Holographic RG flows for Kondo-like impurities

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Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

Motivation for study within gauge/gravity duality:

1. New applications of gauge/gravity duality to condensed matter physics:
   - Magnetic impurity coupled to strongly correlated electron system
   - Entanglement entropy, quantum quenches

2. Model for a RG flow with dynamical scale generation (as in QCD)

3. Example for holographic g-theorem

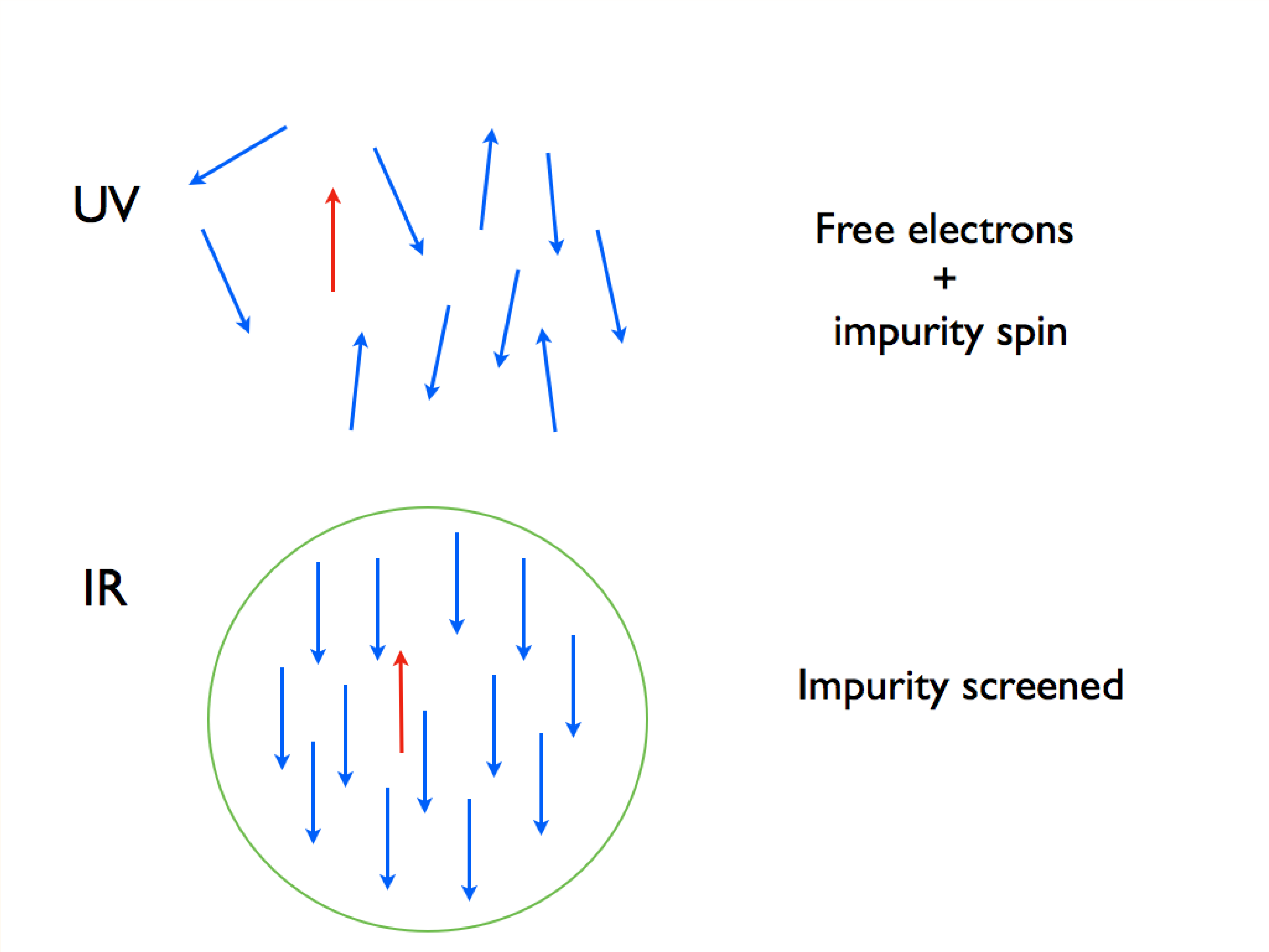
4. Relation to Sachdev-Ye-Kitaev model

Overview

- Kondo effect: Physics and bCFT realization
- Top-down model I: with A. O'Bannon et al
- Top-down model II: with C. Melby-Thompson, C. Northe
- Bottom-up model and applications

- – D3/D5/D7 Model J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086
  - – Entanglement entropy J.E., Flory, Newrzella 1410.7811, JHEP 1501 (2015) 058
    J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu 1511.03666, Fortsch.Phys. 64 (2016)
  - – Two-point functions J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu JHEP 1703 (2017) 039 , PRD 96 (2017) no.2, 021901
  - – Quantum quenches J.E., Flory, Newrzella, Wu JHEP 1704 (2017) 045
  - – Different setting: D1/D5 system with defect
    J.E., Melby-Thompson, Northe JHEP 05 (2020) 075

Kondo effect



Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

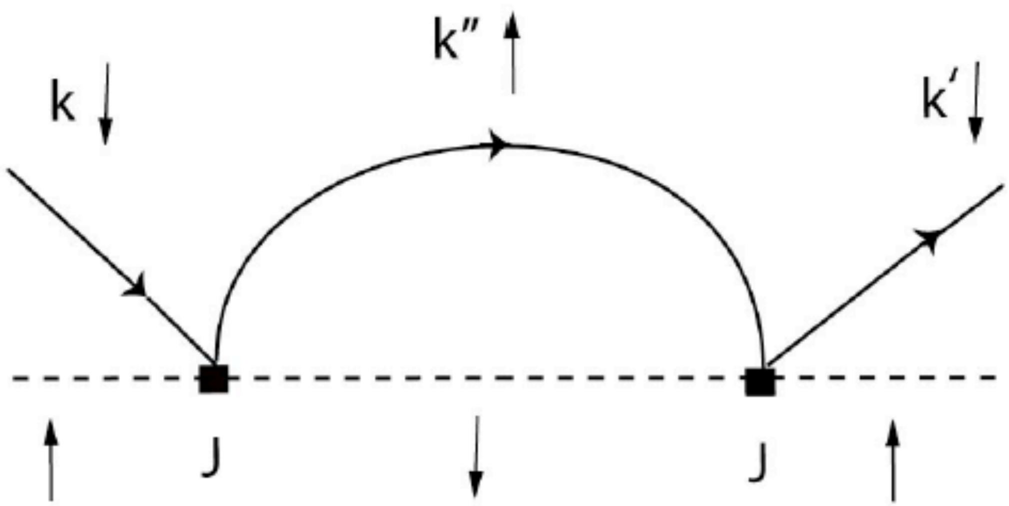
Hamiltonian:

$$H = \frac{v_F}{2\pi}\psi^\dagger i\partial_x\psi + \lambda_K v_F \delta(x)\vec{S}\cdot\vec{J}\,, \quad \vec{J} = \psi^\dagger \frac{1}{2}\vec{T}\psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Large N Kondo model Read, Newns, Coleman, ... 80's

Scattering with magnetic impurities



$$\rho \sim \rho_0 \left(1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|}\right)$$

Antiferromagnetic coupling $\kappa < 0$

Logarithmic behaviour at low temperatures

J. Kondo 1964

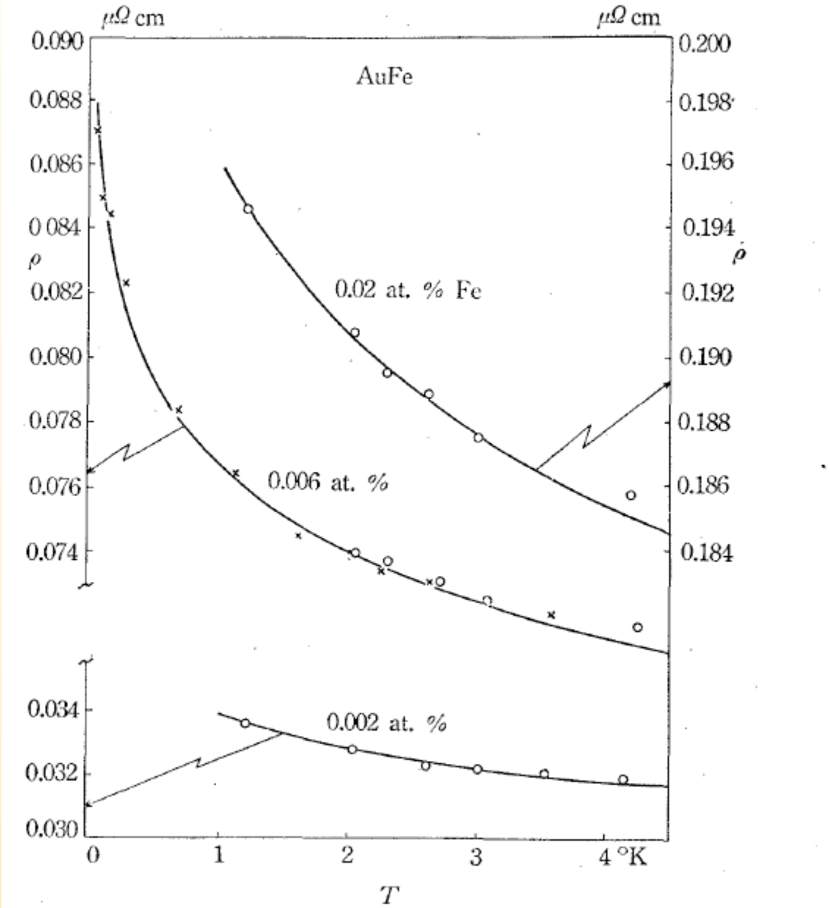


Fig. 1. Comparison of experimental and theoretical ρ–T curves for dilute AuFe alloys.

