The anomalous magnetic moment of the muon: a bit of past, a bit of present, and a lot of future

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"gee minus two"

The theory of (g_μ-2) is particle physics in a nutshell. It is an interesting, exciting and difficult subject [...] at the cutting edge of current research in particle physics, and any deviation [...] might be interpreted as a signal of an as-yet-unknown new physics.

- A. Vainshtein

IFAE
EXCELENCIA SEVERO OCHOA
BIST Barcelona Institute of Science and Technology
UAB
Universitat Autònoma de Barcelona

Outline

The anomalous magnetic moment of the muon

- gyromagnetic ratio: g-factor

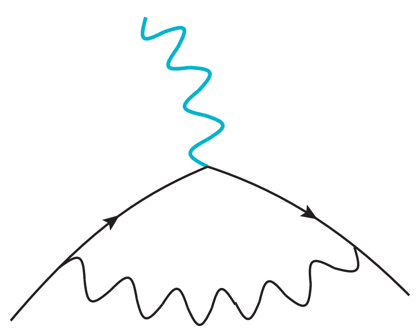
$$\vec{\mu} = g \frac{e}{2m} \cdot \vec{S}$$

spin $\frac{1}{2}$ $\rightarrow$ Dirac theory: $g = 2$
QFT (Rad. Corr): $g \neq 2$

- Deviation from the Dirac value $g = 2$ is:

$$a_\mu = \frac{g_\mu - 2}{2}$$

$$a_\mu^{\rm QED,LO} = \alpha/2\pi \sim 1.16 \times 10^{-3}$$



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- Dirac (g=2) + Experiment confirm → Relativistic theory

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- Experiment ($g \neq 2$, ~0.1%) + theory
- Lamb Shift experiment + theory

} QED $\Downarrow$ Gauge theories (SM)

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} QED ⇓ Gauge theories (SM)

Now:

- 3σ in muon g factor
- proton radius puzzle

} New paradigm: ~~QED~~ ?

- Imagine: 3σ precession perihelion Mercury → ~~RG~~?

The anomalous magnetic moment of the muon: the experiment

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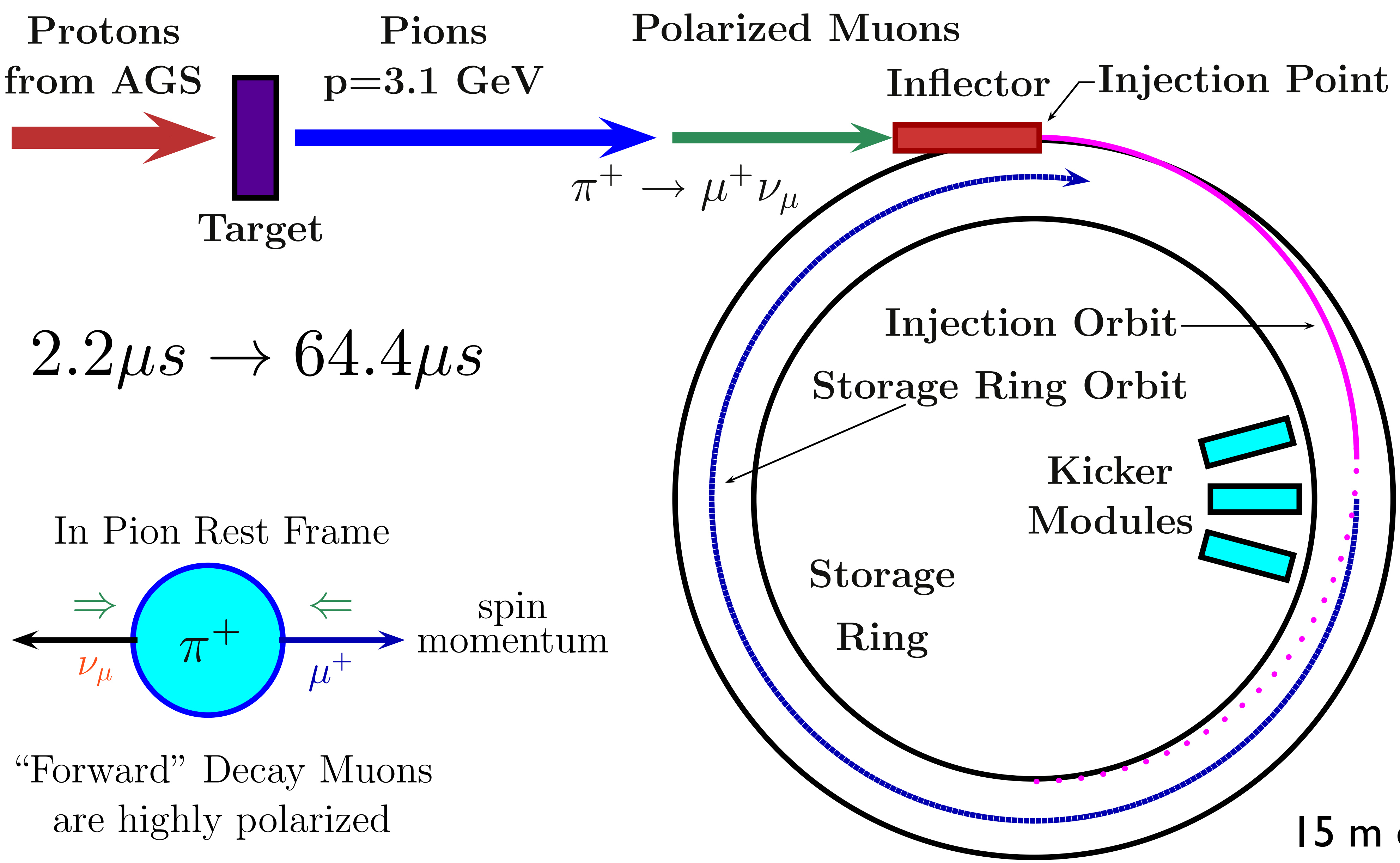


$$2.2\mu s \rightarrow 64.4\mu s$$

15 m diameter

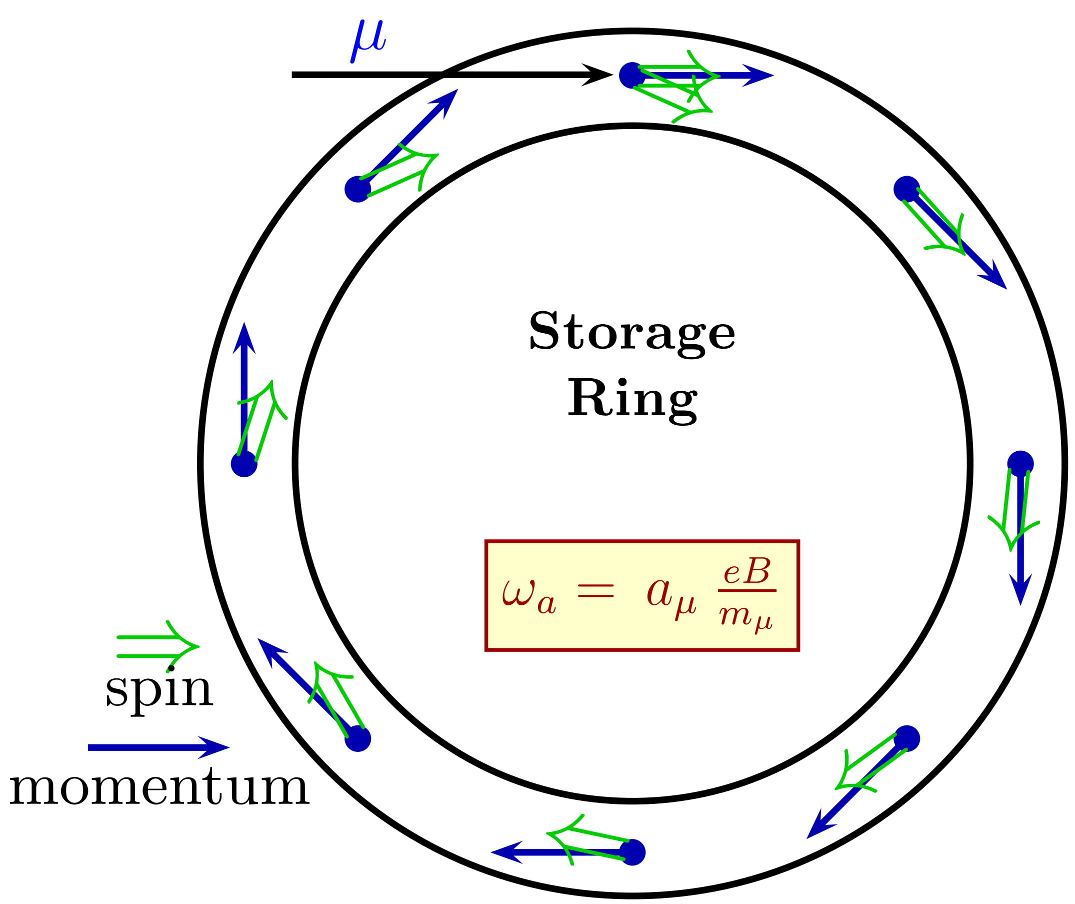
$$1.45T$$

The anomalous magnetic moment of the muon: the experiment

$$\omega_c = \frac{eB}{m\gamma}$$ cyclotron precession

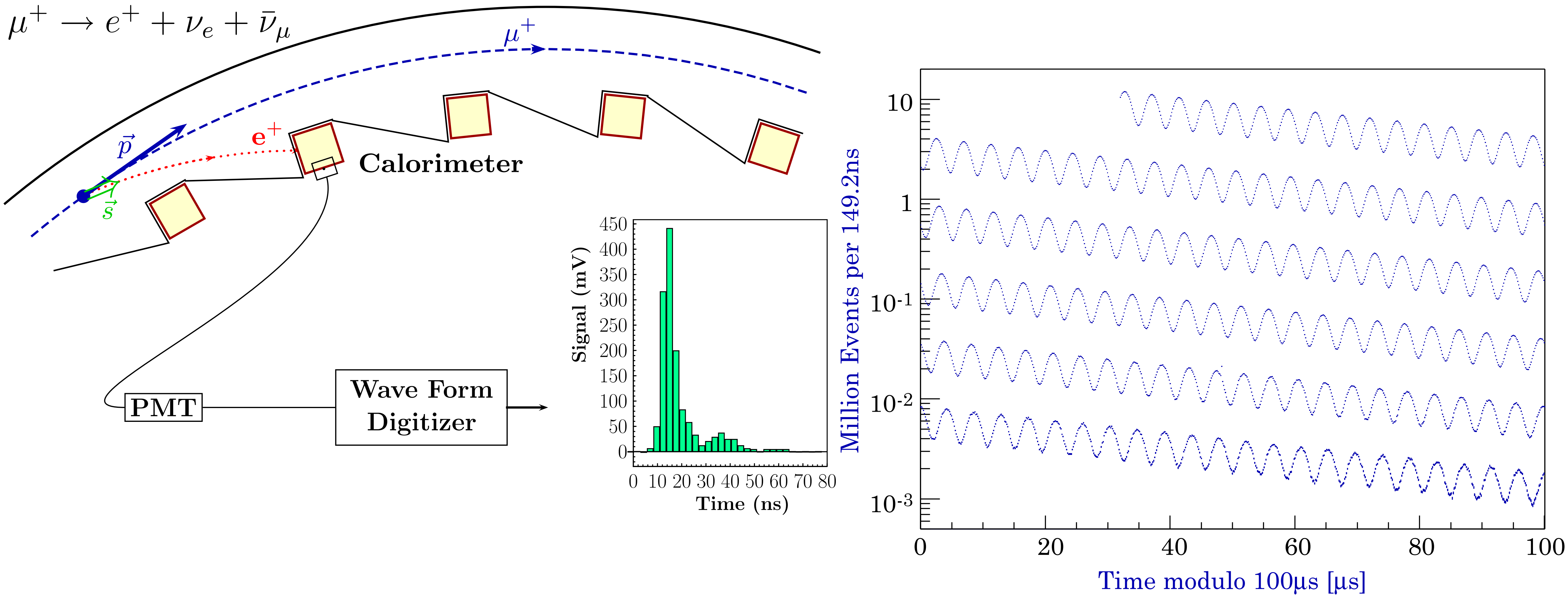
$$\omega_s = \frac{geB}{2m} + (1-\gamma)\frac{eB}{m\gamma}$$

spin precession (Larmor)



$$\omega_a = \omega_s - \omega_c = \left(\frac{g-2}{2}\right)\frac{eB}{m} = a_\mu \frac{eB}{m}$$

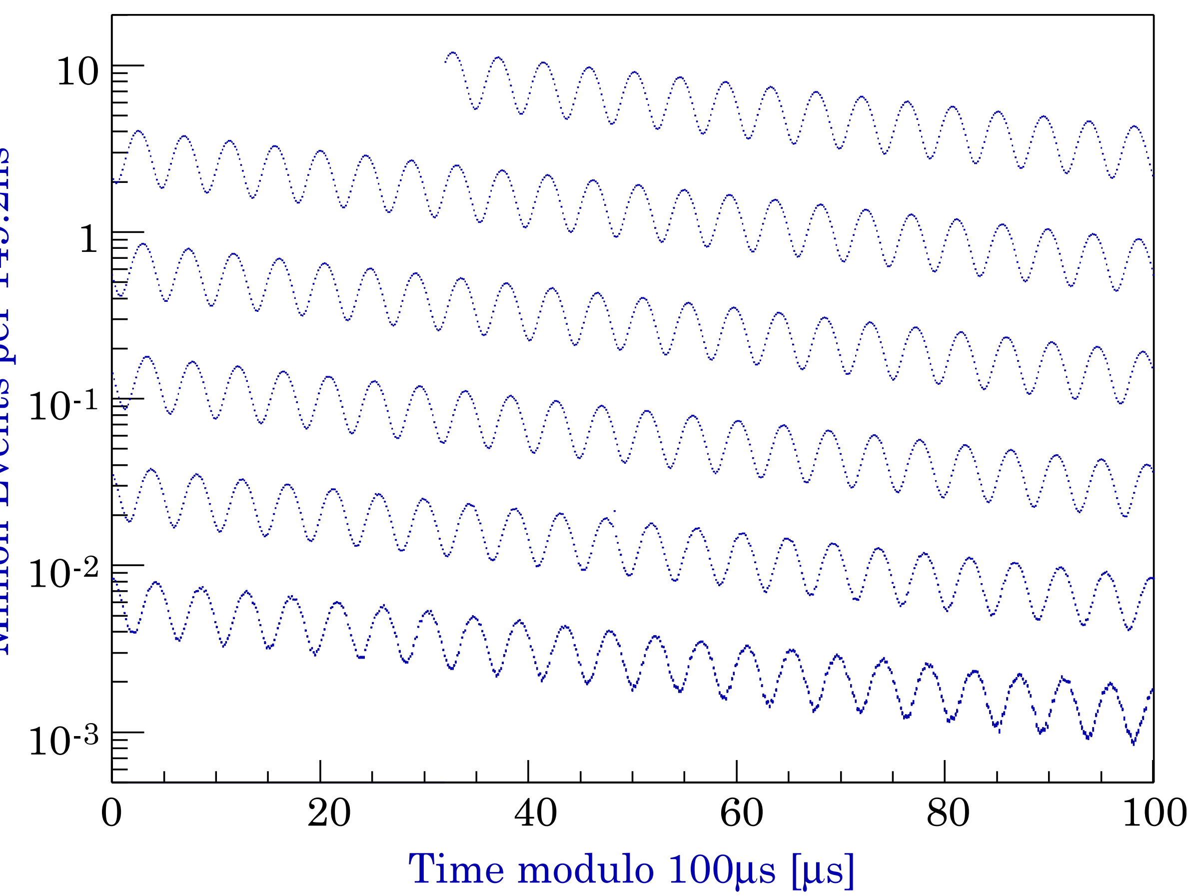
The anomalous magnetic moment of the muon: the experiment



$$N(t) = N_0(E) exp\left(\frac{-t}{\gamma\tau_\mu}\right)\left[1 + A(E)\sin(\omega_a t + \phi(E))\right]$$

