

Discussing the light-meson DAs

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Outline

1 Prologue

2 Meson transition form factors and DAs

- $\gamma^* \gamma \rightarrow \pi$
- $\gamma^* \gamma \rightarrow \eta, \eta'$ transition form factors
- $\gamma^* \gamma \rightarrow f_0(980)$
- $\gamma^* \gamma^* \rightarrow \eta'$

3 Summary

Factorization

$$\mathcal{M}(Q^2) = T_H(x_1, \dots; \dots; Q^2, \mu_{F1}^2, \dots) \otimes \prod_i \Phi_i(x_i; \dots; \mu_{Fi}^2)$$

$x_i \dots$ momentum fraction, $\mu_{F(i)}^2 \dots$ factorization scale(s)

e.g.

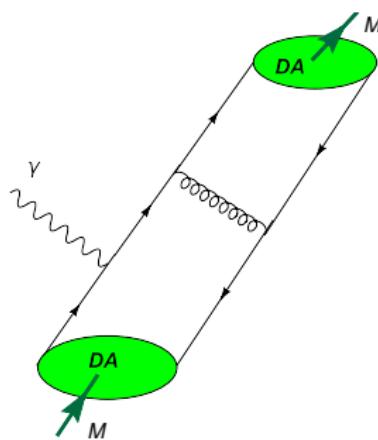
$$\begin{aligned} T_H(x; Q^2, \mu_F^2) &= T_H^{(0)}(x, Q^2) + \frac{\alpha_s(\mu_R^2)}{4\pi} T_H^{(1)}(x, Q^2, \mu_F^2) \\ &\quad + \frac{\alpha_s^2(\mu_R^2)}{(4\pi)^2} T_H^{(2)}(x, Q^2, \mu_R^2, \mu_F^2) + \dots \end{aligned}$$

$$\begin{aligned} \Phi(x; \mu_F^2, \mu_0^2) &= \Phi^{(0)}(x, \mu_F^2, \mu_0^2) + \frac{\alpha_s(\mu_F^2)}{4\pi} \Phi^{(1)}(x, \mu_F^2, \mu_0^2) \\ &\quad + \frac{\alpha_s^2(\mu_F^2)}{(4\pi)^2} \Phi^{(2)}(x, \mu_F^2, \mu_0^2) + \dots \end{aligned}$$

Elementary hard-scattering amplitudes to NLO

Meson em form factor

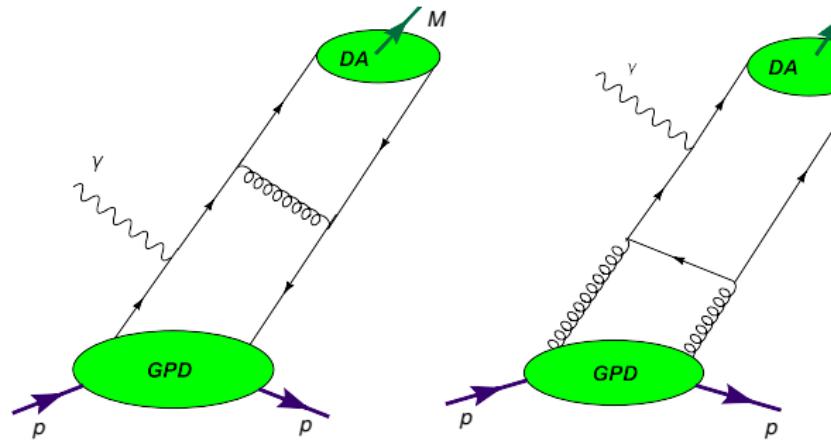
$$\gamma^*(q\bar{q}) \rightarrow (q\bar{q})$$



NLO: [...]

