Quarkonium LCDAs from NRQCD

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Based on
Geoffrey Bodwin (ANL), HSC (TUM), June-Haak Ee, Jungil Lee (KU), PRD95 (2017) 054018, PRD96 (2017) 116014
Nora Brambilla, HSC, Wai Kin Lai (TUM), Vladyslav Shtabovenko (Zhejiang, KIT), Antonio Vairo (TUM), PRD100, 054038 (2019)

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OUTLINE

- NRQCD matching of quarkonium LCDAs
- Evolution of quarkonium LCDAs
- Applications

HEAVY QUARKONIUM IN NRQCD

- NRQCD provides a description of a heavy quarkonium state as nonrelativistic Fock state expansion

$$|H\rangle = O(1)|Q\bar{Q}\rangle + O(v)|Q\bar{Q}g\rangle + O(v^2)|Q\bar{Q}gg\rangle + \cdots$$

Caswell, Lepage, PLB167, 437 (1986)
Bodwin, Braaten, Lepage, PRD51, 1125 (1995), PRD55, 5853 (1997)

$v^2 \approx 0.3$ for charmonia,
$v^2 \approx 0.1$ for bottomonia.

- At leading order in v, the leading Fock state is given by $Q\bar{Q}$ in a color-singlet state.
- At higher orders in v, the Fock states can involve $Q\bar{Q}$ in color-octet states.

HARD EXCLUSIVE PROCESSES IN NRQCD

- Amplitudes for exclusive production processes of quarkonium H in NRQCD factorization

$$i\mathcal{M} = \sum_n c_n \langle H | \mathcal{O}_n | 0 \rangle$$

Bodwin, Braaten, Lepage, PRD51, 1125 (1995), PRD55, 5853 (1997)

- The short-distance coefficients c_n correspond to production amplitudes of heavy quark and antiquark.
- The c_n involve the hard scale Q and the heavy-quark mass m. They can be computed from perturbative matching.
- $\langle H | \mathcal{O}_n | 0 \rangle$ are local NRQCD operator matrix elements that have known scalings in v.

QUARKONIUM LCDA

- Analogous factorization formula can be written for quarkonium LCDA

Jia and Yang, NPB814 (2009) 217

$$f_H \phi_H(x) = \sum_n \tilde{c}_n(x) \langle H | \mathcal{O}_n | 0 \rangle$$

- $\tilde{c}_n(x)$ can be determined as functions of x from perturbative matching calculations.

- Production amplitudes at leading power in m/Q are given by convolution of LCDA and the hard part $T_H(x)$. Hence,

$$c_n = \int dx\, T_H(x) \tilde{c}_n(x) + \text{corrections of subleading powers in } m/Q$$

to all orders in α_s and v.

QUARKONIUM LCDA

- Quarkonium LCDAs from NRQCD allow us to compute exclusive production amplitudes at leading power in m/Q, which reproduce fixed-order calculations at large Q.
- The corrections of subleading powers in m/Q can be obtained from fixed-order calculations.
- We can resum logarithms of Q/m at leading power in m/Q.
- Quarkonium LCDAs from NRQCD involves singular distributions, which originate from matching nonlocal QCD operators to local NRQCD operators. This complicates solving evolution equations.

DEFINITIONS

- We use the following definition for quarkonium LCDAs.

$$\mathcal{Q}[\Gamma](\omega) = \bar{Q}(\omega\bar{n}/2)[\omega\bar{n}/2, -\omega\bar{n}/2]\not{\bar{n}}\Gamma Q(-\omega\bar{n}/2)$$

$J^{PC}=0^{-+}$ $(\eta_c,\ \eta_b)$

$$: \langle H|\mathcal{Q}[\gamma_5](\omega)|0\rangle = -if_P\bar{n}\cdot P\int dx e^{i\omega\bar{n}\cdot P(x-1/2)}\phi_P(x)$$

$J^{PC}=1^{--}$ $(J/\psi,\ \Upsilon)$

$$: \langle H|\mathcal{Q}[1](\omega)|0\rangle = -if_V m_H\bar{n}\cdot\epsilon^*\int dx e^{i\omega\bar{n}\cdot P(x-1/2)}\phi_V^{\parallel}(x)$$

$$\langle H|\mathcal{Q}[\gamma_\perp](\omega)|0\rangle = -if_V^{\perp}\bar{n}\cdot P\epsilon_\perp^*\int dx e^{i\omega\bar{n}\cdot P(x-1/2)}\phi_V^{\perp}(x)$$

