JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Mainz Institute for Theoretical Physics

Light-Cone Distribution Amplitudes of Hadrons in QCD and their Applications

Discussion session on dimeson LCDAs, January 23,2020

A.Khodjamirian:

– dimeson DAs: definitions, properties and inputs

– use in LCSRs for $B \to 2\pi$ form factors

– open problems and outlook

K. Keri Vos:

– applications to heavy meson nonleptonic decays

Dipion LCDAs and LCSRs for $B \to \pi\pi$ form factors

Alexander Khodjamirian

UNIVERSITÄT SIEGEN
Theoretische Physik 1

DFG Collaborative Research Center

Distribution amplitude (LCDA) of a single pion

- twist-2 LCDA:

$$\langle \pi^+(p)|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle = -if_\pi p_\mu \int\limits_0^1 du e^{iu\,p\cdot x}\varphi_\pi(u,\mu) + twist4 + ...$$

- expansion in Gegenbauer polynomials

$$\varphi_\pi(u,\mu) = 6u(1-u)\Big[1 + \sum_{n=2,4,..} a_n(\mu) C_n^{3/2}(2u-1)\Big],$$

$$a_n \sim \langle \pi(p)|\bar{u}C_n^{3/2}(D)\gamma_\mu\gamma_5 d|0\rangle, \quad a_n(\mu) \sim [Log(\mu/\Lambda_{QCD})]^{-\gamma_n} \to 0 \quad \text{at } \mu \to \infty$$

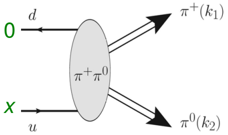
[ERBL evolution]

- twist-3 and twist-4 LCDAs are well elaborated

Dipion light-cone distribution amplitudes

- Prehistory: "wave functions of meson pairs" in exclusive processes at large Q^2 with large inv. masses, *[A.Grozin, (1982-1985)]*
- History: dipion LCDAs introduced to describe $\gamma^*(Q^2)\gamma \to 2\pi$, $\gamma^*(Q^2)N \to 2\pi N$,..., at large Q^2 and small dipion mass

[M. Diehl, T. Gousset, B. Pire and O. Teryaev, (1998)
D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes and J. Horejsi, (1994)
M. V. Polyakov, (1999)] , M. V. Polyakov, C. Weiss (1999)



- twist-2 DAs: analog for $I=0$; scalar dipion; symmetry relations etc.

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\parallel}^{I=1}(u,\zeta,k^2)\,,$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\perp}^{I=1}(u,\zeta,k^2)$$

- the "angular" variable: $\zeta = k_1^+/k^+,\ 1-\zeta = k_2^+/k^+\,,\ \zeta(1-\zeta) \geq \frac{m_\pi^2}{k^2}$.

$q\cdot\bar{k} = \frac{1}{2}(2\zeta-1)\lambda^{1/2}(p^2,q^2,k^2)\,,$ in dipion c.m. $(2\zeta-1) = (1-4m_\pi^2/k^2)^{1/2}cos\theta_\pi$.

What do we know about 2-pion DAs ?

- Gegenbauer$\otimes$ partial wave expansion *[M.Polyakov(1999)]*

$$\Phi_\perp(u,\zeta,k^2) = \frac{6u(1-u)}{f_{2\pi}^\perp} \sum_{n=0,2,..}^{\infty} \sum_{\ell=1,3,..}^{n+1} B_{n\ell}^\perp(k^2) C_n^{3/2}(2u-1)\beta_\pi P_\ell^{(0)}\left(\frac{2\zeta-1}{\beta_\pi}\right)$$

- $B_{n\ell}^{\|(\perp)}(k^2)$ are complex functions (not numbers !) $\quad C_\ell^{1/2} \sim P_\ell$
  $B_{n\ell}^{\|}(k^2) \sim \langle 2\pi|\bar{u}D^n\gamma^\mu d|0\rangle,\ \ell \leq n+1$, $B_{01}(k^2) = F_\pi(k^2)$

