Gravitational form factors and light-cone distributions for pseudoscalar mesons

Kazuhiro Tanaka (Juntendo U/KEK)

KT, PRD98, 034009 ('18)
Y. Hatta, A. Rajan, KT, JHEP1812, 008
KT, JHEP1901, 120
work in progress

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Operator product expansion for B-meson distribution amplitude and dimension-5 HQET operators

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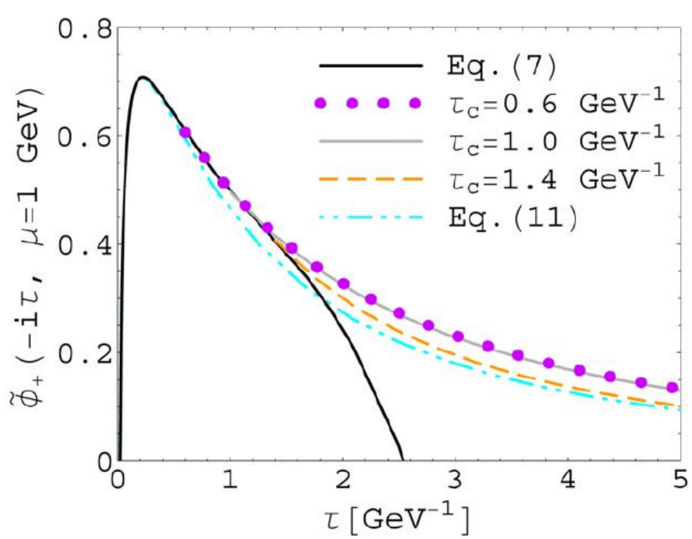
$$\tilde{\phi}_+(t,\mu) = 1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 2L + \frac{5\pi^2}{12}\right) - it\frac{4\bar{\Lambda}}{3}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 4L - \frac{9}{4} + \frac{5\pi^2}{12}\right)\right]$$
$$- t^2\bar{\Lambda}^2\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + \frac{16}{3}L - \frac{35}{9} + \frac{5\pi^2}{12}\right)\right] - \frac{t^2\lambda_E^2(\mu)}{3}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 2L - \frac{2}{3} + \frac{5\pi^2}{12}\right)\right.$$
$$\left. + \frac{\alpha_s C_G}{4\pi}\left(\frac{3}{4}L - \frac{1}{2}\right)\right] - \frac{t^2\lambda_H^2(\mu)}{6}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + \frac{2}{3} + \frac{5\pi^2}{12}\right) - \frac{\alpha_s C_G}{8\pi}(L-1)\right],$$



$$L \equiv \ln[i(t - i0)\mu e^{\gamma_E}]$$

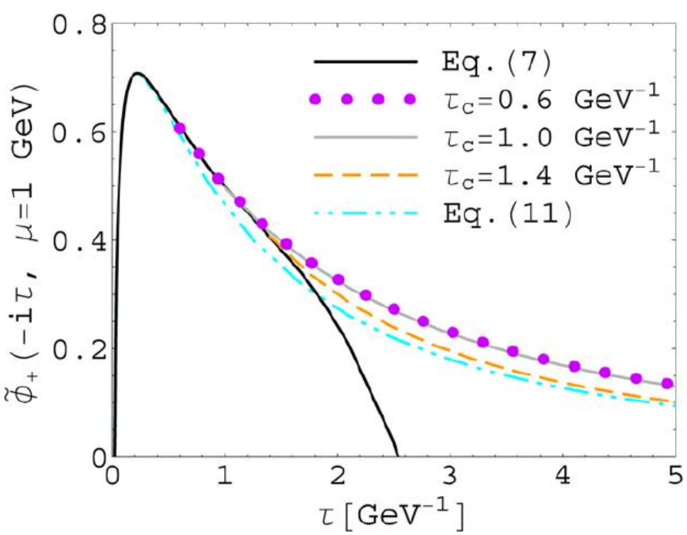
$$\int_0^\infty d\omega\, e^{-\omega\tau} N \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} = \frac{N}{(\tau\omega_0 + 1)^2}.$$

Table 1

Parameters of the model function (12) for different values of τ_c, determined by continuity at $\tau = \tau_c$ with the OPE-based LCDA (7) for $\mu = 1$ GeV, and the results of the inverse moment $\lambda_B^{-1}(\mu)$ at $\mu = 1$ GeV, with the first and second numbers in the parentheses denoting the contributions from the first and the second terms in the RHS of (13).

τ_c [GeV^{-1}]	$\lambda_E^2 = 0.11$ GeV^2, $\lambda_H^2 = 0.18$ GeV^2			$\lambda_E^2 = \lambda_H^2 = 0$		
	N	ω_0 [GeV]	λ_B^{-1} [GeV^{-1}]	N	ω_0 [GeV]	λ_B^{-1} [GeV^{-1}]
0.4	0.816	0.257	3.11 (0.23 + 2.88)	0.832	0.301	2.69 (0.23 + 2.46)
0.6	0.850	0.306	2.70 (0.35 + 2.35)	0.899	0.394	2.19 (0.35 + 1.84)
0.8	0.852	0.308	2.69 (0.47 + 2.22)	0.966	0.461	1.99 (0.46 + 1.53)
1.0	0.858	0.313	2.66 (0.58 + 2.08)	1.11	0.572	1.79 (0.56 + 1.23)
1.2	0.910	0.349	2.51 (0.67 + 1.84)	1.55	0.839	1.56 (0.64 + 0.92)
1.4	1.09	0.456	2.22 (0.76 + 1.46)	4.43	1.95	1.32 (0.71 + 0.61)
1.6	1.81	0.777	1.87 (0.83 + 1.04)	9.82	−4.55	1.11 (0.77 + 0.34)

$$\tilde{\phi}_+(t,\mu) = 1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 2L + \frac{5\pi^2}{12}\right) - it\frac{4\bar{\Lambda}}{3}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 4L - \frac{9}{4} + \frac{5\pi^2}{12}\right)\right]$$
$$- t^2\bar{\Lambda}^2\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + \frac{16}{3}L - \frac{35}{9} + \frac{5\pi^2}{12}\right)\right] - \frac{t^2\lambda_E^2(\mu)}{3}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 2L - \frac{2}{3} + \frac{5\pi^2}{12}\right)\right.$$
$$\left. + \frac{\alpha_s C_G}{4\pi}\left(\frac{3}{4}L - \frac{1}{2}\right)\right] - \frac{t^2\lambda_H^2(\mu)}{6}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + \frac{2}{3} + \frac{5\pi^2}{12}\right) - \frac{\alpha_s C_G}{8\pi}(L-1)\right],$$



$$L \equiv \ln[i(t - i0)\mu e^{\gamma_E}]$$

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Nuclear Physics B 879 (2014) 110–142

QCD sum rules for quark–gluon three-body components in the B meson ☆

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[b] Department of Physics, Juntendo University, Inzai, Chiba 270-1695, Japan
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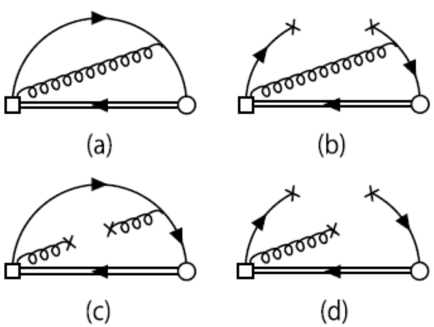

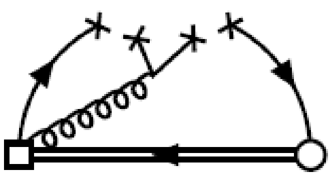

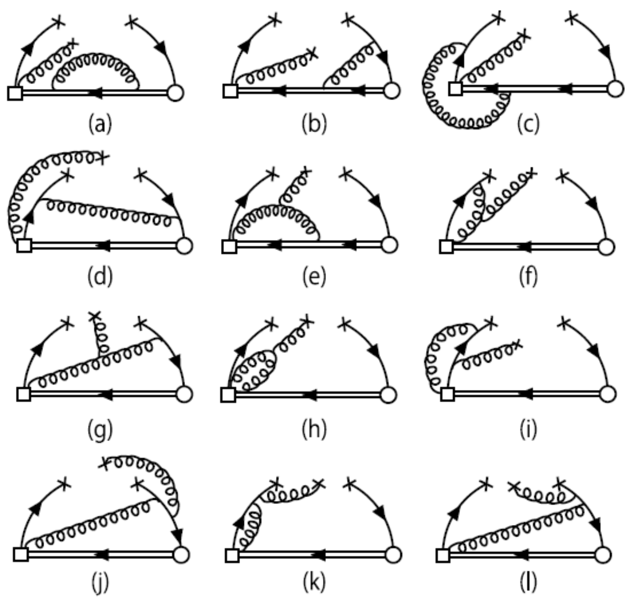

$$\lambda_E^2(\mu) = 0.11 \pm 0.06 \text{ GeV}^2,$$
$$\lambda_H^2(\mu) = 0.18 \pm 0.07 \text{ GeV}^2,$$

$$\lambda_E^2(1 \text{ GeV}) = 0.03 \pm 0.02 \text{ GeV}^2,$$
$$\lambda_H^2(1 \text{ GeV}) = 0.06 \pm 0.03 \text{ GeV}^2.$$

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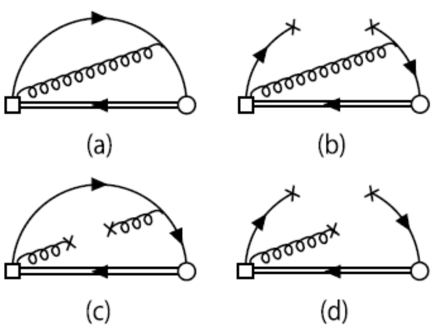

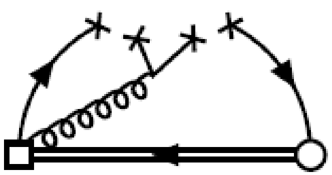

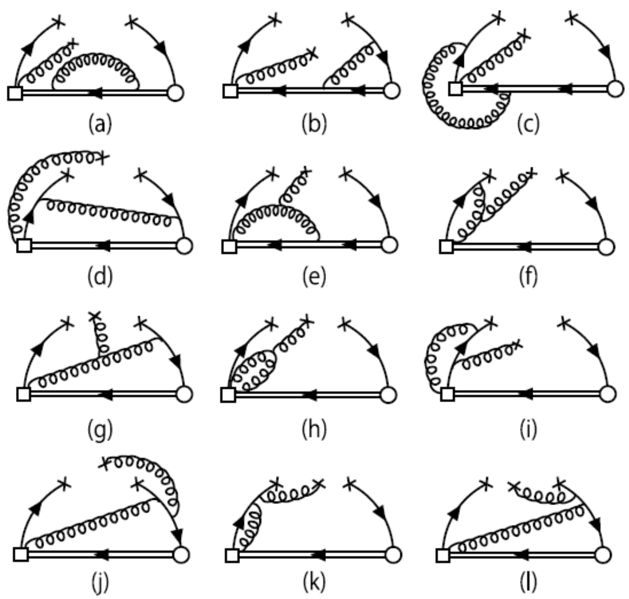

$$\lambda_E^2(\mu) = 0.11 \pm 0.06 \text{ GeV}^2,$$
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$\dfrac{\lambda_E^2}{\lambda_H^2}$ **comming soon**

Gravitational form factors and light-cone distributions for pseudoscalar mesons

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work in progress

1. Definitions QCD energy-momentum tensor

2. Access through hard processes

OPE in DVCS

3. Constraints from operator relations

Twist-3 FF

4. Constraints from trace anomaly

Twist-4 FF

3-loop renormalization

hadron mass composition

5. Prospects

heavy meson

LCSRs

(Belinfante-improved) energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4} F^2 \; + \; (\text{ghost}) \; + \; (\text{gauge fix})$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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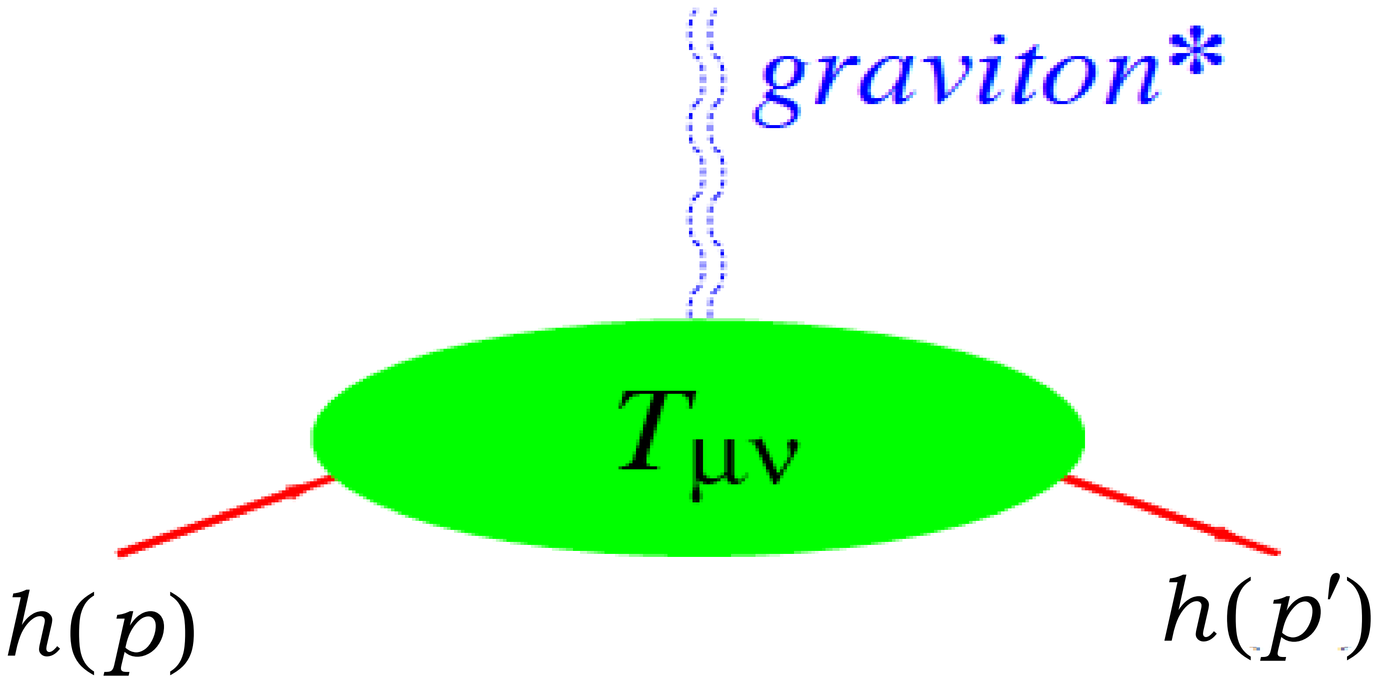

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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$$\langle N(p')|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

Ji, PRL78, 610 ('97)

Polyakov, Schweitzer, IJMPA33, 1830025('18)

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4} F^2$$

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$$\langle h(p')| T_{q,g}^{\mu\nu} | h(p)\rangle = \frac{1}{2}\Theta_{2\,q,g}(t) P^\mu P^\nu + \frac{1}{2}\Theta_{1\,q,g}(t)\left(g^{\mu\nu}\Delta^2 - \Delta^\mu\Delta^\nu\right) + \Lambda^2 \bar{C}_{q,g}(t) g^{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle N(p)|T^{\mu\nu}|N(p)\rangle = 2p^\mu p^\nu$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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$$\partial_{\mu}T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle N(p)|T^{\mu\nu}|N(p)\rangle = 2p^{\mu}p^{\nu}$$

$$\frac{1}{2}\Big(A_q(0) + B_q(0) + A_g(0) + B_g(0)\Big) = \frac{1}{2}$$

$$\frac{\langle N(p)\,S|J^i|N(p)\,S\rangle}{\langle N(p)\,S|N(p)\,S\rangle} = \frac{1}{2}S^i$$

$$B_q(0) + B_g(0) = 0$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^{\rho}T^{\mu\sigma} - x^{\sigma}T^{\mu\rho}$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \; + \; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^{2}$$

