(Selected) Introduction to Flavor Physics

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58. International Winter Meeting on Nuclear Physics, Bormio
January 19, 2020

What is flavor?

The standard model distinguishes particles by their interactions with

the strong force (QCD)

the weak force

the electro-magnetic force

These are described by the gauge groups

$$\mathsf{SU}(3)_{\mathrm{C}} \times \mathsf{SU}(2)_{\mathrm{L}} \times \mathsf{U}(1)_{\mathrm{Y}}$$

$\mathsf{SU}(3)_{\mathrm{C}}$: color; particles are members of

triplets, which are charged
or singlets, which are neutral.

$\mathsf{SU}(2)_{\mathrm{L}}$: weak isospin; particles are members of

doublets, which are charged, $T_3 = \pm\frac{1}{2}$ (left-handed, "L")
or singlets, which are neutral (right-handed, "R").

$\mathsf{U}(1)_{\mathrm{Y}}$: weak hypercharge

not electric charge: $Q = T_3 + Y$

What is flavor?

rep.	SU(2)$_\mathrm{L}$	SU(3)$_\mathrm{C}$		
$(\mathbf{3},\mathbf{2})_Y$	left-handed	quarks	$Y = +1/3$	$Q = \begin{matrix} +2/3 \\ -1/3 \end{matrix}$
$(\mathbf{3},\mathbf{1})_Y$	right-handed	quarks	$Y = \begin{matrix} +4/3 \\ -2/3 \end{matrix}$	$Q = \begin{matrix} +2/3 \\ -1/3 \end{matrix}$
$(\mathbf{1},\mathbf{2})_Y$	left-handed	leptons	$Y = -1$	$Q = \begin{matrix} 0 \\ -1 \end{matrix}$
$(\mathbf{1},\mathbf{1})_Y$	right-handed	leptons	$Y = \begin{matrix} 0 \\ -2 \end{matrix}$	$Q = \begin{matrix} 0 \\ -1 \end{matrix}$

- left- and right-handed **up quarks** ($Q = +2/3$) in three colors
- left- and right-handed **down quarks** ($Q = -1/3$) in three colors
- left- and right-handed **electrons** ($Q = -1$)
- left-handed **electron neutrinos** ($Q = 0$)

What is flavor?

So we have

$$\mathsf{u} \qquad \mathsf{d} \qquad \mathsf{e}^- \qquad \nu_\mathsf{e}$$

But we also have

$$\mathsf{c} \qquad \mathsf{s} \qquad \mu^- \qquad \nu_\mu$$

and

$$\mathsf{t} \qquad \mathsf{b} \qquad \tau^- \qquad \nu_\tau$$

which are indistinguishable from the first **generation**
by their $\mathsf{SU}(3)_\mathrm{C} \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_\mathrm{Y}$ quantum numbers

We distinguish them by **flavor**:

up-type quarks: up, charm, top

down-type quarks: down, strange, bottom

charged-leptons: electron, muon, tau

neutrinos: electron neutrino, muon neutron, tau neutrino

What is flavor physics?

study of differences and dynamics between the flavors

Why?

Grand scheme:

- understand flavor structure (hierarchy)
- explain the matter dominance of the universe
  (CP violation)

Nearer goals:

- measure parameters of the standard model flavor sector
- search for physics beyond the standard model
  - new sources of CP violation
  - differences between flavors
    not part of $\mathsf{SU}(3)_{\mathrm{C}} \times \mathsf{SU}(2)_{\mathrm{L}} \times \mathsf{U}(1)_{\mathrm{Y}}$ and
    not owing to mass differences

From the foundation up ...

We have our **fermions**

$$f = \begin{pmatrix} \mathsf{u}_{\mathrm{L}}^{i} \\ \mathsf{d}_{\mathrm{L}}^{i} \end{pmatrix},\ \mathsf{u}_{\mathrm{R}}^{i},\ \mathsf{d}_{\mathrm{R}}^{i},\ \begin{pmatrix} \nu_{\mathrm{L}}^{i} \\ \mathsf{e}_{\mathrm{L}}^{i} \end{pmatrix},\ \mathsf{e}_{\mathrm{R}}^{i},\ \nu_{\mathrm{R}}^{i}$$

and a Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{Yukawa}} + \mathcal{L}_{\mathrm{kin/int}}.$$

We'll focus on the **Yukawa term**,
which couples fermions to the (scalar) Higgs field

$$\phi \equiv (\mathbf{1}, \mathbf{2})_{1} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

(that is, takes on a vacuum expectation value)

From the foundation up . . .

The only terms allowed by (intrinsic) angular momentum conervation are

$$\mathcal{L}_{\text{Yukawa}} \supset \bar{f} f' \phi.$$

Let's look at the chiral structure (using projection operators P_{R} and P_{L})

$$\begin{aligned}
\bar{f} f' \phi &= \bar{f} \underbrace{(P_{\text{R}} + P_{\text{L}})}_{\mathbb{1}} \underbrace{(P_{\text{R}} + P_{\text{L}})}_{\mathbb{1}} f' \phi \\
&= \bar{f}(P_{\text{R}} P_{\text{R}} + \underbrace{P_{\text{L}} P_{\text{R}}}_{=0} + \underbrace{P_{\text{R}} P_{\text{L}}}_{=0} + P_{\text{L}} P_{\text{L}}) f' \phi \\
&= \overline{P_{\text{L}} f}\, P_{\text{R}} f'\, \phi + \overline{P_{\text{R}} f}\, P_{\text{L}} f'\, \phi \\
&= \overline{f_{\text{L}}} f'_{\text{R}} \phi + \overline{f_{\text{R}}} f'_{\text{L}} \phi
\end{aligned}$$

Our terms must also be neutral by QCD
so we may only couple

quarks with quarks and leptons with leptons

From the foundation up . . .

Let's apply hypercharge conservation:

$$\mathcal{L}_{\text{Yukawa}} \supset \overline{f_{\rm L}} f'_{\rm R} \phi \quad \text{requiring} \quad Y(\overline{f_{\rm L}} f'_{\rm R} \phi) = 0$$

Recall

$$\phi = \begin{pmatrix} 0 \\ v \end{pmatrix} \equiv (\mathbf{1}, \mathbf{2})_{1}$$

$$\begin{pmatrix} \nu^i_{\rm L} \\ \mathsf{e}^i_{\rm L} \end{pmatrix} \equiv (\mathbf{1}, \mathbf{2})_{-1}$$

$$\mathsf{e}^i_{\rm R} \equiv (\mathbf{1}, \mathbf{1})_{-2}$$

$$\begin{pmatrix} \mathsf{u}^i_{\rm L} \\ \mathsf{d}^i_{\rm L} \end{pmatrix} \equiv (\mathbf{3}, \mathbf{2})_{+\frac{1}{3}}$$

$$\mathsf{u}^i_{\rm R} \equiv (\mathbf{3}, \mathbf{1})_{+\frac{4}{3}}$$

$$\mathsf{d}^i_{\rm R} \equiv (\mathbf{3}, \mathbf{1})_{-\frac{2}{3}}$$

So $\mathcal{L}_{\text{Yukawa}}$ contains

$$y^{ij}_{\mathsf{e}} \underbrace{(\bar{\nu}^i_{\rm L}, \bar{\mathsf{e}}^i_{\rm L})}_{+1} \underbrace{\mathsf{e}^j_{\rm R}}_{-2} \underbrace{\begin{pmatrix} 0 \\ v \end{pmatrix}}_{+1} = \underbrace{y^{ij}_{\mathsf{e}} v}_{m^{ij}_{\mathsf{e}}} \; \bar{\mathsf{e}}^i_{\rm L} \mathsf{e}^j_{\rm R}$$

$$y^{ij}_{\mathsf{d}} \underbrace{(\bar{\mathsf{u}}^i_{\rm L}, \bar{\mathsf{d}}^i_{\rm L})}_{-1/3} \underbrace{\mathsf{d}^j_{\rm R}}_{-2/3} \underbrace{\begin{pmatrix} 0 \\ v \end{pmatrix}}_{+1} = m^{ij}_{\mathsf{d}} \; \bar{\mathsf{d}}^i_{\rm L} \mathsf{d}^j_{\rm R}$$

$$y^{ij}_{\mathsf{u}} \underbrace{(\bar{\mathsf{u}}^i_{\rm L}, \bar{\mathsf{d}}^i_{\rm L})}_{-1/3} \underbrace{\mathsf{u}^j_{\rm R}}_{+4/3} \underbrace{\begin{pmatrix} v \\ 0 \end{pmatrix}}_{-1} = m^{ij}_{\mathsf{u}} \; \bar{\mathsf{u}}^i_{\rm L} \mathsf{u}^j_{\rm R}$$

From the foundation up ...