DEFINITIONS

$J^{PC}=0^{++}$: $\langle H|\mathcal{Q}[1](\omega)|0\rangle = f_S \bar{n}\cdot P \int dx e^{i\omega \bar{n}\cdot P(x-1/2)}\phi_S(x)$
$(\chi_{c0},\ \chi_{b0})$

$J^{PC}=1^{++}$: $\langle H|\mathcal{Q}[\gamma_5](\omega)|0\rangle = i f_{3A} m_H \bar{n}\cdot\epsilon^* \int dx e^{i\omega \bar{n}\cdot P(x-1/2)}\phi_{3A}^{\parallel}(x)$
$(\chi_{c1},\ \chi_{b1})$

$$\langle H|\mathcal{Q}[\gamma_\perp\gamma_5](\omega)|0\rangle = i f_{3A}^{\perp} \bar{n}\cdot P\epsilon_\perp^* \int dx e^{i\omega \bar{n}\cdot P(x-1/2)}\phi_{3A}^{\perp}(x)$$

- Definitions for $J^{PC}=1^{+-}$ $(h_c,\ h_b)$ and $J^{PC}=2^{++}$ $(\chi_{c2},\ \chi_{b2})$ states in Wang and Yang, JHEP 1406 (2014) 121.
- In practice, we work in x space using Fourier transform.

MATCHING

- To compute $\tilde{c}_n(x)$ from perturbative matching, we replace the quarkonium state H with a perturbative $Q\bar{Q}$ state, and compute the matrix elements $\langle Q\bar{Q}|\mathcal{Q}[\Gamma](\omega)|0\rangle$.
- At leading order in v, only the $Q\bar{Q}$ with same quantum numbers as the quarkonium can contribute.
- At higher orders in v, higher Fock state contributions like $Q\bar{Q}g$ can appear, if $\mathcal{O}_n$ contains gluon field strengths.

MATCHING

- For J/ψ, the leading-order matrix element is given by

$$\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\chi|0\rangle$$

- At relative order v^2, $\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|0\rangle$ appears.

- At relative order v^3 and higher,

$$\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^4\chi|0\rangle \qquad \langle J/\psi|\psi^\dagger \epsilon^i(\lambda)\sigma^j(-\tfrac{i}{2})^2\overleftrightarrow{\boldsymbol{D}}^{(i}\overleftrightarrow{\boldsymbol{D}}^{j)}\chi|0\rangle$$

$$\langle J/\psi|\psi^\dagger g_s\boldsymbol{B}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle$$

$$\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\tfrac{1}{3}(\overleftrightarrow{\boldsymbol{D}}\cdot g_s\boldsymbol{E}+g_s\boldsymbol{E}\cdot\overleftrightarrow{\boldsymbol{D}})\chi|0\rangle$$

$$\langle J/\psi|\psi^\dagger \boldsymbol{\epsilon}(\lambda)\cdot\tfrac{1}{2}[\boldsymbol{\sigma}\times(\overleftrightarrow{\boldsymbol{D}}\times g_s\boldsymbol{E}-g_s\boldsymbol{E}\times\overleftrightarrow{\boldsymbol{D}})]\chi|0\rangle$$

- Similar operators appear for η_c, without the ϵ.

MATCHING

- For χ_{cJ}, the leading-order matrix element is given by

$$\langle \chi_{c0} | \psi^\dagger \boldsymbol{\sigma} \cdot (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}}) \chi | 0 \rangle$$

- At relative order v^2,

$$\langle \chi_{c0} | \psi^\dagger \boldsymbol{\sigma} \cdot (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}})(-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}})^2 \chi | 0 \rangle$$

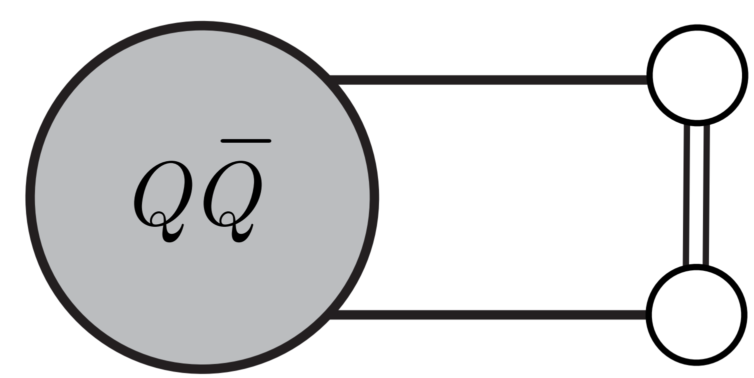
$$\langle \chi_{c0} | \psi^\dagger g_s \boldsymbol{E} \cdot \boldsymbol{\sigma} \chi | 0 \rangle$$

- Similar operators appear for h_c.

