$B_{uds} \to \gamma$ Form Factors in LCSRs @NLO and $1/\lambda_B$

CP3 Origins
Cosmology & Particle Physics

Roman Zwicky
Edinburgh University

CENTRE FOR THEORETICAL PHYSICS · Higgs

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* Janowski, Pullin, RZ ' to appear

Overview

(1) $B_{uds} \to \gamma$ Form Factors light-LCDA with LCSR
(brief remark about off-shell FFs)

photon-LCDA

(2) Use of Equation of Motion for FFs

useful check
of computation &
formalism

(3) Match leading $1/m_b$ terms for FFs with
light-LCDA and B-LCDA

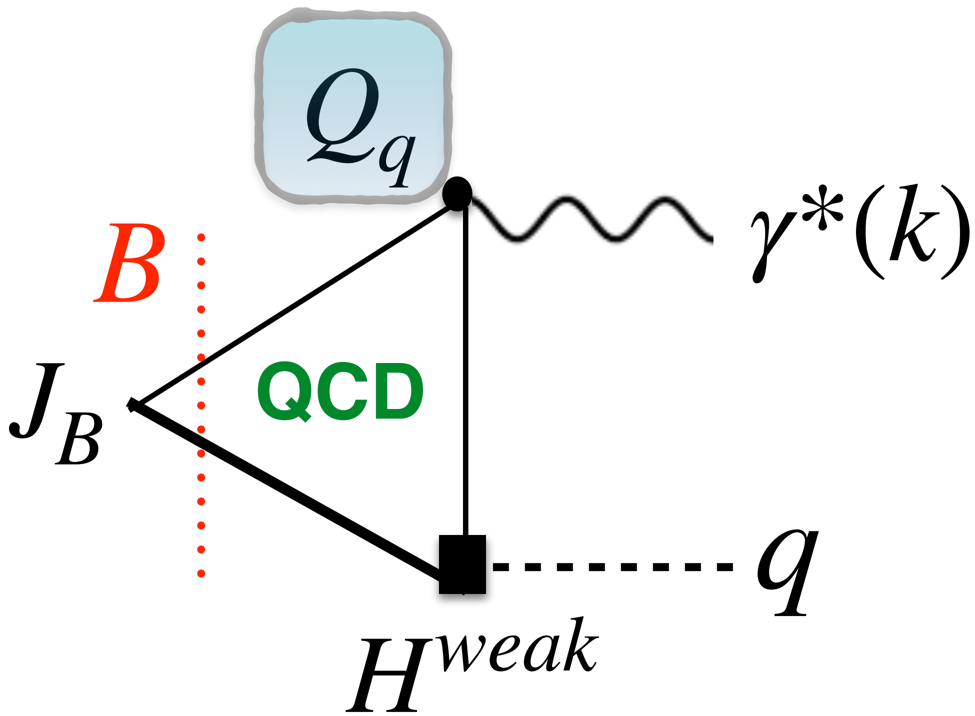
in progress

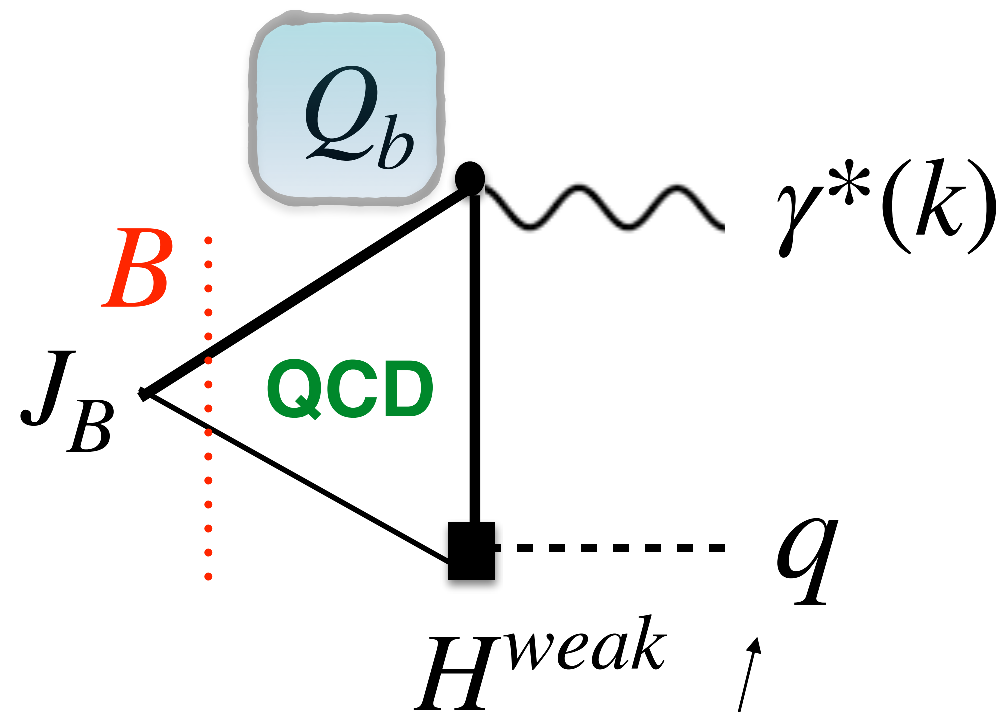
$$F^{B\to\gamma}_{LCSR}(0) \propto \frac{Q_q}{m_b^{1/2}} \qquad F^{B\to\gamma}_{B-LCDA}(0) \propto \frac{Q_q}{m_b^{1/2}}\frac{1}{\lambda_B}$$

$\Longrightarrow$ extract a value for $1/\lambda_B$

Non-technical Prologue

- B→γ^* Form Factors come in various forms
- Two **independent** contributions proportional to parton charges

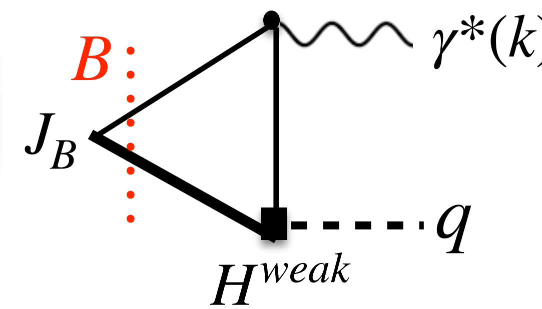




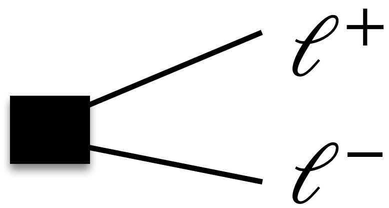
⇒ Form Factor of two momenta: $F = F(q^2, k^2)$