DVMP

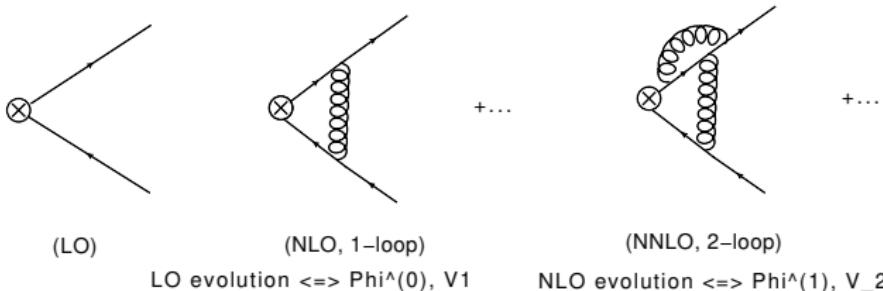
$$\gamma^* q \rightarrow (q\bar{q})q, \gamma^* g \rightarrow (q\bar{q})g$$



NLO: [Belitsky and Müller '01, Ivanov et al '04,]

[Duplančić, Müller, KPK '17]

DAs



$$V = \frac{\alpha_s(\mu_F^2)}{4\pi} V_1 + \frac{\alpha_s^2(\mu_F^2)}{(4\pi)^2} V_2 + \dots$$

$$\frac{\mu_F^2}{\partial \mu_F^2} \Phi = V \otimes \Phi \Leftrightarrow \text{resummation of } (\alpha_s \ln(\mu_F^2/\mu_0^2))^n$$

NLO evolution : [Müller '94]

3-loop kernels: [Braun, Manashov, Moch, Strohmaier '17]

DAs

$$\phi_P(x, \mu_F^2) = 6x(1-x) \left[1 + \sum_{n=2,4,\dots} a_{Pn}(\mu_F^2) C_n^{3/2}(2x-1) \right]$$

$$\int_0^1 dx \phi_P(x, \mu_F^2) = 1$$

$$a_{Pn}(\mu_F^2) = a_{Pn}^{LO}(\mu_F^2) + \frac{\alpha_S(\mu_F^2)}{4\pi} a_{Pn}^{NLO}(\mu_F^2)$$

$$a_{Pn}^{LO}(\mu_F^2) = \left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu_F^2)} \right)^{\gamma_n/\beta_0} a_{Pn}^{LO}(\mu_0^2) \quad (\leq a_{Pn}^{LO}(\mu_0^2))$$

$$a_{Pn}^{NLO}(\mu_F^2) = f(\mu_F^2, \mu_0^2, a_{Pk}(\mu_0^2)) \quad (k \leq n)$$

On literature...

Impact of tff results on literature:

- Round 1: a number of papers trying to accommodate BABAR '09 results, eg. flat DA [Radyuskin '09, Polyakov '09] ...
- Round 2: no definitive proof (neither from experimental nor theoretical side) but BELLE '12 results favoured in the literature

On fits...

How much can the fits tell us?

$$\begin{aligned} Q^2 F_{\pi\gamma}(Q^2) &= 6f_\pi C_\pi \left[1 + \sum_{n=2}^{\infty}' a_n(Q^2) \right. \\ &\quad \left. + \frac{\alpha_S(Q^2)}{\pi} \left(-1.667 + \sum_{n=2}^{\infty}' T_n^{NLO} a_n(Q^2) \right) \right] \end{aligned}$$

On fits...

How much can the fits tell us?

$$\begin{aligned} t = \alpha_S(Q^2) &= 6f_\pi C_\pi \left[1 + \sum_{n=2}^{\infty}' a_n(\mu_0^2) \left(\frac{t}{t_0} \right)^{-\gamma_n/\beta_0} \right. \\ &\quad \left. + \frac{t}{\pi} \left(-1.667 + \sum_{n=2}^{\infty}' T_n^{NLO} a_n(\mu_0^2) \left(\frac{t}{t_0} \right)^{-\gamma_n/\beta_0} \right) \right] \end{aligned}$$

- fractional polynomial in $t = \alpha_S(Q^2)$ variable
 - $0.4 > t > 0.2(0.26)$ for $2 < Q^2 < 34(8)\text{GeV}^2$
- ⇒ large correlations for a_n s



- one (effective) coefficient (a_2) to be determined

On fits...

How much can the fits tell us?