$$\rho \sim \rho_0 \left(1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|}\right)$$

Perturbation theory breaks down at $T_K = |\epsilon - \epsilon_F| e^{1/\kappa}$

T_K: Kondo temperature

$T_K \sim \ \Lambda_{\mathrm{QCD}}$

Large N Kondo model

Gauge/gravity requires large N: Spin group $SU(N)$

Large N Kondo model

Gauge/gravity requires large N: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = Q$

Interaction: $J^a S^a = (\psi^\dagger T^a \psi)(\chi^\dagger T^a \chi) = \mathcal{O}\mathcal{O}^\dagger$, where $\mathcal{O} = \psi^\dagger \chi$

Screened phase has condensate $\langle \mathcal{O} \rangle$

Coleman PRB 35, 5072 (1987)

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192, PRB 58 (1998) 3794

Senthil, Sachdev, Vojta cond-mat/0209144, PRL 90 (2003) 216403

Holographic top-down model I

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

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Results:

- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation, screening
- Holographic superconductor: Condensate forms in AdS_2
- Power-law scaling of resistivity in IR with real exponent
- Holographic entanglement entropy from including backreaction
- Quantum quench: Equilibration dominated by quasinormal modes
- Fano resonance in spectral function (spectral asymmetry)

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \rightarrow$ Chern-Simons A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \rightarrow \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar dual to } \psi^\dagger \chi \end{cases}$

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $Q = \chi^\dagger \chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger \chi$	$\Leftrightarrow$	2d complex scalar Φ

J.E., Melby-Thompson, Northe 2001.04991

Supersymmetric boundary RG flow with $SU(2) \times SU(2)_R$ symmetry

CFT perturbed by boundary operator $\langle \mathrm{Tr}\, \mathcal{P} \exp(i\lambda \int J^a M^a) \rangle$

Brane construction based on D1/D5 system

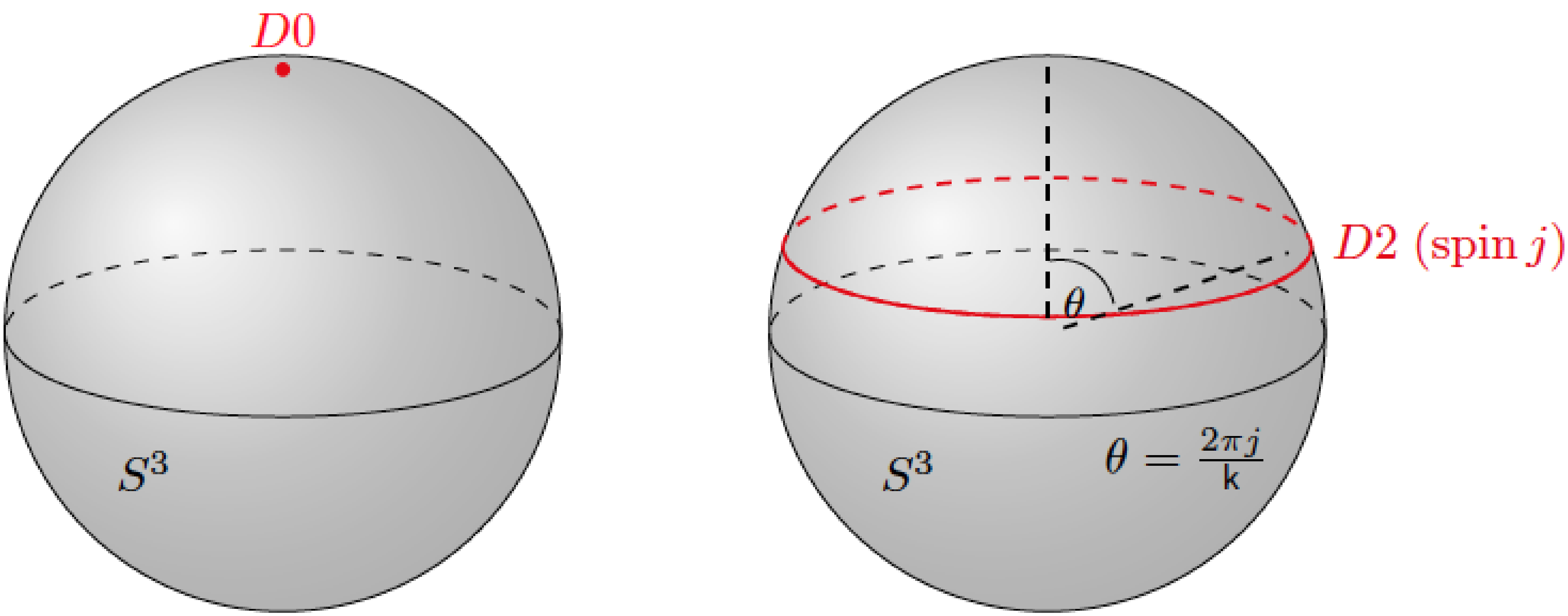
	t	x	z	θ	ϕ	χ	6	7	8	9
D1	X	X								
D5	X	X					X	X	X	X
D3(p,q)	X		X		(X)	(X)				

$$\mathrm{AdS}_3 \quad \times \quad S^3 \quad \times \quad M^4$$

Special case of boundary RG flow in $\widehat{\mathfrak{su}(2)}_k$ WZW model, a conformal non-linear sigma model on S^3 with k units of NS-NS three-form flux $H^{(3)}$.

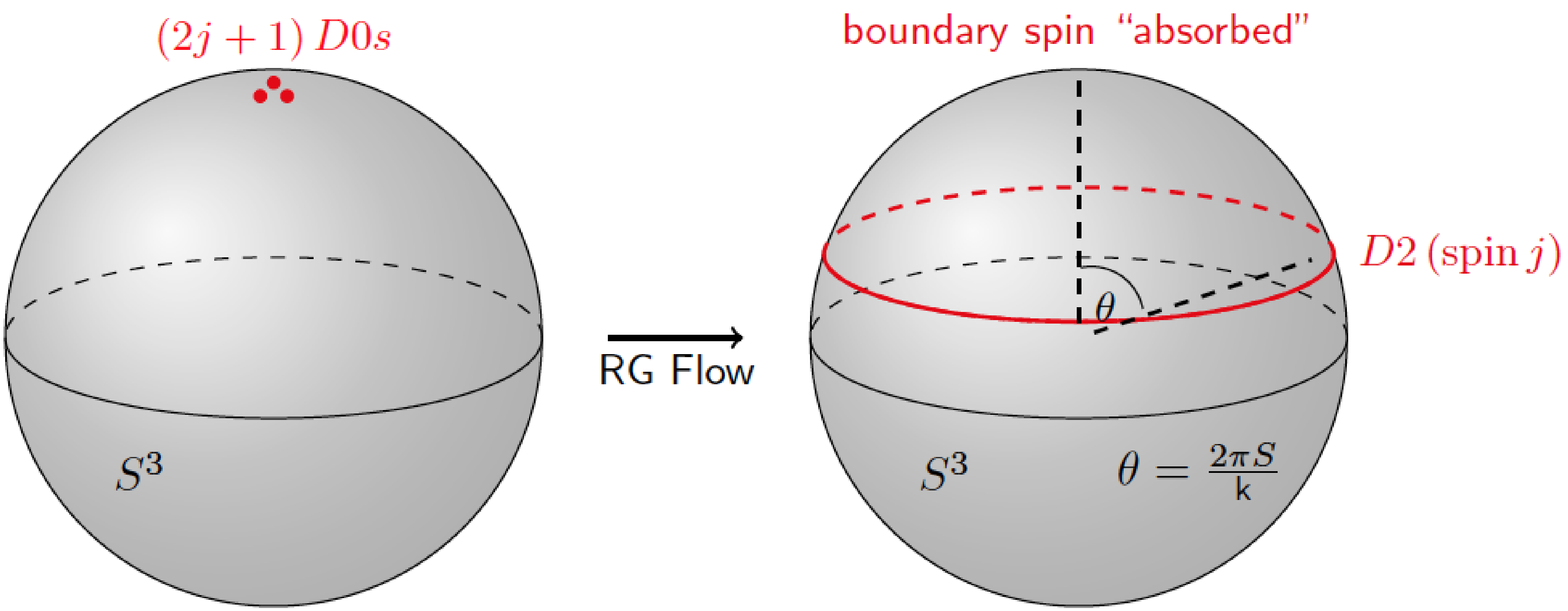
Conformal boundary conditions preserving a $\widehat{\mathfrak{su}(2)}_k$ are classified by spin j, $0 \leq 2j \leq k$.

Boundary conditions: Neumann along the S^2 at $\theta = \frac{2\pi j}{k}$.



bCFT realization of Kondo effect

- Exist RG flows between these two types of boundary condition.
- Adding the spin coupling results in a non-abelian polarization, causing the branes to puff up along the RG flow.
- In Kondo context: "absorption of boundary spin" principle [Affleck+Ludwig]



bCFT realization of Kondo effect

- RG flow can be described as the fusion of a defect RG flow with a fixed boundary state [Gaberdiel+Bachas hep-th/0411067]
- Defects in $\widehat{\mathfrak{su}(2)}_k$ WZW model are Wilson-esque line operators:

$$\mathcal{D} = \mathrm{Tr}_{2j+1} \mathcal{P} \exp\Big(ig \int dt\, \vec{S} \cdot \vec{J}(t) \Big) \, ;$$

  $\vec{J}(z)$ is a chiral $\mathrm{SU}(2)$ current of the WZW model.
- g is marginally relevant and generates a defect flow.
- Chirality of J implies it can be fused with the D0 boundary without generating singularities. This generates D0→D2 RG flow.
- The flows generated by these defects are described by a similar non-abelian polarization process.

