COULOMB DISTORTION IN POLARIZED ELECTRON-NUCLEUS SCATTERING

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The Physics Case of the Weak Charge of Carbon-12

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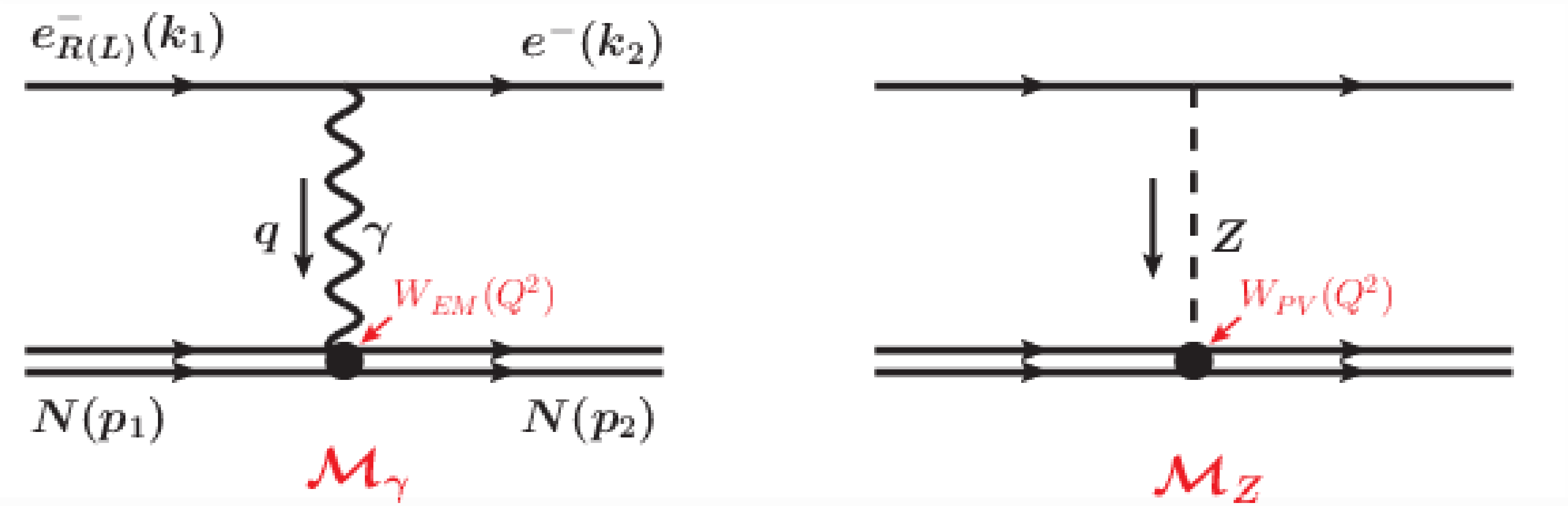
OUTLINE

- PV asymmetry and implications of respective measurements
- Coulomb distortion and related uncertainties
- Beam-normal single-spin asymmetry (SSA) and its interplay with PV measurements
- Perturbative and Coulomb distortion calculations of the beam-normal single-spin asymmetry (SSA)
- Conclusions

INTRODUCTION TO PV ASYMMETRY



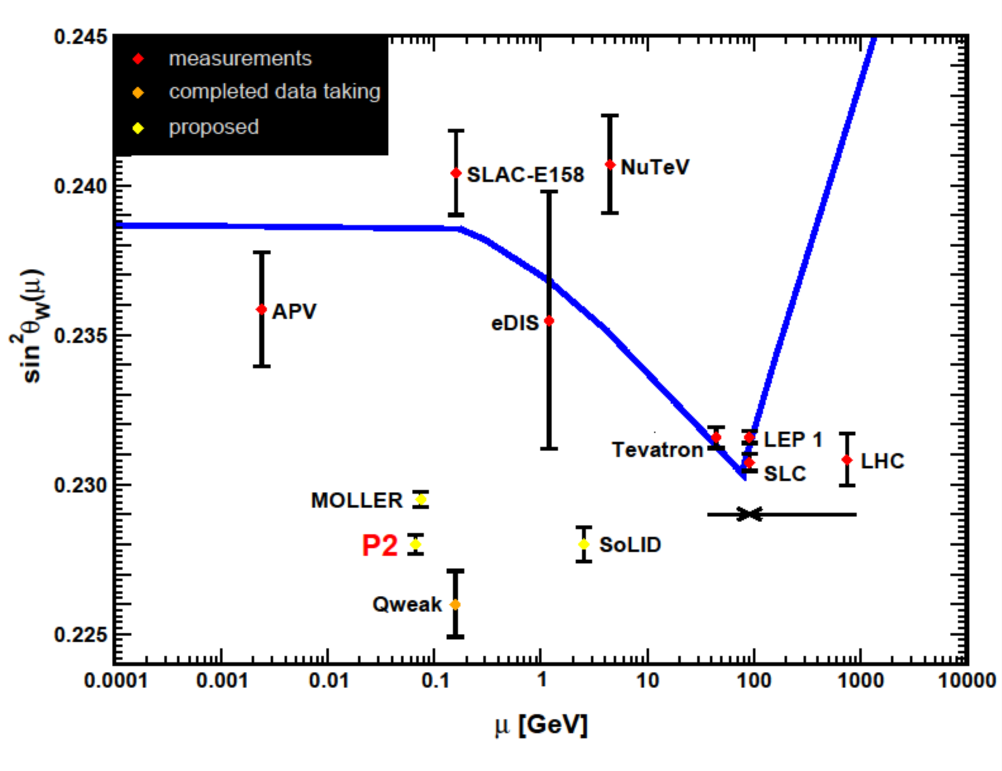
$$A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = -\frac{G_F}{\sqrt{2}} \frac{Q^2}{4\pi\alpha_{EM}} \frac{W_{PV}}{W_{EM}} = A_0 \frac{W_{PV}}{W_{EM}}$$

$$d\sigma_{R(L)} \sim \left| M_\gamma + M_Z^{R(L)} \right|^2$$

Weak charge of nucleus (Z protons, N neutrons):

$$\frac{Q_W^{Z,N}}{Z} = \lim_{Q^2 \to 0} \frac{W_{PV}}{W_{EM}} \Bigg|_{E_{beam}=0}$$

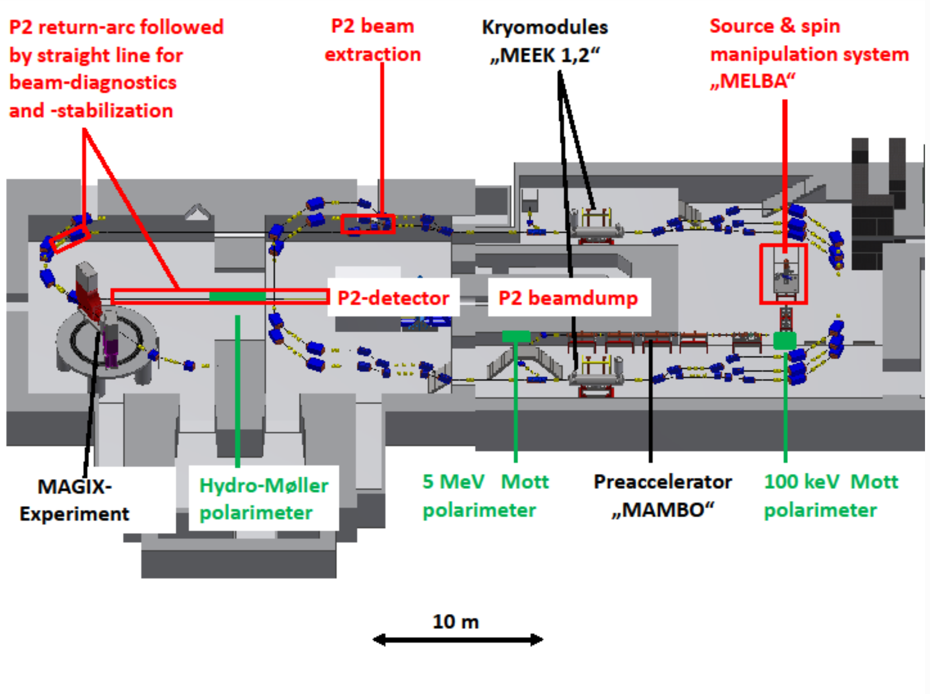
WEAK CHARGE AS A PRECISION TEST OF SM



[Becker et al., EPJA, 2018]

Tree-level: $Q_W^{Z,N} = Z\left(1-4\sin^2\theta_W\right)-N$

P2 @ MESA



[Becker et al., EPJA, 2018]

