Light-cone distribution amplitudes from Euclidean correlation functions

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EPJ **C78** (2018) 217, PRD **98** (2018) 094507

Definition of distribution amplitudes

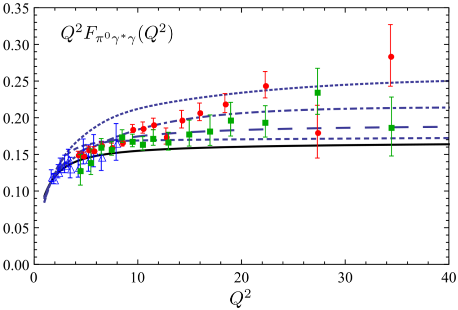
$$|\pi\rangle = |\bar{q}q\rangle + |\bar{q}gq\rangle + \dots$$

- hard exclusive processes are sensitive to
  - Fock states with smallest number of partons
  - the distribution of the momentum within a Fock state at small transverse distances
- this information is contained in light-cone DAs; leading twist DA ϕ_π

$$\langle 0|\bar{u}(z)[z,-z]\not{z}\gamma_5 u(-z)|\pi(p)\rangle = iF_\pi \, p\cdot z \int_0^1 du \, e^{i(2u-1)p\cdot z}\phi_\pi(u,\mu) \qquad \underline{z^2 = 0}$$

- quark and antiquark carry the momentum fraction u and $\bar{u} = 1-u$, respectively
- physical information: *complementary* to PDFs
- lattice technique: very *similar* to PDFs

The BaBar Puzzle



plot taken from PRD **86** (2012) 077504

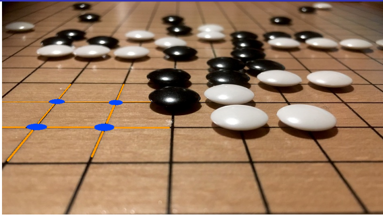
- data from

  CLEO (1998, blue trianlges)
  BaBar (2009, red circles)
  Belle (2012, green squares)
- solid line: result obtained for the asymptotic pion DA $\phi(u) = 6u(1-u)$
- dashed lines: results for various DA models

- BaBar Puzzle: the continuous rising exhibited by the BaBar data seemed to contradict collinear factorization at intermediate momentum transfer
- the Belle data does not support such a conclusion anymore

$\Rightarrow$ additional information from lattice QCD is highly valuable

Lattice QCD in a nutshell



- evaluate pathintegral numerically on a 4D lattice
- the quark fields q live on lattice sites
- the gauge field U is represented by 3×3 matrices on the links between the sites
- after integrating out fermionic degrees of freedom, e.g.,

$$\langle q(x)\bar{q}(y)\rangle = \frac{1}{Z}\int \mathcal{D}U \det(M[U])e^{-S_E[U]}\left(M[U]\right)^{-1}_{xy}$$

$M \equiv$ Dirac matrix

- one considers Euclidean space-time (i.e., imaginary times)
  $\Rightarrow \det(M[U])e^{-S_E[U]}$ can be used as weight in a Monte-Carlo integration
- small problem: we cannot evaluate quark fields at light-like separations

Lattice methods

Problem: on a Euclidean space-time one cannot realize nontrivial lightlike distances

- **traditional solution:** calculate Mellin moments of the DAs ($\hat{=}$ local derivative ops.)
  talk by Gunnar, yesterday (JHEP **1908** (2019) 065, EPJ **A55** (2019) 116)
  - higher moments $\rightarrow$ problems with renormalization (operator mixing)
- **new approach:** relate DAs to correlation functions at spacelike distance

  $\rightarrow$ requires large hadron momenta
  $\rightarrow$ relies heavily on pQCD
  $\rightarrow$ large higher twist contributions

  - **Option 1: use a nonlocal operator** $\langle 0|\bar{q}(z)\Gamma[z,0]q(0)|\pi\rangle$ $\underline{z^2 < 0}$
    Ji, PRL **110** (2013) 262002
  - **Option 2: use two local operators** $\langle 0|\bar{q}(z)\Gamma_1 q(z)\bar{q}(0)\Gamma_2 q(0)|\pi\rangle$ $\underline{z^2 < 0}$
    Braun, Müller, EPJ **C55** (2008) 349
    Ma, Qiu, PRL **120** (2018) 022003
  - . . . (e.g., scalar auxiliary quark, heavy quark, etc.)

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    Ma, Qiu, PRL **120** (2018) 022003
- nice features of **Option 2:**
  - circumvents all problems with renormalization of nonlocal operators
  - off-axis directions possible (no problems with cusp anomalous dimension)

DA $\longleftrightarrow$ correlation function (schematically & oversimplified)

$$\text{DA} \xleftarrow{\text{LaMET, } p_z \to \infty} \text{quasi-DA} \xleftarrow{\text{FT } (z)} \text{lattice data}$$

PRD **91** (2015) 054510, PRD **92** (2015) 014502, NPB **911** (2016) 246, PRD **98** (2018) 054504, and many more...

PRD **95** (2017) 094514 (DAs), NPB **939** (2019) 429 (DAs)

or

$$\text{DA} \xleftarrow{\text{pQCD, } z^2 \to 0} \text{pseudo-DA} \xleftarrow{\text{FT } (\nu = \mathbf{p} \cdot \mathbf{z})} \text{lattice data}$$

PRD **96** (2017) 034025, PRD **96** (2017) 094503, EPJWC **175** (2018) 06032, PRD **100** (2019) 114512, . . .

or

$$\text{DA} \xrightarrow{\text{(analyze directly in position space)}} \text{lattice data}$$

our ansatz: (also works when using the Wilson-line operator)

- parametrize DA (& higher twist effects) and fit directly to the lattice data

recently: JHEP **10** (2019) 137 (qPDF), PRD **100** (2019) 034516 (qPDF),

PRD **100** (2019) 114512 (pPDF), PRD **99** (2019) 074507 (latt-σ)

- basic idea very similar to the socalled "lattice cross section" approach for PDFs

cf. arXiv:2001.04960 (pion PDF)

[also called "factorizable matrix elements", PoS LATTICE2018 (2018) 018 (nice review by C. J. Monahan)]