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Ji, PRL78, 610 ('97)

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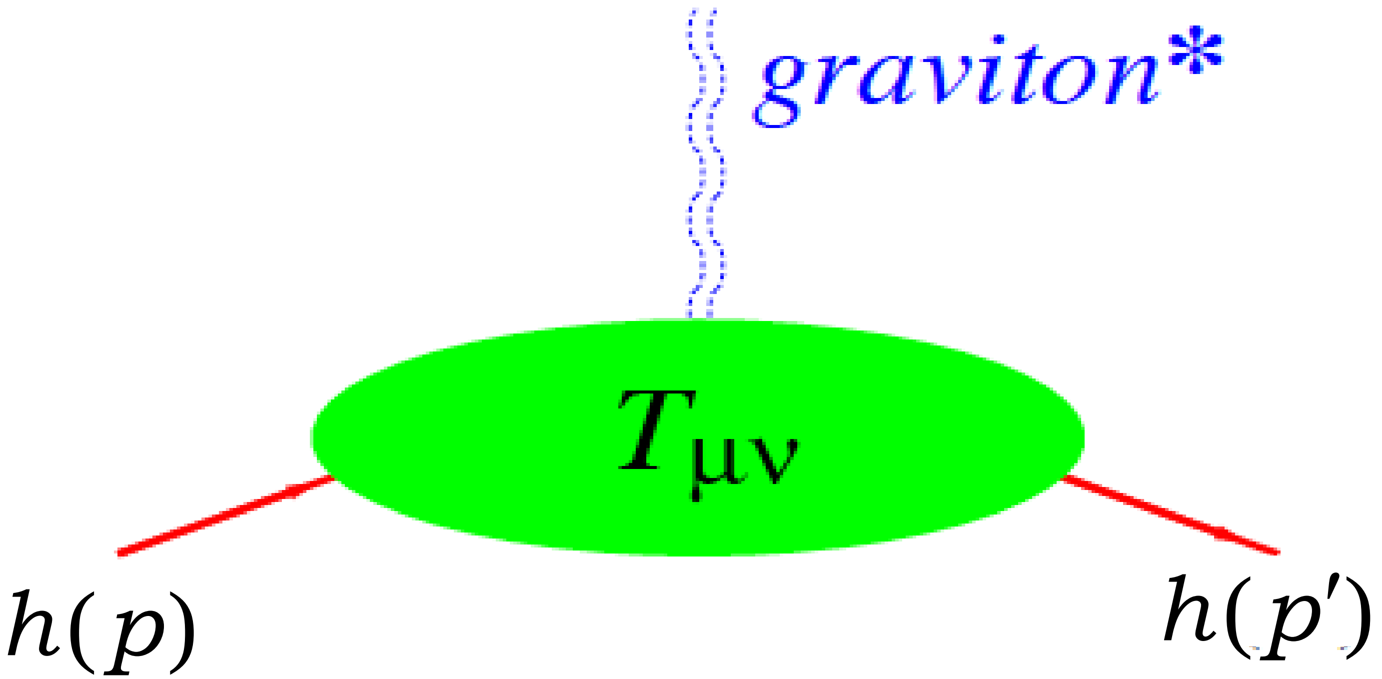
$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\Theta_{2q}\left(0\right) + \Theta_{2g}\left(0\right) = 4$$

$$\langle p | T^{\mu\nu} | p \rangle = 2 p^{\mu} p^{\nu}$$

$$\langle h(p') | T^{\mu\nu}_{q,g} | h(p) \rangle = \frac{1}{2}\Theta_{2\,q,g}(t) P^{\mu}P^{\nu} + \frac{1}{2}\Theta_{1\,q,g}(t)\left(g^{\mu\nu}\Delta^{2} - \Delta^{\mu}\Delta^{\nu}\right) + \Lambda^{2}\bar{C}_{q,g}(t) g^{\mu\nu}$$



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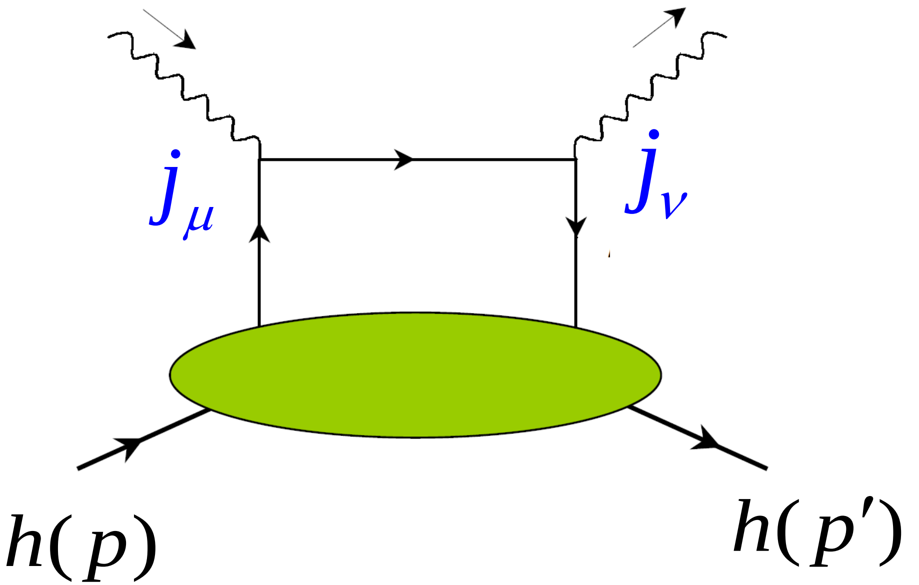


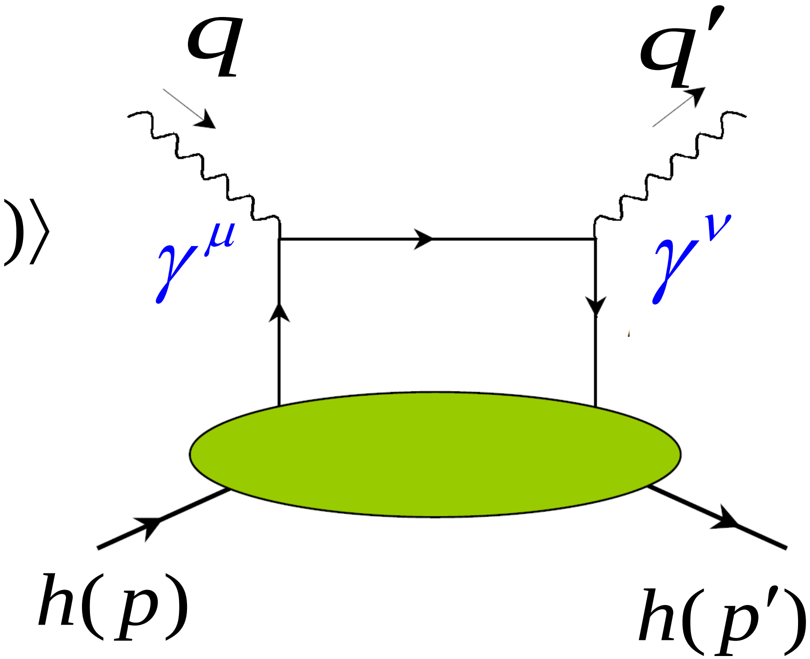
$$j_\mu(x)\,j_\nu(0) \sim \sum_i C_i(x) O_i(0)$$

$$C^q_{\mu\nu;\alpha\beta}(x) T_q^{\alpha\beta}$$

$$C^g_{\mu\nu;\alpha\beta}(x) T_g^{\alpha\beta}$$

DVCS

$$\int d^4x e^{iq'\cdot x} \langle h(p') | \mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x) | h(p) \rangle$$



$$\bar{P} = \frac{p+p'}{2}, \quad \bar{q} = \frac{q+q'}{2}, \quad \Delta = p' - p = q - q',$$

$$t = \Delta^2, \quad \xi = \frac{-\bar{q}^2}{2\bar{P}\cdot\bar{q}}, \quad \eta = \frac{-\Delta\cdot n}{2\bar{P}\cdot n} = \frac{-\Delta\cdot\bar{q}}{2\bar{P}\cdot\bar{q}} + O(\text{twist-4}),$$

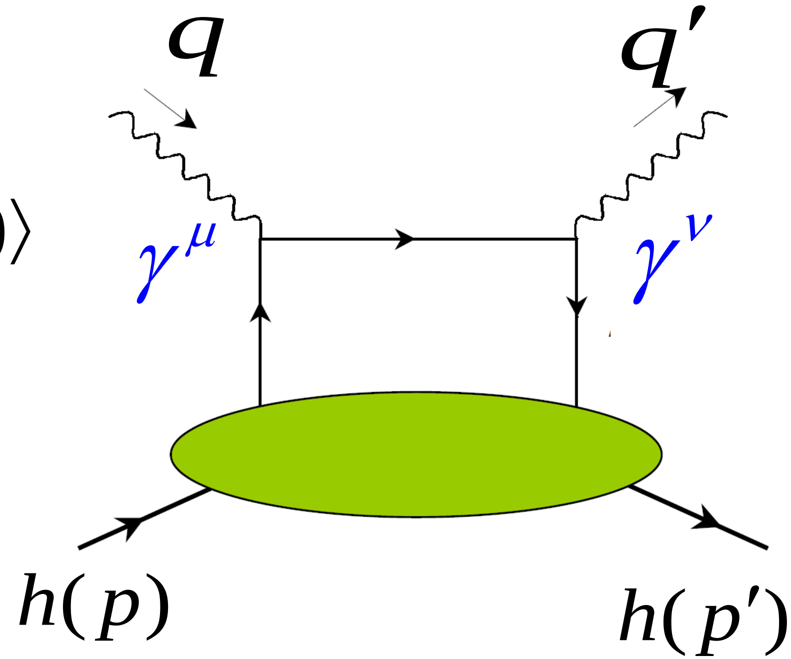
$$n_\mu = -\frac{2\xi}{\bar{q}^2}\left(\bar{q}_\mu + \xi\bar{P}_\mu\right), \quad \tilde{n}_\mu = \bar{P}_\mu, \qquad \left(n^2 = \tilde{n}^2 = 0, \quad n\cdot\tilde{n} = 1\right)$$

generalized Bjorken kinematics:

$$|\bar{q}^2| \to \infty, \quad |\bar{P}\cdot\bar{q}| \to \infty, \quad |\Delta\cdot\bar{q}| \to \infty, \quad \Delta^2 = \text{finite} \quad (\xi \text{ and } \eta \text{ fixed})$$

DVCS

$$\int d^4x e^{iq'\cdot x}\langle h(p')|\,\mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x)\,|h(p)\rangle$$



$$\bar{P} = \frac{p+p'}{2}, \quad \bar{q} = \frac{q+q'}{2}, \quad \Delta = p' - p = q - q',$$

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$$n_\mu = -\frac{2\xi}{\bar{q}^2}\left(\bar{q}_\mu + \xi\bar{P}_\mu\right), \quad \tilde{n}_\mu = \bar{P}_\mu, \qquad \left(n^2 = \tilde{n}^2 = 0, \quad n\cdot\tilde{n} = 1\right)$$

generalized Bjorken kinematics: light-cone expansion

$$|\bar{q}^2| \to \infty, \quad |\bar{P}\cdot\bar{q}| \to \infty, \quad |\Delta\cdot\bar{q}| \to \infty, \quad \Delta^2 = \text{finite} \quad (\xi \text{ and } \eta \text{ fixed})$$

$$\int d^4x e^{iq'\cdot x}\langle h(p')|\,\mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x)\,|h(p)\rangle$$

$$= \mathcal{T}_{\mu\nu}^{(1)} \frac{ie_q^2}{\bar{q}^2}\int dx \left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2} - \frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2} \right) \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} < h(p')\left|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})\right| h(p) > \Big|_{z^+=\vec{z}_\perp=0}$$

$$+\mathcal{T}_{\mu\nu}^{(2)} \frac{ie_q^2}{\bar{q}^2}\int dx \left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2} + \frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2} \right) \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} < h(p')\left|\bar{q}(-\frac{z}{2})\gamma^+\gamma_5 q(\frac{z}{2})\right| h(p) > \Big|_{z^+=\vec{z}_\perp=0}$$

$$+ O(\text{twist-3})$$

$$\mathcal{T}_{\mu\nu}^{(1)} = \tilde{n}_\mu\left(\bar{q}_\nu + \xi\tilde{n}_\nu\right) + \tilde{n}_\nu\left(\bar{q}_\mu + \xi\tilde{n}_\mu\right) - g_{\mu\nu}\bar{q}\cdot\tilde{n} = \frac{\bar{q}^2}{2\xi}\left(g_{\mu\nu} - \frac{q'_\mu q_\nu}{q\cdot q'}\right) + 2\xi\left(\bar{P}_\mu - \frac{\bar{P}\cdot q}{q\cdot q'}q'_\mu\right)\left(\bar{P}_\nu - \frac{\bar{P}\cdot q'}{q\cdot q'}q_\nu\right)$$

$$\mathcal{T}_{\mu\nu}^{(2)} = i\varepsilon_{\mu\alpha\nu\rho}\bar{q}^\alpha\tilde{n}^\rho = i\varepsilon^{\lambda\beta\rho\sigma}\bar{P}_\rho\bar{q}_\sigma\left(g_{\mu\lambda} - \frac{\bar{P}_\mu q_\lambda}{\bar{P}\cdot q}\right)\left(g_{\nu\beta} - \frac{\bar{P}_\nu q'_\beta}{\bar{P}\cdot q'}\right)$$

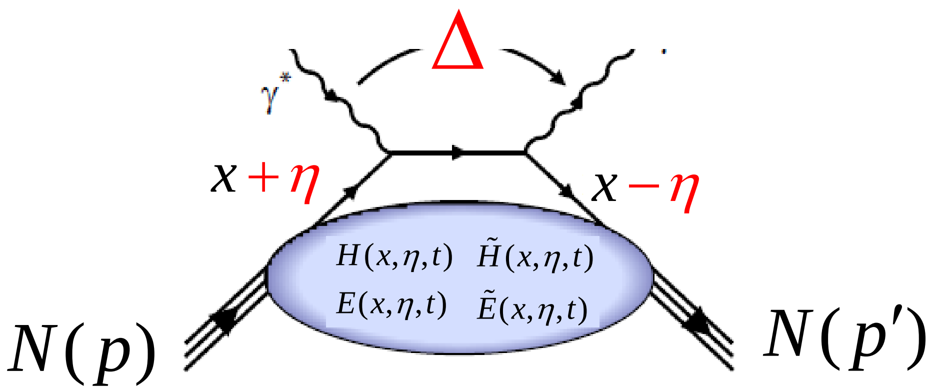
$$q^\mu\mathcal{T}_{\mu\nu}^{(1)} = q'^\nu\mathcal{T}_{\mu\nu}^{(1)} = q^\mu\mathcal{T}_{\mu\nu}^{(2)} = q'^\nu\mathcal{T}_{\mu\nu}^{(2)} = 0\,,$$