The anomalous magnetic moment of the muon: the experiment

$$N(t) = N_0(E) exp\left(\frac{-t}{\gamma\tau_\mu}\right)[1 + A(E)\sin(\omega_a t + \phi(E))] \quad \Rightarrow \quad \omega_a$$



Magnetic field B?

$$a_\mu = \frac{\omega_a/\omega_p}{\lambda_+ - \omega_a/\omega_p} = \frac{R}{\lambda_+ - R}$$

(free-proton precession frequency)

$$R = 0.0037072047(26)$$

$$\lambda_+ = \mu_{\mu^+}/\mu_p = 3.183345137(85)$$

(muonium $\mu^+ e^-$ hyperfine level structure measurements)

The anomalous magnetic moment of the muon

- The E821 experiment at BNL

Bennet et al, PRD73,072003 (2006)

$$a_{\mu+}^{\rm exp} = 11\,659\,204(6)(5) \times 10^{-10}$$ [2000]

$$a_{\mu-}^{\rm exp} = 11\,659\,215(8)(3) \times 10^{-10}$$ [2001]

- Assuming CPT invariance

Bennet et al, PRD73,072003 (2006)

$$a_{\mu}^{\rm exp} = 11\,659\,209.1\,\underbrace{\overset{\text{stat}}{(5.4)}\overset{\text{sys}}{(3.3)}}_{(6.3)} \times 10^{-10}$$

(~1500 citations)

The anomalous magnetic moment of the muon

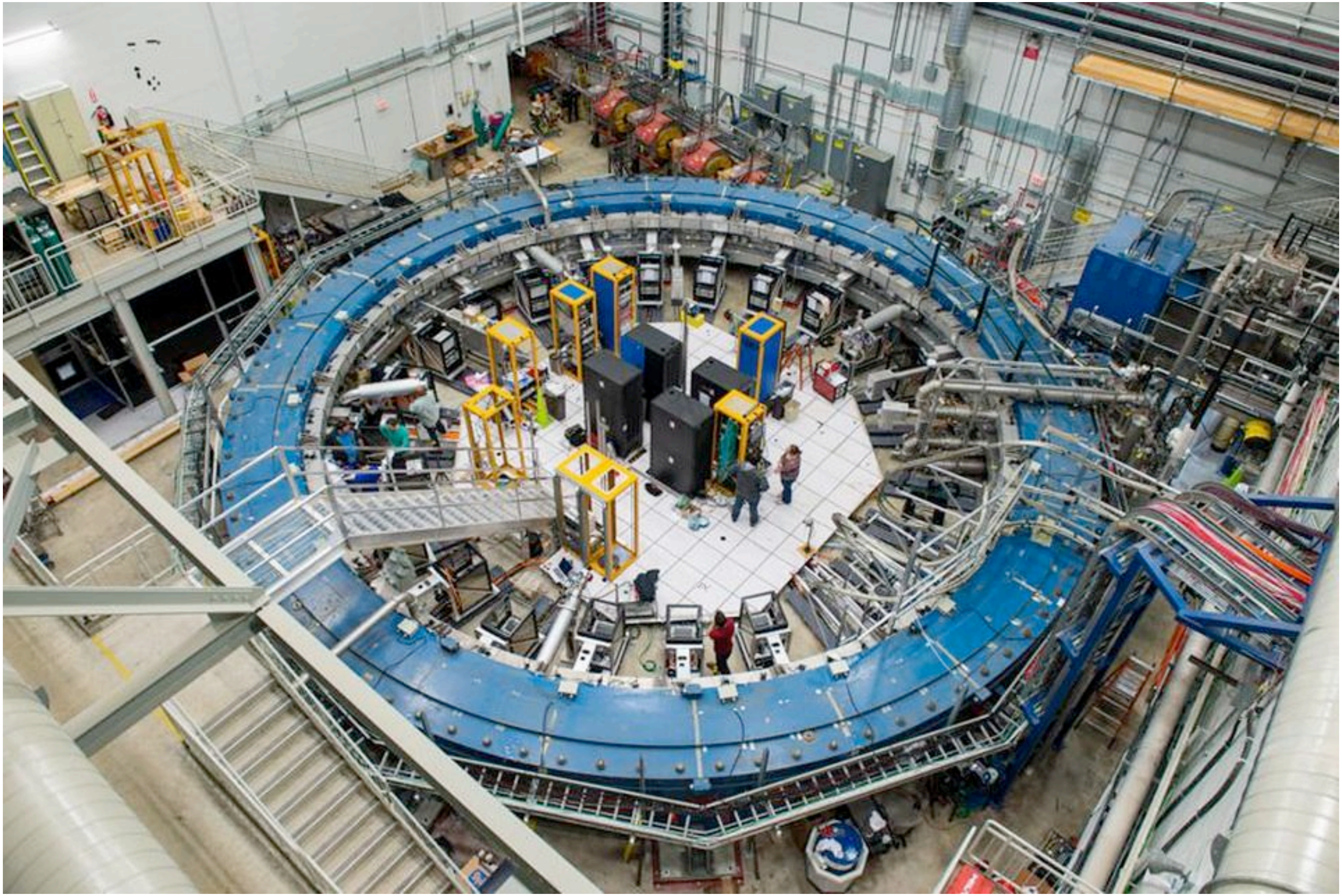
Bennet et al, PRD73,072003 (2006)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11450000(220000)	4300
CERN II	1962–1968	μ^+	11661600(3100)	270
CERN III	1974–1976	μ^+	11659100(110)	10
CERN III	1975–1976	μ^-	11659360(120)	10
BNL	1997	μ^+	11659251(150)	13
BNL	1998	μ^+	11659191(59)	5
BNL	1999	μ^+	11659202(15)	1.3
BNL	2000	μ^+	11659204(9)	0.73
BNL	2001	μ^-	11659214(9)	0.72
Average			11659208.0(6.3)	0.54

Forthcoming exp: FNAL & J-PARC $\sim 1.6 \times 10^{-10}$

http://muon-g-2.fnal.gov/bigmove/gallery.shtml

Muon g-2 Project already calibrated and measuring right now!



The anomalous magnetic moment of the muon: the Standard Model

The anomalous magnetic moment of the muon

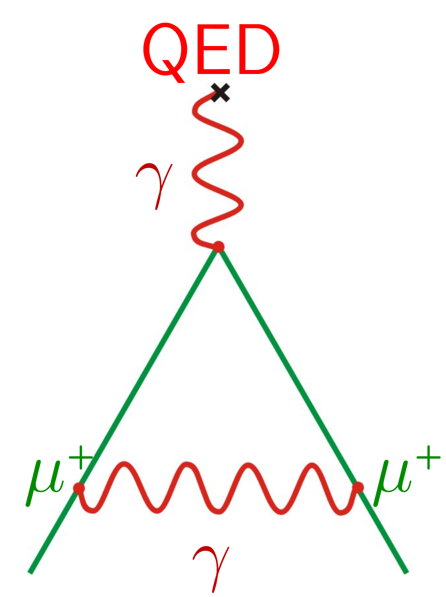
Anomalous magnetic moment a_μ (anomaly):

$$g_\mu = 2\left(1 + a_\mu = \frac{\alpha}{2\pi} + \cdots\right)$$

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Contribution	Result in power of $\frac{\alpha}{\pi}$
$a_\mu^{(2)}$	$0.5\left(\frac{\alpha}{\pi}\right)$

Schwinger 1948



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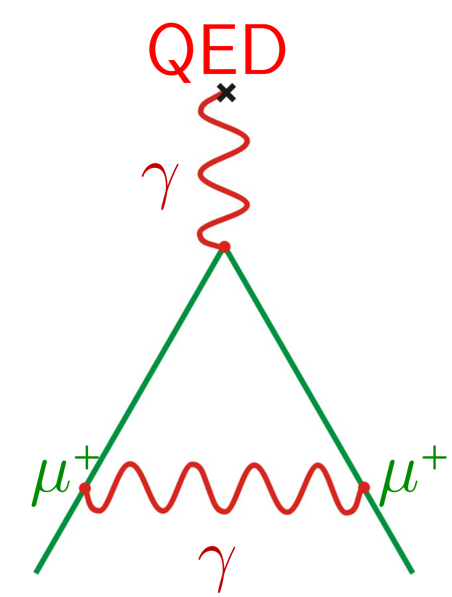
Contribution	Result in power of $\frac{\alpha}{\pi}$
$a_\mu^{(2)}$	$0.5\left(\frac{\alpha}{\pi}\right)$
$a_\mu^{(4)}$	$0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2$
$a_\mu^{(6)}$	$24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3$
$a_\mu^{(8)}$	$130.879\,6(63)\left(\frac{\alpha}{\pi}\right)^4$
$a_\mu^{(10)}$	$753.29(1.04)\left(\frac{\alpha}{\pi}\right)^5$
a_μ^{QED}	$11\,658\,471.885(4) \times 10^{-10}$

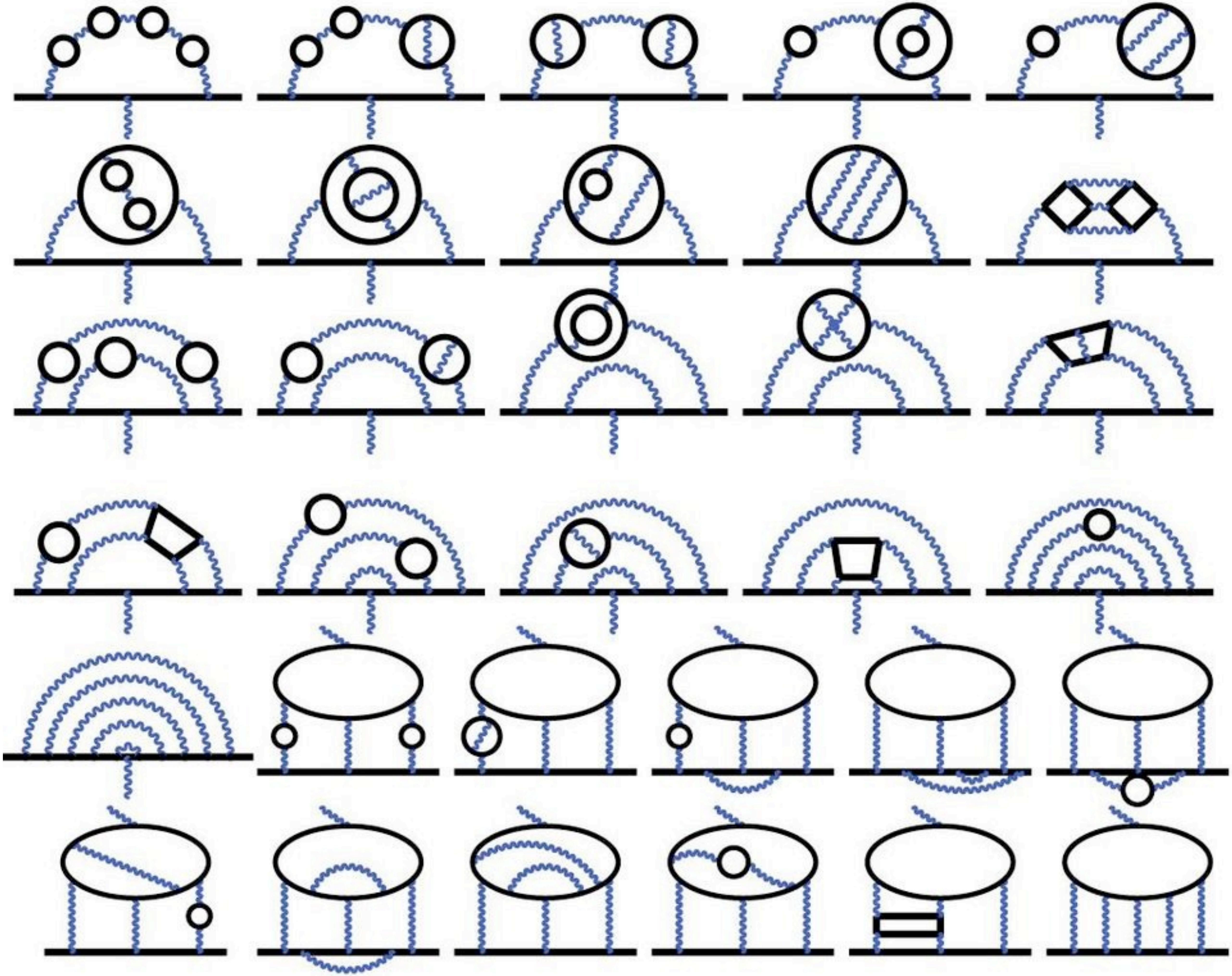
Schwinger 1948

Petermann and Sommerfield 1958

Laporta and Remiddi 1996

Kinoshita *et al* 2012





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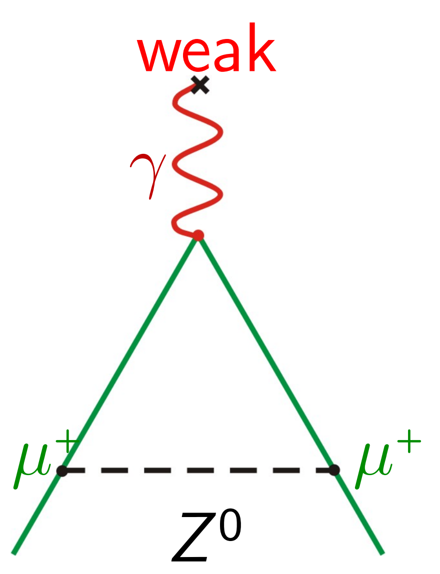
QED(leptons)	11658471.885 ± 0.004	Kinoshita *et al* 2012, Remiddi
HVP(leading order)	690.8 ± 4.7	Davier *et al* 2011
HVP(NLO)	-9.93 ± 0.07	Hagiwara *et al* 2009
HVP(NNLO)	1.22 ± 0.01	Kurz *et al* 2014
HLBL (+NLO)*	11.7 ± 4.0	Jegerlehner, Nyffeler 2009
EW (2 loop)	15.4 ± 0.1	Czarnecki 2003, Gnendinger 2013
Total	11659179.1 ± 6.2	

* NLO: Colangelo et al 2014



$$a_\mu^{(2)\mathrm{EW}}(W) \sim +39 \times 10^{-10}$$

$$a_\mu^{(2)\mathrm{EW}}(Z) \sim -19 \times 10^{-10}$$

$$a_\mu^{(4)\mathrm{EW}} \sim -4 \times 10^{-10}$$

(Higgs ∈ error)

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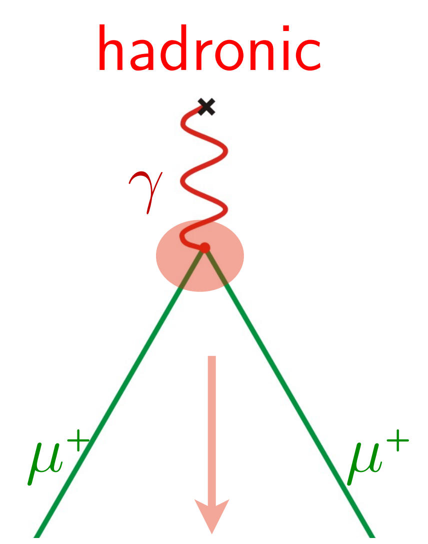