So ... (phew)

$$\mathcal{L}_{\text{Yukawa}} = \sum_{ij} m_{\mathsf{e}}^{ij} \bar{\mathsf{e}}_{\text{L}}^{i} \mathsf{e}_{\text{R}}^{j} + \sum_{ij} m_{\mathsf{d}}^{ij} \; \bar{\mathsf{d}}_{\text{L}}^{i} \mathsf{d}_{\text{R}}^{j} + \sum_{ij} m_{\mathsf{u}}^{ij} \; \bar{\mathsf{u}}_{\text{L}}^{i} \mathsf{u}_{\text{R}}^{j} \quad + \text{h.c.}$$

Let's focus on the quarks: we have two **generic** hermitian 3×3 matrices

$$[M_{\mathsf{d}}]_{ij} \equiv m_{\mathsf{d}}^{ij} \quad \text{and} \quad [M_{\mathsf{u}}]_{ij} \equiv m_{\mathsf{u}}^{ij}$$

which describe the Yukawa interactions of our **gauge interaction eigenstates**

$$\mathcal{L}_{\text{Yukawa}} \supset \overline{D_{\text{L}}^{\text{I}}} M_{\mathsf{d}} D_{\text{R}}^{\text{I}} + \overline{U_{\text{L}}^{\text{I}}} M_{\mathsf{u}} U_{\text{R}}^{\text{I}} \quad + \text{h.c.}$$

with

$$U_{\chi}^{\text{I}} \equiv \left\{ \mathsf{u}_{\chi}^{1}, \mathsf{u}_{\chi}^{2}, \mathsf{u}_{\chi}^{3} \right\} \quad \text{and} \quad D_{\chi}^{\text{I}} \equiv \left\{ \mathsf{d}_{\chi}^{1}, \mathsf{d}_{\chi}^{2}, \mathsf{d}_{\chi}^{3} \right\} \quad \text{with} \quad \chi = \text{L}, \text{R}$$

sometimes written

$$U_{\chi}^{\text{I}} \equiv \left\{ \mathsf{u}_{\chi}', \mathsf{c}_{\chi}', \mathsf{t}_{\chi}' \right\} \quad \text{and} \quad D_{\chi}^{\text{I}} \equiv \left\{ \mathsf{d}_{\chi}', \mathsf{s}_{\chi}', \mathsf{b}_{\chi}' \right\}$$

From the foundation up ...

$$\mathcal{L}_{\text{Yukawa}} \supset \overline{D_{\rm L}^{\rm I}} M_{\sf d} D_{\rm R}^{\rm I} + \overline{U_{\rm L}^{\rm I}} M_{\sf u} U_{\rm R}^{\rm I} \quad + \text{h.c.}$$

We diagonalize the generic mass matrices

$$L_{\sf u} M_{\sf u} R_{\sf u}^\dagger = \mathsf{diag}(m_{\sf u}, m_{\sf c}, m_{\sf t}) \quad \text{and} \quad L_{\sf d} M_{\sf d} R_{\sf d}^\dagger = \mathsf{diag}(m_{\sf d}, m_{\sf s}, m_{\sf b})$$

with unitary rotation matrices

$$L_{\sf u}^\dagger L_{\sf u} = \mathbb{1} \qquad R_{\sf u}^\dagger R_{\sf u} = \mathbb{1} \qquad L_{\sf d}^\dagger L_{\sf d} = \mathbb{1} \qquad R_{\sf d}^\dagger R_{\sf d} = \mathbb{1}$$

(with suggestive variable naming ...)

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} \supset \overline{D_{\rm L}^{\rm I}} M_{\sf d} D_{\rm R}^{\rm I} + \overline{U_{\rm L}^{\rm I}} M_{\sf u} U_{\rm R}^{\rm I} &= \overline{D_{\rm L}^{\rm I}} \mathbb{1} M_{\sf d} \mathbb{1} D_{\rm R}^{\rm I} + \overline{U_{\rm L}^{\rm I}} \mathbb{1} M_{\sf u} \mathbb{1} U_{\rm R}^{\rm I} \\
&= \overline{D_{\rm L}^{\rm I}} L_{\sf d}^\dagger L_{\sf d} M_{\sf d} R_{\sf d}^\dagger R_{\sf d} D_{\rm R}^{\rm I} + \overline{U_{\rm L}^{\rm I}} L_{\sf u}^\dagger L_{\sf u} M_{\sf u} R_{\sf u}^\dagger R_{\sf u} U_{\rm R}^{\rm I} \\
&= \overline{L_{\sf d} D_{\rm L}^{\rm I}} M_{\sf u}^{\rm diag} R_{\sf d} D_{\rm R}^{\rm I} + \overline{L_{\sf u} U_{\rm L}^{\rm I}} M_{\sf d}^{\rm diag} R_{\sf u} U_{\rm R}^{\rm I} \\
&\equiv \overline{D_{\rm L}} M_{\sf u}^{\rm diag} D_{\rm R} + \overline{U_{\rm L}} M_{\sf d}^{\rm diag} U_{\rm R}
\end{aligned}$$

defining the **mass eigenstates**:

$$\begin{aligned}
U_{\rm L} &\equiv L_{\sf u} U_{\rm L}^{\rm I} & U_{\rm R} &\equiv R_{\sf u} U_{\rm R}^{\rm I} & D_{\rm L} &\equiv L_{\sf d} D_{\rm L}^{\rm I} & D_{\rm R} &\equiv R_{\sf d} D_{\rm R}^{\rm I} \\
&= \{{\sf u}_{\rm L}, {\sf c}_{\rm L}, {\sf t}_{\rm L}\} & &= \{{\sf u}_{\rm R}, {\sf c}_{\rm R}, {\sf t}_{\rm R}\} & &= \{{\sf d}_{\rm L}, {\sf s}_{\rm L}, {\sf b}_{\rm L}\} & &= \{{\sf d}_{\rm R}, {\sf s}_{\rm R}, {\sf b}_{\rm R}\}
\end{aligned}$$

From the foundation up ...

Let's now look at the interactions in $\mathcal{L}_{\text{int}}$ (ignoring coupling constants)

We have **neutral-current interactions**:

$$\overline{D_\text{L}^\text{I}}\gamma^\mu D_\text{L}^\text{I} X_\mu + \overline{U_\text{L}^\text{I}}\gamma^\mu U_\text{L}^\text{I} X_\mu = \overline{D_\text{L}} L_\text{d} \gamma^\mu L_\text{d}^\dagger D_\text{L} X_\mu + \overline{U_\text{L}} L_\text{u} \gamma^\mu L_\text{u}^\dagger U_\text{L} X_\mu$$
$$= \overline{D_\text{L}}\gamma^\mu D_\text{L} X_\mu + \overline{U_\text{L}}\gamma^\mu U_\text{L} X_\mu$$

which are unaffected (same holds for right-handed fields).

X_μ: QCD, QED, weak neutral current (Z^0)

And we have the weak **charged-current interaction**:

$$\overline{U_\text{L}^\text{I}}\gamma^\mu D_\text{L}^\text{I} W_\mu^+ + \overline{D_\text{L}^\text{I}}\gamma^\mu U_\text{L}^\text{I} W_\mu^- = \overline{U_\text{L}} L_\text{u} \gamma^\mu L_\text{d}^\dagger D_\text{L} W_\mu^+ + \overline{D_\text{L}} L_\text{d}^\dagger \gamma^\mu L_\text{u} U_\text{L} W_\mu^-$$
$$\equiv \overline{U_\text{L}}\gamma^\mu V_\text{CKM} D_\text{L} W_\mu^+ + \overline{D_\text{L}}\gamma^\mu V_\text{CKM}^\dagger U_\text{L} W_\mu^-$$

which **is** affected.

$$\boxed{V_\text{CKM} \equiv L_\text{u} L_\text{d}^\dagger}$$

is the **Cabibbo-Kobayashi-Maskawa matrix**.

From the foundation up . . .

So we have for the weak charged-current interaction

$$\mathcal{L}_{\rm CC} \propto (\mathsf{u}_{\rm L}, \mathsf{c}_{\rm L}, \mathsf{t}_{\rm L})\, \gamma^{\mu}\, V_{\rm CKM} \begin{pmatrix} \mathsf{d}_{\rm L} \\ \mathsf{s}_{\rm L} \\ \mathsf{b}_{\rm L} \end{pmatrix} W^{+}_{\mu} \quad + \text{h.c.}$$

$$V_{\rm CKM} = \begin{pmatrix} V_{\mathsf{ud}} & V_{\mathsf{us}} & V_{\mathsf{ub}} \\ V_{\mathsf{cd}} & V_{\mathsf{cs}} & V_{\mathsf{cb}} \\ V_{\mathsf{td}} & V_{\mathsf{ts}} & V_{\mathsf{tb}} \end{pmatrix}$$

$V_{\rm CKM}$ is a $3{\times}3$ **complex unitary** matrix

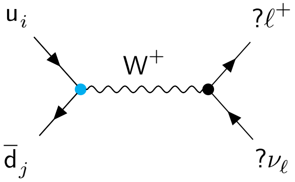
This is the Kobayashi-Maskawa theory.

CKM: Magnitudes

$$V_{\text{CKM}} = \begin{pmatrix} V_{\mathsf{ud}} & V_{\mathsf{us}} & V_{\mathsf{ub}} \\ V_{\mathsf{cd}} & V_{\mathsf{cs}} & V_{\mathsf{cb}} \\ V_{\mathsf{td}} & V_{\mathsf{ts}} & V_{\mathsf{tb}} \end{pmatrix}$$

Let's look at how to measure $|V_{ij}|$

V_{ij} parametrizes a flavor-changing weak interaction:



Leptonic decay of pseudoscalar $\mathsf{P}_{ij}^+ = \mathsf{u}_i\bar{\mathsf{d}}_j$

$$\Gamma(\mathsf{P}_{ij}^+ \to \ell^+\nu_\ell) = \frac{G_F^2}{8\pi} m_P m_\ell^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 f_P^2 \left|V_{ij}\right|^2$$

requires knowledge of **meson decay constant**

$\lvert V_{\mathsf{ud}}\rvert$: π^+	$\lvert V_{\mathsf{us}}\rvert$: K^+	$\lvert V_{\mathsf{ub}}\rvert$: B^+
$\lvert V_{\mathsf{cd}}\rvert$: D^+	$\lvert V_{\mathsf{cs}}\rvert$: D_s^+	$\lvert V_{\mathsf{cb}}\rvert$: B_c^+

✗ top does not hadronize! f_P not usually precise enough to be competitive.