- P2 experiment
  - Electron beam energy E=155 MeV (150 μA)
  - Polarization > 85%
  - High runtime (more than 4000 h/year)
  - Scattering angle 20 ± 10 deg
  - 60 cm liquid hydrogen target
  - Theory + Exp. uncertainty ~1.8%
- C-12 measurement
  - C-12 target
  - Interesting physics case if uncertainty ~0.3%
  - German-Mexican collaboration research grant: theory predictions within the SM, including QED and hadronic uncertainties

WEAK CHARGES

- Nucleus:

$$Q_W^{Z,N} \approx Z\left(1-4\sin^2\theta_W\right)-N$$

- Proton: The weak charge is highly sensitive to the weak mixing angle.

$$Q_W^p \approx 1-4\sin^2\theta_W \approx 0.08 \quad \Rightarrow \quad \Delta\sin^2\theta_W/\sin^2\theta_W \approx 0.09\,\Delta Q_W^p/Q_W^p$$

- C-12: Theoretically easy to handle, significantly reduced beam time.

$$Q_W^{^{12}C} \approx -24\sin^2\theta_W \quad \Rightarrow \quad \Delta\sin^2\theta_W/\sin^2\theta_W = \Delta Q_W^{^{12}C}/Q_W^{^{12}C}$$

- Neutron: Weak interactions probe neutrons inside the nucleus.

$$Q_W^n \approx -1$$

NEUTRON SKIN

PV asymmetry: $A_{PV} = A_0 W_{PV} / W_{EM}$

Response functions:

$$W_{EM}(Q^2) = \int d^3 r \rho_{EM}(\vec{r}) e^{i\vec{q}\cdot\vec{r}} = Z\left(1 - \frac{Q^2}{6} R_{ch}^2 + \ldots\right)$$

$$W_{PV}(Q^2) = \int d^3 r \rho_W(\vec{r}) e^{i\vec{q}\cdot\vec{r}} = Q_W\left(1 - \frac{Q^2}{6} R_W^2 + \ldots\right)$$

RMS radii:

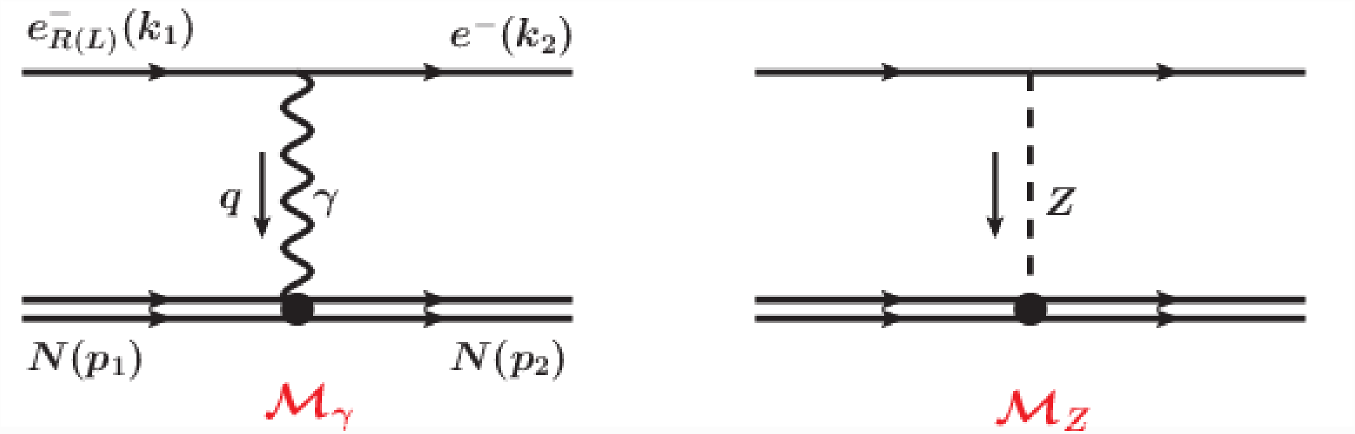
$$R_{ch} = \left(\frac{4\pi}{Z} \int dr\ r^4 \rho_{EM}(r)\right)^{1/2} \qquad R_W = \left(\frac{4\pi}{Q_W} \int dr\ r^4 \rho_W(r)\right)^{1/2}$$

Weak skin: $\Delta R = R_W - R_{ch}$ Neutron skin: $\Delta R_{np} = R_n - R_p$

The neutron weak charge is much larger than that of the proton, so we get access (free from strong interaction uncertainties) to neutron density distribution by studying the PV asymmetry.

[Donnelly, Dubach, Sick, NPA, 1989]

PERTURBATIVE CALCULATION OF PV ASYMMETRY

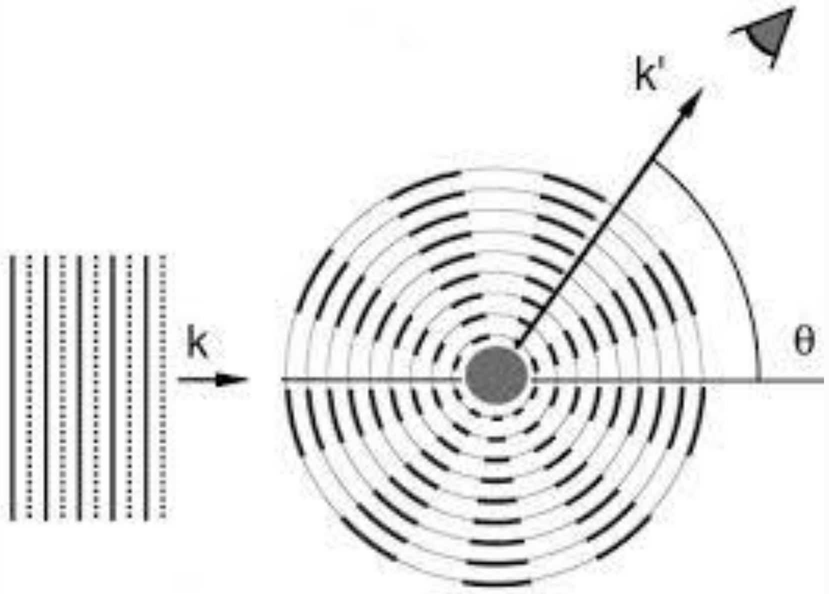


$$A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = A_0\left(Q_W + Q^2 B(E, Q^2)\right)$$

$$d\sigma_{R(L)} \sim \left| M_\gamma + M_Z^{R(L)} + M_{2\gamma} + M_{\gamma Z}^{R(L)} + \ldots \right|^2$$

Perturbative approach based on plane waves fails to describe well scattering off heavy nuclei!

ELECTRON-NUCLEUS SCATTERING



[www.tcm.phy.cam.ac.uk]

- Solve the Dirac equation:

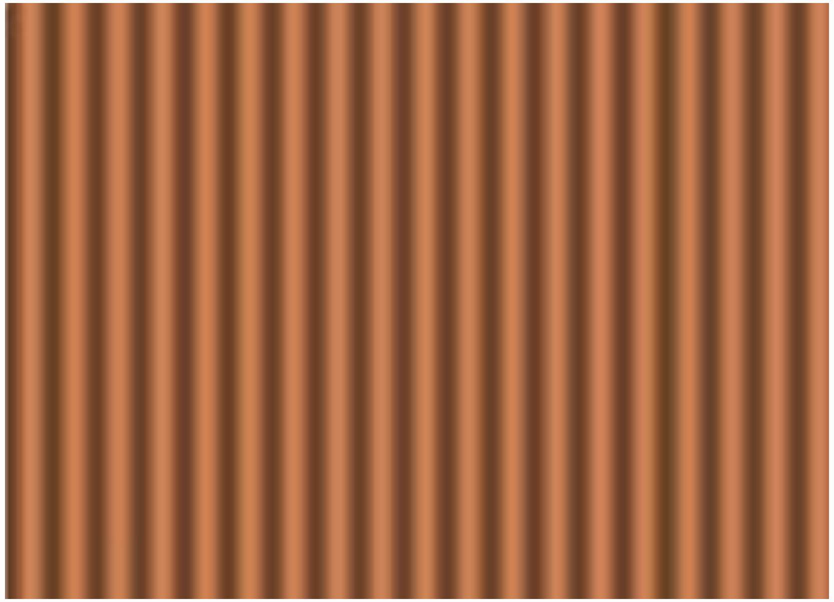
$$\left[\vec{\alpha}\cdot\vec{p}+\beta m_e+V(\vec{r})\right]\psi_\lambda=E\psi_\lambda$$