TREE-LEVEL MATCHING AT LO IN v

- For η_c or J/ψ, NRQCD matrix elements at leading order in v are given by heavy quark and antiquark field operators. Hence, it suffices to work with $Q\bar{Q}$ with vanishing relative momentum.
- In this case, $\langle Q\bar{Q}|\mathcal{Q}[\Gamma](\omega)|0\rangle$ is independent of ω, so in x space, the tree-level LCDAs can only involve the delta function.
- The LCDA has support only for $x = 1/2$, so

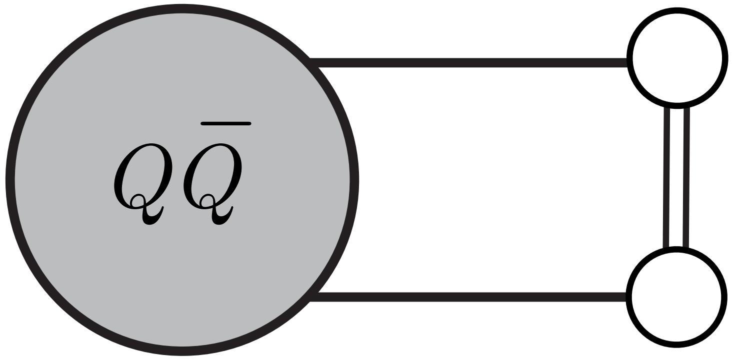
$$\phi_P(x) = \phi_V^{\parallel}(x) = \phi_V^{\perp}(x) = \delta(x - 1/2)$$



TREE-LEVEL MATCHING AT LO IN v

- For h_c or χ_{cJ}, NRQCD matrix elements at leading order in v involve at least one covariant derivative. We expand to linear power in the relative momentum of $Q\bar{Q}$.

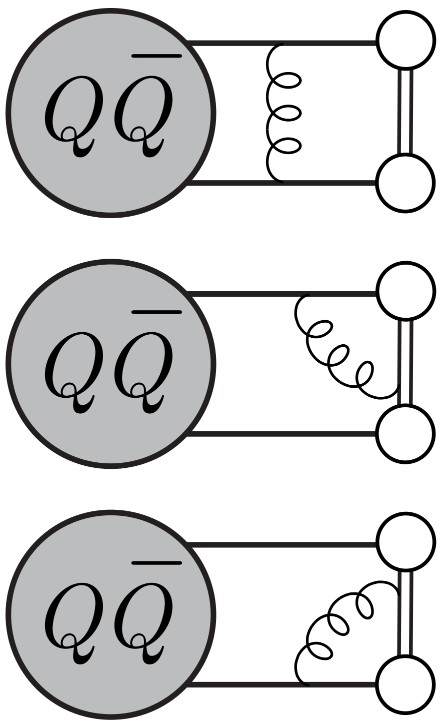
- At tree level, $\langle Q\bar{Q}|\mathcal{Q}[\Gamma](\omega)|0\rangle$ can involve only the exponentials of ω, multiplied by polynomials of ω. In x space, the tree-level LCDAs can only involve the delta function and its derivatives.



$$\phi^{\|}_{3A}(x) = \delta(x - 1/2)$$

$$\phi_S(x) = \phi^{\perp}_{3A}(x) = -\frac{1}{2}\delta'(x - 1/2)$$

ONE-LOOP MATCHING AT LO IN v



- At one loop, quarks and Wilson lines exchange gluons. This introduces more distributions in x, as well as a UV pole multiplied by the evolution kernel convolved with the tree-level LCDA, which is removed by renormalization.

