Electroweak Boxes

Chien-Yeah Seng

Helmholtz-Institut für Strahlen- und Kernphysik
and
Bethe Center for Theoretical Physics,
Universität Bonn

cabrón

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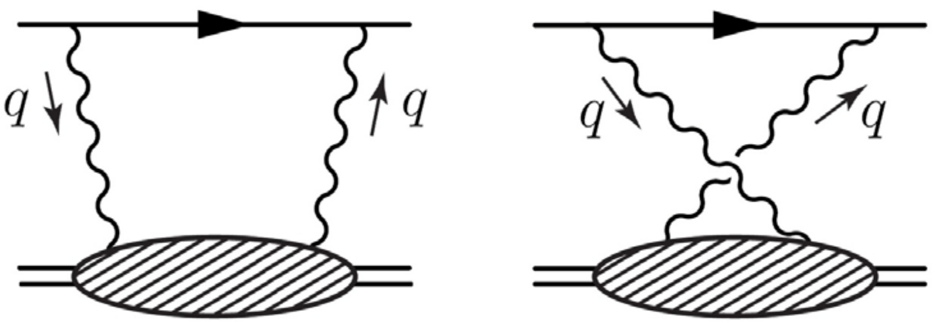
Outline

1. Electroweak Boxes: Overview
2. Dispersive Approach
3. First-Principle Calculation
4. Electroweak Boxes in a Nucleus
5. Summary

1. Electroweak Boxes: Overview

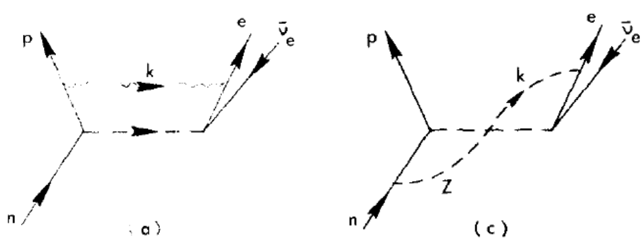
Electroweak Boxes: Overview

- **(Hadronic) Electroweak box diagrams**: Feynman diagrams involving the exchange of a **pair of EW gauge bosons** between a **lepton** and a **QCD bound state**. Appear in many important EW processes.



Forward EW boxes

- Some of their first appearances in history:



Sirlin, Nucl.Phys.B. 71,29 (1974)

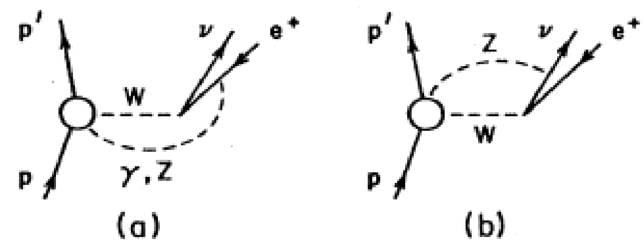


FIG. 6. Box diagrams involving the exchange of γ and Z between hadrons and leptons.

Sirlin, Rev. Mod. Phys. 50, 573 (1978)

Electroweak Boxes: Overview

- General structure of **(forward) EW box diagram amplitude**:

$$\Box \sim \int \frac{d^4 q}{(2\pi)^4} L^{\mu\nu} \frac{1}{q^2 - m_{B_1}^2} \frac{1}{q^2 - m_{B_2}^2} T_{\mu\nu}(p,q)$$

Lepton piece

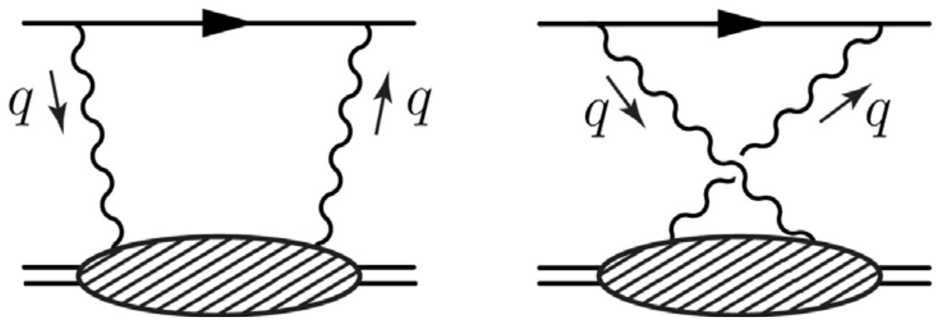
Hadron piece

- "Generalized" **forward Compton tensor**:

$$T^{\mu\nu}(p,q) \quad = \quad \int d^4 x e^{iq\cdot x} \langle N(p)| T[J_1^{\mu}(x) J_2^{\nu}(0)] |N(p)\rangle$$

- Two cases:
  1. When both gauge bosons are heavy: **Perturbative boxes**
  2. When at least one of them is photon: **Non-perturbative boxes**

Electroweak Boxes: Overview



$$\Box \sim \int \frac{d^4q}{(2\pi)^4} L^{\mu\nu} \frac{1}{q^2 - m_{B_1}^2} \frac{1}{q^2 - m_{B_2}^2} T_{\mu\nu}(p,q)$$

- Both bosons are heavy: sensitive only **to large Q²**. **OPE** gives:

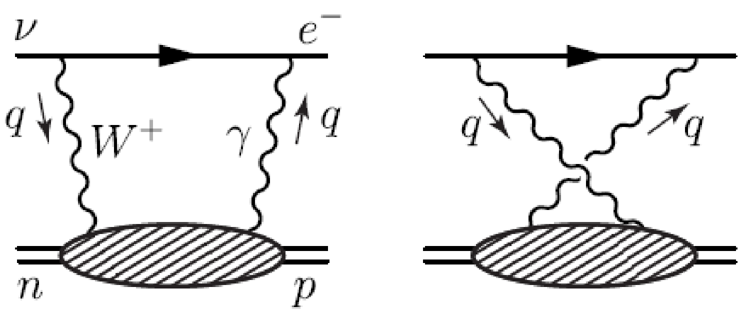
$$\int d^4x e^{iq\cdot x} T[J_1^\mu(x) J_2^\nu(0)] \sim \frac{i}{q^2}[(q^\mu g^{\nu\lambda} - g^{\mu\nu} q^\lambda + q^\nu g^{\mu\lambda}) J'_\lambda(0) + i\varepsilon^{\mu\nu\alpha\lambda} q_\alpha J''_\lambda(0)]$$

- pQCD corrections can also be included.

- When **at least one boson is massless**, then the result is **sensitive to all Q²**, so OPE does not tell the whole story.

