QED radiative corrections for polarized lepton nucleon scattering

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The Physics Case of the Weak Charge of Carbon-12 IF-UNAM

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Overview

(1) Introduction — P2 experiment and the weak mixing angle (see Misha's talk)

(2) the shift of Q^2 due to photon radiation.

(3) $\mathcal{O}(\alpha^2)$ QED corrections

(4) $\mathcal{O}(\alpha)$ Hadronic QED corrections

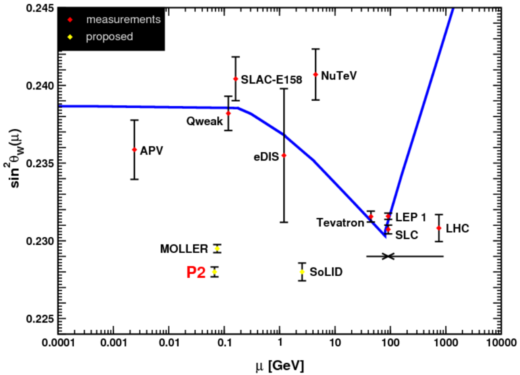
(5) POLARES — an event generator for electron-proton scattering adapted for P2.

P2 experiment @ MESA

- MESA =
  Mainz Energy-recovering Superconducting Accelerator
  A small superconducting accelerator for particle and nuclear physics
- Funded by PRISMA - Cluster of Excellence and
  Collaborative Research Center 1044
  German Science Foundation (DFG)
- P2 (Project 2):
  Parity-violating electron proton scattering
- Other Projects: Search for a dark photon,
  Nuclear physics program

The running of $\sin^2\theta_W$

$$\sin^2\hat{\theta}_W(Q)_{\overline{\text{MS}}} = \kappa(Q)_{\overline{\text{MS}}}\sin^2\theta_W(M_Z)_{\overline{\text{MS}}}$$

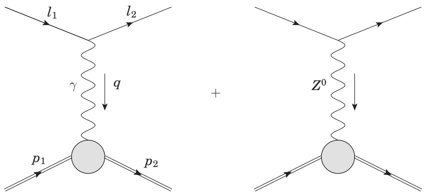


D. Becker, et al., EPJA 2018

→ The future P2 experiment at low momentum transfer will complement other high-precision determinations and may thus help to resolve differences between previous measurements, or find interesting new effects.

How to measure $\sin^2\theta_W$?

→ Extract Q_W^p (weak charge of the proton — the neutral equivalent of the proton's electric charge) in ep scattering with polarized e^- beam and unpolarized proton target. (P2 and Qweak approach)



→ Measure the very small asymmetry $\approx 10^{-8}$ between cross sections for electrons with + and − helicities to filter out the weak interaction

$$A_{\mathrm{PV}} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

In the Born approximation

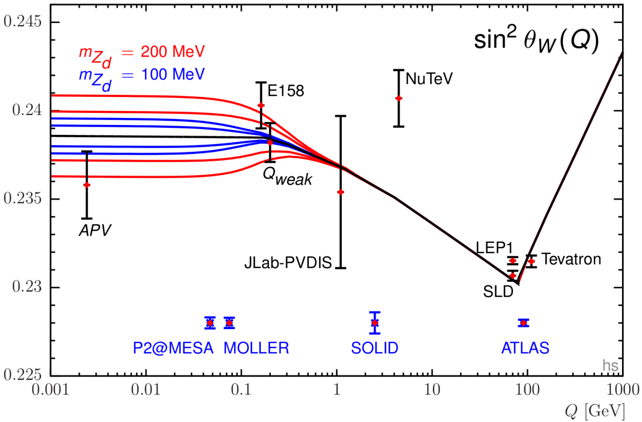
$$A_{PV} \xrightarrow{\text{low } Q^2} -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[Q_W^p - F(Q^2)\right].$$

SM at tree level $Q_W^{p,\text{tree}} \to 1 - 4\sin^2\theta_W^{\text{tree}} \approx 0.07$ (good canditate for New Physics search)

Why measure at low Q^2? → small contribution from $F(Q^2)$ (form factors)
Why measure at low E? → small $\gamma - Z$ box correction.

→ Precision measurements at low-energies are sensitive to high-scale physics.
→ P2 can probe physics at 50 TeV.

New physics at low mass scales — Dark Z boson



→ Models with dark photons predict a small shift of the running weak mixing angle at low mass scales, visible for P2, but not at higher energies. (Hooman Davoudiasl, Hye-Sung Lee, and William J. Marciano 2015)

Radiative Corrections

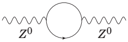
P2 accuracy: $\frac{\Delta A_{\mathrm{PV}}}{A_{\mathrm{PV}}} = 1.7\%$ $\rightarrow$ $\frac{\Delta \sin^2\theta_W}{\sin^2\theta_W} = 0.15\%$

$\rightarrow$ Include full treatment of radiative corrections at $\mathcal{O}(\alpha^2)$ to match experimental precision.