For transverse J/ψ, one-loop correction is given by

$$\phi_V^{\perp(\alpha_s)}(x,\mu_0) = \frac{\alpha_s(\mu_0)C_F}{4\pi}\theta(1-2x)\left\{\left[\frac{8x}{1-2x}\left(\log\frac{\mu_0^2}{m^2(1-2x)^2}-1\right)\right]_+ + \left[\frac{16x(1-x)}{(1-2x)^2}\right]_{++}\right\} + (x\leftrightarrow 1-x),$$

$$\int_0^1 dx\, f(x)[g(x)]_{++} = \int_0^1 dx\,[f(x)-f(1/2)-f'(1/2)(x-1/2)]g(x).$$

All one-loop corrections for $\eta_c,\ J/\psi,\ h_c\ \chi_{cJ}$ LCDAs have been computed in Wang and Yang, JHEP 1406 (2014) 121

RELATIVISTIC CORRECTIONS AT TREE LEVEL

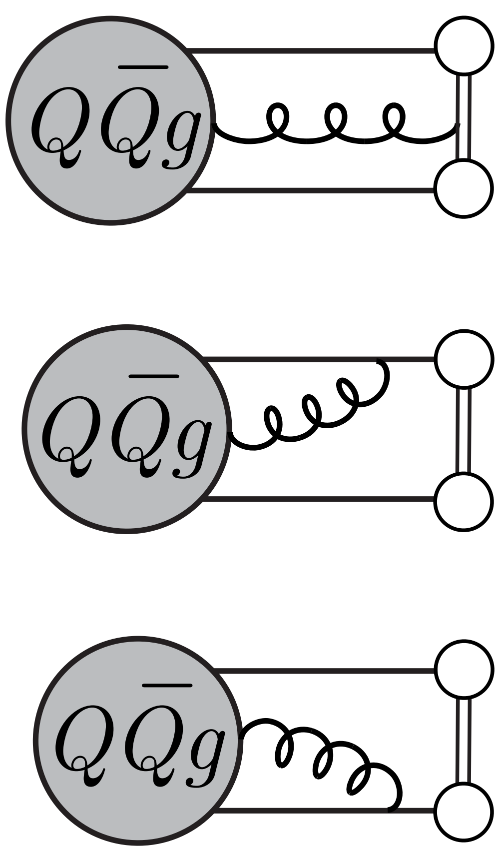
- NRQCD matrix elements at higher orders in v can involve more covariant derivatives and gluon field strengths. For operators with no gluon field strengths, it suffices to work with $Q\bar{Q}$ at higher orders in powers of the relative momentum.
- For η_c or J/ψ, at tree level to relative order v^2,

$$\phi(x) = \delta(x-1/2) + \frac{\langle v_S^2 \rangle}{24} \delta^{(2)}(x-1/2)$$

$$\langle v_S^2 \rangle = \frac{\langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} (-\frac{i}{2} \overleftrightarrow{\boldsymbol{D}})^2 \chi | 0 \rangle}{m^2 \langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle}$$

RELATIVISTIC CORRECTIONS AT TREE LEVEL

- For operators involving gluon field strengths, it is necessary to work with $Q\bar{Q}g$ states at higher orders in powers of relative momenta.



- At tree level, relativistic corrections from these operators are still given in terms of delta functions and its derivatives.

RELATIVISTIC CORRECTIONS AT TREE LEVEL

- For transverse J/ψ, the relativistic corrections to order v^4 are given by

$$\phi_V^\perp(x,\mu_0) = \phi_V^{\perp\,(0)}(x,\mu_0) + \phi_V^{\perp\,(\alpha_s)}(x,\mu_0) + \phi_V^{\perp\,(2)}(x,\mu_0) + \phi_V^{\perp\,(4)}(x,\mu_0),$$

$$\begin{aligned}
\phi_V^{\perp\,(0)}(x,\mu_0) &= \delta(x-1/2), \\
\phi_V^{\perp\,(2)}(x,\mu_0) &= \left[\frac{1}{3}\langle v_S^2\rangle_V + \frac{5}{18}\left(\langle v_S^2\rangle_V\right)^2 - \frac{19}{30}\langle v_S^4\rangle_V \right. \\
&\quad \left. + \frac{1}{8}\langle DE_0\rangle - \frac{1}{5}\langle v_D^2\rangle_V - \frac{1}{8}\langle DE_1\rangle_V\right]\frac{\delta^{(2)}(x-1/2)}{8}, \\
\phi_V^{\perp\,(4)}(x,\mu_0) &= \frac{1}{5}\langle v_S^4\rangle_V \frac{\delta^{(4)}(x-1/2)}{384},
\end{aligned}$$