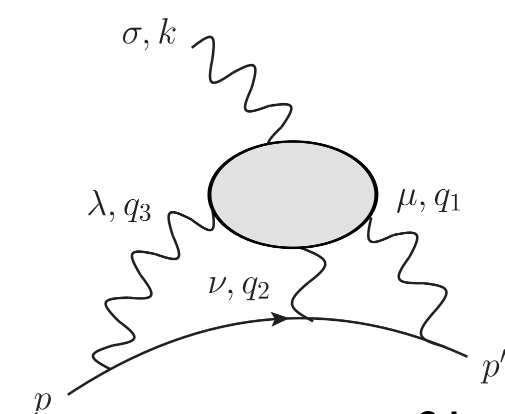

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Forthcoming exp:
FNAL
JPAC

$\sim 1.6 \times 10^{-10}$

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Hints to NP

Very different contributions to a_μ

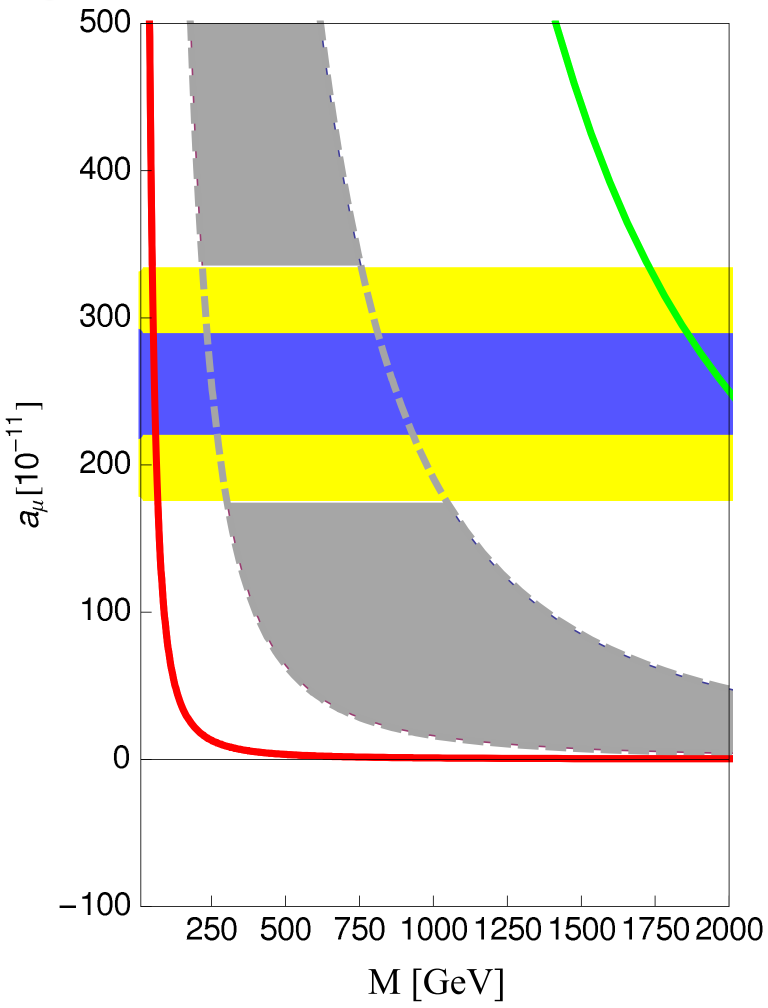
(courtesy by Stöckinger)

[Czarnecki and Marciano '01]

generally: $$C = \frac{\delta m_\mu(\text{N.P.})}{m_\mu}, \quad \delta a_\mu(\text{N.P.}) = \mathcal{O}(C)\left(\frac{m_\mu}{M}\right)^2$$

classify new physics: C **very** model-dependent

Very useful constraints on new physics



$\mathcal{O}(1)$	radiative muon mass generation . . . [Czarnecki,Marciano '01] [Crivellin, Girrbach, Nierste '11][Dobrescu, Fox '10]
$\mathcal{O}(\frac{\alpha}{4\pi}\ldots)$	supersymmetry (tan β), unparticles [Cheung, Keung, Yuan '07] extra dim. (ADD/RS) (n_c). . . [Davioudasl, Hewett, Rizzo '00] [Graesser,'00][Park et al '01][Kim et al '01]
$\mathcal{O}(\frac{\alpha}{4\pi})$	Z', W', UED, Littlest Higgs (LHT). . .

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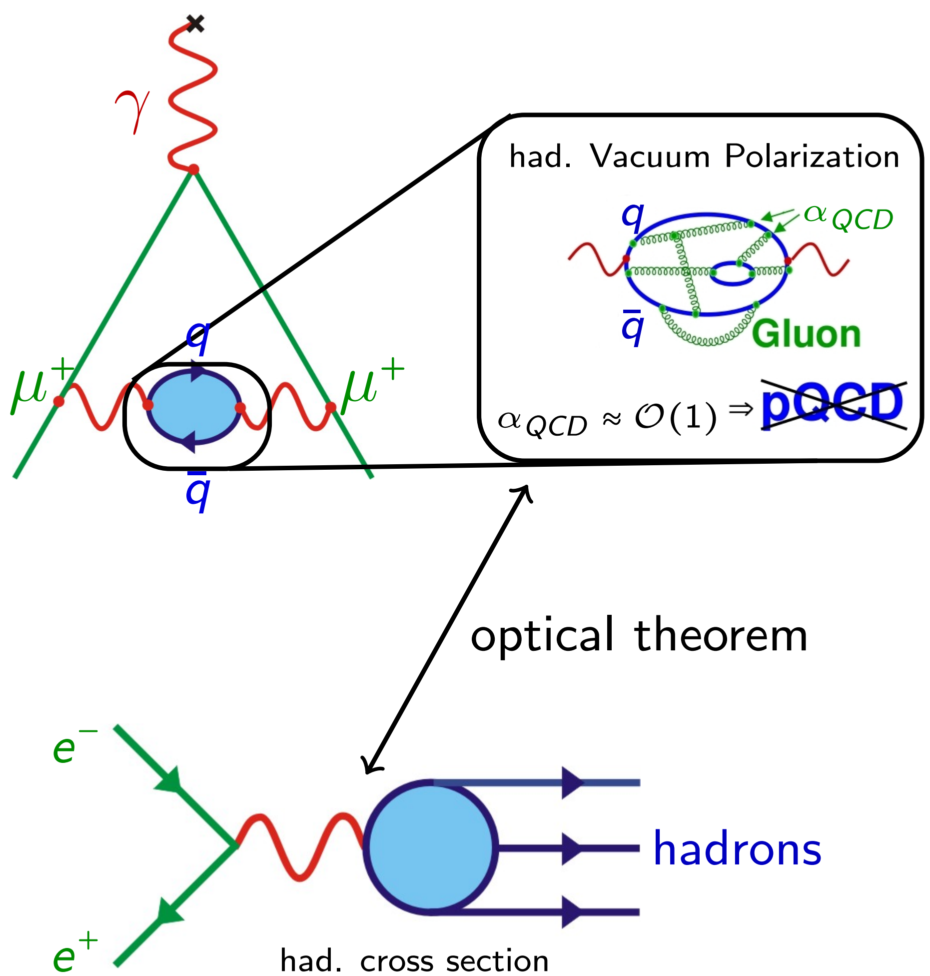
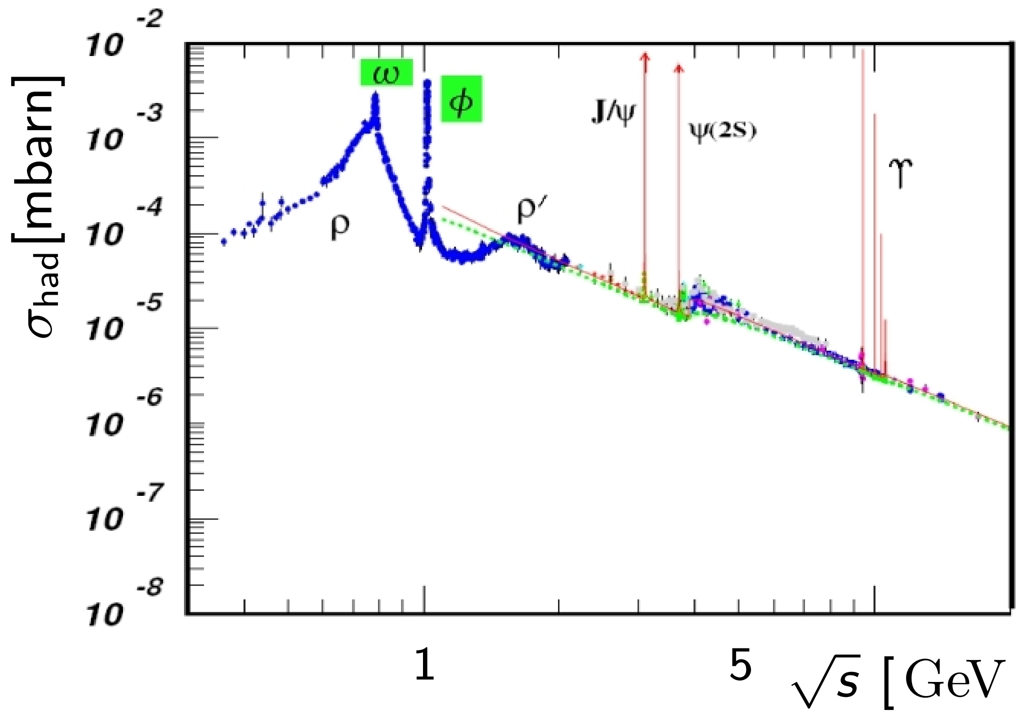
Hadronic Vacuum Polarization





$$a_{\mu,LO}^{\text{had}} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\, \sigma_{\text{had}}(s)$$

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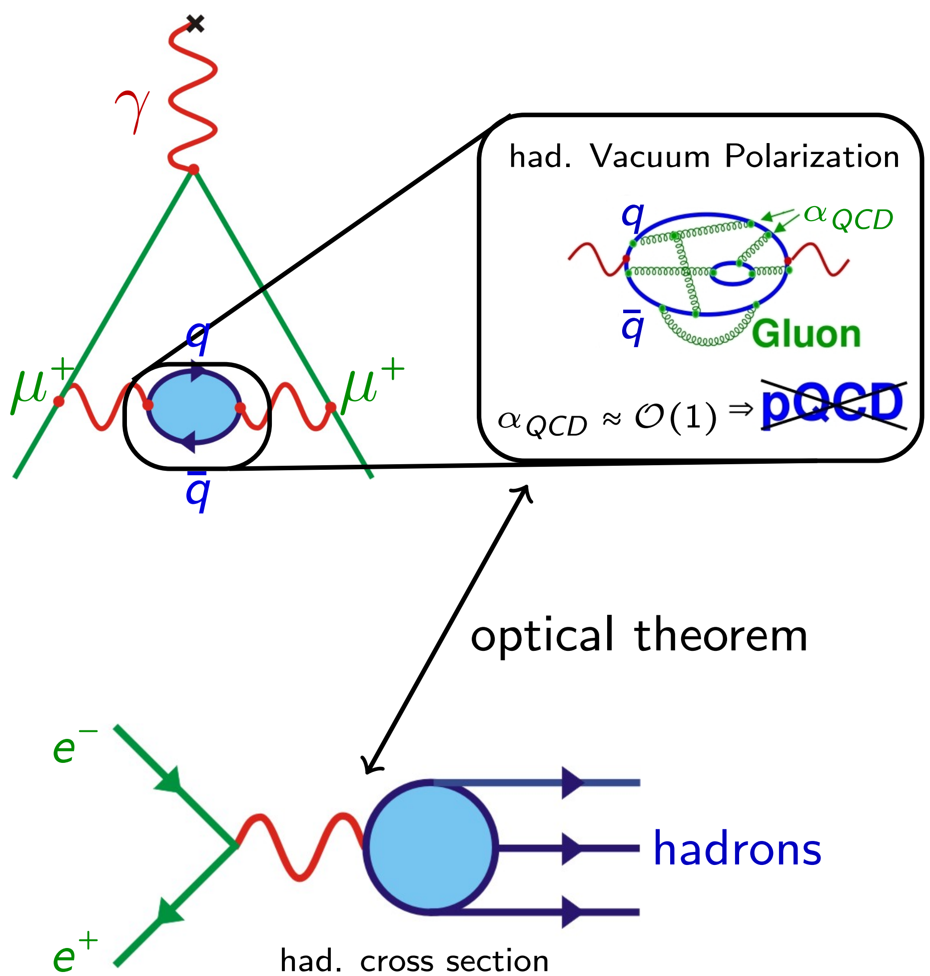

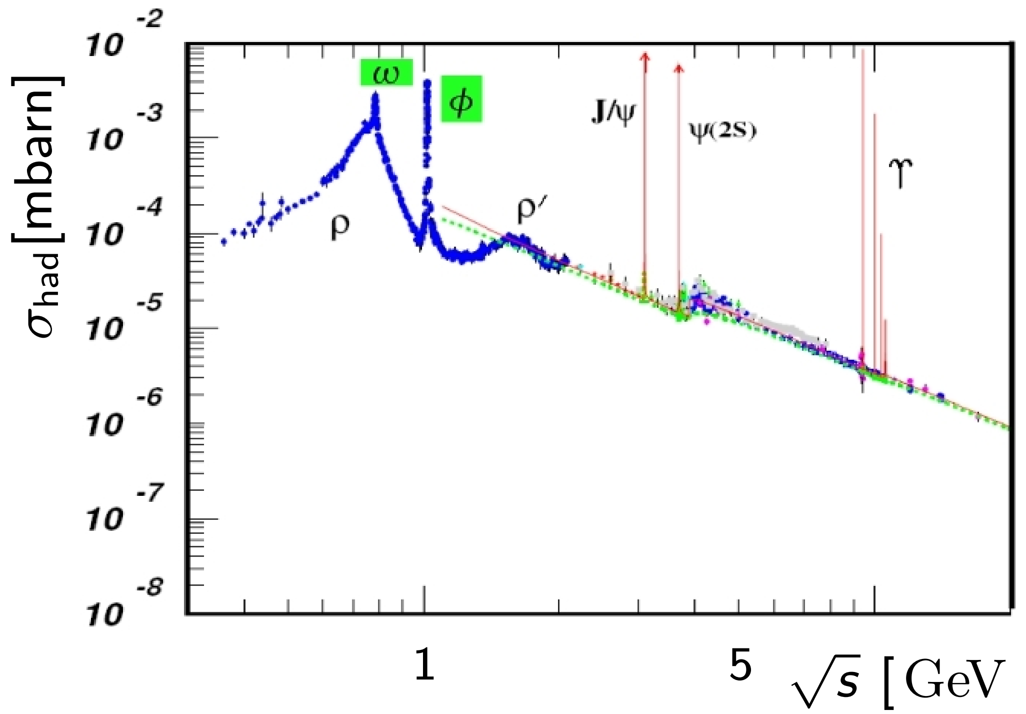

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$\sigma_{\text{had}}(s) \sim 1/s$ & $K(s) \sim 1/s$