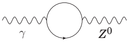
$$A_{PV} = -\frac{G_F Q'^2}{4\sqrt{2}\pi\alpha}\left[\widetilde{Q}^p_W - F(Q'^2)\right].$$

$$\widetilde{Q}^p_W = (\rho + \Delta_e)(1 - 4\kappa(Q'^2)\sin^2\hat{\theta}_W(0) + \Delta_{e'}) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z},$$

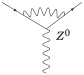
Universal corrections (loop diagrams) $\rightarrow$ $\rho = 1 + \Delta\rho$



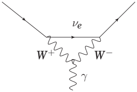
and $\kappa = 1 + \Delta\kappa$



Non-universal corrections (vertex corrections) $\rightarrow$ Δ_e

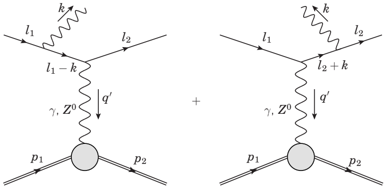


and $\Delta_{e'}$



Form Factors $\rightarrow$ $F(Q'^2) = F_{EM}(Q'^2) + F_{axial}(Q'^2) + F_{strangeness}(Q'^2)$

Shift of momentum transfer due to photon radiation



Shifted kinematics:

$$Q^2 = -(l_1 - l_2)^2 \to Q'^2 = -(l_1 - l_2 - k)^2$$

$\to Q'^2$ can be on average much smaller than Q^2.

The average Shift of momentum transfer squared due to hard-photon bremsstrahlung can be defined as

$$\langle \Delta Q^2 \rangle = \frac{1}{\sigma^{(1)}} \int \frac{\mathrm{d}^4 \sigma_{1\gamma}}{\mathrm{d}E' \mathrm{d}\theta_l \mathrm{d}E_\gamma \mathrm{d}\theta_\gamma} \mathrm{d}E' \mathrm{d}\theta_l \mathrm{d}E_\gamma \mathrm{d}\theta_\gamma \Delta Q^2,$$

with

$$\Delta Q^2 = Q'^2 - Q^2,$$

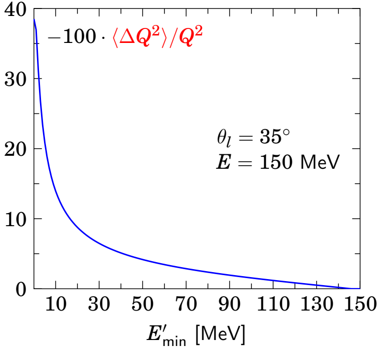
$$\sigma^{(1)} = \sigma^{1\gamma}_{1-\mathrm{loop}}\Big|_{E_\gamma < \Delta} + \sigma^{1\gamma}\Big|_{E_\gamma > \Delta}.$$

Shift of momentum transfer due to photon radiation

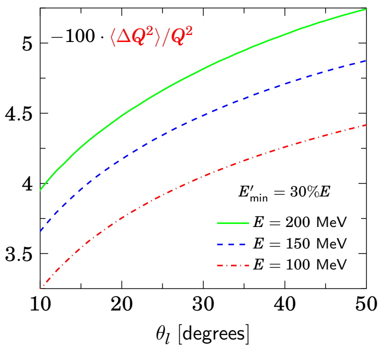
→ Strong dependence on experimental prescriptions for measuring kinematic variables

→ It is important to include full treatment of radiative corrections at the level of the event generator.

→ Dependence on the detector acceptance



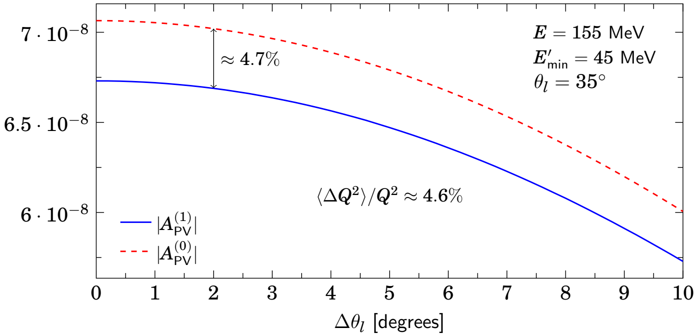
→ Dependence on the scattering angle



Shift of momentum transfer due to photon radiation

→ the shift of Q^2 due to photon bremsstrahlung induces a shift similar in size in the asymmetry.

→ A significant effect is also given by the bin size $\Delta\theta_l$ of the integration over the scattering angle.



The total asymmetry with first order QED corrections compared with the leading order asymmetry.

$\mathcal{O}(\alpha^2)$ QED corrections to the unpolarized cross-section

- The cross-section with $\mathcal{O}(\alpha^2)$ QED corrections is given by

$$\begin{aligned} \mathrm{d}\sigma^{(2)} &= \mathrm{d}\sigma_{0\gamma} \\ &\quad \times \left[1 + \delta^{(1)}_{1-\text{loop}} + \delta^{(2)}_{2-\text{loop}} + \delta^{(1)}_{1s\gamma}(\Delta) + \delta^{(2)}_{2s\gamma}(\Delta) + \delta^{(2)}_{1-\text{loop}+1s\gamma}(\Delta)\right] \\ &\quad + \int_{E_\gamma>\Delta} \mathrm{d}^4\sigma_{1\gamma} \left[1 + \delta^{(2)}_{1-\text{loop}+1h\gamma} + \delta^{(2)}_{1s\gamma+1h\gamma}(\Delta)\right] + \int_{E_\gamma,\, E'_\gamma>\Delta} \mathrm{d}^7\sigma_{2\gamma}. \end{aligned}$$

Δ is the cut-off energy that makes the separation between soft- and hard-photons.