CKM: $\left|V_{\mathsf{ud}}\right|$

neutron β decay gives most precise $\left|V_{\mathsf{ud}}\right|$ measurement

$$\mathsf{n} \to \mathsf{p}\,\mathsf{e}\,\overline{\nu}_{\mathsf{e}} \qquad \Rightarrow \qquad \mathsf{d} \to \mathsf{u}\,\mathsf{e}\,\overline{\nu}_{\mathsf{e}}$$

in (superallowed) nuclear transitions: $0^+ \to 0^+$

$$\boxed{\left|V_{\mathsf{ud}}\right| = 0.974\,20 \pm 0.000\,21}$$

(arXiv:1807.01146)

that's 0.02 % precision!

in free neutron decay:

$$\Gamma_{\mathsf{n}} = \tau_{\mathsf{n}}^{-1} = \frac{G_{\mathrm{F}}^2}{2\pi^3} m_{\mathsf{e}}^5 \underbrace{g_{\mathrm{V}}^2 \left(1 + 3\frac{g_{\mathrm{A}}^2}{g_{\mathrm{V}}}^2\right)}_{\text{axial/vector couplings}} \underbrace{(1+\Delta_{\mathrm{R}}) f_{\mathrm{R}}}_{\text{rad. corr.}} \left|V_{\mathsf{ud}}\right|^2$$

$$\boxed{\left|V_{\mathsf{ud}}\right| = 0.9763 \pm 0.0016}$$

(PDG2019)

CKM: $\left|V_{\mathrm{ud}}\right|$

pion "beta" decay is also not much worse:

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e) \propto G_F^2 \times (\text{phase-space factors}) \times (\text{rad. corr.}) \times \left|V_{\mathrm{ud}}\right|^2$$

$$\boxed{\left|V_{\mathrm{ud}}\right| = 0.9748 \pm 0.0025}$$

(arXiv:hep-ex/0312030,PDG2019)

CKM: $|V_{us}|$

Recall unitarity:

$$V_{CKM}^{\dagger} V_{CKM} = \mathbb{1} \qquad \rightarrow \qquad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Since $|V_{ub}| = \mathcal{O}\left(10^{-3}\right)$

$$|V_{us}| = \sqrt{1 - |V_{ud}|^2} \qquad \text{(unitarity)}$$

We can also measure it from kaon decays

$$\Gamma(K \rightarrow \pi \ell \bar{\nu}_\ell) \propto G_F^2 \times (\text{form factor}) \times |V_{us}|^2 \qquad (K_{\ell 3})$$

$$\frac{\Gamma(K \rightarrow \ell \bar{\nu}_\ell)}{\Gamma(\pi \rightarrow \ell \bar{\nu}_\ell)} \propto \frac{f_K}{f_\pi} \frac{|V_{us}|^2}{|V_{ud}|^2} \qquad (K_{\ell 2})$$

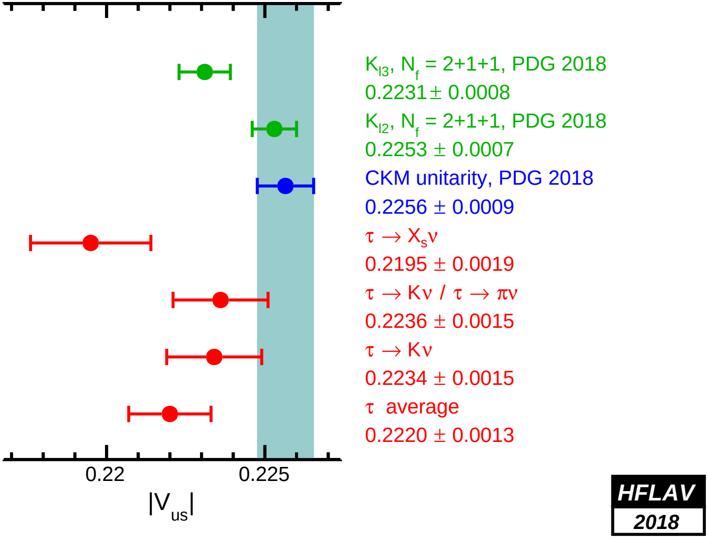
or in hadronic tau decays

$$\frac{\Gamma(\tau \rightarrow \nu_\tau K)}{\Gamma(\tau \rightarrow \nu_\tau \pi)} \propto \frac{f_K}{f_\pi} \frac{|V_{us}|^2}{|V_{ud}|^2}$$

(using f_K / f_π from Lattice QCD)

or by comparing $\Gamma(\tau \rightarrow \nu_\tau X_s)$ to $\Gamma(\tau \rightarrow \nu_\tau X_d)$

CKM: $|V_{us}|$



Tension in the measurements $\Longrightarrow$ are $\tau \to \nu X_s$ and $\tau \to \nu X_d$ correct?

CKM: $|V_{cd}|$ and $|V_{cs}|$

The best determinations of $|V_{cd}|$ and $|V_{cs}|$ are from

Semileptonic D decay

$$\Gamma(\mathrm{D} \to \mathrm{P}_q \ell \bar{\nu}_\ell) \propto \frac{G_F^2}{24\pi^3} \left| f_+^{\mathrm{DP}_q}(0) \right|^2 \left| V_{cq} \right|^2$$

for $\ell = \mathrm{e}, \mu$ and $q = \mathrm{d}, \mathrm{s} \to \mathrm{P_d} \equiv \pi$, $\mathrm{P_s} = \mathrm{K}$

Using Lattice QCD inputs: $f^{\mathrm{DP}_q} \equiv \langle \mathrm{P}_q | \bar{q} \gamma^\mu \mathrm{c} | \mathrm{D} \rangle$

$f_+^{\mathrm{D}\pi}(0) = 0.666 \pm 0.029$ and $f_+^{\mathrm{DK}}(0) = 0.747 \pm 0.019$ (FLAG2016)

$|V_{cd}| = 0.2140 \pm 0.0029_{\mathrm{exp}} \pm 0.0093_{\mathrm{th}}$ and $|V_{cs}| = 0.967 \pm 0.005_{\mathrm{exp}} \pm 0.025_{\mathrm{th}}$

Leptonic D and $\mathrm{D_s}$ decay

$$\mathcal{B}(\mathrm{D}_q \to \ell \bar{\nu}_\ell) = \frac{G_F^2}{8\pi} m_{\mathrm{D}_q} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_{\mathrm{D}_q}^2} \right)^2 \tau_{\mathrm{D}_q} f_{\mathrm{D}_q}^2 \left| V_{cq} \right|^2$$

$f_{\mathrm{D}} = (212.15 \pm 1.45)\,\mathrm{MeV}$ and $f_{\mathrm{D_s}} = (248.83 \pm 1.27)\,\mathrm{MeV}$ (FLAG2016)

$|V_{cd}| = 0.2164 \pm 0.0050_{\mathrm{exp}} \pm 0.0014_{\mathrm{th}}$ and $|V_{cs}| = 1.006 \pm 0.018_{\mathrm{exp}} \pm 0.005_{\mathrm{th}}$

CKM: $|V_{\text{cd}}|$ and $|V_{\text{cs}}|$

Averaging the above results with some other inputs (for $|V_{\text{cd}}|$)

$$\boxed{|V_{\text{cd}}| = 0.218 \pm 0.004} \qquad \text{(PDG2019)}$$

and

$$\boxed{|V_{\text{cs}}| = 0.997 \pm 0.017} \qquad \text{(PDG2019)}$$

and so far, the two-gen quark-mixing matrix is

$$|V_{\text{Cabibbo}}| = \begin{pmatrix} 0.98 & 0.23 \\ 0.22 & 1.00 \end{pmatrix}$$

CKM: $|V_{ub}|$ and $|V_{cb}|$

$|V_{ub}|$ and $|V_{cb}|$ are determined from the semileptonic decays

$$b \to u\ell\bar{\nu}_\ell \quad \text{and} \quad b \to c\ell\bar{\nu}_\ell$$

but of course we don't detect free quarks → two routes:

Inclusive: from $B \to X_u \ell\bar{\nu}_\ell$ and $B \to X_c \ell\bar{\nu}_\ell$

$$|V_{ub}| = (4.25 \pm 0.30) \times 10^{-3} \quad \text{and} \quad |V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$$

Exclusive: where we limit X_u and X_c to specific individual states

$$|V_{ub}| = (3.70 \pm 0.16) \times 10^{-3} \quad \text{and} \quad |V_{cb}| = (39.5 \pm 0.9) \times 10^{-3}$$

Leptonic decays, $B_{(c)} \to \ell\bar{\nu}_\ell$, are the cleanest theoretically
but the experimental inputs are not yet competitive.

Averages:

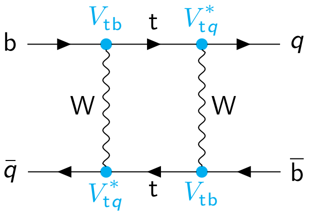
$$\boxed{|V_{ub}| = (3.82 \pm 0.24) \times 10^{-3} \quad \text{and} \quad |V_{cb}| = (41.0 \pm 1.4) \times 10^{-3}}$$

(PDG2019)

CKM: $|V_{td}|$ and $|V_{ts}|$

top does not hadronize . . . so we can't analyze $T \to X_q \ell \bar{\nu}_\ell$

but top quarks dominate in the loops that create $B_{(s)}$ meson mixing



From measurements of $B_{(s)}$ mixing:

$$|V_{td}| = (8.1 \pm 0.5) \times 10^{-3} \quad \text{and} \quad |V_{ts}| = (39.4 \pm 2.3) \times 10^{-3}$$

(PDG2019)

and

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.210 \pm 0.008$$

(PDG2019)

CKM: $|V_{tb}|$

Finally... (phew)

Here we're purely in the realm of top physics

We can look at the decay of the top quark:

$$\frac{\mathcal{B}(t \to W^+ b)}{\sum_q \mathcal{B}(t \to W^+ q)} = \frac{|V_{tb}|^2}{\underbrace{\sum_q |V_{tq}|^2}_{=1 \text{ (unitarity)}}} = |V_{tb}|^2 = \frac{\mathcal{B}(t \to \ell^+ \nu_\ell + \text{b jet})}{\mathcal{B}(t \to \ell^+ \bar{\nu}_\ell + \text{jet})}$$

$$\boxed{|V_{tb}| > 0.975 \text{ at } 95\,\% \text{ CL}}$$

(CMS, PLB736 33 (2014))

One can also measure it in "single top production" at hadron colliders

$$\boxed{|V_{tb}| = 1.019 \pm 0.025}$$

(PDG2019)

CKM: Magnitudes

We can further constrain the CKM matrix (assuming unitarity)

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.224 & 0.974 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

(PDG2019/CKMfitter/UTfit)

What does this mean?

