An invitation to surface operators in the (2,0) theory

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Based on: arXiv:2003.12372 with M. Probst and M. Trépanier

arXiv:2004.04562 with S. Giombi, A. Tseytlin and X. Zhou

arXiv:2009.10732 with M. Probst and M. Trépanier

arXiv:2012.11087 with M. Trépanier

Quantum Field Theory at the Boundary

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Outline

This talk will be comprised of two main parts

- Slow introduction to surface operators in the $\mathcal{N} = (2,0)$ theory.
- Quick review of new BPS observables in the theory and their properties.

Join M. Trépanier at the poster session for further details and to learn of new applications.

6d $\mathcal{N} = (2,0)$ CFT

String/M-theory considerations suggested the existence of these theories.

We know that:

- The largest dimension for which there is superconformal algebra is 6d.
- It is expected that there are theories realizing this symmetry for every ADE algebra.
- No Lagragian is known for any of these theories, no continuous parameters.
- Simplest examples realizing this algebra:
  - The free tensor multiplet with a self-dual field strength, 5 scalars and fermions.
  - M-theory on $AdS_7 \times S^4$ realizing the large N limit.
- When compactified to 5d, gives 5d SYM. In 4d gives class $\mathcal{S}$ theories, including $\mathcal{N} = 4$ SYM.
- We need tools to understand this theory.

Surface operators

- The simplest argument for the $\mathcal{N} = (2,0)$ theory is from M-theory:
  - It describes the low energy dynamics of N M5-branes.
- It is known that M2-branes can end on M5-branes.
- The end-points form a two dimensional surface in 6d.
- In compactifying M-theory, an M2-brane wrapping the compact cycle becomes a fundamental string.
  - M2-brane ending on M5-brane becomes fundamental string ending on D5-brane.
  - Fundamental strings carry electric flux, their endpoint is a Wilson loop in 5d YM.
- The surface operators in 6d are analogous to Wilson loops in gauge theories.

Wilson loops

- Wilson loops are crucial observables in any gauge theory.
- They can serve as an order parameter for confinement and are calculable on the lattice.
- Focusing on Wilson loops in theories with extended supersymmetry (like $\mathcal{N} = 4$ SYM in 4d) we can:
  - Evaluate them perturbatively.
  - Evaluate them at strong coupling via AdS/CFT.
  - Integrability.
  - Relate them to scattering amplitudes.
  - Evaluate the circular and other BPS Wilson loops exactly using localization.
  - Use defect-CFT techniques including OPE for small deformations around a line or circle.

- Are the non-perturbative tools also applicable in 6d?

Surface operators

- Surface operators are crucial observables in $\mathcal{N} = (2,0)$ theory.
- ~~They can serve as an order parameter for confinement and are calculable on the lattice.~~
- Focusing on surface operators in the $\mathcal{N} = (2,0)$ theory in six dimension
  we can:
  - ~~Evaluate them perturbatively.~~
  - Evaluate them at strong coupling via AdS/CFT.
  - ~~Integrability.~~
  - ~~Relate them to scattering amplitudes.~~
  - Evaluate ~~the circular Wilson loop~~ some surfaces exactly using localization?
  - Use defect-CFT techniques including OPE for small deformations around a ~~line or circle~~ plane or sphere.

Surfaces in the free theory

[Henningson Skenderis][Gustavson]

- For the theory with $N = 1$ we define the surface operator explicitly as

$$V_\Sigma = \exp \int_\Sigma \left(iB^+ - n^i \Phi_i \, \mathrm{vol}_\Sigma \right) ,$$

- B^+ is a 2-form with self-dual field strength and Φ_i are 5 scalars.
- Even if free, no easy Lagrangian description because of self-duality constraint.
- For a planar or spherical surface and constant $|n| = 1$, this is globally BPS.
- Can name surfaces with any shape and non-constant $|n| = 1$ “locally BPS”.

Holographic description

[Maldacena]

- A surface operator is captured at large N by an M2-brane in $AdS_7 \times S^4$ ending on the surface operator at the boundary.
- We can sometimes solve for the minimal volume to get the leading large N result.
- Divergences can always be studied by near-boundary analysis.
- n^i determine Dirichlet boundary conditions on S^4 and non-BPS $|n| = 0$ satisfy Neumann boundary conditions.