We can rewrite the tff contribution in terms of a linear fit:

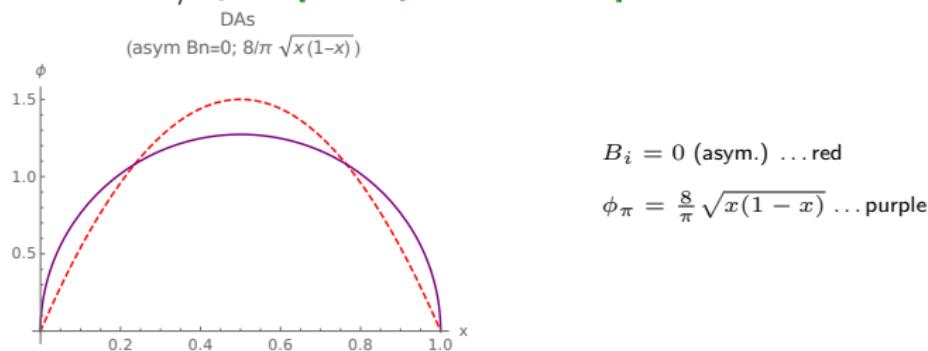
$$F(\textcolor{red}{X(t)}) = a_2 + a_4 \textcolor{red}{X(t)}$$

data range ($0.9 < X(t) < 1.2$) is narrow and away from the ordinate

⇒ strong correlation between a_2 and a_4

Beyond the truncation of the Gegenbauer series

- from AdS/QCD [Brodsky, Teramond '08]:



- parameterizing the deviation from conformal expansion [Ball, Talbot '05]
- summing the series...

Inspiration by DVCS calculation (conformal moment series representation)

- factorization formula in momentum fraction space (x-space):

$$F_{M\gamma}(Q^2; \mu_R^2) = \int dx \quad T_H(x, Q^2, \mu_R^2, \mu_F^2) \quad \Phi(x, \mu_F^2)$$

- ... in terms of **conformal moments** (n-space)

(analogous to Mellin moments in DIS: $x^n \rightarrow C_n^{3/2}(x), C_n^{5/2}(x)$):

$$= \sum_{n=0}^{\infty} T_n(Q^2, \mu_R^2, \mu_F^2) \quad a_n(\mu_F^2)$$

$$T_n(Q^2, \mu_R^2, \mu_F^2)) = \int_0^1 x(1-x)C_n^{3/2}(2x-1) \quad T_H(x, Q^2, \mu_R^2, \mu_F^2)$$

- series summed using **Mellin-Barnes** integral over complex n :

$$= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dn \left[i + \tan\left(\frac{\pi n}{2}\right) \right] T_n(Q^2, \mu_R^2, \mu_F^2) \quad a_n(\mu_F^2)$$

⇒ complete summation (inclusion of NLO evolution easier...) but modeling of meson DA coefficients $a_n(\mu_F^2)$ needed

η, η' transition form factors

Novel features:

① flavour-mixing (singlet-octet mixing)

② $|gg\rangle$ states contribute

\Rightarrow mixing of $q\bar{q}_1$ and gg DAs under evolution

$$\begin{array}{ccc} (\Phi_{M1} \equiv \Phi_{Mq}) & & (\Phi_{Mg}) \\ & \downarrow & \\ \mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix} & = & \begin{pmatrix} V_{qq} & V_{qg} \\ V_{gq} & V_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix} \end{array}$$

① flavour-mixing

[review Feldman '00]



simplest possibility: to take particle dependence and the flavour-mixing to be solely embedded in the decay constants f_M^i

- $\Phi_{Mi} = f_M^i \phi_i$ $i \in \{1, 8\}$
- the decay constants parametrized as

$$f_\eta^8 = f_8 \cos \theta_8 \quad f_\eta^1 = -f_1 \sin \theta_1$$

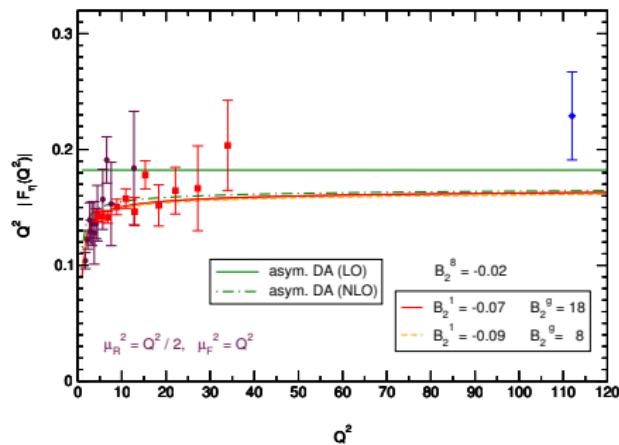
$$f_{\eta'}^8 = f_8 \sin \theta_8 \quad f_{\eta'}^1 = f_1 \cos \theta_1$$

