Strange nucleon form factors from Lattice QCD

Dalibor Djukanovic, Konstantin Ottnad, Harvey Meyer,
Georg von Hippel, **Jonas Wilhelm**, Hartmut Wittig

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ
JG|U
SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL
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Standard Model precision test

- weak mixing angle Θ_W [1]

$$A_{PV} = \frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} \left\{ Q_W(p) - F(E_i, Q^2) \right\}$$
$$Q_W(p) = 1 - 4\sin^2\Theta_w$$

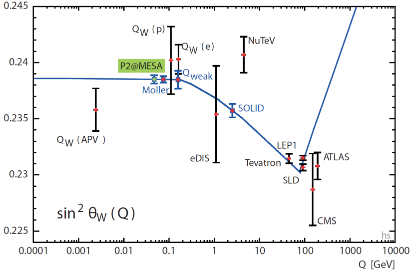
$\rightarrow$ $F(E_i, Q^2)$ contains G_E^s, G_M^s and G_A^s

- $Q_W(p)$ sensitive to Θ_W and possible beyond SM effects [1]

$$\frac{\Delta \sin^2\Theta_W}{\sin^2\Theta_W} \approx 0.09 \, \frac{\Delta Q_W(p)}{Q_W(p)}$$

[1] Becker et al., Eur. Phys. J. A54 (2018) 208

Standard Model precision test



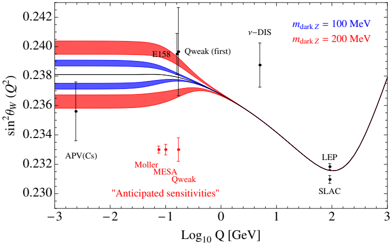


Figure: Running of $\sin^2 \Theta_W$ [1] (left) and influence of Dark Z [2] (right)

[1] Berger et al., JUSTC 46 (2016) 481

[2] Davoudiasl et al., Phys. Rev. D89 (2014) 095006

expectation values

- importance-sampling Monte Carlo

$$\langle O \rangle \approx \frac{1}{N_{\mathrm{cfg}}} \sum_{i=1}^{N_{\mathrm{cfg}}} O[D[U_i]^{-1}, U_i]$$

- generation of ensembles

$$dP(U) = \frac{1}{Z} e^{-S_{QCD}^{Lat.}[U]} \mathcal{D}[U]$$

- Markov Chain $\rightarrow$ HMC algorithm [1]

$$U_1 \rightarrow U_2 \rightarrow U_3 \rightarrow \cdots$$

[1] Duane et al., Phys. Lett. B195 (1987) 216

ensembles

- **C**oordinated **L**attice **S**imulations (CLS)[1,2]
  - $N_f = 2 + 1$ $\mathcal{O}(a)$-improved Wilson fermions
  - tree-level improved Lüscher-Weisz gauge action
  - open boundary conditions in time
  - $\mathrm{tr}\ M_q = \text{const}$

	β	a [fm]	$N_s^3 \times N_t$	m_π[MeV]	m_K[MeV]
H102	3.40	0.08636	$32^3 \times 96$	352	438
H105	3.40	0.08636	$32^3 \times 96$	278	460
C101	3.40	0.08636	$48^3 \times 96$	223	472
N401	3.46	0.07634	$48^3 \times 128$	289	462
N203	3.55	0.06426	$48^3 \times 128$	345	441
N200	3.55	0.06426	$48^3 \times 128$	283	463
D200	3.55	0.06426	$64^3 \times 128$	200	480
N302	3.70	0.04981	$48^3 \times 128$	354	458

[1] Bruno et al., J. High Energ. Phys. (2015) 43
[2] Bruno et al., Phys. Rev. D95 (2017) 074504

primary observables

- nucleon two-point function

$$C_2^N(\vec{p}\,', y_0; \Gamma) = \sum_{\vec{y}} e^{-\mathrm{i}\vec{p}\,'\vec{y}} \; \Gamma_{\beta\alpha} \left\langle N_\alpha(\vec{y}, y_0) \bar{N}_\beta(0) \right\rangle$$