- Identify interaction potential energy:

$$V(r)$$

- Deduce scattering amplitude from asymptotic form of solution for ψ_λ
- Determine cross section

PLANE WAVE APPROACH



[www.tcm.phy.cam.ac.uk]

Expansion of the potential energy:

$$V = V_0 + V_C + O(Z^2\alpha^2)$$

$$V_0 = 0$$

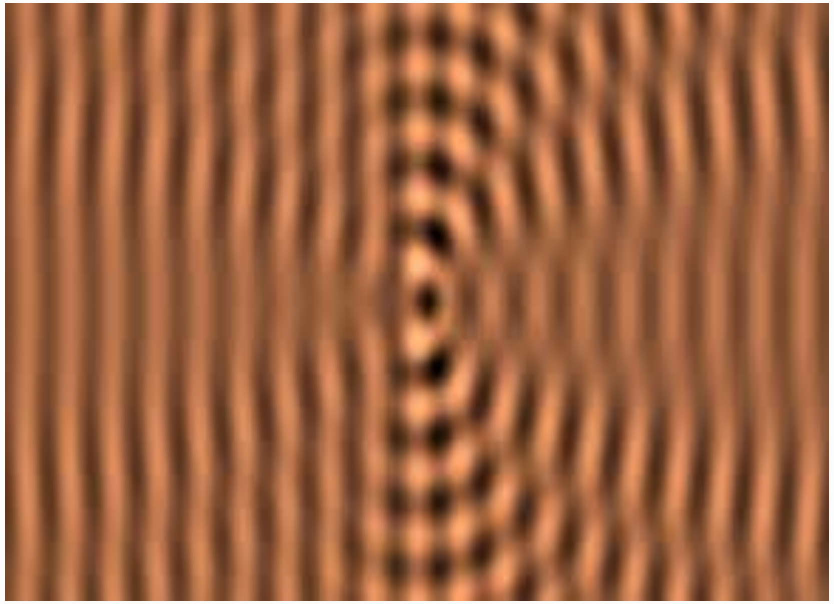
$$V_C = -\frac{1}{4\pi}\frac{Ze^2}{r} = -\frac{Z\alpha}{r}$$

Solution for V_0 is applied at each order:

$$\psi_\lambda \sim e^{i\vec{k}\cdot\vec{r}}$$

Does not work for heavy nuclei that considerably distort the plane wave!

COULOMB DISTORTION



[www.tcm.phy.cam.ac.uk]

Solution contains outgoing spherical waves:

$$\psi_\lambda \sim a_\lambda e^{i\vec{k}\cdot\vec{r}} + u_\lambda \frac{e^{ikr}}{r}$$

Electromagnetic interaction only:

$$V_{EM}(r) = -e^2 \left[\frac{1}{r} \int_0^r dr' r'^2 \rho_{EM}(r') + \int_r^\infty dr' r' \rho_{EM}(r') \right]$$

Effectively, the potential energy is:

$$V(r) = V_{sr}(r) + V_c(r)$$

Numerical calculations are performed using the ELSEPA code by Salvat et al.

[Salvat, Jablonski, Powell, Comp. Phys. Com., 2004]

COULOMB DISTORTION AND PV ASYMMETRY

Massless electron scattering:

$$\left[\vec{\alpha}\cdot\vec{p}+V_{R(L)}(r)\right]\psi_{R(L)} = E\psi_{R(L)}$$

$$V_{R(L)}(r) = V_{EM}(r) \mp V_{PV}(r)$$

[Horowitz, PRC, 1998]

EM potential: $$V_{EM}(r) = -e^2\left[\frac{1}{r}\int_0^r dr' r'^2 \rho_{EM}(r') + \int_r^\infty dr' r' \rho_{EM}(r')\right]$$

Weak potential: $$V_{PV}(r) = -\frac{G_F}{2\sqrt{2}}\rho_W(r)$$

Electromagnetic and weak charge density distributions are the crucial input for determinations of PV asymmetry in Coulomb distortion approach!

MODELS FOR EM CHARGE DISTRIBUTION

- Sum of Gaussians (SG): $$\rho_{ch}(r) = \sum_{i=1}^{12} A_i \left[\exp\left(-\frac{(r-R_i)^2}{\gamma^2} \right) + \exp\left(-\frac{(r+R_i)^2}{\gamma^2} \right) \right]$$

- Fourier-Bessel (FB): $$\rho_{ch}(r) = \theta(R-r) \sum_{v} a_v \, j_0(v\pi r / R)$$

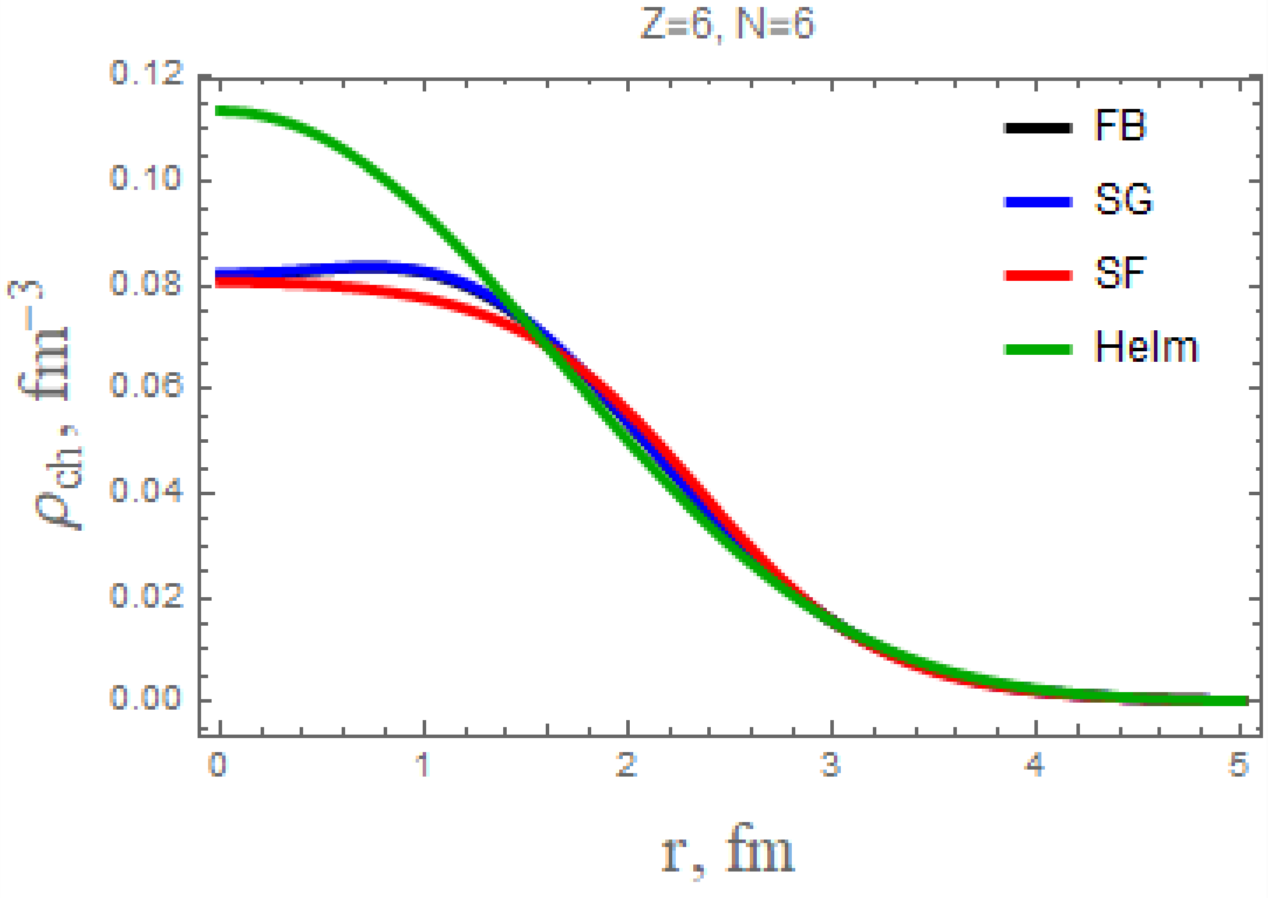
[H. de Vries et al, ADNDT, 1987]

- Helm: $$\rho_{ch}(r) = \frac{1}{2}\rho_0 \left[\left(\mathrm{erf}\left[\frac{r+R_0}{\sqrt{2}\sigma} \right] - \left[\frac{r-R_0}{\sqrt{2}\sigma} \right] \right) + \frac{1}{\sqrt{2\pi}} \frac{\sigma}{r} \left(\exp\left[-\frac{(r+R_0)^2}{2\sigma^2} \right] - \exp\left[-\frac{(r-R_0)^2}{2\sigma^2} \right] \right) \right]$$