D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$ (with $M_4 =$ K3 or T^4):

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								

At low energies, described by a 2d gauge theory with $\mathcal{N} = (4,4)$ susy.

Gauge theory description. $\mathrm{U}(N_1) \times \mathrm{U}(N_5)$ gauge theory with bifundamental hypermultiplet. To get well-behaved CFT, turn on Fayet-Iliopoulos parameters to push us onto the Higgs branch. Gives:

Instanton description. D5 brane has a coupling $\int C^{(2)} \wedge \mathrm{Tr}(F \wedge F)$. This means that D1 branes can be dissolved as $\mathrm{U}(N_5)$ gauge instantons on M_4.

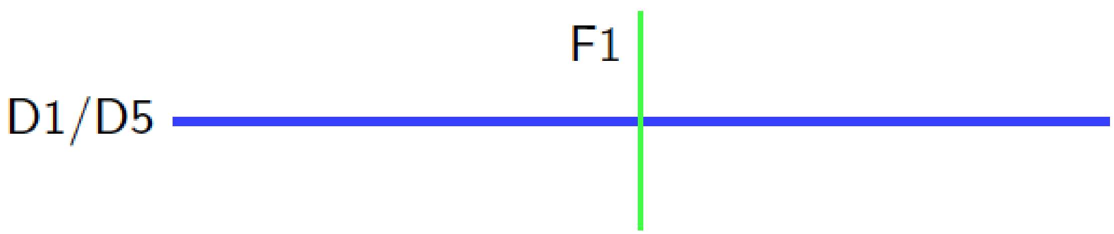
Low energy dynamics described by an $\mathcal{N} = (4,4)$ non-linear sigma model on (a deformation of) the moduli space of instantons on M_4. [Strominger+Vafa '96]

Interfaces in D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$:

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								
F1 (p)	•		•							

- p fundamental strings intersect the D1/D5 system
- preserves $\mathcal{N} = 4$, $d = 1$ supersymmetry
- when string ends on D1/D5 system, realized in gauge theory description as a Wilson line. Sources jump in background electric field, changing the CFT on one side while preserving the central charge. This case is an interface, not a defect.



Interfaces in D1/D5 CFT

- Particularly interested in a Wilson line corresponding to a long string connecting a distant D3 brane to the D1/D5 system.
- Two types of modes: D3-D5 strings and D3-D1 strings. D5-D3 strings give one complex Grassmann mode. D1-D3 strings give a hypermultiplet.
- In CFT, fields corresponding to D1-D5 strings have expectation values, allowing D1-D3 strings to turn into D5-D3 strings and vice versa. As a result they mix and most of them increase in energy.
- After mixing, lowest-lying fermions have Lagrangian [Tong+Wong '14]

$$L_\eta = \eta^\dagger(i\partial_0 + \Omega_A \partial_t Z^A)\eta \tag{3}$$

  where η is in the fundamental of $\mathrm{U}(N)$, Z^A is the coordinate on $\mathcal{M}$, and Ω_A is a $\mathrm{U}(N)$ connection on $M_4 \times \mathcal{M}$.
- This can be rewritten as the insertion of

$$W = \mathrm{Tr}_F \mathcal{P} \exp\left(i \int dt\, \partial_t Z^A \Omega_A(y_0, Z)\right) \tag{4}$$

  with y_0 the location of the Wilson line in M_4.

Gravity dual in probe limit

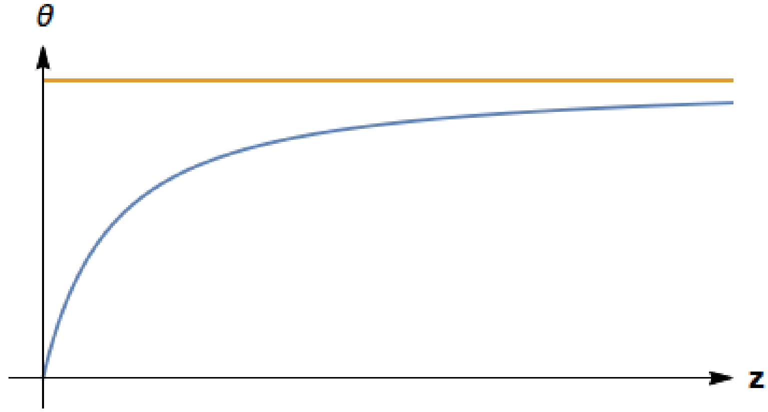
- In D3-brane description,

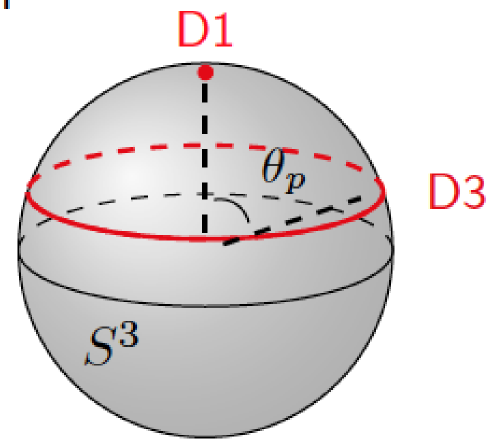
$$I = T_{\text{D3}} \int d^4\xi e^{-\Phi} \sqrt{-\det(\hat{g} + F)} + T_{\text{D3}} \int (C^{(2)} \wedge F + \frac{1}{2} F \wedge F)$$

- Simplest case: when D3 branes carry no D1 charge, solution is given by

$$z = z_0 \frac{\sin\theta}{\theta_p - \theta} \qquad \theta_p = \pi \frac{p}{N_5}$$

where z is the radial coordinate in Poincaré patch.





Action:

$$S = S_{\mathrm{Einstein-Hilbert}} + S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi}\int_{AdS_3} \mathrm{Tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right),$$

$$S_{AdS_2} = -N\int dx dt dz\,\delta(x)\sqrt{-g}\left[\frac{1}{4}\mathrm{T}r f^{mn} f_{mn} + g^{mn}\left(D_m\Phi\right)^{\dagger} D_n\Phi + V(\Phi^{\dagger}\Phi)\right]$$

$$V(\Phi) = M^2\Phi^{\dagger}\Phi$$

Metric: BTZ black hole

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = \frac{1}{z^2}\left(\frac{dz^2}{h(z)} - h(z)\,dt^2 + dx^2\right),$$

$$h(z) = 1 - z^2/z_H^2\,, \qquad T = 1/(2\pi z_H)$$

AdS$_2$ gauge field: asymptotically $a_t = \frac{Q/N}{z} + \mu$

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

Berkooz, Sever, Shomer hep-th/0112264

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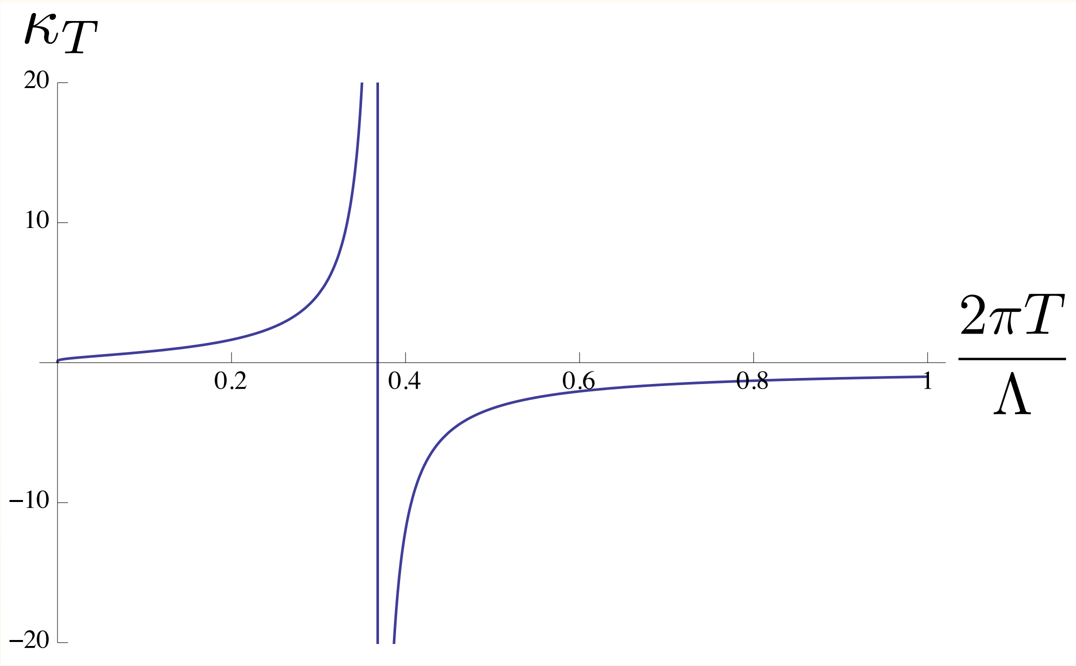
Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