$\rightarrow$ energies (masses at $\vec{p}\,' = 0$) and overlap factors

- nucleon three-point function

$$C_{3,J_\mu}^N(\vec{q}, z_0; \vec{p}\,', y_0; \Gamma_\nu) = \sum_{\vec{y},\vec{z}} e^{\mathrm{i}\vec{q}\vec{z}} e^{-\mathrm{i}\vec{p}\,'\vec{y}} \; (\Gamma_\nu)_{\beta\alpha} \left\langle N_\alpha(\vec{y}, y_0) J_\mu(\vec{z}, z_0) \bar{N}_\beta(0) \right\rangle$$

$\rightarrow$ energies, overlap factors and matrix elements

form factors

- construct a ratio

$$R_{J_\mu}(\vec{q};\vec{p}';\Gamma_\nu) = \frac{C^N_{3,J_\mu}(\vec{q},z_0;\vec{p}',y_0;\Gamma_\nu)}{C^N_2(\vec{p}',y_0)}\sqrt{\frac{C^N_2(\vec{p}',y_0)C^N_2(\vec{p}',z_0)C^N_2(\vec{p}'\text{-}\vec{q},y_0\text{-}z_0)}{C^N_2(\vec{p}'\text{-}\vec{q},y_0)C^N_2(\vec{p}'\text{-}\vec{q},z_0)C^N_2(\vec{p}',y_0\text{-}z_0)}}$$

$$\overset{\text{s.d.}}{=} M^1_{\nu\mu}(\vec{q},\vec{p}')G_1(Q^2) + M^2_{\nu\mu}(\vec{q},\vec{p}')G_2(Q^2) + \text{exc. states}$$

- (overdetermined) system of equations at each Q^2

$$M\,\vec{G} = \vec{R}\ ;\ M = \begin{pmatrix} M^1_1 & M^2_1 \\ \vdots & \vdots \\ M^1_N & M^2_N \end{pmatrix}\ ,\ \vec{G} = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix},\ \vec{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix}$$

- minimizing the least-squares function

$$\chi^2 = (\vec{R} - M\vec{G})^T\ C^{-1}\ (\vec{R} - M\vec{G})$$

form factor parametrization

$$\left\langle N, \vec{p}, s \left| J_\mu(x) \right| N, \vec{p}', s' \right\rangle = \bar{u}^s(\vec{p}) \ \widetilde{J}_\mu \ u^{s'}(\vec{p}') e^{\mathrm{i}q\cdot x}$$

- axial vector current

$$\widetilde{A}_\mu = \gamma_\mu \gamma_5 G_\mathrm{A}(Q^2) + \gamma_5 \frac{q_\mu}{2m} G_\mathrm{P}(Q^2)$$

- vector current

$$\widetilde{V}_\mu = \gamma_\mu F_1(Q^2) + \mathrm{i}\sigma_{\mu\nu} \frac{q^\nu}{2m} F_2(Q^2)$$

$$F_1, F_2 \leftrightarrow G_\mathrm{E}, G_\mathrm{M}$$

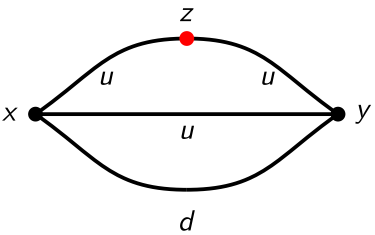
quark-flavor dependence

$$\left\langle N(\vec{y}, y_0) J_\mu^f(\vec{z}, z_0) \bar{N}(\vec{0}, 0) \right\rangle \ , \ f = u, d, s$$

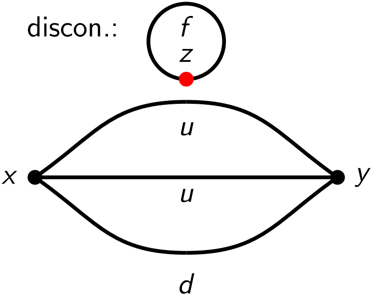
- current operator

$$J_\mu^f(\vec{z}, z_0) = \begin{cases} V_\mu^f(\vec{z}, z_0) = \bar{f}(\vec{z}, z_0)\gamma_\mu f(\vec{z}, z_0) \\ A_\mu^f(\vec{z}, z_0) = \bar{f}(\vec{z}, z_0)\gamma_5\gamma_\mu f(\vec{z}, z_0) \end{cases}$$

connected:



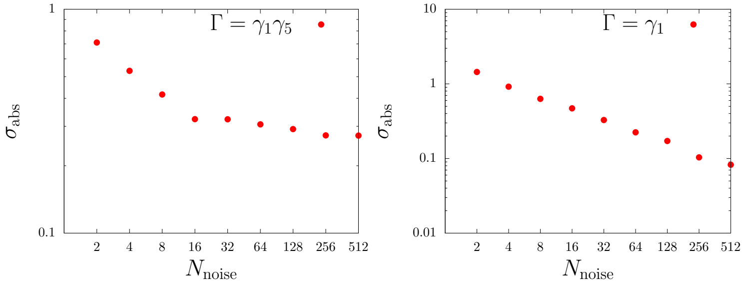
discon.:



quark loops

$$L^s_{\Gamma_\mu}(\vec{q}, z_0) = -\sum_{\vec{z}\in\Lambda} e^{i\vec{q}\cdot\vec{z}} \ \langle tr\,[S^s(z;z)\ \Gamma_\mu]\rangle_G$$
$$= -\sum_{\vec{z}\in\Lambda} e^{i\vec{q}\cdot\vec{z}} \ \left\langle \eta^\dagger(z)\ \Gamma_\mu\ s^s(z)\right\rangle_{G,\eta}$$

- $U(1)$ noise: $\eta(x)^a_\alpha = e^{\mathrm{i}\phi}$, $\phi \in [0, 2\pi)$

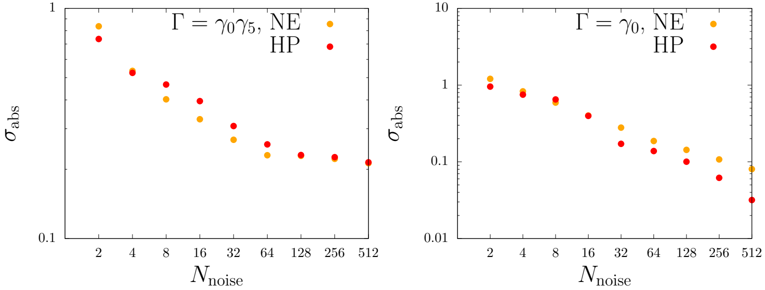


- gauge noise reached for axial current, but not for vector current!

effectiveness of hierarchical probing

- hierarchical probing [1]

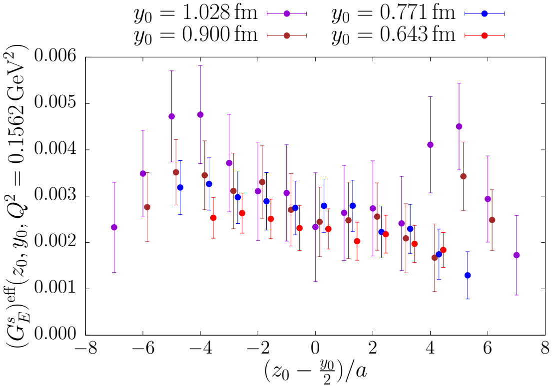
$$\eta_n \to h_n \odot \eta$$



[1] Stathopoulos et al., 1302.4018

excited-state contamination

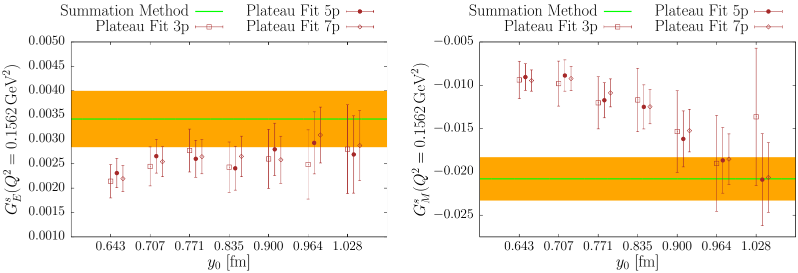
$$G^{\text{eff}}(Q^2, z_0, y_0) = G(Q^2) + A_1(Q^2)\ e^{-\Delta z_0} + B_1(Q^2)\ e^{-\Delta'(y_0 - z_0)} \\ + C_1(Q^2)\ e^{-\Delta z_0 - \Delta'(y_0 - z_0)}$$