- Symmetrized Fermi (SF): $$\rho_{ch}(r) = \rho_0 \frac{\sinh(c/a)}{\cosh(r/a) + \cosh(c/a)}$$

[Piekarewicz et al., PRC, 2016]

CHARGE DISTRIBUTION PARAMETRIZATIONS

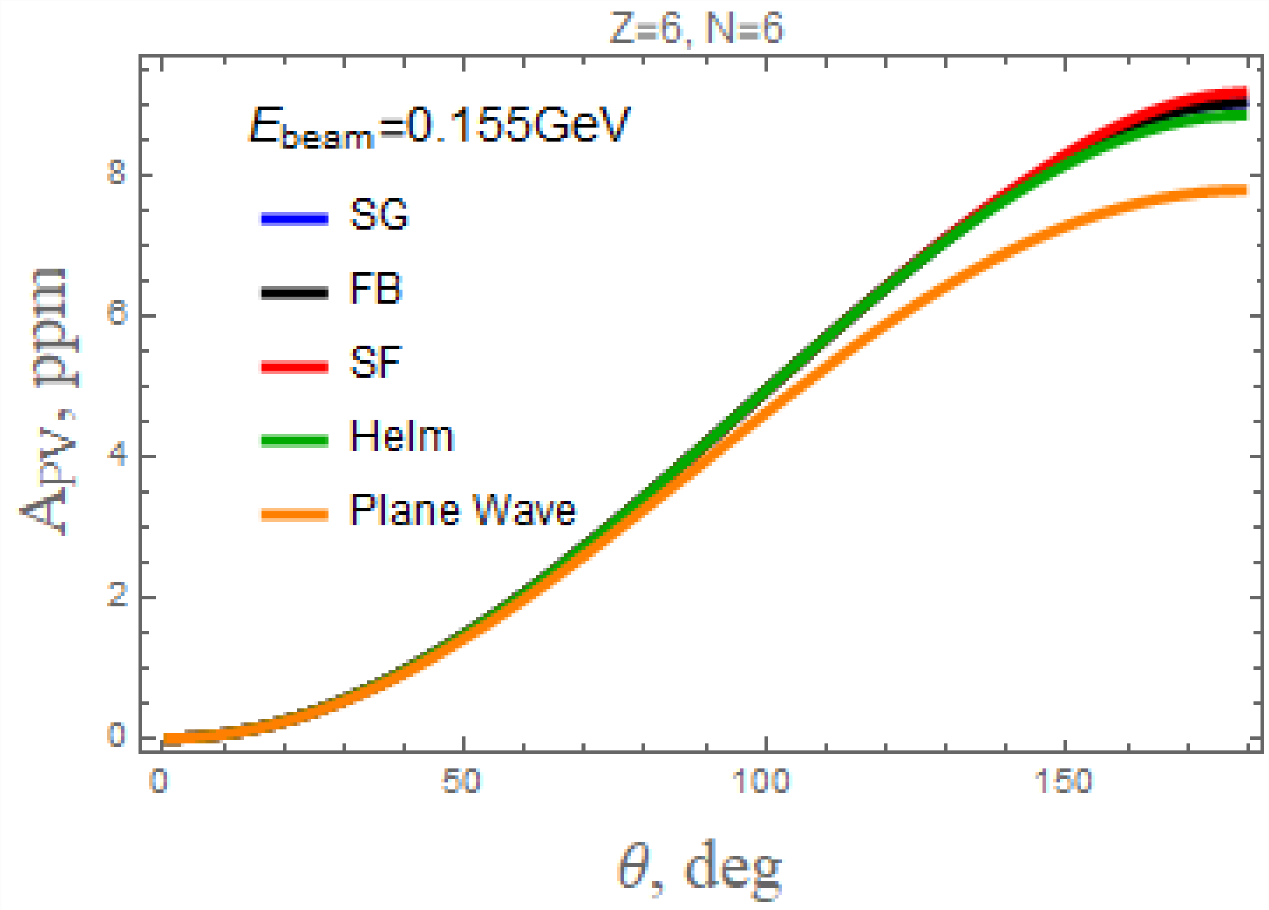


$$R_{ch}^2 = \frac{1}{Z}\int d^3r\ r^2 \rho_{EM}(r)$$

$$R_{ch}^4 = \frac{1}{Z}\int d^3r\ r^4 \rho_{EM}(r)$$

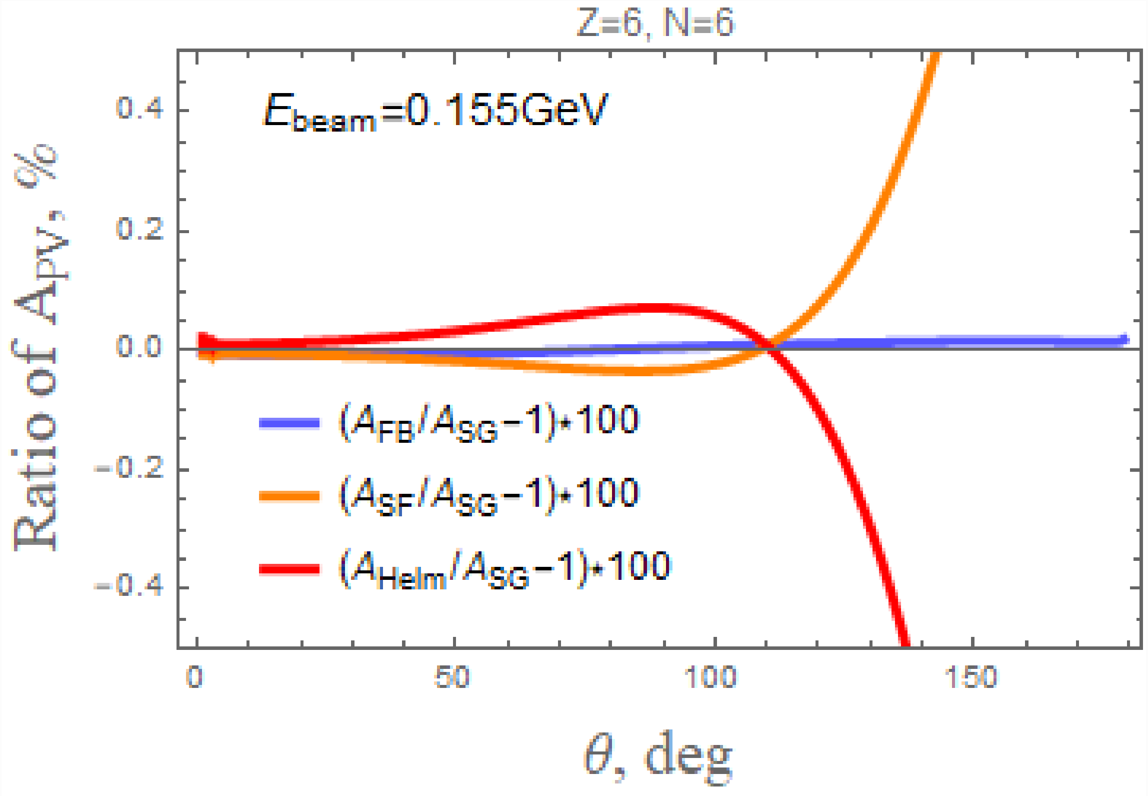
Two parameters of both Helm and SF distributions are adjusted to reproduce the first two moments (R_{ch}^2 and R_{ch}^4) of the SG distribution!

COULOMB DISTORTION FOR C-12 TARGET



Coulomb distortion is clearly significant for scattering off C-12. To see difference between SG, FB, SF and Helm we need to zoom in.

UNCERTAINTY DUE TO EM CHARGE DISTRIBUTION IN C-12



SF and Helm distributions, normalized to reproduce the first two moments (R_{ch}^2 and R_{ch}^4) of the SG, do not bring significant uncertainty in kinematics of interest.