$\Downarrow$

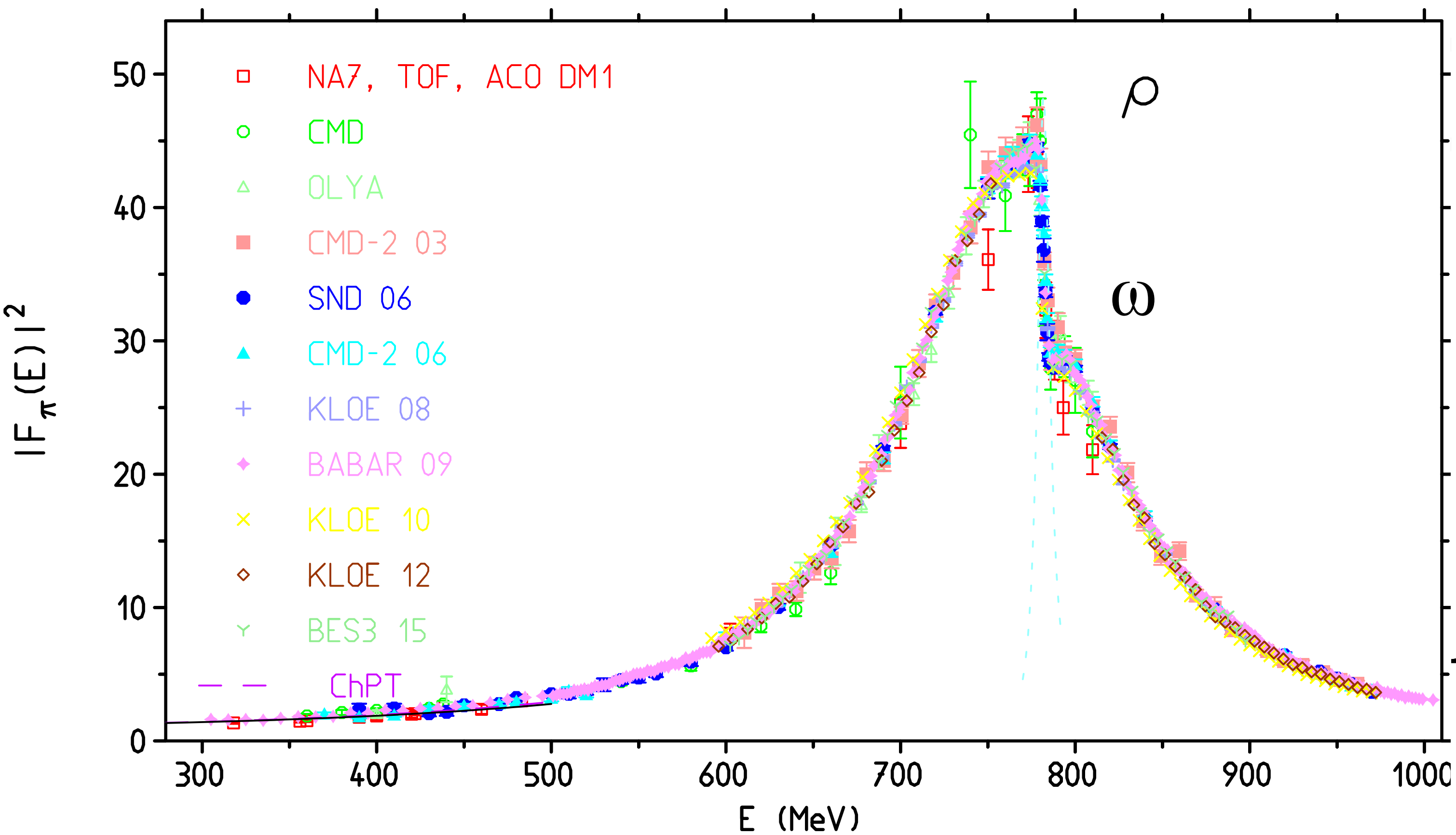
Low energy region important! $\sim 1/s^2$

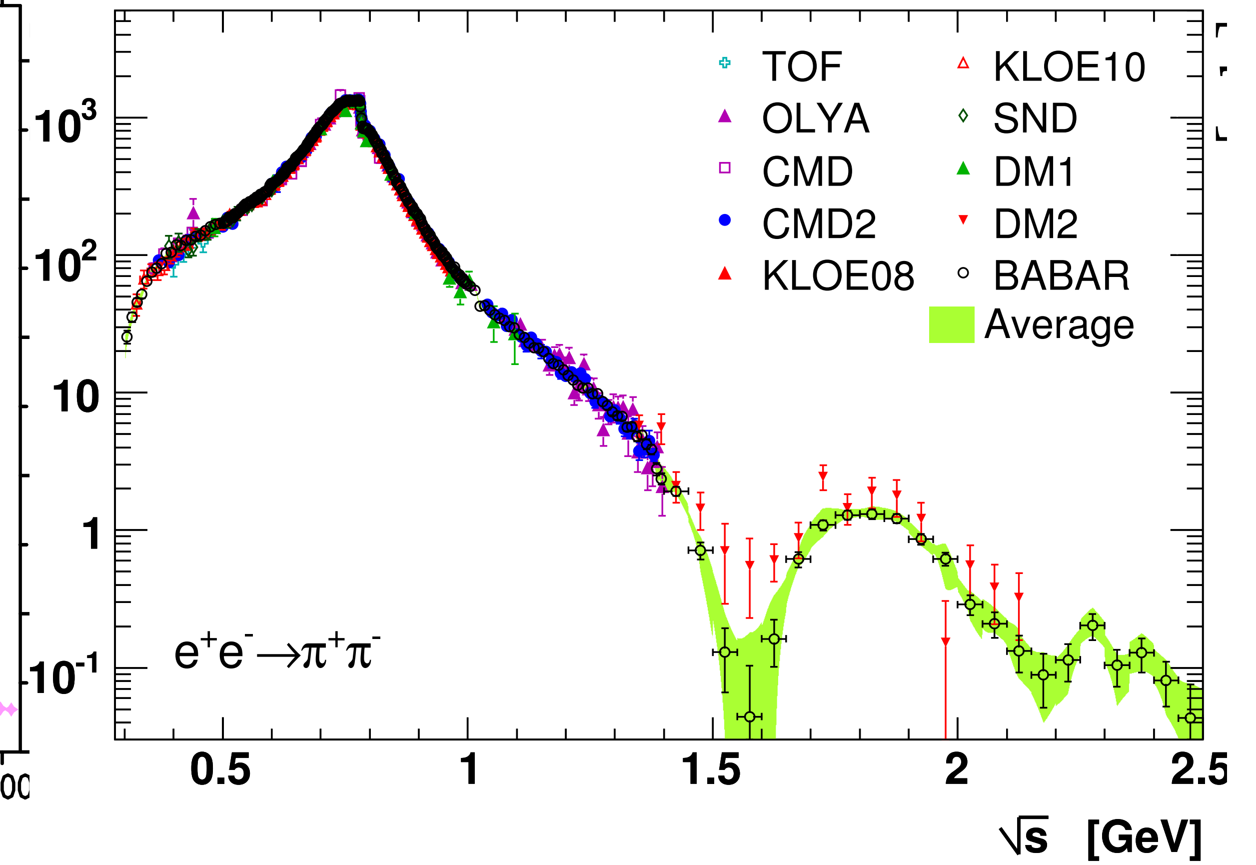
Sum of exclusive σ_{had}

$\Downarrow$

Hadronic contribution of a_μ

Hadronic Vacuum Polarization





- ρ peak
- ρ-ω interference
- Contribution to a_μ(VP): 75%
- Largest error from 1-2GeV

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- BNL E821: $11\,659\,209.1(6.3) \times 10^{-10}$ — Bennet et al, PRD73,072003 (2006)
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NO HLBL

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 41.7(7.9) \times 10^{-10} \Rightarrow 5.3\,\sigma$$ (2σ effect)

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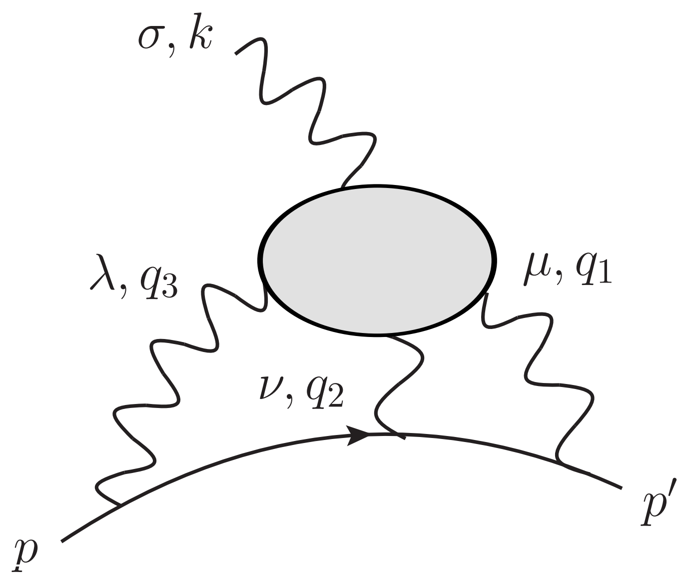
Forthcoming FNAL $\sim 1.6 \times 10^{-10}$ $\Longrightarrow$ from 5σ to 8σ, w/o HLBL: 5σ effect

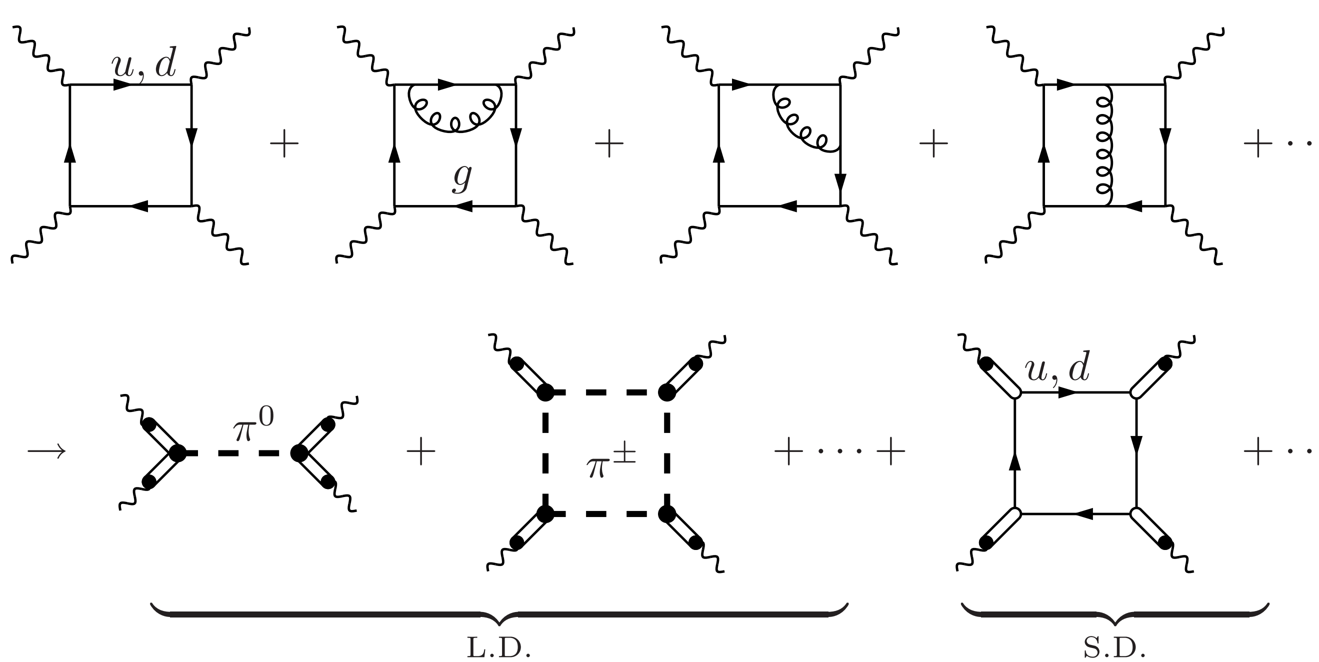
We need to understand such numbers and errors

Hadronic light-by-light scattering in the muon g-2



order O(α^3) hadronic contribution

Multiscale problem

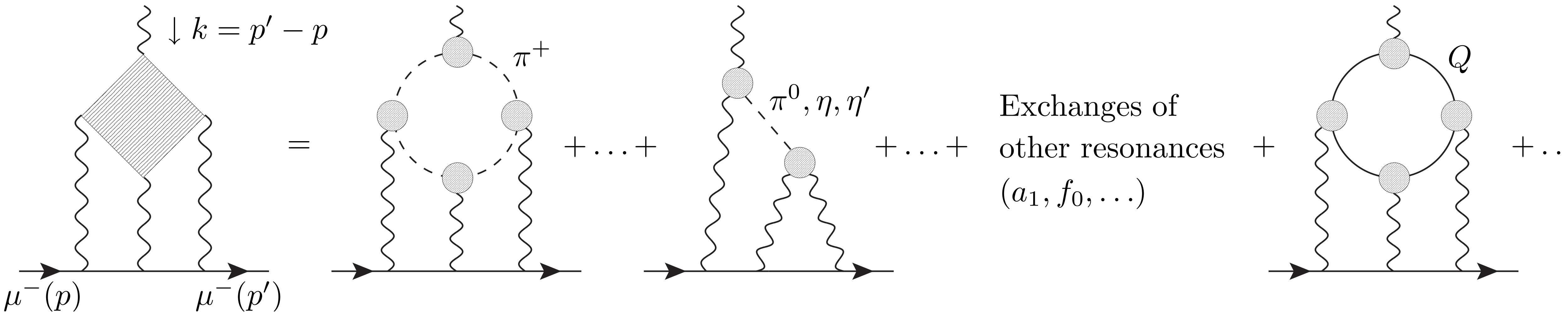


Model at low energies
(with exchange of resonances)

Model at high energies
(quark-loop)

Classification proposal by Eduardo de Rafael '94

Chiral Perturbation Theory counting (p^2)+large-Nc counting



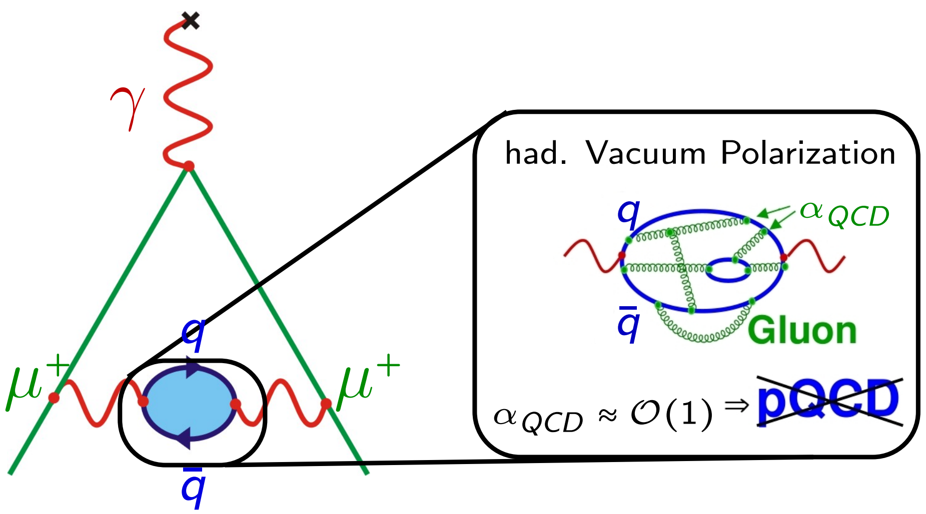
Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

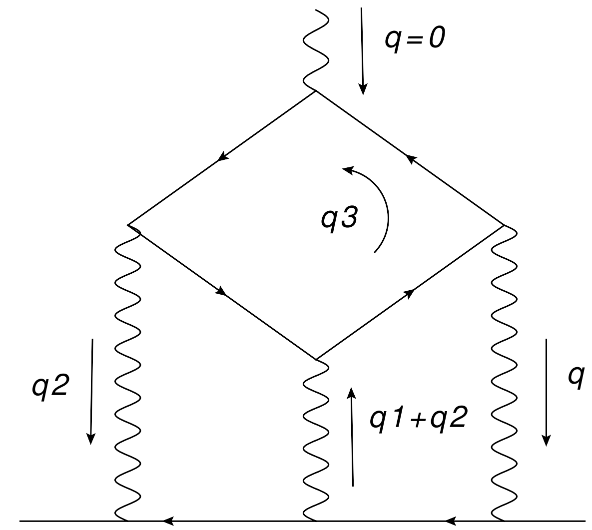
Pesudoscalars: numerically dominant contribution (according to most models)

The anomalous magnetic moment of the muon

Ballpark prediction: Order of magnitude?