- *What is the particle(s) coming from weak vertex?*

Particle(s) from weak vertex with momenta q



- **FCNC** Qb= Qq (need long distance in addition) :



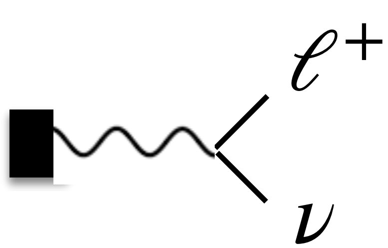
H^{weak} ~ O$_{9,10}$: $B_{d,s} \to \ell^+\ell^-\gamma$ $\quad F(q^2) = F(q^2,0)$

H^{weak} ~ O$_7$: $B_{d,s} \to \ell^+\ell^-\gamma$ $\quad F^*(k^2) = F(0,k^2)$

H^{weak} ~ $\bar{q}\gamma_\mu b_L \partial^\mu a$: $B_{d,s} \to \ell^+\ell^- a$ $\quad F(m_a^2, k^2) \to F^*(k^2)$

o*r dark photon, scalar DM, …*

- **FCCC** Qb ≠ Qq :



H^{weak} ~ $V_{ub}\, \bar{u}\gamma_\mu b_L \ell \gamma^\mu \nu_L$: $B_u \to \ell^+\nu\gamma$

- Physics: helicity suppression of $B \to f_i \bar{f}_j$ relieved in radiative decay!

A few references

- **Charged** $F(q^2) = F(q^2,0)$

  Korchemsky,Pirjol,Yan,'99, Bosch, Lange, Neubert, Paz'01
  Descotes-G. Sachrajda'02,
  Rohrwild,Beneke'11, Braun,Kohodjamirian'12,
  Wang'16, Shen, Wang'18 Braun, Beneke, Ji, Wei'18, …

  Toy model for QCD-factorisation & **extracting 1/λ$_B$** from experiment (BelleII …)

  most attention

- **Neutral** $F(q^2) = F(q^2,0)$

  Krueger, Melikhov'02, Nikitin, Melikhov, '04,'17

  Const. Quark model computations.

- **People who worked** $B_s \to \mu\mu\gamma$

  Guadagnioli,Reboud,Detori,RZ,Nikitin,Melikhov,Hazard,Petrov

  @LHCb soft photon undetected in $B_s \to \mu\mu$ tail

- **Flavoured Axions** $B_s \to \mu\mu a \;\; vs \;\; B_s \to \mu\mu\gamma$

  Albrecht, Stamou,Ziegler, RZ '19,

  In competition with $B \to K\bar{\nu}\nu$ bounds

- **Weak annihilation@LO** $B_q \to V\ell\ell$

  Ali, Braun'95, Khodjamirian, stoll, Wyler'95 Beneke, Feldman, Seidel'01, Lyon, RZ '13,

Form factor Counting

- Off-shell: $q^2, k^2 \neq 0$ like $B \to V$ FF with "$k^2 = m_V^2$" $\Rightarrow$ 7 FFs
- Off-shell but: $q^2 = 0$ like $B \to V$ FF $A_0(0) = A_3(0), T_1(0) = T_2(0)$
- On-shell: $k^2 = 0$, two photon polarisation in vector and tensor 5 FFs

type$\backslash$ J^P	#	1^-	1^+	1^+	0^-
$F^*(q^2,k^2)$	7	$V_\perp^*(q^2,k^2)$	$V_\parallel^*(q^2,k^2)$	$\hat{V}_{\mathbb{L}}^*(q^2,k^2)$	$P^*(q^2,k^2)$
		$T_\perp^*(q^2,k^2)$	$T_\parallel^*(q^2,k^2)$	$T_{\mathbb{L}}^*(q^2,k^2)$	$--$
$F(q^2) \equiv$	4	$V_\perp(q^2)$	$V_\parallel(q^2)$	$V_{\mathbb{L}}(q^2) = V_\parallel(q^2)$	$P(q^2) = 0$
$F^*(q^2,0)$		$T_\perp(q^2)$	$T_\parallel(q^2)$	$T_{\mathbb{L}}(q^2) = T_\parallel(q^2)$	$--$
$F^*(0,k^2)$	5	$V_\perp^*(0,k^2)$	$V_\parallel^*(0,k^2)$	$\hat{V}_{\mathbb{L}}^*(0,k^2) = P^*(0,k^2)$	$P^*(0,k^2)$
		$T_\perp^*(0,k^2)$	$T_\parallel^*(0,k^2) = (1-\hat{k}^2)T_\perp^*(0,k^2)$	$T_{\mathbb{L}}^*(0,k^2)$	$--$

- Useful for $B \to \mu\mu\gamma$ background to flavoured axion searches at LHCb

$$P^{B_s \to \gamma^*}(q^2,k^2) = -2k^2 \left(\frac{1}{m_{B_q}} \frac{f_\phi^{em} A_0^{B_s \to \phi}(q^2)}{(m_\phi^2 - k^2)} + \dots \right)$$

from dispersion representation

QCD sum rules results at LO

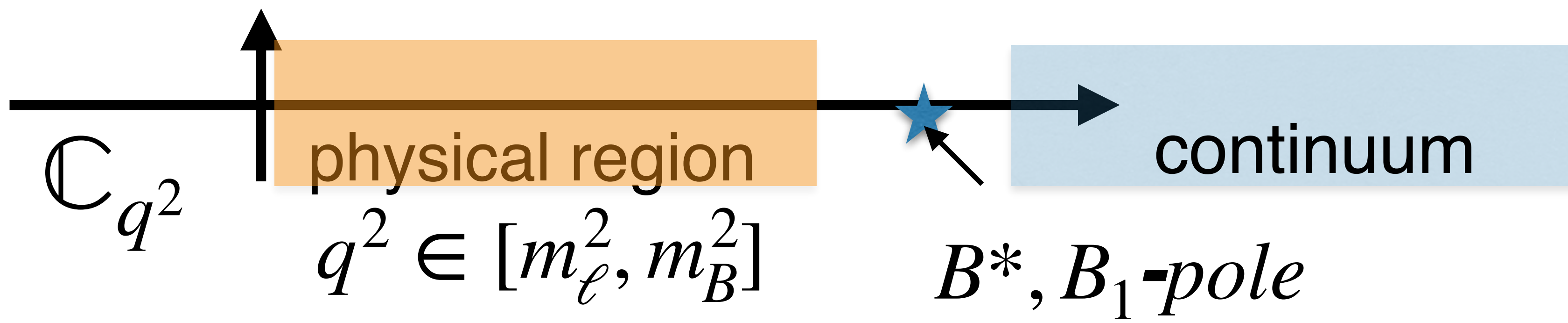
Albrecht, Stamou,Ziegler, RZ '19,

Part 1

On-shell photon form factor @NLO with Light-Cone Sum Rules (LCSR) [photon-DA]