- We can separate therefore the second order cross-section into 3 parts as

$$\sigma^{(2)} = \sigma^{(2)}_{\text{non-rad}} + \sigma^{(2)}_{1h\gamma} + \sigma^{(2)}_{2h\gamma}.$$

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&+ \int_{E_\gamma > \Delta} \mathrm{d}^4\sigma_{1\gamma}\left[1 + \delta^{(2)}_{1-\text{loop}+1h\gamma} + \delta^{(2)}_{1s\gamma+1h\gamma}(\Delta)\right] + \int_{E_\gamma,\, E'_\gamma > \Delta} \mathrm{d}^7\sigma_{2\gamma}.
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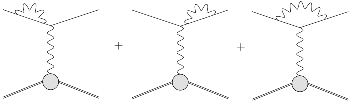
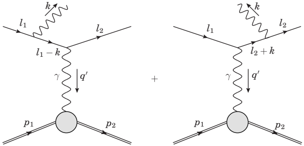
$\sigma^{(2)}_{\text{non-rad}}$

We consider here only corrections to the lepton line.

- $\delta^{(1)}_{1-\text{loop}} \to \delta^{(1)}_{\text{vert}} \equiv \frac{2\,\mathrm{Re}(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vert}})}{|\mathcal{M}_{\text{Born}}|^2} = \frac{\alpha}{\pi}V(Q^2) + \delta_{\text{IR}}$



- $\mathcal{M}_{1\gamma} \to \mathcal{M}_{1s\gamma} = -e\mathcal{M}_{0\gamma}\left(\frac{l_1\epsilon^*}{l_1k} - \frac{l_2\epsilon^*}{l_2k}\right)$
  $\delta^{(1)}_{1s\gamma}(\Delta) = \left(-\frac{\alpha}{4\pi^2}\right)\int_{E_\gamma<\Delta}\frac{\mathrm{d}^3k}{E_\gamma}\left(\frac{l_1}{l_1k} - \frac{l_2}{l_2k}\right)^2 = -\frac{\alpha}{\pi}\left[B_{l_1l_1}(Q^2,\Delta) - B_{l_1l_2}(Q^2,\Delta) + B_{l_2l_2}(Q^2,\Delta)\right] - \delta_{\text{IR}}$

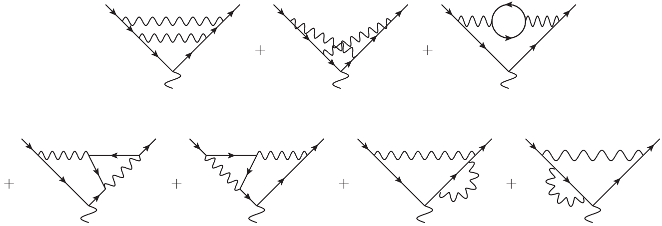


- The total first non-radiative correction is therefore free of IR divergences and is given by

$$\delta^{(1)}_{\text{non-rad}} = \delta^{(1)}_{1-\text{loop}} + \delta^{(1)}_{1s\gamma}(\Delta).$$

$\sigma^{(2)}_{\text{non-rad}}$

- $\delta^{(2)}_{2-\text{loop}} \equiv \frac{|\mathcal{M}_{\text{vert}}|^2}{|\mathcal{M}_{\text{Born}}|^2} + \frac{2\,\text{Re}(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{2-loop}})}{|\mathcal{M}_{\text{Born}}|^2}$



For $Q^2 \gg m_e^2$, $\delta_{2-\text{loop}}$ can be written in a compact form, by keeping the leading term of the expansion in $\frac{m_e^2}{Q^2}$, as

$$\delta^{(2)}_{2-\text{loop}} = \frac{1}{2}\left(\delta^{(1)}_{\text{vert}}\right)^2 + \left(\frac{\alpha}{4\pi}\right)^2\Bigg[-\frac{8}{9}L^3 + \frac{76}{9}L^2 + \left(-\frac{979}{27} + \frac{146\pi^2}{9} + 48\zeta(3)\right)L + \frac{4252}{27} + \frac{15\pi^2}{3} - 16\pi^2\ln(2) - 72\zeta(3) - \frac{64\pi^4}{45} + \mathcal{O}\left(\frac{m_e^2}{Q^2}\right)\Bigg],$$

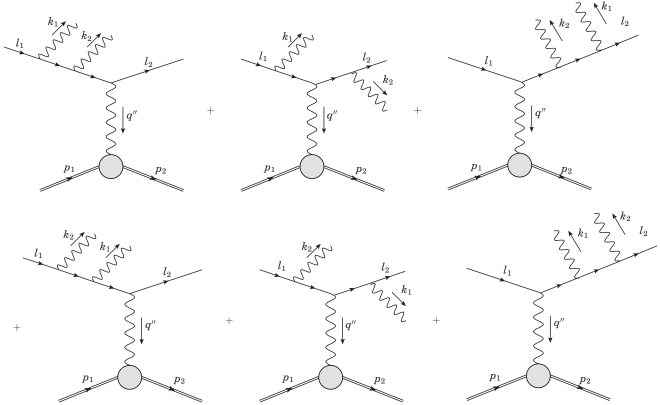
where $L = \ln\left(Q^2/m_e^2\right)$.