Dynamical scale generation

Scale generation

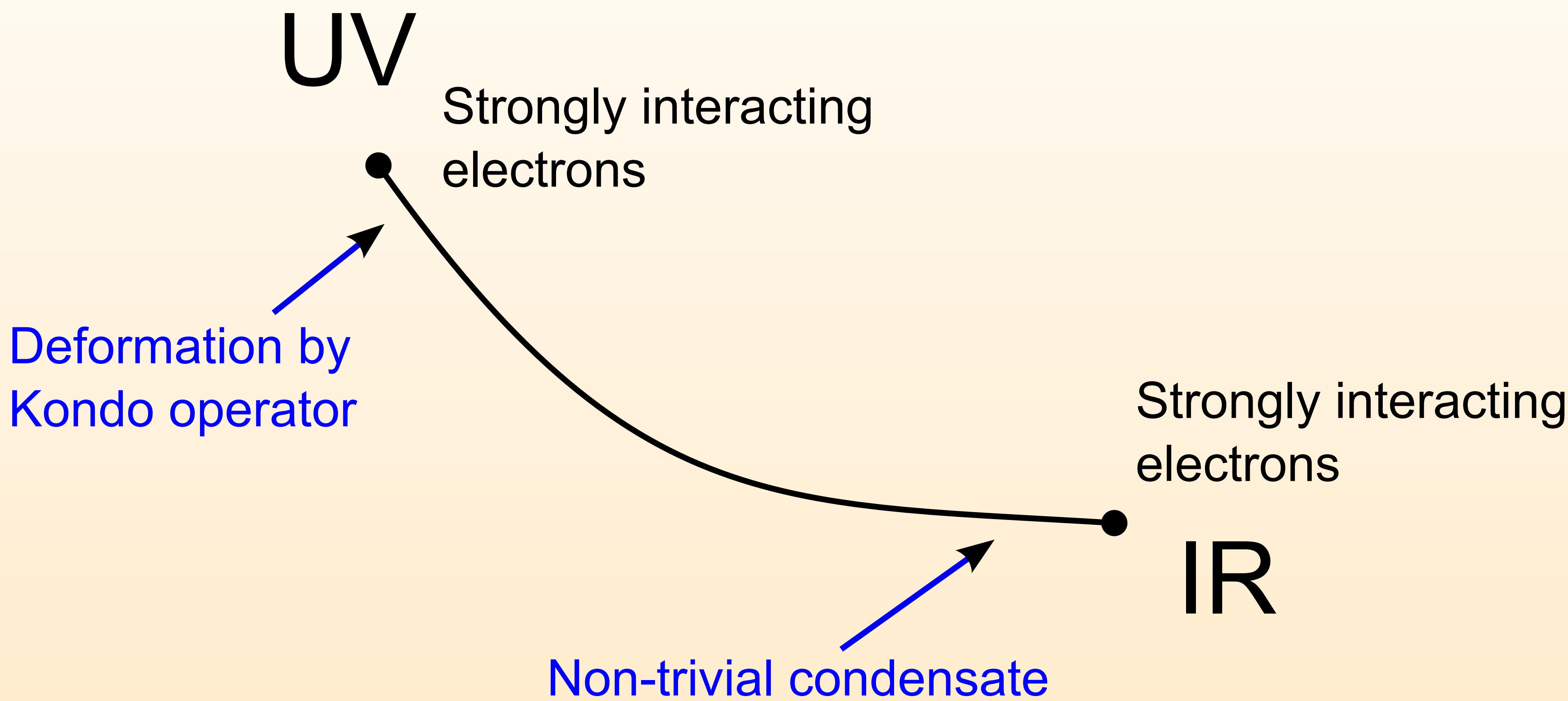


Divergence of Kondo coupling determines Kondo temperature T_K

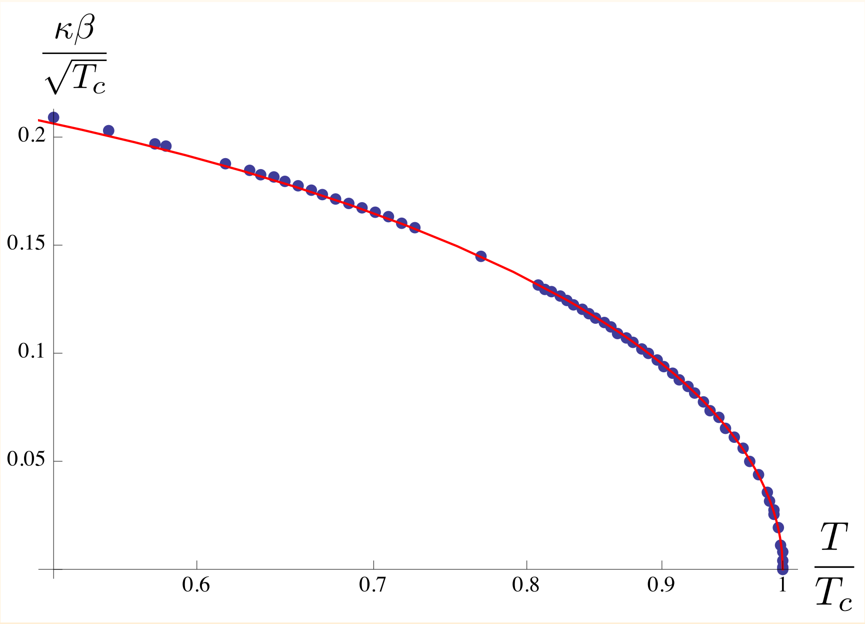
Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

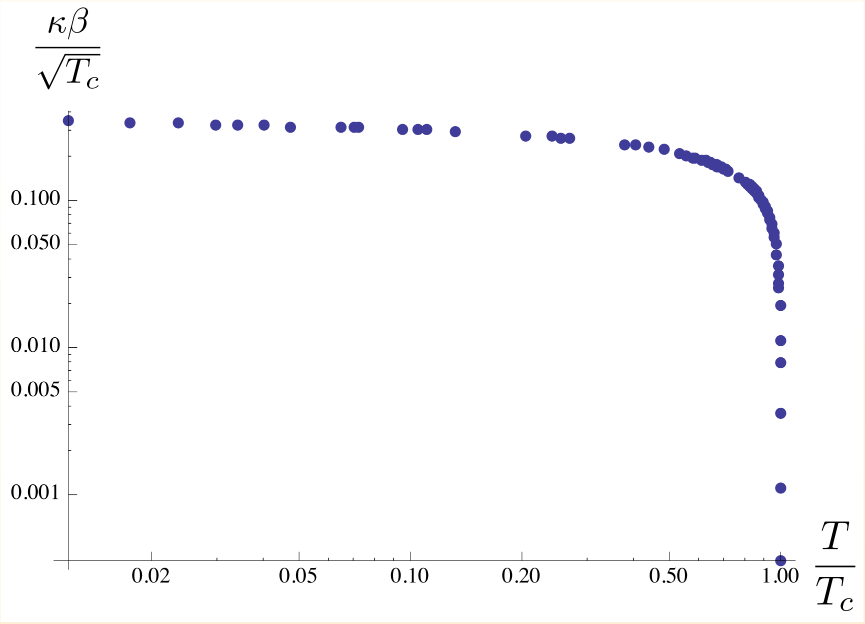
RG flow


UV
Strongly interacting
electrons
Deformation by
Kondo operator
Strongly interacting
electrons
IR
Non-trivial condensate

Normalized condensate $\langle \mathcal{O} \rangle \equiv \kappa\beta$ as function of the temperature



(a)

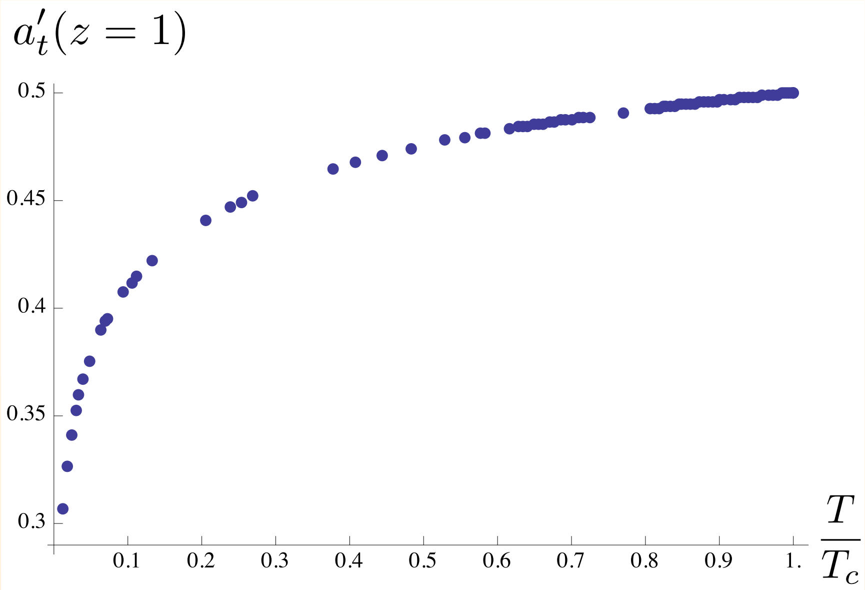


(b)

Mean field transition

$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$

Electric flux at horizon



(a)

$$\sqrt{-g}\,f^{tr}\Big|_{\partial AdS_2} = q$$

charge $q = Q/N$ of 2d gauge field determines impurity representation

Impurity is screened

Quantum quench and time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Quantum quench and time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