[Leutwyler '98, Felmann, Kroll, Stech, '98, '99]

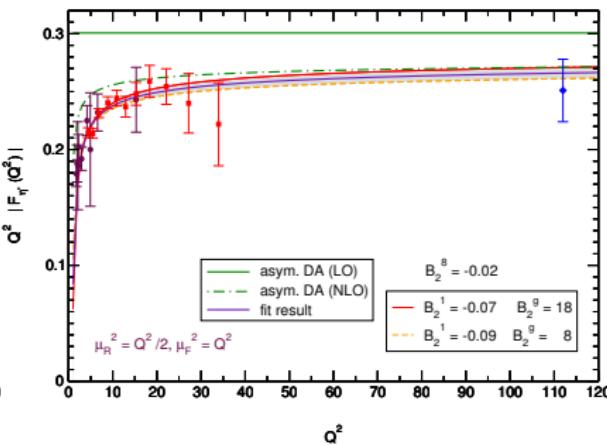
$$\begin{aligned} F_{P\gamma}(Q^2) = & \frac{6f_P^1 C_1}{Q^2} \left\{ 1 + a_2^1(\mu_F^2) + a_4^1(\mu_F^2) + \dots \right. \\ & + \frac{\alpha_s}{\pi} \left[-1.667 + \left(0.403 - 1.389 \ln \left(\frac{Q^2}{\mu_F^2} \right) \right) a_2^1(\mu_F^2) \right. \\ & \quad \left. + \left(2.482 - 2.022 \ln \left(\frac{Q^2}{\mu_F^2} \right) \right) a_4^1(\mu_F^2) + \dots \right] \\ & + \frac{\alpha_s}{\pi} \left[\left(-1.801 + 0.463 \ln \left(\frac{Q^2}{\mu_F^2} \right) \right) a_2^g(\mu_F^2) \right. \\ & \quad \left. + \left(-2.510 + 0.519 \ln \left(\frac{Q^2}{\mu_F^2} \right) \right) a_4^g(\mu_F^2) + \dots \right] \\ & \left. + \mathcal{O}(\alpha_s^2) \right\} \end{aligned}$$

Experimental situation and fits for η and η' tff

CLEO '97, BABAR '11, BABAR '06



CLEO '97, BABAR '11, BABAR '06



CLEO '97 (purple), BABAR '11 (red),

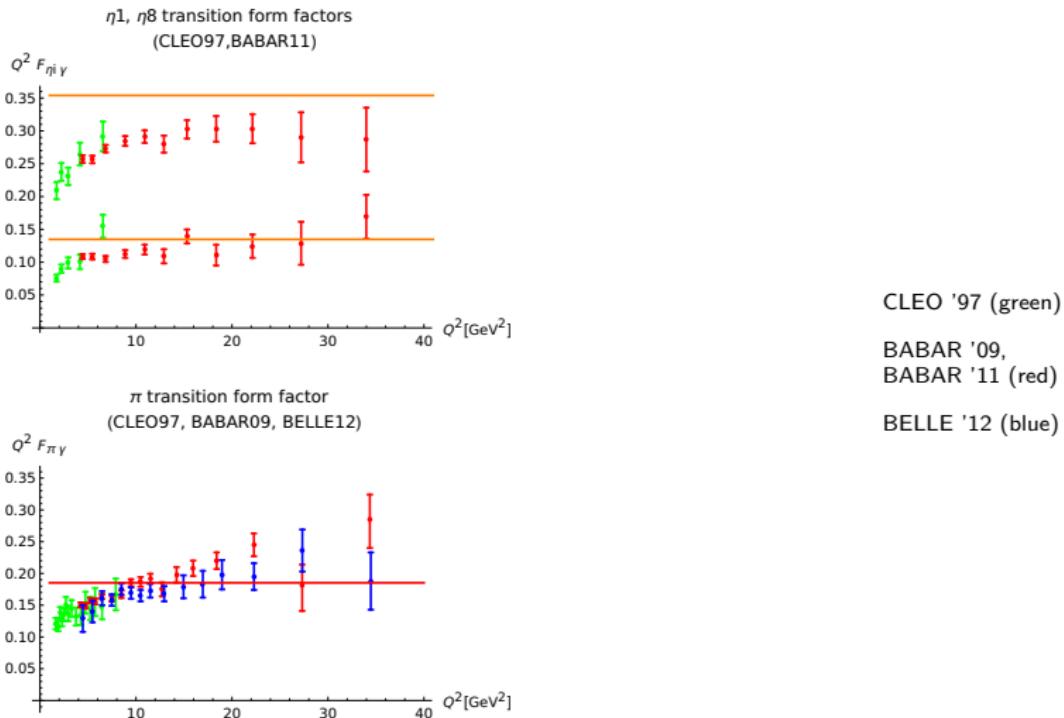
BABAR '06 (blue) ... timelike! (not used in fits)