Jarlskog defined the angle between two matrices as

$$\cos\Theta_{AB} \equiv \frac{\text{Tr}(ABAB)}{\text{Tr}(AABB)}$$

(analogous to between two vectors)

Using our Yukawa-coupling mass matrices M_{u} and M_{d} we get

$$\Theta \approx \sqrt{|V_{\text{cb}}|^2 + |V_{\text{ts}}|^2} \approx 3^\circ$$

The misalignment of the quark sectors is not that large!

Unitarity Triangles

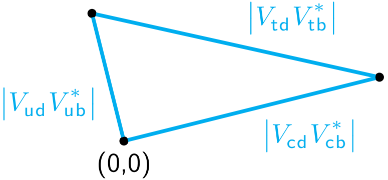
Let's look again at the unitarity of the CKM matrix:

$$V_{\mathrm{CKM}}^{\dagger} V_{\mathrm{CKM}} = \mathbb{1} \quad \rightarrow \quad \sum_k V_{ik}^{\dagger} V_{kj} = \sum_k V_{ki}^{*} V_{kj} = \delta_{ij}$$

and focus on the off-diagonal constraints, for example

$$V_{\mathsf{ud}} V_{\mathsf{ub}}^{*} + V_{\mathsf{cd}} V_{\mathsf{cb}}^{*} + V_{\mathsf{td}} V_{\mathsf{tb}}^{*} = 0$$

we can visualize this as a triangle in the complex plane



Unitarity Triangles

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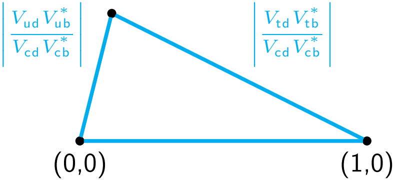
and focus on the off-diagonal constraints, for example

$$\frac{V_{\mathsf{ud}} V_{\mathsf{ub}}^{*}}{V_{\mathsf{cd}} V_{\mathsf{cb}}^{*}} + \frac{V_{\mathsf{td}} V_{\mathsf{tb}}^{*}}{V_{\mathsf{cd}} V_{\mathsf{cb}}^{*}} + 1 = 0$$

we can visualize this as a triangle in the complex plane



for better visualization, we divide out the most precisely known leg

this is a **unitarity triangle**

Unitarity Triangles

Of course, these triangles are only triangles
if the CKM matrix elements have relative complexity;

otherwise they collapse down into lines. Unitarity lines.

Let's look at why they are indeed triangles

by proving the CKM matrix is inherently complex

CKM Parameter Counting

An $N \times N$ complex matrix has $\boldsymbol{2N^2}$ **parameters**:

$\boldsymbol{N^2}$ **magnitudes** and $\boldsymbol{N^2}$ **phases**

Unitarity: $V^\dagger V = \mathbb{1} \Longrightarrow N^2$ constraints — removes N^2 parameters

$\boldsymbol{N^2}$ **parameters**

Unitarity $\supset$ orthogonality $\rightarrow$ $\frac{1}{2}N(N-1)$ of the parameters are magnitudes

$\boldsymbol{\frac{1}{2}N(N-1)}$ **magnitudes** and $\boldsymbol{\frac{1}{2}N(N+1)}$ **phases**

we can remove one phase for each q_{L} and $q_{\mathrm{R}} \rightarrow 2N$ re-phasings
but one overall phase can't be wiped out this way $\rightarrow$ remove only $(2N-1)$:

$\boldsymbol{\frac{1}{2}N(N-1)}$ **magnitudes** and $\boldsymbol{\frac{1}{2}(N-2)(N-1)}$ **phases**

the "magnitudes" are products of sines and cosines of

quark mixing angles

CKM Parameter Counting

So an N-generation CKM matrix has

		$N=1$	$N=2$	$N=3$
mixing angles	$\frac{1}{2}N(N-1)$	0	1	3
complex phases	$\frac{1}{2}(N-2)(N-1)$	0	0	**1**

$$\boxed{\text{CP Violation} \implies \text{complex couplings} \iff N \geq 3}$$

Our 3-generation CKM matrix is often parametrized as

$$\boxed{V_{\mathrm{CKM}} = R(\theta_{23}) \times \mathrm{diag}(e^{-i\delta_{\mathrm{CP}}}, 1, 1) \times R(\theta_{13}) \times \mathrm{diag}(e^{i\delta_{\mathrm{CP}}}, 1, 1) \times R(\theta_{12})}$$

explicitly labeling the complex phase as the CP phase.

The Wolfenstein Parametrization

To show how the unitarity triangles illustrate existence of a complex phase, let's reparametrize the CKM matrix

$$V_{\mathrm{CKM}} = R(\theta_{23}) \times \mathsf{diag}(e^{-i\delta_{\mathrm{CP}}}, 1, 1) \times R(\theta_{13}) \times \mathsf{diag}(e^{i\delta_{\mathrm{CP}}}, 1, 1) \times R(\theta_{12})$$

using four different parameters

$$\begin{aligned} \sin\theta_{12} &\equiv \lambda \\ \sin\theta_{13} &\equiv A\lambda^2 \\ \sin\theta_{13} e^{i\delta_{\mathrm{CP}}} &\equiv A\lambda^3(\rho + i\eta) \\ \rightarrow \delta_{\mathrm{CP}} &= \arg(\rho + i\eta) \end{aligned}$$

plug it in . . .

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad + \quad \mathcal{O}\left(\lambda^4\right)$$

CKM Phase in the Unitarity Triangle

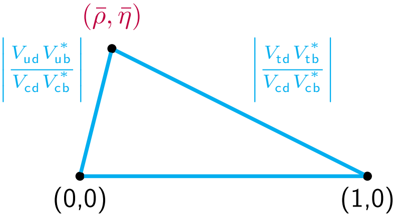
$$V_{\text{CKM}} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

let's plug this into our unitarity triangle

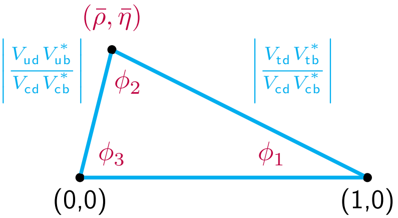
$$\frac{V_{\mathsf{ud}}V^*_{\mathsf{ub}}}{V_{\mathsf{cd}}V^*_{\mathsf{cb}}} \approx \frac{[1-\frac{1}{2}\lambda^2][A\lambda^3(\rho+i\eta)]}{[-\lambda][A\lambda^2]} = -(1-\tfrac{1}{2}\lambda^2)(\rho+i\eta)$$

$$\equiv -(\bar\rho + i\bar\eta)$$

So we have



CKM Phase in the Unitarity Triangle



We measure the angles . . .

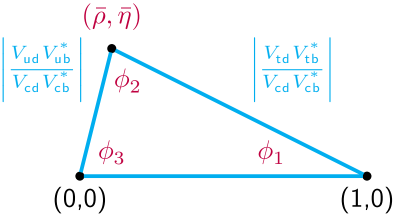
. . . and of course anything we measure, we name.

unfortunately two sets of names are used

$$\alpha = \phi_2 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \beta = \phi_1 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\gamma = \phi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

CKM: Phase

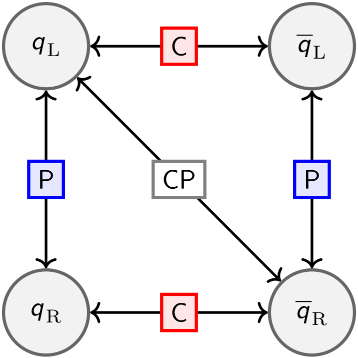


But to explain how we measure the angles

let's first look at the concept of

CP violation

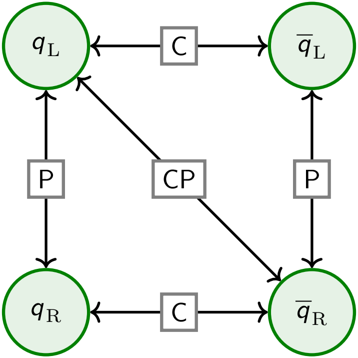
What is CP?



Charge conjugation mirrors charges:
it exchanges particle and antiparticle.

Parity inversion mirrors spacial coordinates:
it exchanges left and right handedness.

What is CP conservation?

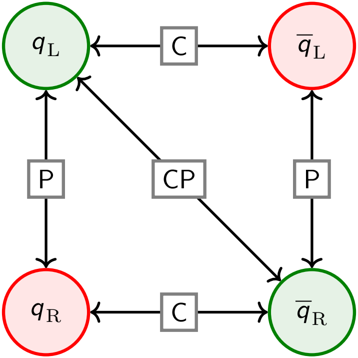


The electromagnetic & strong forces conserve C, P, and CP:

$$\Gamma_{\mathrm{L}} = \Gamma_{\mathrm{R}} = \overline{\Gamma}_{\mathrm{L}} = \overline{\Gamma}_{\mathrm{R}}$$

$$\underbrace{\frac{\Gamma_{\mathrm{L}} - \Gamma_{\mathrm{R}}}{\Gamma_{\mathrm{L}} + \Gamma_{\mathrm{R}}}}_{\mathcal{A}^{\mathrm{P}}} = \underbrace{\frac{\Gamma_{\mathrm{L}} - \overline{\Gamma}_{\mathrm{L}}}{\Gamma_{\mathrm{L}} + \overline{\Gamma}_{\mathrm{L}}}}_{\mathcal{A}^{\mathrm{C}}} = \underbrace{\frac{\Gamma_{\mathrm{L}} - \overline{\Gamma}_{\mathrm{R}}}{\Gamma_{\mathrm{L}} + \overline{\Gamma}_{\mathrm{R}}}}_{\mathcal{A}^{\mathrm{CP}}} = 0$$

What is CP violation?