Matrix elements $\leftrightarrow$ DAs

$$\mathbb{T}_{\rm XY}(p\cdot z, z^2) = \langle 0 | J_X^\dagger(\tfrac{z}{2}) J_Y(-\tfrac{z}{2}) | \pi^0(p)\rangle$$

$$J_S = \bar{q}\,u\,, \quad J_P = \bar{q}\gamma_5 u\,, \quad J_V^\mu = \bar{q}\gamma^\mu u \equiv J_{V^\mu}\,, \quad J_A^\mu = \bar{q}\gamma^\mu\gamma_5 u \equiv J_{A^\mu}$$

$$\mathbb{T}_{\rm SP} = T_{\rm SP}$$

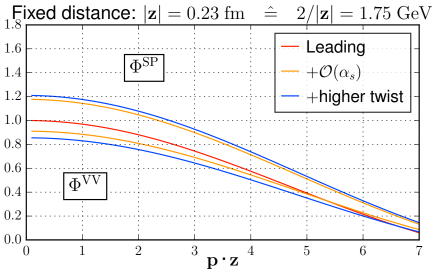
$$\mathbb{T}^{\mu\nu}_{\rm VV} = \frac{i\varepsilon^{\mu\nu\rho\sigma} p_\rho z_\sigma}{p\cdot z} T_{\rm VV}$$

$$\mathbb{T}^{\mu\nu}_{\rm VA} = \frac{p^\mu z^\nu + z^\mu p^\nu - g^{\mu\nu} p\cdot z}{p\cdot z} T_{\rm VA} + \frac{p^\mu z^\nu - z^\mu p^\nu}{p\cdot z} T^{(2)}_{\rm VA} + \frac{2 z^\mu z^\nu - g^{\mu\nu} z^2}{z^2} T^{(3)}_{\rm VA} + \frac{2p^\mu p^\nu - g^{\mu\nu} p^2}{p^2} T^{(4)}_{\rm VA} + g^{\mu\nu} T^{(5)}_{\rm VA}$$

- similar for PS, AA, AV
- q is an auxiliary quark $q \neq u, d$, but $m_q = m_u = m_d$

Matrix elements $\leftrightarrow$ DAs

$$T_{\mathrm{XY}}(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4} \underbrace{\int_0^1 du\, e^{i(u-1/2)p\cdot z}\phi_\pi(u) + \mathcal{O}(\alpha_s) + \text{higher twist}}_{\equiv\, \Phi^{\mathrm{XY}}(p\cdot z, z^2)}$$



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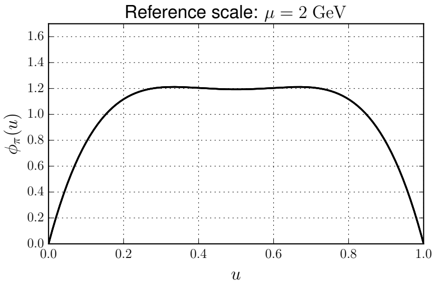

- twist 4 effects estimated using asymptotic shape for chiral-odd twist three DAs $\rightarrow$ one parameter $\delta_2^\pi = 0.17\,\mathrm{GeV}^2$ (at $\mu = 2\,\mathrm{GeV}$, QCD sum rule estimate)

Matrix elements ↔ DAs

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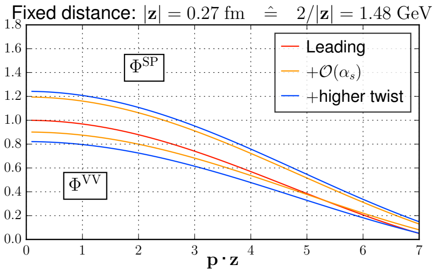

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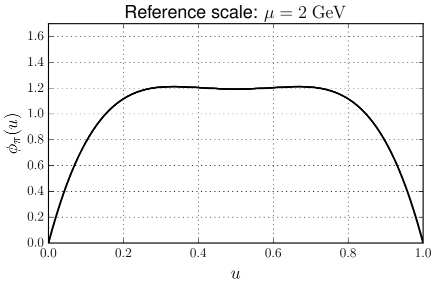

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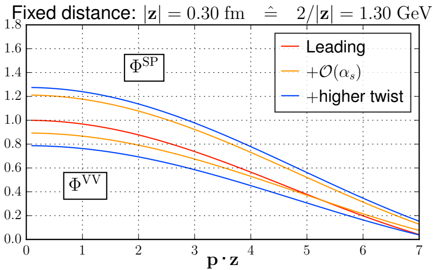

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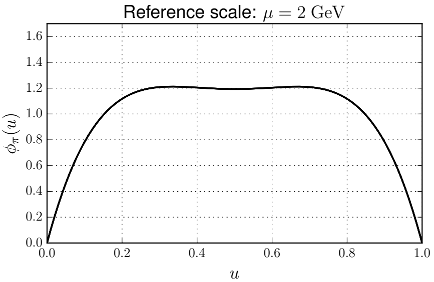

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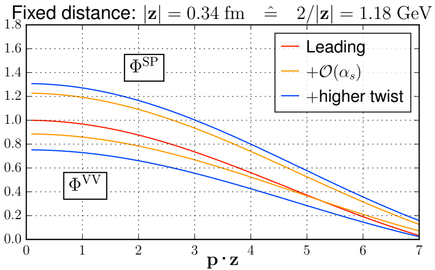

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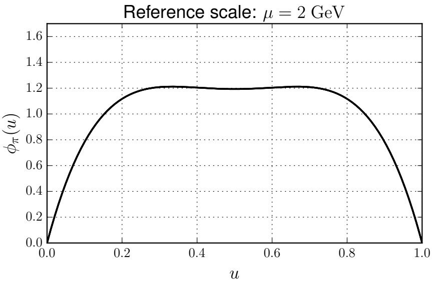

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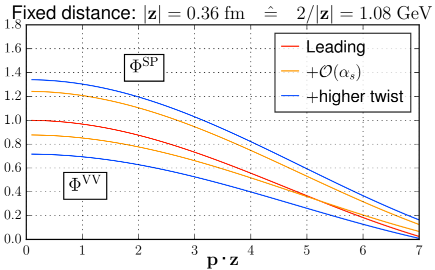

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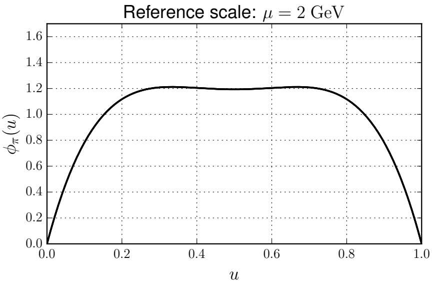

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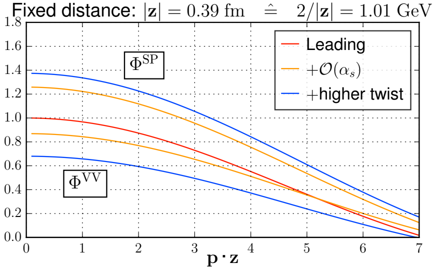

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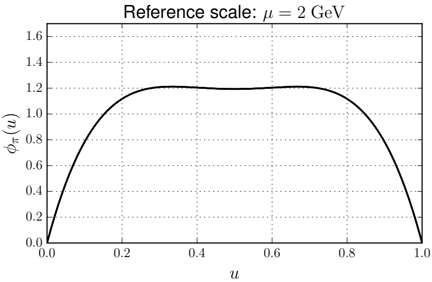

