Meson electromagnetic form factors from Lattice QCD

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HPQCD collaboration

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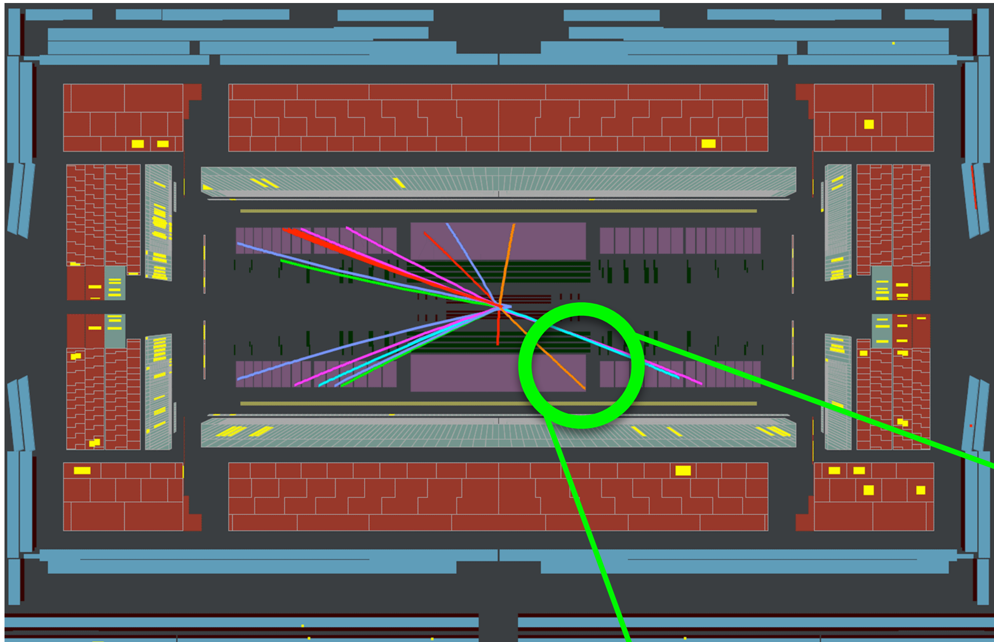
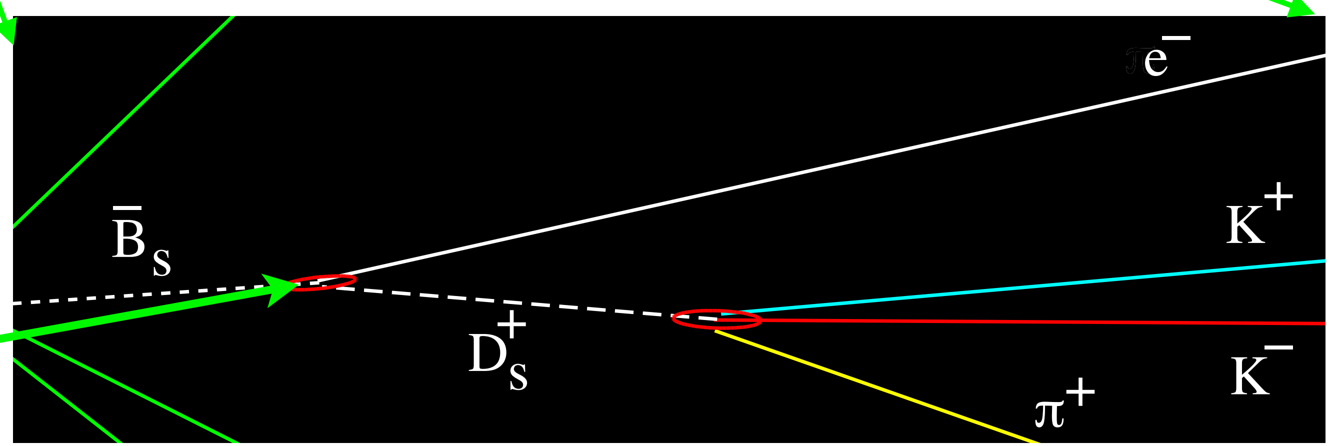
Understanding the strong force is key to testing the SM - binds quarks into hadrons that we see in experiment



Connecting observed hadron properties to those of quarks requires full nonperturbative treatment of Quantum Chromodynamics - lattice QCD

ATLAS@ LHC

Can calculate hadron masses and rates for simple weak/em decays and transitions to test Standard Model

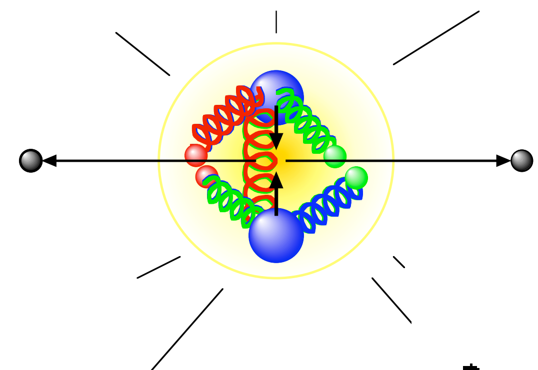


Electromagnetic transitions provide good tests/probes of hadron structure free of CKM issues. Lattice QCD calculations can be compared directly to experiment for simple cases e.g. (to be discussed here)

1) Vector meson annihilation to a photon (lepton pair)

$$\Gamma(V \to e^+e^-) = \frac{4\pi}{3}\alpha^2 Q^2 \frac{f_V^2}{M_V}$$

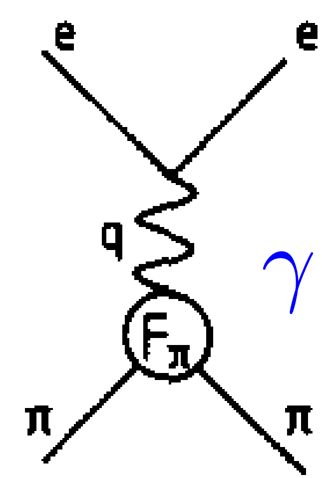
decay constant



2) Scattering of a meson from a photon as a function of Q^2

$$\frac{d\sigma}{dq^2} \propto |F(q^2)|^2$$

form factor



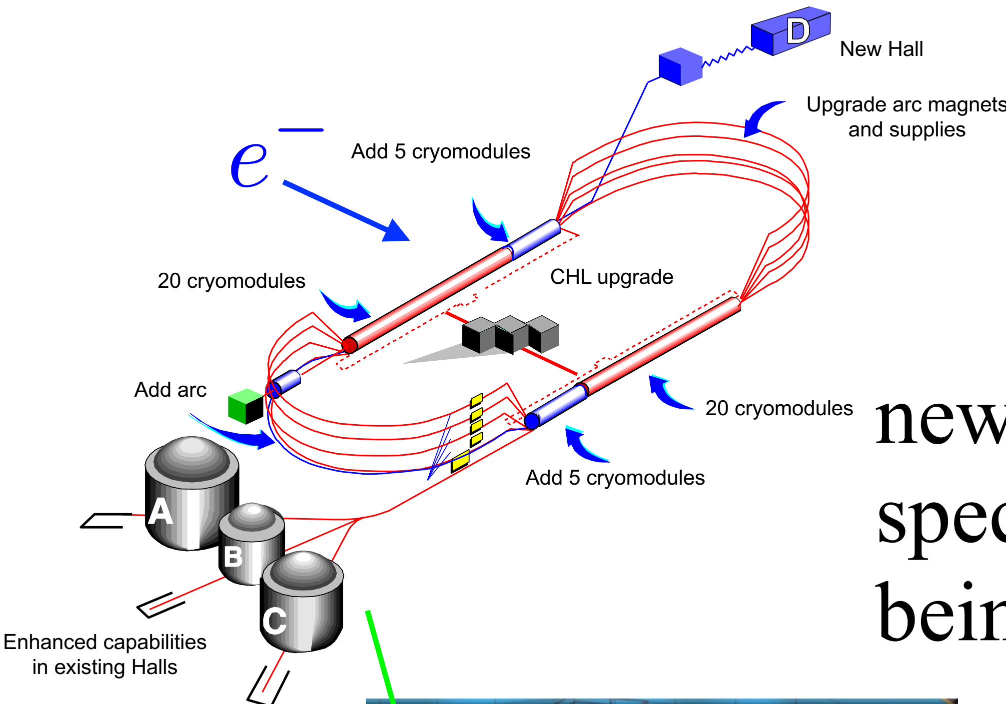
Lattice QCD can also provide decay constants for mesons that cannot be accessed experimentally (e.g. neutral pseudo scalars such as η_c) and for mesons that do not exist experimentally (e.g. η_s)

The same is true for form factors hence Lattice QCD can provide additional “data” with which non-lattice theorists can test model relationships between form factors and decay constants.

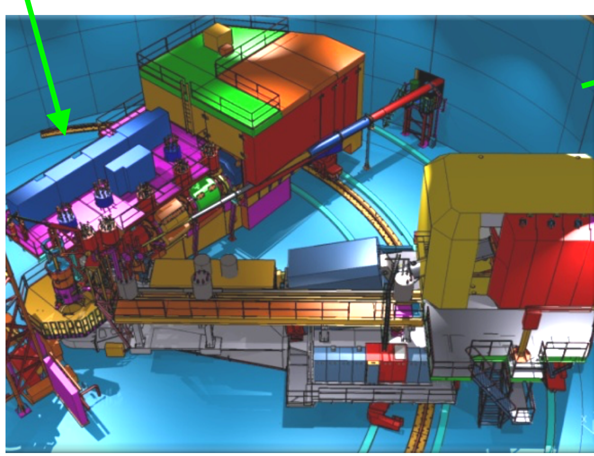
Meanwhile, a key aim of lattice QCD calculations is to map out the space-like form factor $Q^2F(Q^2)$ from low to high Q^2 ahead of experiment.

