Defects and brane transport in Abelian gauged linear sigma models

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Mainz, 27.09.2021

Introduction

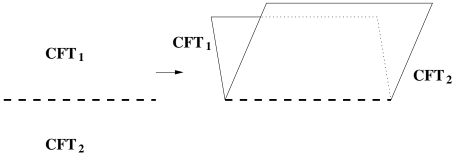
- Gauged linear sigma model: 2 dimensional SUSY Abelian gauge theory.
- Several phases: Landau-Ginzburg phase, geometric phase.
- **Question:** How can we transport "data" from one phase to another? From UV to IR?
- **Data?:** For example boundary conditions/D-branes.
- **Tool:** Defects.

2101.12314 with Fabian Klos and Daniel Roggenkamp

2109.04124 with Lukas Krumpeck and DR

Defects

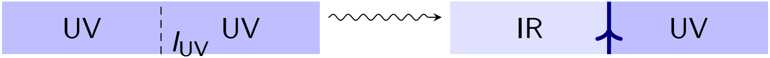
- One dimensional lines which separate two possibly different 2d CFTs/TFTs
- On the defect, there are in general some additional defect degrees of freedom that couple to the bulk.
- Defects can be regarded as boundaries of a folded theory



- But they are more than just boundaries for folded theories.
- They can be moved, merged, intersect ...

Defects and flows, general theories

- Very special class of defects: Flow defects (RG domain wall).
- Defect that separates UV and IR theory
- Obtain them by starting with an initial (UV) theory and restricting the perturbation to a subdomain $U \subset \Sigma$.
- Defect will build up at the boundary of the subdomain ∂U.

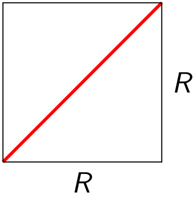


- Functors between category of boundary conditions

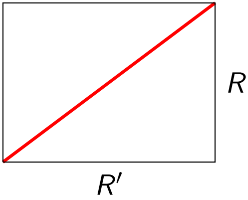
- Merging RG defect with UV boundary condition $\rightarrow$ IR boundary condition.

RG defect in folded picture

- Sigma model with some target geometry
- Toy example: free boson on a circle S^1, radius R
- Folding: Identity defect $\rightarrow$ Diagonal brane on torus $S^1 \times S^1$

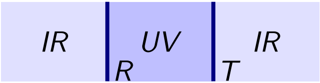


- Deformation of radius $\Rightarrow$ Deformed identity



Features of RG defects

- RG defects are not topological. Fusion with other defects is highly singular.
- Favorable situations: SUSY and topological subsectors
- Fusion in one direction yields identity:



$$R \otimes T = id_{IR}$$

- . . . and a projector in the other direction

$$T \otimes R = P_{UV}$$

Gauged linear sigma models

- UV theory: $G = U(1)^k$ gauge theory, charged matter multiplets Y_i, superpotential, $N = (2, 2)$ supersymmetry
- Potential for scalars

$$U = \sum_{i=1}^{n} \left| \sum_{a=1}^{k} Q_i^a \sigma_a y_i \right|^2 + \frac{e^2}{2} \sum_{a=1}^{k} \left(\sum_{i=1}^{n} Q_i^a |y_i|^2 - r^a \right)^2 + \sum_{i=1}^{n} \left| \frac{\partial W}{\partial y_i}(y_1, \ldots, y_n) \right|^2 . \qquad (1)$$

- Classical vacuum manifold: $U = 0/_{gauge-transformations}$
- Depends on r^a.
- Here: $G = U(1)$,
- Examples with geometric phase:
  Fields P, X_i, $Q(X_i) = 1, Q(P) = -d$,
  homogeneous superpotential $W = PG(X_i)$.
- $r \gg 0 \rightarrow$ Geometric phase:
  $G(X_i) = 0$ hypersurface in projective space
- $r \ll 0 \rightarrow$: stringy Landau Ginzburg phase:
  $P = 1$, Landau-Ginzburg orbifold, orbifold group $\mathbb{Z}_d$