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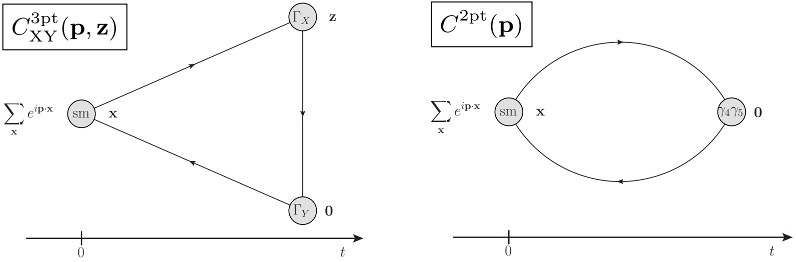
Obtaining the matrix elements from Lattice



$$\frac{\mathbb{T}_{\text{XY}}(p\cdot z, z^2)}{F_\pi} = \frac{Z_X(\mu)Z_Y(\mu)}{Z_A}\,\frac{C^{3\text{pt}}_{\text{XY}}(\mathbf{p},\mathbf{z})e^{\frac{i}{2}\mathbf{p}\cdot\mathbf{z}}}{C^{2\text{pt}}(\mathbf{p})}E(\mathbf{p}) + \text{excited states}$$

- the Z_X is the renormalization factor for the respective current (nonperturbatively calculated in RI′-MOM → conversion to $\overline{\text{MS}}$ in 3-loop PT)
- we set both, the renormalization and the factorization scale to $\mu = 2/|\mathbf{z}|$
- phase factor shifts the currents to the symmetric position

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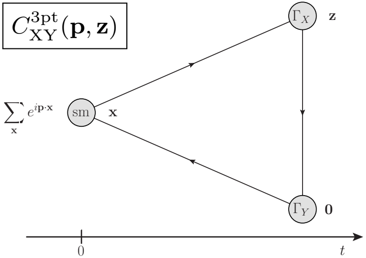
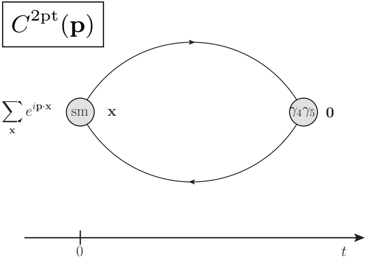




$$\frac{\mathbb{T}_{\rm XY}(p\cdot z, z^2)}{F_\pi} = \frac{Z_X(\mu)Z_Y(\mu)}{Z_A}\,\frac{C^{\rm 3pt}_{\rm XY}(\mathbf{p},\mathbf{z})e^{\frac{i}{2}\mathbf{p}\cdot\mathbf{z}}}{C^{\rm 2pt}(\mathbf{p})}E(\mathbf{p}) + \text{excited states}$$

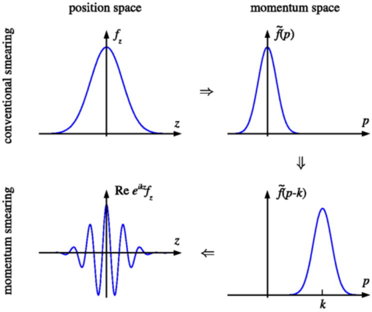
- smearing: momentum smearing
  $\rightarrow$ improved overlap with hadrons at large momentum

PRD **93** (2016) 094515

- new: we use stochastic estimation
  $\rightarrow$ get a volume average at the cost of some stochastic noise
  $\rightarrow$ much smaller statistical error

Momentum smearing

plot taken from PRD **93** (2016) 094515



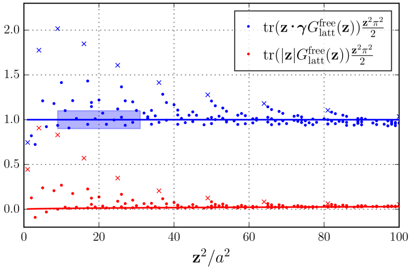
- idea: smear the quark fields such that they carry momentum
- can be achieved by appropriate phase factors
- $\Rightarrow$ leads to larger overlap with hadrons carrying momentum
- essential ingredient for many lattice QCD calculations

Discretization effects of the free Wilson propagator

propagator comparison:

free Wilson vs. free continuum

- large effects in chiral even (blue, $\propto \not{z}$) and chiral odd (red, $\propto \mathbb{1}$) part
- in continuum: chiral odd part strongly suppressed
- **problem on lattice:** large artefacts from terms removing the doublers



solution:

1. use observables, where the chiral odd part does not contribute at tree-level

$$\frac{1}{2}\left(T_{\mathrm{SP}}+T_{\mathrm{PS}}\right), \qquad \frac{1}{2}\left(T_{\mathrm{VA}}+T_{\mathrm{AV}}\right), \qquad \frac{1}{2}\left(T_{\mathrm{VV}}+T_{\mathrm{AA}}\right)$$

2. introduce correction factor for chiral even part
3. most important: ignore distances where the correction $> 10\%$ or $|\mathbf{z}| < 3a$

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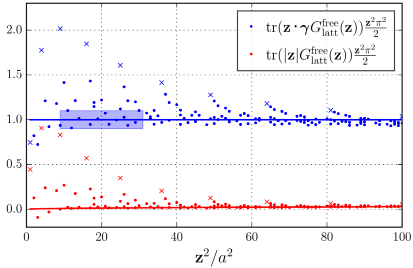


note:

1. upper limit of range determined by $\mu = 2/|\mathbf{z}| \geq 1\,\mathrm{GeV}$
   $\Rightarrow a \rightarrow a/2$ shifts the upper limit by a factor 4 to the right
2. discretization effects are strongest along the axes (crosses)
   $\rightarrow$ similar for Wilson-line operators?
   in case of the pseudo-DA formalism: reduced matrix elements might be beneficial

Numerical study

Simulation details: from PRD **98** (2018) 094507

- mass-degenerate $N_f = 2$ nonperturbatively improved Wilson (clover) fermions and Wilson gluon action
- $L^3 \times T = 32^3 \times 64$
- coupling parameter $\beta = 5.29 \mathrel{\hat{=}}$ lattice spacing $a \approx 0.071\,\text{fm} = (2.76\,\text{GeV})^{-1}$
- mass parameter $\kappa = 0.13632 \mathrel{\hat{=}}$ pion mass $m_\pi = 0.10675(59)/a \approx 295\,\text{MeV}$
- 12 momenta in different directions with $0.54\,\text{GeV} \leq |\mathbf{p}| \leq 2.03\,\text{GeV}$

