+\sqrt{z\bar z})}{z\bar z} + \frac{z\bar z\big((z+\bar z)(3-2z\bar z+z^2\bar z^2) - 6 + 6z\bar z - 4z^2\bar z^2\big)\log z\bar z}{(1-z\bar z)^3}\right]$$



$$\sim \frac{\lambda^{1/2}}{N^2}$$

Consistency checks

Consistency checks

- It has the correct Disc $\mathcal{F}(z,\bar{z},\omega)$

Consistency checks

- It has the correct Disc $\mathcal{F}(z, \bar{z}, \omega)$
- It is consistent with localization.

Consistency checks

- It has the correct Disc $\mathcal{F}(z,\bar{z},\omega)$
- It is consistent with localization.
- It satisfies the Ward identities:

$$\left(\partial_z + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z,\bar{z},\omega)\Big|_{z=\omega} = 0$$

Consistency checks

- It has the correct Disc $\mathcal{F}(z, \bar{z}, \omega)$
- It is consistent with localization.
- It satisfies the Ward identities:

$$\left(\partial_z + \frac{1}{2}\partial_\omega\right)\mathcal{F}(z, \bar{z}, \omega)\Big|_{z=\omega} = 0$$

- It has sensible expansions around $z \sim 0$ and $z \sim 1$

Conclusions and future directions

Conclusions and future directions

- We bootstrapped $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$ at strong coupling, where $\mathcal{O}_J \sim \mathrm{Tr}\phi^J$. See our paper for $J = 2, 3, 4$.

Conclusions and future directions

- We bootstrapped $\langle\!\langle \mathcal{O}_J \mathcal{O}_J \rangle\!\rangle$ at strong coupling, where $\mathcal{O}_J \sim \text{Tr}\phi^J$. See our paper for $J = 2, 3, 4$.
- One could try to perform the explicit holographic calculation.

Conclusions and future directions

- We bootstrapped $\langle\!\langle \mathcal{O}_J \mathcal{O}_J \rangle\!\rangle$ at strong coupling, where $\mathcal{O}_J \sim \text{Tr}\phi^J$. See our paper for $J = 2, 3, 4$.
- One could try to perform the explicit holographic calculation.
- It might be convenient to go to Mellin space $\mathcal{M}(s, t)$.

Conclusions and future directions

- We bootstrapped $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$ at strong coupling, where $\mathcal{O}_J \sim \text{Tr}\phi^J$. See our paper for $J = 2, 3, 4$.
- One could try to perform the explicit holographic calculation.
- It might be convenient to go to Mellin space $\mathcal{M}(s, t)$.
- Can we go to higher orders? Flat space limit?

Conclusions and future directions

- We bootstrapped $\langle\langle\, \mathcal{O}_J \mathcal{O}_J \,\rangle\rangle$ at strong coupling, where $\mathcal{O}_J \sim \text{Tr}\phi^J$. See our paper for $J = 2, 3, 4$.
- One could try to perform the explicit holographic calculation.
- It might be convenient to go to Mellin space $\mathcal{M}(s, t)$.
- Can we go to higher orders? Flat space limit?
- Defects in ABJM? in $6d$ $(2, 0)$?

Thank you!

Defects in $(2, 0)$ theories

Defects in $(2, 0)$ theories

What about $\langle\!\langle \mathcal{O}_J \mathcal{O}_J \rangle\!\rangle$ in $6d$ $(2, 0)$ theories?

Defects in $(2,0)$ theories

What about $\langle\!\langle \mathcal{O}_J \mathcal{O}_J \rangle\!\rangle$ in $6d$ $(2,0)$ theories?

In [Drukker, Giombi, Tseytlin, Zhou (2020)] they studied

$$\Big(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\Big)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\alpha=1/\chi} = 0\,, \qquad \Big(-\frac{1}{2}\bar{\chi}\partial_{\bar{\chi}} + \bar{\alpha}\partial_{\bar{\alpha}}\Big)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=1/\bar{\chi}} = 0\,. \qquad (4.18)$$

for operators on the defect.

Defects in $(2,0)$ theories

What about $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$ in $6d$ $(2,0)$ theories?

In [Drukker, Giombi, Tseytlin, Zhou (2020)] they studied

$$\left(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\alpha=1/\chi} = 0\,, \qquad \left(-\frac{1}{2}\bar{\chi}\partial_{\bar{\chi}} + \bar{\alpha}\partial_{\bar{\alpha}}\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=1/\bar{\chi}} = 0\,. \qquad (4.18)$$

for operators on the defect.

We have a bootstrap problem

Defects in $(2,0)$ theories

What about $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$ in $6d$ $(2,0)$ theories?

In [Drukker, Giombi, Tseytlin, Zhou (2020)] they studied

$$\Big(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\Big)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\alpha=1/\chi} = 0\,, \qquad \Big(-\frac{1}{2}\bar{\chi}\partial_{\bar{\chi}} + \bar{\alpha}\partial_{\bar{\alpha}}\Big)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=1/\bar{\chi}} = 0\,. \qquad (4.18)$$

for operators on the defect.

We have a bootstrap problem

- Obtain the Ward identities for $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$

Defects in $(2,0)$ theories

What about $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$ in $6d$ $(2,0)$ theories?

In [Drukker, Giombi, Tseytlin, Zhou (2020)] they studied

$$\left(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\alpha=1/\chi} = 0\,, \qquad \left(-\frac{1}{2}\bar{\chi}\partial_{\bar{\chi}} + \bar{\alpha}\partial_{\bar{\alpha}}\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=1/\bar{\chi}} = 0\,. \qquad (4.18)$$

for operators on the defect.

We have a bootstrap problem

- Obtain the Ward identities for $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$
- Calculate superconformal blocks

Defects in $(2,0)$ theories

What about $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$ in $6d$ $(2,0)$ theories?

In [Drukker, Giombi, Tseytlin, Zhou (2020)] they studied

$$\left(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\alpha=1/\chi} = 0\,, \qquad \left(-\frac{1}{2}\bar{\chi}\partial_{\bar{\chi}} + \bar{\alpha}\partial_{\bar{\alpha}}\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=1/\bar{\chi}} = 0\,. \tag{4.18}$$

for operators on the defect.

We have a bootstrap problem

- Obtain the Ward identities for $\langle\langle \mathcal{O}_J \mathcal{O}_J \rangle\rangle$
- Calculate superconformal blocks
- Apply Disc to the holographic spectrum