Hadronic Vacuum Polarization vs Hadronic Light-by-Light





$$a_\mu^{\rm ferm}({\rm LO}) = I(m_q)\left(\frac{\alpha}{\pi}\right)^2 \longleftrightarrow a_\mu^{HVP} \sim 690 \times 10^{-10}$$

$$I(m_q) = \int_{4m_q^2}^{\infty} \frac{\rho_q(s)K(s)}{s} ds$$

$$\rho_q(s) = \frac{1}{3}\sqrt{1-\frac{4m_q^2}{s}}\left(1+\frac{2m_q^2}{s}\right)$$

(see Pivovarov '03)

$$m_q \sim 0.160 - 0.180 \text{ GeV}$$

(Identify the constituent quark mass from HVP)

The anomalous magnetic moment of the muon

Quark models for a Ballpark prediction

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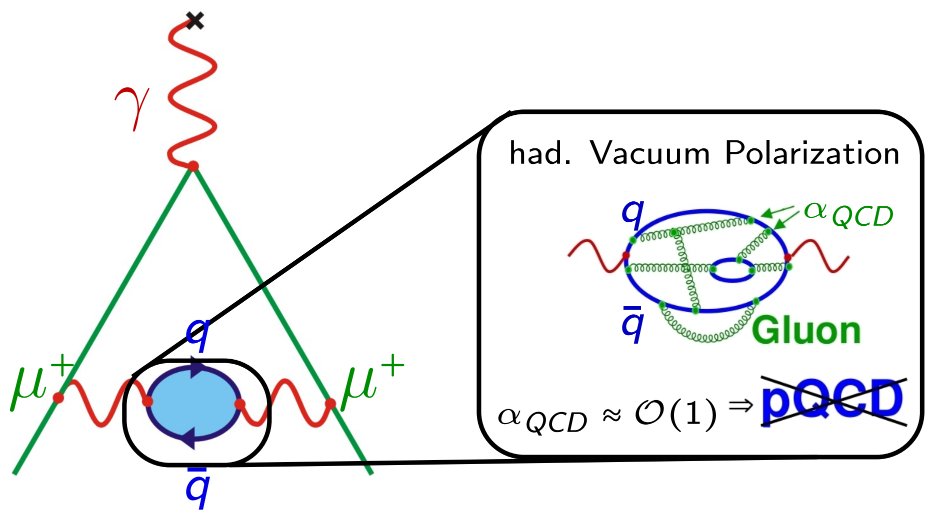
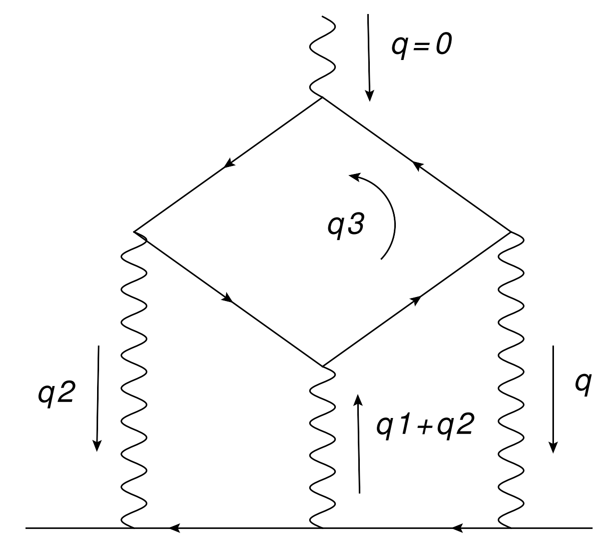




$$a_\mu^{HLBL}(M(Q)) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right)\left[\left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right)\frac{m_\mu^2}{M(Q)^2} + \mathcal{O}\left(\frac{m_\mu^4}{M(Q)^4}\log^2\frac{m_\mu^2}{M(Q)^2}\right)\right]$$

Laporta and Remiddi 1996

$a_\mu^{HLBL} \sim 14 \times 10^{-10}$ Pivovarov '03

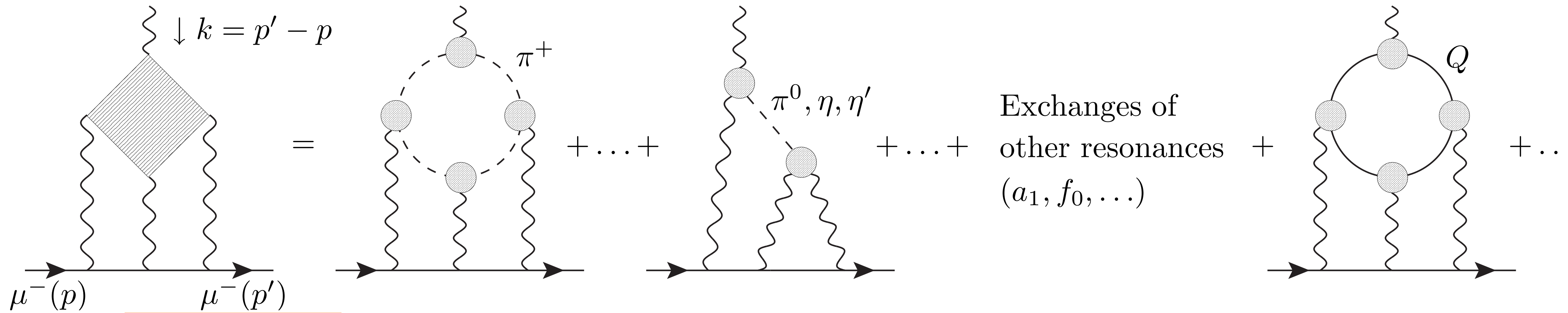
$a_\mu^{HLBL} < 15.9 \times 10^{-10}$ Erler and Toledo Sanchez '06 $m_q \sim 0.160 - 0.180$ GeV

$a_\mu^{HLBL} = 11.8 - 14.8 \times 10^{-10}$ Boughezal and Melnikov, '11

$a_\mu^{HLBL} = [10.5(2.0) \div 15.0(2.5)] \times 10^{-10}$ P.M, Vanderhaeghen 2012

The anomalous magnetic moment of the muon

Reference numbers



						$a_\mu \times 10^{11}$
JN:	$+116(40)$	$-19(13)$	$+99(16)$	$+15(7)$	$+21(3)$	Large N_c, '09
PdRV:	$+105(26)$	$-19(19)$	$+114(13)$	$+8(12)$	0	Average, '09

JN: Jegerlehner and Nyffeler, Phys. Rep. 477 (2009) 1-110
PdRV: Prades, de Rafael, and Vainshtein, arXiv:0901.0306 (Glasgow White Paper)

- use the same model from Knecht and Nyffeler '01 and inputs for the PS (issue of pion-pole vs pion-exchange, i.e., how to correctly implement QCD constraints)
  - errors summed linearly in JN and in quadrature in PdRV
- lack of systematic error (large-Nc model, see P.M. and Vanderhaeghen '12)
- the model neither reproduce the new experimental data on PSTFF (see P.M in arXiv:1407.4021) nor the $\pi^0 \to e^+e^-$ (see P. Sanchez-Puertas in arXiv:1407.4021)
- On top, double counting (or correct overlap) + missing pieces (higher states...)

- All in all, need for more calculations, closer to data (if possible)

The anomalous magnetic moment of the muon

- Main current strategies
  - Lattice QCD
  - Data driven approaches

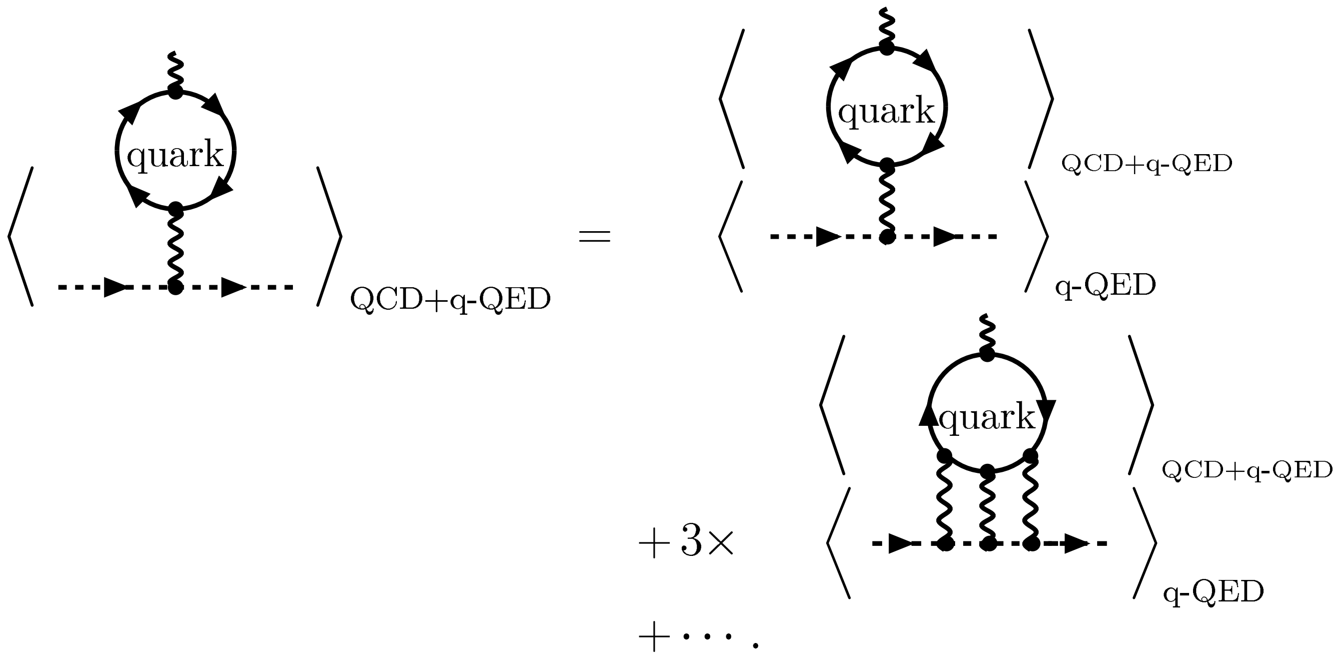
The anomalous magnetic moment of the muon

- Main current strategies
  - Lattice QCD
  - Data driven approaches

The anomalous magnetic moment of the muon

lattice QCD

Blum et al '14,'16,'17



- Next step:
  - physical values
  - larger volume
  - controlled extrapolation to Q^2=0

Very promising!
confronted with other approaches

PRL **118,** 022005 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Connected and Leading Disconnected Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment with a Physical Pion Mass

Thomas Blum,[1,2] Norman Christ,[3] Masashi Hayakawa,[4,5] Taku Izubuchi,[6,2]
Luchang Jin,[3,*] Chulwoo Jung,[6] and Christoph Lehner[6]

We find $a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$, where the error is statistical only. The finite-volume and finite lattice-spacing errors could be quite large and are the subject of ongoing research. The omitted disconnected graphs, while expected to give a correction of order 10%, also need to be computed.

The anomalous magnetic moment of the muon

- Main current strategies
  - Lattice QCD
  - Data driven approaches:
    - Dispersion relations for the low-energy region
    - Hadronic models for the different contributions

The anomalous magnetic moment of the muon

Dispersion relations for the low-energy region

$$\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3) = i^3 \int d^4x \int d^4y \int d^4z\, e^{-i(x\cdot q_1+y\cdot q_2+z\cdot q_3)} \langle 0|T\{j^\mu(x)j^\nu(y)j^\lambda(z)j^\sigma(0)\}|0\rangle$$

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots ,$$

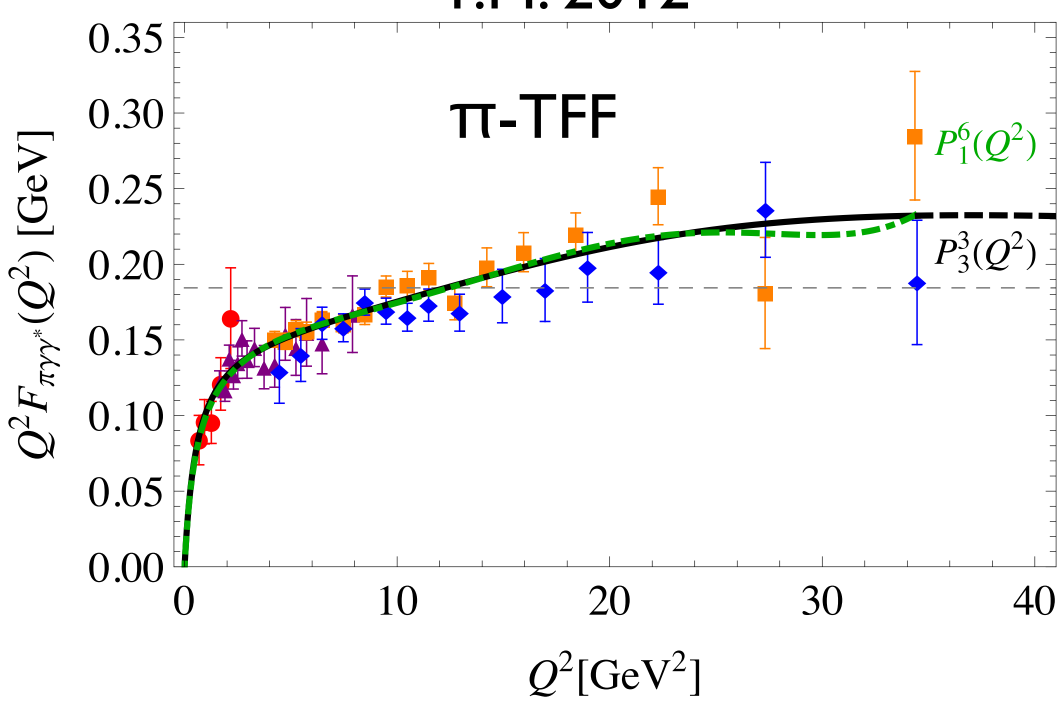
(helicity amplitude decomposition)