DA parametrizations: at the scale $\mu = 2\,\text{GeV}$

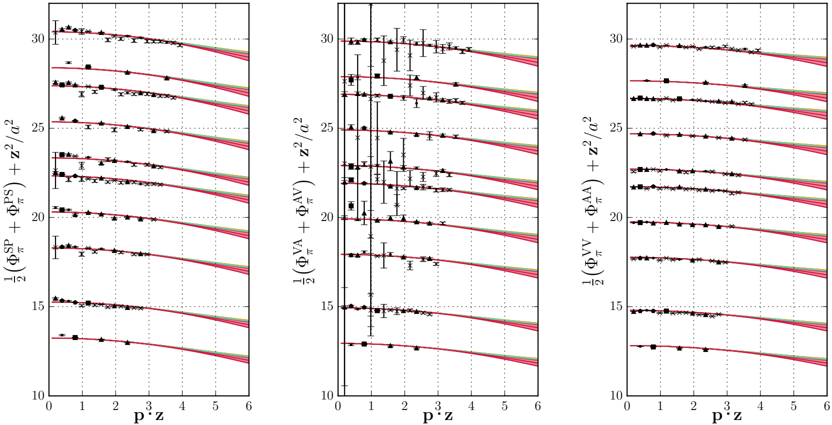
- Expansion in orthogonal (Gegenbauer) polynomials (truncated at $n = 2$ or $n = 4$)

$$\phi_\pi(u, \mu) = 6u(1-u) \sum_{n=0,2,\ldots}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u-1)\,, \qquad a_0^\pi = 1 \text{ (normalization)}$$

- alternatively we try

$$\phi_\pi(u, \mu) \propto \Big[u(1-u)\Big]^\alpha\,, \qquad \text{normalized to one}$$

Combined fit to all channels (Legacy Plot)



- two parameters: α, δ_2^π
- two parameters: a_2^π, δ_2^π
- three parameters: a_2^π, a_4^π, δ_2^π $\leftarrow$ yields unreasonable values for a_4^π

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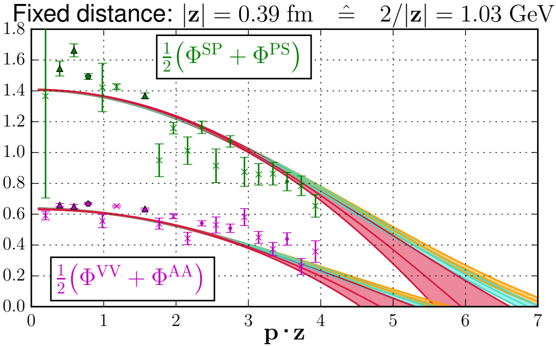

- splitting between SP+PS and VV+AA data is consistent with the pQCD expectation
- "jumping" of the points shows large discretization effects
- probably 2 loop perturbative effects are crucial

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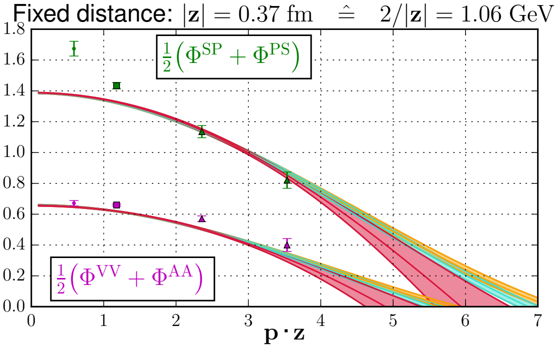

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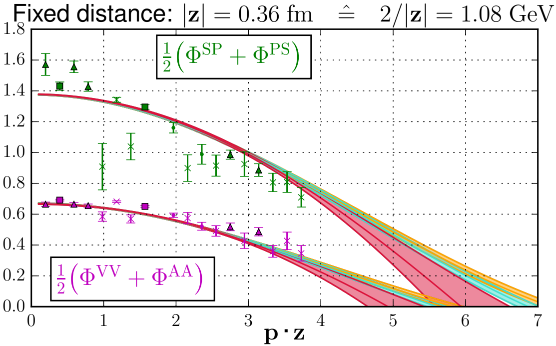

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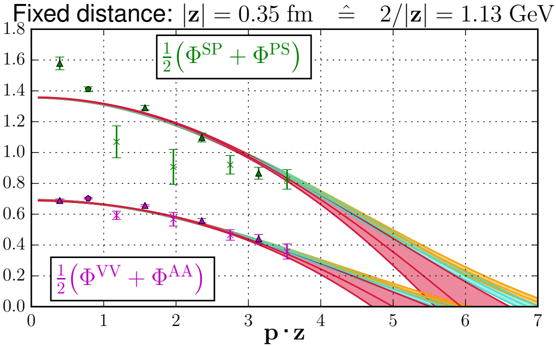

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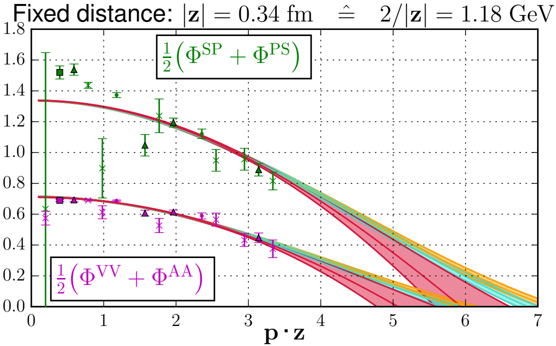

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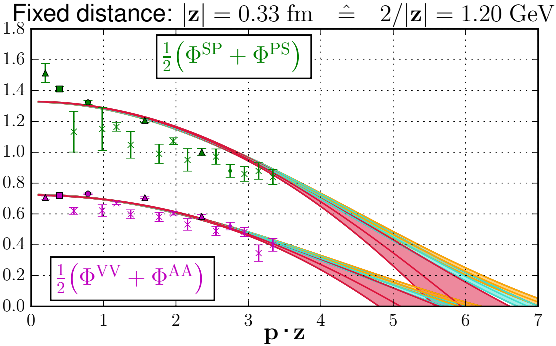

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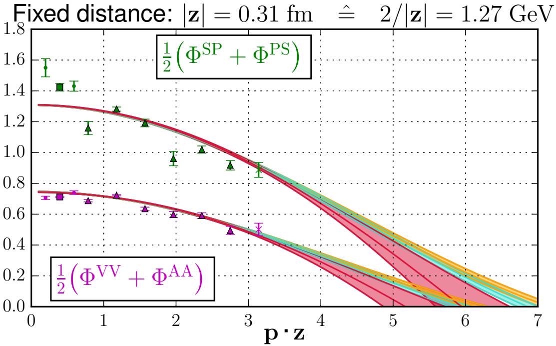

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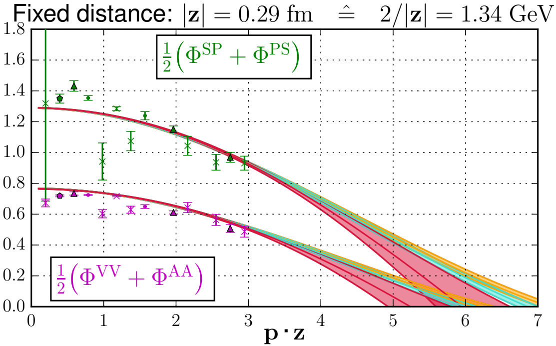