- ERBL renormalization of B_{nl}, the same as for $a_n^{(\pi)}$
- normalization conditions $\rightarrow$ pion timelike form factors ,

$$\int_0^1 du \left\{ \begin{array}{c} \Phi_{\|}^{l=1}(u,\zeta,k^2) \\ \Phi_{\perp}^{l=1}(u,\zeta,k^2) \end{array} \right. = (2\zeta-1) \left\{ \begin{array}{c} F_\pi^{em}(k^2) \\ F_\pi^{t}(k^2) \end{array} \right. \quad \begin{array}{c} \text{pion e.m. form factor} \\ \text{pion “tensor” form factor} \end{array}$$

- $F_\pi^{em}(0) = 1$, , “tensor” charge of the pion $F_\pi^t(0) = 1/f_{2\pi}^\perp$
- due to accurate data on $F_\pi(k^2)$ we know the asymptotic DA
  $\Phi_{\|}(z,\zeta,k^2) = 6z(1-z)(2\zeta-1)F_\pi(k^2)$
- less known is $\Phi_\perp$ and nonasymptotic effects in both DAs

What do we know about LCDAs

[M.Polyakov(1999)]

- the soft pion limit relates $B_{nl}(0)$ to $a_n^{(\pi)}$:

$$a_n^{(\pi)} = \sum_{\ell=1}^{n+1} B_{n\ell}^{\|}(0)$$

- Omnes representation for the k^2-dependence

$$B_{nl}^I(w^2) = \sum_{k=0}^{N-1} \frac{w^{2k}}{k!} \frac{d^k}{dw^{2k}} B_{nl}^I(0) + \frac{w^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\tan \delta_l^I(s) \mathrm{Re}\, B_{nl}^I(s)}{s^N(s - w^2 - i0)} . \tag{5.8}$$

The solution of such a type of dispersion relation was found long ago by Omnes [17] and has the exponential form

$$B_{nl}^I(w^2) = B_{nl}^I(0) \exp \left\{ \sum_{k=1}^{N-1} \frac{w^{2k}}{k!} \frac{d^k}{dw^{2k}} \log B_{nl}^I(0) + \frac{w^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_l^I(s)}{s^N(s - w^2 - i0)} \right\} . \tag{5.9}$$

- approixmating the P-wave phase shift with ρ resonance yields nontrivial relations between $B_{n\ell=1}$ and a_n^ρ
- instanton vacuum model for the coefficients,
  $n = 0, 2, 4$, valid at small $k^2 \sim 4m_\pi^2$ *[M. V. Polyakov and C. Weiss, (1999)]*

$$B_{01}^{\perp}(k^2) = 1 + \frac{k^2}{12M_0^2}, \; B_{21}^{\perp}(k^2) = \frac{7}{36}\left(1 - \frac{k^2}{30M_0^2}\right), \; B_{23}^{\perp}(k^2) = \frac{7}{36}\left(1 + \frac{k^2}{30M_0^2}\right), \ldots$$

Dipion LCDAs in LCSRs for $B \to 2\pi$ form factors

[Ch. Hambrock, AK, 1511.02509 [hep-ph]]

[S.Cheng, AK, J.Virto 1709.01173[hep-ph]]

- the method: similar to the LCSRs for $B \to \pi$ form factors,
- we consider only $\bar{B}^0 \to \pi^+\pi^0\ell^-\nu_\ell$, isospin 1, $L = 1, 3, , ..$
- only LO, twist-2 approximation for dipion DAs available
- problems to be addressed:
  - how important are $L > 1$ partial waves of 2π state in $B \to \pi\pi$?
  - $B \to \rho$ dominance in the P-wave?

The method of LCSRs

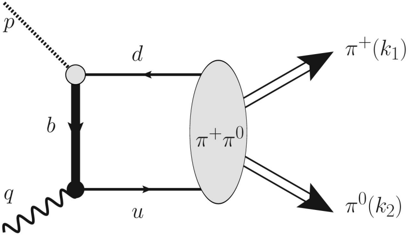
- The correlation function: $k = k_1 + k_2,\ \bar{k} = k_1 - k_2$

$$\Pi_\mu(q, k_1, k_2) = \\ = i \int d^4x\, e^{iqx} \langle \pi^+(k_1)\pi^0(k_2) | T\{\bar{u}(x)\gamma_\mu(1-\gamma_5) b(x), i m_b \bar{b}(0)\gamma_5 d(0)\} | 0 \rangle \\ = i\epsilon_{\mu\alpha\beta\rho} q^\alpha k_1^\beta k_2^\rho\, \Pi^{(V)} + q_\mu \Pi^{(A,q)} + k_\mu \Pi^{(A,k)} + \bar{k}_\mu \Pi^{(A,\bar{k})},$$