The weak force **maximally** violates C and P and **nearly** conserves CP:

$$\Gamma_\mathrm{R} = \overline{\Gamma}_\mathrm{L} = 0 \qquad\qquad \Gamma_\mathrm{L} \neq \overline{\Gamma}_\mathrm{R}$$

$$\underbrace{\frac{\Gamma_\mathrm{L} - \cancel{\Gamma_\mathrm{R}}}{\Gamma_\mathrm{L} + \cancel{\Gamma_\mathrm{R}}}}_{\mathcal{A}^\mathrm{P}} = \underbrace{\frac{\Gamma_\mathrm{L} - \cancel{\overline{\Gamma}_\mathrm{L}}}{\Gamma_\mathrm{L} + \cancel{\overline{\Gamma}_\mathrm{L}}}}_{\mathcal{A}^\mathrm{C}} = 1 \qquad\qquad \left|\underbrace{\frac{\Gamma_\mathrm{L} - \overline{\Gamma}_\mathrm{R}}{\Gamma_\mathrm{L} + \overline{\Gamma}_\mathrm{R}}}_{\mathcal{A}^\mathrm{CP}}\right| = 10^{-3} \text{ to } 10^{-1}$$

What causes direct CP asymmetry?

A process' rate is proportional to the square of its amplitude:

$$\Gamma \propto |A|^2, \quad \text{and} \quad \overline{\Gamma} \propto |\overline{A}|^2 \quad \text{so} \quad \boxed{\mathcal{A}^{\mathrm{CP}} = \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2}}$$

Let us suppose there are several contributing subprocesses: $|A_1| > |A_2| > \ldots$

$$A = |A_1| e^{i \arg A_1} + |A_2| e^{i \arg A_2} + \ldots$$
$$\overline{A} = |\overline{A}_1| e^{i \arg \overline{A}_1} + |\overline{A}_2| e^{i \arg \overline{A}_2} + \ldots$$

And that

$$|A_i| = |\overline{A}_i|$$
$$\arg A_i \equiv \delta_i + \phi_i$$
$$\arg \overline{A}_i \equiv \delta_i - \phi_i$$

$\delta_i \equiv$ CP-even phase ("strong phase")

$\phi_i \equiv$ CP-odd phase ("weak phase")

What causes direct CP asymmetry?

For clarity, let's limit to two amplitudes contributing to, e.g., a decay:

$$A = |A_1|e^{i\delta_1}e^{i\phi_1} \quad + |A_2|e^{i\delta_2}e^{i\phi_2}$$
$$\overline{A} = |A_1|e^{i\delta_1}e^{-i\phi_1} + |A_2|e^{i\delta_2}e^{-i\phi_2}$$

$$|A|^2 = |A_1|^2 + |A_2|^2 + |A_1||A_2|\left(e^{i\Delta\delta}e^{-i\Delta\phi} + e^{-i\Delta\delta}e^{i\Delta\phi}\right)$$

$$|\overline{A}|^2 = |A_1|^2 + |A_2|^2 + |A_1||A_2|\left(e^{i\Delta\delta}e^{i\Delta\phi} + e^{-i\Delta\delta}e^{-i\Delta\phi}\right)$$

with

$$\Delta\delta \equiv \delta_2 - \delta_1 \qquad \Delta\phi \equiv \phi_2 - \phi_1$$

and

$$\mathcal{A}^{\rm CP} = \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \frac{-2|A_1||A_2|\sin\Delta\delta\sin\Delta\phi}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos\Delta\delta\cos\Delta\phi}$$

$$= -2\tfrac{|A_2|}{|A_1|}\sin\Delta\delta\sin\Delta\phi + \mathcal{O}\left(\tfrac{|A_2|^2}{|A_1|^2}\right) \qquad \text{for } |A_2| \ll |A_1|$$

What causes direct CP asymmetry?

$$\mathcal{A}^{\mathrm{CP}} \approx -2\,\frac{|A_2|}{|A_1|}\,\sin\Delta\delta\,\sin\Delta\phi$$

there is CP violation **if and only if**

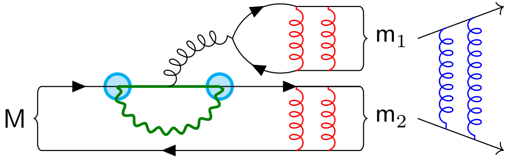
$\frac{|A_2|}{|A_1|} \neq 0 \longrightarrow$ more than one process contributes

$\sin\Delta\delta \neq 0 \longrightarrow$ nonzero CP-even phase difference betwen them.

$\sin\Delta\phi \neq 0 \longrightarrow$ nonzero CP-odd phase difference between them.

Where do CP-even and CP-odd phases come from?

Let's focus on mesons decaying into other mesons: $\mathsf{M} \to \mathsf{m}_1\mathsf{m}_2\ldots$



CP-even phases can arise from

hadronization: $(q\bar{q}) \to$ m

final-state interactions

loops

These phases are the same for a process and its CP conjugate

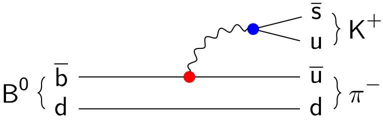
CP-odd phases come from

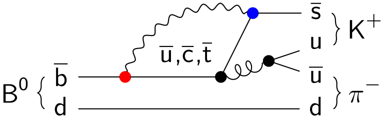
complex couplings $\Longrightarrow$ **CKM Matrix Elements V_{ij}**

which are *complex-conjugated* by CP conjugation:

$$x = |x|e^{+i\phi_x} \to |x|e^{-i\phi_x} = \overline{x}$$

An example: $B^0 \to K^+\pi^-$





$$\equiv T \times V_{ub}^{*} V_{us} \qquad\qquad \equiv P \times V_{tb}^{*} V_{ts}$$

$$\equiv |T|\left|V_{ub}^{*} V_{us}\right| \underbrace{e^{i\delta_T}}_{\text{CP even}} e^{i\arg(V_{ub}^{*} V_{us})} \qquad \equiv |P|\left|V_{tb}^{*} V_{ts}\right| \underbrace{e^{i\delta_P}}_{\text{CP even}} e^{i\arg(V_{tb}^{*} V_{ts})}$$

$$A = |T|\left|V_{ub}^{*} V_{us}\right| e^{i\delta_T} e^{+i\arg(V_{ub}^{*} V_{us})} + |P|\left|V_{tb}^{*} V_{ts}\right| e^{i\delta_P} e^{+i\arg(V_{tb}^{*} V_{ts})}$$

$$\overline{A} = |T|\left|V_{ub}^{*} V_{us}\right| e^{i\delta_T} \underbrace{e^{-i\arg(V_{ub}^{*} V_{us})}}_{\text{CP odd}} + |P|\left|V_{tb}^{*} V_{ts}\right| e^{i\delta_P} \underbrace{e^{-i\arg(V_{tb}^{*} V_{ts})}}_{\text{CP odd}}$$

$$\boxed{\mathcal{A}^{\mathrm{CP}}(B^0 \to K^+\pi^-) = -2\,\frac{|P|\left|V_{tb}^{*} V_{ts}\right|}{|T|\left|V_{ub}^{*} V_{us}\right|}\ \sin\Delta\delta\ \sin\arg(V_{ub} V_{us}^{*} V_{tb}^{*} V_{ts})}$$

$$\sin\arg(V_{ub} V_{us}^{*} V_{tb}^{*} V_{ts}) = \frac{\mathrm{imag}(V_{ub} V_{us}^{*} V_{tb}^{*} V_{ts})}{\left|V_{ub} V_{us}^{*} V_{tb}^{*} V_{ts}\right|} = \sin\delta_{\mathrm{CP}}$$

CPV Measurement Examples

This asymmetry has been measured very clearly

$$A^{\mathrm{CP}}(\mathrm{B}^0 \to \mathrm{K}^+\mathrm{K}^-) = (-8.3 \pm 0.4)\,\% \qquad \text{(PDG2019)}$$

But it wasn't the first measurement of CP violation

(Indirect) CPV was first seen in $\mathrm{K}^0_\mathrm{L} \to \pi\pi$ in 1964.

Direct CPV was first seen in $\mathrm{K}^0_\mathrm{L} \to \pi\ell\bar{\nu}_\ell$ in the 1990s

$$A_\mathrm{L} \equiv \frac{\Gamma(\mathrm{K}^0_\mathrm{L} \to \pi^+\ell^-\bar{\nu}_\ell) - \Gamma(\mathrm{K}^0_\mathrm{L} \to \pi^-\ell^+\nu_\ell)}{\Gamma(\mathrm{K}^0_\mathrm{L} \to \pi^+\ell^-\bar{\nu}_\ell) + \Gamma(\mathrm{K}^0_\mathrm{L} \to \pi^-\ell^+\nu_\ell)} = (3.32 \pm 0.06) \times 10^{-3}$$

(PDG2019)

(Direct) CP violation was first observed in B^0_s mesons via

$$A^{\mathrm{CP}}(\mathrm{B}^0_\mathrm{s} \to \mathrm{K}^-\pi^+) = (22.1 \pm 1.5)\,\% \qquad \text{(PDG2019)}$$

(Direct) CP violation was first observed in the charm quark sector via

$$\Delta A^{\mathrm{CP}} \equiv A^{\mathrm{CP}}(\mathrm{D}^0 \to \mathrm{K}^+\mathrm{K}^-) - A^{\mathrm{CP}}(\mathrm{D}^0 \to \pi^+\pi^-) = (-0.161 \pm 0.029)\,\%$$

(PDG2019)

Measuring ϕ_1 (β) abd ϕ_2 (α)