$$\begin{aligned}
\langle v_S^n\rangle_V &= \frac{1}{m^4}\frac{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}})^n\chi|0\rangle}{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}, \\
\langle v_D^2\rangle_V &= \frac{1}{m^2}\frac{\langle V|\psi^\dagger \epsilon^i(\lambda)\sigma^j(-\frac{i}{2})^2\overleftrightarrow{\boldsymbol{D}}^{(i}\overleftrightarrow{\boldsymbol{D}}^{j)}\chi|0\rangle}{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}, \\
\langle DE_0\rangle_V &= \frac{1}{m^3}\frac{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\frac{1}{3}(\overleftrightarrow{\boldsymbol{D}}\cdot g_s\boldsymbol{E} + g_s\boldsymbol{E}\cdot\overleftrightarrow{\boldsymbol{D}})\chi|0\rangle}{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}, \\
\langle DE_1\rangle_V &= \frac{1}{m^3}\frac{\langle V|\psi^\dagger \boldsymbol{\epsilon}(\lambda)\cdot\frac{1}{2}[\boldsymbol{\sigma}\times(\overleftrightarrow{\boldsymbol{D}}\times g_s\boldsymbol{E} - g_s\boldsymbol{E}\times\overleftrightarrow{\boldsymbol{D}})]\chi|0\rangle}{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}.
\end{aligned}$$

Brambilla, HSC, Lai, Shtabovenko, Vairo, PRD100, 054038 (2019)

NRQCD MATRIX ELEMENTS

- We also need the nonperturbative NRQCD matrix elements to determine the quarkonium LCDAs.

- For η_c or J/ψ, potential-model calculations are available for $\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\chi|0\rangle$, $\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|0\rangle$, and $\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^4\chi|0\rangle$.
  Bodwin, HSC, Kang, Lee, Yu, PRD77, 094017 (2008)

- For χ_{cJ} and h_c, $\langle \chi_{c0}|\psi^\dagger \boldsymbol{\sigma}\cdot(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})\chi|0\rangle$ is known from potential models. For $\langle \chi_{c0}|\psi^\dagger \boldsymbol{\sigma}\cdot(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|0\rangle$ and $\langle \chi_{c0}|\psi^\dagger g_s\boldsymbol{E}\cdot\boldsymbol{\sigma}\chi|0\rangle$, constraints from decay rates exist.
  Brambilla, Chen, Jia, Shtabovenko, Vairo, PRD97, 096001 (2018)

- Some (but not all) of the f_H can be determined from measured decay rates and lattice measurements.

DECAY CONSTANTS

- The decay constants f_H also have NRQCD expansions. For J/ψ, $f_V^{\parallel}$ is available from the EM current.

$$f_V^{\parallel} = \frac{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}{2m}\Bigg(1 - 8\frac{\alpha_s(\mu_0)C_F}{4\pi} - \frac{2}{3}\langle v_S^2\rangle_V + \frac{7}{12}\langle v_S^4\rangle_V - \frac{5}{8}\langle DE_0\rangle_V - \frac{1}{2}\langle v_D^2\rangle_V + \frac{1}{2}\langle B\rangle_V - \frac{1}{4}\langle DE_1\rangle_V + O(v^5)\Bigg),$$

$$f_V^{\perp}(\mu_0) = \frac{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}{2m}\Bigg[1 - \frac{\alpha_s(\mu_0)C_F}{4\pi}\left(\log\frac{\mu_0^2}{m^2} + 8\right) - \frac{5}{6}\langle v_S^2\rangle_V + \frac{19}{24}\langle v_S^4\rangle_V - \frac{7}{8}\langle DE_0\rangle_V + \frac{1}{2}\langle v_D^2\rangle_V + \langle B\rangle_V - \frac{1}{4}\langle DE_1\rangle_V + O(v^5)\Bigg].$$

$$\langle B\rangle_V = \frac{1}{m^2}\frac{\langle V|\psi^\dagger g_s \boldsymbol{B}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}{\langle V|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}$$

Wang and Yang, JHEP 1406 (2014) 121

Brambilla, Chen, Jia, Shtabovenko, Vairo, PRD97, 096001 (2018)

COMPARISON WITH SUM RULES

- Sum rules also give predictions for moments

$$\langle \xi^n \rangle = \int_0^1 dx\, \phi(x)(2x-1)^n$$

- In NRQCD these moments are given by ratios of NRQCD matrix elements. Good agreements with sum rules are found for lowest moments.