$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p) \right]$$

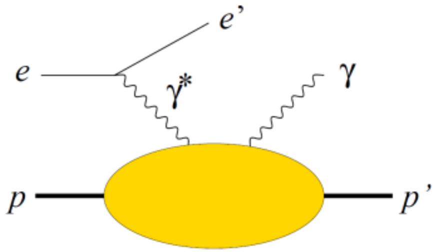
$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+ \gamma_5 q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz} \langle N(p')|\psi^\dagger(0)\psi(z^-)|N(p)\rangle$$



JLab, HERMES, COMPASS,…

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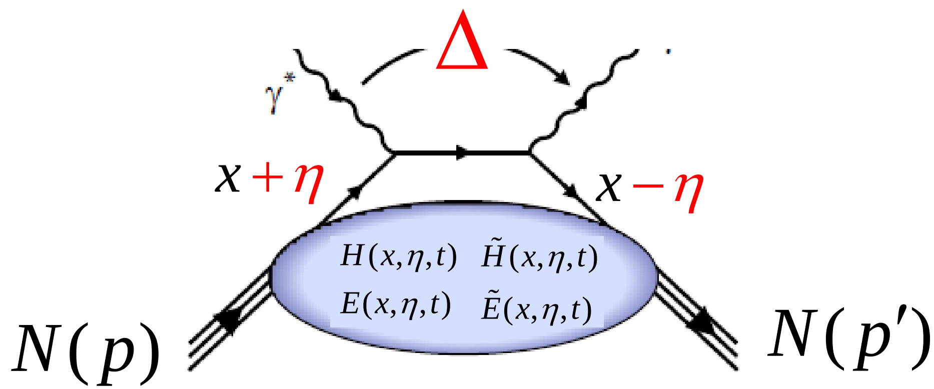

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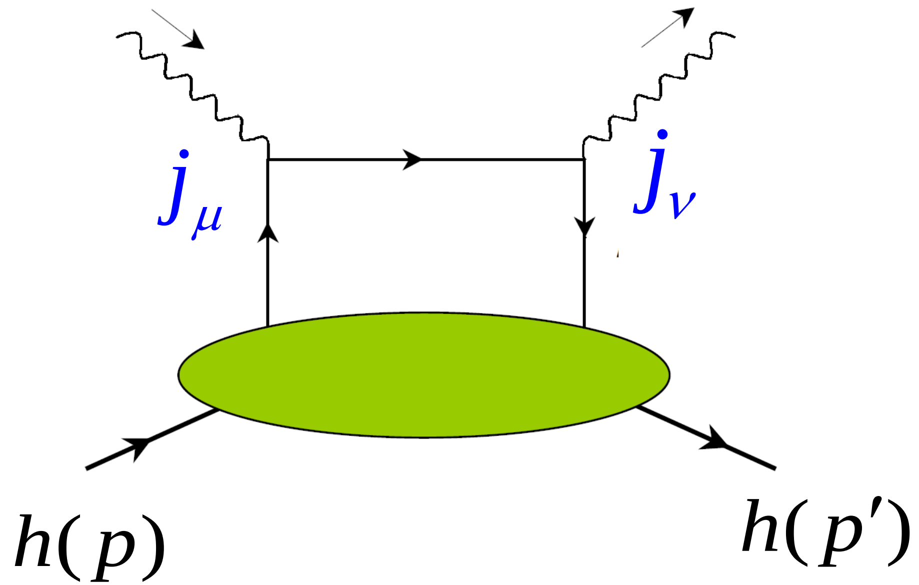

$$j_\mu(x)\, j_\nu(0) \sim \sum_i C_i(x) O_i(0)$$

$$C^q_{\mu\nu;\alpha\beta}(x) T_q^{\alpha\beta}$$

$$C^g_{\mu\nu;\alpha\beta}(x) T_g^{\alpha\beta}$$

$$\int d^4x e^{iq'\cdot x}\langle h(p')|\,\mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x)\,|h(p)\rangle$$

$$= \mathcal{T}_{\mu\nu}^{(1)} \frac{ie_q^2}{\bar{q}^2}\int dx \left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2} - \frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2} \right) \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} < h(p')\left|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})\right| h(p) \, >\Big|_{z^+=\vec{z}_\perp=0}$$

$$+\mathcal{T}_{\mu\nu}^{(2)} \frac{ie_q^2}{\bar{q}^2}\int dx \left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2} + \frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2} \right) \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} < h(p')\left|\bar{q}(-\frac{z}{2})\gamma^+\gamma_5 q(\frac{z}{2})\right| h(p) \, >\Big|_{z^+=\vec{z}_\perp=0}$$

$$+ O(\text{twist-3})$$

$$\mathcal{T}_{\mu\nu}^{(1)} = \tilde{n}_\mu\left(\bar{q}_\nu + \xi\tilde{n}_\nu\right) + \tilde{n}_\nu\left(\bar{q}_\mu + \xi\tilde{n}_\mu\right) - g_{\mu\nu}\bar{q}\cdot\tilde{n} = \frac{\bar{q}^2}{2\xi}\left(g_{\mu\nu} - \frac{q'_\mu q_\nu}{q\cdot q'}\right) + 2\xi\left(\bar{P}_\mu - \frac{\bar{P}\cdot q}{q\cdot q'}q'_\mu\right)\left(\bar{P}_\nu - \frac{\bar{P}\cdot q'}{q\cdot q'}q_\nu\right)$$

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$$q^\mu\mathcal{T}_{\mu\nu}^{(1)} = q'^\nu\mathcal{T}_{\mu\nu}^{(1)} = q^\mu\mathcal{T}_{\mu\nu}^{(2)} = q'^\nu\mathcal{T}_{\mu\nu}^{(2)} = 0\,,$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p') | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | N(p) \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p') | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | N(p) \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

$$\left(t = \Delta^2 \to 0 \,, \quad \eta = \frac{-\Delta \cdot n}{2\bar{P} \cdot n} \to 0 \right)$$

$$H^q(x,0,0) = q(x)$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p) \right]$$

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$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = \cdots \quad , \quad \int_{-1}^{1} dx\, x E^q(x,\eta,t) = \cdots$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p') | \bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2}) | N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

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$$\frac{1}{2P^+} \left\langle N(p') \middle| \bar{q}(0) \gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| N(p) \right\rangle = \bar{u}(p') \gamma^+ u(p) \int_{-1}^{1} dx x H^q(x,\eta,t) + \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} u(p) \int_{-1}^{1} dx x E^q(x,\eta,t)$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

$$\frac{1}{2P^+}\left\langle N(p')\left|\bar{q}(0)\gamma^+ i\overleftrightarrow{D}^+ q(0)\right|N(p)\right\rangle = \bar{u}(p')\gamma^+u(p)\int_{-1}^{1} dx\,xH^q(x,\eta,t) + \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\int_{-1}^{1} dx\,xE^q(x,\eta,t)$$

$\parallel$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p') \,|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|\, N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

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$$\parallel$$

$$\frac{1}{P^+}\left\langle N(p')\middle|T_q^{++}(0)\middle| N(p)\right\rangle$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \;+\; F^{\mu\rho}F_\rho{}^\nu + \frac{g^{\mu\nu}}{4}F^2$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')\,|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|\, N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

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$$=$$

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$$\langle N(p')\,|\,T^{\mu\nu}_{q,g}\,|\,N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p) \right]$$

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$$\parallel$$

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$$\langle N(p')|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

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$$\|$$

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$$\int_{-1}^{1} dx\,xH^q(x,\eta,t) = A_q(t)+4\eta^2D_q(t)\,, \qquad \int_{-1}^{1} dx\,xE^q(x,\eta,t) = B_q(t)-4\eta^2D_q(t)$$

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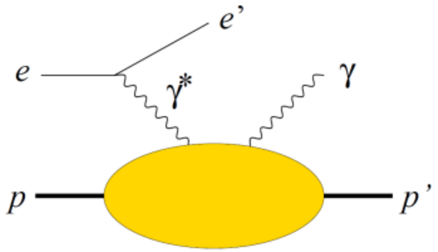

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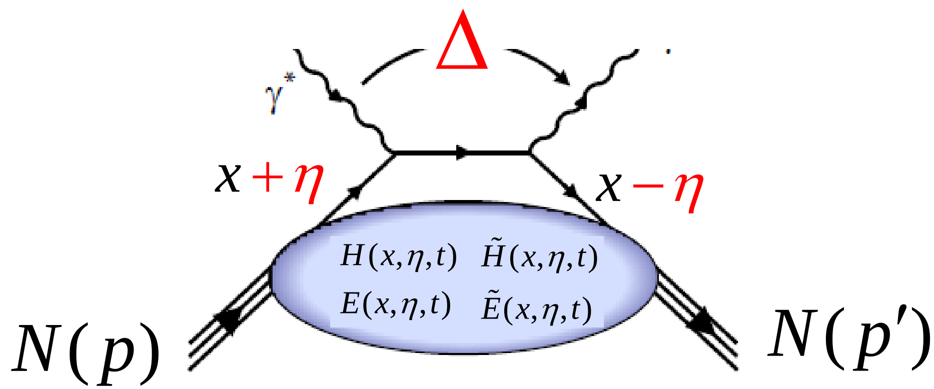
JLab, HERMES, COMPASS,...

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+\gamma_5 q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

GPD



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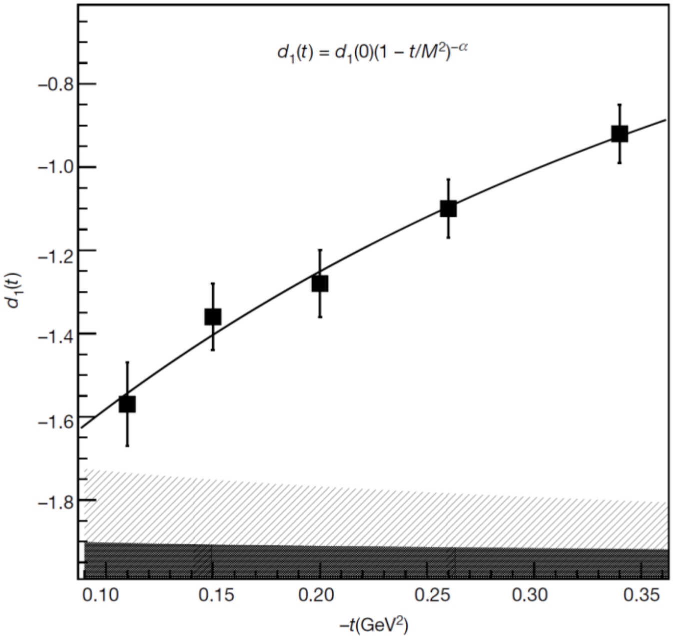
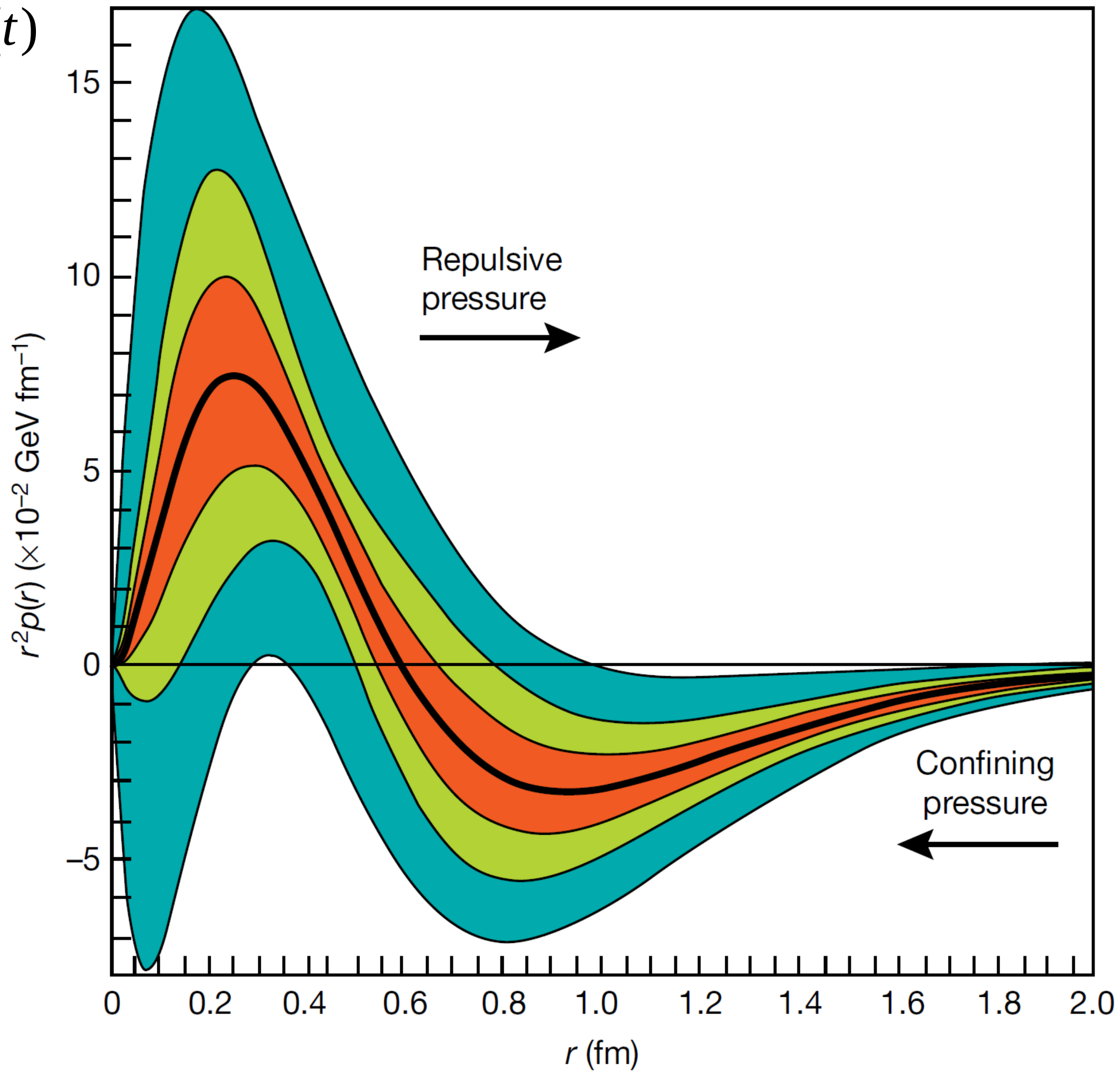
V. D. Burkert et al, Nature 557 ('18) 396

$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t)$$

$$5\sum_q e_q^2 D_q(t) \sim d_1(t)$$





$$\langle N(p')|T^{ik}(0)|N(p)\rangle \sim \left(\Delta^i\Delta^k - \delta^{ik}\boldsymbol{\Delta}^2\right)D(t), \quad T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\, p(r)$$

