Nucleosynthesis in heavy-ion collisions at the LHC via the Saha equation

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Bormio 58th winter meeting on nuclear physics,
20th -24th january, Bormio, Italy

in collaboration with:

V. Vovchenko, K. Gallmeister and J. Schaffner-Bielich

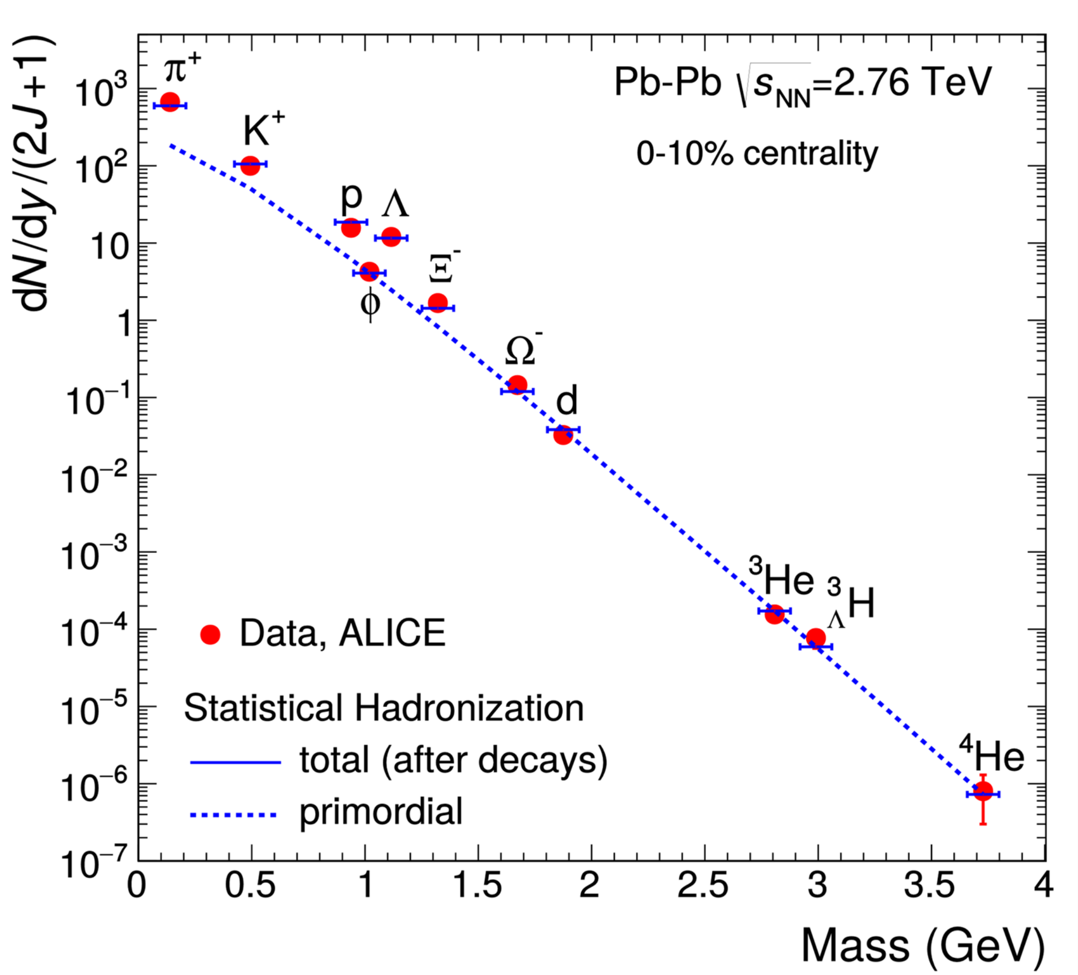
Big-Bang nucleosynthesis

Estimates

Numerical solution

arXiv:1903.10024
Phys.Lett. B800 (2020) 135131

ALICE data



$$T_{\mathrm{ch}} = 156.5\,\mathrm{MeV}$$

$$T_{\mathrm{kin}} \sim 115\,\mathrm{MeV}$$

Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561 (2018) 321

binding energies:

$$^2\mathrm{H},\ \ ^3\mathrm{He},\ \ ^4\mathrm{He} : 2.22,\ \ 7.72,\ \ 28.3\,\mathrm{MeV} \qquad ^3_\Lambda\mathrm{H} : 130\,\mathrm{keV}$$

primordial nucleosynthesis: network

- Deuterium

$$p(n,\gamma)\mathrm{D}$$

$$p+n\to \mathrm{D}+\gamma$$
$$\mathrm{D}+\gamma\to p+n$$

- Helium

$$\mathrm{D}(\mathrm{D},\gamma)^4\mathrm{He} \qquad {}^3\mathrm{He}(n,\gamma)^4\mathrm{He}$$
$$\mathrm{T}(p,\gamma)^4\mathrm{He} \qquad {}^3\mathrm{He}(\mathrm{D},p)^4\mathrm{He}$$
$$\mathrm{T}(\mathrm{D},n)^4\mathrm{He} \qquad {}^3\mathrm{He}({}^3\mathrm{He},2p)^4\mathrm{He}$$

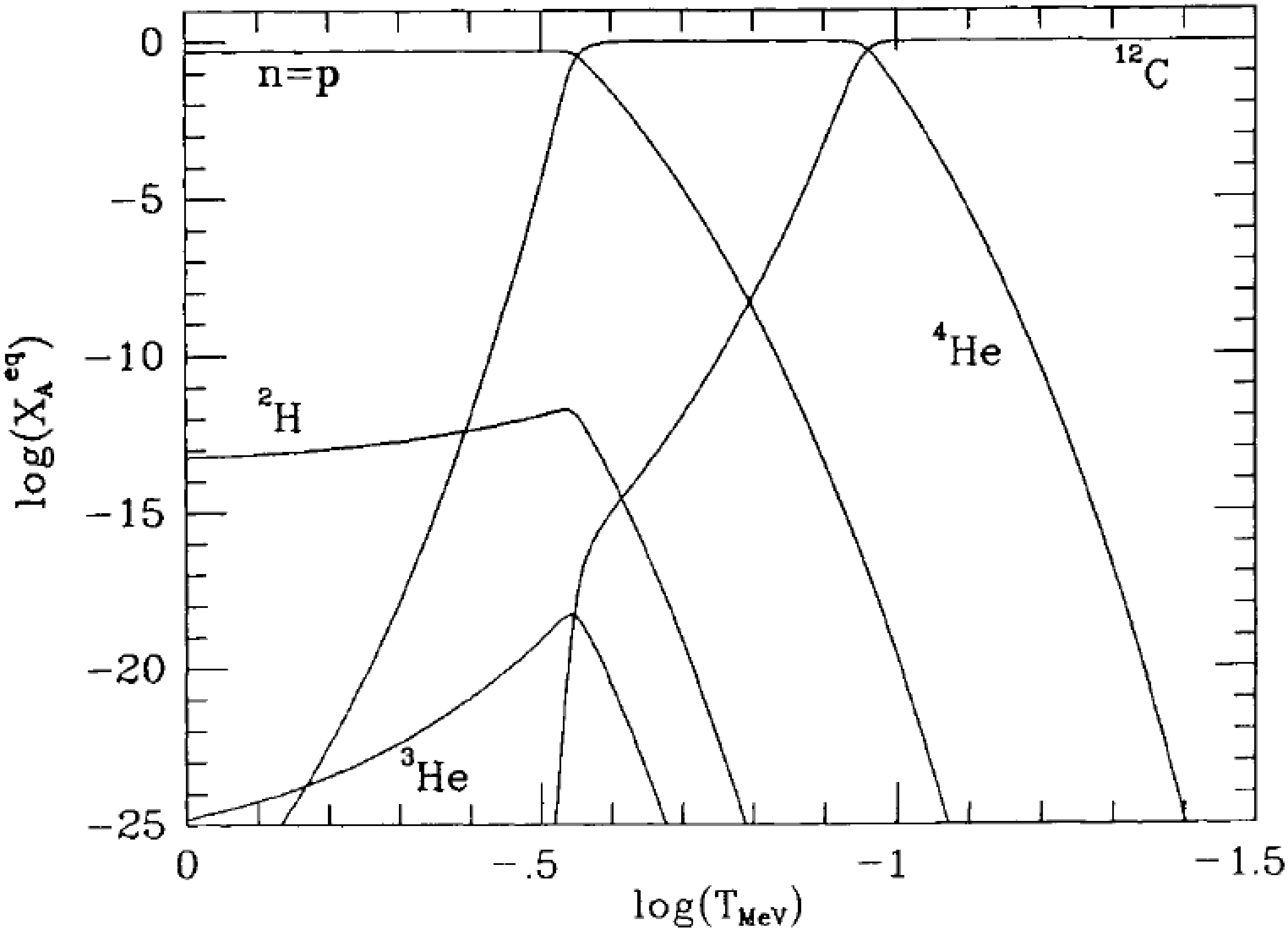
- here now:

$$\pi+A\leftrightarrow \pi+A'+m_1p+n_1n$$

e.g.

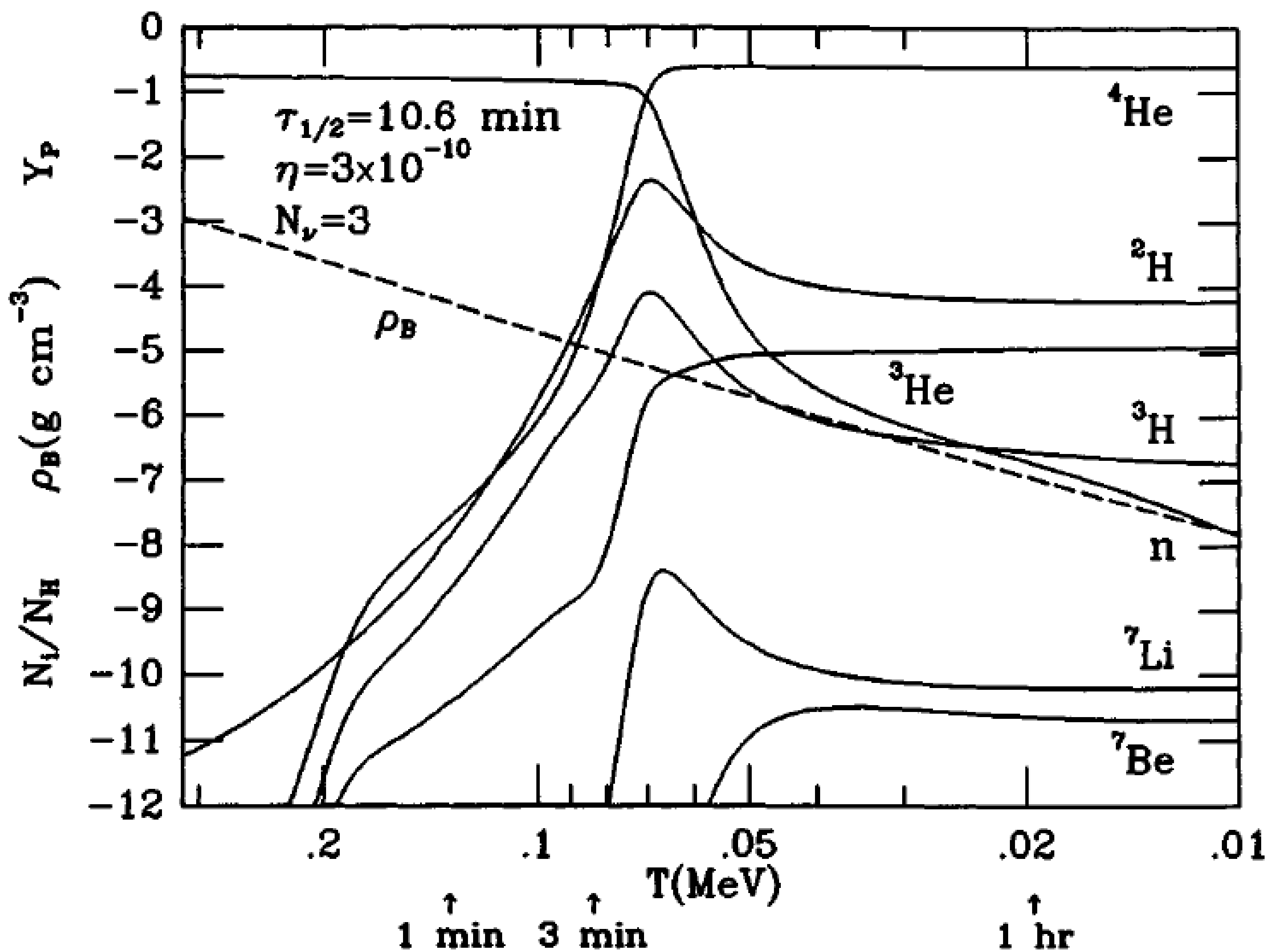
$$\pi+\mathrm{D}\leftrightarrow \pi+p+n$$