- the invariant amplitudes $\Pi^{(V),(A,q),..)}(p^2, q^2, k^2, q\cdot\bar{k})$, $p = (k+q)$
- OPE valid at $q^2 \ll m_b^2$ (b-quark virtual)

  $k^2 \ll m_b^2$ (2-pion system produced near the LC)
- LO diagram:

  $\langle b(x)\bar{b}(0)\rangle \to S_b(x, 0)$
- vacuum $\to$ on-shell dipion hadronic matrix elements of nonlocal $\bar{u}(x)d(0)$ operators



Result for the correlation function in twist-2 approx.

- at LO, twist-2 accuracy:

$$\Pi_\mu(q,k_1,k_2) = i\sqrt{2}m_b \int\limits_0^1 \frac{du}{(q+uk)^2-m_b^2}\Big\{\Big[(q\cdot\bar{k})k_\mu - \Big((q\cdot k)+uk^2\Big)\bar{k}_\mu$$

$$+i\epsilon_{\mu\alpha\beta\rho}q^\alpha k_1^\beta k_2^\rho\Big]\frac{\Phi_\perp(u,\zeta,k^2)}{2\zeta-1} - m_b k_\mu \Phi_\parallel(u,\zeta,k^2)\Big\}\,.$$

- read off invariant amplitudes: $\Pi^{(V)}$, $\Pi^{(A,k)}$, $\Pi^{(A,\bar{k})}$, $\Pi^{(A,q)} = 0$
- transform to a form of dispersion integral in the variable p^2:
  $s(u) = \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{u}$

$$\Pi^{(r)}(p^2,q^2,k^2,\zeta) = \sum_{i=\parallel,\perp} f_i^{(r)}(p^2,q^2,k^2,\xi)\int\limits_{m_b^2}^{\infty}\frac{ds}{s-p^2}\left(\frac{du}{ds}\right)\Phi_i(u(s),\zeta,k^2)\,.$$

Hadronic dispersion relation

- the ground B-meson state contribution:

$$\Pi_\mu(q,k_1,k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2 - p^2} + ... ,$$

- expansion of $B \to \pi\pi$ matrix element in form factors:

$$\begin{aligned} i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = & -F_\perp(q^2,k^2,\zeta)\,\frac{4}{\sqrt{k^2\lambda_B}}\, i\epsilon^{\mu\alpha\beta\gamma}\, q_\alpha\, k_{1\beta}\, k_{2\gamma} \\ & + F_t(q^2,k^2,\zeta)\,\frac{q^\mu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k^\mu - \frac{k\cdot q}{q^2}q^\mu\right) \\ & + F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}^\mu - \frac{4(q\cdot k)(q\cdot \bar{k})}{\lambda_B}\,k^\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q^\mu\right), \end{aligned}$$

- quark-hadron duality in the B-channel, $\Rightarrow$ effective threshold s_0, Borel transformation , $p^2 \to M^2$

LCSRs for the form factors at twist-2 in LO

- in both sum rules only the chiral-odd twist-2 DA contributes:

$$\frac{F_\perp(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2 (1-2\zeta)} \int\limits_{u_0(s_0)}^{1} \frac{du}{u} \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}}\,,$$

$$\frac{F_\parallel(q^2,k^2,\zeta)}{\sqrt{k^2}} = \frac{m_b}{\sqrt{2} f_B m_B^2 (1-2\zeta)} \int\limits_{u_0(s_0)}^{1} \frac{du}{u^2} \left(m_b^2 - q^2 + k^2 u^2\right) \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2\bar{u} + k^2}{uM^2}}$$