Kumano, Song, Teryaev, PRD97, 014020 ('18) ,1711.08088

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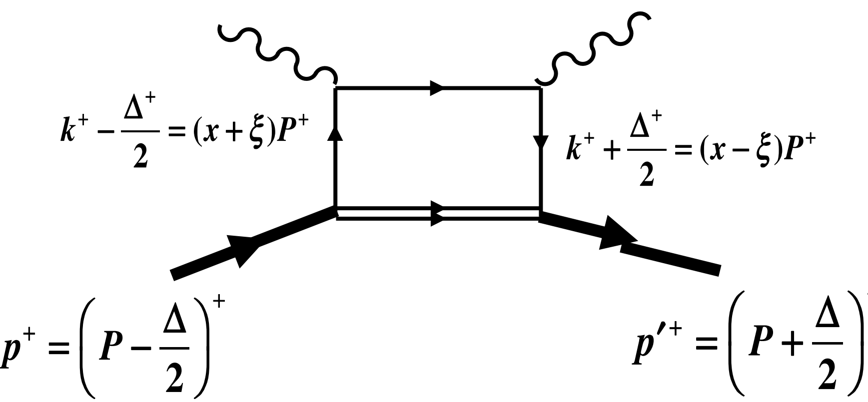
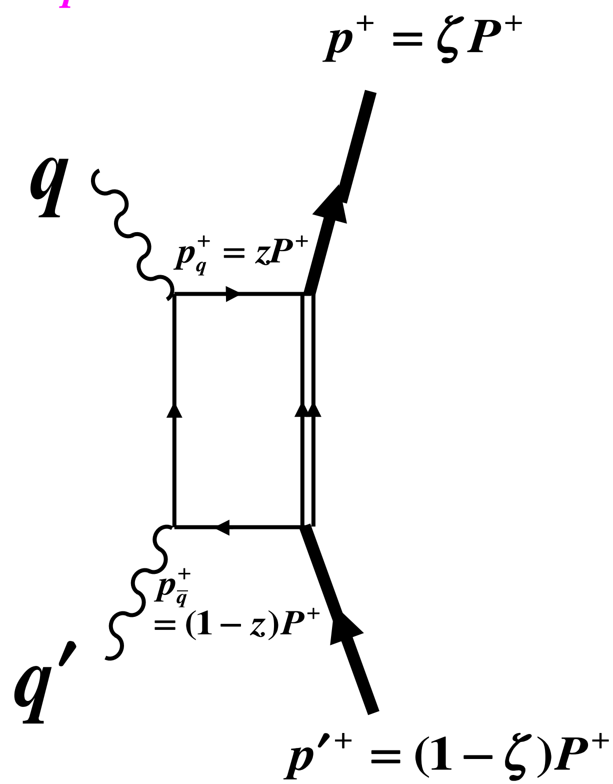
$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

s-t crossing





Kumano, Song, Teryaev, PRD97, 014020 ('18) ,1711.08088

Spacelike gravitational form factors and radii for pion

$$F(s)=\Theta_1(s),\,\Theta_1(s),\quad F(t)=\int_{4m_\pi^2}^{\infty} ds\,\frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r)=\frac{1}{(2\pi)^3}\int d^3q\,e^{-i\vec{q}\cdot\vec{r}}F(q)=\frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\,e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2\rangle_{\text{mass}}}=0.32\sim0.39\ \text{fm},\quad \sqrt{\langle r^2\rangle_{\text{mech}}}=0.82\sim0.88\ \text{fm}$$

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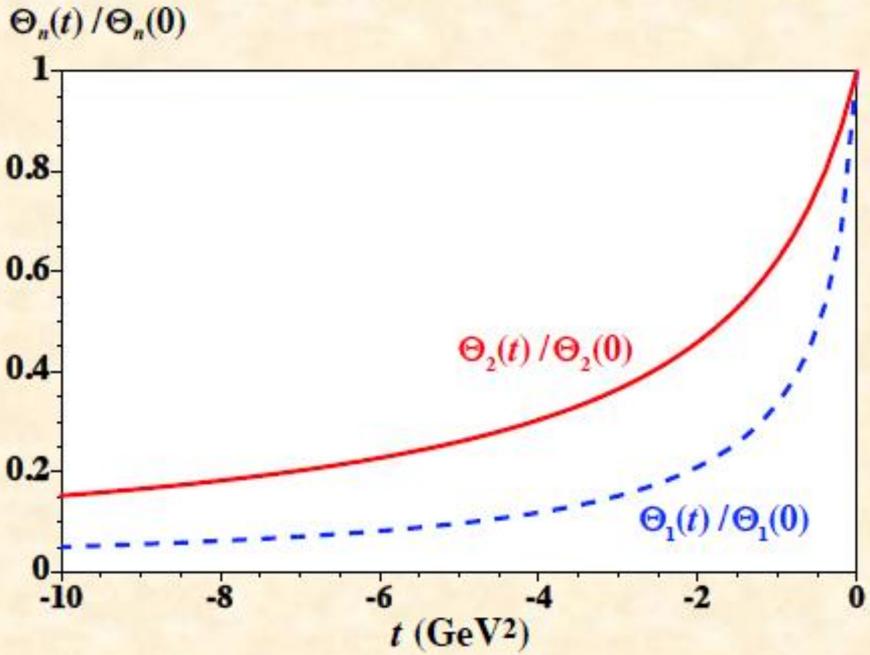

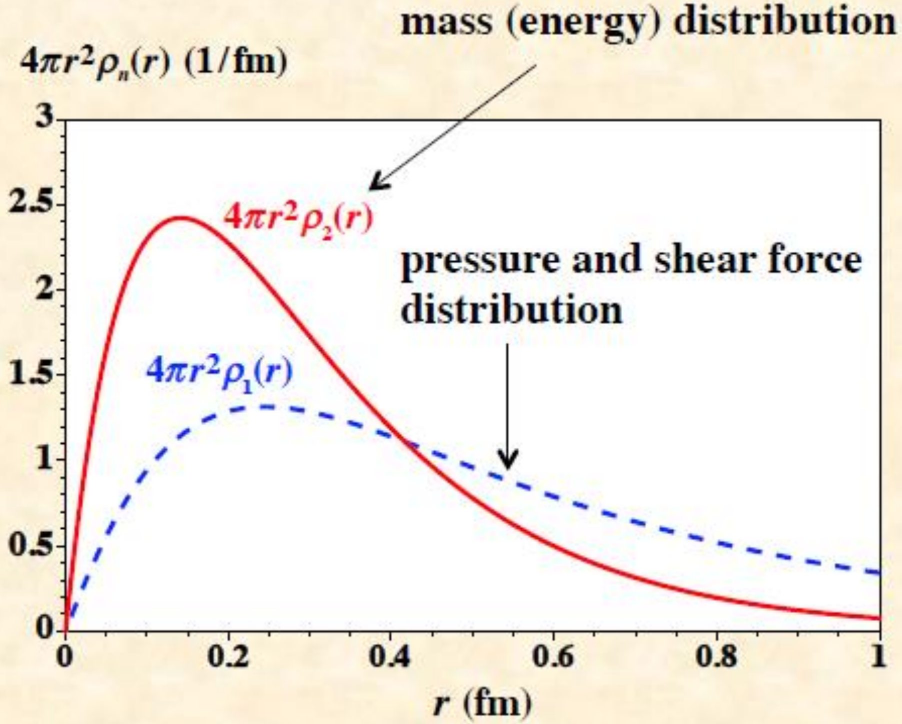

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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Ji, PRL78, 610 ('97)

Polyakov, Schweitzer, IJMPA33, 1830025('18)

$$\langle h(p')|T^{\mu\nu}_{q,g}|h(p)\rangle = \frac{1}{2}\Theta_{2\,q,g}(t)P^{\mu}P^{\nu} + \frac{1}{2}\Theta_{1\,q,g}(t)\left(g^{\mu\nu}\Delta^2 - \Delta^{\mu}\Delta^{\nu}\right) + \Lambda^2\bar{C}_{q,g}(t)g^{\mu\nu}$$

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KT, PRD98, 034009

$$\langle p' \,|\, T_q^{\mu\nu} \,|\, p\rangle = \frac{1}{4}\langle p' \,|\, \bar{q}\left(-i\overleftarrow{\partial}^{\mu} + i\overrightarrow{\partial}^{\mu} + 2gA^{\mu}\right)\gamma^{\nu} q \,|\, p\rangle + \left(\mu \leftrightarrow \nu\right)$$

$$-i\partial^{\mu} q(x) = \left[\hat{P}^{\mu}, q(x)\right] \qquad A^{\mu}(z^-) = \frac{1}{2}\int_{-\infty}^{\infty} dz'^- \, \mathrm{sgn}(z'^- - z^-) F^{\mu+}(z'^-)$$

KT, PRD98, 034009

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intermediated states: "partonic"

KT, PRD98, 034009

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$$A_q(t) + \eta^2 D_q(t) \simeq \langle x\rangle F_1^q(t)\,, \quad B_q(t) - \eta^2 D_q(t) \simeq \langle x\rangle F_2^q(t)$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p') | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | N(p) \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

$$\left(t = \Delta^2 \to 0 \, , \quad \eta = \frac{-\Delta \cdot n}{2\bar{P} \cdot n} \to 0 \right)$$

$$H^q(x,0,0) = q(x)$$

$$\int_{-1}^{1} dx H^q(x,\eta,t) = F_1^q(t), \qquad \int_{-1}^{1} dx E^q(x,\eta,t) = F_2^q(t)$$

$$\int_{-1}^{1} dx x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t) \, , \qquad \int_{-1}^{1} dx x E^q(x,\eta,t) = B_q(t) - 4\eta^2 D_q(t)$$

KT, PRD98, 034009

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KT, PRD98, 034009

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$$-\frac{P^{+}\eta\Delta_{\perp}^{\mu}\bar{u}(p')u(p)}{M} D_q(t) \simeq \langle N(p') | \bar{q} g A_{\perp}^{\mu}\gamma^{+} q | N(p)\rangle$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} d\lambda\, \mathrm{sgn}(\lambda) n_{\alpha} \langle N(p') | g F_a^{\mu\alpha}(\lambda n)\bar{q}(0) t^a \gamma^{+} q(0) | N(p)\rangle$$

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Ji, PRL78, 610 ('97)
Polyakov, Schweitzer, IJMPA33, 1830025('18)

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KT, PRD98, 034009

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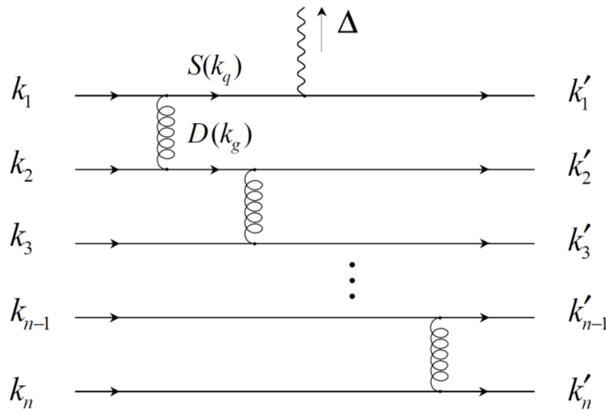
$t\to\infty$

$$A_q(t)\sim\frac{1}{t^2}\,,\qquad D_q(t)\sim\frac{1}{t^3}$$



KT, PRD98, 034009

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$$\frac{1}{2}\Theta_{2q}(t) - 2\eta^2\Theta_{1q}(t) \simeq 2\langle x\rangle F_v(t)$$

$$2\eta\Delta_\perp^\mu\Theta_{1q}(t) \simeq \langle h(p')|\bar{q}gA_\perp^\mu \not{n} q|h(p)\rangle$$

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$t \to \infty$

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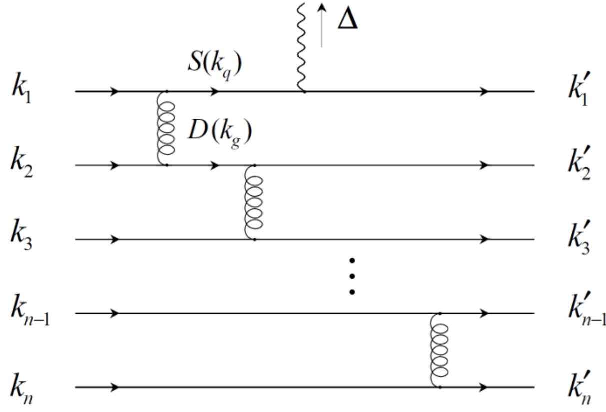

Kumano, Song, Teryaev, PRD97, 014020 ('18) ,1711.08088

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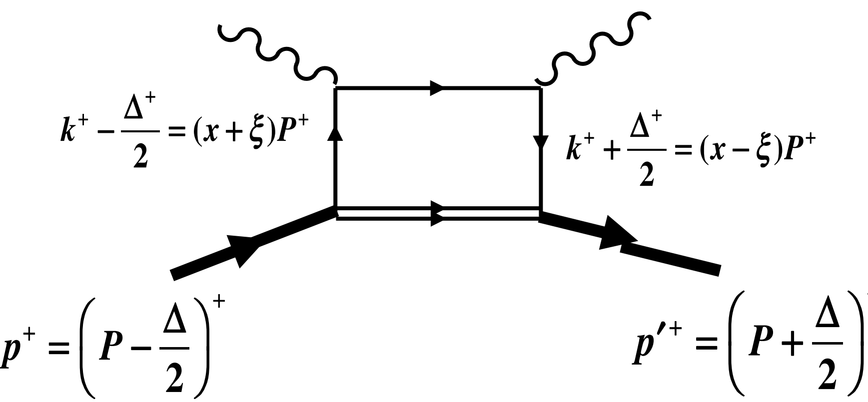

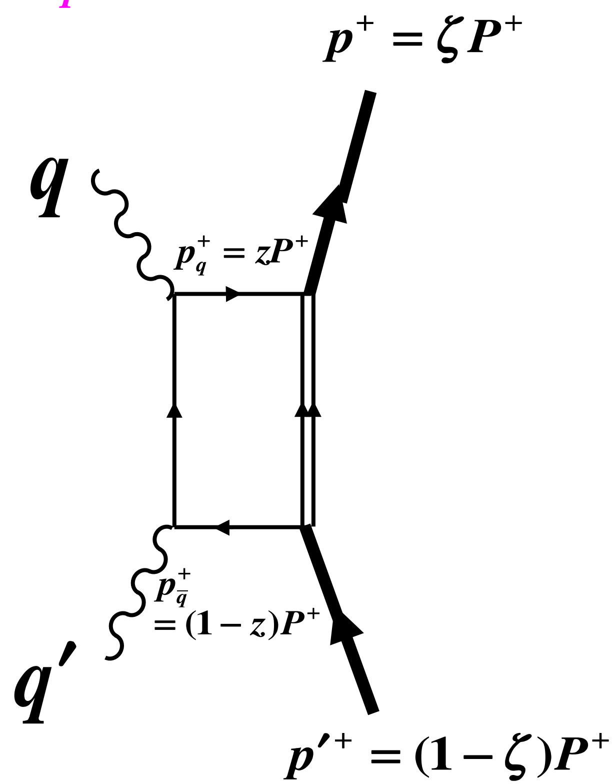

Spacelike gravitational form factors and radii for pion

$$F(s)=\Theta_1(s),\ \Theta_1(s),\quad F(t)=\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r)=\frac{1}{(2\pi)^3}\int d^3q e^{-i\vec{q}\cdot\vec{r}} F(q)=\frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\, e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

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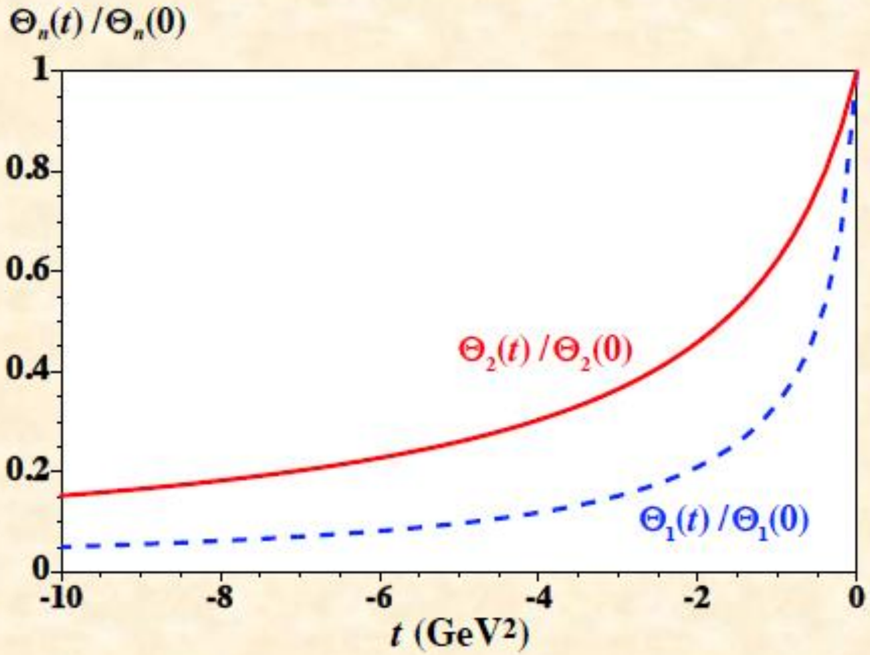