R. J. Hill, 2017, arXiv:1605.0261

P. Mastrolia and E. Remiddi, 2003, hep-ph/0307295

$\sigma^{(2)}_{\text{non-rad}}$

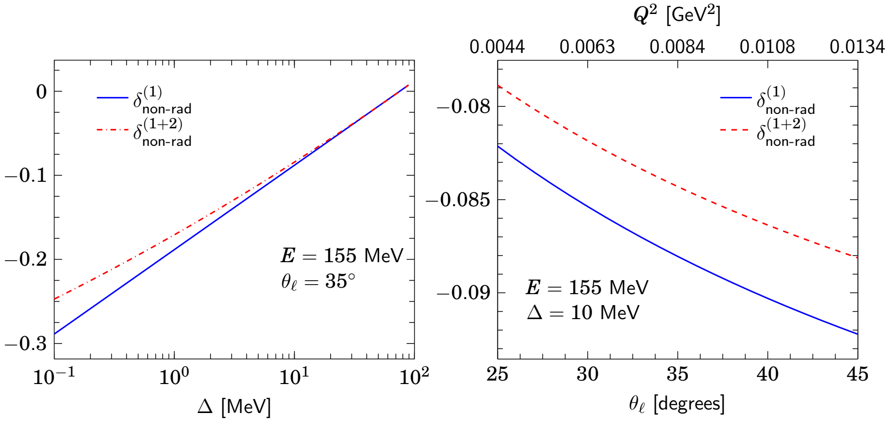
- $\delta^{(2)}_{2s\gamma} = \frac{1}{2!}\left(-\frac{\alpha}{4\pi^2}\right)^2 \int_{E_\gamma,\, E'_\gamma<\Delta} \frac{\mathrm{d}^3k_1}{E_\gamma}\frac{\mathrm{d}^3k_2}{E'_\gamma}\left(\frac{l_1}{l_1k_1} - \frac{l_2}{l_2k_1}\right)^2\left(\frac{l_1}{l_1k_2} - \frac{l_2}{l_2k_2}\right)^2 = \frac{1}{2!}\left(\delta^{(1)}_{1s\gamma}\right)^2$
  - In contrast, if the integration is done restricting the sum of the photon energies, $E_\gamma + E'_\gamma < \Delta$, then the soft-photon correction factor $\delta^{(2)}_{2s\gamma}$ contains an additional term $-\frac{\alpha^2}{3}(L-1)^2$.



$\sigma^{(2)}_{\text{non-rad}}$

- For a soft photon in the final state the 1-loop and one soft-photon correction factorizes as $\delta^{(2)}_{1-\text{loop}+1s\gamma} \to \delta^{(1)}_{1-\text{loop}}\delta^{(1)}_{1s\gamma}$.
- The total second-order is then also free of IR divergences and given by

$$\delta^{(2)}_{\text{non-rad}} = \delta^{(2)}_{2-\text{loop}} + \delta^{(2)}_{2s\gamma} + \delta^{(2)}_{1-\text{loop}+1s\gamma}.$$



$\sigma^{(2)}_{1h\gamma}$

- The cross-section for one radiated photon is given by

$$\mathrm{d}^4\sigma_{1\gamma} = \frac{\mathrm{d}^4\Gamma_{1\gamma}}{4M|\vec{l}_1|}\overline{|\mathcal{M}_{1\gamma}|^2}.$$

- The differential phase-space from this equation is given by

$$\mathrm{d}^4\Gamma = \int \frac{1}{(2\pi)^5}\mathrm{d}^4 l_2 \mathrm{d}^4 k \mathrm{d}^4 p_2 \delta(l_2^2 - m_e^2)\delta(k^2)\delta(p_2^2 - M^2) \times \delta^4\left(l_1 + p_1 - l_2 - p_2 - k\right),$$

  where a successive integration is carried out over the delta functions.

- The matrix element squared is calculated using the Mathematica package *Feyncalc*.

- Integrating over the last delta function we can choose to express one variable in terms of the remaining four. The cross-section for one hard radiated photon becomes

$$\sigma_{1h\gamma} = \frac{1}{32(2\pi)^4 M|\vec{l}_1|}\int_{E'_{\mathrm{min}}}^{E'_{\mathrm{max}}} \mathrm{d}E' \int_{\theta_l^{\mathrm{min}}}^{\theta_l^{\mathrm{max}}} \mathrm{d}\cos\theta_l \int_{\Delta}^{E_\gamma^{\mathrm{max}}} \mathrm{d}E_\gamma \int_{\theta_\gamma^{\mathrm{min}}}^{\theta_\gamma^{\mathrm{max}}} \mathrm{d}\cos\theta_\gamma \frac{\overline{|\mathcal{M}|^2}}{\sin\theta_l \sin\theta_\gamma \sin\phi_\gamma}\Theta\left(1 - \frac{A^2}{B^2}\right),$$

  where $\sin\phi_\gamma = \sqrt{1 - A^2/B^2}$, with $A = A(E', \theta_l, E_\gamma, \theta_\gamma)$ and $B = B(E', \theta_l, E_\gamma, \theta_\gamma)$.

$\sigma^{(2)}_{1h\gamma}$

- The IR divergent contribution for one hard-photon and one soft-photon can be factorized as before

$$\mathcal{M}^{\text{IR-div}}_{2\gamma} = -e\left[\mathcal{M}_{1\gamma}(k_2)\left(\frac{l_1\epsilon^*}{l_1k_1} - \frac{l_2\epsilon^*}{l_2k_1}\right)\right],$$

- The cross section for two photons in the final state is given by

$$\mathrm{d}^7\sigma_{2\gamma} = \frac{\mathrm{d}^7\Gamma_{2\gamma}}{2\cdot 4M|\vec{l_1}|}\overline{|\mathcal{M}_{2\gamma}|^2}.$$

Two identical particles in the final state → symmetrization factor.