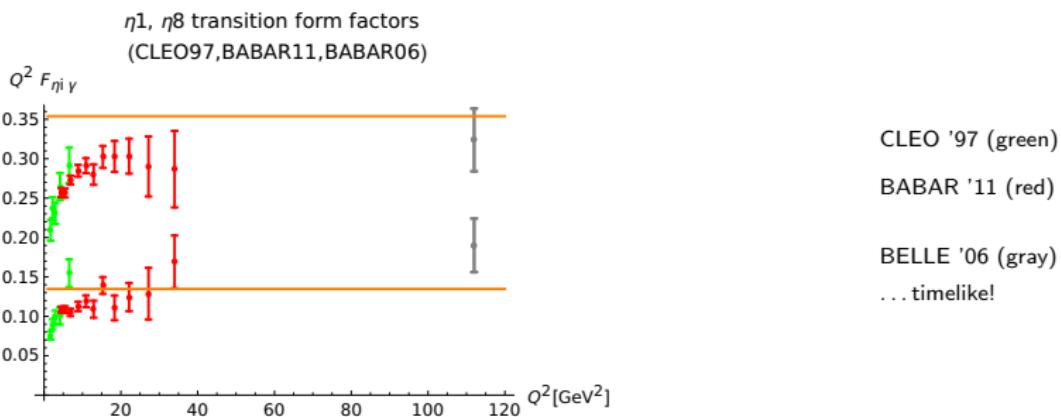
best fit:

$$a_2^8 = -0.05 \pm 0.02 \quad a_2^1 = -0.12 \pm 0.01 \quad a_2^g = 0.63 \pm 0.17$$

[Kroll, KPK '13]

Flavour singlet and flavour nonsinglet analysis





On fits...

How much can the fits tell us?

- We can rewrite the flavour-octet contribution in terms of a linear fit:

$$F_8(X_8(t)) = a_2^8 + a_4^8 X_8(t)$$

data range $0.9 < X_8(t) < 1.2$ is narrow and away from the ordinate
 \Rightarrow strong correlation between a_2^8 and a_4^8 ,
only effective a_2^8 to be determined

- We can rewrite the flavour-singlet contribution in terms of a linear fit:

$$F_1(X_1(t)) = a_2^1 + a_2^g X_1(t)$$

data range $-0.0029 < X_1(t) < 0.032$ (for $2 < Q^2 < 34 \text{ GeV}^2$) is near the ordinate
 \Rightarrow moderate correlation between a_2^1 and a_2^g ,
both a_2^1 and a_2^g to be determined

Scalar transition form factor

- BELLE 2015 data on the scalar $f_0(980)$ (and tensor $f_2(1270)$) meson transition form factor
- [Kroll '16]: $f_0(980)$ mainly $s\bar{s}$ state considered in modified approach with quark transverse moment effects included
- we discuss here standard hard-scattering approach with gluons included (NLO taken from DVCS)

● PSEUDOSCALAR CASE

$$\phi_{Pq}(x, \mu_F^2) = 6x(1-x) \left[1 + \sum_{n=2,4,\dots} a_{Pn}^q(\mu_F^2) C_n^{3/2}(2x-1) \right]$$

$$\phi_{Pg}(x, \mu_F^2) = 30x^2(1-x)^2 \sum_{n=2,4,\dots} a_{Pn}^g(\mu_F^2) C_{n-1}^{5/2}(2x-1)$$

$$\int_0^1 dx \phi_{Pq}(x, \mu_F^2) = 1, \quad \gamma_0^q = 0 \Rightarrow a_{P0}^q = \text{cte} \rightarrow 1$$