- an additional relation between the axial-current form factors:

$$\frac{1}{\sqrt{\lambda_B}}(m_B^2 - q^2 - k^2) F_0(q^2,k^2,\zeta) = F_t(q^2,k^2,\zeta) + 2\frac{\sqrt{k^2}\sqrt{q^2}(2\zeta-1)}{\sqrt{\lambda_B}} F_\parallel(q^2,k^2,\zeta)\Big]\,.$$

- a sum rule for F_t is obtained from a slightly different correlation function

Sum rules for partial waves

- The form factors expanded in partial waves:

$$F_{\perp,\parallel}(q^2,k^2,\zeta) = \sum_{\ell=1}^{\infty} \sqrt{2\ell+1}\, F^{(\ell)}_{\perp,\parallel}(q^2,k^2)\, \frac{P^{(1)}_{\ell}(\cos\theta_\pi)}{\sin\theta_\pi}\,,$$

$\zeta \sim \cos\theta$, $P_l^{(m)}$ -the (associated) Legendre polynomials

- sum rules for separate partial waves

$$F^{(\ell)}_{\perp}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2} f^{\perp}_{2\pi}} \frac{\sqrt{\lambda_B} m_b}{m_B^2 f_B} e^{m_B^2/M^2} \sum_{n=0,2,..} \sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B^{\perp}_{n\ell'}(k^2) J^{\perp}_{n}(q^2,k^2,M^2,s_0^B)\,,$$

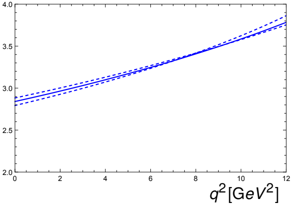
$$F^{(\ell)}_{\parallel}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2} f^{\perp}_{2\pi}} \frac{m_b^3}{m_B^2 f_B} e^{m_B^2/M^2} \sum_{n=0,2,4,..} \sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B^{\perp}_{n\ell'}(k^2) J^{\parallel}_{n}(q^2,k^2,M^2,s_0^B)\,,$$

- $I_{\ell\ell'}$ - integrals over Legendre polynomials,
- $J_n^{\perp,\parallel}$ - the Borel-weighted integrals over $C_n^{3/2}(2u-1)$
- in the limit of asymptotic DA, ($B_{01} \neq 0$, $B_{n>0,\ell} = 0$),
  only P-wave form factors are $\neq 0$

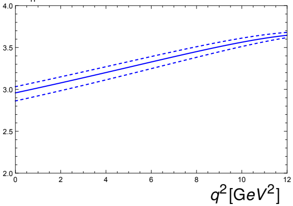
Numerical results

- *P*-wave form factors: (only twist-2)

$F_{\perp}^{(\ell=1)}(q^2, k^2_{min})$

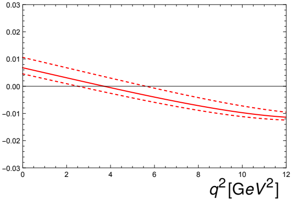


$F_{\parallel}^{(\ell=1)}(q^2, k^2_{min})$

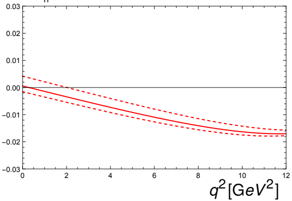


- *P*-wave dominance: ratios of *F*- and *P*-wave form factors

$R_{\perp}^{(\ell=3)}(q^2, k^2_{min})$



$R_{\parallel}^{(\ell=3)}(q^2, k^2_{min})$



- - - - uncertainties from the variation of M^2.

How much $B \to \rho$ contributes to the $B \to 2\pi$?

- dispersion relations for the $B \to \pi\pi$ *P*-wave ($\ell = 1$) form factors:

$$\frac{\sqrt{3}F_{\perp}^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\,\frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + \ldots$$

and

$$\frac{\sqrt{3}F_{\parallel}^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}(m_B + m_\rho)A_1^{B\to\rho}(q^2) + \ldots$$

$$\Gamma_\rho(k^2) = \frac{m_\rho^2}{k^2}\left(\frac{k^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2}\theta(k^2 - 4m_\pi^2)\Gamma_\rho^{tot}\,,$$