- The separation between soft- and a hard-photon phase space regions is done as

$$\int_0^{E_\gamma^{\text{max}}}\mathrm{d}E_\gamma\int_0^{E_\gamma^{\text{max}}}\mathrm{d}E'_\gamma = \int_0^{\Delta}\mathrm{d}E_\gamma\int_0^{\Delta}\mathrm{d}E'_\gamma + 2\int_\Delta^{E_\gamma^{\text{max}}}\mathrm{d}E_\gamma\int_0^{\Delta}\mathrm{d}E'_\gamma + \int_\Delta^{E_\gamma^{\text{max}}}\mathrm{d}E_\gamma\int_\Delta^{E_\gamma^{\text{max}}}\mathrm{d}E'_\gamma .$$

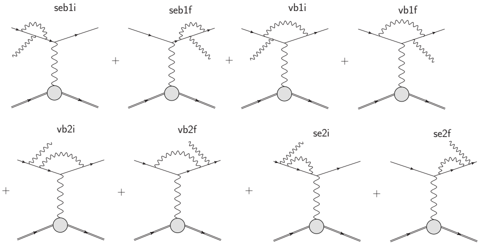
Only the second term on the right-hand side of contributes to the part of the total cross section considered here, where we require one hard and one soft photon.

- The infrared divergence can be factorized, resulting in

$$\sigma^{(2)}_{1s\gamma+1h\gamma}(\Delta) = 2\int_\Delta^{E_\gamma^{max}}\int_0^{\Delta}\left(\mathrm{d}^7\sigma_{2\gamma}\right)_{1\gamma\to 0} = \delta^{(1)}_{1s\gamma}(\Delta)\int_\Delta^{E_\gamma^{max}}\mathrm{d}^4\sigma_{1\gamma} + 2\int_\Delta^{E_\gamma^{max}}\int_0^{\Delta}\left(\mathrm{d}^7\sigma_{2\gamma}\right)^{\text{IR-finite}}_{1\gamma\to 0} .$$

$$\sigma^{(2)}_{1h\gamma}$$

- $\delta^{(2)}_{1-\text{loop}+1h\gamma} = \delta_{\text{se}1\gamma} + \delta_{\text{v}1\gamma} + \delta_{\text{se}2\gamma} + \delta_{\text{v}2\gamma}$
- $\sigma^{(2)}_{1s\gamma+1h\gamma}(\Delta) + \delta^{(2)}_{1-\text{loop}+1h\gamma}$ = IR-finite result



λ^2 [GeV^2]	m_e^2	10^{-4}	10^{-8}	10^{-12}
$\sigma^{(2)}_{1h\gamma}$	3946.51 ± 0.87	3945.74 ± 0.86	3946.30 ± 0.81	3946.2 ± 0.82

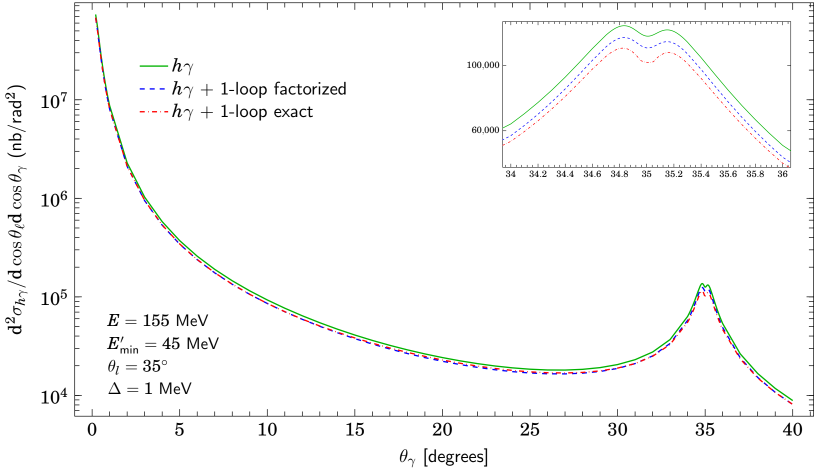
Δ_ϵ	0	10^2	10^4	10^6
$\sigma^{(2)}_{1h\gamma}$	3946.51 ± 0.87	3946.25 ± 0.85	3945.82 ± 0.84	3943.52 ± 0.87

μ^2	1	10^4	10^8	10^{12}
$\sigma^{(2)}_{1h\gamma}$	3946.51 ± 0.87	3945.44 ± 0.84	3946.45 ± 0.88	3945.82 ± 0.86

$\sigma^{(2)}_{1h\gamma}$

- We can approximate the correction for one hard-photon and 1-loop by assuming that they factorize. This amounts to the replacement

$$\delta^{(2)}_{\text{1-loop}+1h\gamma} \to \delta^{(1)}_{\text{1-loop}}.$$



$\sigma_{2h\gamma}^{(2)}$

- The cross section for two photons in the final state is given, as previously shown, by

$$\mathrm{d}^7\sigma_{2\gamma} = \frac{\mathrm{d}^7\Gamma_{2\gamma}}{2\cdot 4M|\vec{l}_1|}\overline{|\mathcal{M}_{2\gamma}|^2}.$$

- The differential phase-space is given in this case by

$$\mathrm{d}^7\Gamma = \int \frac{1}{(2\pi)^8}\,\mathrm{d}^4l_2\,\mathrm{d}^4p'\,\mathrm{d}^4k_1\,\mathrm{d}^4k_2\,\delta(l_2^2-m_\ell^2)\delta(p'^{\,2}-M^2)\delta(k_1^2)\delta(k_2^2) \\ \times\delta^4\left(l_1+p-l_2-p'-k_1-k_2\right)\,,$$

where a successive integration is carried out over the delta functions, as for one radiated photon.