● SCALAR CASE

$$\phi_{Sq}(x, \mu_F^2) = 6x(1-x) \left[a_{S1}^q(\mu_F^2) C_1^{3/2}(\dots) + \sum_{n=3,5,\dots} a_{Sn}^q(\mu_F^2) C_n^{3/2}(\dots) \right]$$

$$\phi_{Pg}(x, \mu_F^2) = 30x^2(1-x)^2 \left[a_{S1}^g(\mu_F^2) + \sum_{n=3,5,\dots} a_{Sn}^g(\mu_F^2) C_n^{3/2}(\dots) \right]$$

$$\int_0^1 dx \phi_{Pg}(x, \mu_F^2) = a_{S1}^g(\mu_F^2), \quad \gamma_1^+ = 0 \Rightarrow a_{S1}^+ = \text{cte}$$

$$a_{Mn}^q(\mu_F^2) = a_{Mn}^+(\mu_F^2) + \rho_n^- a_{Mn}^-(\mu_F^2)$$

$$a_{Mn}^g(\mu_F^2) = \rho_n^+ a_{Mn}^+(\mu_F^2) + a_{Mn}^-(\mu_F^2)$$

$$a_{Mn}^\pm(\mu_F^2) = \left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu_F^2)} \right)^{\frac{\gamma_n^\pm}{\beta_0}} a_{Mn}^\pm$$

● SCALAR CASE

$$\phi_{Sq}(x, \mu_F^2) = 6x(1-x) \left[a_{S1}^q(\mu_F^2) C_1^{3/2}(\dots) + \sum_{n=3,5,\dots} a_{Sn}^q(\mu_F^2) C_n^{3/2}(\dots) \right]$$

$$\phi_{Pg}(x, \mu_F^2) = 30x^2(1-x)^2 \left[a_{S1}^g(\mu_F^2) + \sum_{n=3,5,\dots} a_{Sn}^g(\mu_F^2) C_n^{3/2}(\dots) \right]$$

$$\int_0^1 dx \phi_{Pg}(x, \mu_F^2) = a_{S1}^g(\mu_F^2), \gamma_1^+ = 0 \Rightarrow a_{S1}^+ = \text{cte}$$

$$a_{Mn}^q(\mu_F^2) = a_{Mn}^+(\mu_F^2) + \rho_n^- a_{Mn}^-(\mu_F^2)$$

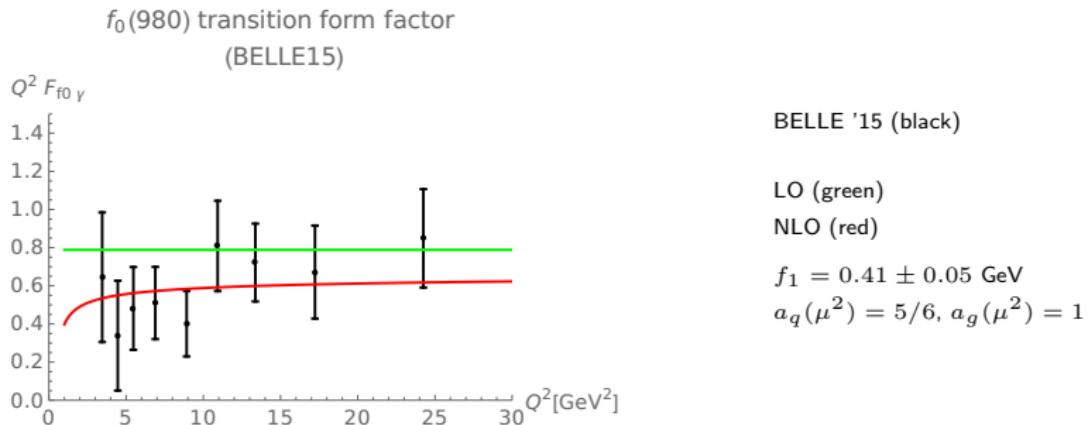
$$a_{Mn}^g(\mu_F^2) = \rho_n^+ a_{Mn}^+(\mu_F^2) + a_{Mn}^-(\mu_F^2)$$

$$a_{Mn}^\pm(\mu_F^2) = \left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu_F^2)} \right)^{\frac{\gamma_n^\pm}{\beta_0}} a_{Mn}^\pm$$