Let's return to the CKM matrix

We measure ϕ_1 in $\mathrm{b} \to \mathrm{c\bar{c}s}$ decays
by looking at the **time-dependent CP asymmetry** of

$$A^{\mathrm{CP}}(\mathrm{B}^0 \to f; t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)$$

with

$$S_f \equiv \frac{2\,\mathrm{Im}(\lambda_f)}{1 + |\lambda_f|^2}, \qquad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \qquad \lambda_f \propto e^{-2\phi_1}$$

averaging measurements with different final states, f:

$$\boxed{\sin 2\phi_1 = 0.691 \pm 0.017} \qquad \text{(HFLAV)}$$

We measure ϕ_2 in $\mathrm{b} \to \mathrm{u\bar{u}d}$ decays ...similar to ϕ_1, but a bit more complicated
from analysis of B decays to $\pi\pi$, $\rho\pi$, $\rho\rho$

$$\boxed{\alpha = (84.5^{+5.9}_{-5.2})^\circ} \qquad \text{(HFLAV)}$$

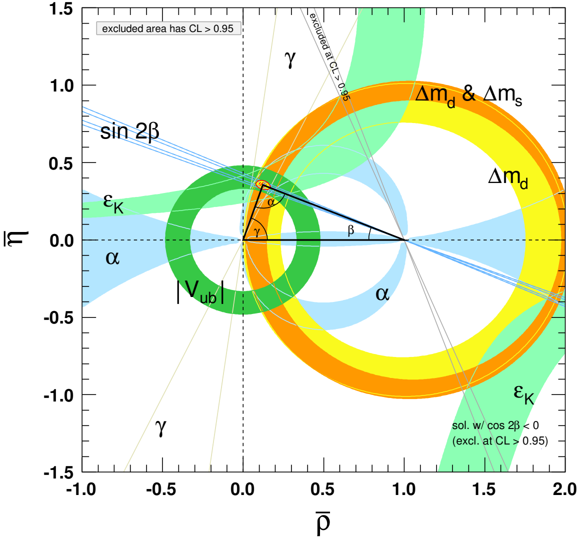
Measuring ϕ_3 (γ)

We measure ϕ_3 via $B^{\pm} \to DK^{\pm}$ decays:

$$\boxed{\gamma = (73.5^{+4.2}_{-5.1})^\circ} \qquad \text{(PDG2019)}$$

Putting it all together

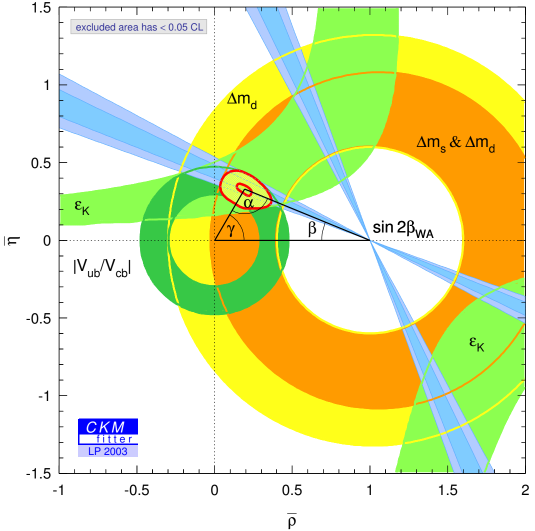
We can combine all the information on the angles (and related measurements)



CKMfitter 2018

Putting it all together

We can combine all the information on the angles (and related measurements)



CKMfitter 2003

So. . . CP violation. . . so what?

We've observed CP violation in s (K), c (D), and b (B) decays

Is it just a curiosity to measure?

No: we need CP violation in the world.

We observe that the universe is CP **asymmetric**:

$$\mathcal{A}^{\mathrm{CP}}_{\mathrm{univ}} \equiv \frac{\overbrace{\text{original matter} - \text{original antimatter}}^{\text{observable matter (baryons)}}}{\underbrace{\text{original matter} + \text{original antimatter}}_{\text{observable radiation (photons)}}} \geq \mathcal{O}\left(10^{-10}\right)$$

known from measurement of the visible baryon density
and the cosmic-microwave-background radiation.

The existence of our matter-dominated universe violates CP!

Does the standard model explain this asymmetry?

SM CP violation comes from V_{CKM}—from a mismatch between M_{u} and M_{d}

Jarlskog showed that this mismatch can be quantified; and

$$\mathcal{A}^{\text{CP}}_{\text{univ}} \approx 2\, m_{\mathsf{t}}^4\, m_{\mathsf{b}}^4\, m_{\mathsf{c}}^2\, m_{\mathsf{s}}^2\, |J|/\Lambda^{12} \approx \left(\frac{7\,\text{GeV}}{\Lambda}\right)^{12} |J|$$

Jarlskog's $|J|$ separates out the V_{CKM}-dependent part

$$\begin{aligned} |J| &= \text{the area of any unitarity triangle (with side normalization)} \\ &= \left|\text{imag}(V_{ij} V_{il}^* V_{kj}^* V_{kl})\right| \approx \sin\theta_{12} \sin\theta_{23} \sin\theta_{13} \sin\delta_{\text{CP}} \\ &= (3.2 \pm 0.1) \times 10^{-5} \end{aligned}$$

[CKMfitter 2018]

Remaning ingredient: $\Lambda \equiv$ temperature at which asymmetry is created

$$\Lambda \gtrsim 100\,\text{GeV}$$

So

$$\boxed{\left(\mathcal{A}^{\text{CP}}_{\text{univ}}\right)_{\text{SM}} \lesssim \mathcal{O}\left(10^{-19}\right) \ll \mathcal{O}\left(10^{-10}\right) \lesssim \left(\mathcal{A}^{\text{CP}}_{\text{univ}}\right)_{\text{obs}}}$$

New sources of CP violation

$$\left(\mathcal{A}^{\mathrm{CP}}_{\mathrm{univ}}\right)_{\mathrm{SM}} \ll \left(\mathcal{A}^{\mathrm{CP}}_{\mathrm{univ}}\right)_{\mathrm{obs}}$$

so look for sources of CP violation beyond the standard model

look where the standard model expects CP conservation

We can do this, for example, in

charm meson decays , and

τ lepton decays

(Examples chosen with bias from my own work.)

Of course:

new sources of CP violation are
signs of physics beyond the standard model!

CP violation in D $\to \pi\pi$ decays

Initial states:

$$D^+ = c\bar{d}, \quad D^0 = c\bar{u}, \qquad \overline{D}^0 = \bar{c}u, \quad D^- = \bar{c}d$$

all spinless; charmed; with isospin $= \frac{1}{2}$ (carried by the light quark)

Final states:

$$\pi^+\pi^- \qquad \pi^0\pi^0 \qquad \pi^+\pi^0$$

all spinless; charmless; superpositions of total isospin $= 0, 2$

So the decays involve

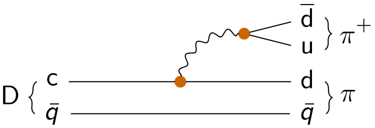
$$|\Delta C| = 1 \quad \text{and} \quad \boxed{|\Delta I| = \tfrac{1}{2} \text{ or } \tfrac{3}{2}}$$

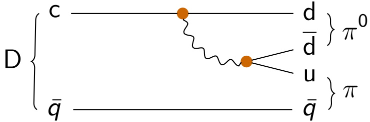
We can decompose the decay amplitudes by isospospin transition:

$$\begin{aligned} A(D^0 \to \pi^+\pi^-) &= \sqrt{2}A_{\frac{3}{2}} + \sqrt{2}A_{\frac{1}{2}} \\ A(D^0 \to \pi^0\pi^0) &= 2A_{\frac{3}{2}} - A_{\frac{1}{2}} \\ A(D^+ \to \pi^+\pi^0) &= 3A_{\frac{3}{2}} \end{aligned}$$

CP violation in $D \to \pi\pi$ decays

There are tree-level amplitudes:

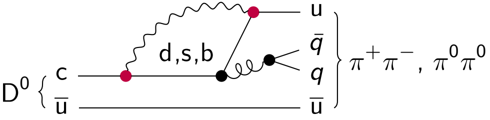




$D^+ \to \pi^+\pi^0$, $D^0 \to \pi^+\pi^-$ $D^+ \to \pi^+\pi^0$, $D^0 \to \pi^0\pi^0$

$$\propto V_{cd}^* V_{ud} \qquad (\bar{d}c)(\bar{u}d) \to |\mathbf{\Delta I}| = \tfrac{1}{2}, \tfrac{3}{2}$$

And loop-level amplitudes:



$D^0 \to \pi^+\pi^-$, $D^0 \to \pi^0\pi^0$

$$\propto V_{cs}^* V_{us} \qquad \sum_q (\bar{u}c)(\bar{q}q) \to |\mathbf{\Delta I}| = \tfrac{1}{2}, \not{\tfrac{3}{2}}$$

CP violation in D $\to \pi\pi$ decays

$$A_{\frac{1}{2}} = T_{\frac{1}{2}} \times V_{cd}^{*} V_{ud} \quad + \quad P_{\frac{1}{2}} \times V_{cs}^{*} V_{us}$$
$$A_{\frac{3}{2}} = T_{\frac{3}{2}} \times V_{cd}^{*} V_{ud}$$

$$A(D^0 \to \pi^+\pi^-) \equiv A_{+-} = \sqrt{2}A_{\frac{3}{2}} + \sqrt{2}A_{\frac{1}{2}}$$
$$A(D^0 \to \pi^0\pi^0) \equiv A_{00} = 2A_{\frac{3}{2}} - A_{\frac{1}{2}}$$
$$A(D^+ \to \pi^+\pi^0) \equiv A_{+0} = 3A_{\frac{3}{2}}$$

$$\boxed{\mathcal{A}_{+0}^{\mathrm{CP}} = \mathcal{A}_{\frac{3}{2}}^{\mathrm{CP}} = 0}$$

$$\boxed{\mathcal{A}_{\frac{1}{2}}^{\mathrm{CP}} \text{ can be nonzero}}$$

$$\boxed{\mathcal{A}_{\frac{1}{2}}^{\mathrm{CP}} \neq 0 \leftrightarrow \mathcal{A}_{+-}^{\mathrm{CP}} \neq 0,\ \mathcal{A}_{00}^{\mathrm{CP}} \neq 0}$$