Driving improved theory for form factors:

Jefferson Lab 12 GeV upgrade - new expts E12-06-101 + E12-09-11 to determine π /K form factors to 6 GeV^2



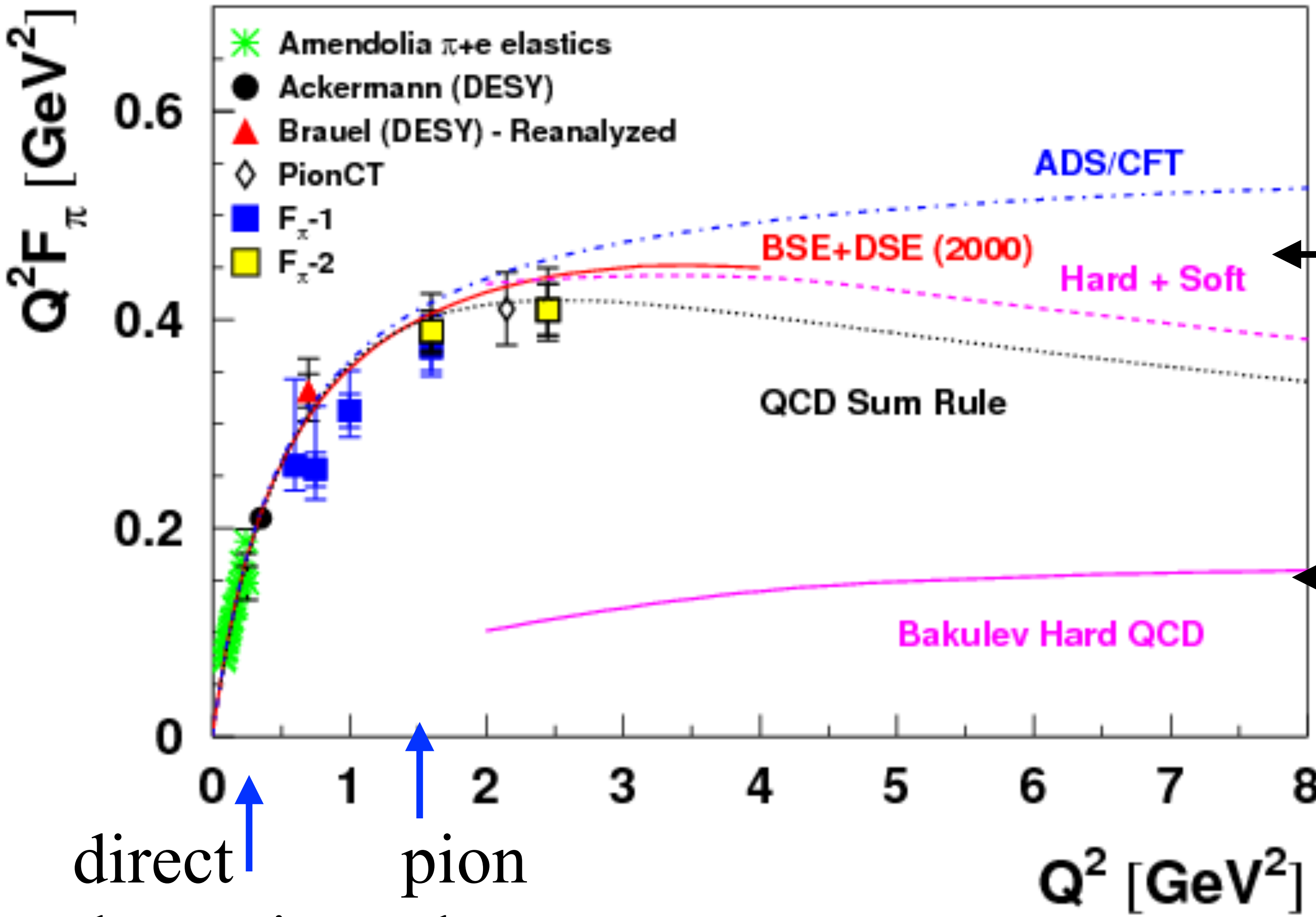
new spectrometer being built



Hall C, JLAB

Current experimental status for π -

for K direct determn. to 0.1 GeV²



variety of (non-lattice) theory predictns

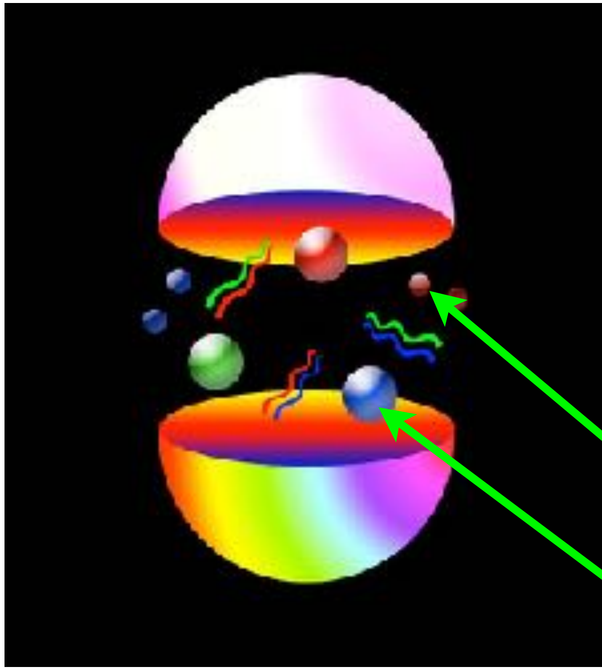
asymptotic form

direct determination

pion electro-production

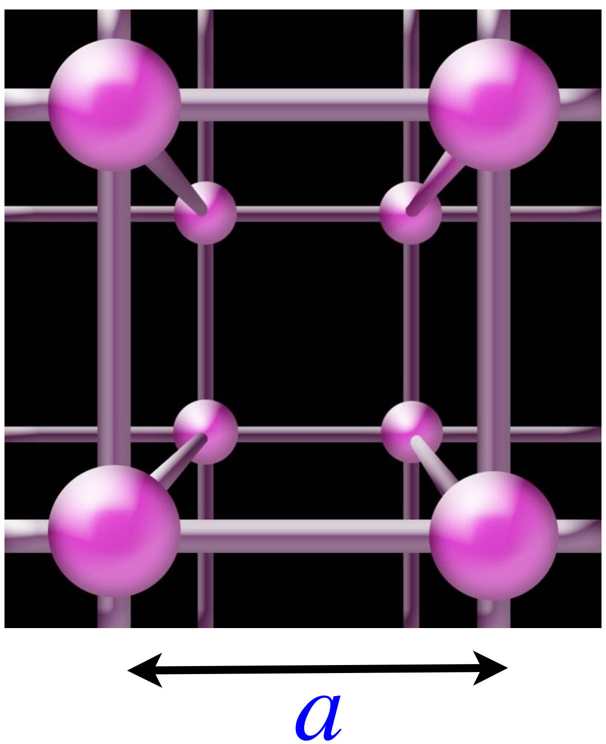
Lattice QCD should be able to do better …

Lattice QCD: fields defined on 4-d discrete space-(Euclidean) time. Lagrangian parameters: $\alpha_s, m_q a$



1) Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s, (c) sea quarks)

2) Calculate valence quark propagators and combine for “hadron correlators” . Fit for hadron masses and amplitudes



• Determine a to convert results in lattice units to physical units. Fix m_q from hadron mass

numerically extremely challenging

• cost increases as $a \rightarrow 0, m_{u/d} \rightarrow$ phys and with statistics, volume.

Quark formalisms

Many ways to discretise Dirac Lagrangian onto lattice.
All should give same answers.

Issues are: Discretisation errors at power a^n
Numerical speed of matrix inversion
Chiral symmetry
Fermion doubling

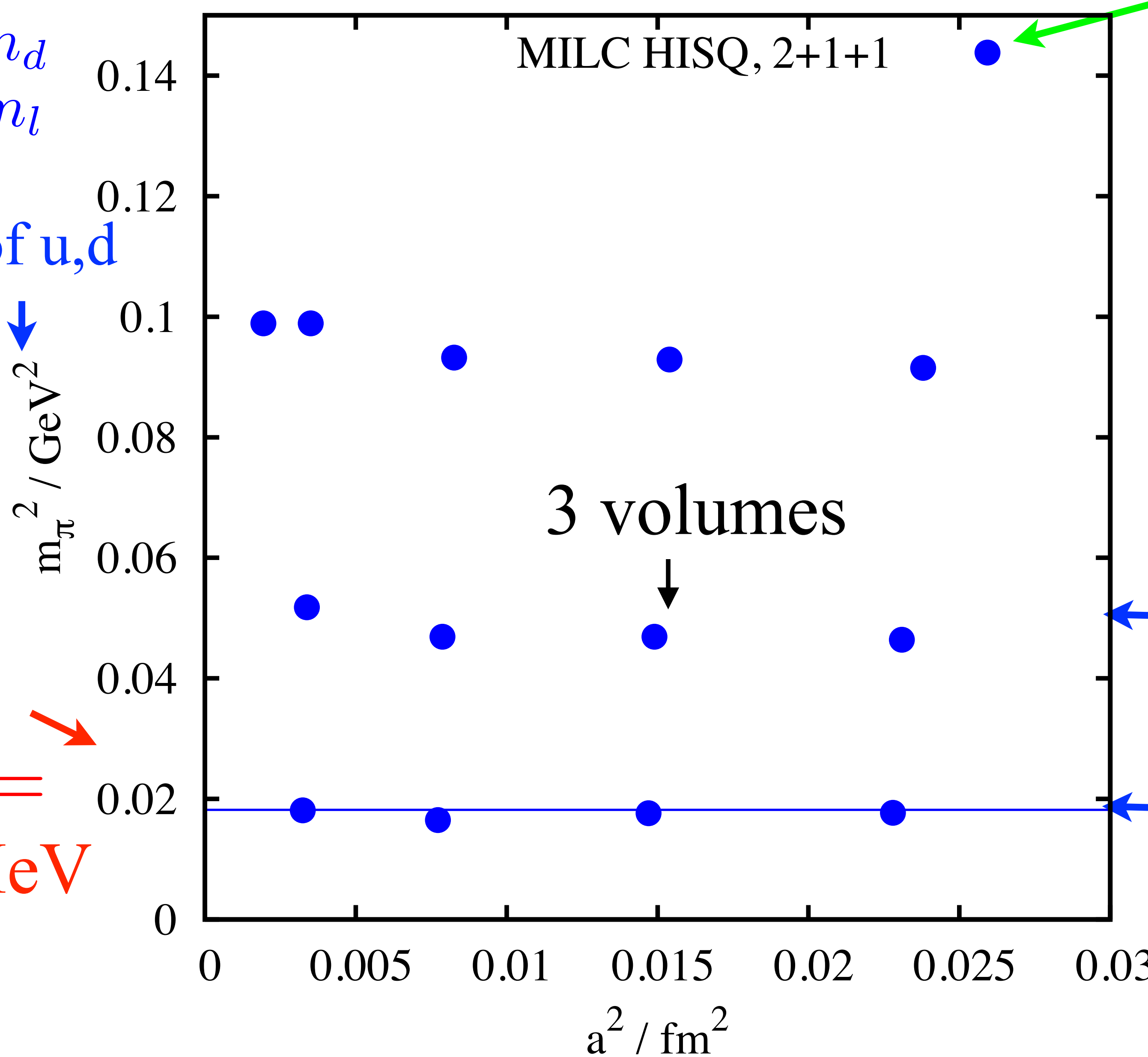
We use Highly Improved Staggered Quarks (HISQ) for u, d, s and c. Also (with extrapolation) for b.
Disc. errors $\alpha_s^2 a^2, a^4$. Numerically fast. Chiral symm.
Some complications from doublers ('tastes') which appear as discretisation effects.