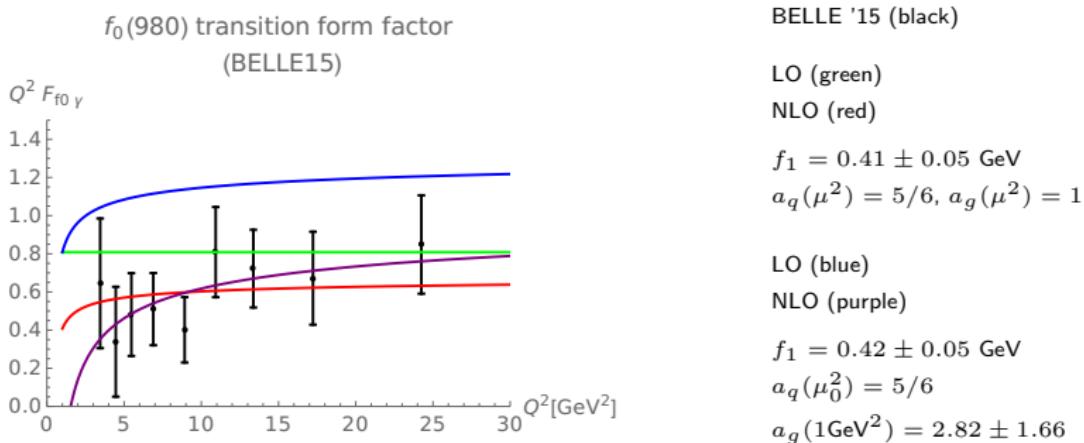
choice:

$a_{S1}^+ = 1/\rho_1^+ = 5/6$ $a_{S1}^- = 0$	\Rightarrow $a_{S1}^q(\mu_F^2) = 1/\rho_1^+$ $a_{S1}^g(\mu_F^2) = 1$
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$f_0(980)$ transition form factor (experiment and fits)



$f_0(980)$ transition form factor (experiment and fits)



$$\gamma^* \gamma^* \rightarrow \eta'$$

BaBar 2018

 \Rightarrow [Ji, Vladimirov '19; Kroll, P-K '19, ...] $\gamma^*(q_1) \gamma^*(q_2) \rightarrow M:$

$$Q_1^2 = -q_1^2 \quad Q_2^2 = -q_2^2$$
$$\bar{Q}^2 = \frac{Q_1^2 + Q_2^2}{2} \quad \omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2} \quad (0 \leq \omega \leq 1)$$

expansion in ω :

$$F_{M\gamma*}(\bar{Q}^2, \omega) \sim \sum_m \omega^m \sum_{l \leq m} a_{Ml} T_l$$

[Diehl, Kroll, Vogt '01; Melić, Müller, P-K '02] ($M = \pi$), [Kroll, P-K '19]
($M = (\eta,)\eta'$)

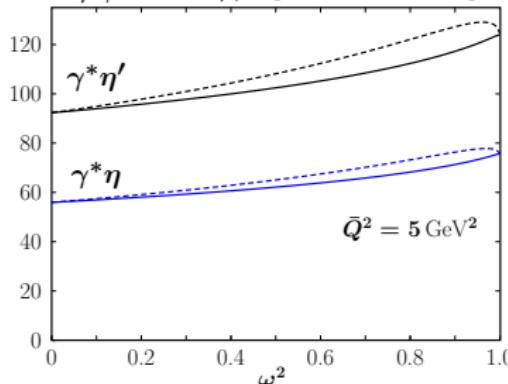
$$\begin{aligned} & F_{P\gamma^*}^1(\bar{Q}^2, \omega) \\ = & \frac{4f_P^1 C_1}{2\bar{Q}^2} \left\{ \left(1 - \frac{\alpha_s}{\pi} \right) \right. \\ & + \omega^2 \left[0.2 + 0.343 a_2^1 \right. \\ & \quad \left. + \frac{\alpha_s}{\pi} (-0.333 + 0.143 a_2^1 - 0.397 a_2^g) \right] \\ & + \omega^4 \left[0.086 + 0.229 a_2^1 + 0.104 a_4^1 \right. \\ & \quad \left. + \frac{\alpha_s}{\pi} (-0.187 + 0.183 a_2^1 - 0.326 a_2^g \right. \\ & \quad \left. + 0.201 a_4^1 - 0.166 a_4^g) \right] + \mathcal{O}(\omega^6) \Big\}_{(\mu_F^2=2\bar{Q})} \end{aligned}$$