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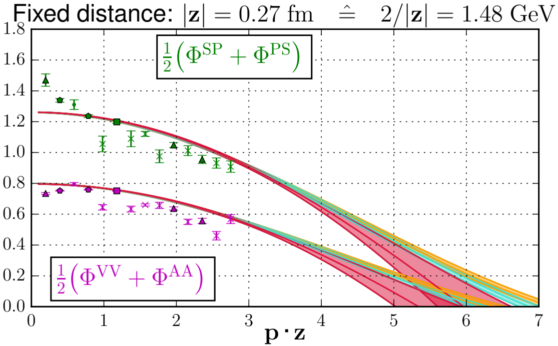

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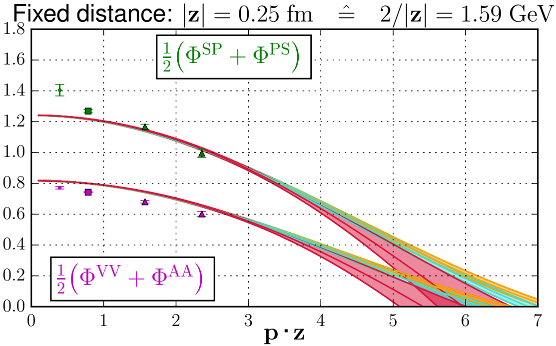
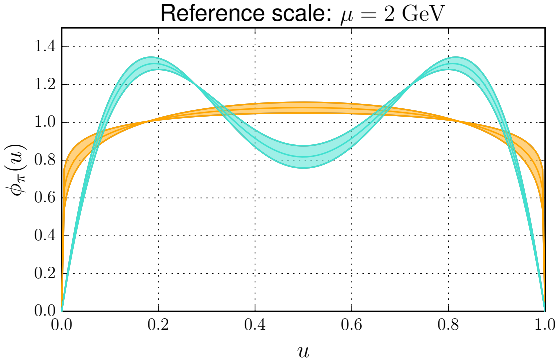
Combined fit to all channels



- splitting between SP+PS and VV+AA data is consistent with the pQCD expectation
- "jumping" of the points shows large discretization effects
- probably 2 loop perturbative effects are crucial

Result for DAs

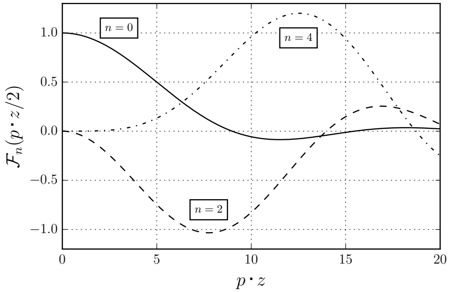


- errorbands show only the statistical error
- parameters: $\alpha = 0.13(5)$, $\delta_2^\pi = 0.223(4)$ GeV^2 $\quad a_2^\pi = 0.30(3)$, $\delta_2^\pi = 0.223(4)$ GeV^2
- both agree perfectly well with our data: **Why?**
- only relevant information from DA for our data points is a_2^π and $a_2^\pi = 0.31(3)$
- **Disclaimer:** current systematic uncertainty for a_2^π, δ_2^π is at least $\approx 50\%$ (fit range variation, estimate for two-loop correction)

Whats the problem with a_4^π?

$$\phi_\pi(u,\mu) = 6u(1-u) \sum_{n=0,2,\dots}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u-1)$$

$$\Rightarrow \quad \Phi^{\mathrm{XY}} = \sum_{n=0,2,\dots}^{\infty} a_n^\pi(\mu)\mathcal{F}_n(p\cdot z/2) + \mathcal{O}(\alpha_s) + \text{higher twist}$$



Expansion in conformal partial waves $\mathcal{F}_n$

- one needs $|p\cdot z| \gtrsim 5$ to constrain a_4^π to reasonable values
- to discriminate between DAs on last slide: $|p\cdot z| \gtrsim 8$?

Summary (so far)

- we have analysed Euclidean correlation functions with two *local* currents
- global fit to multiple channels yields qualitatively reasonable results (universality)
- first determination of HT normalization δ_2^π from lattice QCD (in the ballpark of QCD sum rule estimates)
- statistical accuracy very good for a_2^π and δ_2^π

BUT:

- systematic uncertainty for a_2^π and δ_2^π is very large (discretization effects, two-loop perturbative correction not taken into account)
- with current data no determination of a_4^π possible

Next steps:

- goto smaller lattice spacings ($a \approx 0.04\,\mathrm{fm}$ would be nice)
- perturbative two-loop calculation for coefficient functions
- to be sensitive to a_4^π: goto larger momenta ($|\mathbf{p}| > 3\,\mathrm{GeV}$ would be nice)

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Next steps:

- goto smaller lattice spacings ($a \approx 0.04\,\mathrm{fm}$ would be nice) ✓
- perturbative two-loop calculation for coefficient functions ✓ (for VV)
- to be sensitive to a_4^π: goto larger momenta ($|\mathbf{p}| > 3\,\mathrm{GeV}$ would be nice) not yet

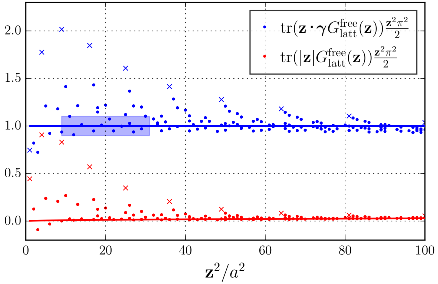
Ensemble details (CLS ensemble J501)

- $N_f = 2 + 1$ nonperturbatively improved Wilson (clover) fermions and Wilson gluon action
- $L^3 \times T = 64^3 \times 192$
- coupling parameter $\beta = 3.85 \hat{=}$ lattice spacing $a \approx 0.039\,\text{fm} = (5.06\,\text{GeV})^{-1}$
- mass parameter:
  - $\kappa_\ell = 0.1369032 \hat{=}$ pion mass $m_\pi \approx 333\,\text{MeV}$
  - $\kappa_s = 0.136749715 \hat{=}$ kaon mass $m_K \approx 445\,\text{MeV}$
- currently: 4 momenta with $|\mathbf{p}| = 0.86\,\text{GeV}$ and $|\mathbf{p}| = 1.72\,\text{GeV}$ (space diagonal direction)
- data stored for $z_i = -8a, \ldots, 8a$, i.e., 17^3 data points in position space

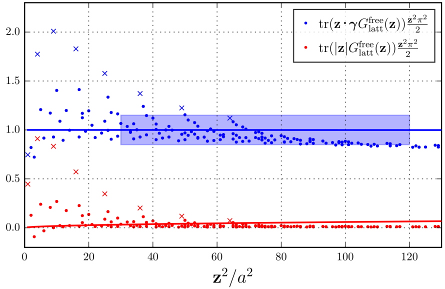
- planned: $|\mathbf{p}| = 2.58\,\text{GeV}$
- test runs with $|\mathbf{p}| = 3.44\,\text{GeV} \rightarrow$ no signal possible (prohibitively expensive...)