Electroweak Boxes: Overview

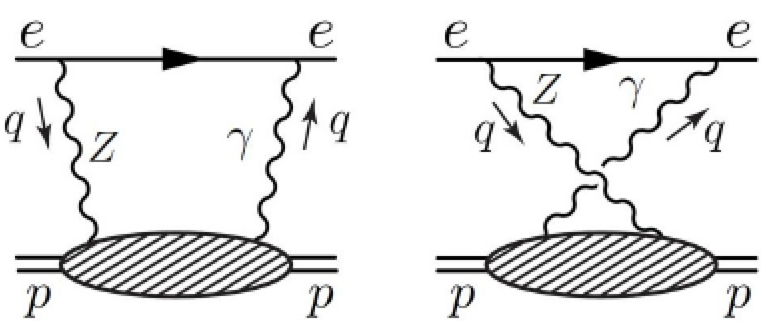
- Examples of **Non-Perturbative EW Boxes**:
  - (1) **γW-box** in **neutron/nuclear beta decay**:



$$|V_{ud}|^2 = \frac{2984.432(3)\ \mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

Super-allowed beta decay

  - (2) **γZ-box** in **P-odd lepton-nucleon/nucleus scattering**:



$$Q_W^p = (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z},$$

Proton weak charge

They represent one of the main contributors of theoretical uncertainty in their respective processes!

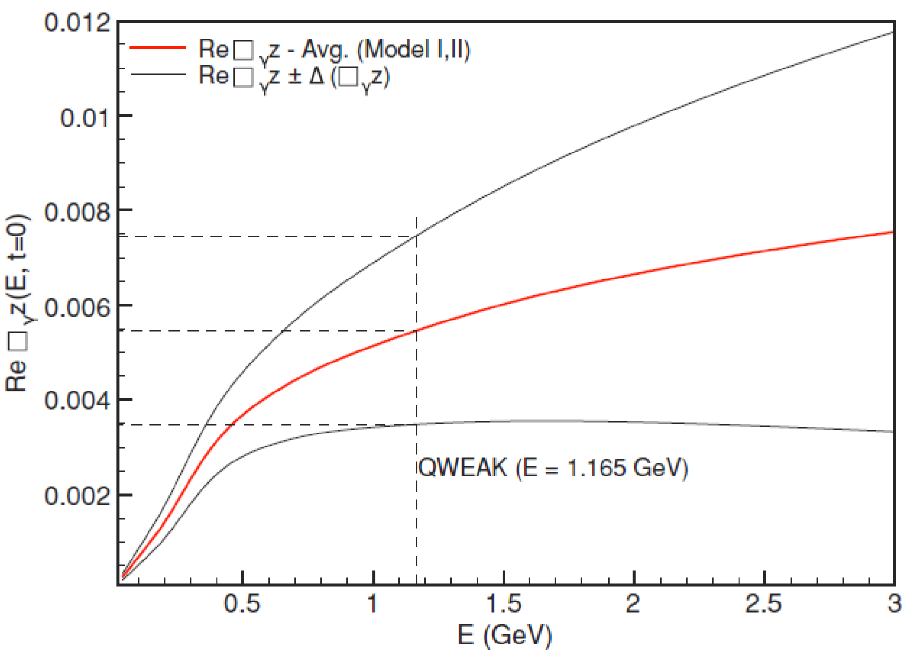
Electroweak Boxes: Overview

- Further decomposition into "**vector**" and "**axial**" box according to parity:

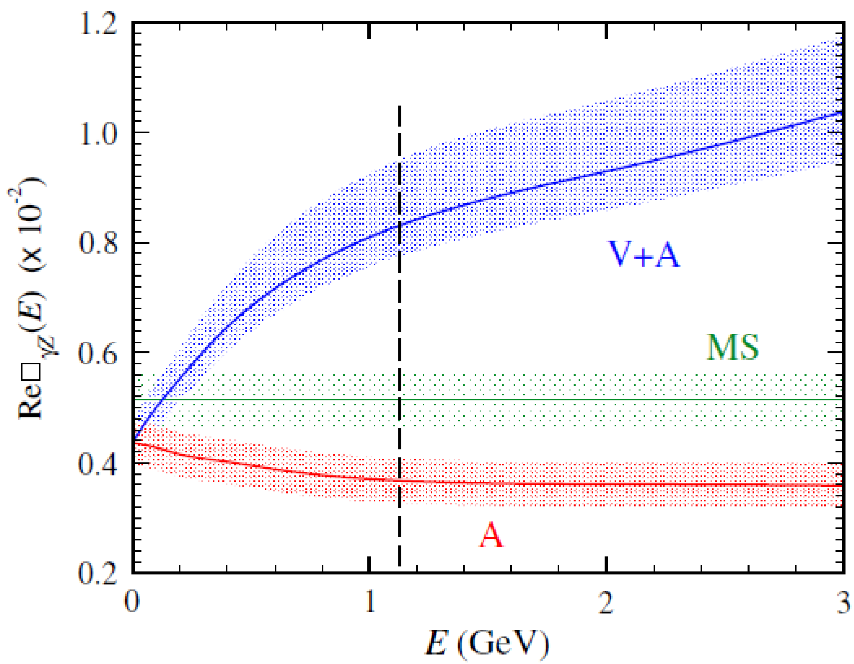
$$T^{\mu\nu} = \int d^4x e^{iq\cdot x} \langle N(p)| T[J^\mu_{em}(x) J^\nu_B(0)] |N(p)\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{p\cdot q} T_2 + i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_\beta}{2p\cdot q} T_3 + \ldots$$

- Example in **γZ box**:



Gorchtein et al, Phys.Rev.C84, 015502 (2011)



Blunden et al, Phys.Rev.Lett.,107,081801(2011)

Electroweak Boxes: Overview

- The axial box is more relevant in:
  - ep-scattering at very low energy
  - Studies of "model-dependent" radiative corrections (RC) in beta decay

$$T^{\mu\nu}(p,q) = \int d^4x e^{iq\cdot x} \langle N(p)| T[J_1^\mu(x) J_2^\nu(0)] |N(p)\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{p\cdot q} T_2 - i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_\beta}{2p\cdot q} \boxed{T_3} + \ldots$$

$$\Box \sim \mathrm{Re} \int \frac{d^4q}{(2\pi)^4} \frac{m_B^2}{m_B^2 + Q^2} \frac{Q^2 + \nu^2}{(Q^2)^2} \frac{T_3(\nu, Q^2)}{\nu}$$

$$\nu = p\cdot q / m_N$$

2. Dispersive Approach

CYS, M.Gorchtein, H.H.Patel and M.J.Ramsey-Musolf, Phys.Rev.Lett. 121 (2018) no. 24, 241804

CYS, M.Gorchtein and M.J.Ramsey-Musolf, arXiv:1812.03352

Dispersive Approach

- T_3 depends **on virtual intermediate states**: theoretical modeling is less transparent
- **Dispersive treatments** to box diagrams are developed since the last ten years, relating the former to matrix elements of **on-shell intermediate states**