Quantum quench and time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

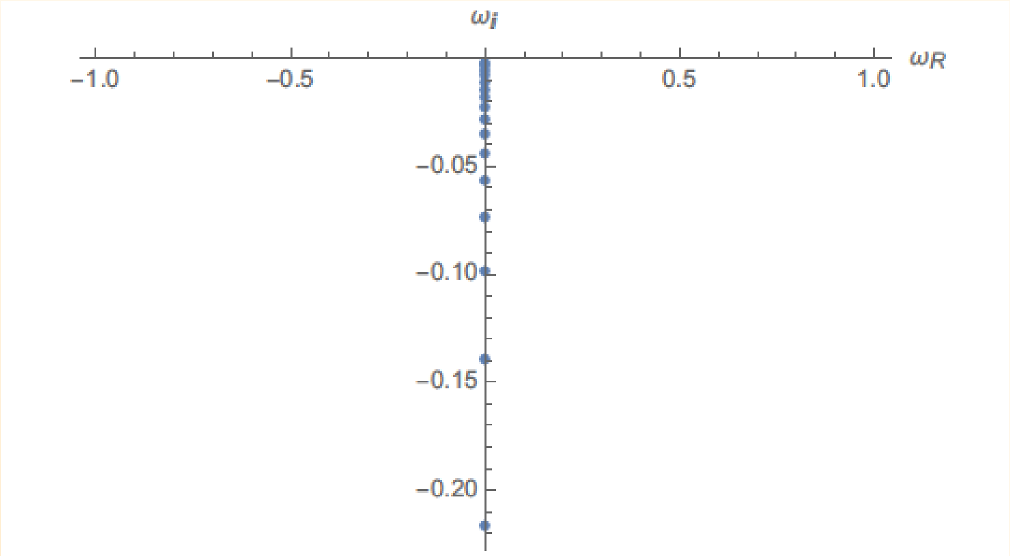
Timescales governed by quasinormal modes (QNM)

Complex eigenfrequencies of fluctuations in gravity system

Complex eigenfrequencies ω_P of gravitational system determine time evolution

The ω_P also determine the poles in the Green's functions

In condensed phase:

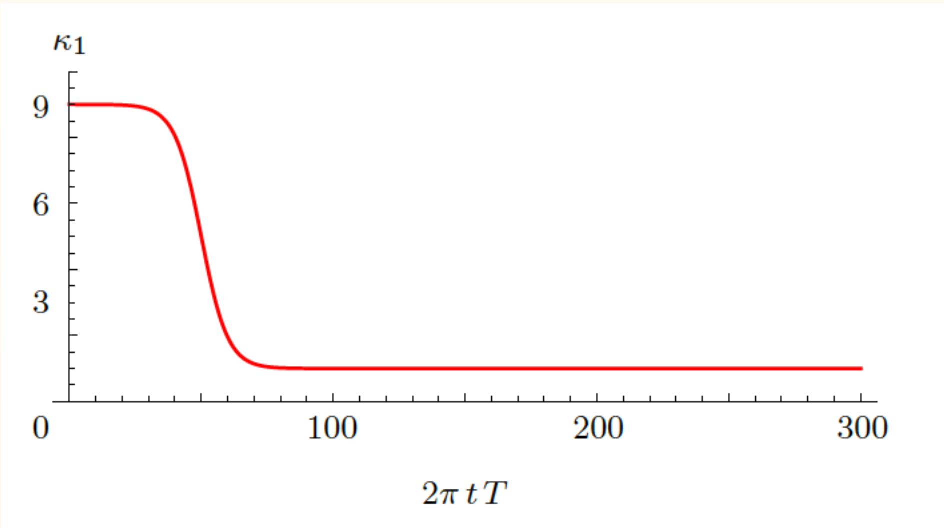


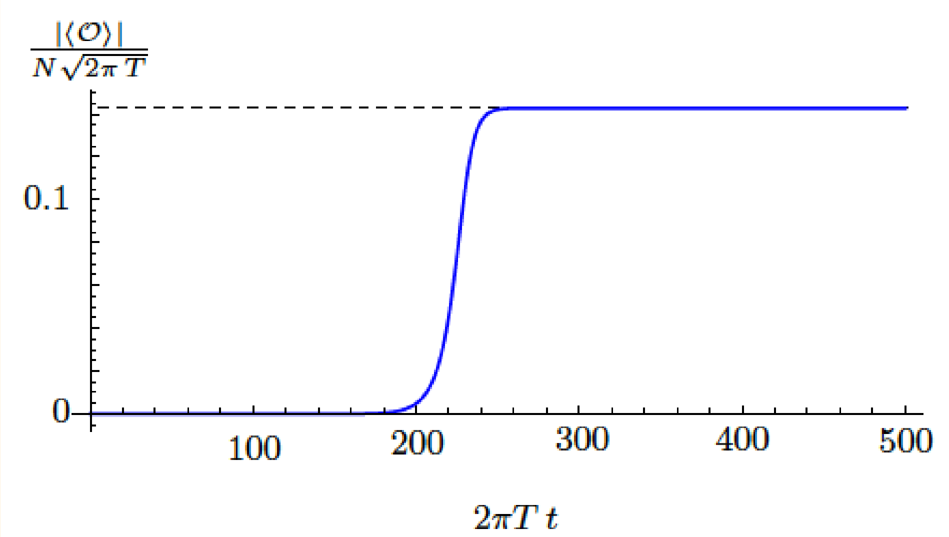
Quasinormal modes on negative imaginary axis, $\omega_{\rm pole} \propto -i \langle \mathcal{O} \rangle^2$

Kondo resonance

Quantum quench in Kondo model within gauge/gravity duality

J.E., Flory, Newrzella, Strydom, Wu JHEP (2017)





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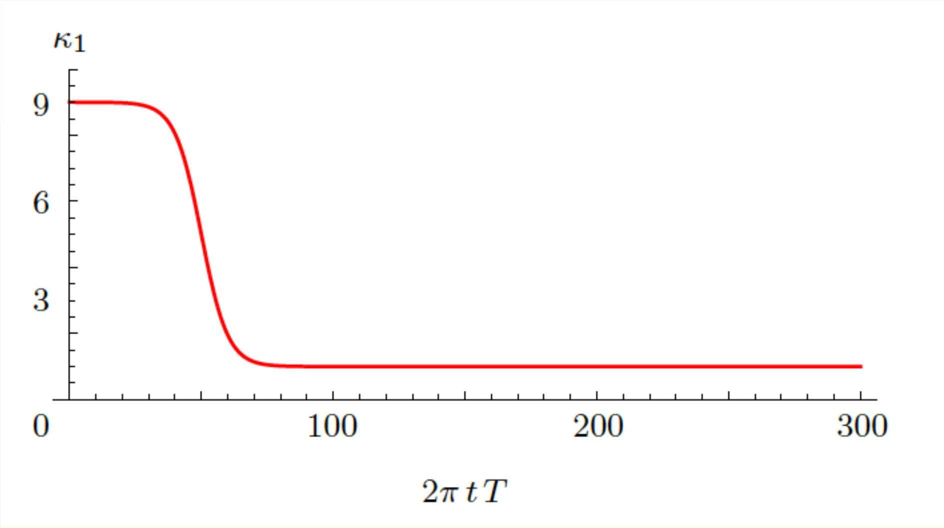
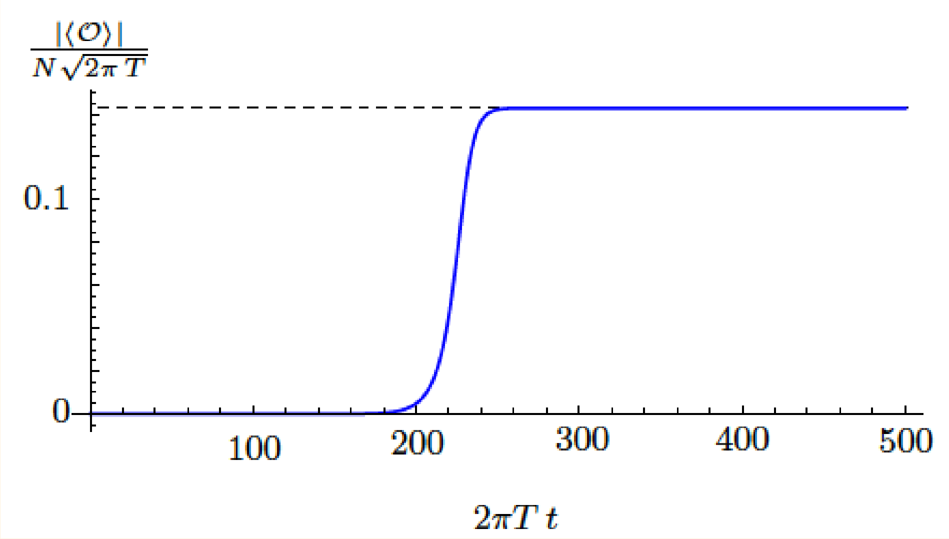
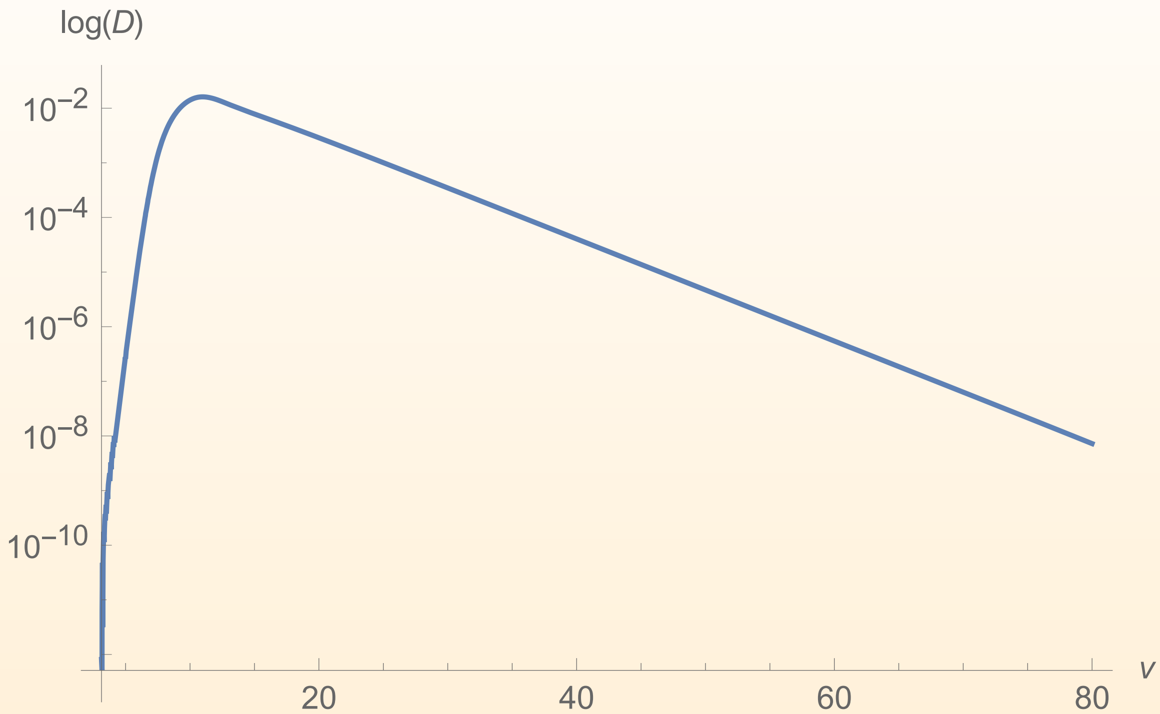