Calculation 1: Anomalies

[Deser Schwimmer]

- Calculating correlation functions of local operators gives rise to logarithmic divergences, which account for anomalous dimensions.
- Smooth line operators have finite expectation values.
- Cusped Wilson loops have anomalies and the anomaly of the almost straight cusp is related to the Bremsstrahlung function.
- surface operators again have logarithmic divergences, signalling anomalies.

$$\log \langle V_\Sigma \rangle \sim \frac{1}{4\pi} \log \epsilon \int_\Sigma \mathrm{vol}_\Sigma \left[a_1 \mathcal{R}^\Sigma + a_2 \left(H^2 + 4 \,\mathrm{tr}\, P \right) + b \,\mathrm{tr}\, W + c \,(\partial n)^2 \right].$$

  - R^Σ is the Ricci scalar on Σ.
  - H is the mean curvature
  - P the pullback of the Schouten tensor.
  - W is the pullback of the Weyl tensor.
  - $(\partial n)^2 = \partial_m n^i \partial^m n^i$ are couplings to scalar fields.

$$\log \langle V_\Sigma \rangle \sim \frac{1}{4\pi} \log \epsilon \int_\Sigma \mathrm{vol}_\Sigma \left[a_1 \mathcal{R}^\Sigma + a_2 \left(H^2 + 4 \,\mathrm{tr}\, P \right) + b \,\mathrm{tr}\, W + c \,(\partial n)^2 \right].$$

- Under conformal transformations ϵ scales.
- For a spherical surface or radius R in flat space

$$\langle V_{S^2} \rangle \sim \frac{1}{R^{2a_1+4a_2}}\,.$$

- In analogy to local operators, where

$$\langle \mathcal{O}(x)\mathcal{O}(0) \rangle \sim \frac{1}{|x|^{2\Delta_{\mathcal{O}}}}\,.$$

- The anomaly coefficients a_1, a_2, b and c can be different for different types of surface operators (different theories, different representations), but do not depend on the geometry.
- In the same way that $\Delta_{\mathcal{O}}$ can have a classical value and quantum corrections, so the anomaly coefficients have classical and subleading in N terms.

Free field theory

[Henningson Skenderis][Gustavson][ND, Probst Trépanier]

- Expand to quadratic order

$$V_\Sigma = \exp \int_\Sigma \left(iB^+ - n^i \Phi_i \, \mathrm{vol}_\Sigma \right) .$$

- We use (in flat space)

$$\langle \Phi_i(x) \Phi_j(y) \rangle = \frac{\delta_{ij}}{\pi^2 |x-y|^4} ,$$

$$\left\langle B^+_{\mu\nu}(x) B^+_{\rho\sigma}(y) \right\rangle = \frac{\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}}{4\pi^2 |x-y|^4} .$$

- To regularize the integration we displace the two surfaces a distance η along a unit normal vector field ν.

- The 2-form contributes

$$-\frac{1}{2\pi\epsilon^2} - \frac{H \cdot \nu}{4\pi\epsilon} - \frac{1}{16\pi} \left(-2R^\Sigma + 3 \left(H^2 + 4 \, \mathrm{tr}\, P \right) \right) \log \epsilon + \dots$$

- And the scalar

$$\frac{1}{2\pi\epsilon^2} + \frac{H \cdot \nu}{4\pi\epsilon} + \frac{1}{16\pi} \left(2R^\Sigma - \left(H^2 + 4 \, \mathrm{tr}\, P \right) + 4 \left(\partial n \right)^2 \right) \log \epsilon + \dots$$

- Together

$$\log\langle V_\Sigma\rangle = \frac{1}{4\pi}\log\epsilon\int_\Sigma \mathrm{vol}_\Sigma\Big[R^\Sigma - \big(H^2 + 4\,\mathrm{tr}\,P\big) + (\partial n)^2\Big] + \ldots$$

- Therefore ($b^{(1)}$ requires curved space propagators)

$$a_1^{(1)} = +1\,,\qquad a_2^{(1)} = -1\,,\qquad c^{(1)} = +1\,.$$

Holographic calculation

[Graham Witten]

- We generalized Graham and Witten to include non-trivial scalar couplings.
- One needs to study the near boundary equations of motion for an M2-brane in $AdS_7 \times S^4$ ending on the surface operator.
- The result is

$$a_1^{(N)} = 0\,, \qquad a_2^{(N)} = -N\,, \qquad b^{(N)} = 0\,, \qquad c^{(N)} = +N\,.$$