Discretization effects

old data

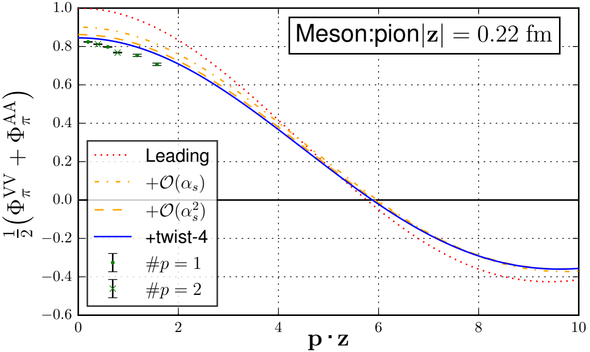


new data (CLS ensemble J501)



- relevant for discretization effects: distance measured in units of the lattice spacing
- upper limit for $|\mathbf{z}|$ due to $\mu = \frac{2}{|\mathbf{z}|} \gtrsim 1\,\mathrm{GeV}$ less problematic
- note: only points plotted, where we have data

The new data



- smaller a $\rightarrow$ smaller discretization effects $\rightarrow$ much less "jumping" of points
- improved statistics due to:
  - larger lattice volume
  - forward-backward averaging implemented
- statements concerning universality still hold (multichannel fit possible)
- **But:** 2-loop coefficient function for VV+AA available
  $\rightarrow$ concentrate on this case in the following

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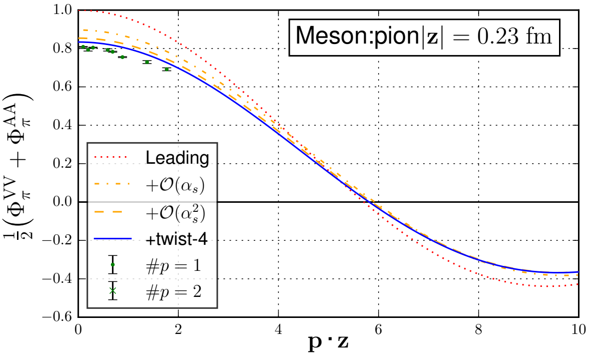

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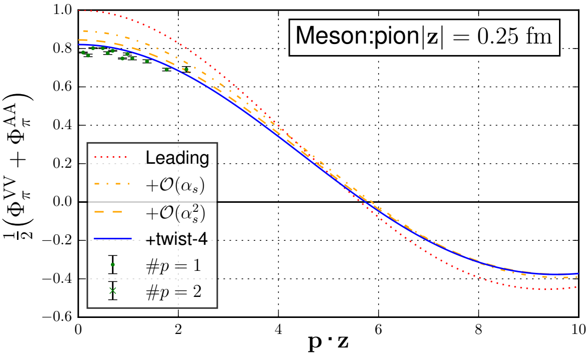

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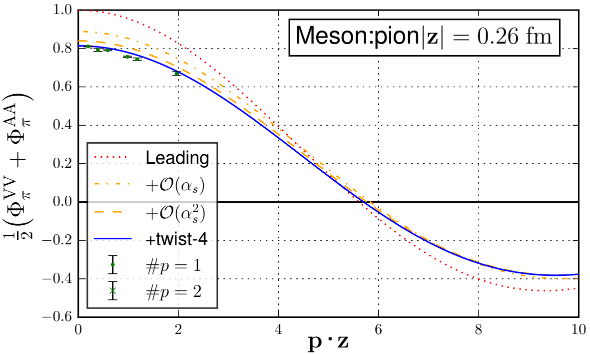

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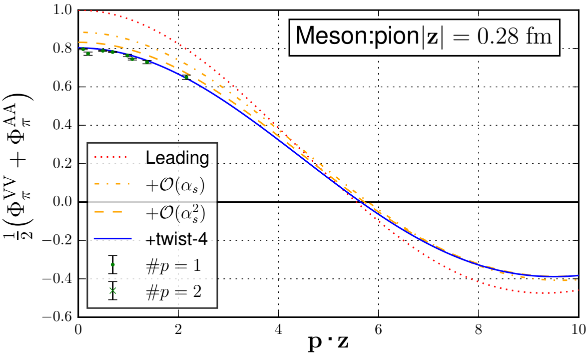

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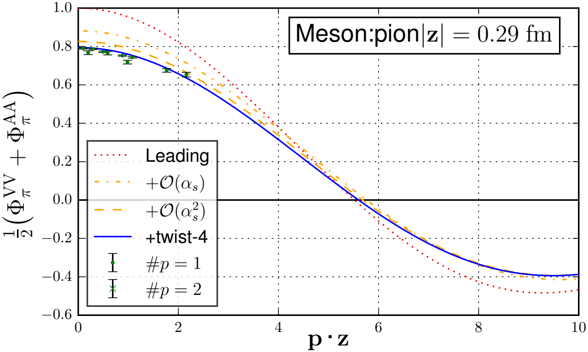

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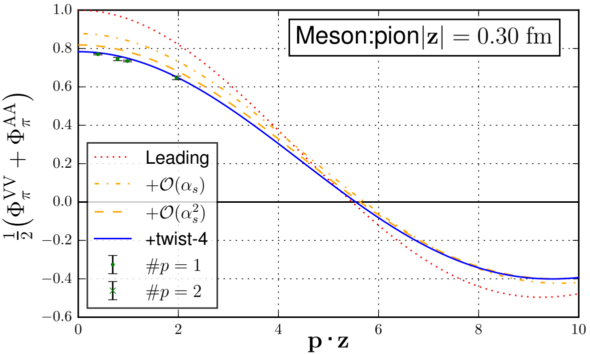

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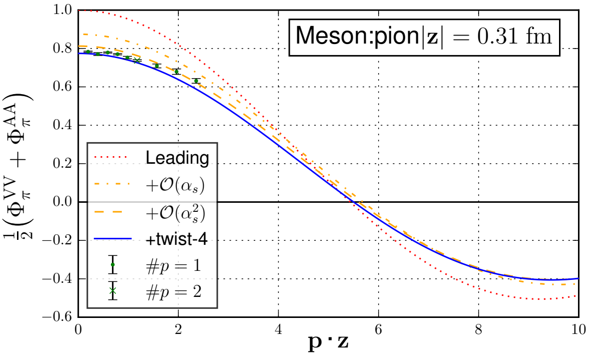