Formation of screening cloud:

Exponential fall-off of number of degrees of freedom at impurity

Screening happens exponentially fast

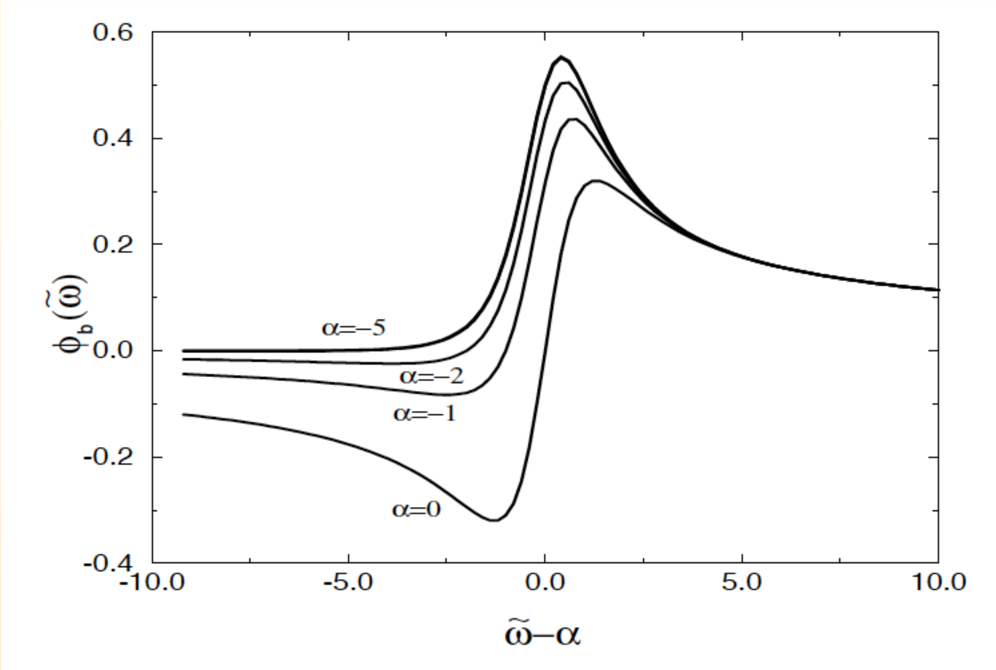


Flux at horizon (proportional to number of impurity degrees of freedom) as function of time

Kondo model: Two-point functions $\langle \mathcal{O}\mathcal{O}^\dagger \rangle$ at $T = 0$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192: Large N Kondo model for weakly coupled electrons

Spectral asymmetry ω_s: Particle-hole symmetry broken



$-\mathrm{Im}G^R$ for bosonic $\langle \mathcal{O}\mathcal{O}^\dagger \rangle$

Example of Fano resonance

A related but different Fano resonance is observed in holographic model

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A related but different Fano resonance is observed in holographic model

Fano (1961):

A discrete set of resonant states interacts with a continuum of states

Example: Light scattering off an atom

Spectral function:

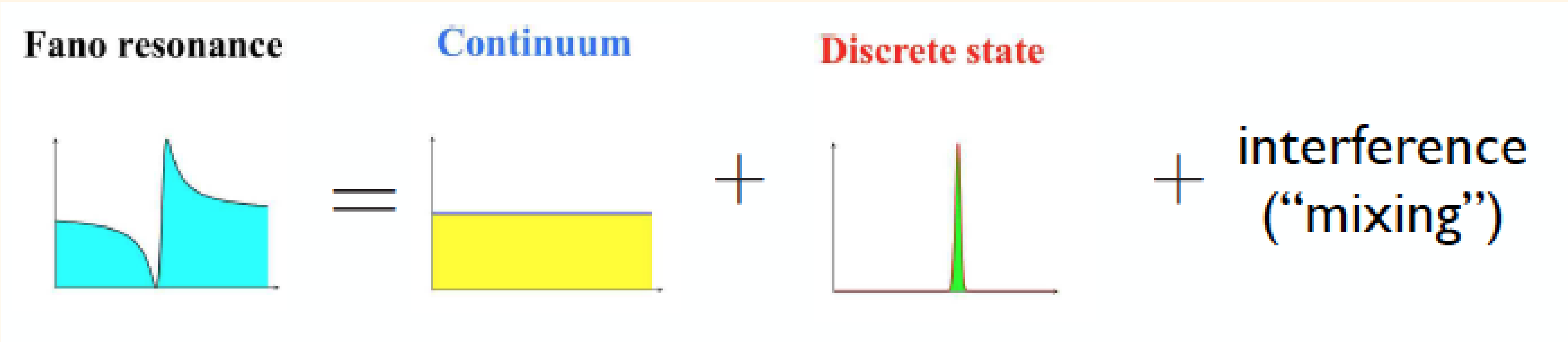
$$\rho_{\text{Fano}}(\omega) = \frac{(\omega - \omega_0 + \frac{\Gamma}{2}q)^2}{(\omega - \omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

q: Fano asymmetry parameter

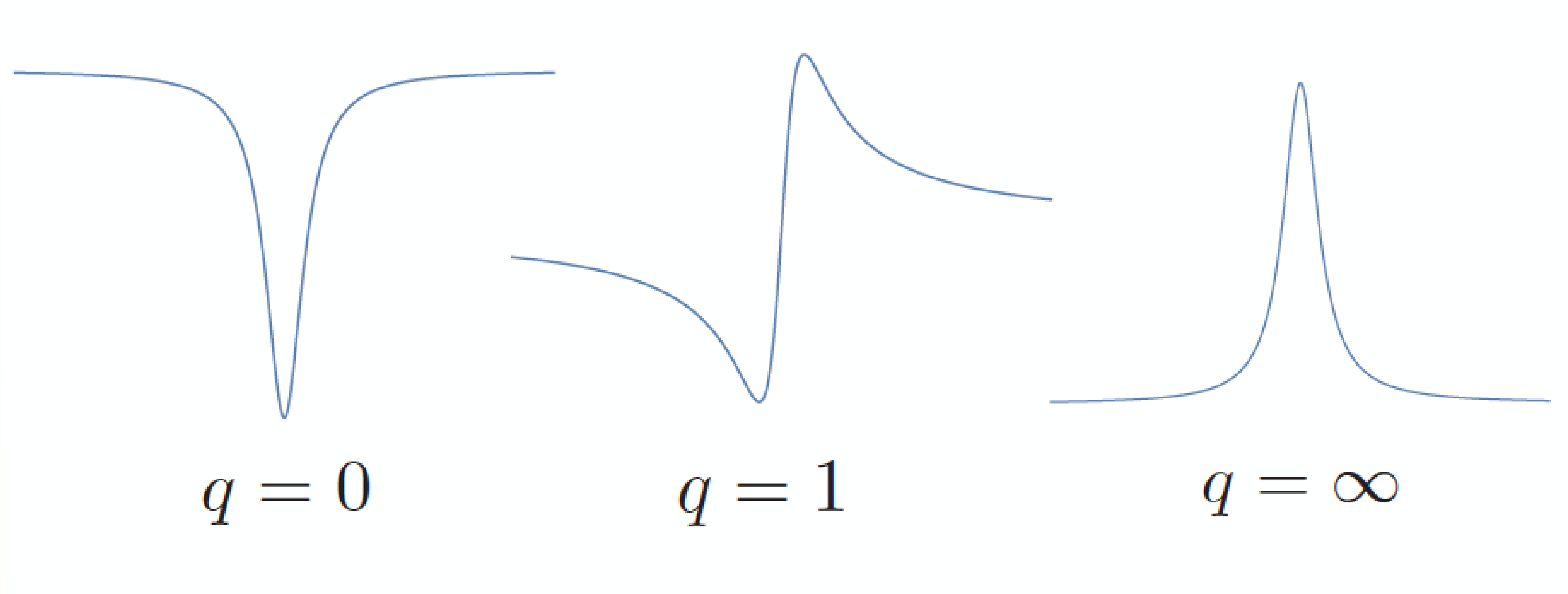
$$q^2 \propto \frac{\text{Probability of resonant scattering}}{\text{Probability of non} - \text{resonant scattering}}$$