CP violation in D $\to \pi\pi$ decays

$$\mathcal{A}^{\rm CP}_{\frac{3}{2}} = \mathcal{A}^{\rm CP}(\mathrm{D}^+ \to \pi^+\pi^0) \neq 0 \longrightarrow \text{physics beyond the standard model}$$

also, if $\quad \mathcal{A}^{\rm CP}(\mathrm{D}^0 \to \pi^+\pi^-) \neq 0 \quad$ or $\quad \mathcal{A}^{\rm CP}(\mathrm{D}^0 \to \pi^0\pi^0) \neq 0$

but $\quad \mathcal{A}^{\rm CP}_{\frac{1}{2}} = 0$

$\longrightarrow$ physics beyond the standard model (also in $|\Delta I| = \frac{3}{2}$)

$$\mathcal{A}^{\rm CP}_{\frac{1}{2}} = \frac{\mathcal{A}^{\rm CP}_{+-}\mathcal{B}_{+-} + \mathcal{A}^{\rm CP}_{00}\mathcal{B}_{00}\frac{p_{+-}}{p_{00}} - \frac{2}{3}\mathcal{A}^{\rm CP}_{+0}\mathcal{B}_{+0}\frac{\tau_0}{\tau_+}\frac{p_{+-}}{p_{+0}}}{\mathcal{B}_{+-} + \mathcal{B}_{00}\frac{p_{+-}}{p_{00}} - \frac{2}{3}\mathcal{B}_{+0}\frac{\tau_0}{\tau_+}\frac{p_{+-}}{p_{+0}}}$$

D lifetime ratio:	$\frac{\tau_0}{\tau_+} = (39.4 \pm 0.3)\,\%$	[PDG'18]
Breakup mometum ratio:	$\frac{p_{+-}}{p_{00}} = 1 - 0.1\,\%$	[PDG'18]
Breakup mometum ratio:	$\frac{p_{+-}}{p_{+0}} = 1 - 0.3\,\%$	[PDG'18]

CP violation in D $\to \pi\pi$ decays

$$\mathcal{A}^{\rm CP}_{\frac{3}{2}} = \mathcal{A}^{\rm CP}(\mathrm{D}^+ \to \pi^+\pi^0) \neq 0 \longrightarrow \text{physics beyond the standard model}$$

also, if $\mathcal{A}^{\rm CP}(\mathrm{D}^0 \to \pi^+\pi^-) \neq 0$ or $\mathcal{A}^{\rm CP}(\mathrm{D}^0 \to \pi^0\pi^0) \neq 0$

but $\mathcal{A}^{\rm CP}_{\frac{1}{2}} = 0$

$\longrightarrow$ physics beyond the standard model (also in $|\Delta I| = \frac{3}{2}$)

$$\mathcal{A}^{\rm CP}_{\frac{1}{2}} = \frac{\mathcal{A}^{\rm CP}_{+-}\mathcal{B}_{+-} + \mathcal{A}^{\rm CP}_{00}\mathcal{B}_{00}\frac{p_{+-}}{p_{00}} - \frac{2}{3}\mathcal{A}^{\rm CP}_{+0}\mathcal{B}_{+0}\frac{\tau_0}{\tau_+}\frac{p_{+-}}{p_{+0}}}{\mathcal{B}_{+-} + \mathcal{B}_{00}\frac{p_{+-}}{p_{00}} - \frac{2}{3}\mathcal{B}_{+0}\frac{\tau_0}{\tau_+}\frac{p_{+-}}{p_{+0}}}$$

Branching fractions:

$$\mathcal{B}_{+-} = (1.407 \pm 0.025) \times 10^{-3}$$ [PDG'18]

$$\mathcal{B}_{00} = (8.22 \pm 0.25) \times 10^{-4}$$ [PDG'18]

$$\mathcal{B}_{+0} = (1.17 \pm 0.06) \times 10^{-3}$$ [PDG'18]

CP violation in $D \to \pi\pi$ decays

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CP asymmetries:

$\mathcal{A}^{\rm CP}(\mathrm{D}^0 \to \pi^+\pi^-) = (0.13 \pm 0.14)\,\%$ [PDG'18]

$\mathcal{A}^{\rm CP}(\mathrm{D}^0 \to \pi^0\pi^0) = (0.0 \pm 0.6)\,\%$ [PDG'18]

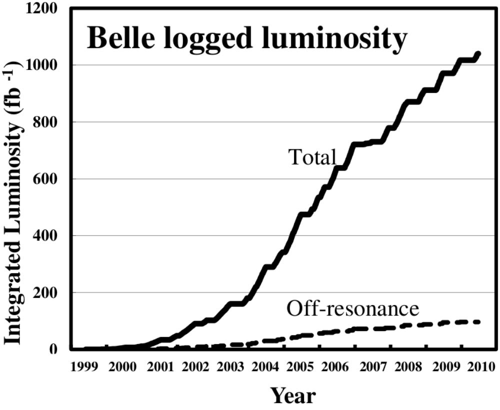
$\mathcal{A}^{\rm CP}(\mathrm{D}^+ \to \pi^+\pi^0) = (2.9 \pm 2.9)\,\%$ [CLEO, PRD81,052013; 2010]

$\mathcal{A}^{\mathrm{CP}}(\mathrm{D}^+ \to \pi^+\pi^0)$ at Belle

Belle was an experiment at the KEKB asymmetric e^+e^- collider in Japan.

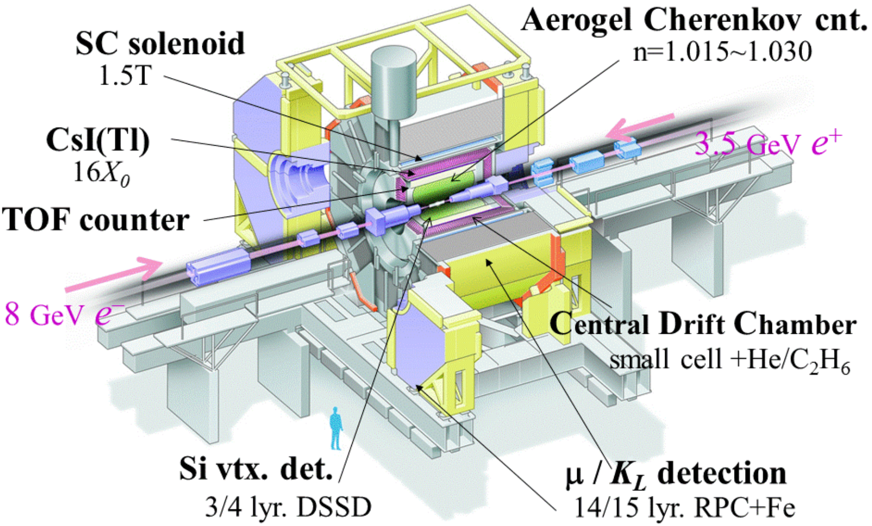
$$e^+ \ (3.5\,\mathrm{GeV}) \ \longrightarrow \ \sqrt{s} \approx \text{mass of } \Upsilon(4S) \ \longleftarrow \ (8.0\,\mathrm{GeV}) \ e^-$$

Over a decade, Belle collected approx. $1\,\mathrm{ab}^{-1}$ of integrated luminonsity.



$\Upsilon(4S)$	$711.0\,\mathrm{fb}^{-1}$
Off resonance	$122.4\,\mathrm{fb}^{-1}$
$\Upsilon(5S)$	$121.4\,\mathrm{fb}^{-1}$
$\Upsilon(2S)$	$24.9\,\mathrm{fb}^{-1}$
$\Upsilon(1S)$	$5.7\,\mathrm{fb}^{-1}$
$\Upsilon(3S)$	$2.9\,\mathrm{fb}^{-1}$

$\mathcal{A}^{\mathrm{CP}}(\mathrm{D}^+ \to \pi^+\pi^0)$ at Belle



$\mathcal{A}^{\mathrm{CP}}(\mathsf{D}^+ \to \pi^+\pi^0)$ at Belle: measurement principle

- Out of the Belle data, we select for events with $\mathsf{D}^\pm \to \pi^\pm\pi^0$
- We fit to the $\mathsf{D}^\pm$ mass distributions to get the numbers of signal events
- We calculate the asymmetry.

But

$$\frac{N(\mathsf{D}^+ \to \pi^+\pi^0) - N(\mathsf{D}^- \to \pi^-\pi^0)}{N(\mathsf{D}^+ \to \pi^+\pi^0) + N(\mathsf{D}^- \to \pi^-\pi^0)} \equiv A^{\mathrm{raw}}_{\pi\pi} \approx A^{\mathrm{CP}}_{+0} + A^{\mathrm{FB}}_{\mathsf{D}} + A^{\mathrm{det}}_{\pi}$$

$$\frac{N(\mathsf{D}^+ \to \pi^+\mathsf{K}^0_\mathsf{S}) - N(\mathsf{D}^- \to \pi^-\mathsf{K}^0_\mathsf{S})}{N(\mathsf{D}^+ \to \pi^+\mathsf{K}^0_\mathsf{S}) + N(\mathsf{D}^- \to \pi^-\mathsf{K}^0_\mathsf{S})} \equiv A^{\mathrm{raw}}_{\pi\mathsf{K}} \approx A^{\mathrm{CP}}_{\pi\mathsf{K}} + A^{\mathrm{FB}}_{\mathsf{D}} + A^{\mathrm{det}}_{\pi}$$

So

$$A^{\mathrm{CP}}(\mathsf{D}^+ \to \pi^+\pi^0) = A^{\mathrm{raw}}_{\pi\pi} - A^{\mathrm{raw}}_{\pi\mathsf{K}} + \underbrace{A^{\mathrm{CP}}(\mathsf{D}^+ \to \pi^+\mathsf{K}^0_\mathsf{S})}_{(0.363 \pm 0.115)\,\%\ \text{[Belle]}}$$

$\mathcal{A}^{\mathrm{CP}}(\mathrm{D}^+ \to \pi^+\pi^0)$ at Belle: event selection

To reconstruct $\mathrm{D}^\pm \to \pi^\pm\pi^0$, we require

- a good charged track, identifiable as a pion
- two good photons consistent with coming from a neutral pion
- $m(\pi^\pm\pi^0)$ within 200 MeV of the nominal $\mathrm{D}^\pm$ mass (1870 MeV)

To reconstruct $\mathrm{D}^\pm \to \pi^\pm\mathrm{K}^0_\mathrm{S}$, we require

- a good charged track, identifiable as a pion
- two further good charged pions, consistent with coming from a K^0_S
- $m(\pi^\pm\mathrm{K}^0_\mathrm{S})$ within 80 MeV of the nominal $\mathrm{D}^\pm$ mass

We divide events into two groups:

- "Tagged" events: $\mathrm{D}^* \to \pi^0\mathrm{D}^+$,
  with $|m(\pi^0\mathrm{D}^+) - m(\mathrm{D}^+)|$ consistent with nominal mass difference.