WEAK DENSITY DISTRIBUTION

- No neutron skin:

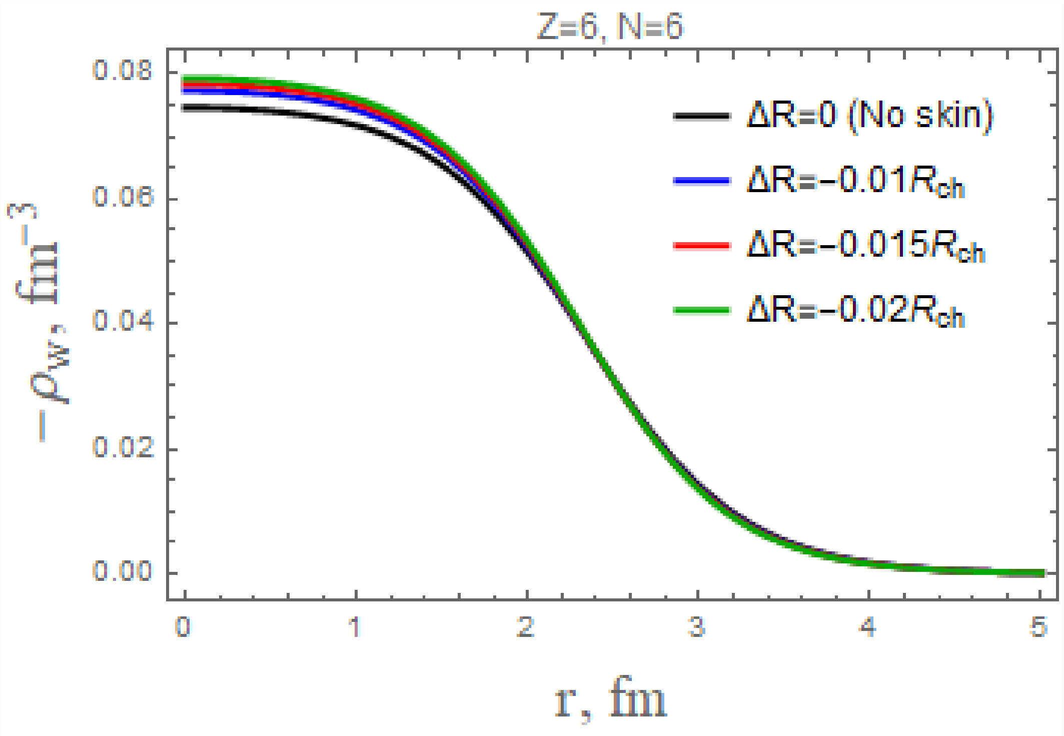
$$\rho_w(r) = \left[\left(1-4\sin^2\theta_W\right) - \frac{N}{Z}\right]\rho_{ch}(r) = \frac{Q_W^{Z,N}}{Z}\rho_{ch}(r)$$

- Model neutron skin using 2p symmetrized Fermi model:

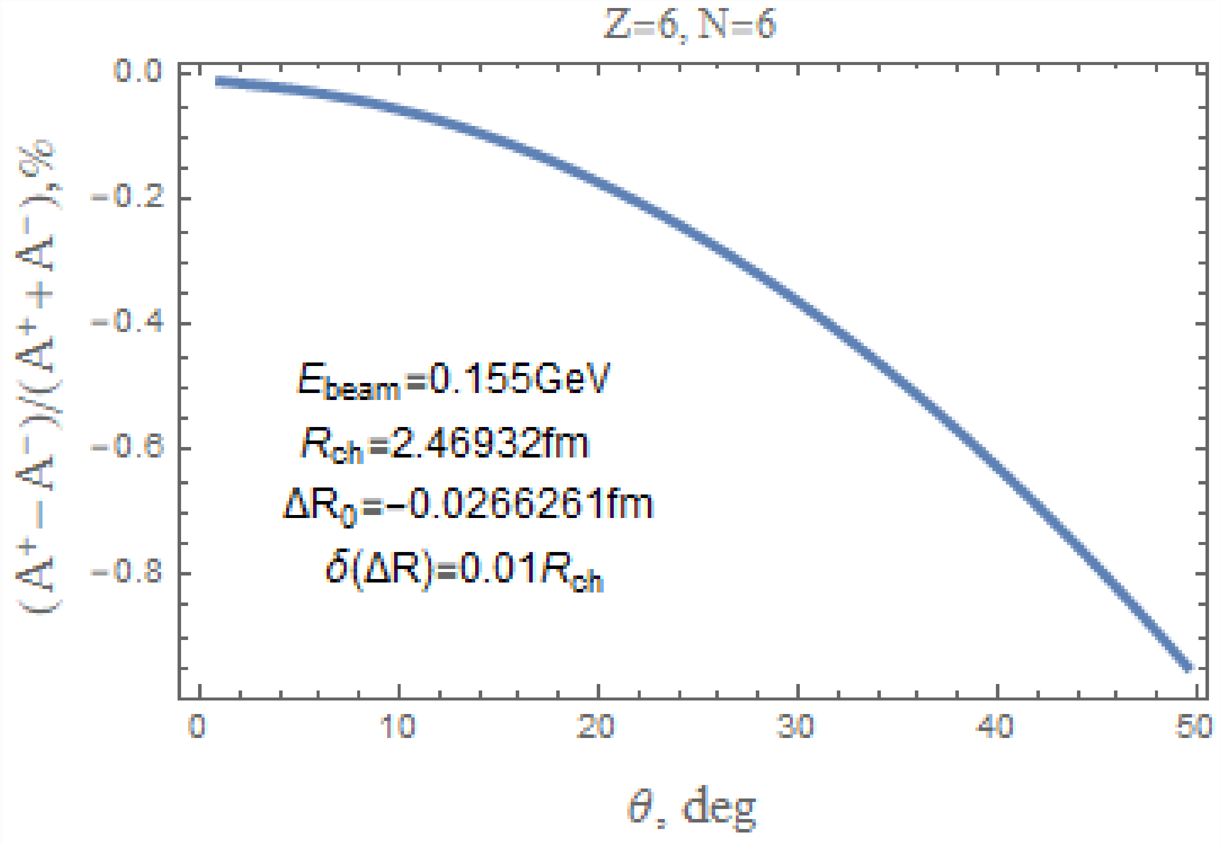
$$\rho_w(r) = \rho_{ch}(r)\frac{Q_W^{Z,N}}{Z} + \frac{1}{Z}\left(\tilde{\rho}(r) - \rho_{ch}(r)\right)$$

with $\int d^3r\left(\tilde{\rho}(r) - \rho_{ch}(r)\right) = 0$

$$\tilde{\rho}(r) = \tilde{\rho}_0 \frac{\sinh\left(\tilde{c}/a\right)}{\cosh\left(r/a\right) + \cosh\left(\tilde{c}/a\right)}$$



UNCERTAINTY DUE TO WEAK SKIN



$$A^+ = A_{PV}(\Delta R = \Delta R_0 + \delta R)$$

$$A^- = A_{PV}(\Delta R = \Delta R_0 - \delta R)$$

[Horowitz, PRC, 1998]

Knowledge of weak skin with accuracy better than ~1% of R_{ch} is must for $sin^2\theta_w$ extraction on C-12 at MESA.

Motivation for additional (backward) measurement of PV asymmetry at MESA?

CONTRIBUTION FROM WEAK SKIN

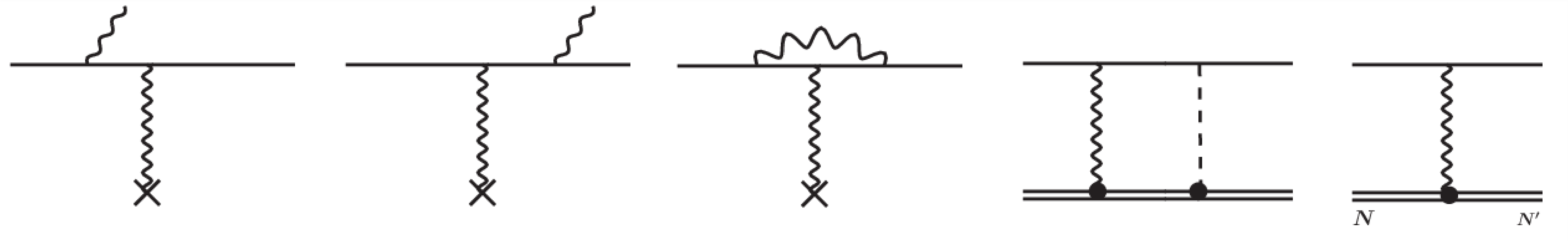
$$A_{PV}(\theta) = A_{PV}^{ns}(\theta) + x(\theta)\left(\frac{\Delta R}{R_{ch}}\right)^2$$

$$A_{PV}(\theta = 25°) = 0.386\,ppm + 9.887\,ppm\left(\frac{\Delta R}{R_{ch}}\right)^2$$

$$A_{PV}(\theta = 90°) = 4.177\,ppm + 1270\,ppm\left(\frac{\Delta R}{R_{ch}}\right)^2$$

$$A_{PV}(\theta = 135°) = 7.4\,ppm + 5269\,ppm\left(\frac{\Delta R}{R_{ch}}\right)^2$$