Non-geometric examples

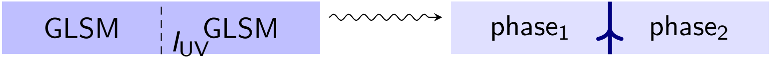
- Fields P, X, superpotential $W = P^{d-n}X^d$
- charges of fields $Q(X) = d - n, \quad Q(P) = -d$
- $r >> 0$: X must not vanish, gets expectation value, LG model with $W \sim P^{d-n}$, gauge symmetry broken to $\mathbb{Z}_{d-n}$
- $r << 0$: X must not vanish, gets expectation value, LG model with $W \sim X^d$, gauge symmetry broken to $\mathbb{Z}_d$
- Quantum effects: r gets renormalized.
- UV phase: $W \sim X^d$.
- IR phase: $W \sim P^{d-n}$ and n massive vacua on the Coulomb branch.
- RG flow drives the model to the IR phase.
- Both phases can be realized within the GLSM. Clingempeel-leFloch-Romo

Setting and strategy

- Consider gauged linear sigma models with different phases.
- Go to the topological sector, B-twist
- Decouple gauge degrees of freedom.
- GLSM $\rightarrow$ $U(1)$ equivariant LG model.
- We want to connect the phases with defects.
- Branes in a geometric phase: Derived category of coherent sheaves.
- Branes in LG phase: Category of matrix factorizations of the superpotential.
- We can transport D-branes between phases using defects.

GLSM situation

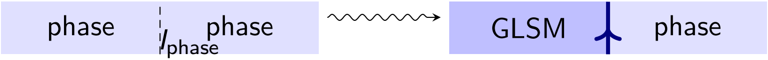
- Same UV theory, two different limits or phases
- Perturb differently on the two sides of the UV identity
- Embed two different phases into the GLSM



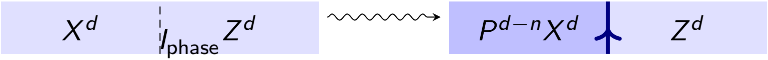
- If we manage to do this, we have constructed the desired functors between phases.
- This involves understanding the identity, as well as how to go to the phases.
- Factorize this defect:

Transition defects T^i

- Some preliminary considerations:
- Starting point: (known) Identity defect in a phase.
- "Lift" on one side to GLSM



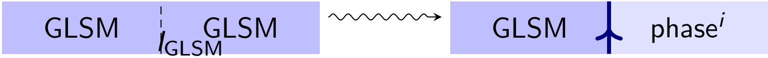
- LG example



- $U(1)$ vs $\mathbb{Z}_d$-invariance $\rightarrow$ Lift involves a choice $a \in \mathbb{Z}$, transition defects T^i_a
- In general, there can be many lifts.

Flowing from the GLSM identity

- Other starting point: GLSM identity defect



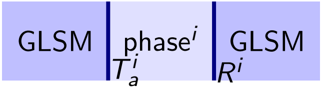
- This constructs the right transition defects T_a^i.
- ..... once we understand the identity defect of the GLSM
- .....and how to push it to a phase on one side.
- Indeed, we can then check that these T_a^i factorize the phase transition

together with suitable R^i.

Further properties

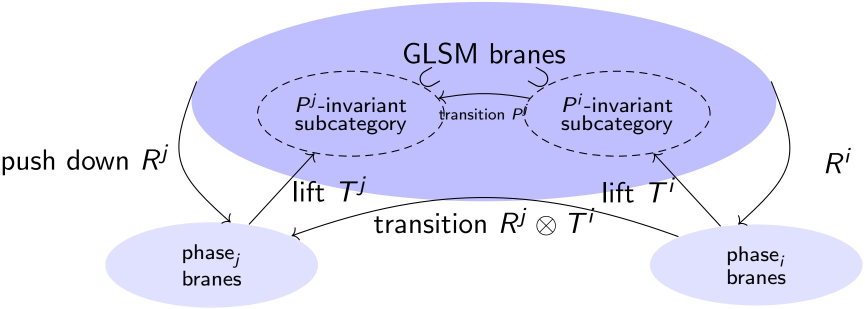
- For a fixed phase i, R^i and T^i_a can be used to embed the phase into the GLSM
- $R^i \otimes T^i_a = id^i$

- $T^i_a \otimes R^i = P^i_a$



- P^i_a is a projector and realizes the brane category of the phase inside the GLSM.