On-shell photon FF computed with photon-meson DA Light-Cone Sum Rules (LCSR)

- **LCSR** cover $q^2 < 12 GeV^2$, extension possible if residue pole known



$$\langle \gamma | O_\mu^V | \bar{B}_q \rangle = P_\mu^\perp V_\perp(q^2) - P_\mu^\parallel V_\parallel(q^2)$$
$$\langle \gamma | O_\mu^T | \bar{B}_q \rangle = P_\mu^\perp T_\perp(q^2) - P_\mu^\parallel T_\parallel(q^2)$$
**

vector: $O_\mu^V \equiv -2/e m_{B_q} \bar{q}\gamma_\mu b_L$

tensor: $O_\mu^T \equiv 2/e \bar{s} i q^\nu \sigma_{\mu\nu} b_R$

2-photon pol. $P_\mu^\perp \equiv \varepsilon_\mu(\epsilon^*, p_B, k)$, $\quad P_\mu^\parallel \equiv i\,(p_B \cdot k\, \epsilon_\mu^* - k_\mu\,(p_B \epsilon^*))$

a) Algebraic constraint: $T_\perp(0) = T_\parallel(0)$(analogous to $T_1(0)=T_2(0)$)

b) Signs of FF depends on convention of cov.-derivative!!!

c) Other FF notation $F_V = V_\perp$, $\quad F_A = V_\parallel$

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* principle we have all off-shell results @NLO not in form of dispersion relation yet

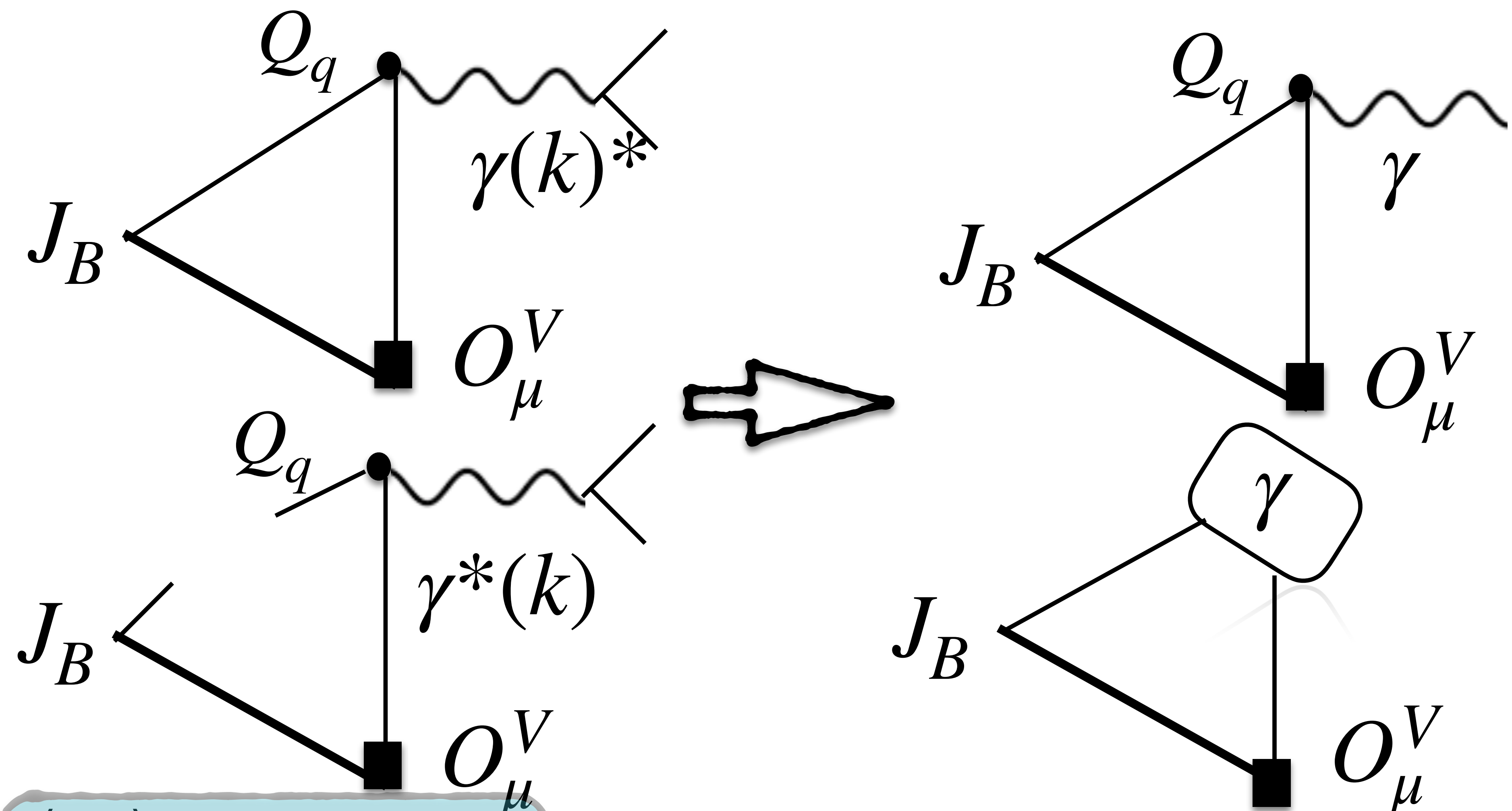
** charged case contact term ~ΔQ - need to include further photon emission e.g. lepton

Aspect of Computation

- Information on FF contained 3-pt function

$$\int_{x,y} e^{-ip_B\cdot x} e^{ik\cdot y}\langle 0\,|\,Tj_\rho(y)J_{B_q}(x)O^V_\mu(0)\,|\,0\rangle$$

Off-shell $k^2 \neq 0$ photon | **On-shell** $k^2 = 0$ photon | **Twist= dim-spin**



1

2,3,4,..