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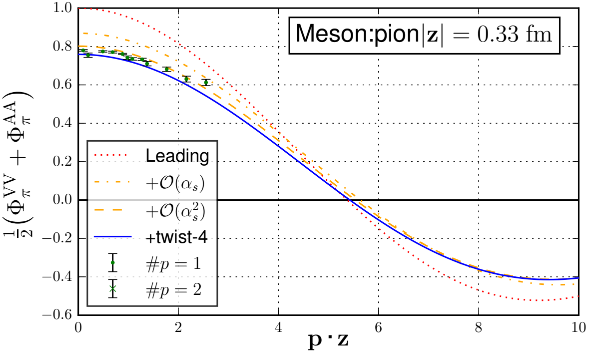

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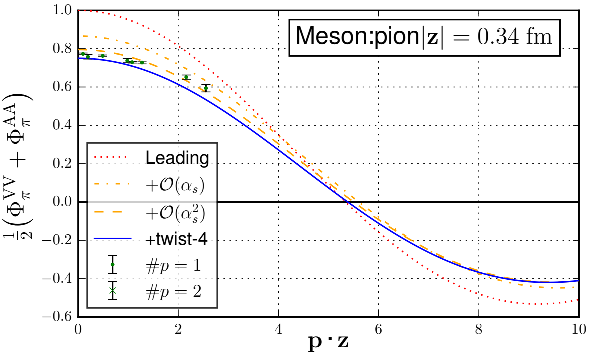

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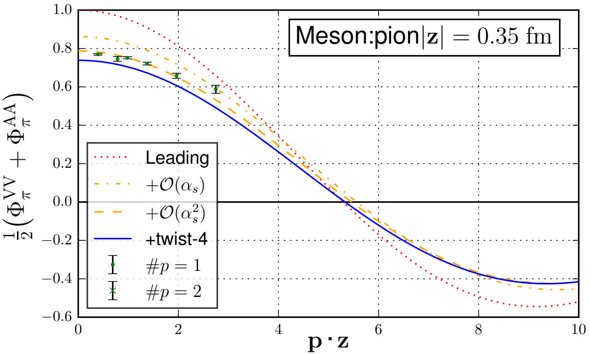

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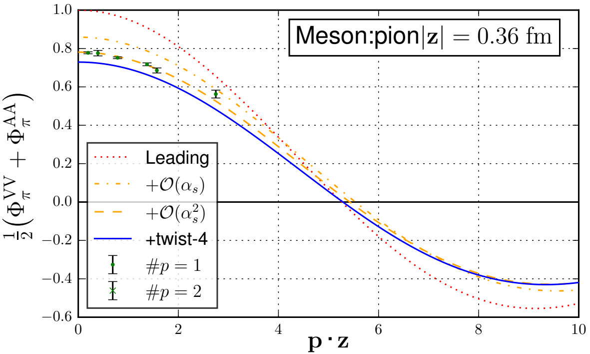

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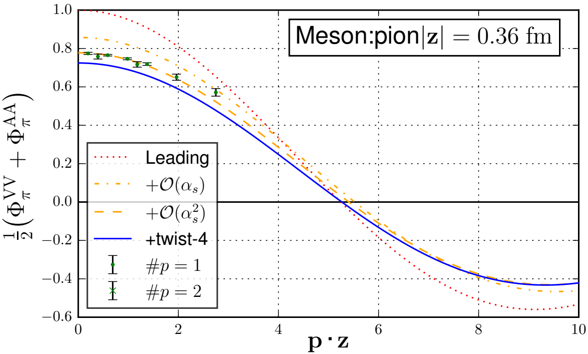

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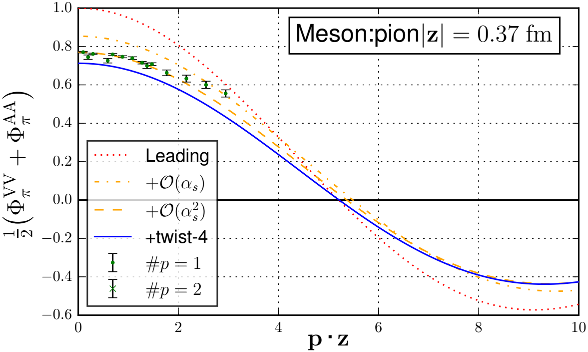

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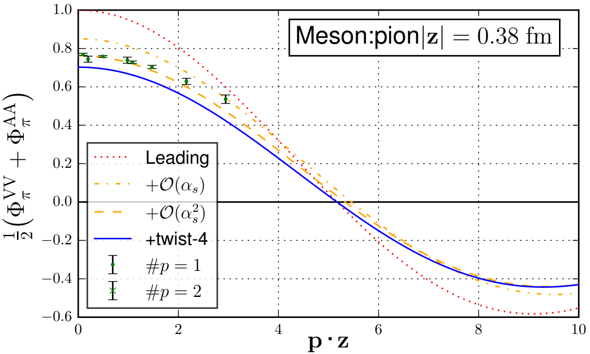

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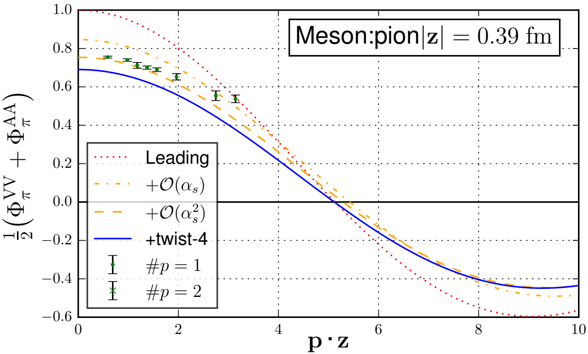

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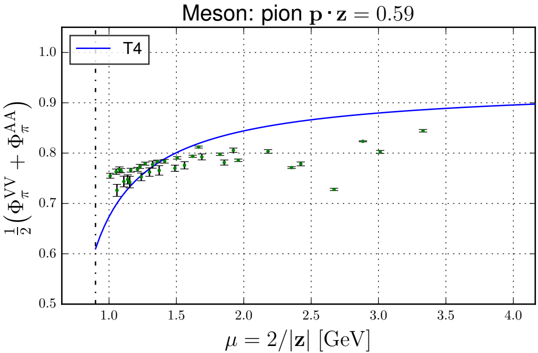
From a different angle...

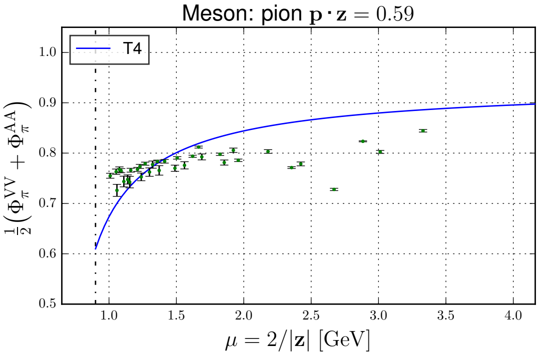


- do we need even higher loop orders?
- are we sensitive to twist-6 contributions?