HPQCD: hep-lat/0610092

Example parameters for '2nd generation' calculations now being done with staggered quarks.

$m_u = m_d$
$= m_l$

mass of u,d quarks



real world $m_{\pi^0} =$ 135 MeV

"2nd generation" lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation

E.Follana, et al, HPQCD, hep-lat/0610092.

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

Volume:

$m_\pi L > 3$

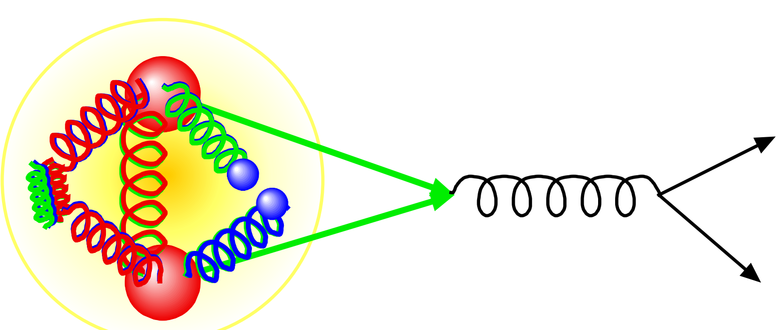
Hadron correlation functions ('2point functions') give masses and decay constants.

$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \xrightarrow{T\ \text{large}} A_0 e^{-m_0 T}$$

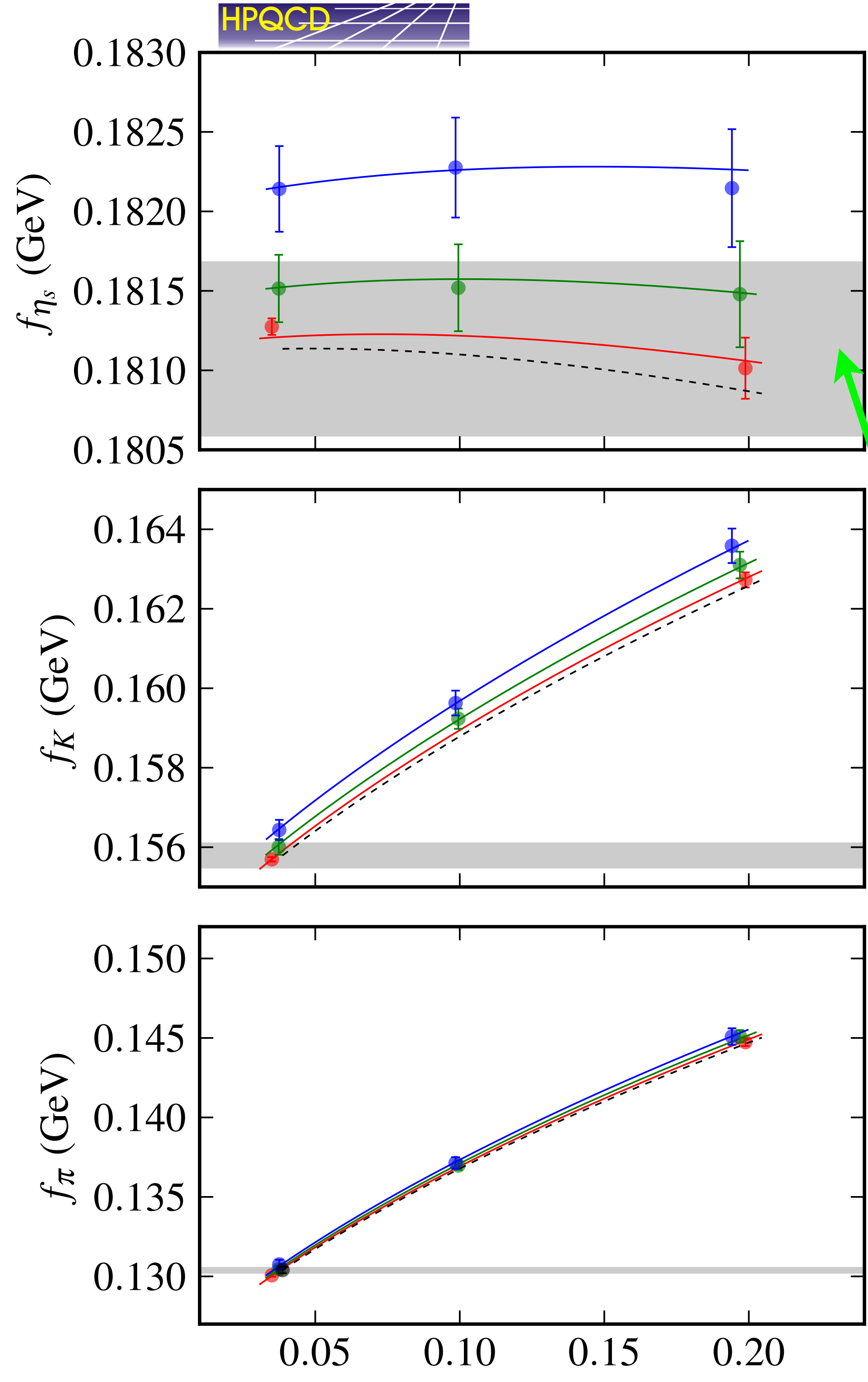
masses of all hadrons with quantum numbers of H

$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$

decay constant parameterises amplitude to annihilate - a property of the meson calculable in QCD.



1% accurate experimental info. for f and m for many mesons! Need accurate determination from lattice QCD to match



Light meson decay constants

R. Dowdall et al, HPQCD, 1303.1670.

Use w_0 to fix lattice spacing, with value of w_0 fixed from f_π.
PCAC reln for HISQ means no renormln factors needed.
Very small discretisation effects.

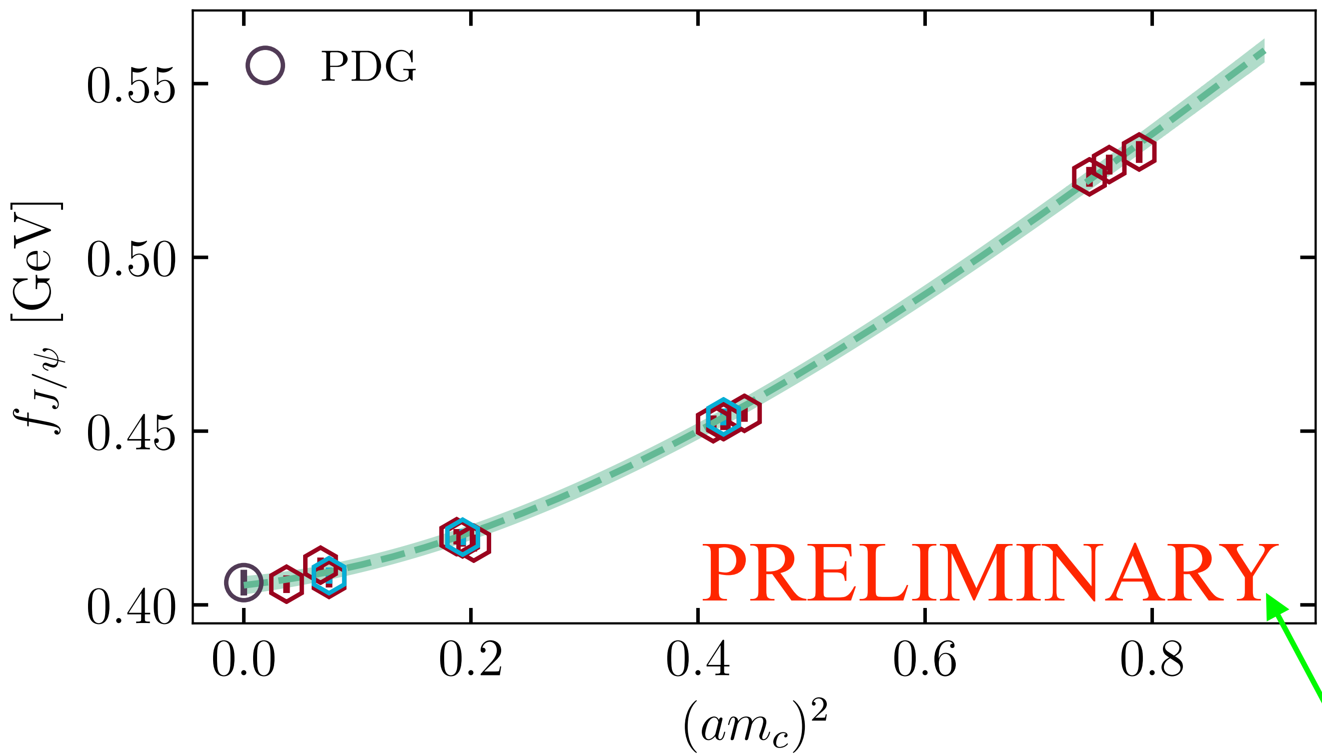
η_s meson defined within lattice QCD as $s\overline{s}$ meson that cannot mix. Can calculate its decay constant very accurately with full chiral analysis.