Summary

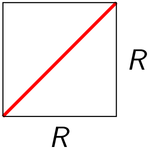

GLSM branes
P^j-invariant subcategory
transition P^j
P^i-invariant subcategory
push down R^j
R^i
lift T^j
lift T^i
transition $R^j \otimes T^i$
phase$_j$ branes
phase$_i$ branes

Comparison

- Brane transport in GLSMs was discussed before Herbst-Hori-Page, Hori-Romo, Knapp-Romo-Scheidegger, Clingempeel-le Floch-Romo,
- There: Analysis of gauge sector, boundary potentials, amplitudes on hemispheres (...)
- Derivation of the “Grade restriction rule”: Smooth transport of branes has to go through a ”window” in the GLSM.
- Our work uses completely different arguments.
- The results on D-brane transport are in agreement.
- We construct a concrete defect: Explicit functor.
- At the heart of the whole construction is the identity defect of the GLSM.

Identity defect, simplified version

- Free boson: diagonal brane on torus $S^1 \times S^1$



- We want: Theories described by polynomial rings, $\mathbb{C}[x]$
- "doubled geometry" described by $\mathbb{C}[x, y]$.
- diagonal: mod out by the ideal generated by $(x - y)$, to get $M_{id} = \mathbb{C}[x, y]/\langle x - y \rangle$
- $0 \to \mathbb{C}[x, y] \xrightarrow{\text{x-y}} \mathbb{C}[x, y] \to M_{id} \to 0$
- "Branes": Described by polynomial ring modulo some ideal, here: polynomial ring in the y variable, e.g. $\mathbb{C}[y]/\langle y^n \rangle$
- $\mathbb{C}[x, y]/\langle x - y \rangle \otimes_{\mathbb{C}[y]} \mathbb{C}[y]/\langle y^n \rangle = \mathbb{C}[x]/\langle x^n \rangle$

Identity defect in LG

- Defects in LG models are matrix factorizations of the difference of the superpotential of the two theories.

$$P : P_1 \underset{p_0}{\overset{p_1}{\rightleftarrows}} P_0$$

- $p_1 p_0 = W_1(X_i) - W_2(X_i')$
- Replace this by the module $M_P = \text{coker } p_1$.
- The matrix factorization provides a two periodic free resolution of M_P.
- Eg single variable case
  Identity defect is a MF of $W(X) - W(X')$ with $p_1 = X - X'$.
- Straight forward generalization to many variables.

Identity defect and orbifolds

- Orbifold group G, G finite group

$$I_{\text{orb.}} = \bigoplus_{g \in G} {}_g I_{\text{non-orb.}}, \qquad {}_g I_{\text{non-orb.}} : \text{ symmetry defect}$$

- Standard orbifold construction as in string theory for D-branes.
- ${}_g I$ is a defect implementing a symmetry transformation.
- Example $W = X^d$, symmetry group $\mathbb{Z}_d$, generator $X \to \eta X$, $\eta = e^{2\pi i/d}$.
- ${}_g I$ as explicit MF: $p_1 = (X - \eta Y)$,
- ... and $p_0 = (X^d - Y^d)/(X - \eta Y)$

GLSM identity defect

- Problem to solve: How to deal with continuous orbifold groups?
- Introduce two new defect fields α and α^{-1}.
- To formulate the matrix factorization, replace $X - Y$ by $X - \alpha^{Q_X} Y$.
- Altogether, consider the module