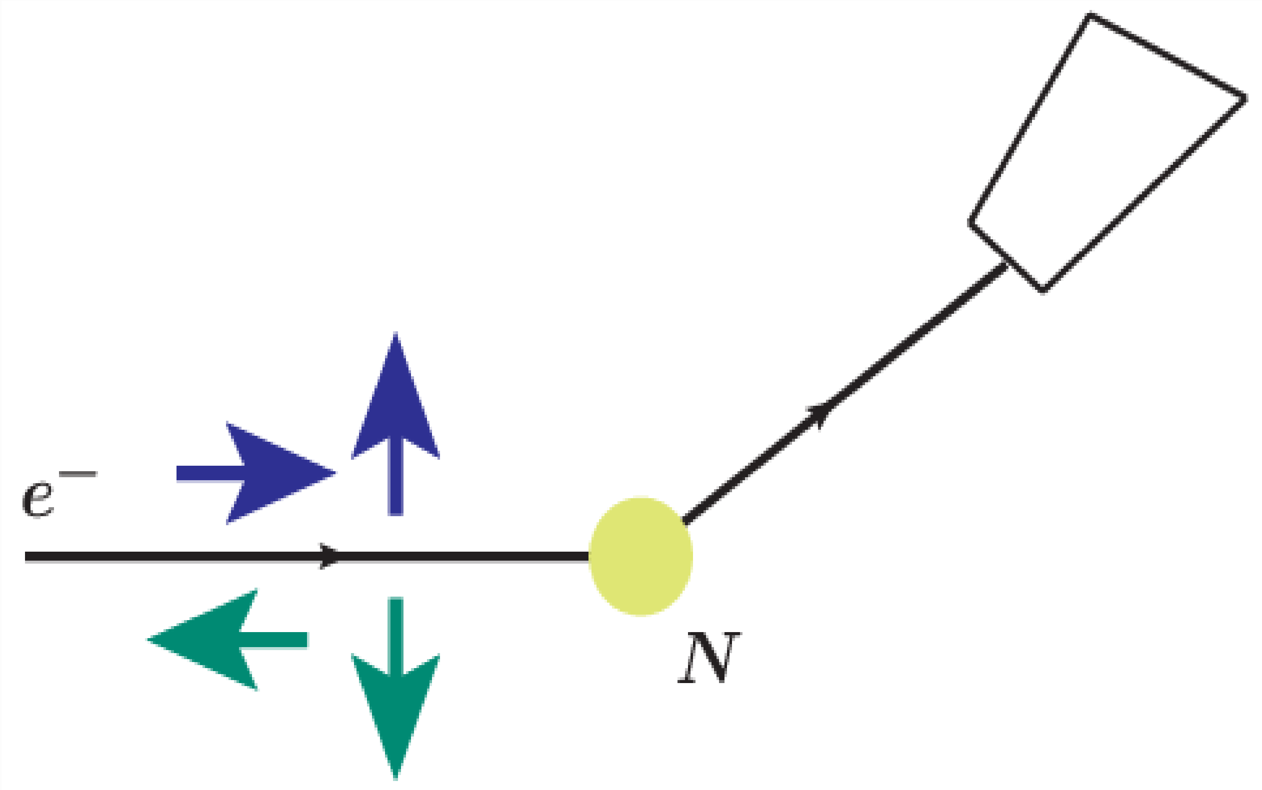
OTHER EFFECTS TO CONSIDER



Not yet considered:

- Radiative corrections: bremsstrahlung + loops, need quantitative estimate of target recoil
- Dispersive corrections: gamma-Z box (need to avoid double counting)
- Inelastic contributions: depends on experimental conditions

INTERPLAY BETWEEN PV AND BEAM-NORMAL ASYMMETRIES



$$A_{PV} = \frac{d\sigma_{\rightarrow} - d\sigma_{\leftarrow}}{d\sigma_{\rightarrow} + d\sigma_{\leftarrow}}$$

$$B_n = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

- PV measurements provide high precision test of the SM, therefore of significant experimental interest. Beam-normal asymmetry may be measured using the same apparatus.
- Transverse component of beam polarization can provide considerable background contribution in PV measurements.
- Lack of understanding of one of the observables casts doubt on the other. It is crucial that theory is able to describe both.

BEAM-NORMAL ASYMMETRY IN E-N SCATTERING

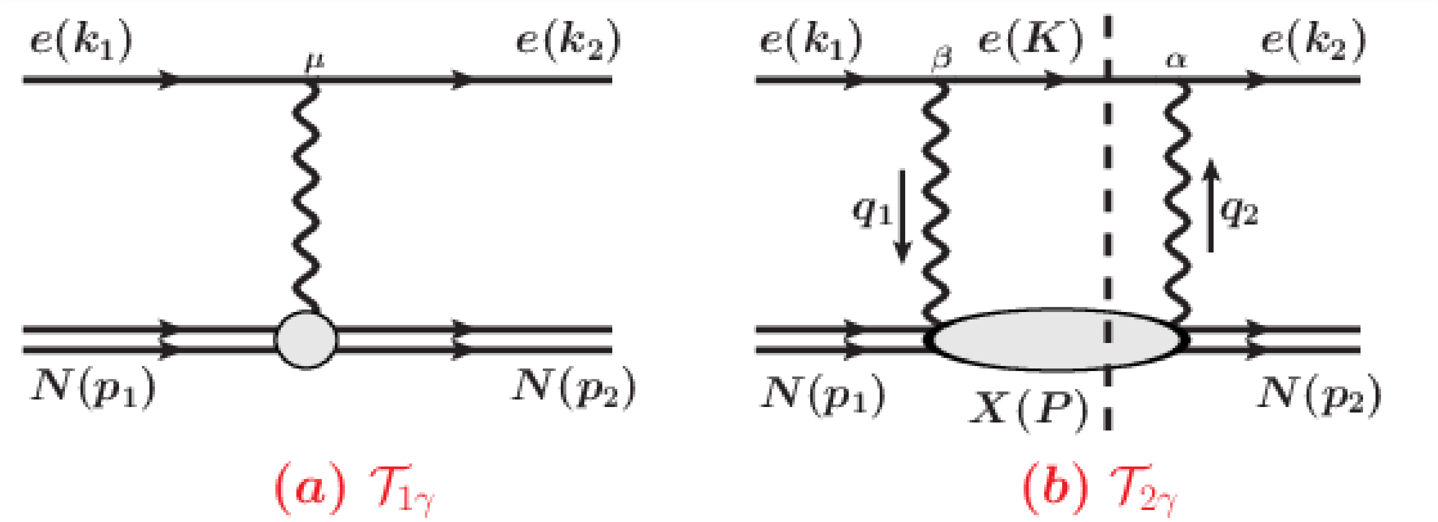
$$B_n \approx \frac{\mathrm{Im}\left[T_{1\gamma}^{\dagger}\mathrm{Abs}\left(T_{2\gamma}\right)\right]}{|T_{1\gamma}|^2} \sim \frac{m_e}{E_{beam}} Z\alpha$$

[de Rujula, Kaplan, de Rafael, NPB, 1971]

$A_{PV} \sim 1$ ppm, $B_n \sim 10$ ppm for proton target and E_{beam}=0.5 GeV

Small transverse component of the electron spin can lead to a substantial systematic effect on the PV asymmetry!