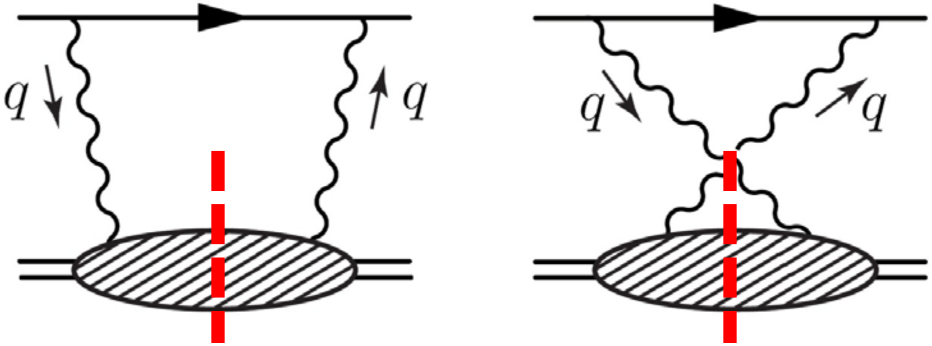
$$T^{\mu\nu}(p,q) = \int d^4x e^{iq\cdot x} \langle N(p)| T[J_1^\mu(x) J_2^\nu(0)] |N(p)\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{p\cdot q} T_2 - i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_\beta}{2p\cdot q} T_3 + \ldots$$

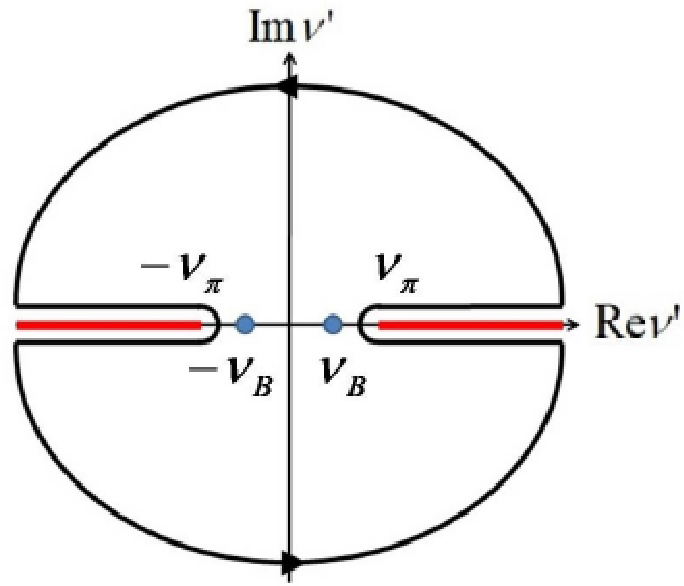
Hadronic tensor in inclusive scattering:

$$W^{\mu\nu}(p,q) = \frac{1}{4\pi} \int d^4x e^{iq\cdot x} \langle N(p)| [J_1^\mu(x), J_2^\nu(0)] |N(p)\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{p\cdot q} F_2 - i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_\beta}{2p\cdot q} F_3 + \ldots$$

Dispersive Approach

- **Dispersion relation:**





$$T_3^N(\omega, Q^2) = -4i\omega \int_0^1 dx \frac{F_3^N(x, Q^2)}{1 - \omega^2 x^2}.$$

$$\omega = 1/x_B = 2p\cdot q/Q^2$$

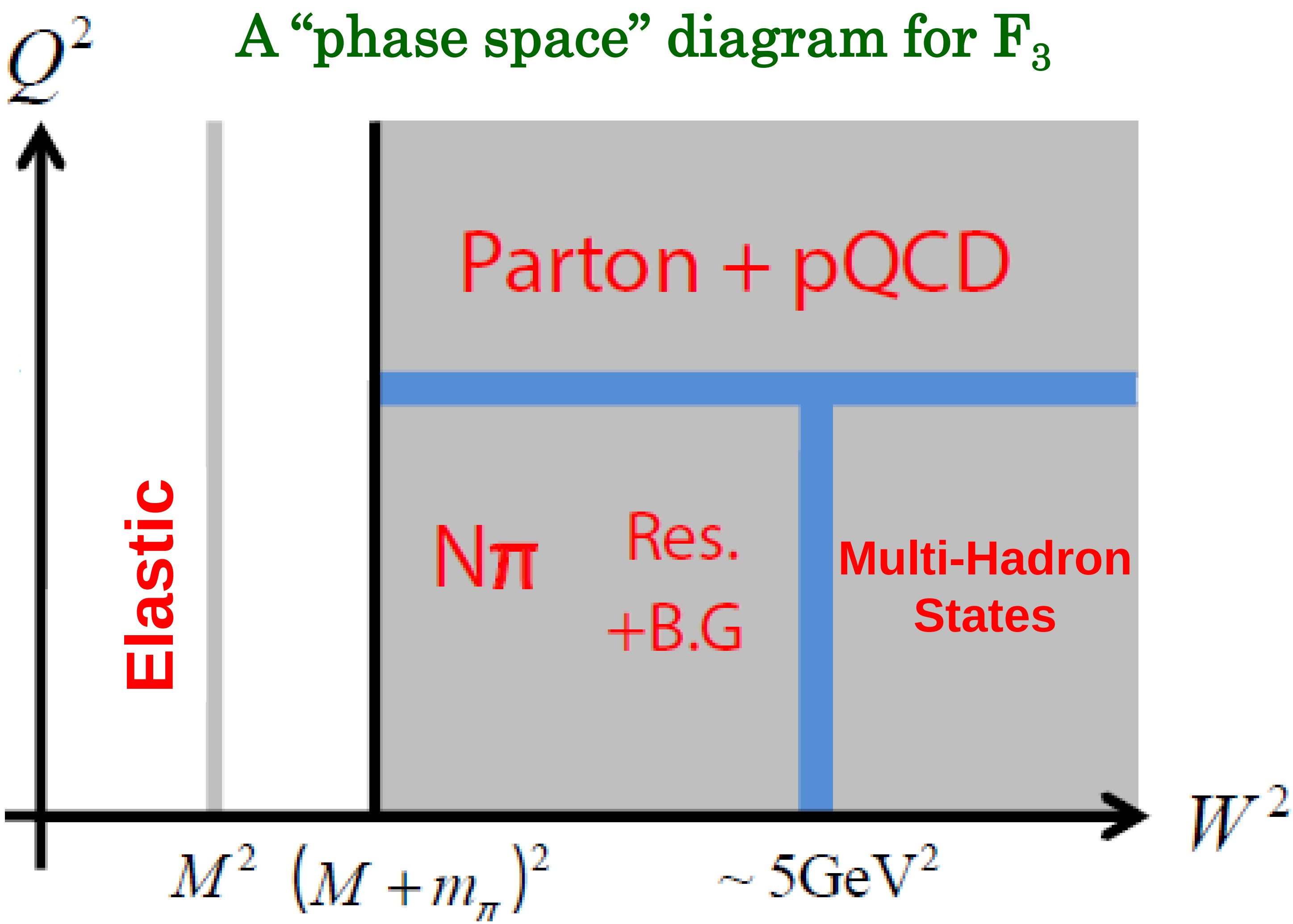
- Box diagrams are expressed in terms of the "**First Nachtmann moment**" of F_3:

$$\Box \sim \int_0^\infty \frac{dQ^2}{Q^2} \frac{m_B^2}{m_B^2 + Q^2} M_1[F_3^N]$$