excited-state contamination

- plateau method $y_0 \sim 1\,\mathrm{fm}$
- summation method $y_0 \in [0.5, 1.3]$ fm

$$S(Q^2, y_0) = \sum_{z_0=1}^{y_0-1} G^{\mathrm{eff}}(Q^2, z_0, y_0)$$
$$= K(Q^2) + (y_0 - 1)G(Q^2) + \mathcal{O}\left(e^{-\Delta y_0}, e^{-\Delta' y_0}\right)$$



z-expansion

- fit the Q^2-dependence [1,2]

$$G(Q^2) = \sum_{k=0}^{\infty} a_k z(Q^2)^k$$

$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut}}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut}}} \quad , \quad t_{cut} = (2m_K)^2$$

- Gaussian priors

$$\tilde{a}_k = 0 \pm 5 \max\{|a_0|, |a_1|\} \;\forall\; k > 1$$

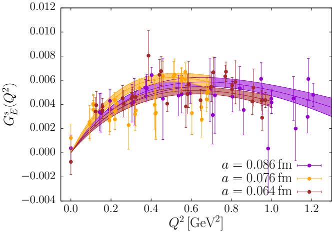
- at $Q^2 = 0$: $a_0^E = 0$, $a_0^M = \mu^s$, $a_0^A = g_A^s$
- charge radius

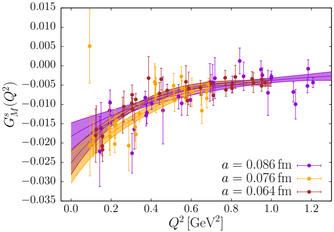
$$r^2 \equiv -6 \left. \frac{dG}{dQ^2} \right|_{Q^2=0} = -\frac{3}{2t_{cut}} a_1$$

[1] R. Hill et al., Phys. Rev. D82 (2010) 113005
[2] Z. Epstein et al., Phys. Rev. D90 (2014) 074027

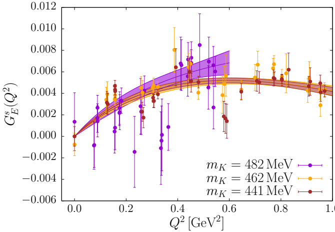
strange vector form factors

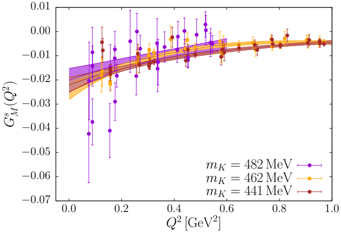
- $m_K \approx 460\,\mathrm{MeV}$





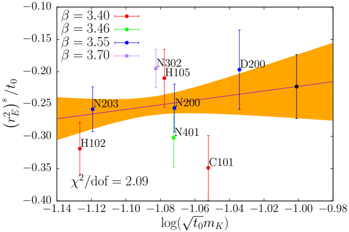
- $\beta = 3.55$

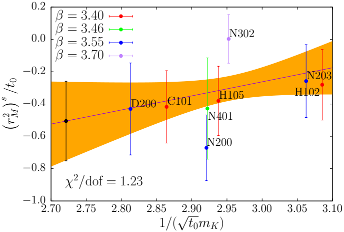


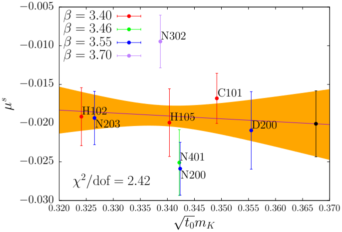


extrapolation

- SU(3) HBChPT [1]







$$(r_E^2)^s(m_K) = (r^2)_E^s + a_1 \log(m_K)$$
$$\mu^s(m_K) = \mu^s + a_2 m_K$$
$$(r_M^2)^s(m_K) = (r^2)_M^s + a_3/m_K$$