- Comparing to the free field calculation we find that in both cases $a_2 = -c$.
- In fact we can prove this using the displacement operator, as well as prove $b = 0$.
- Exact expressions for a_1 and a_2 exist for all N and any representation. For the fundamental rep they are [Wang][Chalabi, Estes, Jensen, Krym O'Bannon, Robinson, Rogers, Sisti][ND, Giombi Tseytlin,Zhou]

$$a_1^{(N)} = \frac{1}{2} - \frac{1}{2N}\,, \qquad a_2^{(N)} = -N + \frac{1}{2} + \frac{1}{2N}\,.$$

does not contribute
to the anomaly
flat space
unconstrained

defect CFT (dCFT) interlude

- A CFT in D dimensions has an $SO(D+1,1)$ symmetry group ($SO(D,2)$ in Minkowski space).
- For $D = 1,2$ the group is enlarged to the Virasoro group.
- A flat (or spherical) dimension d defect or operator preserves an $SO(d+1,1) \times SO(D-d)$ subgroup.
- Most familiar examples:
  - Local operator, $d = 0$: Preserves $\mathbb{R}_+ \times SO(D)$.
  - Boundary, or codimension-1 defect, $d = D-1$: Preserves $SO(D-1,1)$.
- Operators localized on the submanifold are classified by representation of this group.
- The Ward identities of the subgroup still enforce the usual behavior of n-point functions. So the 2-point function is

$$\langle\!\langle \mathcal{O}(x)\mathcal{O}(y) \rangle\!\rangle \sim \frac{1}{|x-y|^{2\Delta}}$$

Displacement operator

- The displacement operator can be define via

$$\partial_\mu T^{\mu n'}(x) = \mathbb{D}^{n'}(x)\delta^{D-d}(x)$$

- It captures the breaking of translation invariance of the theory in the presence of the defect.
- We can define it like this even if we do not have a Lagrangian for the theory.
- It has a protected dimension $d+1$.
- This definition also makes its normalization well defined, so we can define $c_{\mathbb{D}}$ via

$$\left\langle\!\!\left\langle \mathbb{D}^{m'}(\sigma)\mathbb{D}^{n'}(0) \right\rangle\!\!\right\rangle = \frac{C_{\mathbb{D}}\delta^{mn}}{\pi^2|\sigma|^{2d+2}},$$

- In the case of the Wilson loop, $c_{\mathbb{D}}$ is called the Bremsstrahlung function and determines the radiation of an accelerating charged particle.
- In $\mathcal{N}=4$ SYM, it is known exactly for all N and λ.

Displacement operator and anomalies

[Bianchi, Lemos Madalena, Meineri][Bianchi Lemos] [ND, Probst Trépanier]

- We know that the dispalcement 2-point function is

$$\left\langle\!\left\langle \mathbb{D}^{m'}(\sigma)\mathbb{D}^{n'}(0) \right\rangle\!\right\rangle = \frac{C_{\mathbb{D}}\delta^{m'n'}}{\pi^2|\sigma|^6},$$

- Now consider a small deformation of the plane by an arbitrary normal vector field $\xi^{n'}$. At quadratic order we find

$$\log\langle V\rangle \sim \frac{1}{2}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \langle\!\langle \mathbb{D}_{n'}(\sigma)\mathbb{D}_{m'}(\tau)\rangle\!\rangle\, \xi^{n'}(\sigma)\xi^{m'}(\tau) d^2\sigma\, d^2\tau$$

- Expanding $\tau = \sigma + \eta$, this gives the anomaly density

$$\mathcal{A} = \frac{C_{\mathbb{D}}}{2\pi^2}\int_{\mathbb{R}^2}\frac{\delta_{n'm'}}{|\eta|^6}\xi^{n'}(\sigma)\left[\cdots + \frac{1}{24}\eta^n\eta^m\eta^p\eta^q\partial_n\partial_m\partial_p\partial_q\xi^{m'}(\sigma) + \dots\right]d^2\eta\,.$$

- Doing the angular integration and integrating $|\eta|$ by parts, we find a log divergence multiplying the trace of the second fundamental form squared

$$-\frac{C_{\mathbb{D}}}{64\pi}\int_{\mathbb{R}^2}\partial^n\partial^m\xi^{n'}\partial_n\partial_m\xi^{n'}d^2\sigma = -\frac{C_{\mathbb{D}}}{64\pi}\int_{\Sigma}\mathrm{I\!I}^2\,\mathrm{vol}_{\Sigma} = -\frac{C_{\mathbb{D}}}{64\pi}\int_{\Sigma}\left(H^2 - R^{\Sigma}\right)\mathrm{vol}_{\Sigma}\,.$$