familiar from B→π,ϕ LCSR-FF

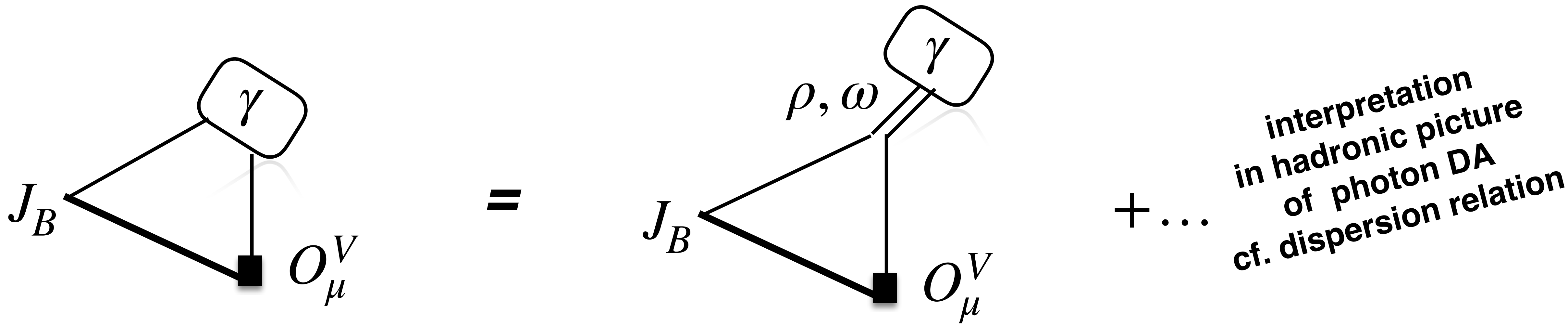
$\frac{\langle \bar{q}q\rangle}{k^2}$ condensate

Photon DA

Aspects of the photon light-cone distribution amplitude

Balitsky, Braun, Kolesnichenko '88   Ball, Braun, Kivel '02

- twist-2 $$\langle \gamma | \bar{q}(0)\sigma_{\alpha\beta} q(z) | 0 \rangle = ie Q_q f_\gamma \epsilon^*_{[\alpha} k_{\beta]} \int_0^1 du e^{i\bar{u}kz} \phi_\gamma^{t=2}(u) + \ldots$$

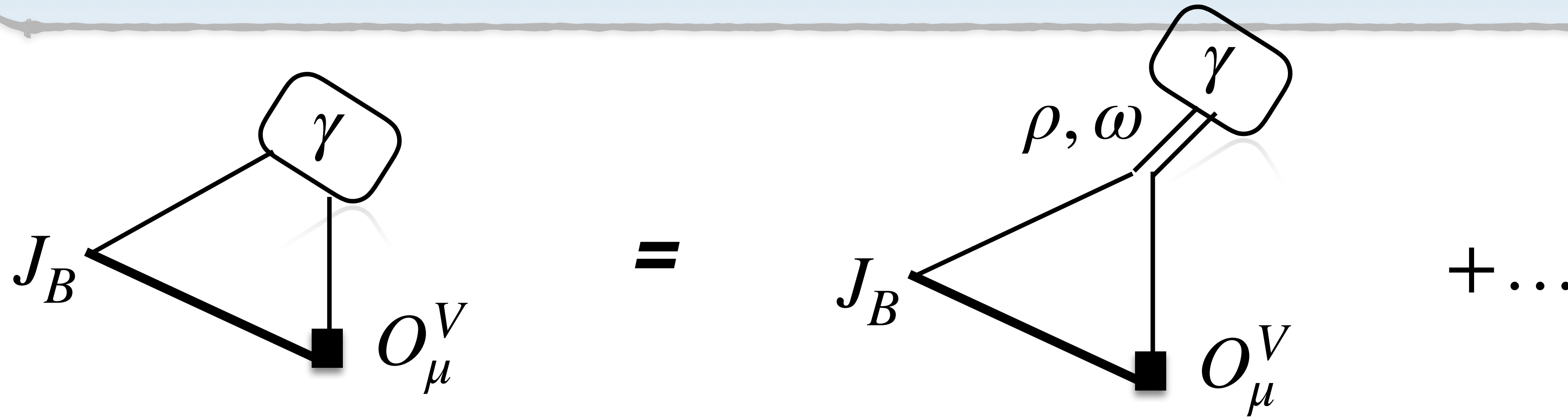


- Analogue of decay constant f_π is "f_γ" $= \langle \bar{q} q \rangle \chi$, where χ is magnetic susceptibility of quarks w.r.t B-field (SR & lattice similar size)

- Photon-DA taken to be asymptotic form $\phi_\gamma(u) = 6u\bar{u}$
  - Gegenbauer moments "undetermined ", sum rule not stable BBK'02
  - cf. instanton model Polyakov et al indicate small corrections)

Aspects of the photon light-cone distribution amplitude

- Interpretation $\langle \gamma | \bar{q}(0)\sigma_{\alpha\beta} q(z) | 0 \rangle = ieQ_q f_\gamma \epsilon^*_{[\alpha} k_{\beta]} \int_0^1 du e^{i\bar{u}kz} \phi_\gamma^{t=2}(u) + \ldots$



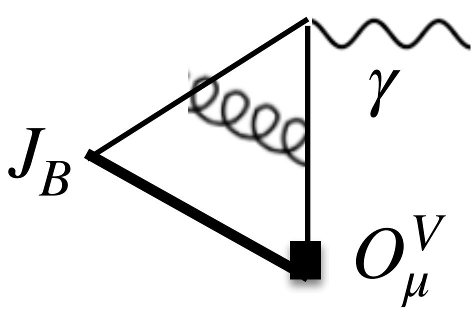
- Analogue of decay constant f_π is $"f_\gamma" = \langle \bar{q}q \rangle \chi$, where χ is vacuum susceptibility of quarks w.r.t B-field (SR & lattice similar res.)