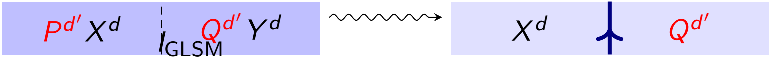
$$M_I = C_{(X,P)(Y,Q)}[\alpha, \alpha^{-1}]/(P - \alpha^{Q_P} Q, X_i - \alpha^{Q_i} Y_i, \alpha\alpha^{-1} - 1)$$

where $C_{(X,P)(Y,Q)} = \mathbb{C}[X_1, \ldots, X_N, P, Y_1, \ldots, Y_N, Q]/(W(P, X_i) - W(Q, Y_i))$

- This is the identity defect of the GLSM. It acts on Branes as identity.

Descending from the GLSM identity

- This defect acts as identity on the GLSM brane category.
- Starting from it, we can construct the transition defects.
- Example: 2 different LG phases:



- Flow defect between different LG phases: $P = Y = 1$
- In addition, a cutoff for the α variable has to be specified.
- The cutoff is part of the data specifying the defect.

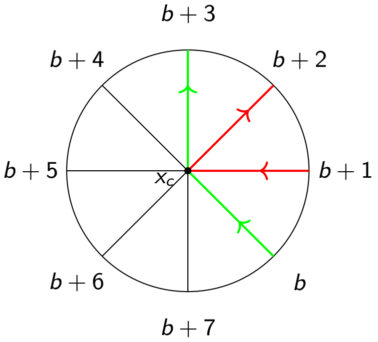
$$M_I = C_{(X,P)(Y,Q)}[\alpha, \alpha^{-1}]/(P - \alpha^{-d} Q, X - \alpha^{d'} Y, \alpha\alpha^{-1} - 1)$$

$$\to M_F = \alpha^N C_{(X,.)(.,Q)}[\alpha^{-1}]/(1 - \alpha^{-d} Q, X - \alpha^{d'} 1)$$

- Resulting defects reproduce known results on RG flows between SUSY $N = 2$ minimal models.

Mirror perspective: A-branes in LG models

- LG orbifold $X^d/\mathbb{Z}_d$ is mirror to LG model with $W = X^d$.
- B-branes get mapped to A-branes
- A-branes: described by straight lines emanating from a critical point, reality condition on W. Hori, Iqbal, Vafa
- RG flow: relevant perturbation by lower order polynomial

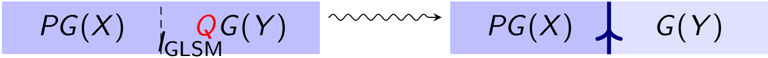


RG flows: Mirror perspective

- Under a perturbation, the critical point splits up and some (elementary) branes decouple.
- The defect describing the flow contains precisely the information on which "wedges" decouple.
- Here: We obtained a lift of known flows to the GLSM model.

Transition to a geometric phase

- GLSM superpotential $W = PG(X_i)$.
- The transition defects LG-GLSM can be constructed as before.



- This defines a matrix factorization of $PG(X) - G(Y)$.

$$T_N : T_1 \underset{t_0}{\overset{t_1}{\rightleftarrows}} T_0$$

- Applying T_N to LG branes, we lift them to "grade restricted" Herbst-Hori-Page branes of the GLSM.
- To go to the geometric phase, apply Knörrer periodicity. "Integrate out" the field P and restrict to $G = 0$.
- Result: Semi-twisted double complex, complex of matrix factorizations.
- Some steps:

$$T_s = T_s^0 \oplus PT_s^1 \oplus P^2 T_s^2 \oplus \ldots, \quad t_s = t_s^0 + Pt_s^1$$

Conclusions

- Discussion of functors between brane categories in different phases of a GLSM.
- Functors are given in terms of defects, e.g. T between phase and GLSM.
- Construction relies on rigidity of SUSY and defect constructions.
- Provides an alternative point of view on brane transport between phases.
- Possible applications in many classes of examples.