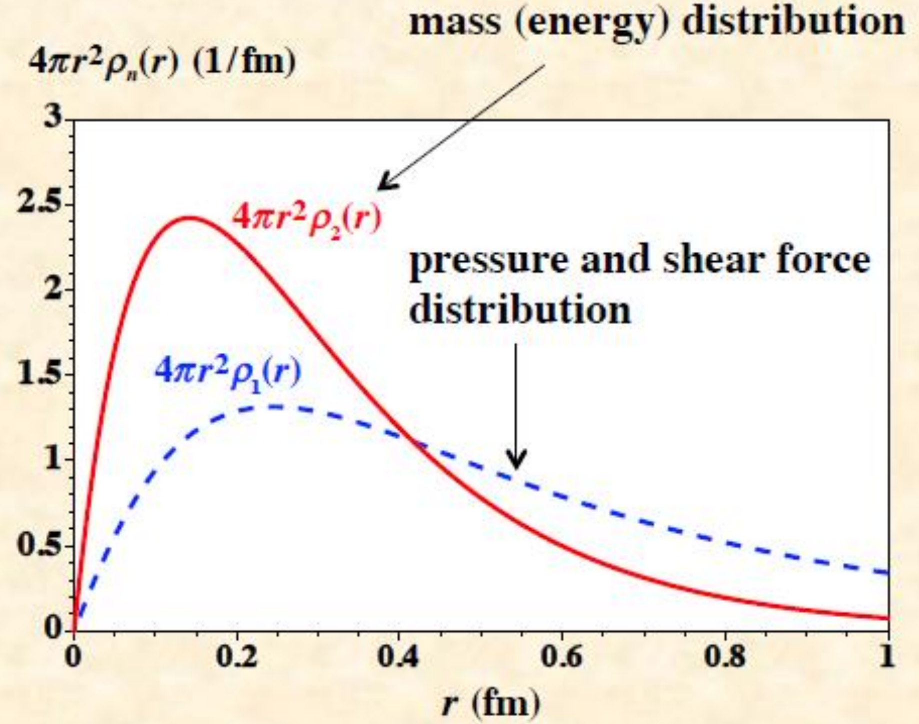

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Ji, PRL78, 610 ('97)

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V. D. Burkert et al, Nature 557 ('18) 396

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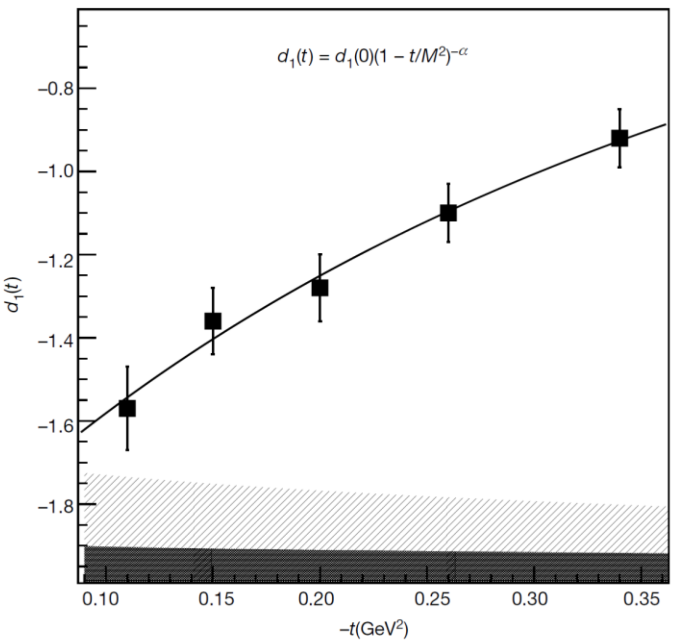

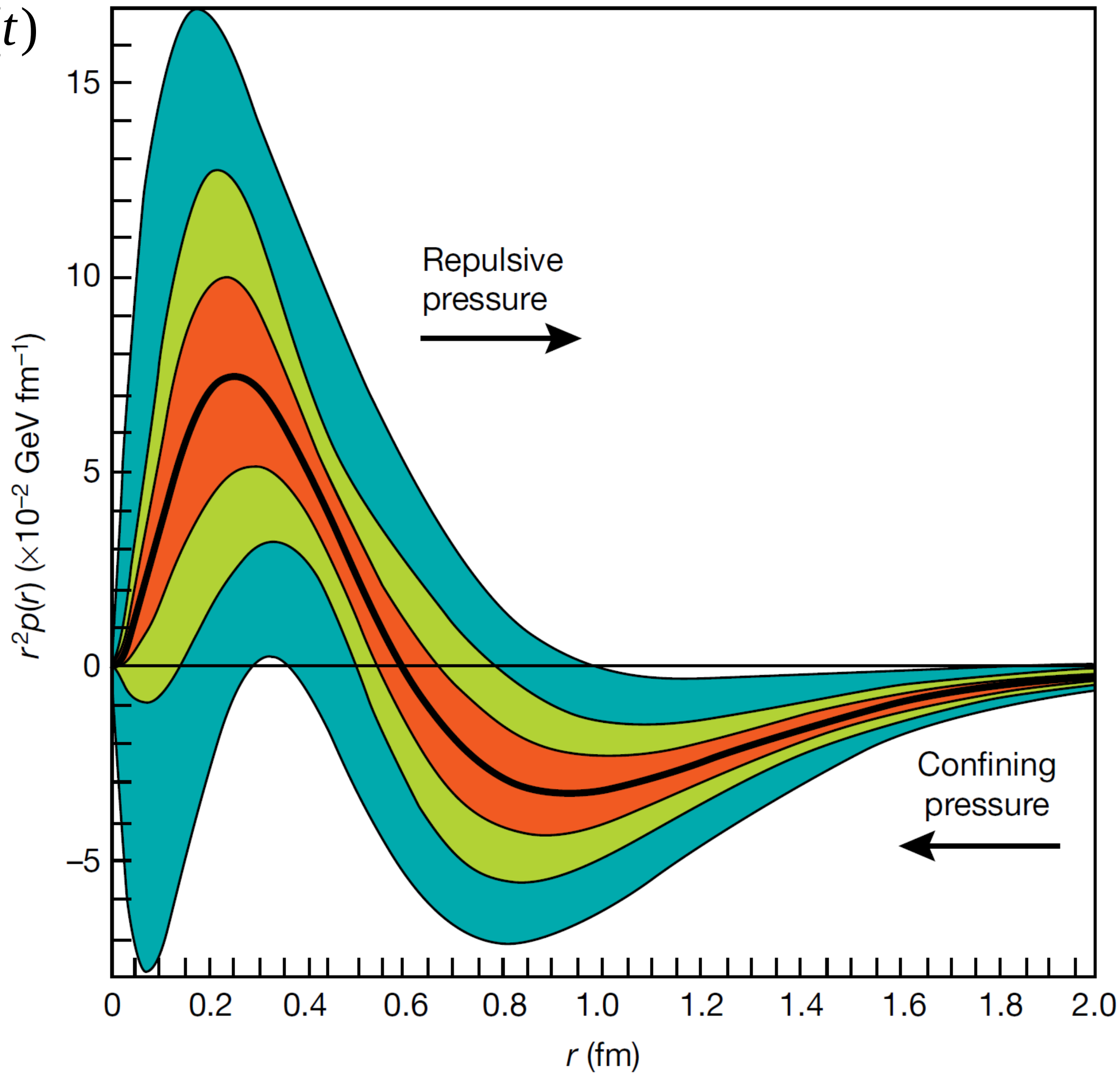

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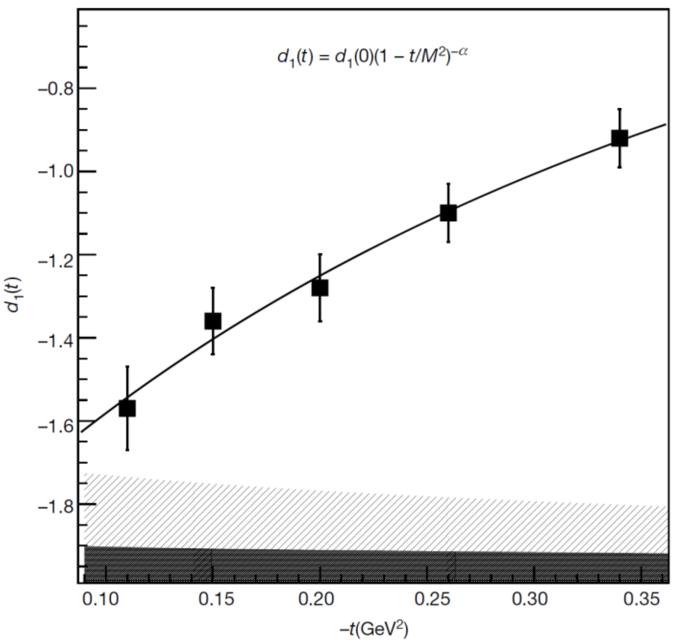

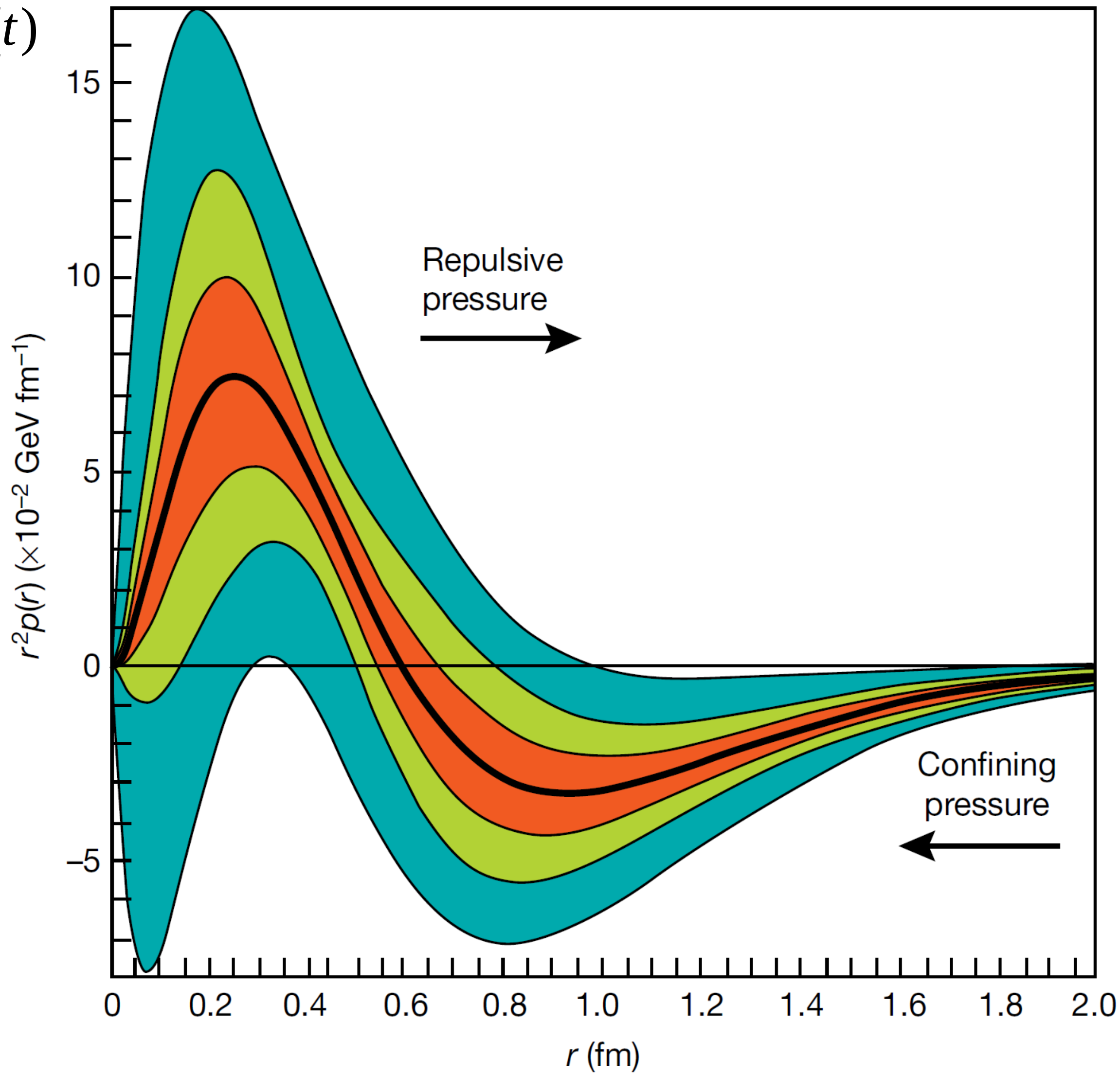

$$\langle N(p')|T_q^{ik}(0)|N(p)\rangle \sim \left(\Delta^i\Delta^k - \delta^{ik}\boldsymbol{\Delta}^2\right)D_q(t) - 4M^2\delta^{ik}\bar{C}_q(t)$$

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KT, PRD98, 034009

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KT, PRD98, 034009

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KT, PRD98, 034009

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$$\bar{C}_q(0)$$

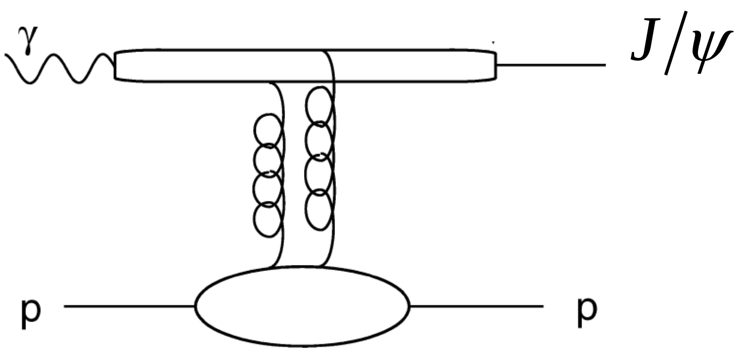
$$\bar{C}_g(0)$$

nucleon's transverse spin sum rule: $$\frac{1}{2} = J_q + J_g$$

$$J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g}) + \frac{p^3}{2(p^0 + M)}\bar{C}_{q,g}$$

Y. Hatta, KT, S. Yoshida, JHEP1302, 003

$\gamma p \to J/\psi\ p$ **near threshold** JLab, EIC



$$\bar{C}_g \ \left(= -\bar{C}_q\right)$$

Y. Hatta, D. Yang, PR98, 074003
Y. Hatta, A. Rajan, D. Yang, PRD100, 014032

$\left(i\not{D}-m\right)q=0$ $\qquad D_\nu F^{\mu\nu}=g\bar{q}\gamma^\mu q$ $\qquad$ KT, PRD98, 034009

$$\partial_\nu T_q^{\mu\nu}=-\bar{q}gF^{\mu\nu}\gamma_\nu q, \qquad \partial_\nu T_g^{\mu\nu}=-F_a^{\mu\nu}D_{ab}^{\rho}F_{\rho\nu}^{b}$$

$$\Delta^\mu\Lambda^2\bar{C}_q(t)=\langle h(p')|\bar{q}igF^{\mu\nu}\gamma_\nu q|h(p)\rangle$$

$$\Delta^\mu\Lambda^2\bar{C}_g(t)=\langle h(p')|F_a^{\mu\nu}iD_{ab}^{\rho}F_{\rho\nu}^{b}|h(p)\rangle$$

$$\bar{C}_q(0)$$

$$\bar{C}_g(0)$$

$$(i\not{D}-m)q=0 \qquad D_\nu F^{\mu\nu}=g\bar{q}\gamma^\mu q$$

KT, PRD98, 034009

$$\partial_\nu T_q^{\mu\nu}=-\bar{q}gF^{\mu\nu}\gamma_\nu q, \qquad \partial_\nu T_g^{\mu\nu}=-F_a^{\mu\nu}D_{ab}^\rho F_{\rho\nu}^b$$