Higher twist formula (VV): twist 4

$$\frac{z^2}{4}\int_0^1 du\, \cos[(u-\tfrac{1}{2})p\cdot z]\left[\frac{80}{3}\delta_2^\pi u^2\bar{u}^2 + \frac{m_\pi^2}{12}u^2\bar{u}^2\left[42u\bar{u} - 13 + 18a_2^\pi(7-30u\bar{u})\right]\right]$$

From a different angle...



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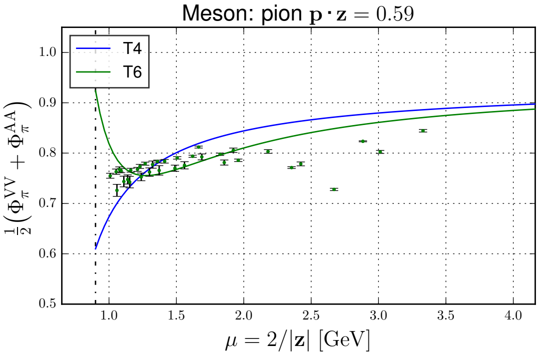
Higher twist formula (VV): twist 4 (most important part)

$$\frac{z^2}{4}\int_0^1 du\,\cos[(u-\tfrac{1}{2})p\cdot z]\left[\frac{80}{3}\delta_2^\pi u^2\bar{u}^2 + \dots\right]$$

From a different angle...



- do we need even higher loop orders?
- are we sensitive to twist-6 contributions?

Higher twist formula (VV): twist 4 (most important part) + twist 6

$$\frac{z^2}{4}\int_0^1 du\, \cos[(u-\tfrac{1}{2})p\cdot z]\left[\frac{80}{3}\left(\delta_2^\pi + \delta_4^{\pi,VV} z^2\right)u^2\bar{u}^2 + \dots\right]$$

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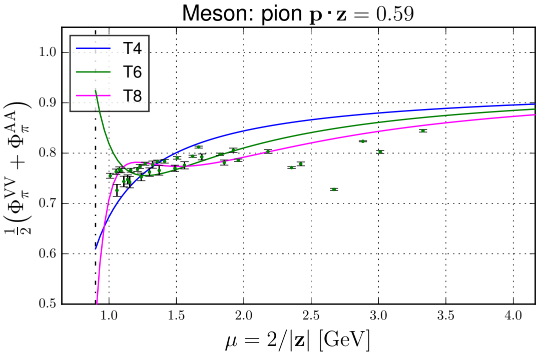
From a different angle...

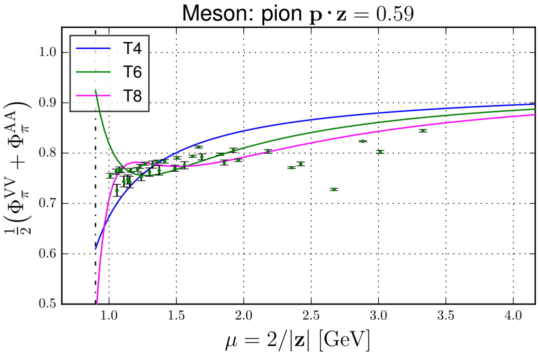


- do we need even higher loop orders?
- are we sensitive to twist-6 contributions?

Higher twist formula (VV): twist 4 (most important part) + twist 6 + twist 8

$$\frac{z^2}{4}\int_0^1 du\, \cos[(u-\tfrac{1}{2})p\cdot z]\left[\frac{80}{3}\left(\delta_2^\pi + \delta_4^{\pi,VV} z^2 + \delta^{\pi_6,VV} z^4\right)u^2\bar{u}^2 + \dots\right]$$

From a different angle...



- do we need even higher loop orders?
- are we sensitive to twist-6 contributions?
- fitted twist 6 term $>$ twist 4 term at roughly $\mu = 1.5\,\text{GeV}$
- $\Rightarrow$ only allow for $\mu > 1.5\,\text{GeV}$?
- $\Rightarrow$ almost no sensitivity for DAs anymore! (even larger momenta necessary)

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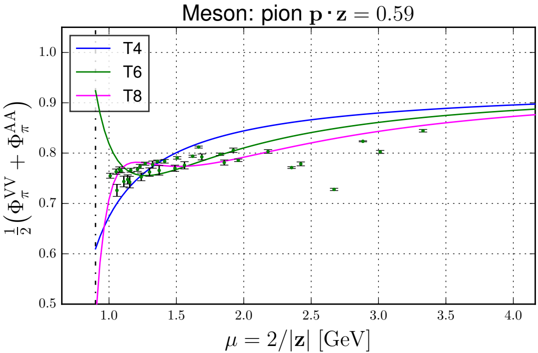
From a different angle...



other possible explanations:

- remaining descretization artifact
- volume effect
- ...

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Preliminary conclusions and outlook

we have a window problem

- discretization effects $\Rightarrow$ large distances preferable
- sensitivity to DAs/PDFs $\Rightarrow$ large distances required (or even larger momenta)
- controlling higher twist $\Rightarrow$ only at relatively small distances possible
- perturbation theory applicable $\Rightarrow$ requires quite small distances

note: this problem is also present in the quasi-/pseudo-DA/PDF approach

solutions?

- we need a better treatment of discretization effects
  (maybe "reduced" matrix elements similar to pseudo-DA/PDF approach helpful)
- data at even larger hadron momenta would be helpful
- higher twist: get better parametrization for large distance behavior from EFT?
  (maybe one can simultaneously address volume effects)
- two-loop PT for all channels (and eventually three-loop)

Preliminary conclusions and outlook (II)

- sensitivity to higher moments of DAs/PDFs very limited
  - a_2^π seems to be possible
  - a_4^π very challenging
  - (probably also true for other position space methods)
- systematic uncertainties are very large
  (discretization, perturbative expansion, higher twist effects, volume effects)
- we are not even talking about extrapolation to the continuum, to physical quark masses, and to infinite volumes yet
- $\Rightarrow$ for the time being the moments method is the choice for quantitative results

cf. the talk by Gunnar, yesterday

This sounds so depressing...

Lets look on the bright side!

- we have two-loop coefficient functions for the VV channel
- statistics are very good
  - it is possible to identify systematic problems
  - one has a chance to analyze systematic effects
  - we might be able to find better ways to control the systematics
- there are large higher twist contributions

  $\Rightarrow$ we can study higher twist effects!
- a systematic uncertainty of $\gtrsim 50\%$ for δ_2^π is not too bad
  - considering its the very first determination from lattice QCD
  - once we understand the systematics: perfect playground to study higher twist effects