Central result!!!

$$M_1[F_3^N] = \int_0^1 dx \Pi(x, Q^2) F_3^N(x, Q^2), \qquad \Pi(x, Q^2) = \frac{4}{3} \frac{1 + 2\sqrt{1 + 4m_N^2 x^2/Q^2}}{(1 + \sqrt{1 + 4m_N^2 x^2/Q^2})^2}$$

Dispersive Approach



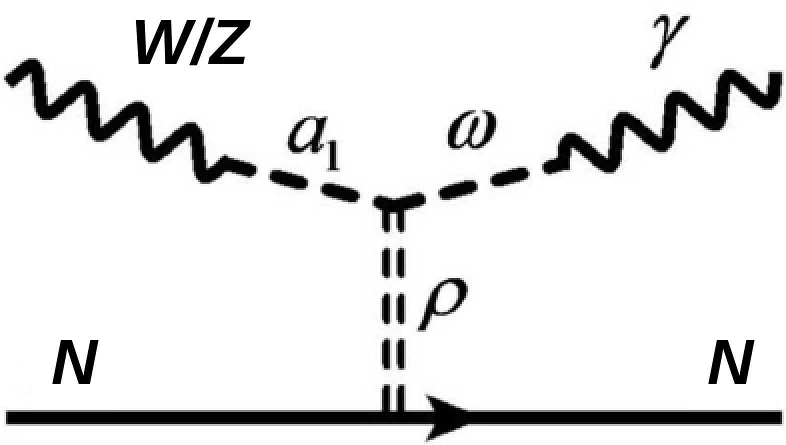
Dispersive Approach

$$F_3^N = F_{3,\mathrm{el}}^N + F_{3,\mathrm{inel}}^N$$

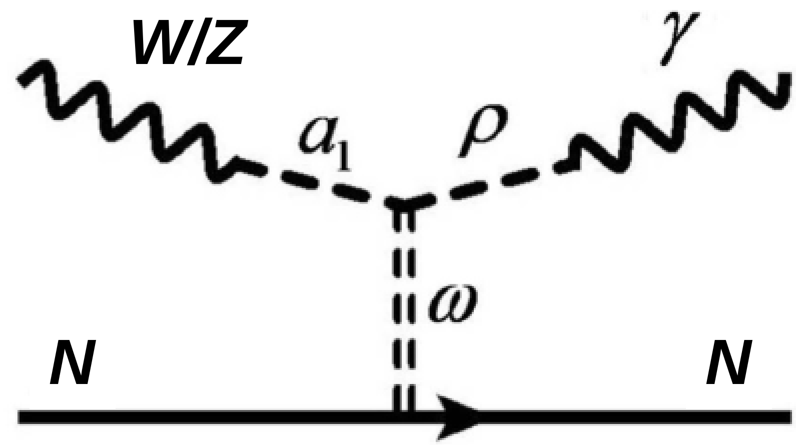
$$F_{3,\mathrm{inel}}^N = \begin{cases} F_{3,\mathrm{DIS}}^N & Q^2 > 2\mathrm{GeV}^2 \\ F_{3,N\pi}^N + F_{3,\mathrm{res}}^N + F_{3,\mathbb{R}}^N & Q^2 < 2\mathrm{GeV}^2 \end{cases}$$

Elastic, DIS, Nπ, resonances: rather straightforward

Multi-hadron states: Regge model + VDM

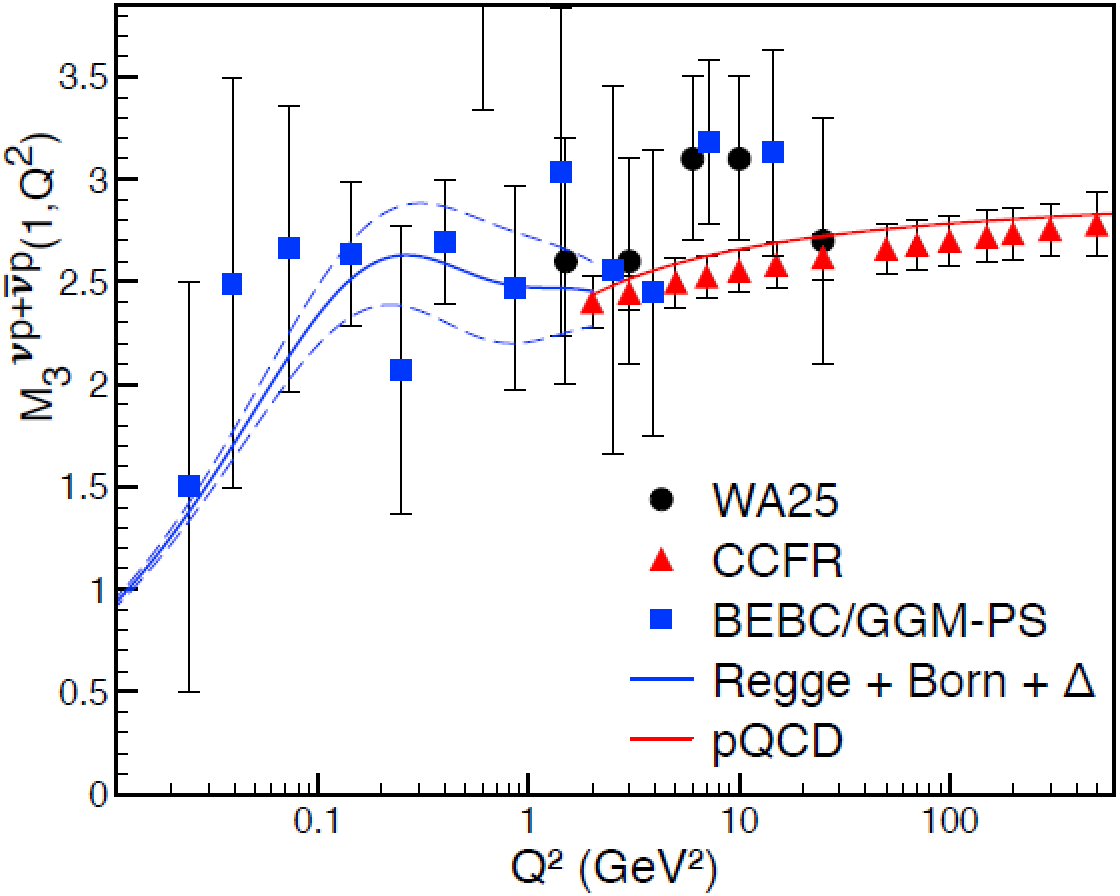


Isoscalar EM current



Isovector EM current

Dispersive Approach



Matching the 1st Nachtmann moment of the **isovector** piece **to ν p/νbar p scattering data**