- In this case we can express one variable in terms of the remaining seven. The cross-section for two hard radiated photon then becomes

$$\sigma_{2h\gamma} = \frac{E_\gamma E'_\gamma|\vec{l}_2|}{128(2\pi)^7M|\vec{l}_1|}\int\limits_{E'_{\text{min}}}^{E'_{\text{max}}}\mathrm{d}E'\int\limits_{\cos\theta_{l,\text{min}}}^{\cos\theta_{l,\text{max}}}\mathrm{d}\cos\theta_l\int\limits_{\Delta}^{E_{\gamma,\text{max}}}\mathrm{d}E_\gamma\int\limits_{\Delta}^{E'_{\gamma,\text{max}}}\mathrm{d}E'_\gamma \\ \times\int\limits_{\cos\theta'_{\gamma,\text{min}}}^{\cos\theta'_{\gamma,\text{max}}}\mathrm{d}\cos\theta'_\gamma\int\limits_{\phi'_{\gamma,\text{min}}}^{\phi'_{\gamma,\text{max}}}\mathrm{d}\phi'_\gamma\int\limits_{\cos\theta_{\gamma,\text{min}}}^{\cos\theta_{\gamma,\text{max}}}\mathrm{d}\cos\theta_\gamma\frac{\overline{|\mathcal{M}_{2\gamma}|^2}}{|\alpha_1\cos\phi_\gamma-\alpha_2\sin\phi_\gamma|}\Theta\left(1-\frac{\alpha_3^2}{\alpha_1^2+\alpha_2^2}\right)\,.$$

$\sigma_{2h\gamma}^{(2)}$

- In the previous expression ϕ_γ is a function of the other variables and is given by

$$\phi_\gamma = \arcsin \frac{\alpha_3}{\sqrt{\alpha_1^2 + \alpha_2^2}} - \arcsin \frac{\alpha_2}{\sqrt{\alpha_1^2 + \alpha_2^2}}.$$

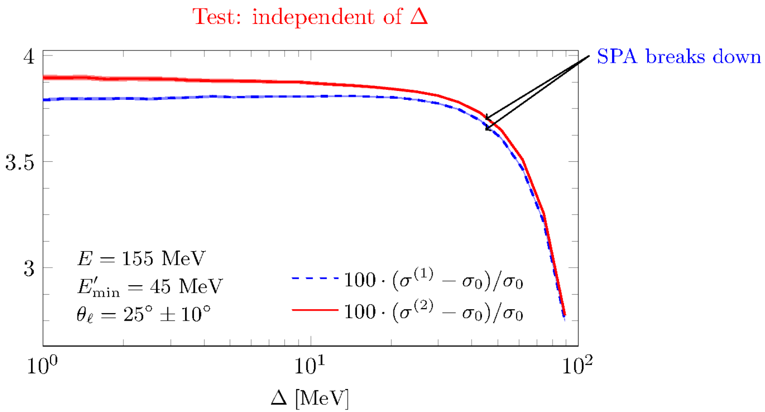
- Integration limits for angles and energies follow from the condition that the arguments of the $\arcsin$-functions in the expression for ϕ_γ have to be between -1 and 1.

- These conditions are enough to achieve a high efficiency of the numerical integration.

- It turns out, however, that collinear poles in the differential cross section render a naive approach numerically unstable. In order to deal with this problem we have used a partial fractioning to separate the collinear poles

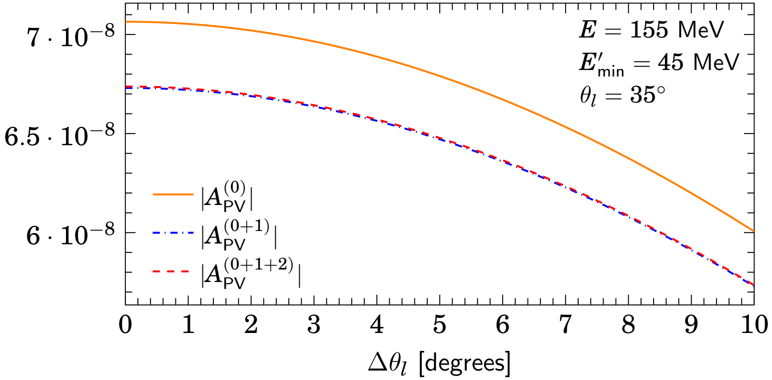
$$\mathrm{d}^7\sigma_{2\gamma} \propto \frac{1}{(l_1k_1)\cdot(l_1k_2)\cdot(l_2k_1)\cdot(l_2k_2)} \to \frac{A}{l_1k_1} + \frac{B}{l_1k_2} + \frac{C}{l_2k_1} + \frac{D}{l_2k_2}\,.$$

$\mathcal{O}(\alpha^2)$ QED corrections to the unpolarized cross section



→ Can be used to decide which is the best value for Δ.
→ Soft photon approximation (SPA) breaks down if the cut-off Δ is too big and numerical uncertainties become too large if Δ is too small.
→ A nice plateau can be found between 1 and 10 MeV.

$\mathcal{O}(\alpha^2)$ QED corrections to the asymmetry (P2 kinematics)

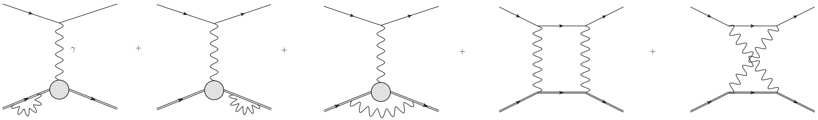


the shift of Q^2 is a kinematical effect included in 1γ radiation $\rightarrow$ very small $\mathcal{O}(\alpha^2)$ corrections to the asymmetry.