- Since the integral of R^{Σ} vanishes, we identify

$$a_2 = -\frac{C_{\mathbb{D}}}{16}\,.$$

- The displacement operator is part of a mulitplet

$$\partial_\mu T^{\mu m'} = \mathbb{D}^{m'} \delta^4(x),$$

$$\partial_\mu J^\mu = \mathbb{Q} \delta^4(x),$$

$$\partial_\mu j^{\mu i' 5} = \mathbb{O}^{i'} \delta^4(x),$$

- and

$$\langle\!\langle \mathbb{O}_{i'}(\sigma) \mathbb{O}_{j'}(0) \rangle\!\rangle = \frac{C_{\mathbb{O}} \delta_{i'j'}}{\pi^2 |\sigma|^4}.$$

- We can follow the same logic to relate $c_{\mathbb{O}}$ to the coefficient of the $(\partial n)^2$ anomaly

$$c = C_{\mathbb{O}}\,.$$

- Using SUSY ward identities we can relate

$$C_{\mathbb{D}} = 16 C_{\mathbb{O}}\,.$$

- This proves

$$c = -a_2\,.$$

BPS surface operators

[ND, Trépanier]

- The theory has 32 supercharges.
- 'Locally BPS" operators have a half-rank projector at every point along the surface.
  - Generally more than one half-rank projector equation will only have a trivial solution.
- Want compatible equations at all points along the surface.
  - The plane has the same equation at all points, hence globally 1/2 BPS.
- Found 4 classes (and several subclasses) of geometries that allow for BPS observables, with appropriate choice of "scalar coupling" n^i:
  - Type-$\mathbb{R}$: Any curve in $\mathbb{R}^5$ times a line.
  - Type-$\mathbb{C}$: Any holomorphic curve in $\mathbb{R}^6 \sim \mathbb{C}^3$
  - Type-$\mathbb{H}$: Any surface in $\mathbb{R}^4 \subset \mathbb{R}^6$.
  - Type-S: Any surface in $S^3 \subset \mathbb{R}^6$.
- Holographic description in terms of generalized calibrating forms in subspaces of $AdS_7 \times S^4$

Type-$\mathbb{R}$

A curve in $\mathbb{R}^5$ extended over x^6:

$$x^I(u) \subset \mathbb{R}^5 \ (I = 1, \ldots, 5) \quad \text{and} \quad x^6 = v\,,$$

$$n^I(u,v) = \frac{\partial_u x^I}{|\partial_u x|}\,.$$

- Restricting the curve to $\mathbb{R}$, we get a plane.
- With a curve in $\mathbb{R}^2$ can make a crease.
- They are uplifts of “Zarembo Wilson loops” in $\mathcal{N} = 4$ SYM.
- No anomaly. Presumably expectation value vanishes.

Type-$\mathbb{C}$

We choose a complex structure such that $\mathbb{R}^6 \to \mathbb{C}^3$ and take any surface which is holomorphic, with fixed $n^I = \delta^{I5}$.

- Generally preserve two supercharges.
- In $\mathbb{C}^2$ preserves 4.
- In $\mathbb{C}$, it's just the plane, so 16.

- The anomaly is

$$\int_\Sigma \mathcal{A}_\Sigma^{\mathbb{C}} \, \mathrm{vol}_\Sigma = a_1 \chi(\Sigma) \, .$$

- Easy to construct holographic 3-surfaces dual to them.

Type-$\mathbb{H}$

Arbitrary surface in $\mathbb{R}^4 \subset \mathbb{R}^6$. n^I chosen from the tangent space by projecting to the self-dual part, which is in S^2

$$n^I = \epsilon^{ab} \eta^I_{\mu\nu} \partial_a x^\mu \partial_b x^\nu \,.$$

- Generally preserve a single supercharge.
- Restricting to $\mathbb{R}^3$ preserve two supercharges.
- Lagrangian manifolds: $n^I \in S^1$ preserve two supercharges.

- Anomaly related to degree of Gauss map.
- Holographic description in terms of surfaces calibrated with respect to a G_2-structure on $AdS_5 \times S^2$.