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 m_N E}{\pi(1+Q^2/m_W^2)^2}\left[xy^2F_1^{\nu(\bar{\nu})} + \left(1-y-\frac{m_N xy}{2E}\right)F_2^{\nu(\bar{\nu})} \pm x\left(y-\frac{y^2}{2}\right)F_3^{\nu(\bar{\nu})}\right]$$

Isoscalar piece is then deduced using **Regge model+VDM**

Dispersive Approach

- **Recent success** of such treatment:
  - Reduced hadronic uncertainty in the **determination of V_{ud}**:

DR+data

$$\Delta_R^V : \quad 0.024(8) \longrightarrow 0.02361(38) \longrightarrow 0.02467(22)$$

1986 2006 2018

CYS, M.Gorchtein, H.H.Patel and M.J.Ramsey-Musolf, Phys.Rev.Lett. 121 (2018) no. 24, 241804

  - Same method applied to the **axial γZ-box in ep-scattering**:

DR DR+data

$$\Box_{\gamma Z}^A(0) : \quad 0.052(5) \longrightarrow 0.0044(4) \longrightarrow 0.0045(2)$$

2003 2011 2019

J.Erler, M.Gorchtein, O.Koshchii, CYS, H.Spiesberger, in preparation

- **Possible issues**:
  - Quality of the neutrino data?
  - Residual model-dependence?

which leads to the following discussions

3. First-Principle Calculation

CYS and Ulf-G. Meissner, arXiv:1903.07969

First-Principle Calculation

$$\Box \sim \int_0^\infty \frac{dQ^2}{Q^2}\frac{m_B^2}{m_B^2+Q^2}M_1[F_3^N]$$

- Recall the that we are interested in $M_1[F_3^N]$ as a function of Q^2 . Neutrino data helps identifying **dominant contributors at different Q^2** :

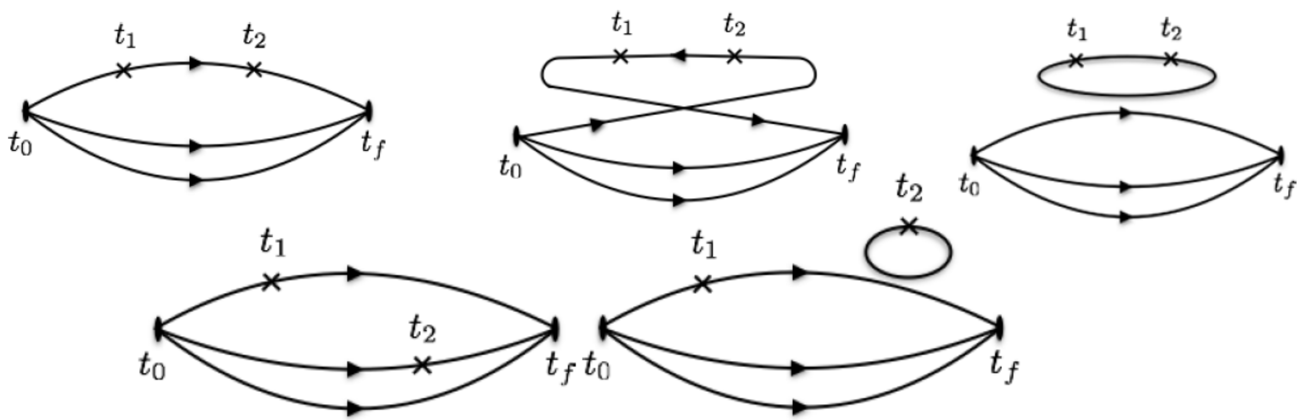
$$M_1[F_3^N] = \begin{cases} \approx \text{elastic} + \Delta & Q^2 < 0.1\text{GeV}^2 \\ \text{multi} - \text{hadron states} & 0.1\text{GeV}^2 < Q^2 < 2\text{GeV}^2 \\ \text{DIS} & Q^2 > 2\text{GeV}^2 \end{cases}$$

- Therefore, to remove the hadronic uncertainties in the box diagrams, we need to have a good handle of the **first Nachtmann moment of F_3 at moderate Q^2**.
- Question: is there a way to calculate $M_1[F_3^N]$ from **FIRST-PRINCIPLE**?

First-Principle Calculation

$$W^{\mu\nu} = \frac{1}{4\pi}\sum_X (2\pi)^4\delta^4(p+q-p_X)\,\langle N(p)|\,J^{\mu}_{em}\,|X\rangle\,\langle X|\,J^{\nu}_{A}\,|N(p)\rangle$$

- Difficult because it involves **a sum of all on-shell intermediate states.**
- Recently-developed techniques in lattice calculation of PDFs (quasi-PDF, pseudo-PDF, lattice cross-section etc) do not apply because they rely on OPE that holds only at large Q^2.
- We wish to **avoid direct calculations of four-point functions** (noisy contractions, complicated finite-volume effect…)



J. Liang, K-F. Liu and Y-B. Yang, EPJ Web Conf. 175 (2018) 14014

First-Principle Calculation

- A more promising approach is through the **Feynman-Hellmann theorem (FHT)**:

$$\frac{dE_{n,\lambda}}{d\lambda} = \left\langle n_\lambda \middle| \frac{\partial H_\lambda}{\partial \lambda} \middle| n_\lambda \right\rangle$$

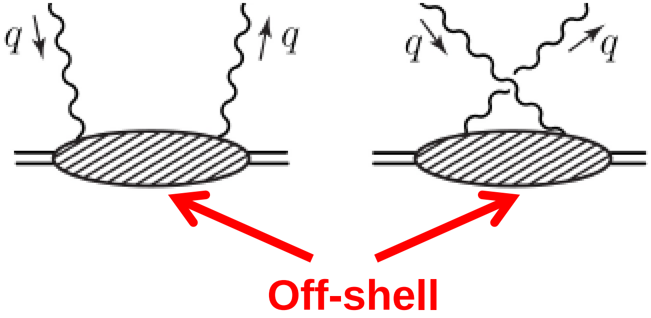
- **Shift in energy level→ matrix element**. Extraction of energy levels on lattice are more straightforward, avoid complicated contraction diagrams.
- **Momentum transfer** could be introduced through **periodic external potential**.
- Shows great potential in studies of:
  - Nucleon axial charge and sigma term
  - EM form factors
  - Compton amplitude
  - P-even structure functions
  - Hadron resonances
  - ……