$$M_{\eta_s} = 688.5(2.2)\ \text{MeV}$$

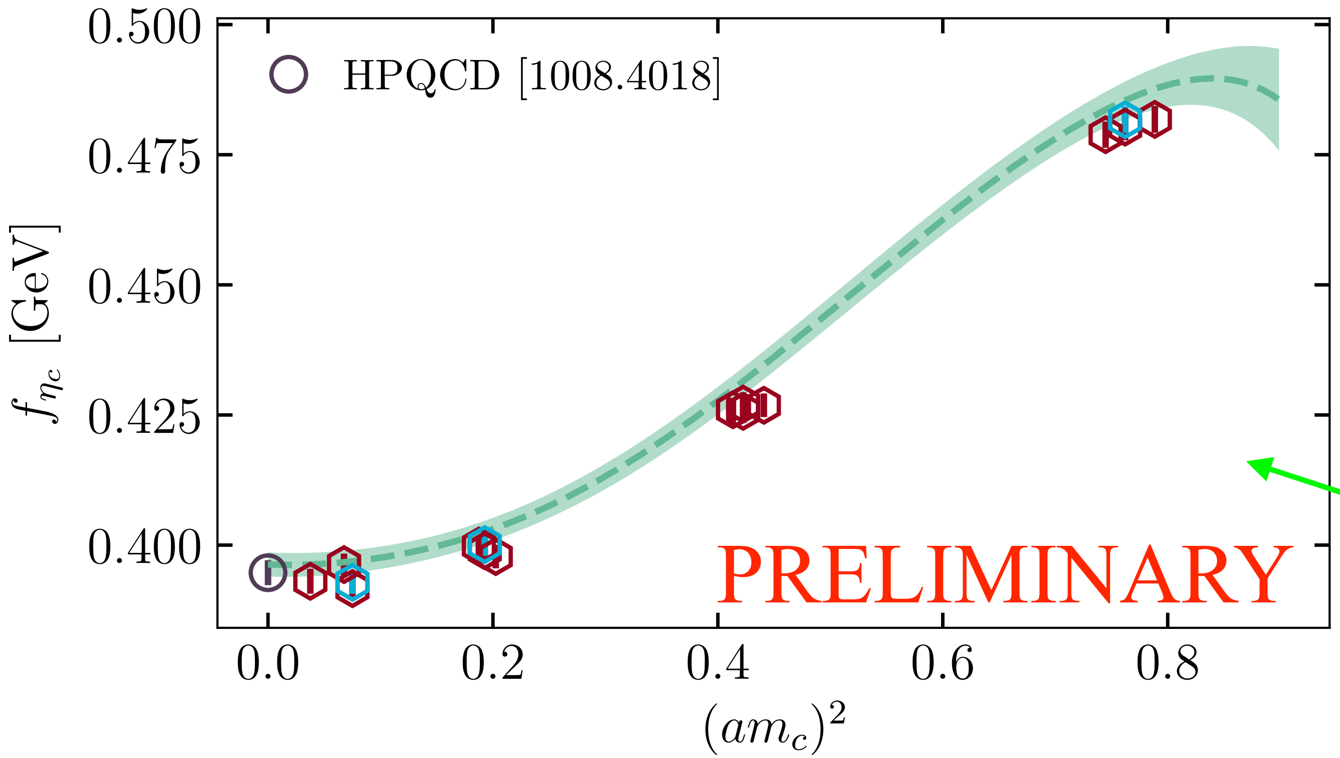
$$f_{\eta_s} = 181.14(55)\ \text{MeV}$$

Heavy meson decay constants

D. Hatton et al, HPQCD, in prep.



Improved analysis of $c\bar{c}$ case nearly complete (inc. impact of c electric charge). Good agreement with expt. for J/ψ .

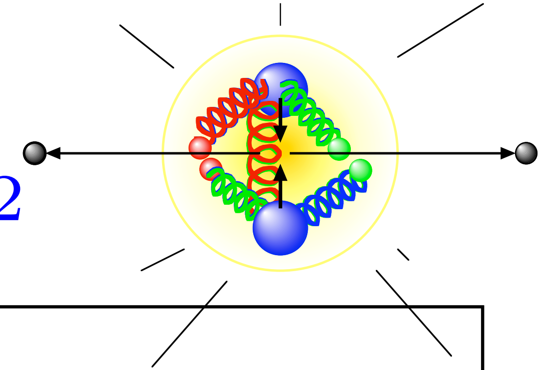


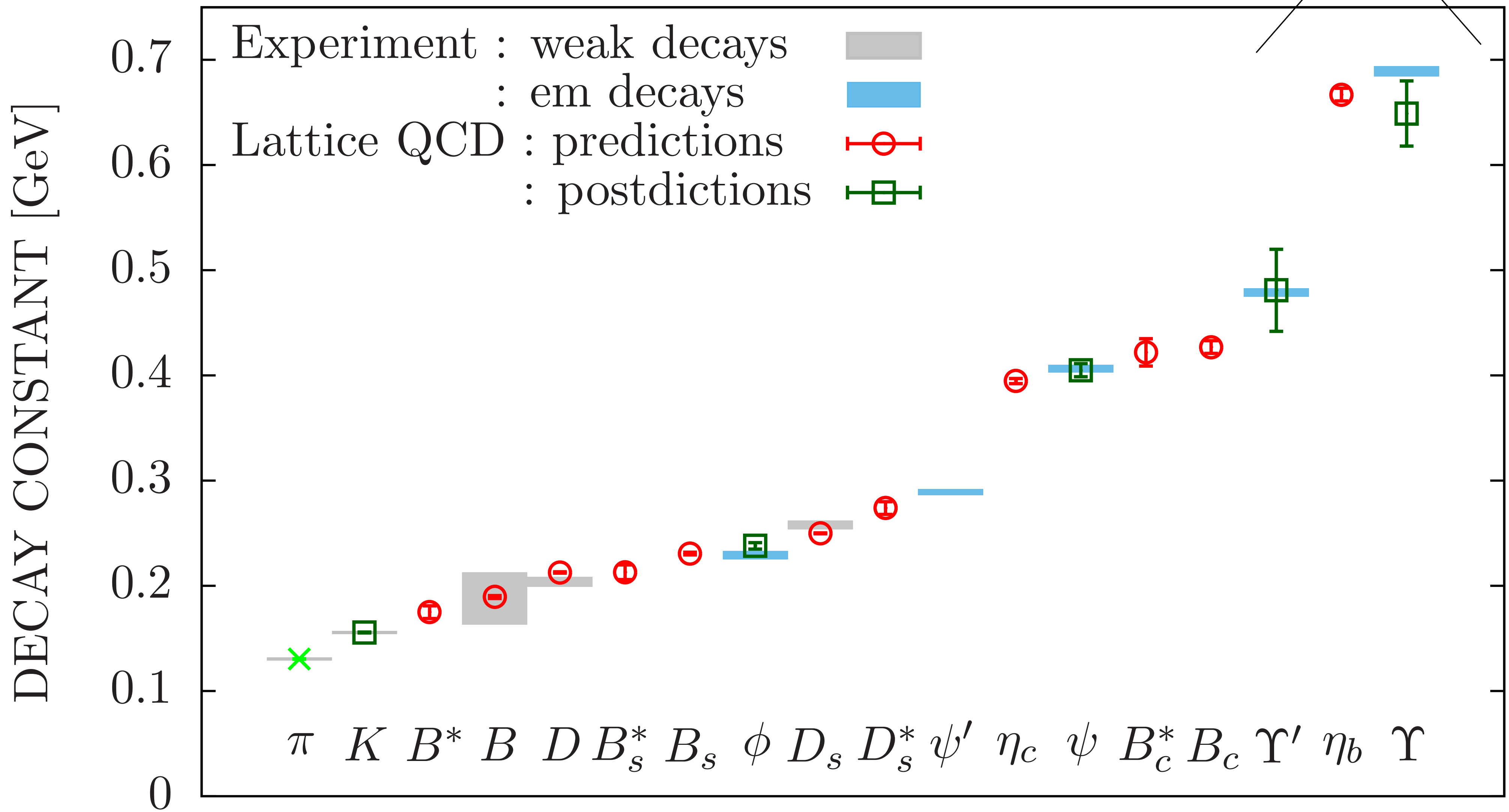
Allows accurate determination of η_c decay constant.

Summary of meson decay constants

Parameterises hadronic information needed for annihilation rate to W or photon:

$\Gamma \propto f^2$



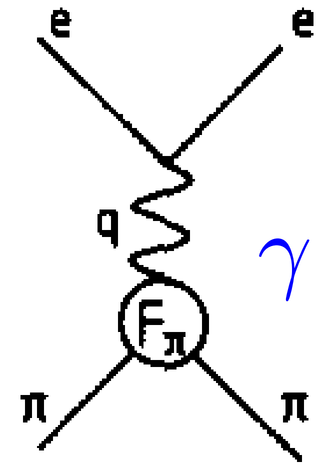


HPQCD 1208.2855, 1312,5264, 1408.5768, 1503.05762, 1703.05552, FNAL/MILC 1712.09262

Form factors give more info. on structure. Electromagnetic form factors provide test for those for weak decays

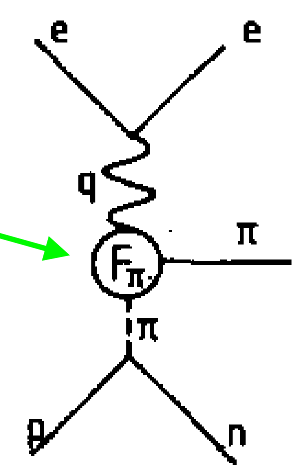
Simplest example is that of the meson electromagnetic form factor at space-like q^2:

$$\frac{d\sigma}{dq^2} \propto |F(q^2)|^2$$



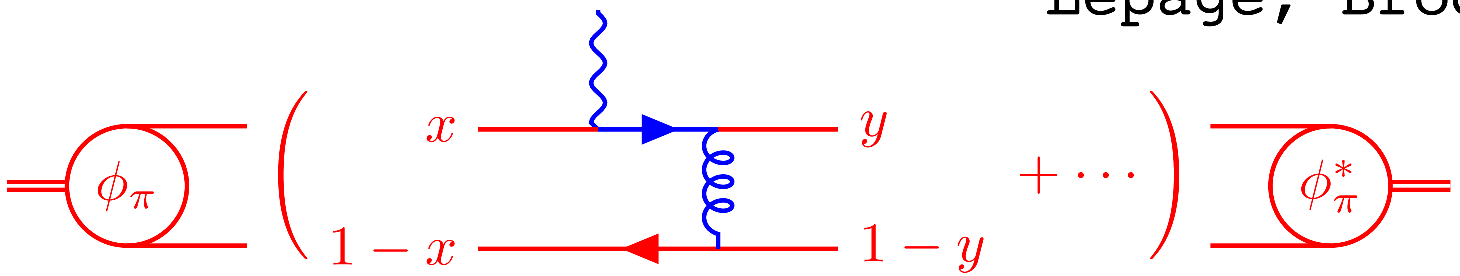
small Q^2 ($=-q^2$): direct πe scattering (NA7, 1986, up to 0.26 GeV^2); determine rms radius of electric charge distn. Many lattice QCD tests of this. e.g. HPQCD, 1511.07382

large Q^2 : use electroproduction on a proton. Done up to Q^2= 2.5 GeV^2 . Key expt for JLAB upgrade: extend to 6 GeV^2. Pert. QCD prediction at very high Q^2. Lattice QCD?