BEAM-NORMAL SSA CALCULATION USING PLANE WAVES



$$B_n = \frac{\mathrm{Im}\left[T_{1\gamma}^{\dagger}\,\mathrm{Abs}\left(T_{2\gamma}\right)\right]}{|T_{1\gamma}|^2}$$

$$\mathrm{Abs}\left(T_{2\gamma}\right) \sim \int \frac{d^3\vec{K}^*}{2E^*}\,\frac{l^{\alpha\beta}W_{\alpha\beta}}{Q_1^2 Q_2^2}$$

Spin-independent part of the two-photon exchange (TPE) hadronic tensor. Can be split into 2 pieces: elastic and inelastic.

$$l^{\alpha\beta} = \bar{u}(k_2)\gamma^{\alpha}(\not{K} + m_e)\gamma^{\beta}u(k_1); \quad W_{\alpha\beta} = W_{\alpha\beta}^{el} + W_{\alpha\beta}^{in}$$

INELASTIC TPE PARAMETRIZATION

Inelastic contribution (everything but the nucleus in the intermediate state). Realistic estimate is possible for the case of nearly forward electron scattering:

[Afanasev, Merenkov, PLB, 2004]

[Gorchtein, Horowitz, PRC, 2008]

$$W_{\alpha\beta}^{in} = \frac{W^2 - M^2}{4\pi M\alpha}\left[-g_{\alpha\beta} + \frac{\bar{p}_\beta q_{1\alpha} + \bar{p}_\alpha q_{2\beta}}{\bar{p}\cdot\bar{q}} - \frac{q_1\cdot q_2}{(\bar{p}\cdot\bar{q})^2}\bar{p}_\alpha\bar{p}_\beta\right]\sigma_{\gamma N}(W^2)$$

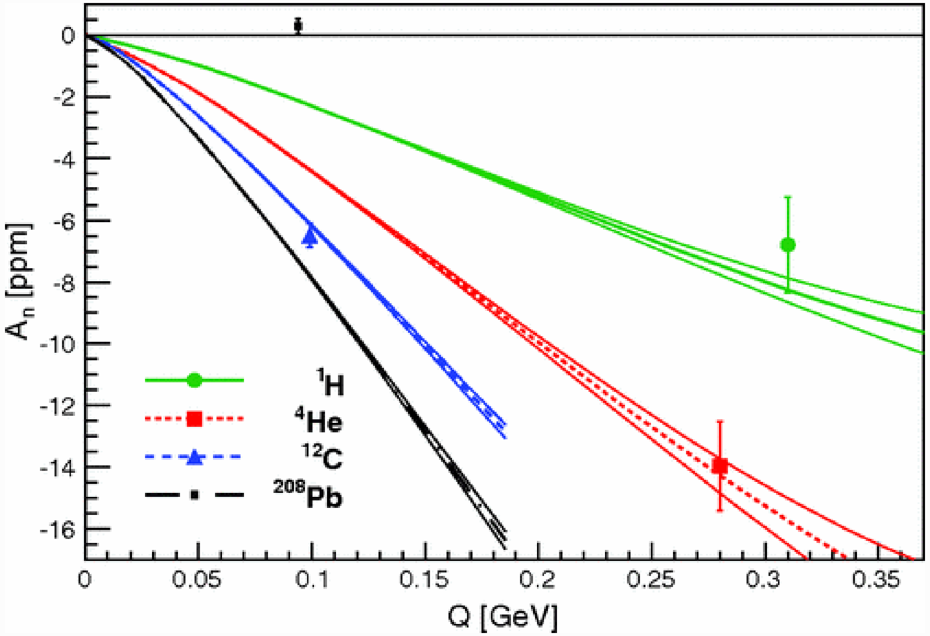
$$B_n^{in} = -\frac{m_e M}{2\pi^2 E_{beam}}\sqrt{\frac{Q^2}{\left(\left(M^2-s\right)-sQ^2\right)}}\frac{1}{W_{EM}(Q^2)}\int_{\omega_\pi}^{E_{beam}} d\omega\,\omega\sigma_{\gamma N}(\omega)\ln\left[\frac{Q^2}{m_e^2}\frac{(E_{beam}-\omega)^2}{\omega^2}\right]\exp[-BQ^2/2]$$

$$T_{eN} = \frac{e^2}{Q^2}\bar{u}(k_2)\left(mA_1 + \not{p}A_2\right)u(k_1) \quad\Rightarrow\quad B_n^{in} = -\frac{m_e}{\sqrt{s}}\tan\left(\frac{\theta_{cm}}{2}\right)\frac{\operatorname{Im}A_1^{in}}{W_{EM}(Q^2)}$$

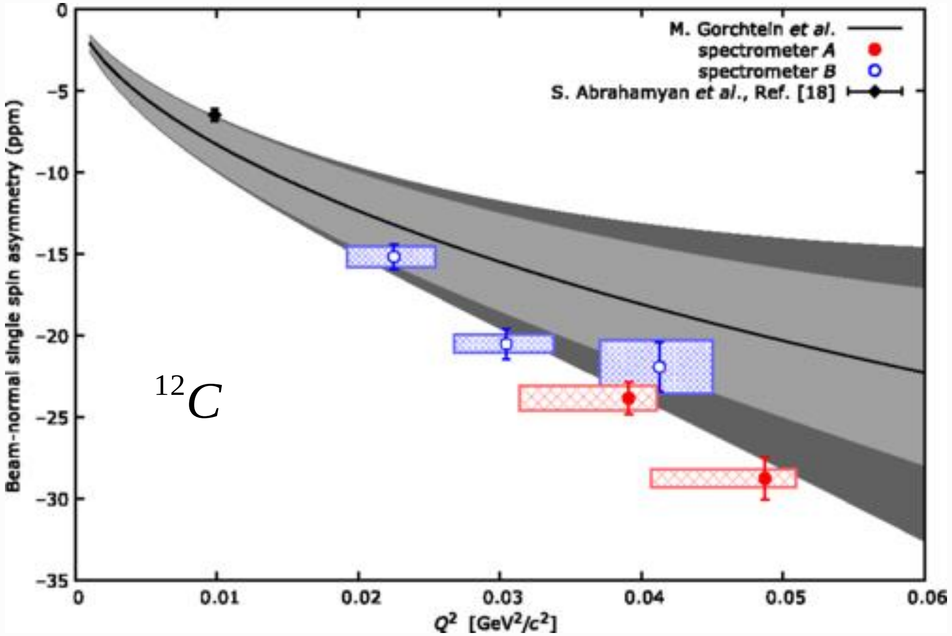
$$\operatorname{Im}A_1^{in}(E_{beam},q^2) = \frac{1}{2\pi^2}\frac{M}{E_{beam}}\int_{\omega_\pi}^{E_{beam}} d\omega\,\omega\sigma_{\gamma N}(\omega)\ln\left[\frac{Q^2}{m_e^2}\frac{(E_{beam}-\omega)^2}{\omega^2}\right]\exp[-BQ^2/2]$$

$$\omega = \frac{W^2 - M^2}{2M}$$

THEORY VS EXPERIMENT



[Abrahamyan *et al.* (HAPPEX and PREX Collaborations), PRL, 2012]



[Esser *et al.*, PRL, 2018]

- The approach works well for light nuclei and forward scattering angles
- Fails completely for lead ⇒ calculation seems to miss important properties of heavy nucleus

ESTIMATE OF B_n INCLUDING COULOMB DISTORTION

The Dirac equation:

$$\left[\vec{\alpha}\cdot\vec{p}+\beta m_e+V(r)\right]\psi = E\psi$$

Assuming that V(r) = Vc(r):

$$\psi = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}e^{i\vec{k}\cdot\vec{r}} + \frac{1}{\sqrt{2}}\begin{pmatrix} f(\theta)-g(\theta)e^{-i\varphi} \\ f(\theta)+g(\theta)e^{i\varphi}\end{pmatrix}\frac{e^{ikr}}{r}$$

$f(\theta)$ and g(θ) are the direct and spin-flip scattering amplitudes:

$$f(\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}\left\{(l+1)\left[\exp(2i\delta_{\kappa=-l-1})-1\right]+l\left[\exp(2i\delta_{\kappa=l})-1\right]\right\}P_l(\cos\theta)$$

$$g(\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}\left[\exp(2i\delta_{\kappa=l})-\exp(2i\delta_{\kappa=-k-1})\right]P_l^1(\cos\theta)$$

ESTIMATE OF B_n INCLUDING COULOMB DISTORTION

Polarized cross section:

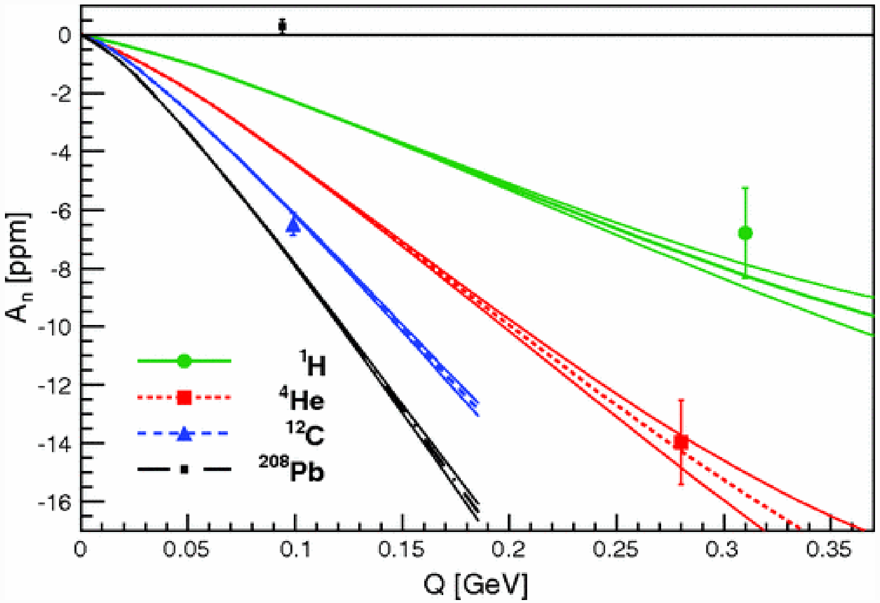
$$d\sigma(\varphi) \sim \left[|f|^2 + |g|^2 + i(fg^* - f^*g)\sin\varphi \right]$$