- Photon-DA taken to be asymptotic form $\phi_\gamma(u) = 6u\bar{u}$

(Gegenbauer moments “undetermined ” , small
cf. instanton model Polyakov et al (no indication large effects))

Computation per se

- Computed PT (t=1) & twist NLO and twist 3,4 LO in α_s



Main work
PT @ NLO

3-pt function NLO
3 off-shell momenta & massive and massless lines
Master Integrals (fully analytic) in terms of O(10^3) Goncharov fits

Mastrolia, Primo, Schubert, di Vita ’17

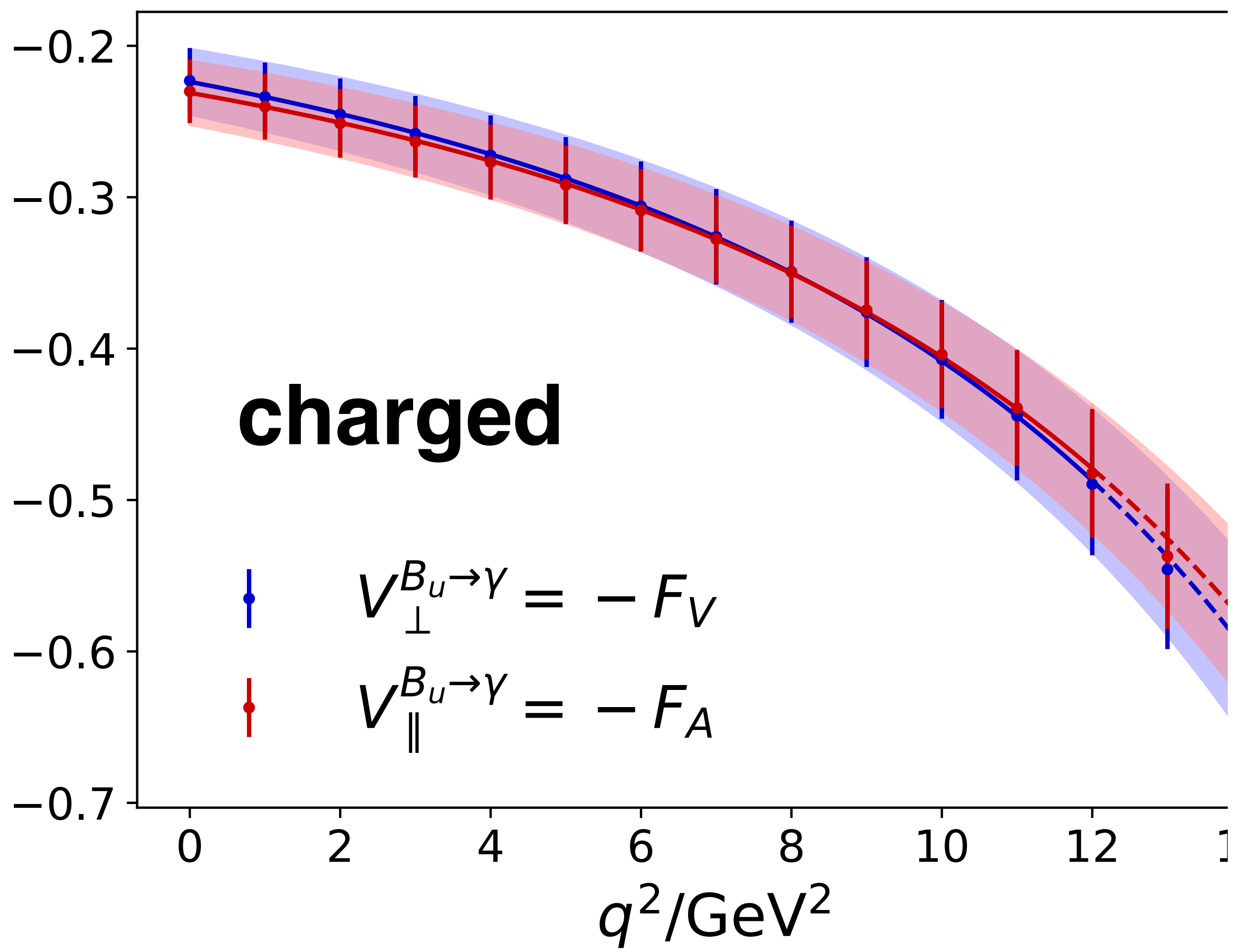
⇨consistency checks

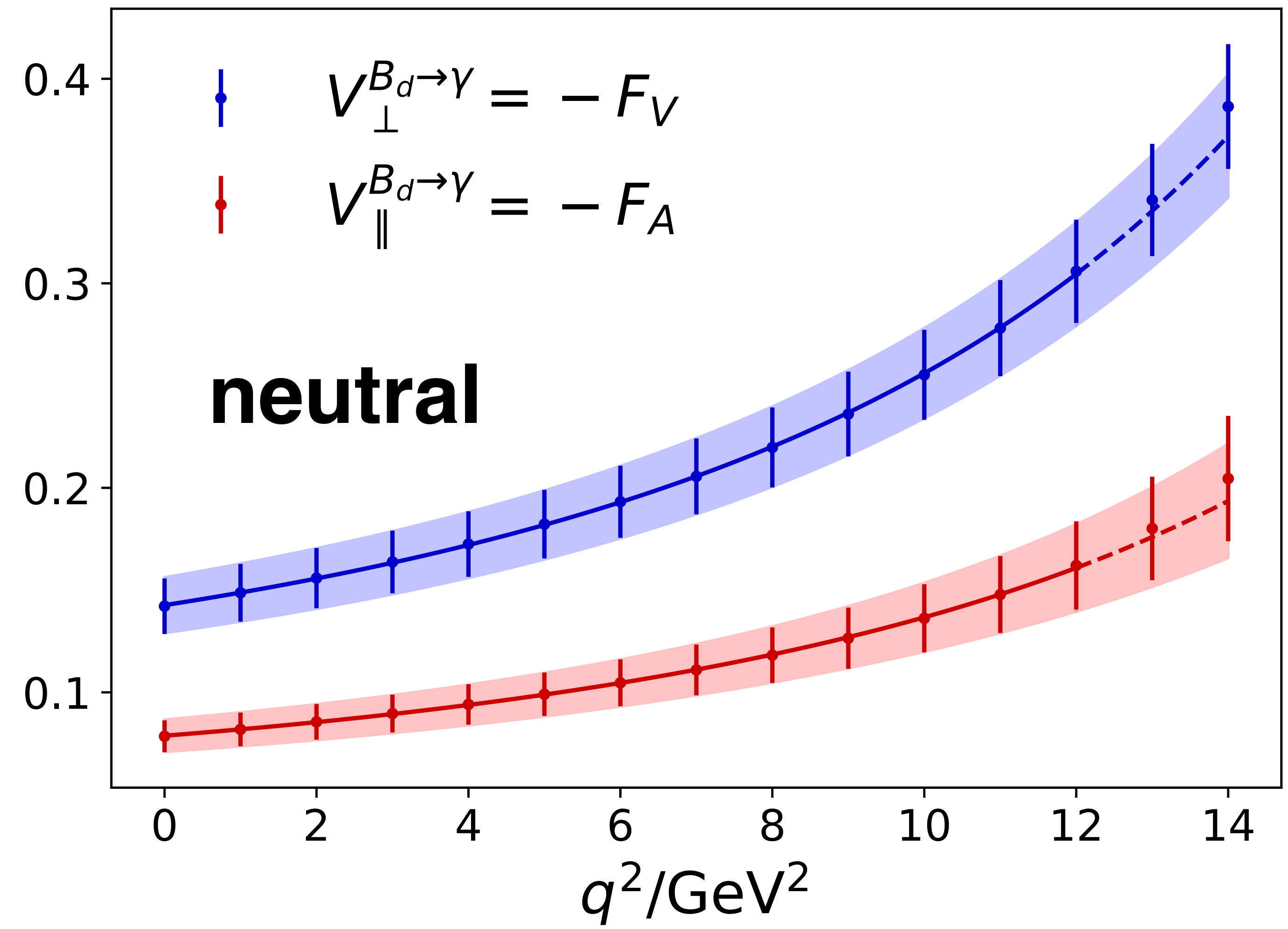
1) **Algebraic identity:** $T_\perp(0) = T_{||}(0)$ (checks γ_5-implementation) ✅