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$$\bar{C}_q(0)$$

trace anomaly separately for q, g

$$\bar{C}_g(0)$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

KT, JHEP1901, 120

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\langle N(p')|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

Ji, PRL78, 610 ('97)

Polyakov, Schweitzer, IJMPA33, 1830025('18)

$$\langle h(p')|T^{\mu\nu}_{q,g}|h(p)\rangle = \frac{1}{2}\Theta_{2\,q,g}(t)P^\mu P^\nu + \frac{1}{2}\Theta_{1\,q,g}(t)\left(g^{\mu\nu}\Delta^2 - \Delta^\mu\Delta^\nu\right) + \Lambda^2\bar{C}_{q,g}(t)g^{\mu\nu}$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

$$g_{\mu\nu}\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\Big[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\Big]u(p)$$

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$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

$$g_{\mu\nu}\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu}T_q^{\mu\nu} = m\bar{q}q, \qquad g_{\mu\nu}T_g^{\mu\nu} = 0, \qquad A_{q,g}(0) + 4\bar{C}_{q,g}(0) \simeq 0$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \; + \; F^{\mu\rho}F_{\rho}^{\ \nu} + \frac{g^{\mu\nu}}{4}F^2$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

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$$g_{\mu\nu}T^{\mu\nu}_{q} \neq m\bar{q}q, \qquad g_{\mu\nu}T^{\mu\nu}_{g} \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \; + \; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

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Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

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trace anomaly separately for q, g

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right]|N(p)\rangle$$

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_{g}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right)|N(p)\rangle$$

$$C_A = N_c\ , \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

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$$g_{\mu\nu}T^{\mu\nu}_{q} \neq m\bar{q}q, \qquad g_{\mu\nu}T^{\mu\nu}_{g} \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

trace anomaly separately for q, g

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right]|N(p)\rangle$$

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_{g}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right)|N(p)\rangle$$

$$g_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

$$g_{\mu\nu}\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu}T_q^{\mu\nu} \neq m\bar{q}q, \qquad g_{\mu\nu}T_g^{\mu\nu} \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

trace anomaly separately for q, g

$$g_{\mu\nu}\langle N(p)|T_q^{\mu\nu}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right]|N(p)\rangle$$

$$g_{\mu\nu}\langle N(p)|T_g^{\mu\nu}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right)|N(p)\rangle$$

heavy Q decouple from $g_{\mu\nu}T_q^{\mu\nu}$ **:** $\bar{Q}Q = -\dfrac{1}{m_Q}\dfrac{\alpha_s}{12\pi}F^2 + \cdots$

$$\bar{C}_q(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0}\right) + \frac{1}{4}\left(\frac{2n_f}{3\beta_0} + 1\right)\frac{\langle N(p)|m\bar{q}q|N(p)\rangle}{2M^2}$$

$$-\frac{4C_F A_q(\mu_0) + n_f\left(A_q(\mu_0) - 1\right)}{4(4C_F + n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}}$$

$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f}{4\beta_0}\left(-\frac{34C_A}{27} - \frac{49C_F}{27}\right) + \frac{\beta_1 n_f}{6\beta_0^2}\right] + \cdots$$

$$\bar{C}_q(0) \Longleftrightarrow A_q(0)$$

$$\bar{C}_g(0) \Longleftrightarrow A_g(0)$$

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$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle N(p')|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|N(p)\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p) \right]$$

$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

$$\left(t = \Delta^2 \to 0\,, \quad \eta = \frac{-\Delta\cdot n}{2\bar{P}\cdot n} \to 0 \right)$$

$$H^q(x,0,0) = q(x)$$

$$\int_{-1}^{1} dx H^q(x,\eta,t) = F_1^q(t), \qquad \int_{-1}^{1} dx E^q(x,\eta,t) = F_2^q(t)$$

$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t)\,, \qquad \int_{-1}^{1} dx\, x E^q(x,\eta,t) = B_q(t) - 4\eta^2 D_q(t)$$

$$\bar{C}_q(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F+n_f}+\frac{2n_f}{3\beta_0}\right)+\frac{1}{4}\left(\frac{2n_f}{3\beta_0}+1\right)\frac{\langle N(p)|m\bar{q}q|N(p)\rangle}{2M^2}$$

$$-\frac{4C_F A_q(\mu_0)+n_f\left(A_q(\mu_0)-1\right)}{4(4C_F+n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}}$$

$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f}{4\beta_0}\left(-\frac{34C_A}{27}-\frac{49C_F}{27}\right)+\frac{\beta_1 n_f}{6\beta_0^2}\right]+\cdots$$

$$\bar{C}_q(0) \Longleftrightarrow A_q(0)$$

$$\bar{C}_g(0) \Longleftrightarrow A_g(0)$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

Cf: Polyakov, Son, JHEP 1809, 156

$$\bar{C}_q(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F+n_f}+\frac{2n_f}{3\beta_0}\right)+\frac{1}{4}\left(\frac{2n_f}{3\beta_0}+1\right)\frac{\langle N(p)|m\bar{q}q|N(p)\rangle}{2M^2}$$

$$-\frac{4C_F A_q(\mu_0)+n_f\left(A_q(\mu_0)-1\right)}{4(4C_F+n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}}$$

$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f}{4\beta_0}\left(-\frac{34C_A}{27}-\frac{49C_F}{27}\right)+\frac{\beta_1 n_f}{6\beta_0^2}\right]+\cdots$$

$$\bar{C}_q(0) \Longleftrightarrow A_q(0)$$

$$\bar{C}_g(0) \Longleftrightarrow A_g(0)$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\bar{C}_q(\mu)\Big|_{n_f=3} = -0.146 + 0.306\frac{\langle N(p)|m\bar{q}q|N(p)\rangle}{2M^2}$$

$$-0.25\left(A_q(\mu_0) - 0.36\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{81}}$$

$$+\alpha_s(\mu)\left(-0.01 + 0.08\frac{\langle N(p)|m\bar{q}q|N(p)\rangle}{2M^2}\right)$$

$$\bar{C}_q(0) \Longleftrightarrow A_q(0)$$

$$\bar{C}_g(0) \Longleftrightarrow A_g(0)$$

Y. Hatta, A. Rajan, KT,
JHEP1812, 008

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu}T^{\mu\nu}_{q} \neq m\bar{q}q, \qquad g_{\mu\nu}T^{\mu\nu}_{g} \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

trace anomaly separately for q, g

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right]|N(p)\rangle$$

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_{g}|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right)|N(p)\rangle$$

$$g_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q$$

Y. Hatta, A. Rajan, KT, JHEP1812, 008

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

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$$g_{\mu\nu}T^{\mu\nu}_q \neq m\bar{q}q, \qquad g_{\mu\nu}T^{\mu\nu}_g \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

trace anomaly separately for q, g

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_q|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right]|N(p)\rangle$$

$$g_{\mu\nu}\langle N(p)|T^{\mu\nu}_g|N(p)\rangle = \frac{1}{2M^2}\langle N(p)|\frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right)|N(p)\rangle$$

$$g_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q$$

$$\langle N(p)|T^{\mu\nu}|N(p)\rangle = 2p^\mu p^\nu$$

$$2M^2 = \langle N(p) | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q \right) | N(p) \rangle \simeq \langle N(p) | \frac{\beta(g)}{2g} F^2 | N(p) \rangle$$

$$2M^2 = \langle N(p) | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q \right) | N(p) \rangle \simeq \langle N(p) | \frac{\beta(g)}{2g} F^2 | N(p) \rangle$$

$$2M^2 = g_{\mu\nu} \langle N(p) | T_q^{\mu\nu} | N(p) \rangle + g_{\mu\nu} \langle N(p) | T_g^{\mu\nu} | N(p) \rangle$$

$$2M^2 = \langle N(p) | \left(\frac{\beta(g)}{2g} F^2 + (1+\gamma_m(g)) m\bar{q}q \right) | N(p) \rangle \simeq \langle N(p) | \frac{\beta(g)}{2g} F^2 | N(p) \rangle$$

$$2M^2 = g_{\mu\nu} \langle N(p) | T_q^{\mu\nu} | N(p) \rangle + g_{\mu\nu} \langle N(p) | T_g^{\mu\nu} | N(p) \rangle$$

$$\frac{\alpha_s}{4\pi} \frac{n_f}{3} F^2 \qquad \frac{\alpha_s}{4\pi} \left(-\frac{11C_A}{6} F^2 \right)$$

$$2M^2 = \langle N(p)| \left(\frac{\beta(g)}{2g} F^2 + \big(1+\gamma_m(g)\big) m\bar{q}q \right) | N(p)\rangle \simeq \langle N(p)| \frac{\beta(g)}{2g} F^2 | N(p)\rangle$$

$$2M^2 = g_{\mu\nu}\langle N(p)| T_q^{\mu\nu} | N(p)\rangle + g_{\mu\nu}\langle N(p)| T_g^{\mu\nu} | N(p)\rangle$$

$$\frac{\alpha_s}{4\pi}\frac{n_f}{3}F^2 \qquad \frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2\right)$$

Hatta, A.Rajan, KT, JHEP1812, 008

$$\left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{17C_A n_f}{27}+\frac{49C_F n_f}{54}\right)F^2 \qquad \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{28C_A n_f}{27}-\frac{17C_A^2}{3}+\frac{5C_F n_f}{54}\right)F^2$$

$$2M^2 = \langle N(p)|\left(\frac{\beta(g)}{2g}F^2 + (1+\gamma_m(g))m\bar{q}q\right)|N(p)\rangle \simeq \langle N(p)|\frac{\beta(g)}{2g}F^2|N(p)\rangle$$

$$2M^2 = g_{\mu\nu}\langle N(p)|T_q^{\mu\nu}|N(p)\rangle + g_{\mu\nu}\langle N(p)|T_g^{\mu\nu}|N(p)\rangle$$

$$\frac{\alpha_s}{4\pi}\frac{n_f}{3}F^2 \qquad \frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2\right)$$

Hatta, A.Rajan, KT, JHEP1812, 008

$$\left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{17C_An_f}{27}+\frac{49C_Fn_f}{54}\right)F^2 \qquad \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{28C_An_f}{27}-\frac{17C_A^2}{3}+\frac{5C_Fn_f}{54}\right)F^2$$

KT, JHEP1901, 120

$$\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\frac{52\zeta(3)}{9}-\frac{401}{324}\right)C_AC_F + \left(\frac{134}{27}-4\zeta(3)\right)C_A^2+\left(\frac{2407}{1458}-\frac{16\zeta(3)}{9}\right)C_F^2\right\} + n_f^2\left(-\frac{697C_A}{729}-\frac{169C_F}{1458}\right)\right]F^2$$

$$\left(\frac{\alpha_s}{4\pi}\right)^3\left[n_f\left(\left(\frac{1123}{162}-\frac{52\zeta(3)}{9}\right)C_AC_F + \left(4\zeta(3)+\frac{293}{36}\right)C_A^2+\frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916}-\frac{361C_F}{729}\right)-\frac{2857C_A^3}{108}\right]F^2$$

$$2m_h^2 = \langle h(p) | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q \right) | h(p) \rangle$$

$$2m_\pi^2 = \langle \pi | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{q}q \right) | \pi \rangle$$

$$2m_\pi^2 = \langle \pi | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q \right) | \pi \rangle$$

$$2 f_\pi^2 m_\pi^2 = -\left(m_u + m_d\right) \langle 0 | \left(\bar{u}u + \bar{d}d \right) | 0 \rangle$$

$$2m_\pi^2 = \langle \pi | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{q}q \right) | \pi \rangle$$

$$2 f_\pi^2 m_\pi^2 = -\left(m_u + m_d\right) \langle 0 | \left(\bar{u}u + \bar{d}d\right) | 0 \rangle$$

$$\left| \pi \right\rangle \to \left| \pi \right\rangle_0 + \left| \pi \right\rangle_1 + \cdots$$

$$m_\pi^2 = {}_0\left\langle \pi \left| m\bar{q}q \right| \pi \right\rangle_0$$

Gasser, Leutwyler ('82)

$$2m_\pi^2 = \langle \pi | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{q}q \right) | \pi \rangle$$

$$2 f_\pi^2 m_\pi^2 = -\left(m_u + m_d\right) \langle 0 | \left(\bar{u}u + \bar{d}d\right) | 0 \rangle$$

$$\left| \pi \right\rangle \rightarrow \left| \pi \right\rangle_0 + \left| \pi \right\rangle_1 + \cdots$$

$$m_\pi^2 = {}_0\left\langle \pi \left| m\bar{q}q \right| \pi \right\rangle_0$$

Gasser, Leutwyler ('82)

$${}_0\left\langle \pi \left| F^2 \right| \pi \right\rangle_0 = 0, \qquad \left(1-\gamma_m(g)\right) m_\pi^2 = {}_1\left\langle \pi \left| \frac{\beta(g)}{2g} F^2 \right| \pi \right\rangle_0 + {}_0\left\langle \pi \left| \frac{\beta(g)}{2g} F^2 \right| \pi \right\rangle_1$$

$$\gamma_m(g) = 0.63662\alpha_s + 0.768352\alpha_s^2 + 0.801141\alpha_s^3 \simeq 0.559$$

$$
g_{\mu\nu}T_q^{\mu\nu} = m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar{q}q + \frac{1}{3}n_f F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)m\bar{q}q + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54}\right)F^2\right]
$$

$$
+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729}\right)C_F^2 - \frac{2}{243}(864\zeta(3)+1079)C_A C_F\right) - \frac{8}{729}(972\zeta(3)+143)C_A C_F^2\right.\right.
$$

$$
\left.+\left(\frac{32\zeta(3)}{9} + \frac{6611}{729}\right)C_A^2 C_F - \frac{76}{243}C_F n_f^2 + \frac{8}{729}(648\zeta(3)-125)C_F^3\right\}m\bar{q}q
$$

$$
\left.+\left\{n_f\left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324}\right)C_A C_F + \left(\frac{134}{27} - 4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9}\right)C_F^2\right) + n_f^2\left(-\frac{697C_A}{729} - \frac{169C_F}{1458}\right)\right\}F^2\right]
$$

$$
g_{\mu\nu}T_g^{\mu\nu} = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar{q}q - \frac{11}{6}C_A F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)m\bar{q}q + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54}\right)F^2\right]
$$

$$
+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729}\right)C_F^2 - \frac{2}{243}(4968\zeta(3)+1423)C_A C_F\right) + \left(\frac{32\zeta(3)}{3} - \frac{91753}{1458}\right)C_A C_F^2\right.\right.
$$

$$
\left.+\left(\frac{294929}{1458} - \frac{32\zeta(3)}{9}\right)C_A^2 C_F - \frac{554}{243}C_F n_f^2 + \left(\frac{95041}{729} - \frac{64\zeta(3)}{9}\right)C_F^3\right\}m\bar{q}q
$$