Colangelo et al, 1402.7081
Vanderhaeghen et al, 1403.7503

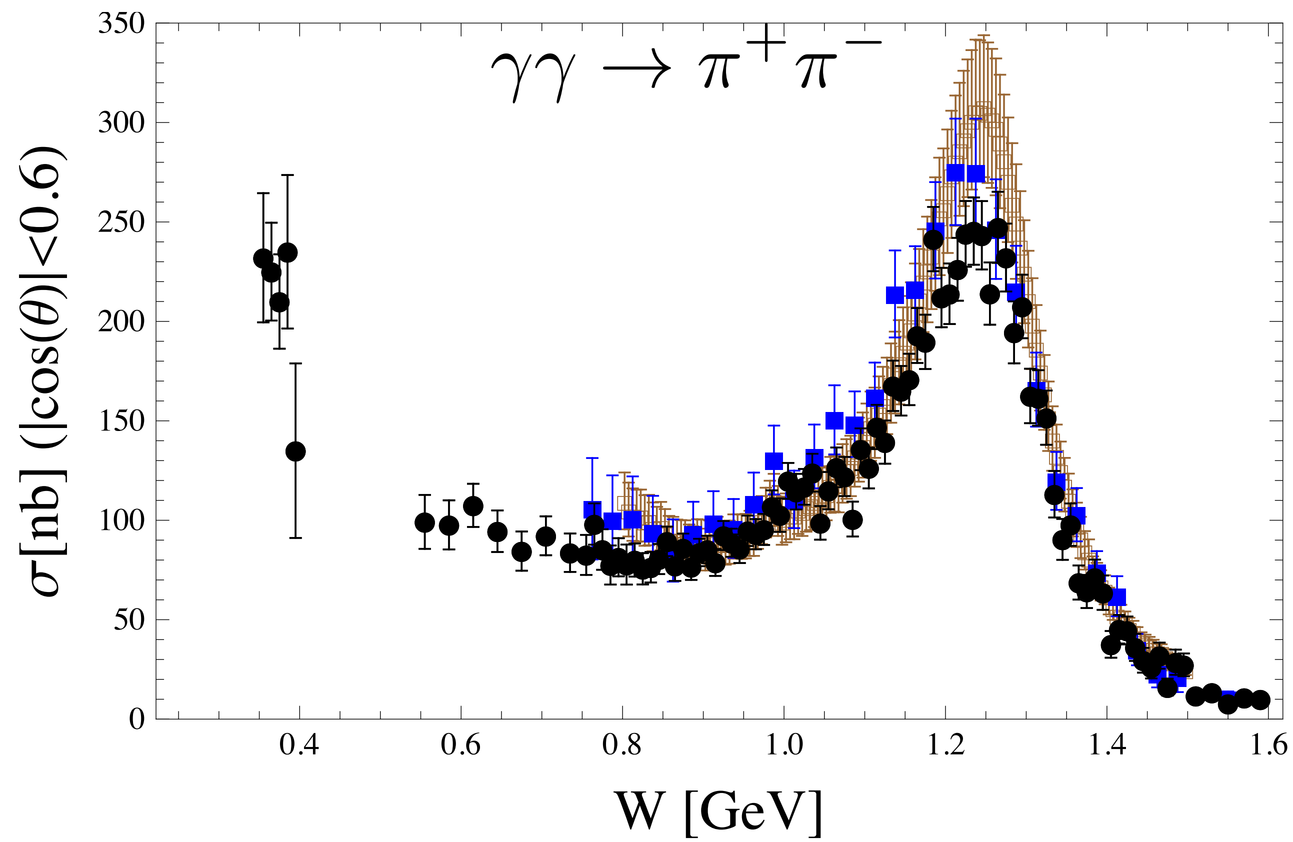
- no intermediate states
- all FF are on-shell, off-shell effects are included in subtraction constants
- need for input: π-TFF and $\gamma\gamma \to \pi^+\pi^-$

[Only subdominant diagrams]

P.M. 2012

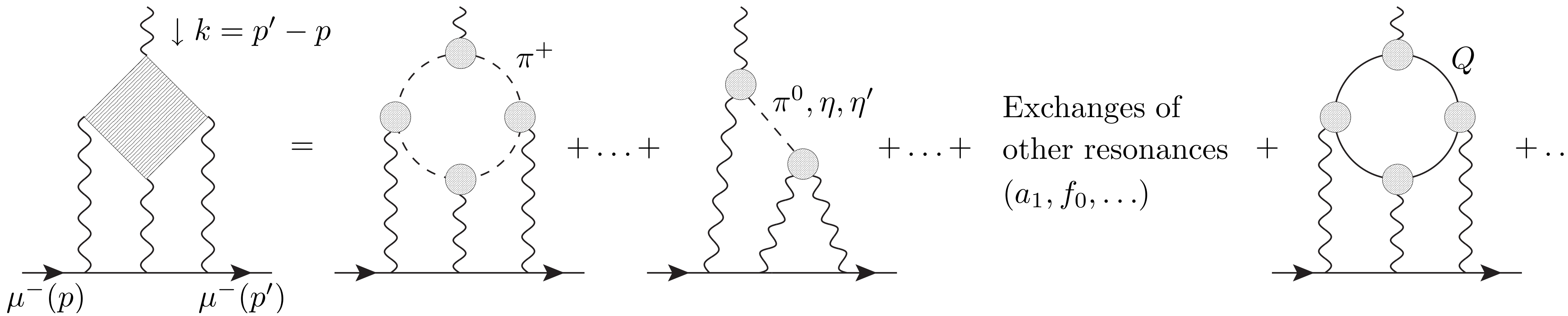


[Mark, Cello, BELLE]



The anomalous magnetic moment of the muon

<u>Hadronic models</u> for the different contributions



$a_\mu^{HLBL} \times 10^{11}$

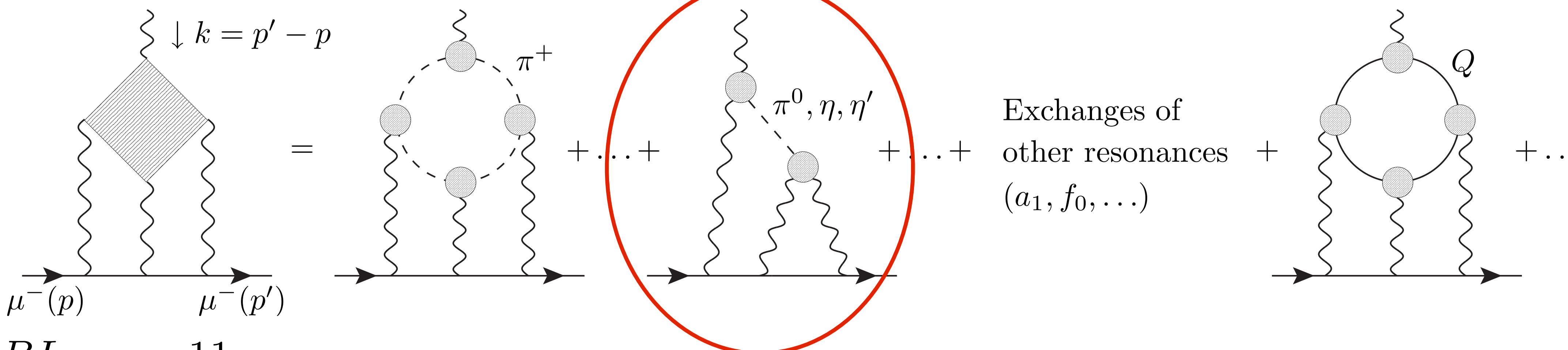
BPP:	$+83(32)$	$-19(13)$	$+85(13)$	$-4(3)$	$+21(3)$	ENJL, '95 '96 '02
HKS:	$+90(15)$	$-5(8)$	$+83(6)$	$+1.7(1.7)$	$+10(11)$	LHS, '98 '02
KN:	$+80(40)$		$+83(12)$			Large N_c, '02
MV:	$+136(25)$	$0(10)$	$+114(10)$	$+22(5)$	0	Large N_c, '04
JN:	$+116(40)$	$-19(13)$	$+99(16)$	$+15(7)$	$+21(3)$	Large N_c, '09
PdRV:	$+105(26)$	$-19(19)$	$+114(13)$	$+8(12)$	0	Average, '09
HK:	$+107$		$+107$			Holographic QCD, '09
DRZ:	$+59(9)$		$+59(9)$			Non-local q.m., '11
EMS:	$+107(20)$	$-19(13)$	$+90(7)$	$+15(7)$	$+21(3)$	Padé-data,'12 '12 '13
GLCR:	$+118(20)$	$-19(13)$	$+105(5)$	$+15(7)$	$+21(3)$	RχT, '14

should add the charm-quark contr. $\sim 2 \times 10^{-11}$

data+systematic error

The anomalous magnetic moment of the muon

The role of experimental data



$a_\mu^{HLBL} \times 10^{11}$

BPP:	$+83(32)$	$-19(13)$	$+85(13)$	$-4(3)$	$+21(3)$	ENJL, '95 '96 '02
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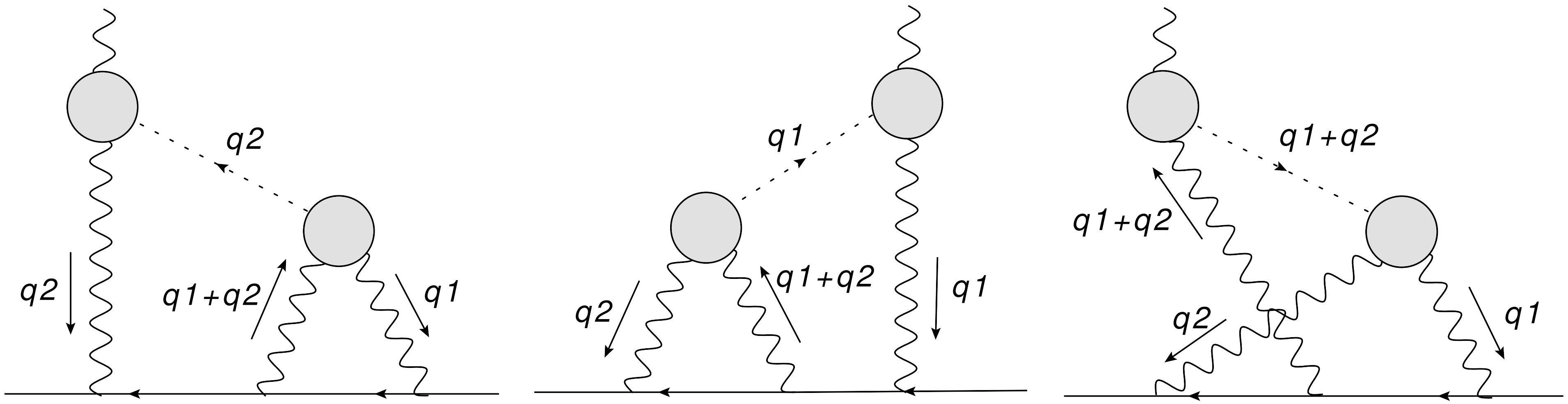
should add the charm-quark contr. $\sim 2 \times 10^{-11}$

The anomalous magnetic moment of the muon

The role of experimental data



From Knecht and Nyffeler, '01:

$$a_\mu^{HLBL;\pi^0} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

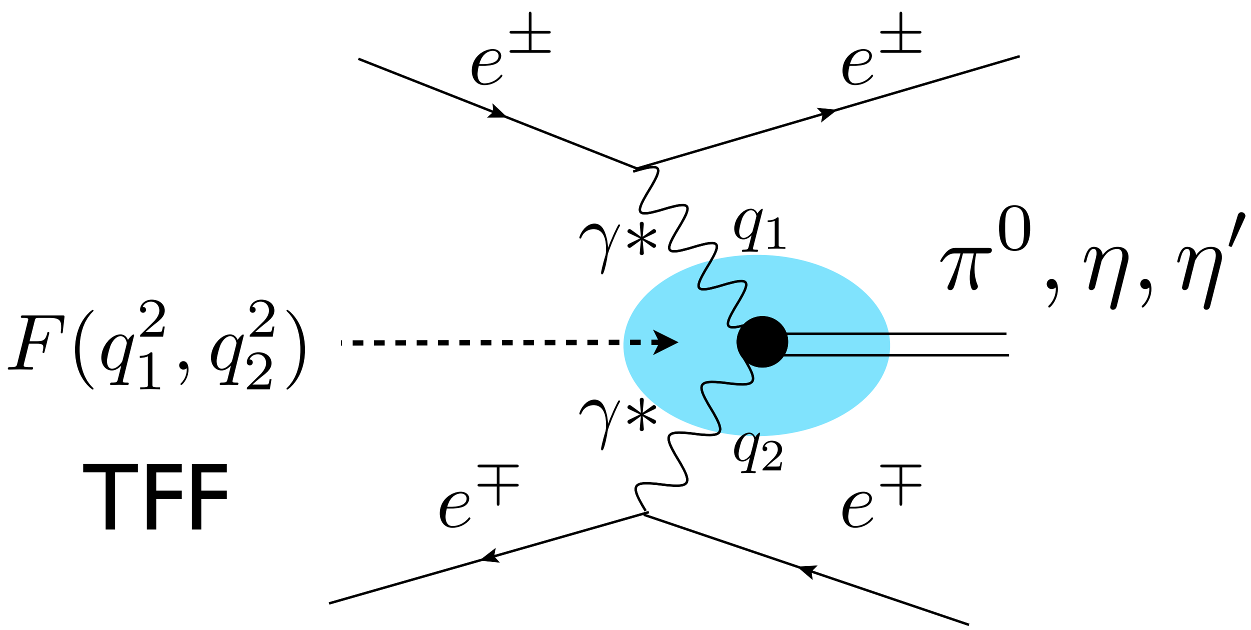
$$\times \left(\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, (q_1+q_2)^2) F_{\pi^0\gamma^*\gamma^*}(q_2^2, 0)}{q_2^2 - M_\pi^2} T_1(q_1, q_2; p) \right.$$

Use data from
the pion Transition Form Factor

$$\left. + \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_\pi^2} T_2(q_1, q_2; p) \right)$$

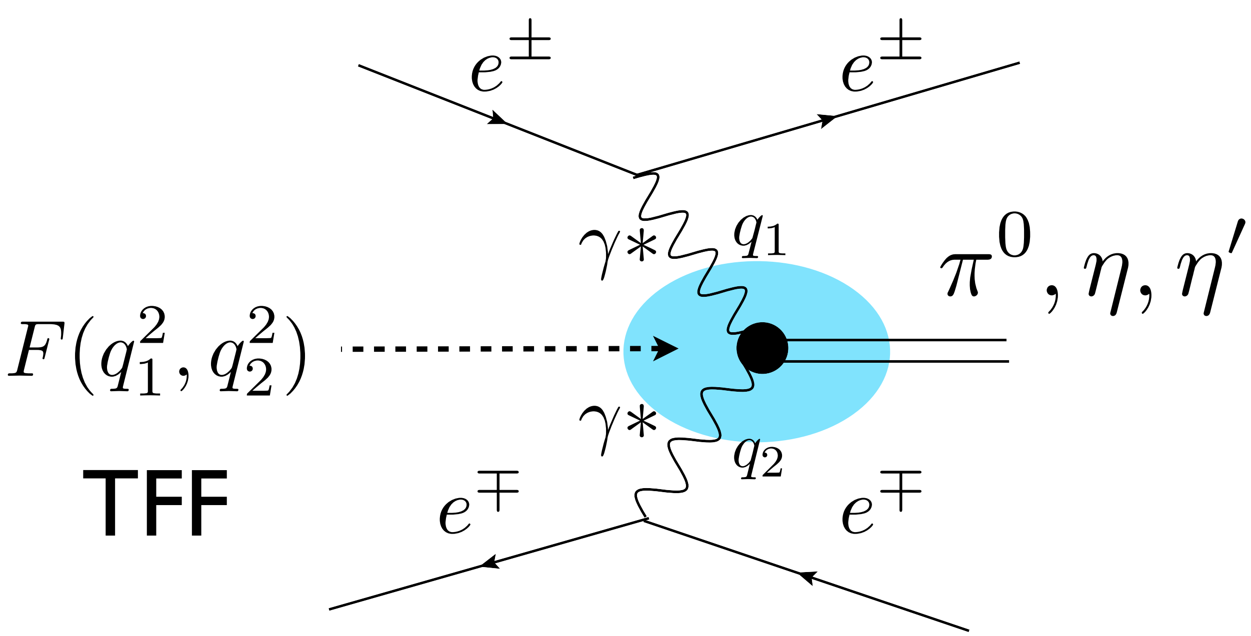
The anomalous magnetic moment of the muon