2) **Renormalisation & scale dep.** ✅

3) **Equation of Motion** (more shortly) ✅

Preliminary plots: vector Form Factors *





Spectator parton dominates

- Charged & neutral **differ** since Qq-part >> Qb-part.

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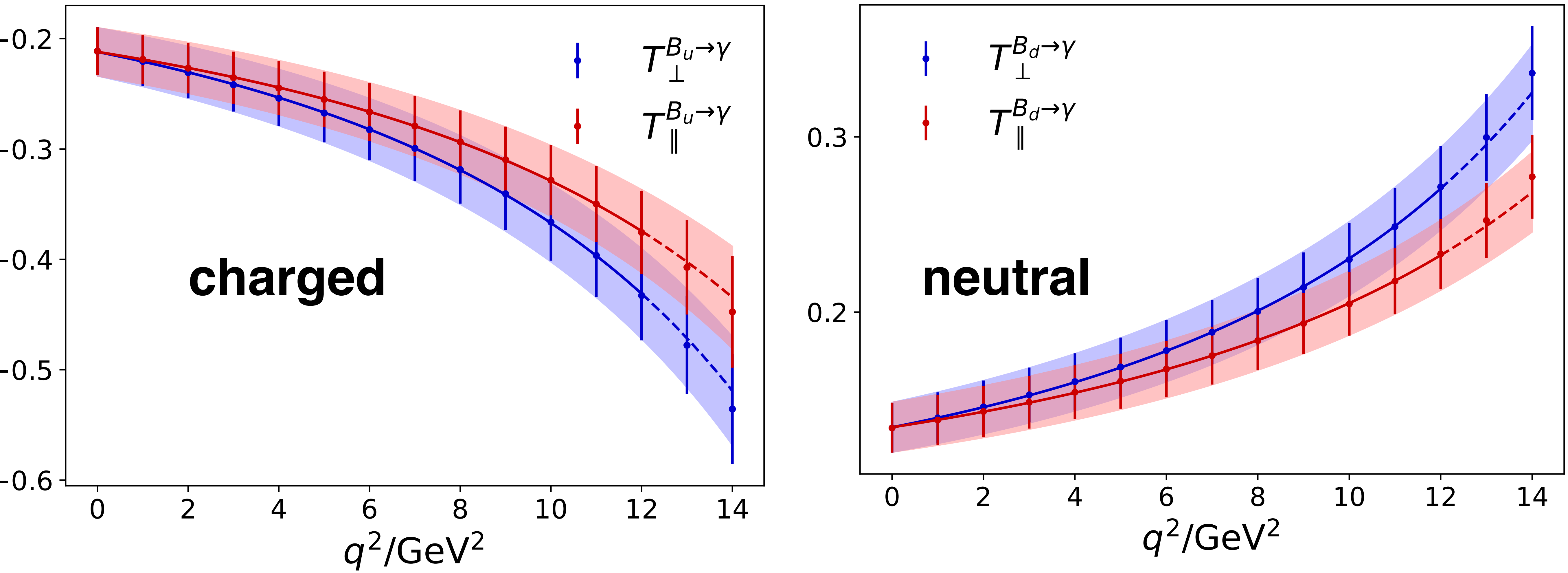
* checking, thorough numerical analysis, Qb NLO not included (Qq NLO is and is larger),

Preliminary plots: tensor Form Factors *



- Recall: **algebraic constraint**: $T_\perp(0) = T_\parallel(0)$ (analogous to $T_1(0)=T_2(0)$)
- Qualitatively similar to vector FFs as D-FF small (recall EOM or LEET)

Breakdown & scale-dependence

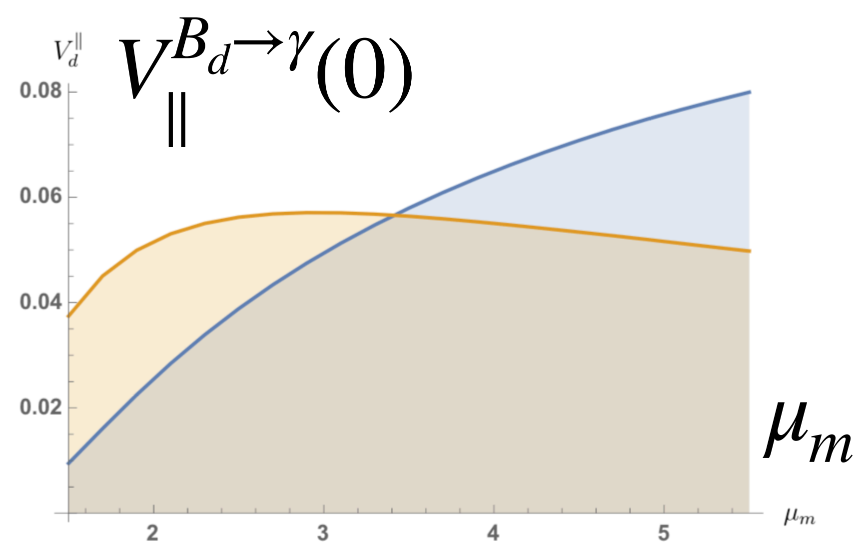
- Breakdown at specific point of charged and neutral vector FF