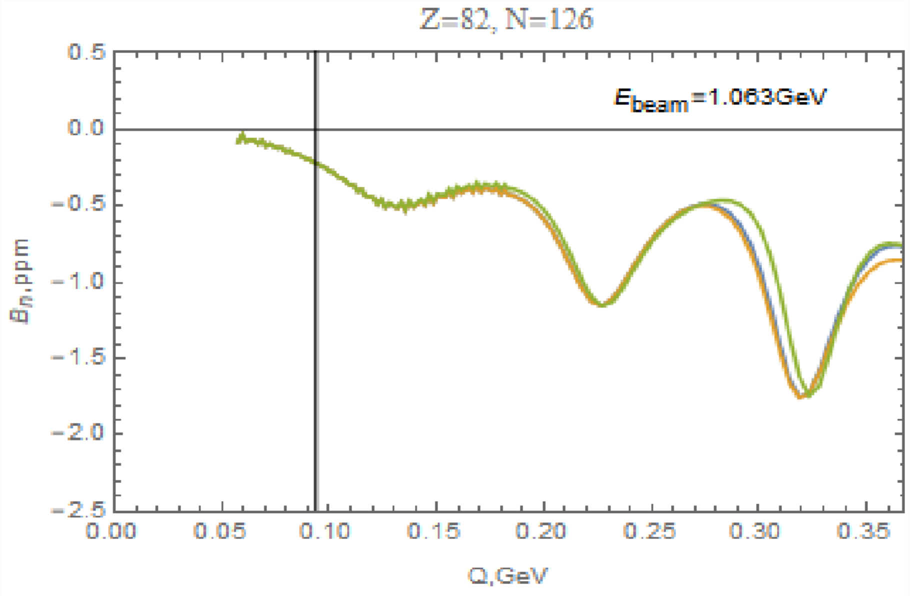
Beam-normal single-spin asymmetry:

$$B_n = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma(\varphi = \pi/2) - d\sigma(\varphi = 3\pi/2)}{d\sigma(\varphi = \pi/2) + d\sigma(\varphi = 3\pi/2)} = i\frac{fg^* - f^*g}{|f|^2 + |g|^2}$$

Also known as the Sherman function or analyzing power

THEORY VS EXPERIMENT ONCE AGAIN





- Coulomb distortion effectively takes into account multiple photon exchanges between the electron and the target nucleus when the latter remains in the ground state
- Previously calculated by E. Cooper and C. Horowitz [Cooper, Horowitz, PRC, 2005]
- The inelastic contribution is dominant
- Effects due to absorptive potential need to be included in Coulomb distortion calculation

WORK IN PROGRESS

Include absorptive potential into the Dirac equation:

$$\left[\vec{\alpha}\cdot\vec{p}+\beta m_e+V_C(r)\mp iV_{abs}(r)\right]\psi^{\pm}=E\psi^{\pm}$$

The absorptive potential is modeled using the plane wave result:

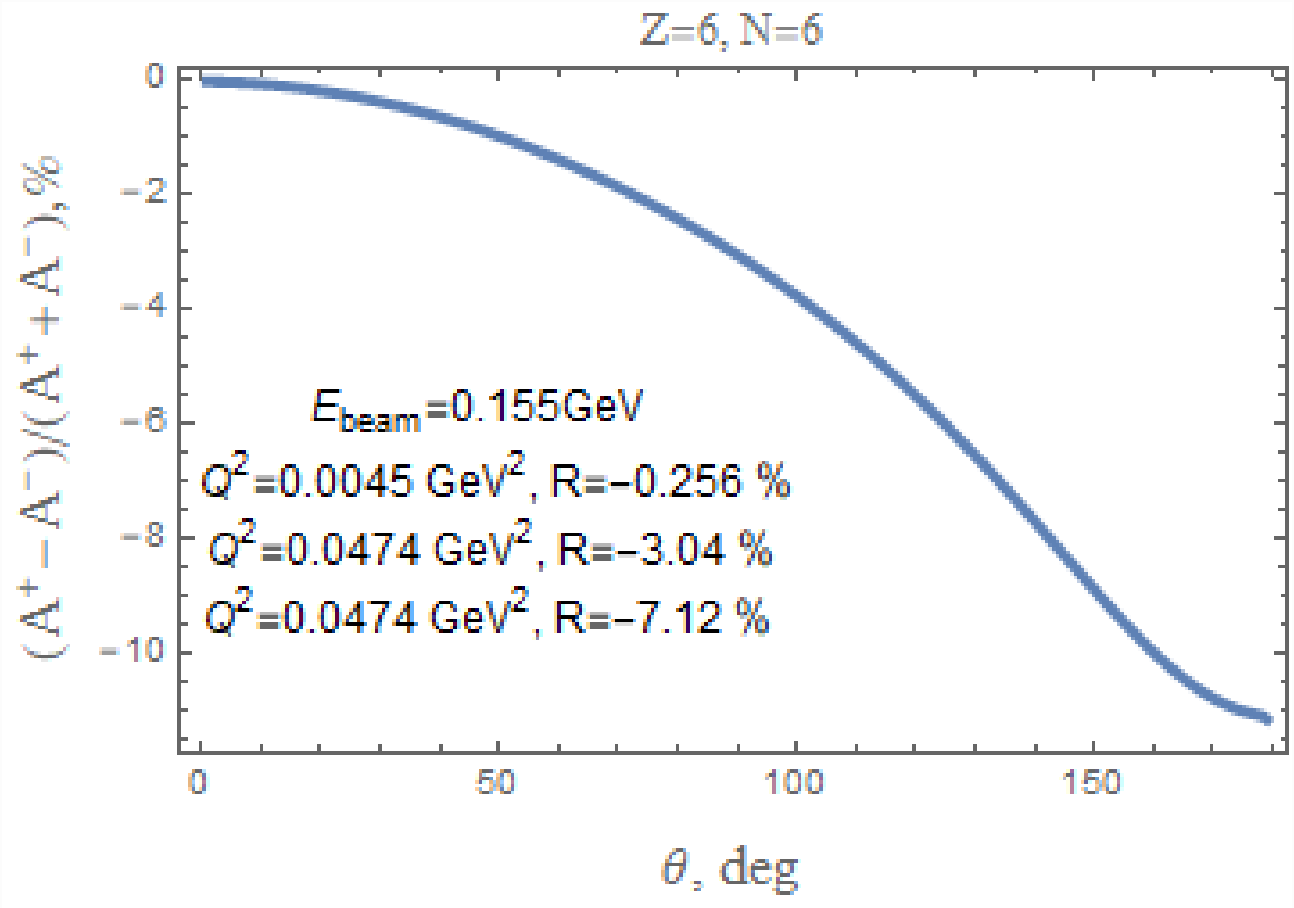
$$V_{abs}(r,E_{beam})=\frac{2}{\pi}C\left(E_{beam}\right)\int_0^{\infty}dq\; j_0(qr)\ln\left[1+\frac{Q^2}{m_e^2}\right]\exp[-BQ^2/2]$$

$$C\left(E_{beam}\right)=\frac{1}{2\pi^2}\frac{M}{E_{beam}}\int_{\omega_\pi}^{E_{beam}}d\omega\;\omega\sigma_{\gamma N}(\omega)$$

CONCLUSION

- Implemented Coulomb distortion formalism to provide PV asymmetry predictions
- Studied effects due to various nuclear charge distribution models on PV asymmetries
- Parametrized neutron skin and its uncertainty using 2p symmetrized Fermi model. Considerable effect on PV asymmetry predictions
- Studied effects due to Coulomb distortion on beam-normal asymmetry
- Absorptive potential implementation is required to describe existing data on beam-normal asymmetry from heavy nuclei

BACK UP SLIDE


Z=6, N=6
(A+ − A−)/(A+ + A−), %
θ, deg
0
−2
−4
−6
−8
−10
0
50
100
150
E_beam=0.155GeV
Q²=0.0045 GeV², R=−0.256 %
Q²=0.0474 GeV², R=−3.04 %
Q²=0.0474 GeV², R=−7.12 %