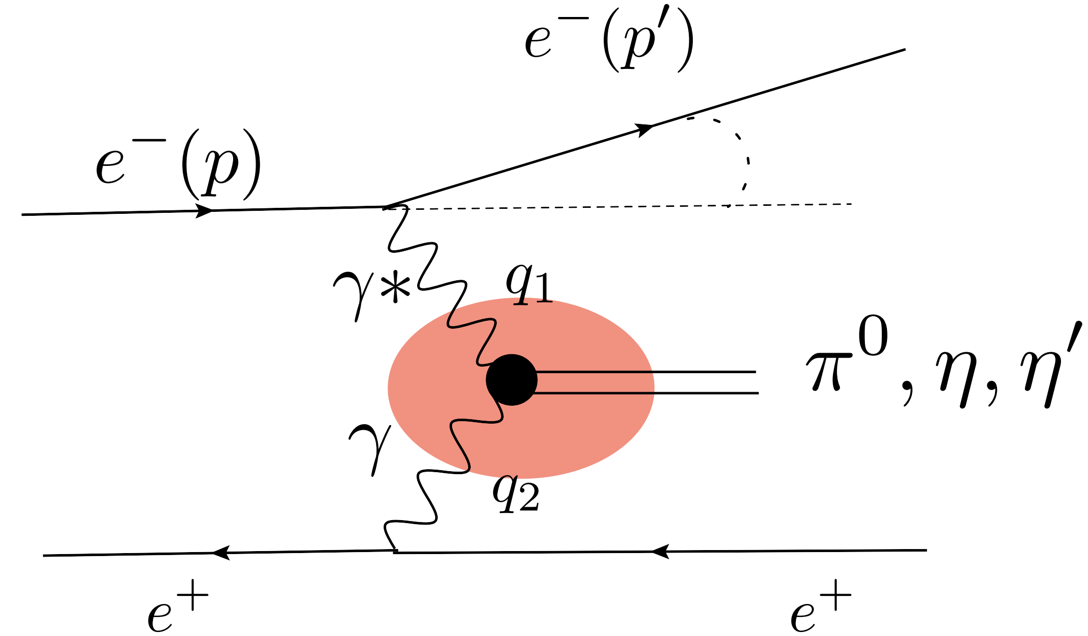
The role of experimental data



The anomalous magnetic moment of the muon

The role of experimental data





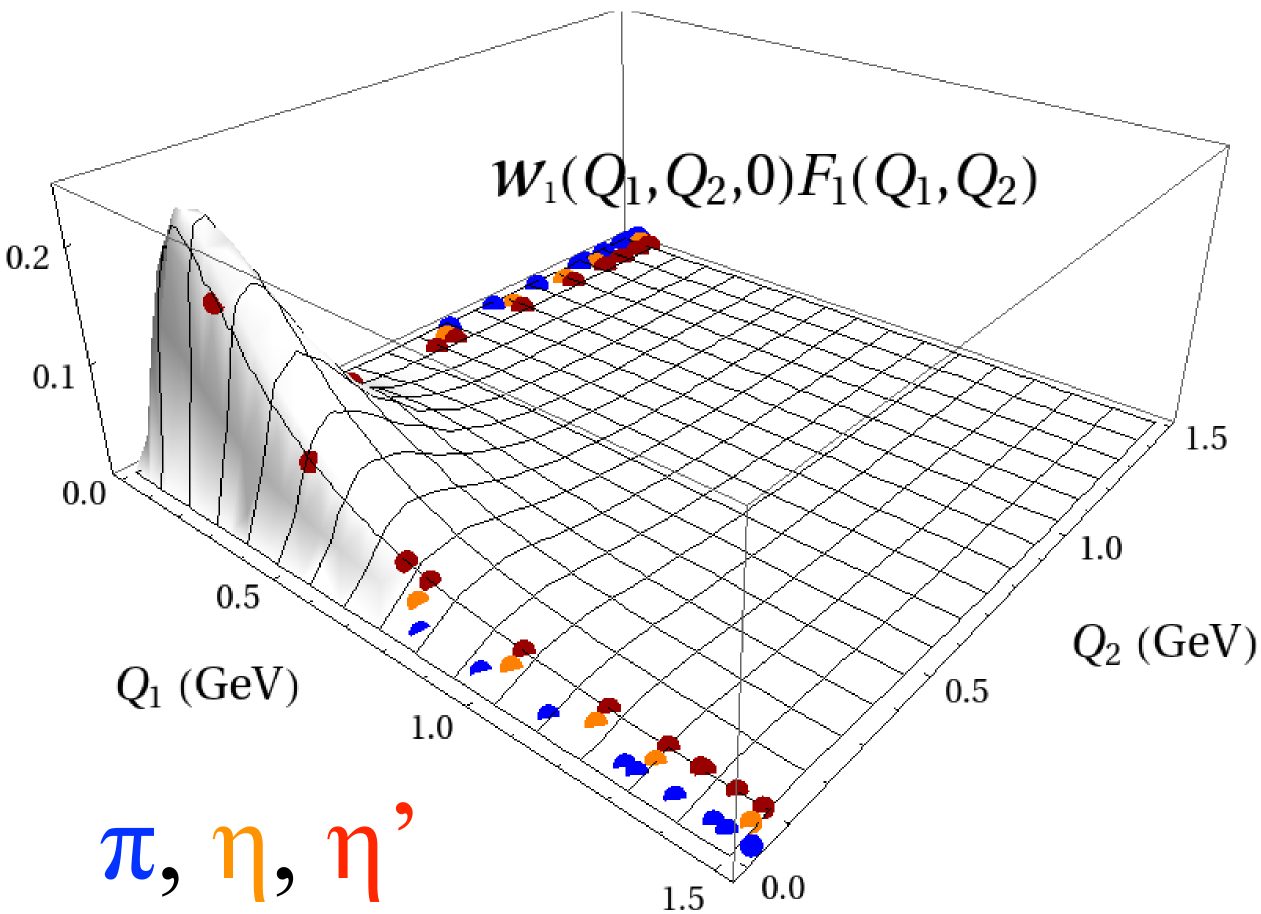
The anomalous magnetic moment of the muon

The role of experimental data

[P.M., Sanchez-Puertas '17]

$$a_\mu^{HLBL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$

Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

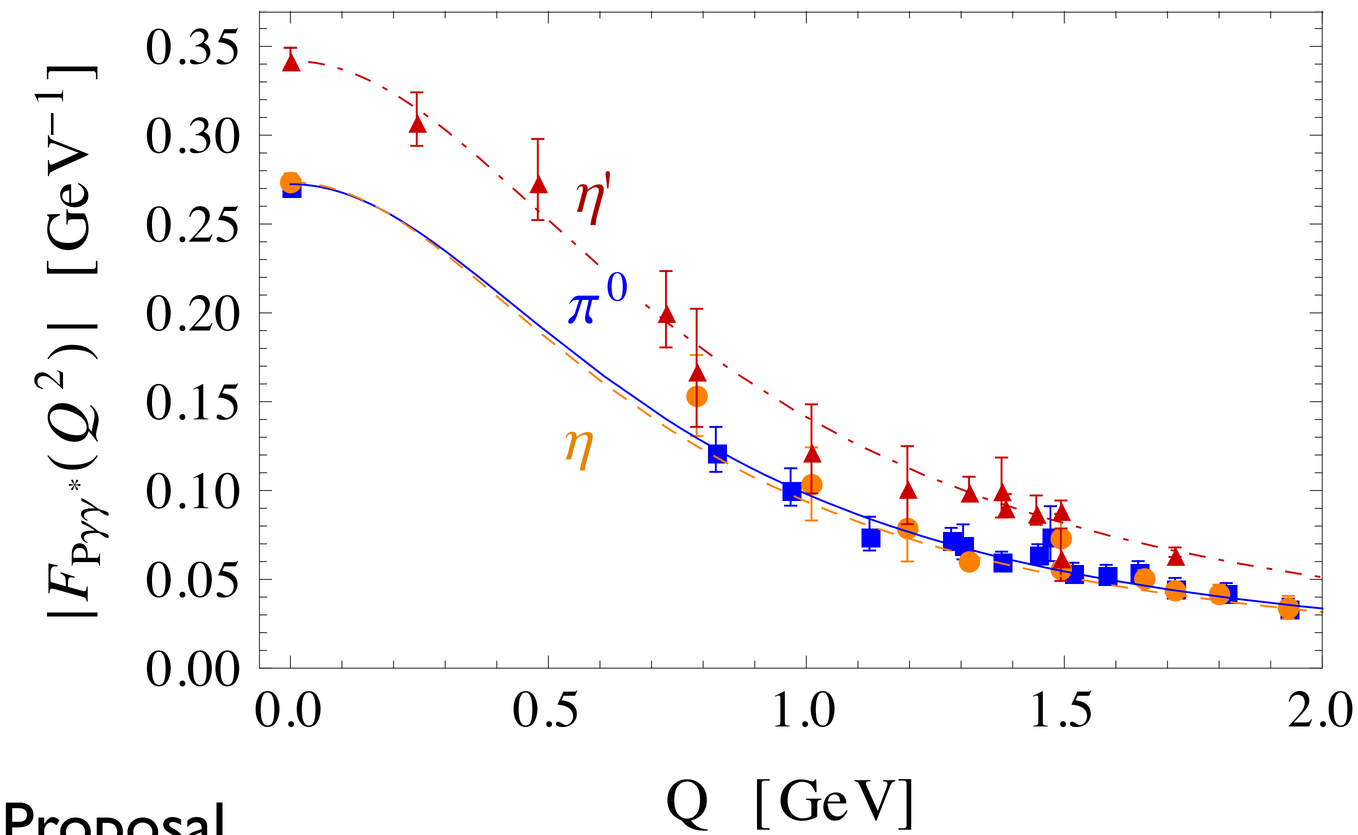


(main energy range from 0 to 1.5 GeV^2)

The anomalous magnetic moment of the muon

The role of experimental data

[P.M., Sanchez-Puertas '17]



At low energies:

$$F_{P\gamma^*\gamma^*}(Q_1^2,Q_2^2) = F_{P\gamma\gamma}(0,0)\left(1 - \frac{b_P}{m_P^2}(Q_1^2+Q_2^2) + \frac{c_P}{m_P^4}(Q_1^4+Q_2^4) + \frac{a_{P;1,1}}{m_P^4}Q_1^2Q_2^2 + \ldots\right)$$

At high energies:

$$\lim_{Q^2\to\infty} F_{P\gamma^*\gamma}(Q^2,0) = P_\infty Q^{-2} + \mathcal{O}(Q^{-4}),$$

$$\lim_{Q^2\to\infty} F_{P\gamma^*\gamma^*}(Q^2,Q^2) = \frac{P_\infty}{3}\left(\frac{1}{Q^2} - \frac{8}{9}\frac{\delta_P^2}{Q^4}\right) + \mathcal{O}(Q^{-6})$$

Proposal
- build a sequence of interpolators based on analyticity and unitary of amplitude: CANTERBURY approximants

$$C_1^0(Q_1^2,Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1+\frac{b_P}{m_P^2}(Q_1^2+Q_2^2)},$$

$$C_2^1(Q_1^2,Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1+\alpha_1(Q_1^2+Q_2^2)+\alpha_{1,1}Q_1^2Q_2^2)}{1+\beta_1(Q_1^2+Q_2^2)+\beta_2(Q_1^4+Q_2^4)+\beta_{1,1}Q_1^2Q_2^2+\beta_{2,1}Q_1^2Q_2^2(Q_1^2+Q_2^2)}.$$

The anomalous magnetic moment of the muon

The role of experimental data

[P.M., Sanchez-Puertas '17]

Using largest set ever:

- Space-like region

$$e^+e^- \to e^+e^- P$$

[L3,CLEO,CELLO,BABAR,BELLE]

- Time-like region

$$P \to \ell^+\ell^-$$

$$P \to \ell^+\ell^-\gamma$$

[NA48,A2,NA62+PDG]

$$P = \pi^0, \eta, \eta'$$
$$\ell = e, \mu$$

[13 different coll.]

$$a_\mu^{\mathrm{HLBL},\pi^0} = 81.8(1.7)[4.0] \cdot 10^{-11}$$

$$+ \quad a_\mu^{\mathrm{HLBL},\eta} = 27.1(1.8)[2.2] \cdot 10^{-11}$$

$$a_\mu^{\mathrm{HLBL},\eta'} = 26.3(1.1)[4.6] \cdot 10^{-11}$$

$$a_\mu^{\mathrm{HLbL};P} = 135(11) \times 10^{-11}$$

adding the rest from *Glasgow Consensus*

$$a_\mu^{\mathrm{HLbL}} = 126(25) \times 10^{-11}$$

vs

$$a_\mu^{\mathrm{HLBL,GC}} = 105(26) \cdot 10^{-11}$$

The anomalous magnetic moment of the muon

Anomalous magnetic moment a_μ (anomaly):

Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	690.8 ± 4.7
HVP(NLO)	-9.93 ± 0.07
HVP(NNLO)	1.22 ± 0.01
HLBL (+NLO)	12.6 ± 2.9
EW	15.4 ± 0.1
Total	11659182.0 ± 5.5

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 27.1(8.4) \times 10^{-10} \Rightarrow 3.2\,\sigma$$

[P.M., Sanchez-Puertas ’17]

Pessimist view

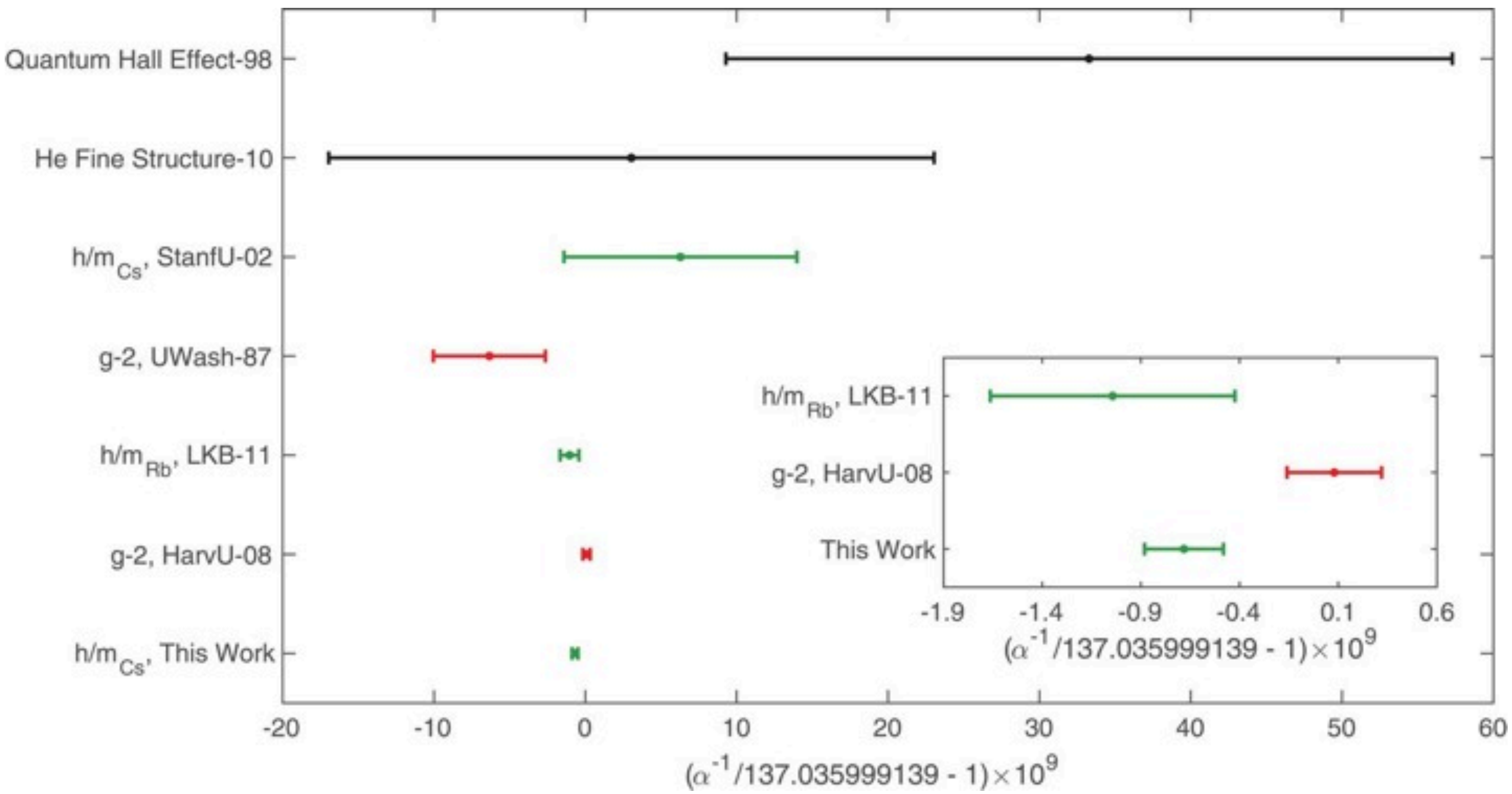
The anomalous magnetic moment of the electron