	$V_\perp(0)^{B_u\to\gamma}(0)$	$V_\perp(0)^{B_d\to\gamma}(0)$
t=1,LO	55%	60%
t=1,NLO	-7%	-7%
t=2, LO	33%	35%
t=2, NLO	-8%	-7%
t=3, LO	1%	1.2%
t=4, LO	-5%	-4.5%

Attention:
Breakdowns slightly misleading as **different parts** vary **under** scale & Borel Mass **variation** but are rather **stable as a sum**.

t=4 sizeable by $\langle\gamma|\bar{q}F_{\mu\nu}q|0\rangle \neq 0$

- Dependance on μ_m
  is much reduced due to NLO



* checking, thorough numerical analysis, Qb NLO not included (Qq NLO is and is larger),

Part 2- Use of Equation of Motion

- Use of EOMs for Form Factors

  Grinstein Pirjol'04 study correction to Isgur-Wise relation at low recoil

  Hambrock, Hiller, Schacht, RZ '13 first application LCSR

  Bharucha, Straub, RZ '15 more systematic exploitation

  Janowskim Pullin, RZ 'in prep full check at NLO

- Essence

  1) check of computation & formalism

  2) can reduce uncertainty of tensor vs vector Form Factors

EOM in QFT $\Leftrightarrow$ relations between correlation functions

- **EOM** (straightforward) on **physical states**: <K*I…IB>:

$$i\partial^\nu(\bar{s}i\sigma_{\mu\nu}(\gamma_5)b) = -(m_s \pm m_b)\bar{s}\gamma_\mu(\gamma_5)b + i\partial_\mu(\bar{s}(\gamma_5)b) - 2\bar{s}i\overleftarrow{D}_\mu(\gamma_5)b\,,$$

- leads to 4 ***equation of motion***

$$\begin{aligned}
&T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0\,,\\
&T_2(q^2) + (m_b - m_s)\mathcal{V}_2(q^2) + \mathcal{D}_2(q^2) = 0\,,\\
&T_3(q^2) + (m_b - m_s)\mathcal{V}_3(q^2) + \mathcal{D}_3(q^2) = 0\,,\\
&(m_b - m_s)\mathcal{V}_P(q^2) + \left(\mathcal{D}_P(q^2) - \frac{q^2}{m_b + m_s}\mathcal{V}_P(q^2)\right) = 0\,.
\end{aligned}$$

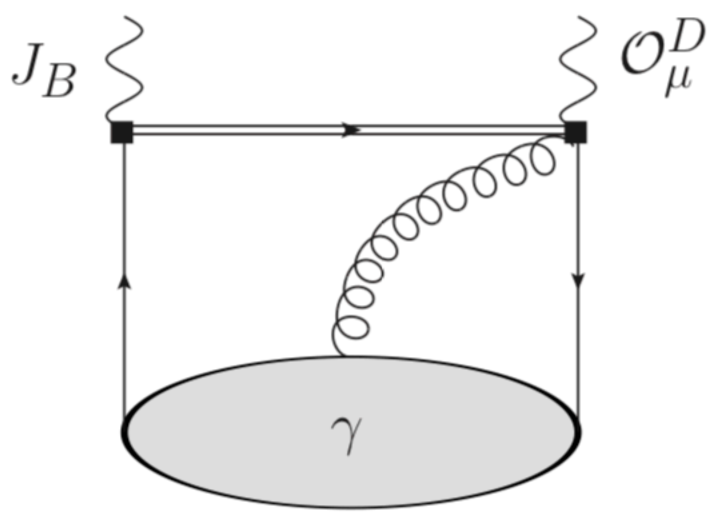
where D_i's are form factors of derivative operator:

$$\langle K^*(p,\eta)|\bar{s}(2i\overleftarrow{D})^\mu(1\pm\gamma_5)b|\bar{B}(p_B)\rangle = P_1^\mu\,\mathcal{D}_1(q^2) \pm P_2^\mu\,\mathcal{D}_2(q^2) \pm P_3^\mu\,\mathcal{D}_3(q^2) \pm P_P^\mu\mathcal{D}_P(q^2)$$

1) EOM as consistency check

$$\boxed{T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0}$$

- **Any form factor** determination has to **obey EOM** ⇒ **consistency check**



new class of diagrams due to covariant derivative operators

- $B \to V$: LCSR checked LO up to **twist 5**, including **O(m$_s$)**-corrections

- $B \to \gamma$: checked @LO up to twist 3-particle: $PT, \chi, f_{3\gamma}, \langle \bar{q}q \rangle$,
  @NLO twist-1 (2-loop diagram) & twist-2

$\Longrightarrow$ non-trivial check Photon-DA classification of Ball, Braun, Kivel '02

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*more involved off-shell photon, further terms

$$\langle T\bar{q}i(D^\nu i\sigma_{\mu\nu} + \overleftrightarrow{D}_\mu)\gamma_5 b(x) j_B(y) j_\rho(0)\rangle + (m_q - m_b)\langle T\bar{q}\gamma_\mu\gamma_5 b(x) j_B(y) j_\rho(0)\rangle =$$
$$-i\delta(x-0)\, g_{\mu\rho}(Q_b + Q_q)\langle T j_B(y) j_B(0)\rangle + i\delta(x-y)\,\langle T V_\mu(y) j_\rho(0)\rangle \,.$$

2) EOM reducing uncertainty

- $B \to V$: $D(q^2)$ small (related to LEET/SCET)

$$T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0$$

0.294   -0.272   -0.022

- Hence if $D(q^2)$ is not pathological (growing evidence)*, the sum rule specific parameters are stabilised by EOM.