Perturbative QCD at very high Q^2: high mom. photon accompanied by high mom. gluon. Hard scattering factorises from 'distribution amplitude' for $q\bar{q}$ in meson.

Lepage, Brodsky 1979



Asymptotic prediction: for meson M

$$Q^2 \to \infty \qquad F_M = \frac{8\pi\alpha_s f_M^2}{Q^2}$$

form factor

decay constant

For lower Q^2:

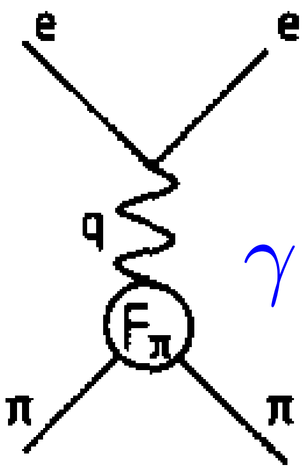
$$F_M(Q^2) = \frac{8\pi\alpha_s(Q/2) f_M^2}{Q^2} \left| 1 + \sum_{n=2}^{\infty} a_n^M(Q/2) \right|^2$$

Higher twist corrns: addnl powers of m^2/Q^2

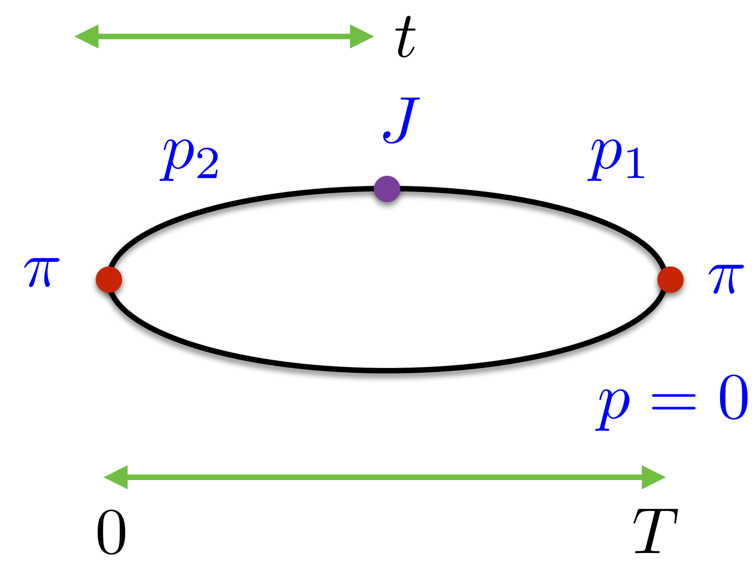
even n for 'symmetric' meson

coeffs of Gegenbauer poly. , run to 0 at high Q^2

Lattice QCD calculation: '3-point functions' needed for form-factors



e.g. for pion to pion transition via vector current J



Need to calculate correlators for multiple T values and 0<t<T and fit as a function of t, T simultaneously with 2pt.

$$C_{3pt} = \sum_{i,j} b_i J_{ij} b_j e^{-E_i t} e^{-E_j (T-t)}$$

$$\langle \pi | V_\mu | \pi \rangle / (2Z\sqrt{E_i E_j})$$

Normln of J must be fixed, here $f_+(0) = 1$ from charge cons.

use Breit frame

$$\vec{p}_1 = -\vec{p}_2$$

to maximise Q^2 for a given (pa). Signal/noise degrades and syst. disc. errors grow with pa.

How to do lattice QCD ff. calculation at high Q^2?

J. Koponen et al, HPQCD, 1701.04250 *updated*

Need a formalism with small discretisation errors that is numerically fast. *HISQ*

Studying η_s and η_c is faster than π and K and with smaller stat. errors, so enabling behaviour to be mapped out to higher Q^2.

Need to test relationship of form factors to decay constants for a range of mesons.

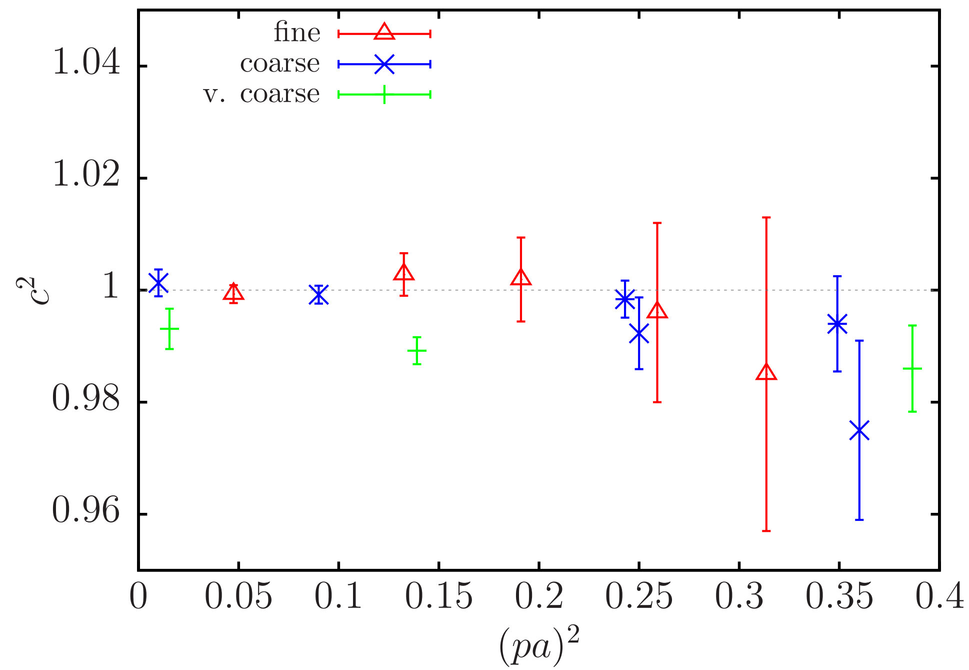
Also need to test relationship of form factors for different mesons containing same struck quarks e.g. K and η_s

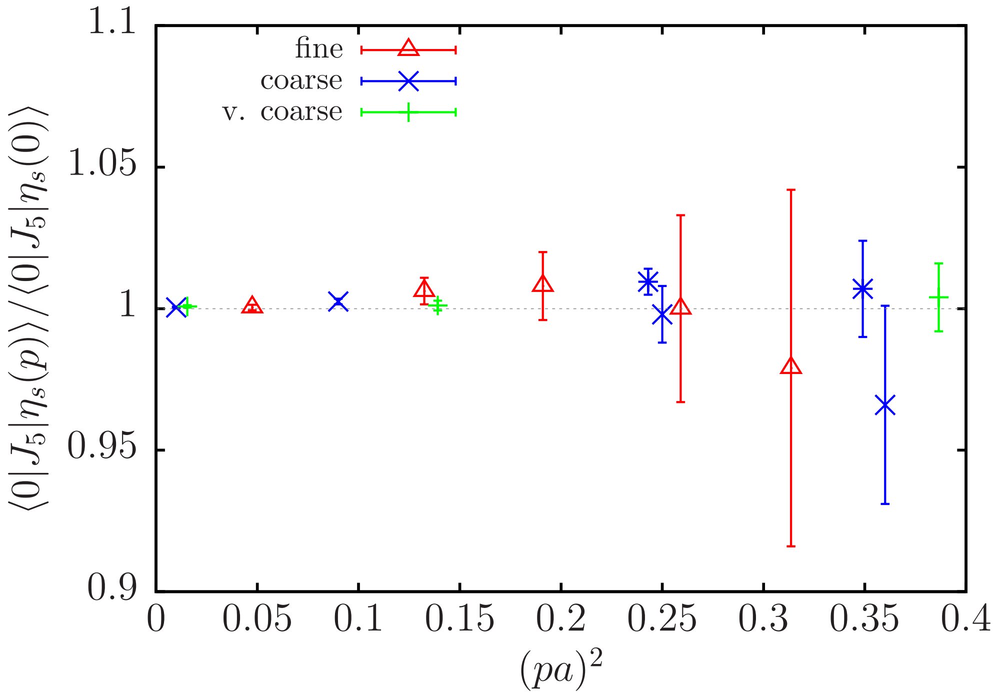
Can also compare form factors for different currents (e.g scalar) vs expectations from perturbative QCD helicity rules