$$
\left.+\left\{n_f\left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9}\right)C_A C_F + \left(4\zeta(3) + \frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916} - \frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right\}F^2\right]
$$

$$g_{\mu\nu}T_q^{\mu\nu} = m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar{q}q + \frac{1}{3}n_f F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)m\bar{q}q + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54}\right)F^2\right]$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729}\right)C_F^2 - \frac{2}{243}(864\zeta(3)+1079)C_A C_F\right) - \frac{8}{729}(972\zeta(3)+143)C_A C_F^2\right.\right.$$

$$\left.+\left(\frac{32\zeta(3)}{9} + \frac{6611}{729}\right)C_A^2 C_F - \frac{76}{243}C_F n_f^2 + \frac{8}{729}(648\zeta(3)-125)C_F^3\right\}m\bar{q}q$$

$$\left.+\left\{n_f\left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324}\right)C_A C_F + \left(\frac{134}{27} - 4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9}\right)C_F^2\right) + n_f^2\left(-\frac{697C_A}{729} - \frac{169C_F}{1458}\right)\right\}F^2\right]$$

$$\frac{1}{2m_\pi^2}\langle\pi|g_{\mu\nu}T_q^{\mu\nu}|\pi\rangle = 0.388889 + 0.12215\alpha_s + 0.124659\alpha_s^2 + 0.0430357\alpha_s^3$$

$$g_{\mu\nu}T_g^{\mu\nu} = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar{q}q - \frac{11}{6}C_A F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)m\bar{q}q + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54}\right)F^2\right]$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729}\right)C_F^2 - \frac{2}{243}(4968\zeta(3)+1423)C_A C_F\right) + \left(\frac{32\zeta(3)}{3} - \frac{91753}{1458}\right)C_A C_F^2\right.\right.$$

$$\left.+\left(\frac{294929}{1458} - \frac{32\zeta(3)}{9}\right)C_A^2 C_F - \frac{554}{243}C_F n_f^2 + \left(\frac{95041}{729} - \frac{64\zeta(3)}{9}\right)C_F^3\right\}m\bar{q}q$$

$$\left.+\left\{n_f\left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9}\right)C_A C_F + \left(4\zeta(3) + \frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916} - \frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right\}F^2\right]$$

$$\frac{1}{2m_\pi^2}\langle\pi|g_{\mu\nu}T_g^{\mu\nu}|\pi\rangle = 0.611111 - 0.12215\alpha_s - 0.124659\alpha_s^2 - 0.0430357\alpha_s^3$$

$$2m_\pi^2 = \langle \pi | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{q}q \right) | \pi \rangle$$

$$2 f_\pi^2 m_\pi^2 = -\left(m_u + m_d\right) \langle 0 | \left(\bar{u}u + \bar{d}d\right) | 0 \rangle$$

$$\left| \pi \right\rangle \to \left| \pi \right\rangle_0 + \left| \pi \right\rangle_1 + \cdots$$

$$m_\pi^2 = {}_0\left\langle \pi \right| m\bar{q}q \left| \pi \right\rangle_0$$

Gasser, Leutwyler ('82)

$${}_0\left\langle \pi \right| F^2 \left| \pi \right\rangle_0 = 0, \qquad \left(1-\gamma_m(g)\right) m_\pi^2 = {}_1\left\langle \pi \right| \frac{\beta(g)}{2g} F^2 \left| \pi \right\rangle_0 + {}_0\left\langle \pi \right| \frac{\beta(g)}{2g} F^2 \left| \pi \right\rangle_1$$

$$\gamma_m(g) = 0.63662\alpha_s + 0.768352\alpha_s^2 + 0.801141\alpha_s^3 \simeq 0.559$$

$$2m_\pi^2 = g_{\mu\nu} \langle \pi | T_q^{\mu\nu} | \pi \rangle + g_{\mu\nu} \langle \pi | T_g^{\mu\nu} | \pi \rangle$$

KT, JHEP1901, 120

$$2m_\pi^2 = \langle \pi | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{q}q \right) | \pi \rangle$$

$$2 f_\pi^2 m_\pi^2 = -\left(m_u + m_d\right)\langle 0 | \left(\bar{u}u + \bar{d}d\right) | 0 \rangle$$

$$|\pi\rangle \to |\pi\rangle_0 + |\pi\rangle_1 + \cdots$$

$$m_\pi^2 = {}_0\langle \pi | m\bar{q}q | \pi \rangle_0$$

Gasser, Leutwyler ('82)

$${}_0\langle \pi | F^2 | \pi \rangle_0 = 0, \qquad \left(1-\gamma_m(g)\right) m_\pi^2 = {}_1\langle \pi | \frac{\beta(g)}{2g} F^2 | \pi \rangle_0 + {}_0\langle \pi | \frac{\beta(g)}{2g} F^2 | \pi \rangle_1$$

$$\gamma_m(g) = 0.63662\alpha_s + 0.768352\alpha_s^2 + 0.801141\alpha_s^3 \simeq 0.559$$

$$2m_\pi^2 = g_{\mu\nu} \langle \pi | T_q^{\mu\nu} | \pi \rangle + g_{\mu\nu} \langle \pi | T_g^{\mu\nu} | \pi \rangle$$

$$= 2m_\pi^2 \times \left(\ 0.479 \ + \ 0.521 \ \right)$$

KT, JHEP1901, 120

$$2M^2 = \langle N(p)| \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{q}q \right) | N(p)\rangle \simeq \langle N(p)| \frac{\beta(g)}{2g} F^2 | N(p)\rangle$$

$$2M^2 = g_{\mu\nu}\langle N(p)|T_q^{\mu\nu}|N(p)\rangle + g_{\mu\nu}\langle N(p)|T_g^{\mu\nu}|N(p)\rangle$$

$$\frac{\alpha_s}{4\pi}\frac{n_f}{3}F^2 \qquad \frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2\right)$$

Hatta, A.Rajan, KT, JHEP1812, 008

$$\left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{17C_A n_f}{27}+\frac{49C_F n_f}{54}\right)F^2 \qquad \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{28C_A n_f}{27}-\frac{17C_A^2}{3}+\frac{5C_F n_f}{54}\right)F^2$$

KT, JHEP1901, 120

$$\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\frac{52\zeta(3)}{9}-\frac{401}{324}\right)C_A C_F + \left(\frac{134}{27}-4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458}-\frac{16\zeta(3)}{9}\right)C_F^2\right\} + n_f^2\left(-\frac{697C_A}{729}-\frac{169C_F}{1458}\right)\right]F^2$$

$$\left(\frac{\alpha_s}{4\pi}\right)^3\left[n_f\left(\left(\frac{1123}{162}-\frac{52\zeta(3)}{9}\right)C_A C_F + \left(4\zeta(3)+\frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916}-\frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right]F^2$$

$$g_{\mu\nu}T_q^{\mu\nu} = m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar{q}q + \frac{1}{3}n_f F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)m\bar{q}q + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54}\right)F^2\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729}\right)C_F^2 - \frac{2}{243}(864\zeta(3)+1079)C_A C_F\right) - \frac{8}{729}(972\zeta(3)+143)C_A C_F^2\right.\right.$$
$$\left.+\left(\frac{32\zeta(3)}{9} + \frac{6611}{729}\right)C_A^2 C_F - \frac{76}{243}C_F n_f^2 + \frac{8}{729}(648\zeta(3)-125)C_F^3\right\}m\bar{q}q$$
$$\left.+\left\{n_f\left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324}\right)C_A C_F + \left(\frac{134}{27} - 4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9}\right)C_F^2\right) + n_f^2\left(-\frac{697C_A}{729} - \frac{169C_F}{1458}\right)\right\}F^2\right]$$

$$g_{\mu\nu}T_g^{\mu\nu} = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar{q}q - \frac{11}{6}C_A F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)m\bar{q}q + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54}\right)F^2\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729}\right)C_F^2 - \frac{2}{243}(4968\zeta(3)+1423)C_A C_F\right) + \left(\frac{32\zeta(3)}{3} - \frac{91753}{1458}\right)C_A C_F^2\right.\right.$$
$$\left.+\left(\frac{294929}{1458} - \frac{32\zeta(3)}{9}\right)C_A^2 C_F - \frac{554}{243}C_F n_f^2 + \left(\frac{95041}{729} - \frac{64\zeta(3)}{9}\right)C_F^3\right\}m\bar{q}q$$
$$\left.+\left\{n_f\left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9}\right)C_A C_F + \left(4\zeta(3) + \frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916} - \frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right\}F^2\right]$$

$$O_1 = F^{\mu\rho} F_{\rho}{}^{\nu} \,, \qquad O_2 = g^{\mu\nu} F^2 \,, \qquad O_3 = i\bar{q}\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} q \,, \qquad O_4 = g^{\mu\nu} m\bar{q}q \,,$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4} F^2 = O_3 + O_1 + \frac{O_2}{4}$$

$$O_1^R = Z_T O_1 + Z_M O_2 + Z_L O_3 + Z_S O_4 \,,$$

$$O_2^R = Z_F O_2 + Z_C O_4 \,,$$

$$O_3^R = Z_\psi O_3 + Z_K O_4 + Z_Q O_1 + Z_B O_2 \,,$$

$$O_4^R = O_4$$

$$g_{\mu\nu}T_q^{\mu\nu} = m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar{q}q + \frac{1}{3}n_f F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)m\bar{q}q + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54}\right)F^2\right]$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729}\right)C_F^2 - \frac{2}{243}(864\zeta(3)+1079)C_A C_F\right) - \frac{8}{729}(972\zeta(3)+143)C_A C_F^2\right.\right.$$

$$\left.+\left(\frac{32\zeta(3)}{9} + \frac{6611}{729}\right)C_A^2 C_F - \frac{76}{243}C_F n_f^2 + \frac{8}{729}(648\zeta(3)-125)C_F^3\right\}m\bar{q}q$$

$$\left.+\left\{n_f\left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324}\right)C_A C_F + \left(\frac{134}{27} - 4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9}\right)C_F^2\right) + n_f^2\left(-\frac{697C_A}{729} - \frac{169C_F}{1458}\right)\right\}F^2\right]$$

$$g_{\mu\nu}T_g^{\mu\nu} = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar{q}q - \frac{11}{6}C_A F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)m\bar{q}q + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54}\right)F^2\right]$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729}\right)C_F^2 - \frac{2}{243}(4968\zeta(3)+1423)C_A C_F\right) + \left(\frac{32\zeta(3)}{3} - \frac{91753}{1458}\right)C_A C_F^2\right.\right.$$

$$\left.+\left(\frac{294929}{1458} - \frac{32\zeta(3)}{9}\right)C_A^2 C_F - \frac{554}{243}C_F n_f^2 + \left(\frac{95041}{729} - \frac{64\zeta(3)}{9}\right)C_F^3\right\}m\bar{q}q$$

$$\left.+\left\{n_f\left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9}\right)C_A C_F + \left(4\zeta(3) + \frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916} - \frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right\}F^2\right]$$

$$2m_h^2 = \langle h(p) | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q \right) | h(p) \rangle$$

$m_Q \gg \Lambda_{\mathrm{QCD}}$

KT, in progress

$$2m_H^2 = \langle H(p)|\left(\frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q + m_Q\bar{Q}Q\right)|H(p)\rangle$$

$$= \langle H(p)|\left(\frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q\right)|H(p)\rangle + m_Q\langle H(p)|\bar{Q}Q|H(p)\rangle$$

$$= \langle H(p)|\left(\frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q\right)|H(p)\rangle + m_Q 2m_H$$

$$Q = e^{-im_Q v\cdot x}\left(1+\frac{i\not{D}_\perp}{2m_Q}+\cdots\right)h_v(x)$$

$$m_H = m_Q + \frac{1}{2m_H}\langle H(p)|\left(\frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q\right)|H(p)\rangle$$

$$\equiv m_Q + \bar{\Lambda}$$

$$\bar{\Lambda} = \frac{1}{2}\langle H(v)|\left(\frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q\right)|H(v)\rangle \qquad \bar{\Lambda} = \frac{iv\cdot\partial\langle 0|\bar{q}h_v|H(v)\rangle}{\langle 0|\bar{q}h_v|H(v)\rangle}$$

KT, in progress

$$\left\langle H(p') \middle| T^{\mu\nu} \middle| H(p) \right\rangle = 2\frac{m_Q}{m_H} P^{\nu} P^{\mu} \xi(v \cdot v')$$
$$+ \left\langle H(p') \middle| \frac{1}{2} \bar{h}_{v'} \gamma^{(\mu} i \overleftrightarrow{D}^{\nu)} h_v \middle| H(p) \right\rangle$$
$$+ \left\langle H(p') \middle| \frac{1}{2} \bar{q} \gamma^{(\mu} i \overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4} F^2 \middle| H(p) \right\rangle$$

$$\langle h(p') \,|\, T^{\mu\nu}_{q,g} \,|\, h(p) \rangle = \frac{1}{2} \Theta_{2\; q,g}(t) P^{\mu} P^{\nu}$$
$$+ \frac{1}{2} \Theta_{1\; q,g}(t) \left(g^{\mu\nu} \Delta^2 - \Delta^{\mu} \Delta^{\nu} \right) + \Lambda^2 \bar{C}_{q,g}(t) g^{\mu\nu}$$

KT, in progress

$$\left\langle H(p')\middle|T^{\mu\nu}\middle|H(p)\right\rangle = 2\frac{m_Q}{m_H}P^\nu P^\mu \xi(v\cdot v')$$
$$+\left\langle H(p')\middle|\frac{1}{2}\bar{h}_{v'}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}h_v\middle|H(p)\right\rangle \quad \propto (1-v\cdot v')$$
$$+\left\langle H(p')\middle|\frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \;+\; F^{\mu\rho}F_\rho{}^\nu + \frac{g^{\mu\nu}}{4}F^2\middle|H(p)\right\rangle$$

$$\langle h(p')\,|\,T^{\mu\nu}_{q,g}\,|\,h(p)\rangle = \frac{1}{2}\Theta_{2\;q,g}(t)P^\mu P^\nu$$
$$+\frac{1}{2}\Theta_{1\;q,g}(t)\left(g^{\mu\nu}\Delta^2 - \Delta^\mu\Delta^\nu\right) + \Lambda^2\bar{C}_{q,g}(t)g^{\mu\nu}$$

KT, PRD98, 034009

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\langle p' | T_q^{\mu\nu} | p\rangle = \frac{1}{4}\langle p' | \bar{q}\left(-i\overleftarrow{\partial}^{\mu} + i\overrightarrow{\partial}^{\mu} + 2gA^{\mu}\right)\gamma^{\nu} q | p\rangle + \left(\mu \leftrightarrow \nu\right)$$

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intermediated states: "partonic"

$$A_q(t) + \eta^2 D_q(t) \simeq \langle x\rangle F_1^q(t)\,, \quad B_q(t) - \eta^2 D_q(t) \simeq \langle x\rangle F_2^q(t)$$

$$-\frac{P^+\eta\Delta_\perp^\mu \bar{u}(p')u(p)}{M} D_q(t) \simeq \langle N(p') | \bar{q} g A_\perp^\mu \gamma^+ q | N(p)\rangle$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} d\lambda\, \mathrm{sgn}(\lambda) n_\alpha \langle N(p') | g F_a^{\mu\alpha}(\lambda n)\bar{q}(0) t^a \gamma^+ q(0) | N(p)\rangle$$

KT, PRD98, 034009

$$\langle h(p')|T_q^{\mu\nu}|h(p)\rangle = \frac{1}{2}\Theta_{2q}(t)P^\mu P^\nu + \frac{1}{2}\Theta_{1q}(t)\left(g^{\mu\nu}\Delta^2 - \Delta^\mu\Delta^\nu\right) + \Lambda^2\bar{C}_q(t)g^{\mu\nu}$$