Ratio of tensor to vector Form Factor with reduced uncertainty)

- $B \to \gamma$: $D(q^2)$ not that small in all cases (due to PT, twist -1) Can understand partly from dispersion relation …

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* converging twist, conformal spin and α_s-expansion

Part 3- Inverse moment B-LCDA

$$\int_0^\infty \frac{d\omega\, \phi_+(\omega)}{\omega} = \boxed{\frac{1}{\lambda_B}}$$

$$\langle 0 \,|\, \bar{q}(z) z_\mu \gamma^\mu \gamma_5 b(0) \,|\, \bar{B}(p_B) \rangle_{z^2=0} = -\, i f_B v z \int_0^\infty d\omega e^{-i\omega t} \phi_+^B(\omega)$$

$t = vz \quad p_B = m_B v$

Grozin Neubert '96 .....

a) **not** related to **local** **operator** (unlike Gegenbauer moments)
$\Rightarrow$ determination difficult (next slide)

b) enters **B-meson DA FF** (also B->ππ) computation at **leading order**
e.g. QCDF, SCET, B-meson LCSR

$$V_\perp^{B\to\gamma}(0) \sim \frac{m_B f_B}{E_\gamma \boxed{\lambda_B(\mu)}}(1 + O(\alpha_s(\mu)) + O(1/m_b))$$

Methods of determination

- This talk: $F^{B\to\gamma}_{LCSR}(0) \propto \frac{Q_q}{m_b^{1/2}}$ $\quad F^{B\to\gamma}_{B-LCDA}(0) \propto \frac{Q_q}{m_b^{1/2}} \frac{1}{\lambda_B}$

  extension of Ball, Kou'03 at LO ($\lambda_B = 600 MeV$ no uncertainty given)

- **Non-local condensate sum rule** $\lambda_B = 460(110) MeV$ Braun,Ivanov,Korchemsky'03

- **SCET/QCDF** $B \to \gamma$ **@NLO** & some NLO in 1/mb vs **Experiment**
  Belle'18 @$771 fb^{-1}$ $\Rightarrow$ $\lambda_B > 240 MeV$ @ $90\,\%\, CL$

…Descotes-G. Sachrajda'02, Rohrwild,Beneke'11, Braun,Khodjamirian'12, Wang'16, Braun, Beneke, Ji, Wei'18,

- **B-meson LCSR = light-meson LCSR** (no $1/m_b$ as ill-behaved)
  @LO $\lambda_B = 460(160) MeV$ Khodjamirian Mannel, Offen '05
  @NLO $\lambda_B = 354(40) MeV$ Wang, Shen '15 (dep. model B-meson DA)

- New proposal for **indirect lattice determination**,
  via **quasi-distribution amplitude** Wang, Wang, Xu, Zhao et al 1908.09933

“Heavy quark limes” in Sum Rules

Shuryak’82

- Possible to extract leading power of quantities

- Concretely our FF assume the form

$$V_\perp(q^2) = \frac{1}{f_B m_B^2} \int_{m_b^2}^{s_0} ds\, e^{(m_B^2 - s)/M^2} \rho_{f_B V_\perp}(s, q^2)$$

- All hadronic quantities need to be replaced/rescaled

$$m_B \to m_b + \bar{\Lambda}\,, \quad s_0 \to (m_b + \omega_0)^2\,, \quad M^2 \to 2 m_b \tau$$

- Scaling agrees with HQET/SCET-literature e.g.

$f_B \sim m_b^{-1/2}$ (e.g. HQET sum rules @NLO *Bagan,Ball,Braun,Dosch’92*)
$f_+^{B\to\pi}(0) \sim m_b^{-3/2}$ (e.g. LCSR twist-2@NLO *Bagan,Ball,Braun,’97*)
$T_\perp^{B\to\gamma}(0) \sim m_b^{-1/2}$ (e.g. LCSR twist-1,2@NLO *Pullin, RZ to appear*)

$$E_\gamma = \frac{m_B^2 - q^2}{2m_B}$$

- B-LCDA approach

$$F_V = -V_\perp = \frac{Q_q f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \left[\xi(E_\gamma) + \frac{Q_b f_B m_B}{2E_\gamma m_b} + \frac{Q_u f_B m_B}{(2E_\gamma)^2}\right],$$

NLO $1/m_b$

… Bosch, Lange, Neubert, Paz'01, Hardmeier, Lunghi, Wyler'01
Descotes-G. Sachrajda'02, Rohrwild,Beneke'11, Braun,Khodjamirian'12,
Wang'16, Shen, Wang'18, Braun, Beneke, Ji, Wei'18,

- Replace $f_B \to m_b^{-1/2} C(m_b/\mu, \alpha_s(m_b))(f_B^{stat}(m_b) + O1/m_b))$ with the latter HQET sum rules* Bagan,Ball,Braun,Dosch'92

Polylogs weight 3

$$V_\perp(q^2) = \frac{1}{f_B^{stat} m_b^{3/2}} \int_0^1 dz\, e^{(\bar{\Lambda} - z\omega_0)/\tau} \underbrace{\rho_{f_B V_\perp}((m_b + \omega_0 z)^2, q^2)}_{\rho_0(z, \ln^{(1,2)}(\omega_0/m_b)) + \frac{1}{m_b}\rho_1}$$

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* includes quark and mixed condensate at LO

Preliminary Discussion & Open questions

- Match and get $\lambda_B(\mu) = f(\mu, \tau, \omega_0)$

- Good stability in Borel parameter τ (Borel window) NLO stabilises it

- NLO: "Stability" in continuum threshold ω_0
  (i.e. cancellation in ratio etc in progress)
  NLO it's too early however Borel stability is a good sign

- LO get higher value than $\lambda_B^{Ball-Kou} = 600 MeV$ due to including condensates

- We do find double logs $\ln^2 \omega_0/m_b$ and they are in B-LCDA there $\ln^2 \omega/m_b$
  in addition $Ei(-\omega_0/\tau)$ with contain further logs (small ones).

$$m_B \to m_b + \bar{\Lambda}\,, \quad s_0 \to (m_b + \omega_0)^2\,, \quad M^2 \to 2 m_b \tau$$

conclusions and summary

- $B_{uds} \to \gamma$ Form Factors from LCSR @NLO on their way at low q^2
  anticipated uncertainties: 10-15%
  with z-expansion fits with error correlations

- $D_{uds} \to \gamma$ will do them as well (can cover less of physical domain)

- Extension to high q^2 possible
  1) If $gBB^*\gamma$ couplings known (pole residues)
  2) Match to upcoming lattice QCD computation e.g. Lehner, Meinel, Detmold,Soni, in prep

- Nice **application** and test of **photon DA** &
  Possibility to extract competitive: $1/\lambda_B$ helpful for B-meson DA computations

- LHCb $B_s \to \mu\mu\gamma$ and BelleII $B_s \to \ell\ell\gamma, B \to \ell^+\nu\gamma$ in the making

Thanks for your attention

BACKUP

- (Comparison vector meson & photon DA @twist 4

$$\varphi_V^{t=4} \sim m_V^2 \ldots + \langle V | \bar{q} G_{\mu\nu} q | 0 \rangle \qquad \varphi_\gamma^{t=4} \sim \langle \gamma | \bar{q} G_{\mu\nu} q | 0 \rangle + \langle \gamma | \bar{q} F_{\mu\nu} q | 0 \rangle$$

sizeable small small sizeable