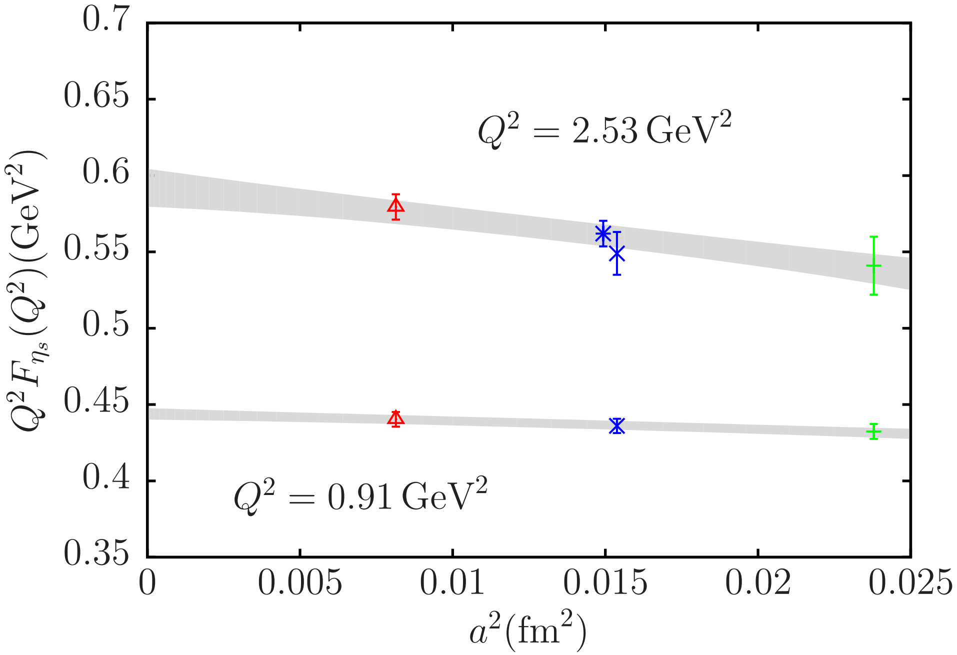
Results for η_s HPQCD, 1701.04250

3 values of a (0.15fm to 0.09fm) + 2 $m_{u/d}^{sea}$ ($m_s/5$ and $m_s/10$)

Use 'twisted boundary conditions' to insert momentum and test discretisation errors as a function of (pa).
Stat. errors are key issue.

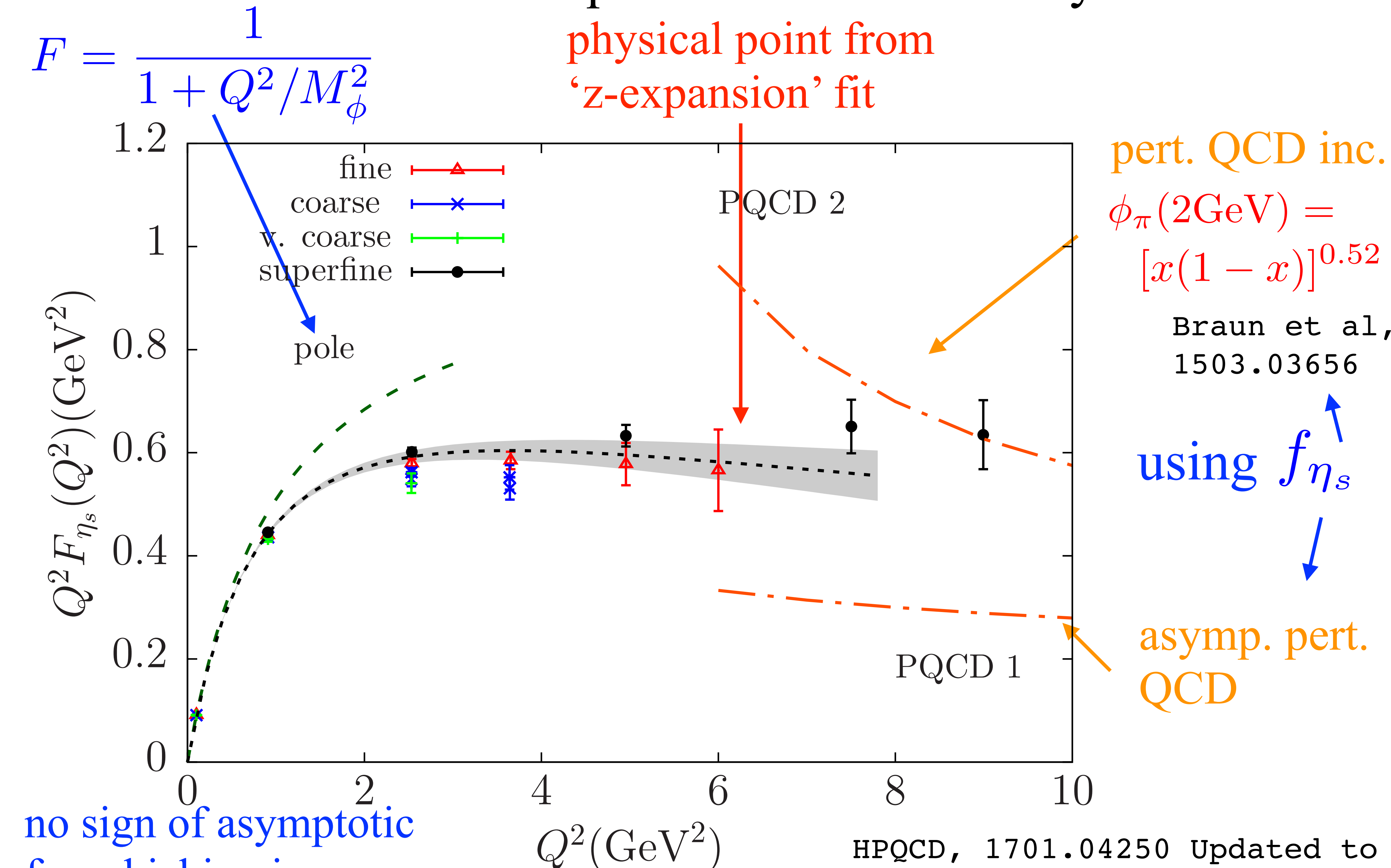






Results - Can reach Q^2 of 6 GeV^2 with small stat. errors

η_s Disc. and sea quark mass effects very small



no sign of asymptotic form kicking in …

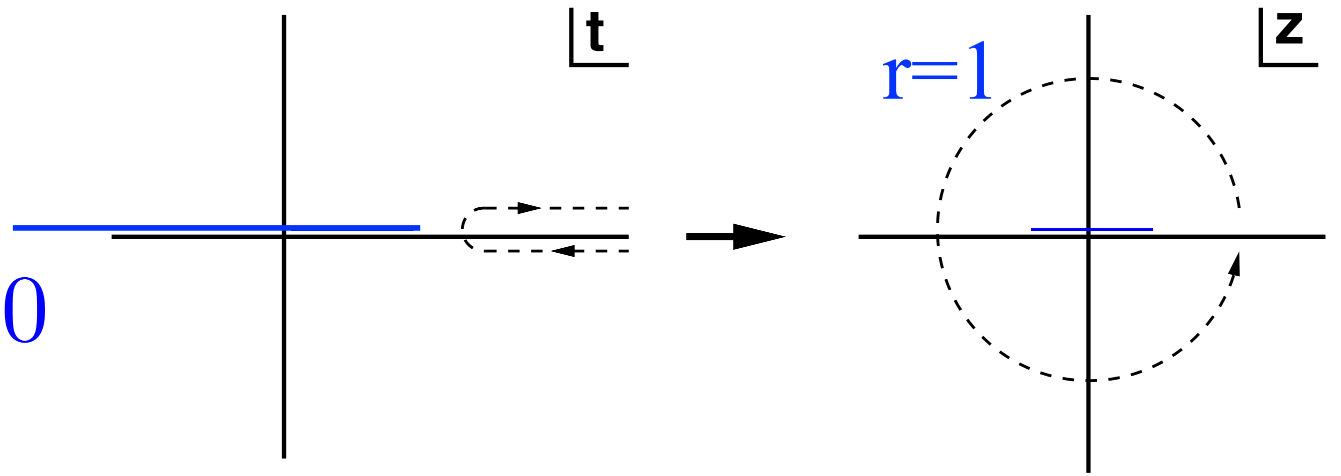
HPQCD, 1701.04250 Updated to add superfine points z-expansion

$$z(t, t_{cut}) = \frac{\sqrt{t_{cut} - t} - \sqrt{t_{cut}}}{\sqrt{t_{cut} - t} + \sqrt{t_{cut}}}$$

$$t = q^2$$

$$t_{cut} = 4M_K^2$$

Maps t region into
-1 < z < 1



$$q^2 = 0 \to z = 0$$

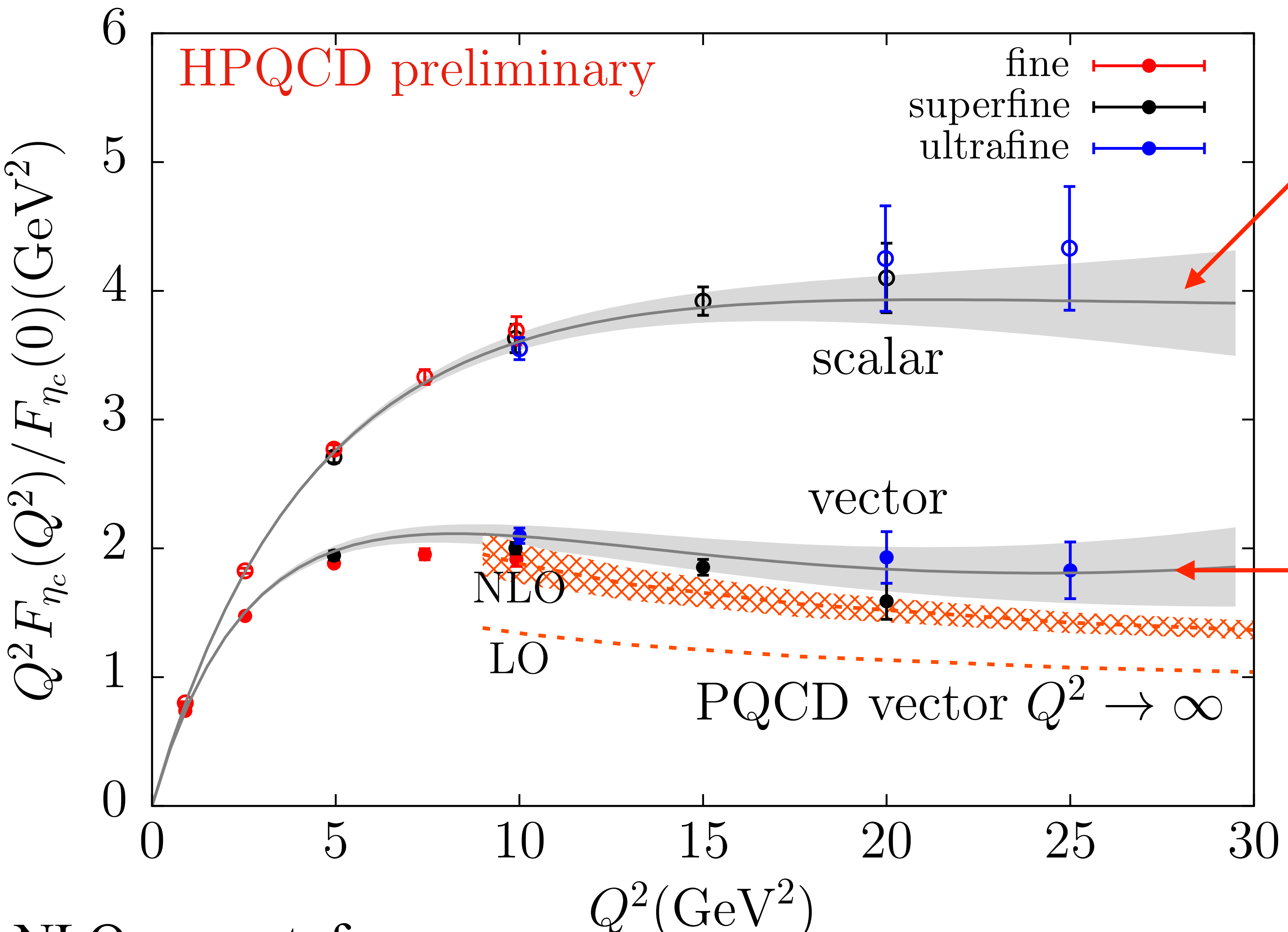
Fit:

$$(1 + Q^2/M_\phi^2)F = 1 + \sum_i z^i A_i \left[1 + B_i(a\Lambda)^i + C_i(a\Lambda)^4 + D_i \frac{\delta m_{sea}}{10}\right]$$

$\Lambda = 1\,\mathrm{GeV}$ no z-independent disc. errors by defn.

Higher Q^2 is possible for heavier quarks - try η_c

HPQCD, LATTICE2018, 1902.03808



pert. QCD : scalar ff should fall more rapidly than $1/Q^2$ - not true

vector ff closer to PQCD at NLO but agreement still not good.

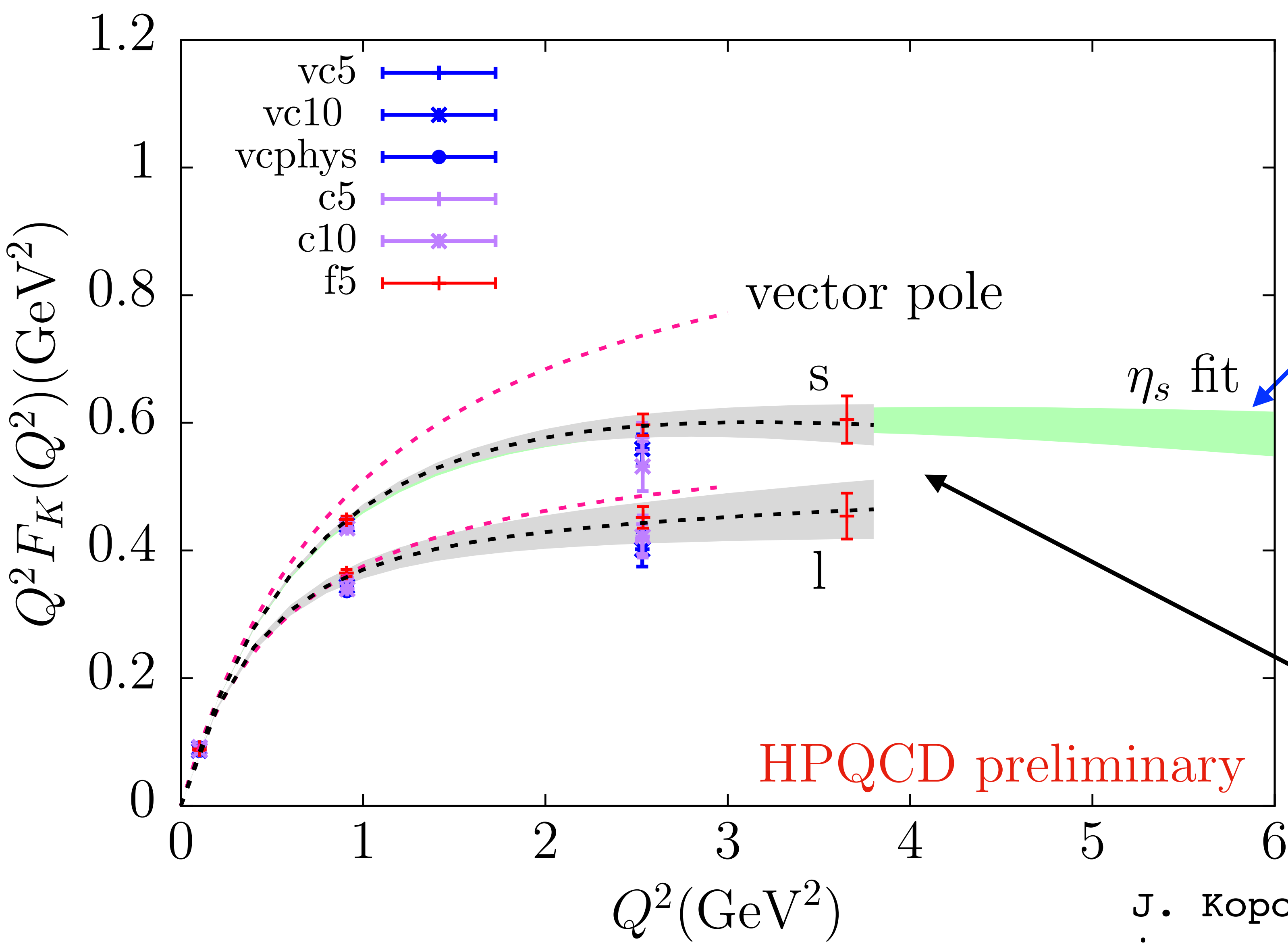
NLO asympt. form :

$8\pi f^2 \alpha_s(Q/2)[1 + 1.18\alpha_s + \ldots]$

Melic et al, hep-ph/9802204

Electromagnetic form factors of the K from lattice QCD

For K must allow for both strange and light (u/d) current interacting with photon - they are quite different



not expected from pert. QCD

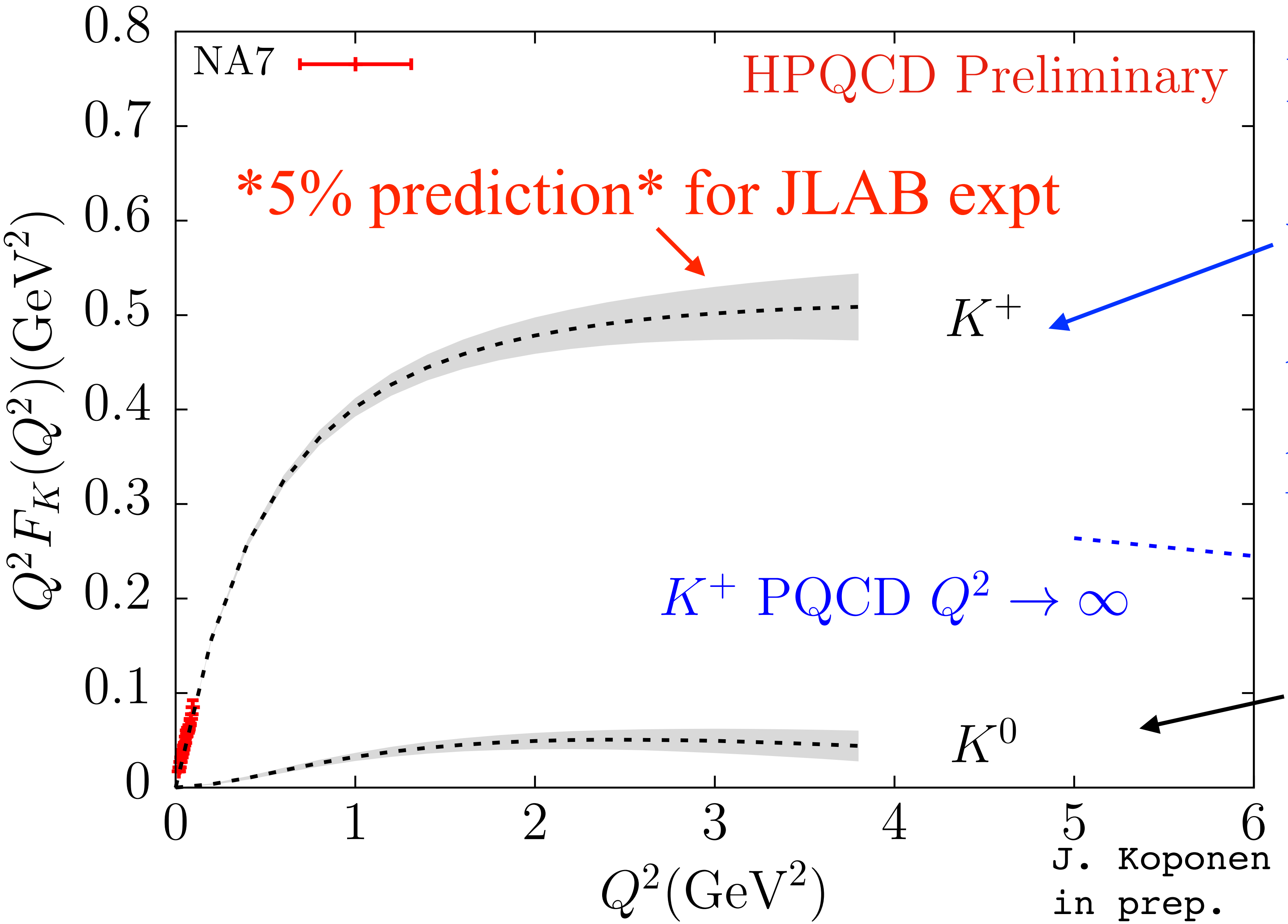
s current: K and η_s agree - spectator irrelevant

physical (a=0, mu/d=phys) result from fit

J. Koponen et al, HPQCD, in progress

Electromagnetic form factors of the K from lattice QCD

K+ form factor is electric charge weighted combination 2/3* s + 1/3*u - first lattice QCD calculation of this



F not changing with f as expected by asympt. pert. QCD

K^0 = s - d - nonzero

J. Koponen et al, HPQCD, in prep.

www.physics.gla.ac.uk/HPQCD

Conclusion

HPQCD

• Lattice QCD gives very accurate map of meson decay constants now. New 1% test against experiment for J/ψ.