Fano resonances

$$\frac{(\omega-\omega_0+\frac{\Gamma}{2}q)^2}{(\omega-\omega_0)^2+\left(\frac{\Gamma}{2}\right)^2} = 1 + \frac{(q^2-1)\left(\frac{\Gamma}{2}\right)^2}{(\omega-\omega_0)^2+\left(\frac{\Gamma}{2}\right)^2} + \frac{2q\frac{\Gamma}{2}(\omega-\omega_0)}{(\omega-\omega_0)^2+\left(\frac{\Gamma}{2}\right)^2}$$



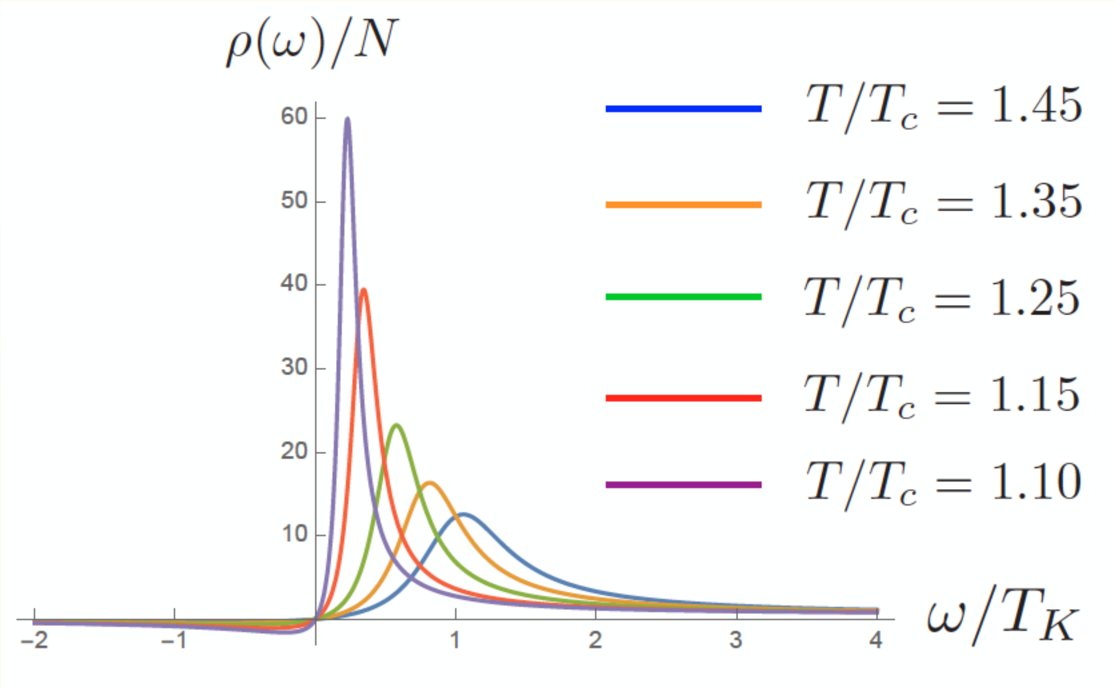
Fano resonance



$$q^2 \propto \frac{\text{Probability of resonant scattering}}{\text{Probability of non}-\text{resonant scattering}}$$

Spectral function $-\mathrm{Im}\langle\mathcal{O}^\dagger\mathcal{O}\rangle$ in normal phase, $\langle\mathcal{O}\rangle = 0$, $T > T_c$

J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu JHEP 1703 (2017) 039 , PRD 96 (2017) 021901