$\langle \xi^n \rangle$	NRQCD	QCD sum rules $\phi_L(\xi,\mu)$	QCD sum rules $\phi_T(\xi,\mu)$
$n=2$	0.057 ± 0.026	0.070 ± 0.007	0.072 ± 0.007
$n=4$	0.010 ± 0.003	0.012 ± 0.002	0.012 ± 0.002
$n=6$		0.0031 ± 0.0008	0.0033 ± 0.0007

Braguta, Likhoded, Luchinsky, PLB646 (2007) 80
Braguta, PRD75, 094016 (2007)
Brambilla, Chen, Jia, Shtabovenko, Vairo, PRD97, 096001 (2018)

EVOLUTION OF QUARKONIUM LCDA

- NRQCD matching calculations for quarkonium LCDAs are usually done at the scale of the heavy quarkonium mass. This is also the scale at which the NRQCD matrix elements are usually determined.
- To resum logarithms we need to solve evolution equations. The standard way is to use Gegenbauer expansion.
- Since quarkonium LCDAs contain singular distributions, Gegenbauer expansions do not converge.

EVOLUTION OF QUARKONIUM LCDA

- Gegenbauer expansion of LCDA:

$$|\phi\rangle = \sum_n |n\rangle\langle n|\phi\rangle$$

$$|n,x\rangle = w(x) C_n^{(3/2)}(2x-1),$$

$$\langle n,x| = N_n C_n^{(3/2)}(2x-1),$$

$$N_n = \tfrac{4(2n+3)}{(n+1)(n+2)},$$

$$w(x) = x(1-x)$$

- Orthonormality: $\langle n|m\rangle = \delta_{nm}$
- Completeness: $\sum_n |n,x'\rangle\langle n,x| = \delta(x-x')$
- Solution to ERBL equation is given by

$$|\phi(\mu)\rangle = \sum_{m,n} |m\rangle\langle m|U|n\rangle\langle n|\phi(\mu_0)\rangle$$

Evolution matrix

- Production amplitude with resummation is given by

$$i\mathcal{A} = \sum_{m,n} \langle T_H|m\rangle\langle m|U|n\rangle\langle n|\phi(\mu_0)\rangle$$

EVOLUTION OF QUARKONIUM LCDA

$$i\mathcal{A} = \sum_{m,n} \langle T_H|m\rangle\langle m|U|n\rangle\langle n|\phi(\mu_0)\rangle$$

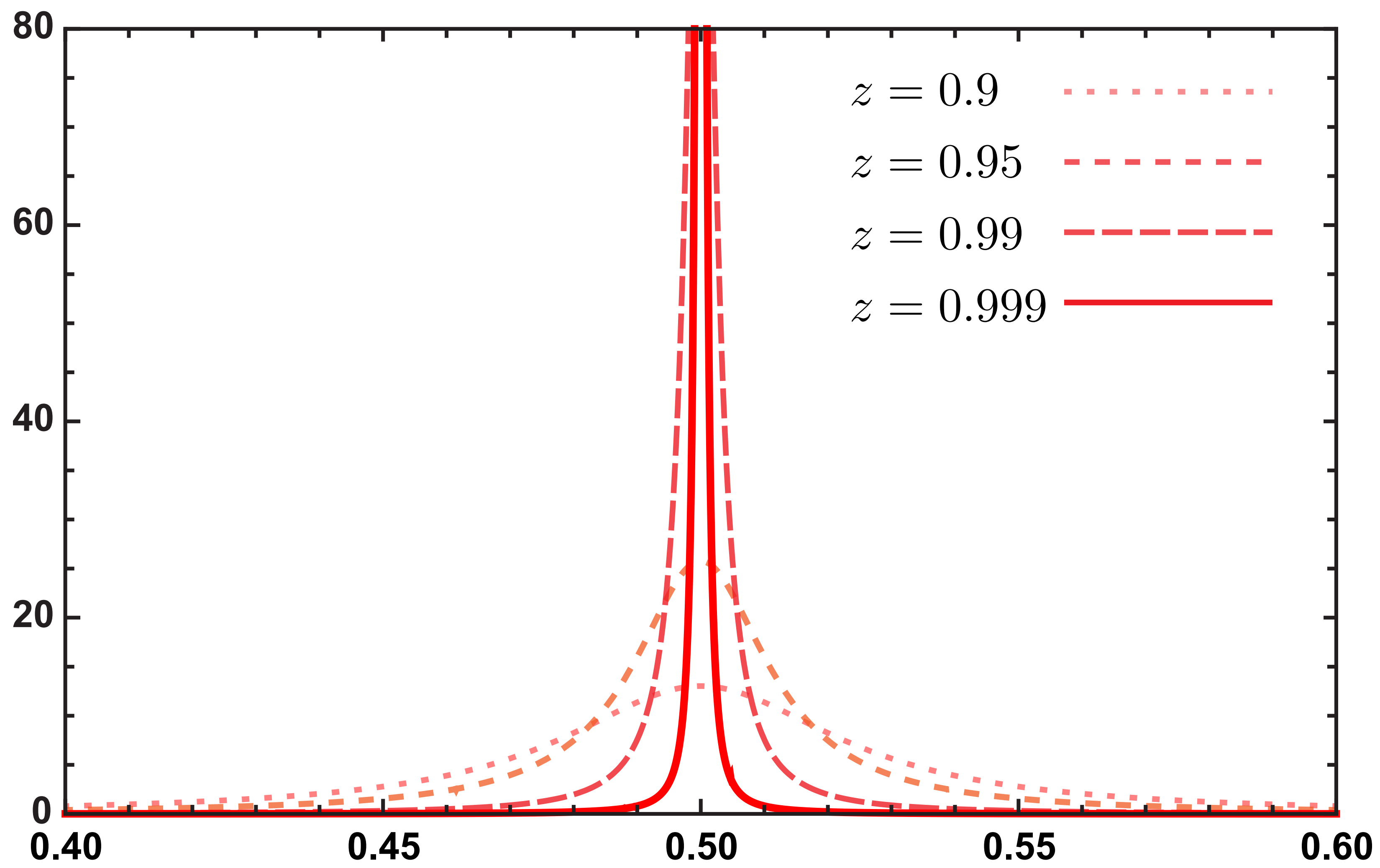
- When $\phi(x,\mu)$ is a singular distribution, **this sum *does not converge*** or *converges very slowly*.
- One way to define that divergent series is to use Abel sum.