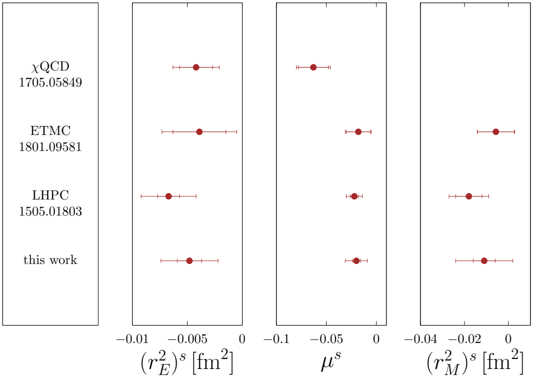
[1] T. R. Hemmert et al., Phys. Rev. C60 (1999) 045501

error budget

	$(r_E^2)^s$ [fm^2]	μ^s	$(r_M^2)^s$ [fm^2]	χ^2/DOF
Main result	-0.0048(11)	-0.020(4)	-0.011(5)	2.09, 2.42, 1.23
Variations:				
Doubling prior width	-0.0059(13)	-0.023(5)	-0.017(9)	1.98, 1.57, 0.75
Plateau method	-0.0038(11)	-0.011(5)	-0.0033(48)	1.91, 1.57, 1.27
Including $\mathcal{O}(a^2)$	-0.0030(13)	-0.016(6)	-0.006(7)	1.34, 2.71, 1.07
No cut in Q^2	-0.0047(8)	-0.017(4)	-0.008(5)	3.71, 2.37, 1.73
Including finite vol.	-0.0051(11)	-0.019(4)	-0.012(5)	1.34, 1.28, 0.42

$$(r_E^2)^s = -0.0048(11)(24)\,\mathrm{fm}^2$$
$$\mu^s = -0.020(4)(11)$$
$$(r_M^2)^s = -0.011(5)(12)\,\mathrm{fm}^2$$

comparison



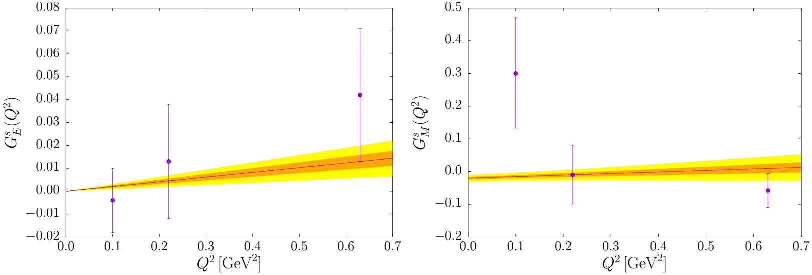
$$(r_E^2)^s = -0.0048(11)(24)\,\mathrm{fm}^2$$
$$\mu^s = -0.020(4)(11)$$
$$(r_M^2)^s = -0.011(5)(12)\,\mathrm{fm}^2$$

comparison

- with experimental determination [1]
- lattice results:

$$G_E^s(Q^2) = -\frac{1}{6}(r_E^2)^s Q^2$$

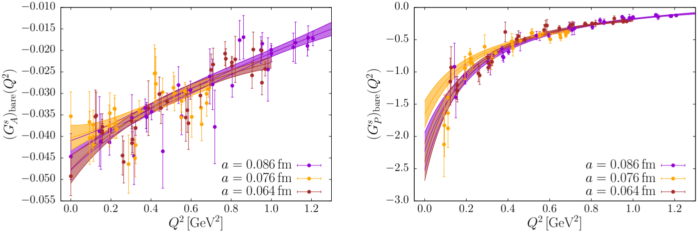
$$G_M^s(Q^2) = \mu^s - \frac{1}{6}(r_E^2)^s Q^2$$



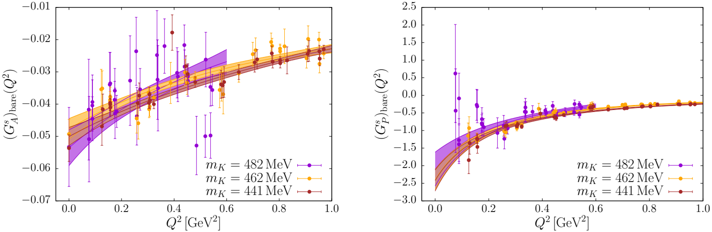
[1] F.E. Maas and K.D. Paschke, Prog. Part. Nucl. Phys. 95 (2017) 209