First-Principle Calculation

Some warm-up:

Kinematics: $q^\mu = (0, \vec{q}) \implies \omega = -\frac{2\vec{p}\cdot\vec{q}}{Q^2}$

"Off-shell condition": $|\omega| < 1 \implies E(\vec{p}\pm\vec{q}) > E(\vec{p})$



- Consider a **periodic potential**: $V(\vec{x}) = V_0 \cos(\vec{q}\cdot\vec{x}) = \frac{1}{2}V_0(e^{i\vec{q}\cdot\vec{x}} + e^{-i\vec{q}\cdot\vec{x}})$

$$V(x)\psi_{\vec{p}}(\vec{x}) \sim \psi_{\vec{p}+\vec{q}}(\vec{x}) + \psi_{\vec{p}-\vec{q}}(\vec{x})$$

- The off-shell condition prohibits mixing of degenerate states through perturbation. Thus, non-degenerate perturbation theory at 1^{st}-order gives:

$$\langle\vec{p}|\, V\, |\vec{p}\rangle \sim \langle\vec{p}|\, \vec{p}\pm\vec{q}\rangle = 0$$

No first-order energy shift!

First-Principle Calculation

Our Strategy:

- Introduce **TWO** periodic source terms, and study the **SECOND ORDER ENRGY SHIFT**:

$$H_\lambda = H_0 + 2\lambda_1 \int d^3x \cos(\vec{q}\cdot\vec{x}) J^2_{em}(\vec{x}) - 2\lambda_2 \int d^3x \sin(\vec{q}\cdot\vec{x}) J^3_A(\vec{x})$$

$$\left(\frac{\partial^2 E_{N,\lambda}(\vec{p})}{\partial\lambda_1\partial\lambda_2}\right)_{\lambda=0} = \frac{iq_x}{Q^2\omega} T_3^N(\omega, Q^2).$$

CYS and U.G-Meissner, hep-ph/1903.07969

- Plugging it into the dispersion relation of T_3:

$$\left(\frac{\partial^2 E_{N,\lambda}(\vec{p})}{\partial\lambda_1\partial\lambda_2}\right)_{\lambda=0} = \frac{4q_x}{Q^2}\int_0^1 dx \frac{F_3^N(x,Q^2)}{1-\omega^2 x^2},$$

Central result!!!

2nd order Energy shift → (FHT) → Generalized Forward Compton tensor → (DR) → Structure Function

First-Principle Calculation

- Lattice momenta are **discrete**:

$$\vec{p} = \frac{2\pi}{L}(n_{px}, n_{py}, n_{pz}), \qquad \vec{q} = \frac{2\pi}{L}(n_{qx}, n_{qy}, n_{qz})$$

- Requiring Q^2 at the hadronic scale and the off-shell condition imply:

$$\frac{4\pi^2}{L^2}(n_{qx}^2 + n_{qy}^2 + n_{qz}^2) \lesssim 1\,\mathrm{GeV}^2$$
$$\frac{2|n_{px}n_{qx} + n_{py}n_{qy} + n_{pz}n_{qz}|}{n_{qx}^2 + n_{qy}^2 + n_{qz}^2} < 1.$$

- **A concrete example**: $L \approx 2.8\mathrm{fm}$ $\quad \vec{q} = \frac{2\pi}{L}(2, 1, 0)$ impose the restriction:

$Q^2 \approx 1\mathrm{GeV}^2$

Allowed values for ω: $|\omega| = 0, \ \frac{2}{5}, \ \frac{4}{5}$

First-Principle Calculation

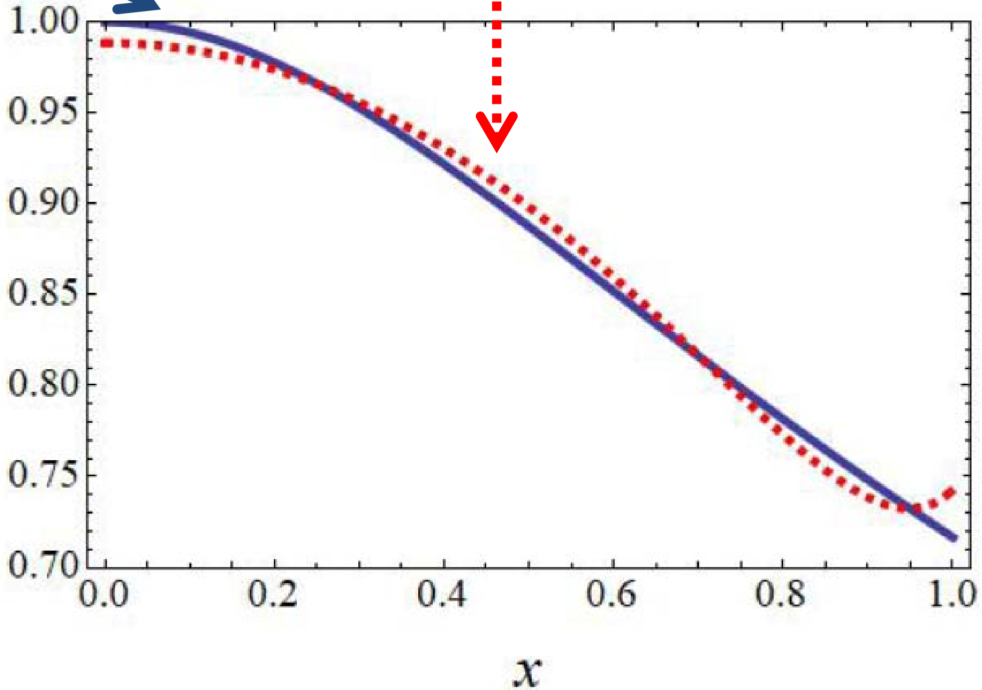
Reconstructing the first Nachtmann moment from energy shifts

$$M_1[F_3^N] = \int_0^1 dx \Pi(x,Q^2) F_3^N(x,Q^2),$$

$$\frac{Q^2}{4q_x}\left(\frac{\partial^2 E_{N,\lambda}}{\partial\lambda_1\partial\lambda_2}\right)_{\lambda=0} = \int_0^1 dx \Lambda(x,\omega) F_3^N(x,Q^2)$$

$$\Lambda(x,\omega) = (1-\omega^2 x^2)^{-1}$$

$$\Pi(x,Q^2) \approx a\Lambda(x,0) + b\Lambda(x,2/5) + c\Lambda(x,4/5)$$



$$a = 3.81529$$
$$b = -3.01135$$
$$c = 0.184491$$

A very good reconstruction!