$$\rho_{\mathrm{peak}} \propto \frac{1}{T - T_c}$$

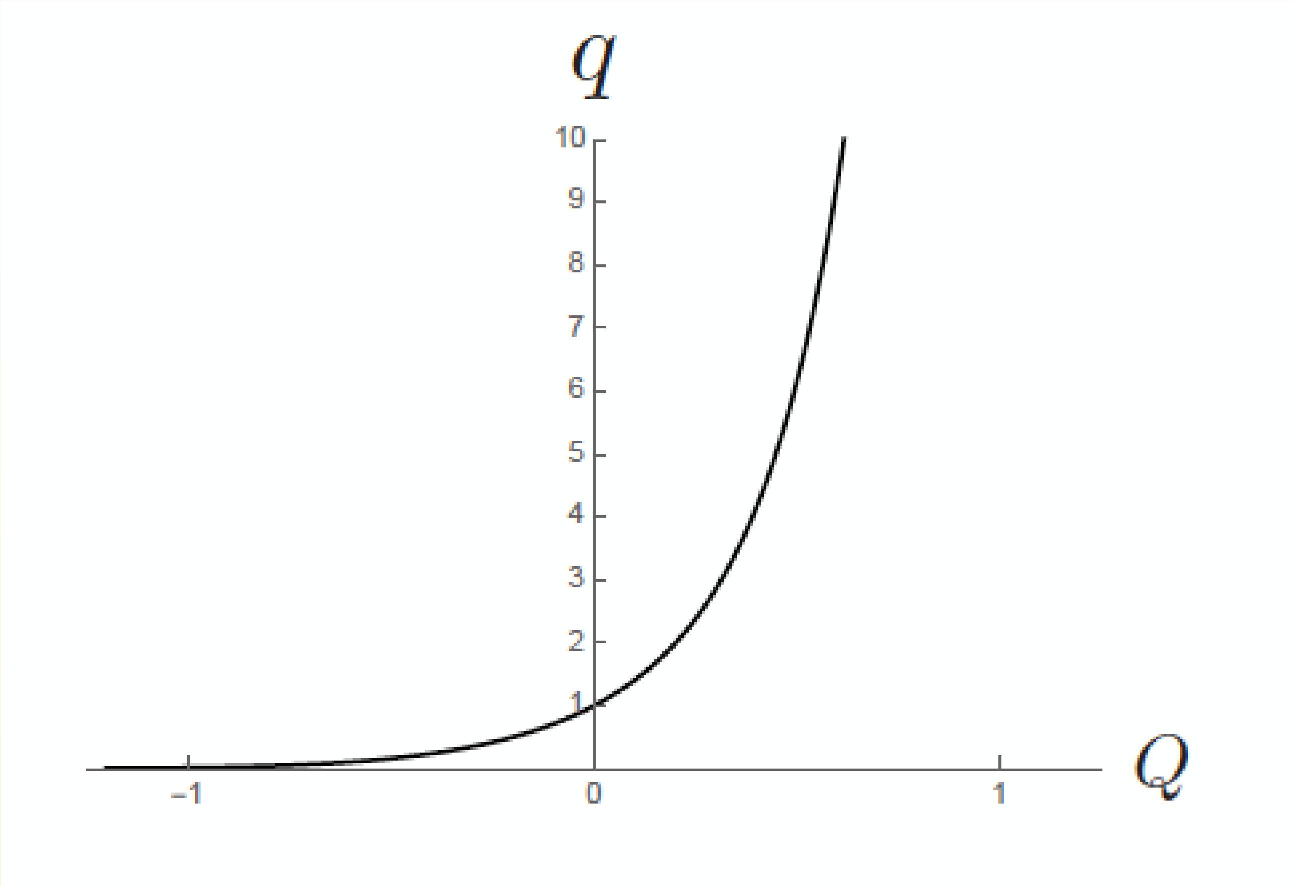
Fano resonance

Here: 0+1 CFT continuum $+$ Resonance with spin impurity $=$ Fano

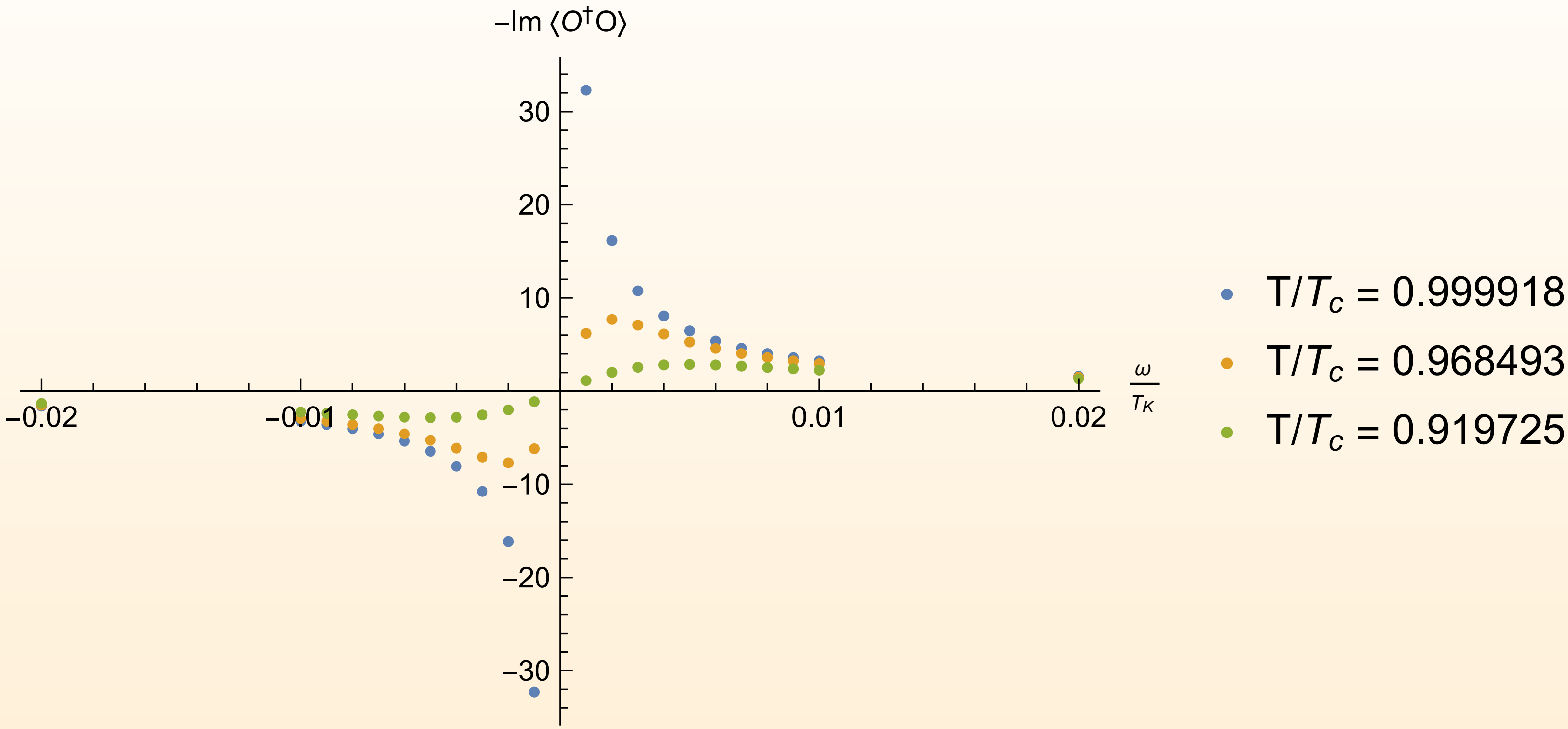
0+1-dimensional conformal symmetry of AdS_2 subspace
broken by double-trace operator of interaction with spin

Dependence of asymmetry parameter q on representation parameter Q

For $T \gtrsim T_c$:



Spectral function $-\mathrm{Im}\langle\mathcal{O}^\dagger\mathcal{O}\rangle$ in condensed phase, $\langle\mathcal{O}\rangle \neq 0$, $T < T_c$

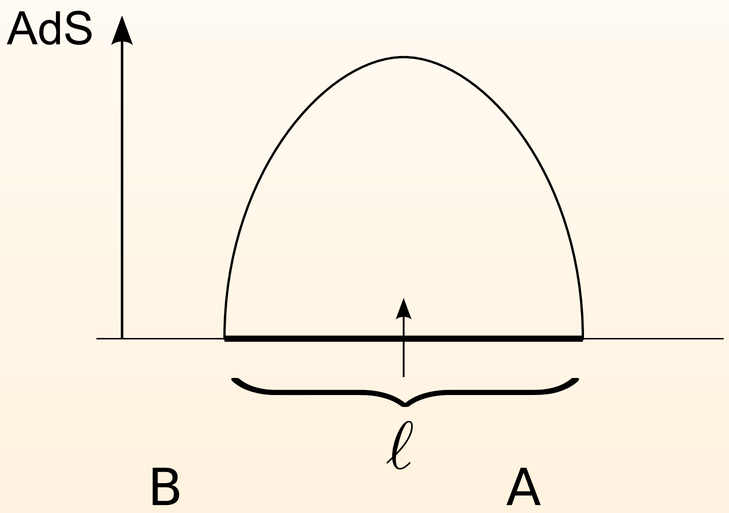


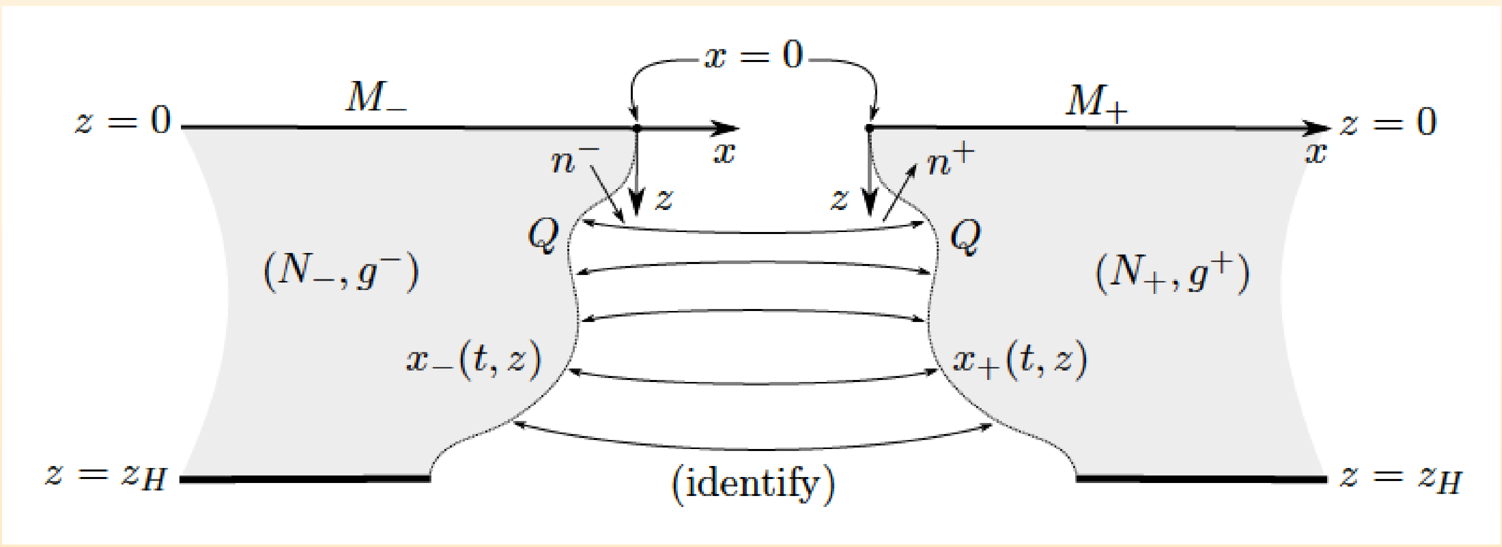
Fano asymmetry parameter $q = 1$ (i.e. no asymmetry)

Poles of retarded Green's function purely imaginary, $\omega \propto -i|\langle\mathcal{O}\rangle|^2$

Manifestation of large N Kondo resonance

$$S_{\text{imp}} = S_{\text{impurity present}} - S_{\text{impurity absent}}$$





Entanglement entropy for magnetic impurity: Comparison to field theory

Field theory result: Sorensen, Chang, Laflorencie, Affleck 2007 , (Eriksson, Johannesson 2011)

$$\Delta S_{\text{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0$$

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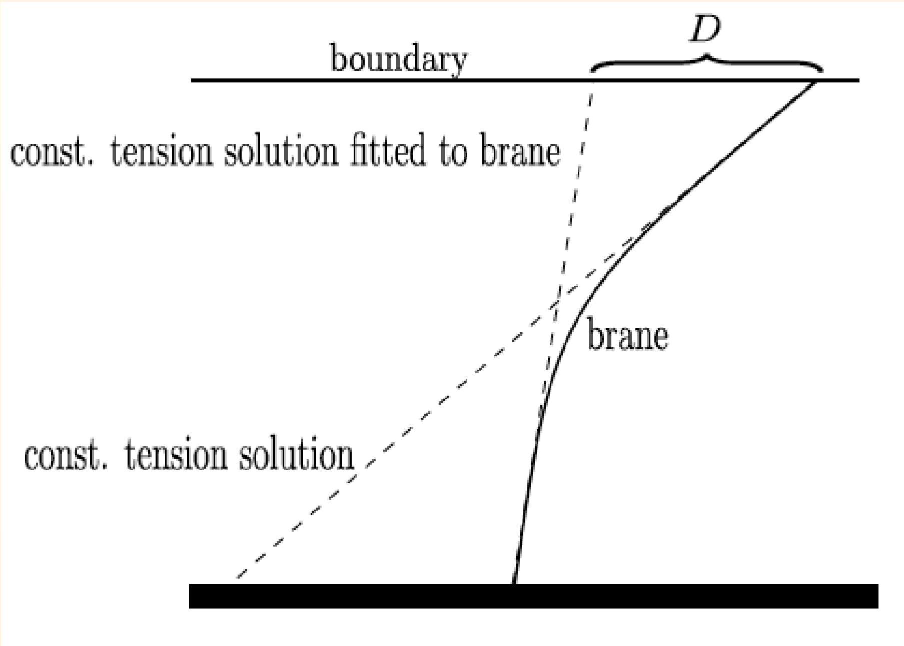
Field theory result: Sorensen, Chang, Laflorencie, Affleck 2007 , (Eriksson, Johannesson 2011)

$$\Delta S_{\mathrm{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0$$

In our gravity approach: Same result if $D \propto \xi_k$



Conclusions and outlook

- Kondo model:
- Magnetic impurity coupled to strongly coupled system
- Two top-down models

- Quantum quenches
  - Dominated by quasinormal modes

- Two-point functions
  - Spectral asymmetry
  - Relation to SYK model

- Entanglement entropy
  - In agreement with g-theorem
  - Reproduces large N field theory result for large ℓ
  - Geometrical realization of Kondo correlation length