- for $\omega = 0$ no dependence on a_i
- for $\omega < 1$ offers insight in the leading Gegenbauer coefficients

[Kroll, P-K '19]:

$\bar{Q}^2 [GeV^2]$	ω	$\bar{Q}^2 F_{\eta'\gamma^*}^{\text{exp}} [MeV]$	$\bar{Q}^2 F_{\eta'\gamma^*} [MeV]$	$\bar{Q}^2 F_{\eta\gamma^*} [MeV]$
6.48	0.000	92.8 ± 13.8	92.7 ± 3.9	56.2 ± 3.3
16.85	0.000	90.1 ± 37.3	93.8 ± 3.9	56.8 ± 3.3
9.55	0.553	78.7 ± 13.5	98.7 ± 4.1	59.9 ± 3.5
26.53	0.436	161.0 ± 44.2	97.7 ± 4.1	59.2 ± 3.5
45.63	0.000	397.4 ± 400.9	94.6 ± 4.0	57.4 ± 3.4

$\bar{Q}^2 F_{\eta' \gamma^*}, \bar{Q}^2 F_{\eta \gamma^*}$ [Kroll, P-K '19]:



solid line

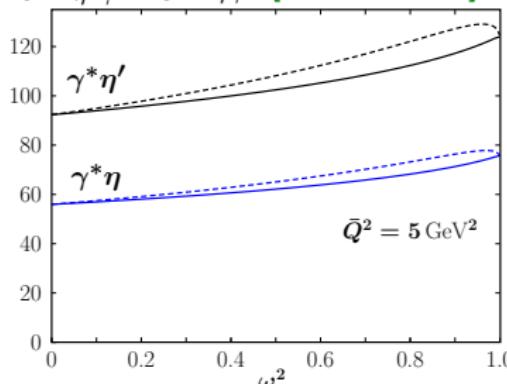
$$a_2^8 = -0.05, a_2^1 = -0.12, a_2^g = 0.63$$

[Kroll, P-K '13]

dashed line

$$a_2^8 = a_2^1 = 0.25, a_2^g = 0.63$$
$$a_4^8 = -0.31, a_4^1 = -0.38$$

$\bar{Q}^2 F_{\eta' \gamma^*}, \bar{Q}^2 F_{\eta \gamma^*}$ [Kroll, P-K '19]:



solid line

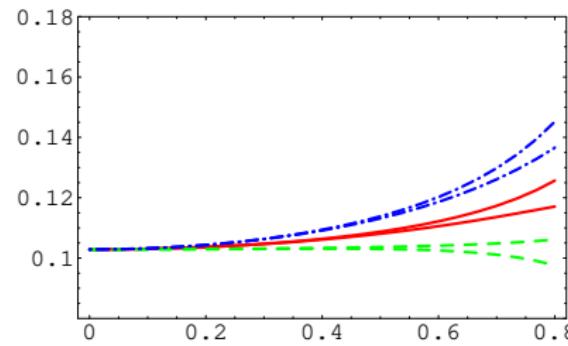
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$$a_2^8 = a_2^1 = 0.25, a_2^g = 0.63$$
$$a_4^8 = -0.31, a_4^1 = -0.38$$

$\bar{Q}^2 F_{\pi \gamma^*}$ [Melić, Müller, P-K '02]:



$$a_2 \in \{0, -0.5, 0.5\} \text{ (red, green, blue)}$$

$$a_4 \in \{-0.25, 0.25\} \text{ (lower, upper)}$$

Conclusions, questions, outlook...

- light-meson DAs were discussed in light of the pQCD twist-2 analysis of several photon-light-meson transition form factors
- Gegenbauer expansion limitations outlined
- many fits inconclusive and cannot be improved significantly but could be tested on other processes (pion em form factor)
- $\gamma^*\gamma^* \rightarrow M$ process and expansion in ω offer new cleaner insight in a_i s
- calculational restrictions on ϕ s and a_i s?
(evolution easier, summation possible . . . ?)
restrictions on a_i s from other sources?
restrictions and possibilities from experiment?
(BELLE II data ?) . . .