- using the definition of $B \to \rho$ FFs:

$$\langle\rho^+(k)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = \epsilon_{\mu\alpha\beta\gamma}\epsilon_\alpha^{*(\rho)}p^\beta k^\gamma\,\frac{2V^{B\to\rho}(q^2)}{m_B + m_\rho}$$
$$-i\epsilon_\mu^{*(\rho)}(m_B + m_\rho)A_1^{B\to\rho}(q^2) + \ldots$$

LCSRs for $B \to \rho$ form factors

e.g., [P. Ball and V. M. Braun, Phys. Rev. D **55** (1997) 5561]

- LCSRs for $B \to \rho$ form factors ($\Gamma_\rho = 0$)
  in terms of the ρ-meson DAs in the twist-2 approximation:

$$V^{B\to\rho}(q^2) = \frac{(m_B + m_\rho)m_b}{2m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int\limits_{u_0}^{1} \frac{du}{u}\, \phi_\perp^{(\rho)}(u)\, e^{-\frac{m_b^2 - q^2\bar{u} + m_\rho^2 u\bar{u}}{uM^2}} ,$$

$$A_1^{B\to\rho}(q^2) = \frac{m_b^3}{2(m_B + m_\rho)m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int\limits_{u_0}^{1} \frac{du}{u^2}\, \phi_\perp^{(\rho)}(u) \left(1 - \frac{q^2 - m_\rho^2 u^2}{m_b^2}\right) e^{-\frac{m_b^2 - q^2\bar{u} + m_\rho^2 u\bar{u}}{uM^2}} .$$

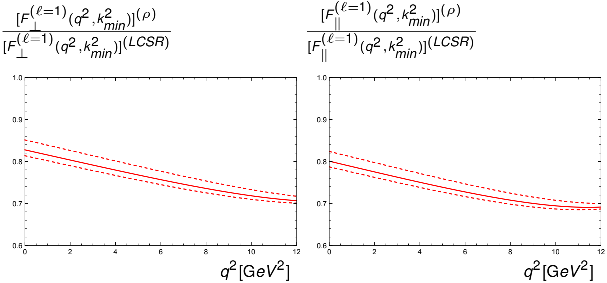
- both sum rules determined by the chiral-odd ρ-meson DA:

$$\langle \rho^+(k)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = -if_\rho^\perp (\epsilon_\mu^{*(\rho)} k_\nu - k_\mu \epsilon_\nu^{*(\rho)}) \int\limits_0^1 du e^{iuk\cdot x} \phi_\perp^{(\rho)}(u) ,$$

- the Gegenbauer polynomial expansion:

$$\phi_\perp^{(\rho)}(u) = 6u(1-u) \left(1 + \sum_{n=2,4,,..} a_n^{(\rho)\perp} C_n^{3/2}(2u-1)\right) ,$$

Numerical estimates



Relative contribution of ρ-meson to the $B \to \pi^+\pi^0$ P-wave form factors

$F_{\perp}^{(\ell=1)}(q^2, k^2_{min})$ *(left panel) and* $F_{||}^{(\ell=1)}(q^2, k^2_{min})$ *(right panel) from LCSRs.*

Dashed lines - the uncertainty due to the variation of the Borel parameter.

Open problems and outlook

- determination of Gegenbauer functions $B_{nl}^{\perp,\parallel}(k^2)$ at $k^2 \lesssim 1$ GeV^2
  from dispersion (Omnes) representations at $k^2 > 4m_\pi^2$, combined with dedicated LCSRs at $k^2 < 0$
- to identify twist-3,4 DAs and their double expansions,
  the methods used for vector meson DAs [V.Braun et al.]
- $\pi^+\pi^-, \pi^0\pi^0$ dipion LCDAs in S- and D-waves,
  involving scalar f_0 and, respectively, tensor f_2 resonances
- to work out dimeson DAs for $K\pi$ state
  a simplified S-wave DA used in [U. G. Meissner, W. Wang, (2014)]
- use of dedicated two-point sum rules,
  e.g., for the normalization of pion tensor form factor
- use of accurate data on $\gamma\gamma^* \to 2\pi$
  cross section and angular distributions (Belle-II, BESS)
- use of other data (electroproduction of pion pairs)

B. Clerbaux and M. V. Polyakov, [hep-ph/0001332]