strange axial vector form factors

- $m_K \approx 460\,\mathrm{MeV}$

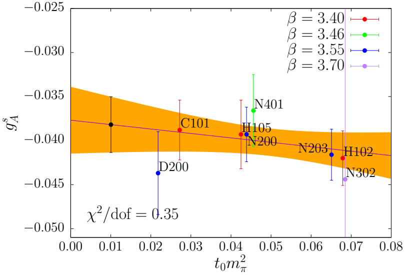


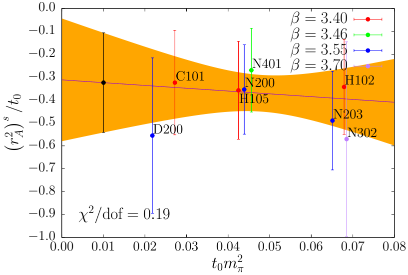
- $\beta = 3.55$



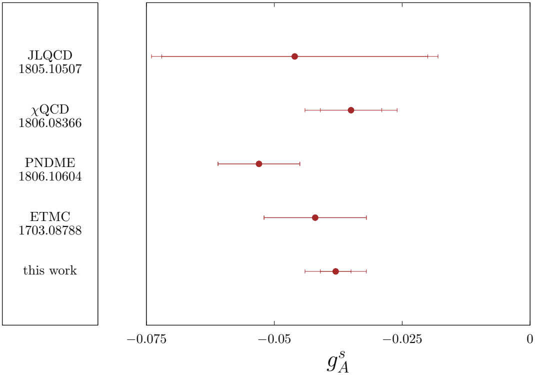
extrapolation

- linear in m_π^2





comparison



$$g_A^s = -0.038(3)(5)$$
$$(r^2)_A^s = -0.007(5)(9)\,\mathrm{fm}^2$$

outlook and related work

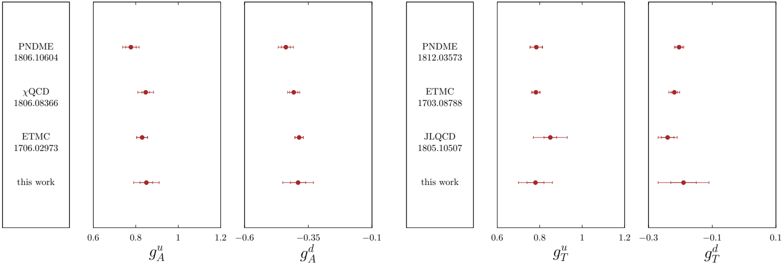
- more configurations and ensembles
- new generation of disconnected loop in progress
- electric and magnetic charge radius and magnetic moment of proton and neutron

$$(r^2)^{p/n} = \frac{1}{2}\left[\frac{1}{3}(r^2)^{u+d-2s} \pm (r^2)^{u-d}\right]$$

$$\mu^{p/n} = \frac{1}{2}\left[\frac{1}{3}\mu^{u+d-2s} \pm \mu^{u-d}\right]$$

outlook and related work

- quark-contributions to g_A and g_T



$$g_A^u = 0.85(3)(5) \qquad g_T^u = 0.78(4)(6)$$

$$g_A^d = -0.39(3)(5) \qquad g_T^d = -0.19(4)(6)$$

Thank you!

Backup Slides

finite-volume corrections

- vector case [1,2]

$$(r_E^2)^s(m_K, a, L) = c_1 + c_2 \log(m_K) + c_3 a^2 + c_4 \sqrt{L}\, e^{-m_K L}$$

$$\mu^s(m_K, a, L) = c_5 + c_6 m_K + c_7 a^2 + c_8 m_K \left(1 - \frac{2}{m_K L}\right) e^{-m_K L}$$

$$(r_M^2)^s(m_K, a, L) = c_9 + \frac{c_{10}}{m_K} + c_{11} a^2 + c_{12} \sqrt{L}\, e^{-m_K L}$$

- axial vector case [3]

$$g_A^s(m_\pi, a, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 \frac{m_\pi^2}{\sqrt{m_\pi L}}\, e^{-m_\pi L}$$