• Lattice QCD calculation of K^+ space-like electro-magnetic form factor underway to make predictions for JLAB expt. - can reach $Q^2=4$ GeV^2 with 5% accuracy.

• Lattice QCD can test behaviour of meson electro-magnetic form factors - predictions from asymptotic pert. QCD do NOT work well.

• Q^2F is flat at large Q^2 (25 GeV^2) but > asymptotic value .

• F depends on struck quark - independent of spectator.

• F does not depend on f in the expected way.

• helicity rules do not seem to work for scalar current.

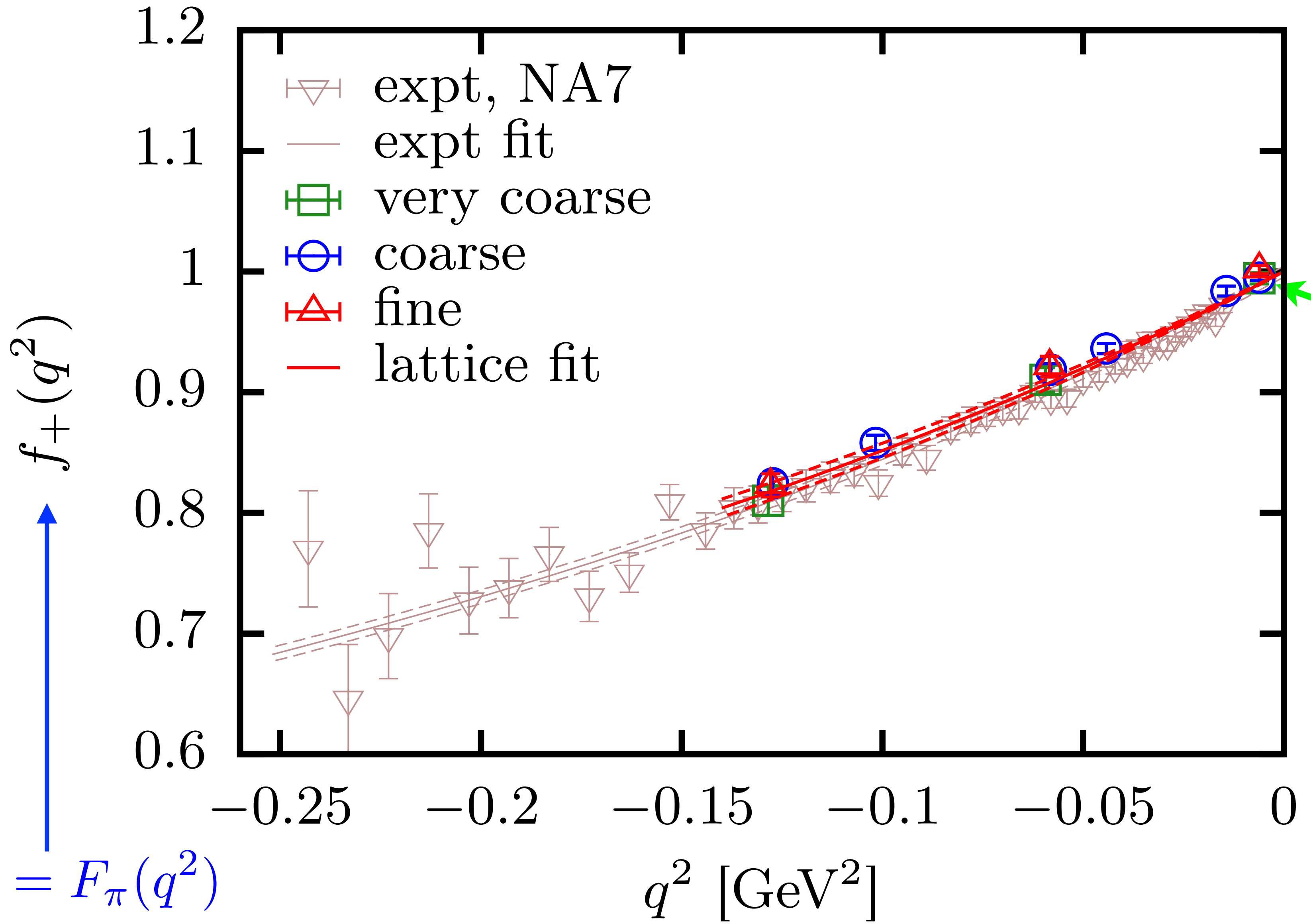
• rethink needed?

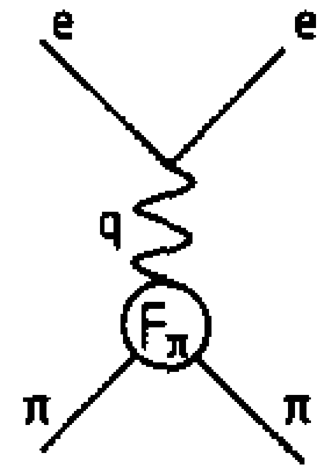
Spares

π electromagnetic form factor at small q^2

$\pi - e$ scattering probes π electric charge distn

$$\langle \pi(p_1)|V_\mu|\pi(p_2)\rangle = f_+(q^2)(p_1+p_2)_\mu$$

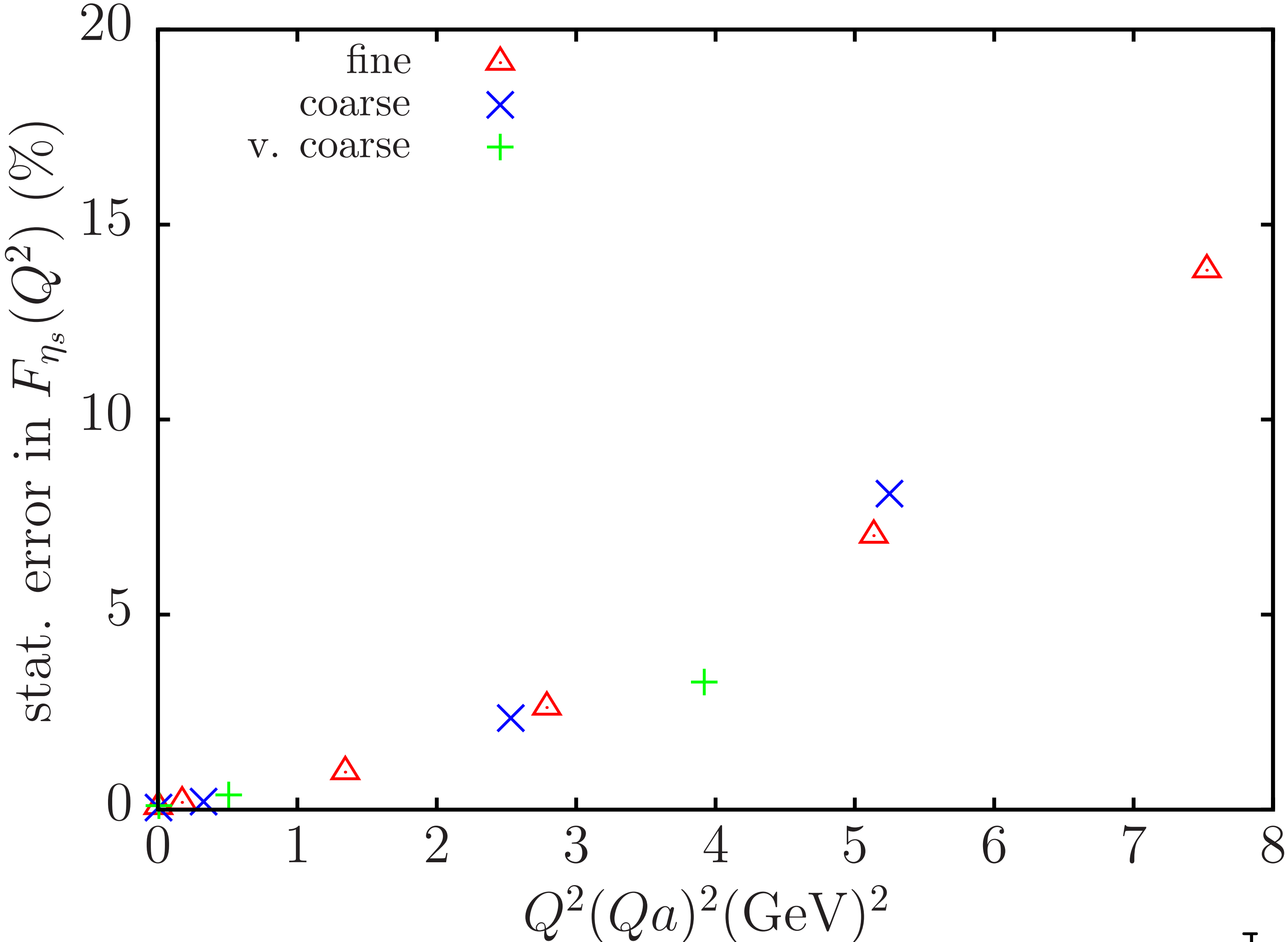




Working at **physical** u/d quark masses on HISQ 2+1+1 configs, lattice QCD raw results on top of experiment

J. Koponen et al, HPQCD, 1511.07382

Stat errors on form factor as a function of Q²



J. Koponen et al, HPQCD, 1701.04250.

Comparison of F/f² for different mesons

Pert. QCD expects the asymptotic value to be the same for all - $8\pi\alpha_s(Q/2)$