New measurement of α_{em}

[Parker et al, Science 360 (2018) 191]

$$\alpha^{-1}(Cs) = 137.035\,999\,046\,(27)$$

2.0×10^{-10} accuracy



[Davoudiasl-Marciano, 1806.10252]

$$\Delta a_e \equiv a_e^{\rm exp} - a_e^{\rm SM} = (-87 \pm 36) \times 10^{-14} \longrightarrow 2.4\sigma$$

(negative deviation)

The anomalous magnetic moment of the muon

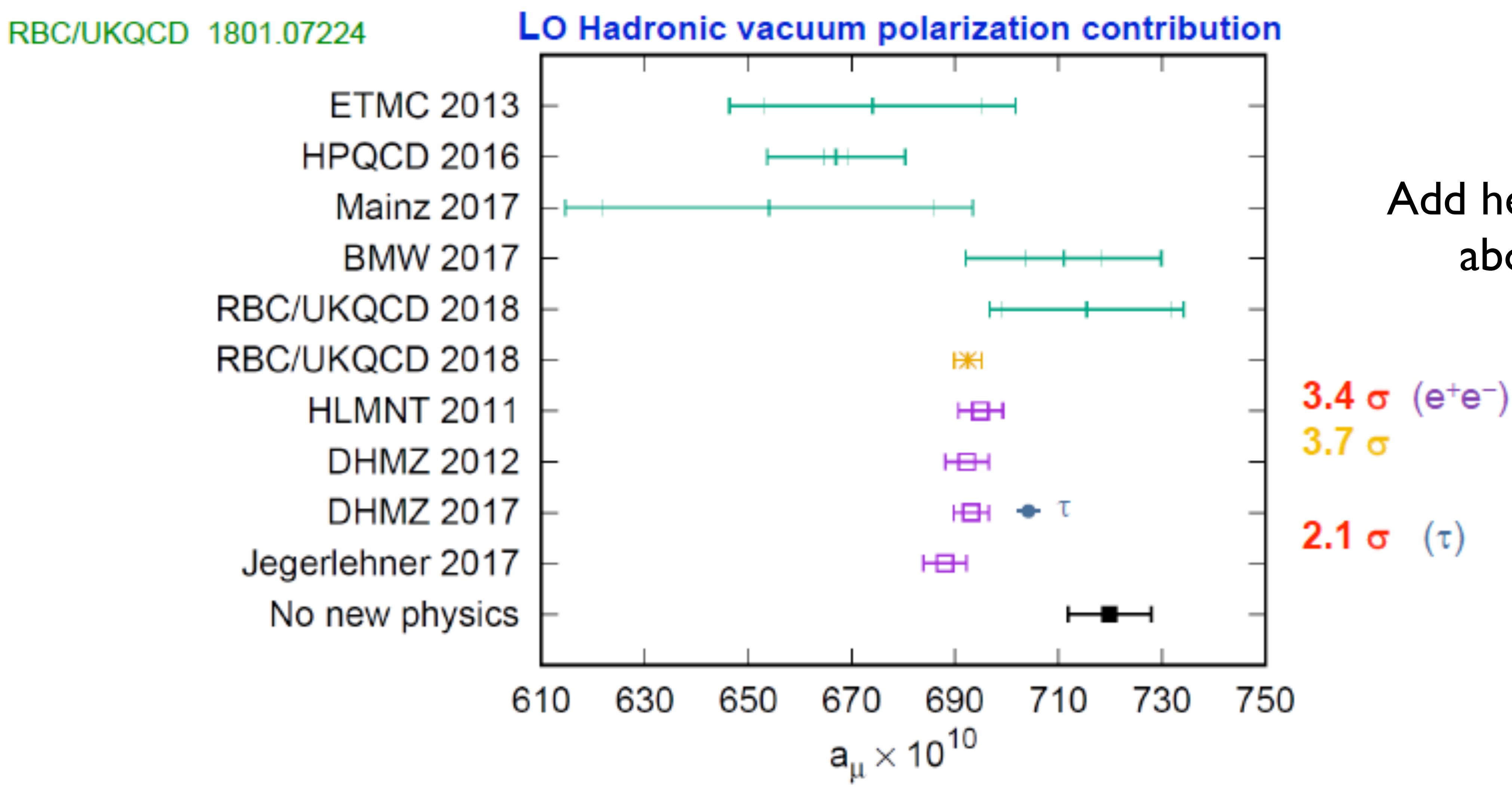
Bennet et al, PRD73,072003 (2006)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11450000(220000)	4300
CERN II	1962–1968	μ^+	11661600(3100)	270
CERN III	1974–1976	μ^+	11659100(110)	10
CERN III	1975–1976	μ^-	11659360(120)	10
BNL	1997	μ^+	11659251(150)	13
BNL	1998	μ^+	11659191(59)	5
BNL	1999	μ^+	11659202(15)	1.3
BNL	2000	μ^+	11659204(9)	0.73
BNL	2001	μ^-	11659214(9)	0.72
Average			11659208.0(6.3)	0.54

Forthcoming exp @FNAL with μ^+ ! ⟶ 2σ !

The anomalous magnetic moment of the muon

No consensus in theory number:



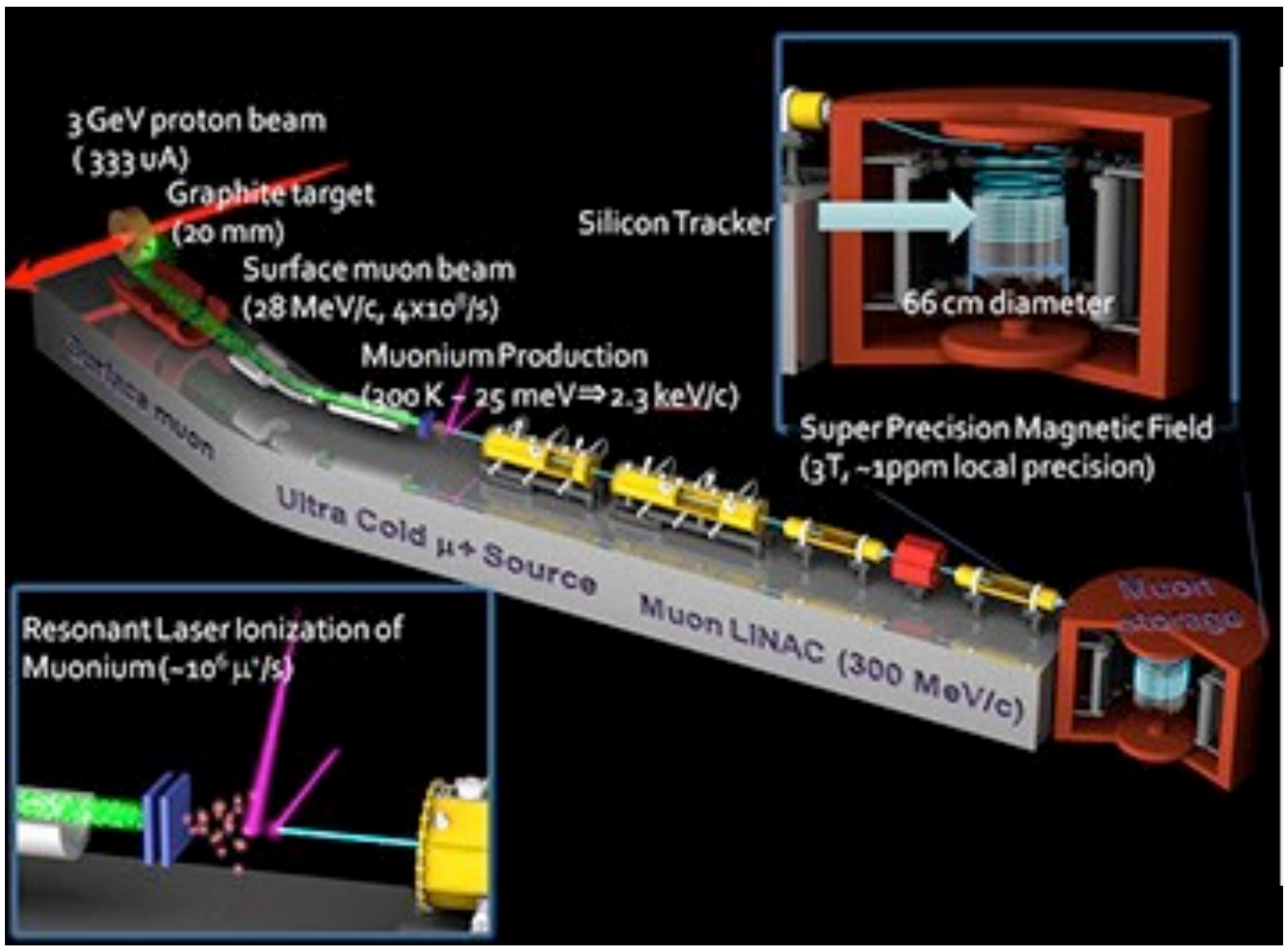
Add here the discussion about the HLBL!

Sociology very difficult!

Optimist view

The anomalous magnetic moment of the muon

Second completely different experiment with cold muons @ JPARC!



hysics.ins-det] 11 Mar 2019

PTEP
Prog. Theor. Exp. Phys. **2015**, 00000 (24 pages)
DOI: 10.1093/ptep/0000000000

A New Approach for Measuring the Muon Anomalous Magnetic Moment and Electric Dipole Moment

M. Abe[1], S. Bae[2,3], G. Beer[4], G. Bunce[5], H. Choi[2,3], S. Choi[2,3], M. Chung[6], W. da Silva[7], S. Eidelman[8,9,10], M. Finger[11], Y. Fukao[1], T. Fukuyama[12], S. Haciomeroglu[13], K. Hasegawa[14], K. Hayasaka[15], N. Hayashizaki[16], H. Hisamatsu[1], T. Iijima[17], H. Iinuma[18], K. Inami[17], H. Ikeda[19], M. Ikeno[1], K. Ishida[20], T. Itahashi[12], M. Iwasaki[20], Y. Iwashita[21], Y. Iwata[22], R. Kadono[1], S. Kamal[23], T. Kamitani[1], S. Kanda[20], F. Kapusta[7], K. Kawagoe[24], N. Kawamura[1], R. Kitamura[14], B. Kim[2,3], Y. Kim[25], T. Kishishita[1], H. Ko[2,3], T. Kohriki[1], Y. Kondo[14], T. Kume[1], M. J. Lee[13], S. Lee[13], W. Lee[26], G. M. Marshall[27], Y. Matsuda[28], T. Mibe[1,29], Y. Miyake[1], T. Murakami[1], K. Nagamine[1], H. Nakayama[1], S. Nishimura[1], D. Nomura[1], T. Ogitsu[1], S. Ohsawa[1], K. Oide[1], Y. Oishi[1], S. Okada[20], A. Olin[4,27], Z. Omarov[25], M. Otani[1], G. Razuvaev[8,9], A. Rehman[29], N. Saito[1,30], N. F. Saito[20], K. Sasaki[1], O. Sasaki[1], N. Sato[1], Y. Sato[1], Y. K. Semertzidis[25], H. Sendai[1], Y. Shatunov[31], K. Shimomura[1], M. Shoji[1], B. Shwartz[9,31], P. Strasser[1], Y. Sue[17], T. Suehara[24], C. Sung[6], K. Suzuki[17], T. Takatomi[1], M. Tanaka[1], J. Tojo[24], Y. Tsutsumi[24], T. Uchida[1], K. Ueno[1], S. Wada[20], E. Won[26], H. Yamaguchi[1], T. Yamanaka[24], A. Yamamoto[1], T. Yamazaki[1], H. Yasuda[28], M. Yoshida[1], and T. Yoshioka[24]

[arXiv:1901.03047]

The anomalous magnetic moment of the muon

- Theory White Paper! (Finally!)
- New measurement of pion FF from BESIII
- New measurement of the neutral pion life time using Primakoff effect @ PrimeEx
- ...

Outlook

- New data required:

  FNL (already started running muons)!

- (my opinion) Theory errors underestimated
  - Lattice QCD will demonstrate (promising, long way)
- New data required (decay constants, masses, form factors, rescattering effects)
  - together with systematics (chiral and large-Nc)
- Missing contributions:

  more on $\gamma\gamma \rightarrow$ hadrons (t-channel), and @mid-large energies

Muchas gracias!

The role of experimental data

Central value:

Model from Knecht and Nyffeler ’01 used in reference numbers

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2 q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2 q_1^2 q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:

$$F_\pi = 92.4 \text{ MeV}$$
$$m_\rho = 769 \text{ MeV}$$
$$m_{\rho'} = 1465 \text{ MeV}$$
$$h_1 = 0\,(\text{ BL limit})$$
$$h_5 = 6.93 \text{ GeV}^4$$
$$h_2 = -10 \text{ GeV}^2$$

$$a_\mu^{\text{HLBL},\pi} = 6.3\times 10^{-10}$$

The anomalous magnetic moment of the muon

The role of experimental data

Central value:

Model from Knecht and Nyffeler '01 used in reference numbers

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:		Preliminary, using new exp data:
$F_\pi = 92.4$ MeV	→	$\Gamma_{\pi^0\to\gamma\gamma}$
$m_\rho = 769$ MeV		$m_\rho = 775$ MeV
$m_{\rho'} = 1465$ MeV	→	curvature TFF
$h_1 = 0$ (BL limit)		$h_1 = 0$ (BL limit)
$h_5 = 6.93$ GeV^4	→	slope TFF
$h_2 = -10$ GeV^2		$h_2 = -10$ GeV^2
$a_\mu^{\mathrm{HLBL},\pi} = 6.3\times10^{-10}$	→	$a_\mu^{\mathrm{HLBL},\pi} = 7.5\times10^{-10}$