Type-S

Take $x^\mu \in S^3$ and n^I according to

$$n^I = \frac{1}{2}\epsilon^{ab}\epsilon^{IJKL}\partial_a x^J \partial_b x^K x^L \,.$$

- Generic surface preserves two combination of Q and S supercharges.
- Restricting to S^2 gives the 1/2 BPS sphere.
- Can construct spherical crease, which is 1/4 BPS.
- Infinitesimal surfaces lead to the same as the $\mathbb{R}^3$ restriction of Type-$\mathbb{H}$.
- Anomaly includes topological part and part related to the area of the surface

$$\int \mathcal{A}^S_\Sigma \, \mathrm{vol}_\Sigma = (a_1 + a_2)\,\chi(\Sigma) + a_2 \frac{\mathrm{vol}(\Sigma)}{2\pi} \,.$$

- Complicated calibration form in $AdS_4 \times S^3$.


propagates

Summary

Type	Geometry	n^I in	SUSYs	Also in	Anomaly
$\mathbb{R}$	$\mathbb{R}\times\gamma,\ \gamma\subset\mathbb{R}^5$	S^4	1 Q		0
	$\mathbb{R}\times\gamma,\ \gamma\subset\mathbb{R}^4$	S^3	1 Q		
	$\mathbb{R}\times\gamma,\ \gamma\subset\mathbb{R}^3$	S^2	2 Q	$\mathbb{H}$	
	$\mathbb{R}\times\gamma,\ \gamma\subset\mathbb{R}^2$	S^1	4 Q	$\mathbb{H}$	
$\mathbb{C}$	$\Sigma\subset\mathbb{C}^3$ (holo.)	point	2 Q		yes
	$\Sigma\subset\mathbb{C}^2$ (holo.)	point	4 Q	$\mathbb{H}$, L	
$\mathbb{H}$	$\Sigma\subset\mathbb{R}^4$	S^2	1 Q		yes
subclass L	Lagrangian	S^1	2 Q		
subclass N	$\Sigma\subset\mathbb{R}^3$	S^2	2 Q	(S)	
S	$\Sigma\subset S^3$	S^3	2 (Q + S)		yes

Special examples with enhanced SUSY

Name	Type	Geometry	n^I in	SUSYs	see
cones	$\mathbb{H}$	over $\gamma \subset S^3$	S^2	Q, S	[Mezei Pufu Wang]
	$\mathbb{H}$, N	over $\gamma \subset S^2$	S^2	$2\mathsf{Q}, 2\mathsf{S}$	
crease	$\mathbb{R}$, $\mathbb{H}$, N	2 half-planes	S^0	$4\mathsf{Q}, 4\mathsf{S} \subset \mathfrak{osp}(4^*\|2)$	[Agmon Wang]
tori	S	$T^2 \subset S^3$	S^4	$2(\mathsf{Q}+\mathsf{S})$	
	$\mathbb{H}$, L	$T^2 \subset \mathbb{R}^4$	S^1	$2\mathsf{Q}, 2\mathsf{S}$	
spheres	$\mathbb{H}$, N	$S^2 \subset \mathbb{R}^3$	S^2	$2\mathsf{Q}, 2\mathsf{S}$	[Berenstein Corrado Fischler Maldacena]
	S	latitude $S^2 \subset S^3$	S^3	$4(\mathsf{Q}+\mathsf{S})$	
	S	large $S^2 \subset S^3$	point	$16(\mathsf{Q}+\mathsf{S}) \subset \mathfrak{osp}(4^*\|2)^2$	
plane	$\mathbb{R}$ $\mathbb{C}$ $\mathbb{H}$ L N	$\mathbb{R}^2$	point	$8\mathsf{Q}, 8\mathsf{S} \subset \mathfrak{osp}(4^*\|2)^2$	[Maldacena]

Conclusion and outlook

- Surface operators are natural observables in the 6d theory.
- They are similar to Wilson loops.
- We can generalize some of the tools used for line operators to study the surface operators.
- We found relations among b, a_2 and c.
- Found very rich examples of BPS observables.

Beyond anomalies

- Most calculations of surface operators lead to logarithmic divergences, which are the surface anomalies.
- Those are like dimensions of local operators.
  - There are more of them: a_1, a_2, b, c.
  - They are known and have only classical, 1-loop and 2-loop corrections.

- We know how to define finite quantities for local operators with arbitrary quantum corrections to their dimensions. In particular their structure constants.
- Likewise can define finite quantities for anomalous surface operators.
- See Maxime's poster.

The end