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$t \to \infty$

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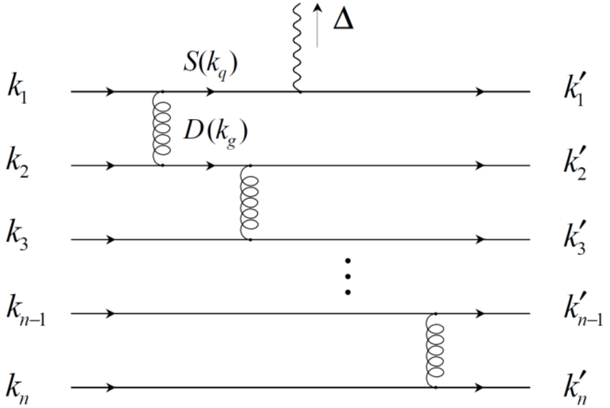

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \ + \ F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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intermediated states: “partonic”

LCSR

$$i\int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T}\, j_5^{\alpha}(0) T_q^{\mu\nu}(x)\big|\pi^{+}(p)\big\rangle \qquad j_5^{\alpha} = \bar{d}\gamma^{\alpha}\gamma_5 u$$

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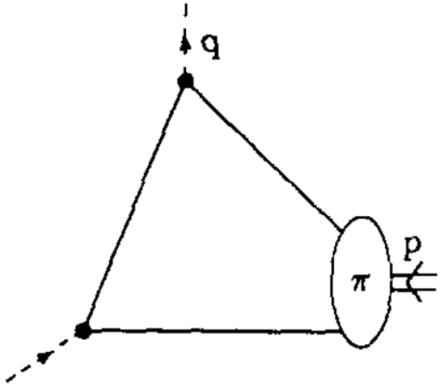

KT, PRD98, 034009

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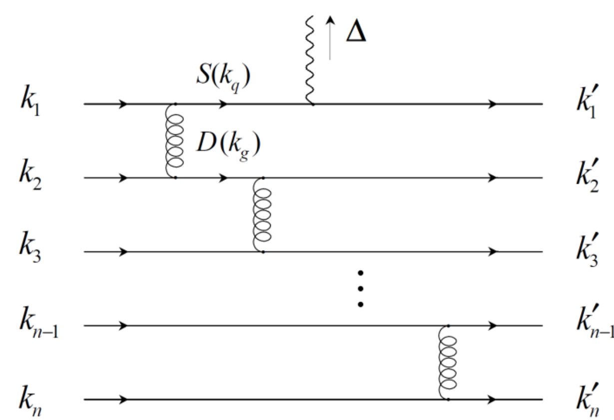

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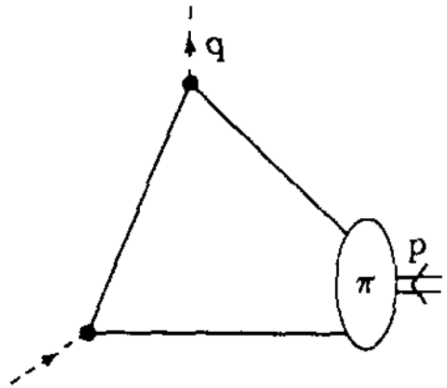

KT, in progress

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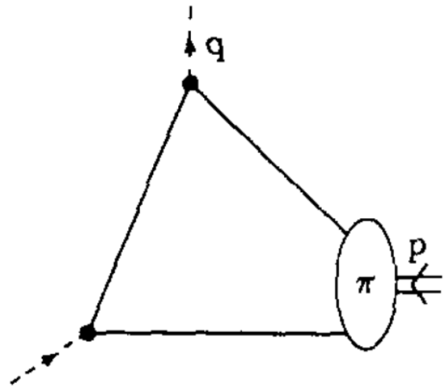

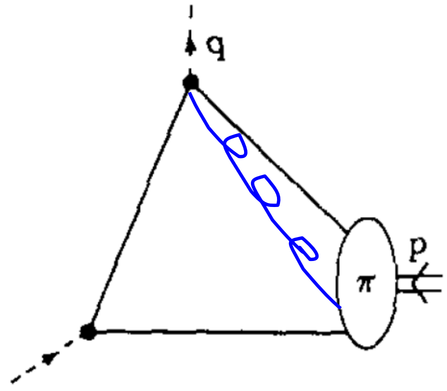

KT, PRD98, 034009

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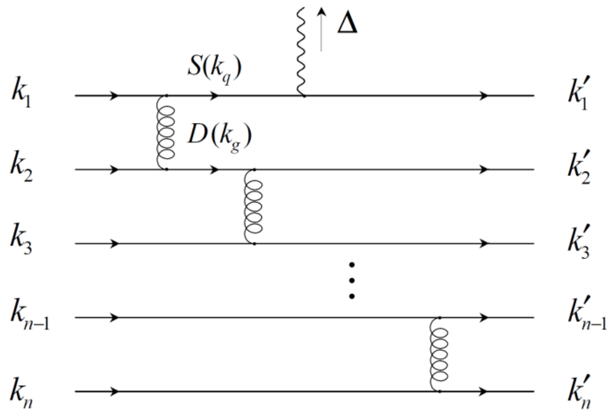

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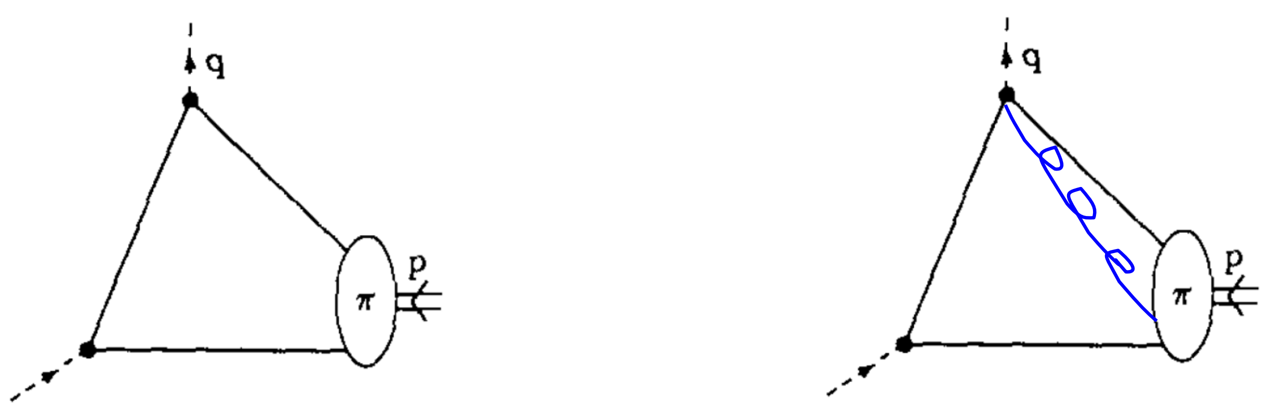
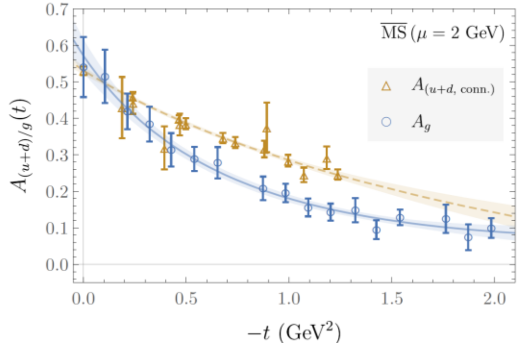
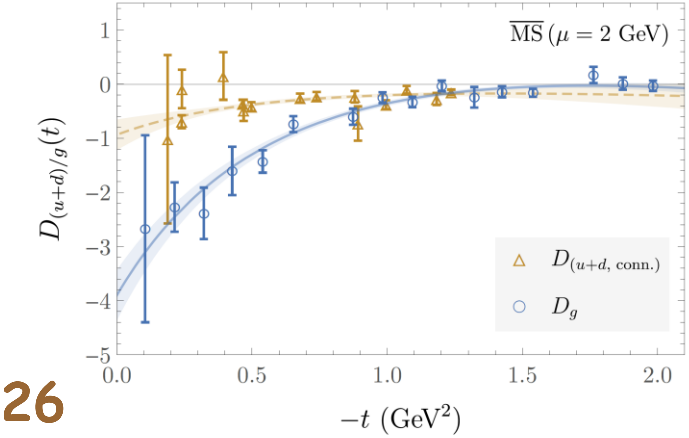
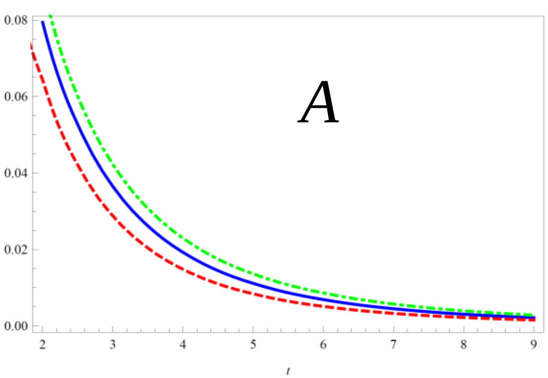


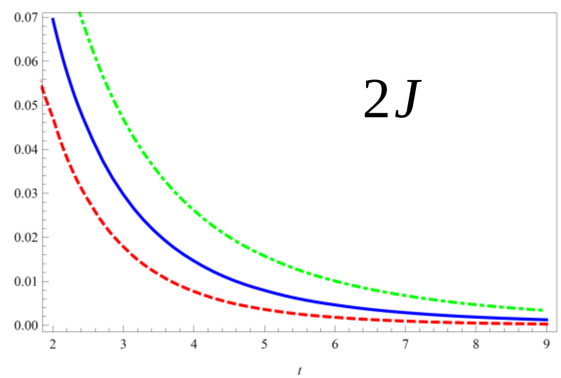
lattice Hagler et al., PRD77, 094502; Shanahan, Detmold, PRD99,014511

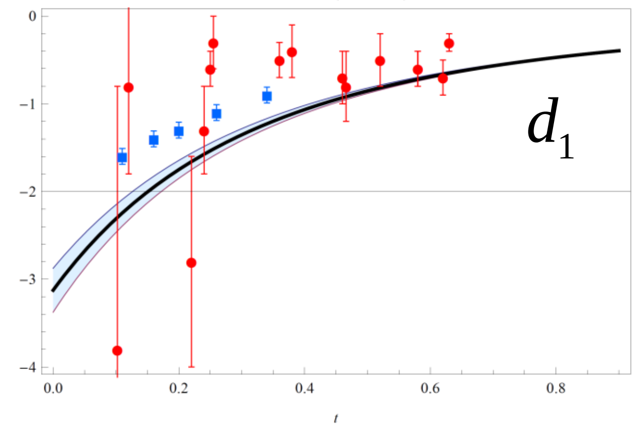




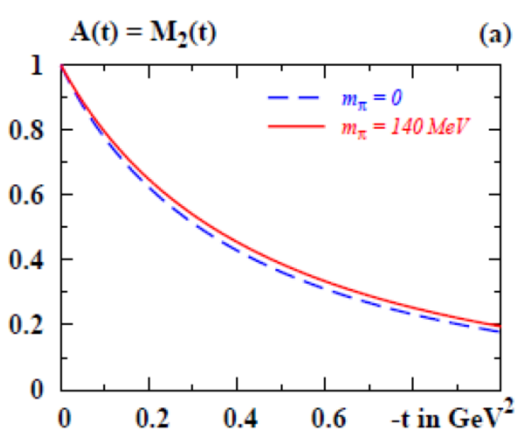
LCSRs for nucleon Anikin, PRD99,094026

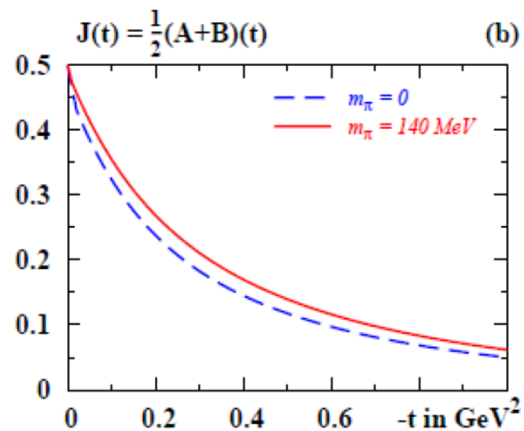


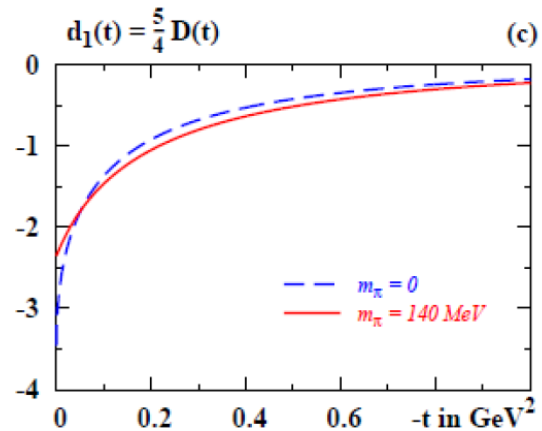




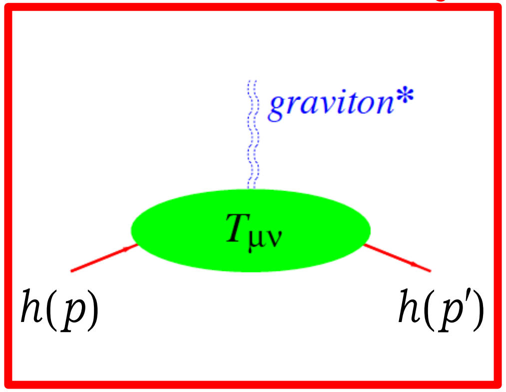
model: e.g. chiral quark soliton model, Goeke et al, PRD75 (2007) 094021







Summary



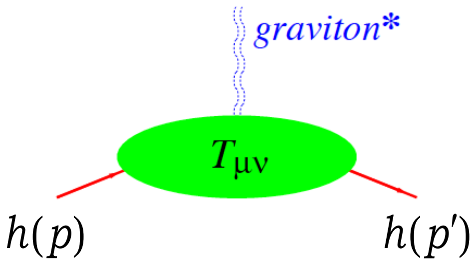
Gravitational form factors can be accessed through OPE in hard processes

mass & energy distribution

$$\langle h(p')\,|\,T_q^{\mu\nu}\,|\,h(p)\rangle = \frac{1}{2}\Theta_{2q}(t)P^\mu P^\nu + \frac{1}{2}\Theta_{1q}(t)\left(g^{\mu\nu}\Delta^2 - \Delta^\mu\Delta^\nu\right) + \Lambda^2\bar{C}_q(t)g^{\mu\nu}$$

force & pressure distribution

Summary



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EOM & RGI

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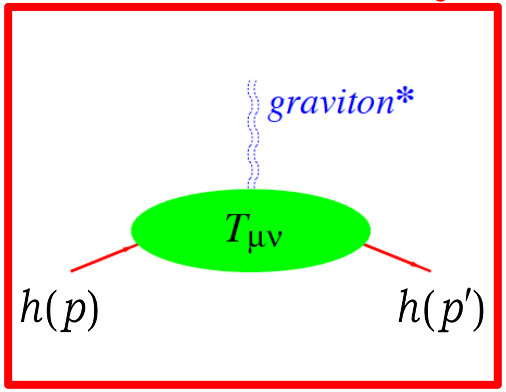
Summary



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$m_q \to 0$ $\quad$ $m_Q \to \infty$ $\quad$ **LCSRs**