- "Untagged" events: events not classifiable as "Tagged"

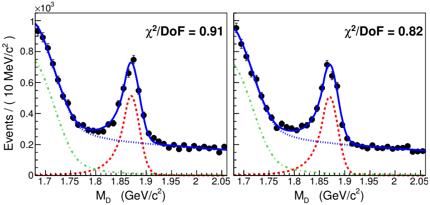
$\mathcal{A}^{\mathrm{CP}}(\mathrm{D}^+ \to \pi^+\pi^0)$ at Belle: the fit

Both tagged and untagged data contain

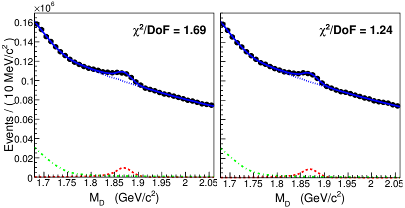
- Signal
- Combinatoric Background
- Misreconstructed D and D_s Background

We fit D^+ and D^- mass distributions in tagged and untagged data separately, with $A^{\mathrm{raw}}_{\pi\pi}$ a free parameter in the fits

Tagged (D^+, D^-)



Untagged (D^+, D^-)



Signal yields: 6632 ± 256 tagged events; $100\,934 \pm 1952$ untagged events.

$$A^{\mathrm{raw}}_{\pi\pi} - A^{\mathrm{raw}}_{\pi \mathrm{K}} = (+2.67 \pm 1.24 \pm 0.20)\ \%$$

$\mathcal{A}^{\mathrm{CP}}(\mathsf{D}^+ \to \pi^+\pi^0)$ at Belle: results

The CP asymmetry is

$$\boxed{A^{\mathrm{CP}}(\mathsf{D}^+ \to \pi^+\pi^0) = (+2.31 \pm 1.24 \pm 0.23)\ \%}$$

including 0.12 % uncertainty from $A^{\mathrm{CP}}(\mathsf{D}^+ \to \pi^+\mathsf{K}^0_\mathsf{S})$.

And we update

$$\mathcal{A}^{\mathrm{CP}}_{\frac{1}{2}} = \frac{\mathcal{A}^{\mathrm{CP}}_{+-}\mathcal{B}_{+-} + \mathcal{A}^{\mathrm{CP}}_{00}\mathcal{B}_{00}\frac{p_{+-}}{p_{00}} - \frac{2}{3}\mathcal{A}^{\mathrm{CP}}_{+0}\mathcal{B}_{+0}\frac{\tau_0}{\tau_+}\frac{p_{+-}}{p_{+0}}}{\mathcal{B}_{+-} + \mathcal{B}_{00}\frac{p_{+-}}{p_{00}} - \frac{2}{3}\mathcal{B}_{+0}\frac{\tau_0}{\tau_+}\frac{p_{+-}}{p_{+0}}}$$

Using the CLEO measurement of $\mathcal{A}^{\mathrm{CP}}_{+0} = (2.9 \pm 2.9)\,\%$

$$\mathcal{A}^{\mathrm{CP}}_{\frac{1}{2}} = (-3.5 \pm 5.4) \times 10^{-3}$$

And with the updated $\mathcal{A}^{\mathrm{CP}}_{+0}$ measurement:

$$\boxed{\mathcal{A}^{\mathrm{CP}}_{\frac{1}{2}} = (-2.6 \pm 3.4) \times 10^{-3}}$$

(PRD97, 011101(R), 2018)

Could there be new physics in CPV in τ decay?

Let's leave the quark sector now:
In the standard model, we expect no direct CP violation in τ decay

But, we do expect CP violation in K decay.

$$\mathcal{A}^{\mathrm{CP}}(\tau^+ \to \overline{\nu}_\tau \mathsf{K}^0 \pi^+) = \frac{\int \eta(t) \left[\Gamma_{\mathsf{K}\to\pi\pi}(t) - \Gamma_{\overline{\mathsf{K}}\to\pi\pi}(t)\right] \mathrm{d}t}{\int \eta(t) \left[\Gamma_{\mathsf{K}\to\pi\pi}(t) + \Gamma_{\overline{\mathsf{K}}\to\pi\pi}(t)\right] \mathrm{d}t}$$

taking into account the time-dependent $[\pi\pi]_{\mathsf{K}}$ detection efficiency, $\eta(t)$.

[JHEP.04.2012.002, 2012]

The BaBar experiment measured this asymmetry:
with their $\eta(t)$, they expected

$$A^{\mathrm{CP}}_{\mathrm{SM}}(\tau^+ \to \overline{\nu}_\tau \mathsf{K}^0 \pi^+) = (0.36 \pm 0.01)\,\%$$

but measured [Phys. Rev. D 85, 031102(R), 2012]

$$A^{\mathrm{CP}}(\tau^+ \to \overline{\nu}_\tau \mathsf{K}^0 \pi^+) = (-0.36 \pm 0.23 \pm 0.11)\,\%$$

At odds with the standard model at 2.8 standard deviations!

τ: Lepton Flavor Violation

Another test for physics beyond the standard model:

look for violation of lepton flavor number

A normal decay: $\tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu$

$$\underbrace{L_\tau(\tau^-)}_{1} = \underbrace{L_\tau(\nu_\tau)}_{1} + \underbrace{L_\tau(\mu^-)}_{0} + \underbrace{L_\tau(\overline{\nu}_\mu)}_{0}$$

$$\underbrace{L_\mu(\tau^-)}_{0} = \underbrace{L_\mu(\nu_\tau)}_{0} + \underbrace{L_\mu(\mu^-)}_{1} + \underbrace{L_\mu(\overline{\nu}_\mu)}_{-1}$$

A LFV'ing decay: $\tau^- \to \mu^- \gamma$

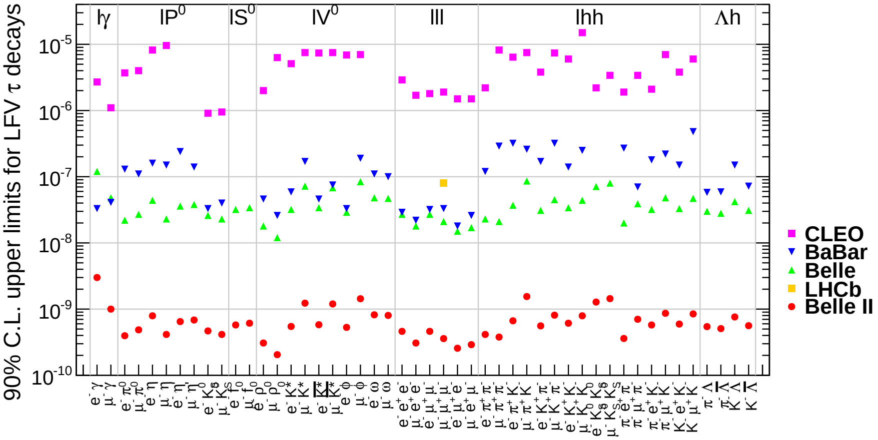
$$\underbrace{L_\tau(\tau^-)}_{1} \neq \underbrace{L_\tau(\mu^-)}_{0} + \underbrace{L_\tau(\gamma)}_{0}$$

$$\underbrace{L_\mu(\tau^-)}_{0} \neq \underbrace{L_\mu(\mu^-)}_{1} + \underbrace{L_\mu(\gamma)}_{0}$$

τ: Lepton Flavor Violation

Another test for physics beyond the standard model:

look for violation of lepton flavor number



Lepton flavor universality violation

A similar search for physics beyond the standard model:

look for differences betweens the lepton generations (in decays)

for example:

$$\frac{\Gamma(\mathrm{B} \to \mathrm{K}\,\mu^+\mu^-)}{\Gamma(\mathrm{B} \to \mathrm{K}\,\mathrm{e}^+\mathrm{e}^-)} \equiv \mathcal{R}_{\mathrm{K}}^{\mu\mathrm{e}}$$

this is predicted and measured in different ranges of $q^2 \equiv m_{\ell\ell}^2$

In the low-q^2 range $[1, 6]$ GeV^2, LHCb measured

$$\mathcal{R}_{\mathrm{K}}^{\mu\mathrm{e}} = 0.745\ {}^{+0.090}_{-0.074}\ \pm 0.036$$ (PRL 113, 151601)

at tension with the SM prediction at 2.6σ:

$$\mathcal{R}_{\mathrm{K}}^{\mu\mathrm{e}} = 1 + (3^{+1.0}_{-0.7}) \times 10^{-4}$$ (JHEP12(2007)040)

(BaBar and Belle have measured values consistent with the SM within 1σ)

Summary

Flavor physics is a rich field, with many activities aiming at

- precisely measuring the (flavor) parameters of the standard model, e.g.
  - $|V_{ij}|$
  - unitarity triangle angles

  by measuring the properties of particles and their decays, e.g.
  - branching fractions
  - CP asymmetries
- searching for signs of new physics
  - observing SM-forbidden CP asymmetries
  - observing SM-forbidden decays
  - over-constraining the SM parameter space

Some topics I did not cover, that I recommend looking up on your own time:

- the kaon sector (ϵ and ϵ')
- neutral meson mixing (K^0, D^0, B^0, B_s)
- rare decay searches
- the lepton flavor sector (neutrinos and the PMNS matrix)