First-Principle Calculation

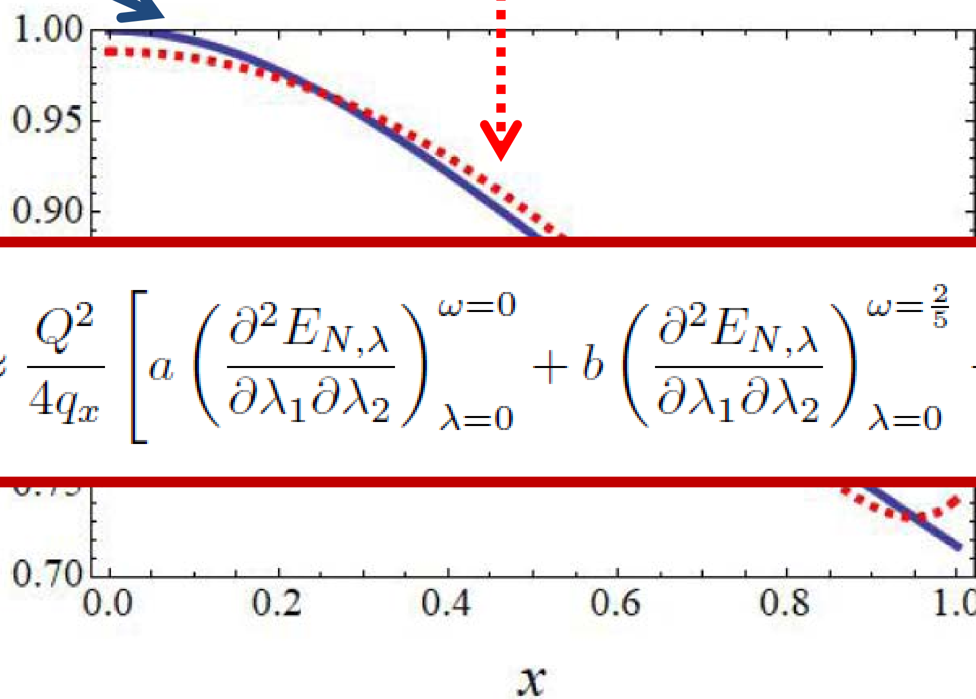
Reconstructing the first Nachtmann moment from energy shifts

$$M_1[F_3^N] = \int_0^1 dx \Pi(x, Q^2) F_3^N(x, Q^2),$$

$$\frac{Q^2}{4q_x}\left(\frac{\partial^2 E_{N,\lambda}}{\partial\lambda_1\partial\lambda_2}\right)_{\lambda=0} = \int_0^1 dx \Lambda(x,\omega) F_3^N(x, Q^2)$$

$$\Lambda(x,\omega) = (1-\omega^2 x^2)^{-1}$$

$$\Pi(x, Q^2) \approx a\Lambda(x, 0) + b\Lambda(x, 2/5) + c\Lambda(x, 4/5)$$



$$a = 3.81529$$
$$b = -3.01135$$
$$c = 0.184491$$

$$M_1[F_3^N] \approx \frac{Q^2}{4q_x}\left[a\left(\frac{\partial^2 E_{N,\lambda}}{\partial\lambda_1\partial\lambda_2}\right)^{\omega=0}_{\lambda=0} + b\left(\frac{\partial^2 E_{N,\lambda}}{\partial\lambda_1\partial\lambda_2}\right)^{\omega=\frac{2}{5}}_{\lambda=0} + c\left(\frac{\partial^2 E_{N,\lambda}}{\partial\lambda_1\partial\lambda_2}\right)^{\omega=\frac{4}{5}}_{\lambda=0}\right]$$

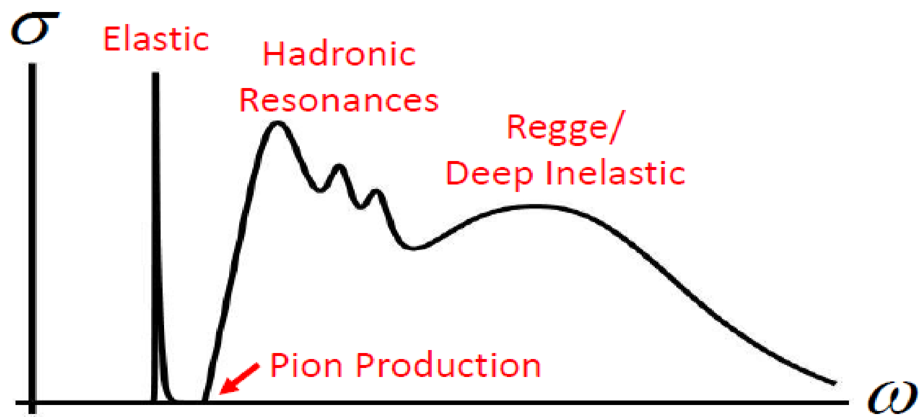
4. Electroweak Boxes in a Nucleus

CYS, M.Gorchtein and M.J.Ramsey-Musolf, arXiv:1812.03352

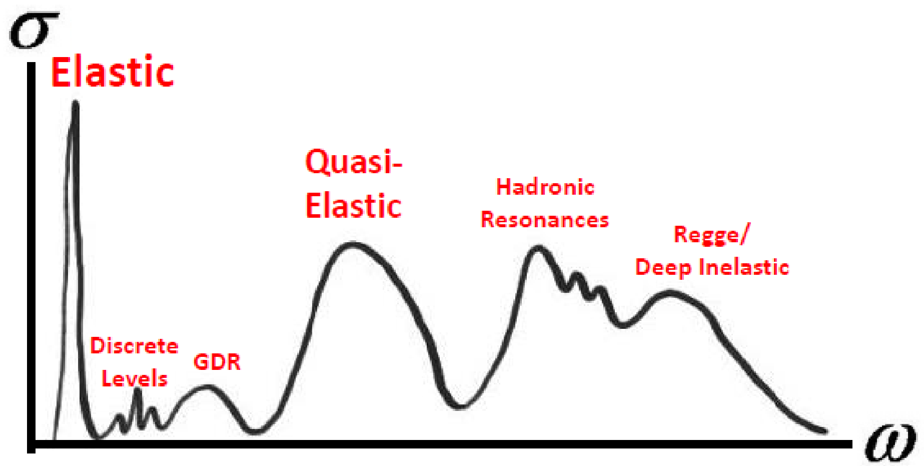
Electroweak Boxes in a Nucleus

- Dispersive treatment can also be applied to **EW boxes in a nucleus**
- However, there is a change in the response structure in the nuclear environment:

"Nucleon" response function



"Nuclear" response function



Electroweak Boxes in a Nucleus

- "**Traditional**" treatment:

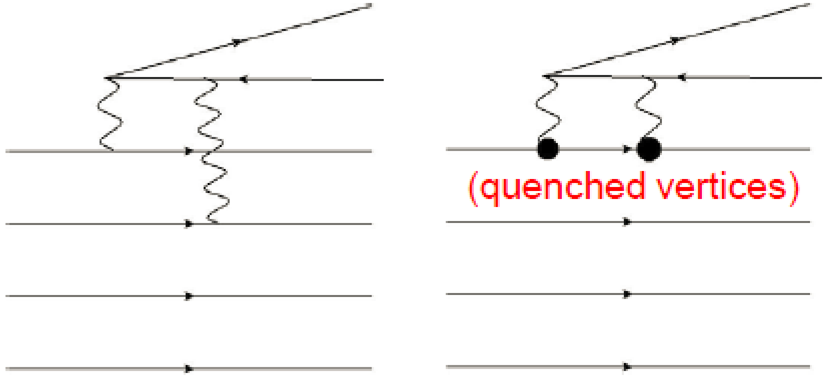
"Nucleus-independent" RC, described before

$$\Box_{\text{nucleus}} = \Box_{\text{nucleon}} + (\Box_{\text{nucleus}} - \Box_{\text{nucleon}})$$

"Nuclear-structure corrections": studied with NR nuclear models

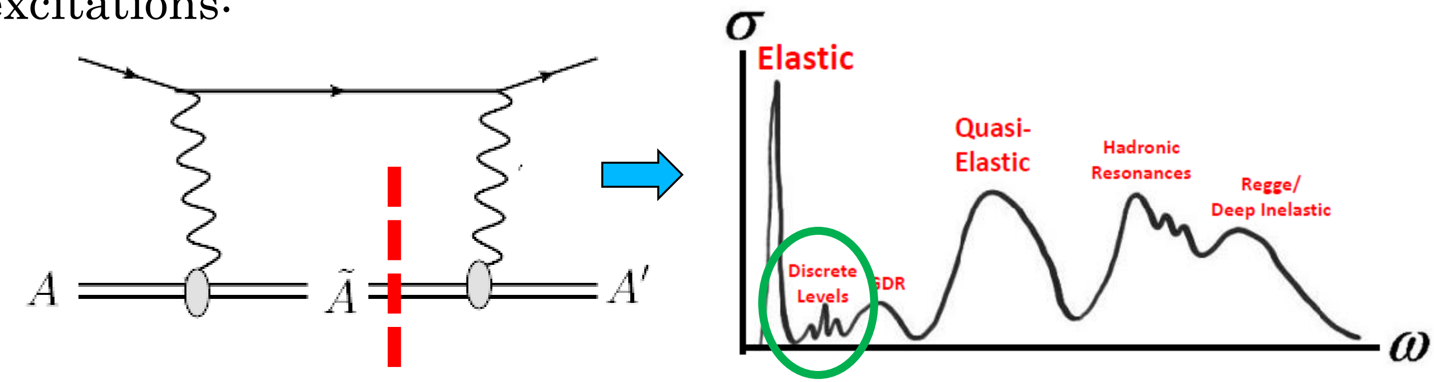
- Concerns: additional **model-dependence** in such separation?

- Some of the nuclear effects studied in beta decay:

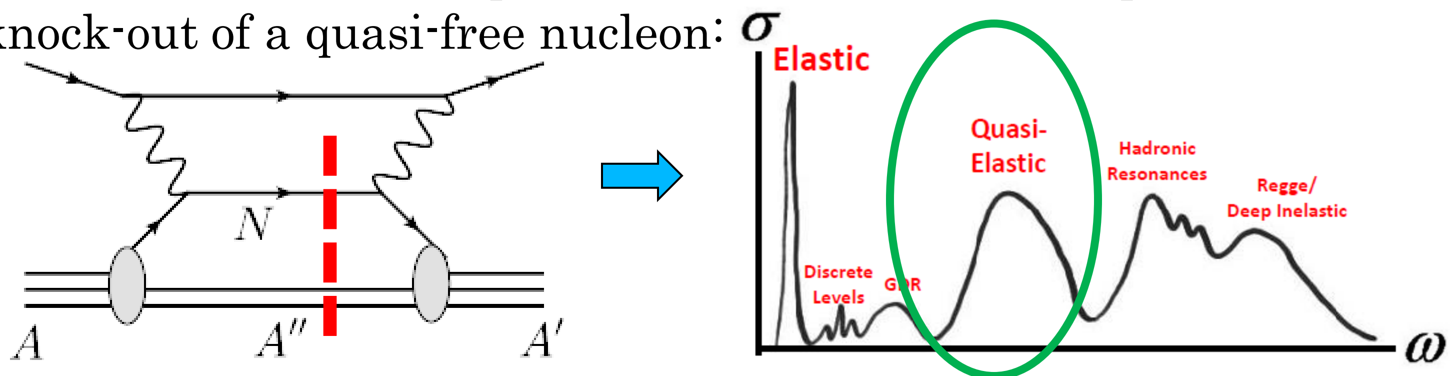


Electroweak Boxes in a Nucleus

- In the **Hardy and Towner (HT)** treatment, (part of) the nuclear-structure effect in nuclear beta decay is incorporated by simply inserting **the "quenched" form factors** of nucleon in a nucleus. This effectively describes the contribution from the lowest-lying nuclear excitations:



- But that does not capture the broader, more important **quasi-elastic peak** in the nuclear response function, which corresponds to the knock-out of a quasi-free nucleon:



Electroweak Boxes in a Nucleus

- We estimate the **quasi-elastic contribution** in a Fermi-gas model.

Quasi-free nucleon

Spectator nucleus

$$|A\rangle = \sqrt{2E_A}\sum_{p\in A}\int \frac{d^3\vec{k}\phi^p_A(k)|p(\vec{k}), A-p(-\vec{k})\rangle}{(2\pi)^3\sqrt{2E_{A-1}\,2E_n}}$$

Fermi distribution

- Still in exploratory phase. More advanced calculations of the QE single-nucleon knock-out contribution using up-to-date nuclear theory are necessary.

Summary

1. I described some general features of forward EW box diagrams for a free nucleon.
2. I explained the dispersion-relation formalism which is a useful starting point in treatments of these diagrams, and described some recent progress based on this method.
3. I outlined a procedure to compute second-order energy shifts on lattice which, combining with dispersion relation, will lead to a first-principle calculation of EW boxes.
4. Dispersion relation provides a universal basis to treat the nucleon and nuclear EW boxes on an equal footing.