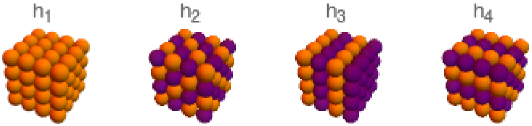
$$(r_A^2)^s(m_\pi, a, L) = c_5 + c_6 m_\pi^2 + c_7 a + c_8 \sqrt{L}\, e^{-m_\pi L}$$

[1] T. R. Hemmert et al., Phys. Rev. C60 (1999) 045501
[2] Phys. Rev. D 96, 114504 (2017)
[3] K. Ottnad et al., Phys. Rev. D100 (2019) 034513

Hadamard Vectors

$$h_j = H_n(:, j) \quad , \quad j = 0, ..., n-1 \ .$$

$$H_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad , \quad H_{2n} = \begin{pmatrix} H_n & H_n \\ H_n & -H_n \end{pmatrix} = H_2 \otimes H_n \quad .$$



renormalization

- $N_f = 3$ periodic boundary condition ensembles
  - $\beta \in \{3.40, 3.46, 3.55\}$
- RI′-MOM scheme in Landau gauge

$$Z_O \left.\langle p \,|O_\Gamma|\, p\rangle\right|_{p^2=\mu^2} = \left.\langle p \,|O_\Gamma|\, p\rangle_{\mathrm{tree}}\right|_{p^2=\mu^2} \quad .$$

  - extrapolation to the chiral limit
  - perturbative subtraction of leading-order lattice artifacts [1]
- conversion to RGI scheme
  - $\overline{\mathrm{MS}}$ as intermediate scheme
  - $\overline{\mathrm{MS}}$ β- and γ-functions
  - fit residual μ-dependence
- convert to $\overline{\mathrm{MS}}$ at $\mu = 2.0\,\mathrm{GeV}$

[1] G. von Hippel et al., PoS (LATTICE2016) 194

renormalization

- start with flavor-diagonal basis

$$O_\Gamma^a(x) = \bar{\psi}(x)\Gamma\lambda^a\psi(x) \quad , \quad \psi = (u,d,s)^T \quad , \quad a \in \{3,8,0\} \ ,$$

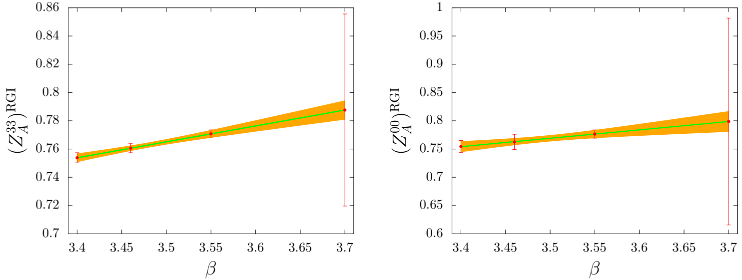
$$Z_\Gamma = \left(\begin{array}{ccc} Z_\Gamma^{33} & 0 & 0 \\ 0 & Z_\Gamma^{88} & 0 \\ 0 & 0 & Z_\Gamma^{00} \end{array}\right) \quad , \quad Z_\Gamma^{33} = Z_\Gamma^{88} \quad , \quad Z_q^i = Z_q \ .$$

- basis transformation

$$\left(\begin{array}{c} O_\Gamma^{u-d}(x)_R \\ O_\Gamma^{u+d}(x)_R \\ O_\Gamma^{s}(x)_R \end{array}\right) = \left(\begin{array}{ccc} Z_\Gamma^{u-d,u-d} & 0 & 0 \\ 0 & Z_\Gamma^{u+d,u+d} & Z_\Gamma^{u+d,s} \\ 0 & Z_\Gamma^{s,u+d} & Z_\Gamma^{s,s} \end{array}\right) \left(\begin{array}{c} O_\Gamma^{u-d}(x) \\ O_\Gamma^{u+d}(x) \\ O_\Gamma^{s}(x) \end{array}\right)$$

renormalization

- linear extrapolation to $\beta = 3.7$
  - error multiplied with factor 10
  - checks performed for g_A^{u-d} in Mainz isovector charges paper [1]
- $\Rightarrow$ consistent results compared to using Z_A^{SF}



[1] K. Ottnad et al., Phys. Rev. D100 (2019) 034513