Hadronic Corrections

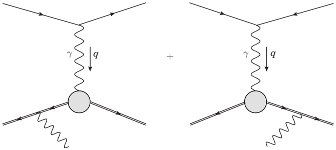
The subtraction of IR divergences is done according to Maximon & Tjon 2000.

- $\delta^{(1)}_{\text{1-loop}} \to \delta^{(1\ell)}_{\text{vert}} + \delta^{(1h)}_{\text{vert}} + \delta^{(1h)}_{\text{box}} \xrightarrow{\text{IR}} \delta^{\text{V}\ell}_{\text{IR}} + \delta^{\text{V}h}_{\text{IR}} + \delta^{\text{box}}_{\text{IR}}$



- $\mathcal{M}^h_{1\gamma} \to \mathcal{M}^h_{1s\gamma} = e\mathcal{M}_{0\gamma}\left(\frac{p_1\epsilon^*}{p_1k} - \frac{p_2\epsilon^*}{p_2k}\right)$

$$\delta^{(1)}_{1s\gamma}(\Delta) = \left(-\frac{\alpha}{4\pi^2}\right)\int_{E_\gamma<\Delta}\frac{\mathrm{d}^3k}{E_\gamma}\left(\frac{l_1}{l_1k} - \frac{l_2}{l_2k} - \frac{p_1}{p_1k} + \frac{p_2}{p_2k}\right)^2$$
$$= -\frac{\alpha}{\pi}\sum_{i,j=1}^{2}\left[B_{l_il_j}(Q^2,\Delta) + B_{p_ip_j}(Q^2,\Delta) + B_{l_ip_j}(Q^2,\Delta)\right] - \delta^{\text{V}\ell}_{\text{IR}} - \delta^{\text{V}h}_{\text{IR}} - \delta^{\text{box}}_{\text{IR}},$$



Hadronic Radiation

- The on-shell electro-magnetic vertex at the hadron side is given by

$$\Gamma_\mu(p_2, p_1) = F_1\left[(p_2 - p_1)^2\right]\gamma_\mu + F_2\left[(p_2 - p_1)^2\right] i\sigma_{\mu\sigma}\frac{(p_2 - p_1)^\sigma}{2M},$$

  where F_1 and F_2 are Pauli and Dirac form factors.

- The hadronic tensor for initial state radiation is given by

$$\mathcal{M}^i_{\mu\nu} = \overline{u}(p_2)\Gamma_\mu(p_2, p_1 - k)\frac{(p_1 - k + M)}{-2p_1 k}\Gamma_\nu(p_1 - k, p_1)u(p_1),$$

  where Γ_μ is now evaluated for off mass-shell values of one of its arguments.

- Same applies for final state radiation.

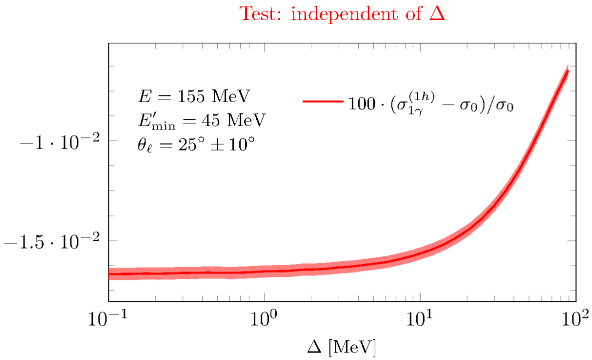
- Using the on-shell vertex structure also for off-shell values allows us to include hard-photon hadronic radiation. M. Vanderhaeghen et al. 2000

Hadronic Radiation

- The cross-section for one photon radiated at the hadron side,

$$\mathrm{d}\sigma_{1\gamma}^{(1h)} = \mathrm{d}\sigma_{0\gamma}(1 + \delta_{1s\gamma}^{(1h,\text{finite})}) + \int_{E_\gamma > \Delta} \mathrm{d}\sigma_{1\gamma}^{(1h)} ,$$

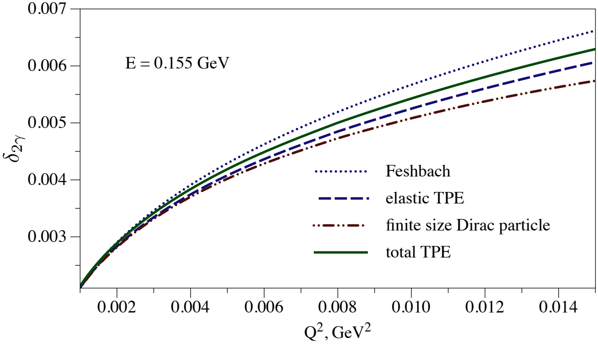
is also independent of the cut-off Δ.



Two-photon exchange

- O. Tomalak and M. Vanderhaeghen, 2015 evaluated the inelastic contribution at low Q^2 by approximating the double virtual Compton scattering with the unpolarized forward virtual Compton scattering.
- For P2 kinematics the inelastic contribution to two-photon exchange is negligible.
- The scattering cross section of relativistic massless electrons on a point-like charged target in Dirac theory (so-called Feschbach term), is a good approximation of the total two-photon exchange contribution.