$$\sum_n |n,x'\rangle\langle n,x| \to \lim_{z\to 1^-} \sum_n z^n |n,x'\rangle\langle n,x|$$

- This replaces the delta function $\delta(x-x')$ with a sequence of ordinary functions.

EVOLUTION OF QUARKONIUM LCDA

- Abel sum of the Gegenbauer expansion of $\delta(x - \frac{1}{2})$



EVOLUTION OF QUARKONIUM LCDA

- Now the series converges and we can compute the resummed amplitude

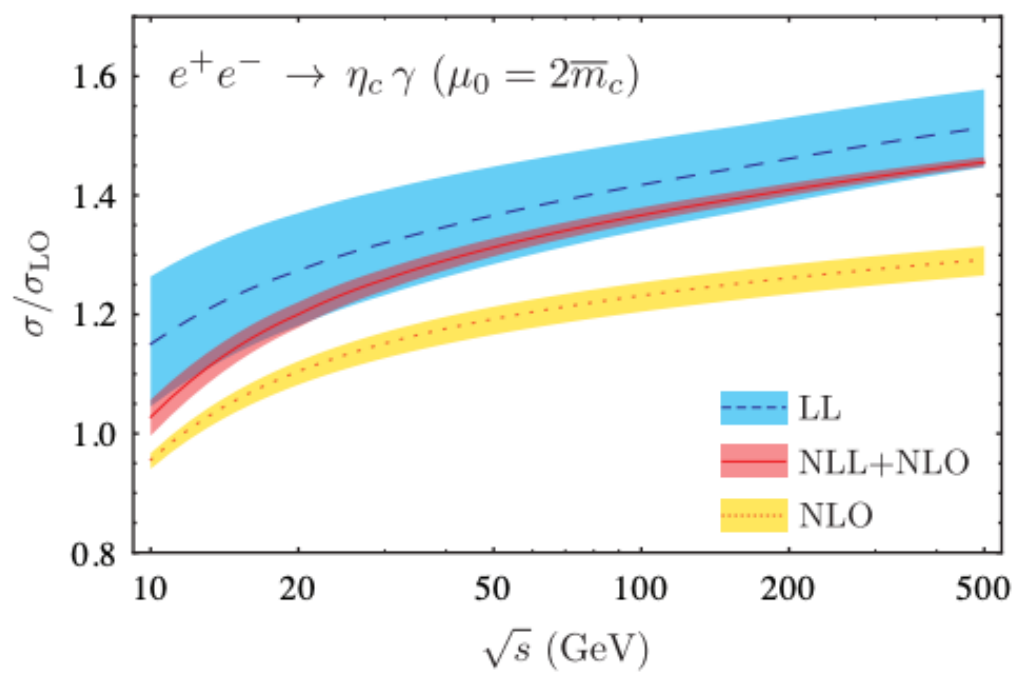
$$i\mathcal{A} = \lim_{z\to 1^-} \sum_{m,n} z^n \langle T_H|m\rangle\langle m|U|n\rangle\langle n|\phi(\mu_0)\rangle$$