Tomalak, private communication



Vacuum Polarization

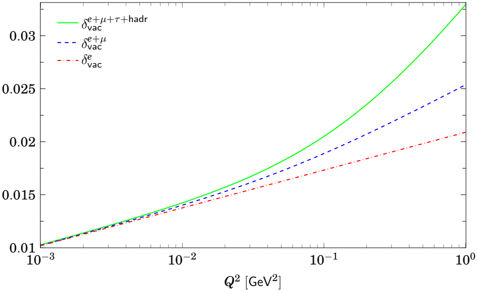
- The vacuum polarization correction can be absorbed in the fine-structure constant

$$\alpha_{\text{eff}}(q^2) = \frac{\alpha}{1 - \Pi(q^2)}.$$

- The contribution from lepton loops is given at first order by

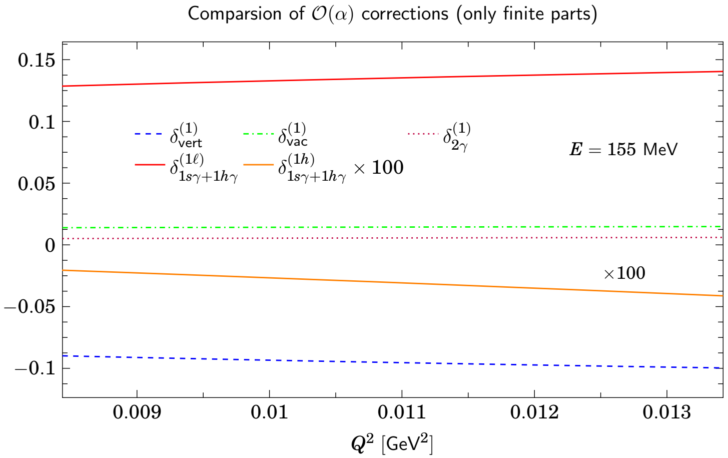
$$\delta^{\text{leptons}}_{\text{vac-pol}} = 2\Pi_{e+\mu+\tau}(q^2) = \delta^{e}_{\text{vac-pol}} + \delta^{\mu}_{\text{vac-pol}} + \delta^{\tau}_{\text{vac-pol}}$$

- The hadronic part of $\Pi(q^2)$ can be extracted from experimental data for the cross section of e^+e^- annihilation into hadrons. (Jegerlehner, 2011)



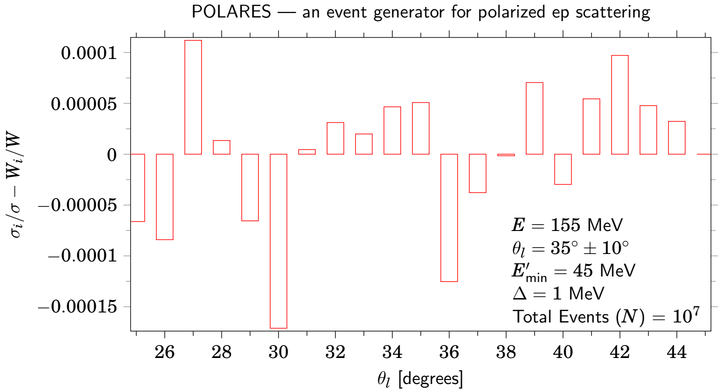
Comparison of $\mathcal{O}(\alpha)$ finite corrections

- The total $\mathcal{O}(\alpha)$ correction for P2 kinematics is $\approx 6\%$.
- Hadronic radiation contributes with $\approx -0.02\%$ and it is therefore negligible.



Conclusion

It is important to include full treatment of radiative corrections at the level of the event generator.



→ A technical comparison between the numerical integration of the cross-section for each bin and the weights produced by the event generator.

Some features of POLARES

- The event generator is written as a C++ library and can be easily combined with the simulation of the experiment.
- The P2 experiment uses a thick target, which causes energy losses. The consequence is that the energy of the scattering process can vary, up to a maximum of 155 MeV.
- POLARES can easily account for that, since it has the following structure
  - for $E \in (E_{\text{min}}, E_{\text{max}})$ initialization
  - while (i<no. of events) events(E)

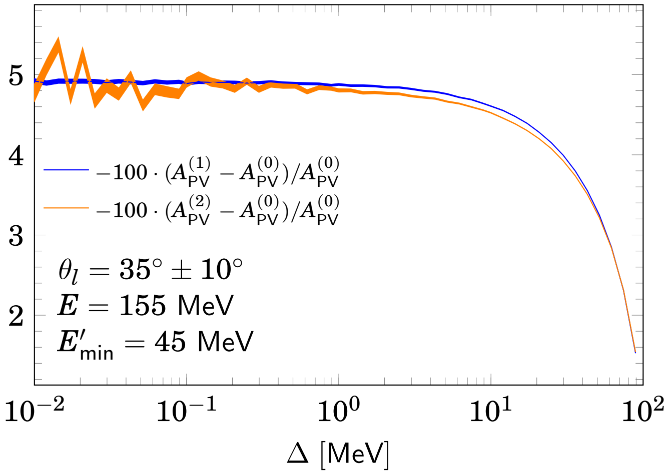
Overview

- Significant $\mathcal{O}(\alpha)$ QED corrections to the asymmetry were found
  $\rightarrow$ need full Monte Carlo treatment
- Very small $\mathcal{O}(\alpha^2)$ corrections to the asymmetry.
- Negligible $\mathcal{O}(\alpha)$ corrections from hadronic radiation.
- A modern, flexible, easy-to-use event generator was developed that will include complete $\mathcal{O}(\alpha^2)$ electroweak corrections.

Thank you for your attention!

Extra Slides

Total Asymmetry



Cross Section dependence on detector acceptance