- Numerical values of the series can be computed efficiently by using Padé approximants.
- This technique have been used for NLL resummation in $H \to J/\psi + \gamma,\ Z \to J/\psi + \gamma$, and $e^+e^- \to \eta_c + \gamma$.

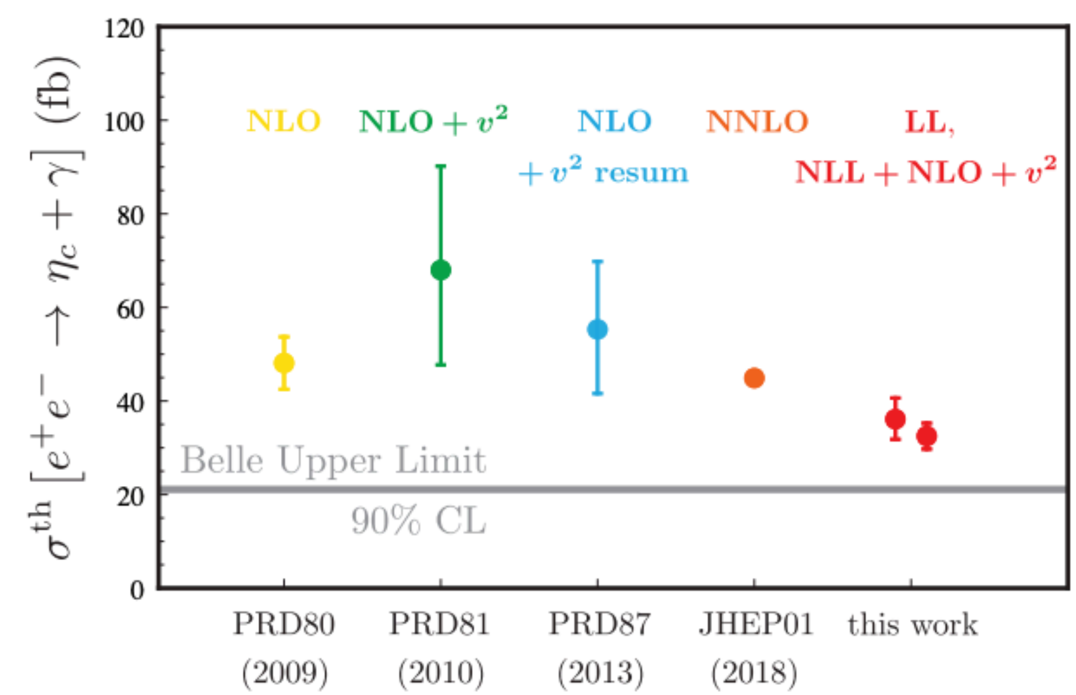
Bodwin, HSC, Ee, Lee, PRD95 (2017) 054018, PRD96 (2017) 116014, PRD97 (2018) 016009
Brambilla, HSC, Lai, Shtabovenko, Vairo, PRD100 (2019) 054038
HSC, Ee, Kang, Kim, Lee, Wang, JHEP 1910 (2019) 162

η_c PRODUCTION IN LEPTON COLLIDERS

- Resummation in the process $e^+e^- \to \eta_c + \gamma$ mildly increases the cross section.
  The theory prediction is above the measured upper limit.



HSC, Ee, Kang, Kim, Lee, Wang, JHEP 1910 (2019) 162



Belle, PRD 98, 092015 (2018)

- This may imply that the color singlet NLP fragmentation could be overestimating the inclusive η_c cross section.

SUMMARY

- Quarkonium LCDAs can be determined in terms of NRQCD local operator matrix elements through perturbative matching. Order-$\alpha_s v^0$ corrections are available, and relativistic corrections are, or will soon become, available. NRQCD matrix elements are generally available to relative order v^2.

- Issues with singular distributions can be circumvented and large logarithms can be resummed. Higher twist contributions can be included from fixed-order calculations. These corrections can be important for charmonium production processes.