primordial nucleosynthesis – nuclear statistical eq.



Kolb, Turner, The Early Universe, 1990

primordial (big bang) nucleosynthesis



Kolb, Turner, The Early Universe, 1990

“Bevalac” nucleosynthesis

VOLUME 43, NUMBER 20 PHYSICAL REVIEW LETTERS 12 NOVEMBER 1979

Evidence for a Soft Nuclear-Matter Equation of State

Philip J. Siemens[a] and Joseph I. Kapusta
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(Received 3 August 1979)

The entropy of the fireball formed in central collisions of heavy nuclei at center-of-mass kinetic energies of a few hundred MeV per nucleon is estimated from the ratio of deuterons to protons at large transverse momentum. The observed paucity of deuterons suggests that strong attractive forces are present in hot, dense nuclear matter, or that degrees of freedom beyond the nucleon and pion may already be realized at an excitation energy of 100 MeV per baryon.

Because of the reaction $d+N \leftrightarrow p+n+N$, where N is a spectator nucleon or cluster, deuterons will be constantly breaking up and reforming. If collisions are frequent enough, the deuterons will quickly reach an equilibrium concentration determined by detailed balancing[4]:

$$\exp(-\mu_d/T)d_d(\vec{R},\vec{P},S_z) = \sum_{s_z} d_p(\vec{R},\vec{P}/2,s_z)d_n(\vec{R},\vec{P}/2,S_z-s_z)\exp[-(\mu_n+\mu_p)/T],$$

“Bevalac” nucleosynthesis

Nuclear Physics **A476** (1988) 718-772
North-Holland, Amsterdam

THE QUANTUM STATISTICAL MODEL OF FRAGMENT FORMATION: Entropy and temperature extraction in heavy ion collisions

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Received 8 October 1986
(Revised 5 October 1987)

Abstract: Quantum statistical model (QSM) calculations of nuclear fragment formation are presented. Various independent methods for extracting the temperature, T, and entropy, S/A, from fragment- and pion yields in heavy-ion collisions are analysed. It is emphasized that stable and unstable medium mass fragments play an important role in determining T and S/A: They alter the relation $S/A(R_{dp})$ dramatically and distort via feeding simple temperature measurements. However, these fragments allow by their very abundance for a variety of new, alternative methods to determine S/A from data on multifragmentation (ratios of complex fragment yields, mass yield curves, and charged-particle multiplicities).

Entropy values deduced from 4π plastic ball data exhibit a strong multiplicity dependence. For large multiplicities the entropy residing in nuclear fragments appears to be independent of the bombarding energy and low in absolute value, $S/A \simeq 3.5$.

The corresponding break-up temperatures of the fragment conglomerate are $T = 12$, 16, and 20 MeV at $E_{lab} = 400$, 650, and 1050 MeV/n, respectively. These values are much smaller (a factor $\frac{1}{3}$) than the temperatures extracted from pion yields. This result can be understood only if the pions are created in the early, hot stage of the collision, while the fragments are formed after an isentropic expansion of the system at small densities where the temperature is low. This would imply that in the late stage of the reaction a large fraction ($\simeq 80\%$) of the available center-of-mass energy resides in (possibly isotropic) flow.

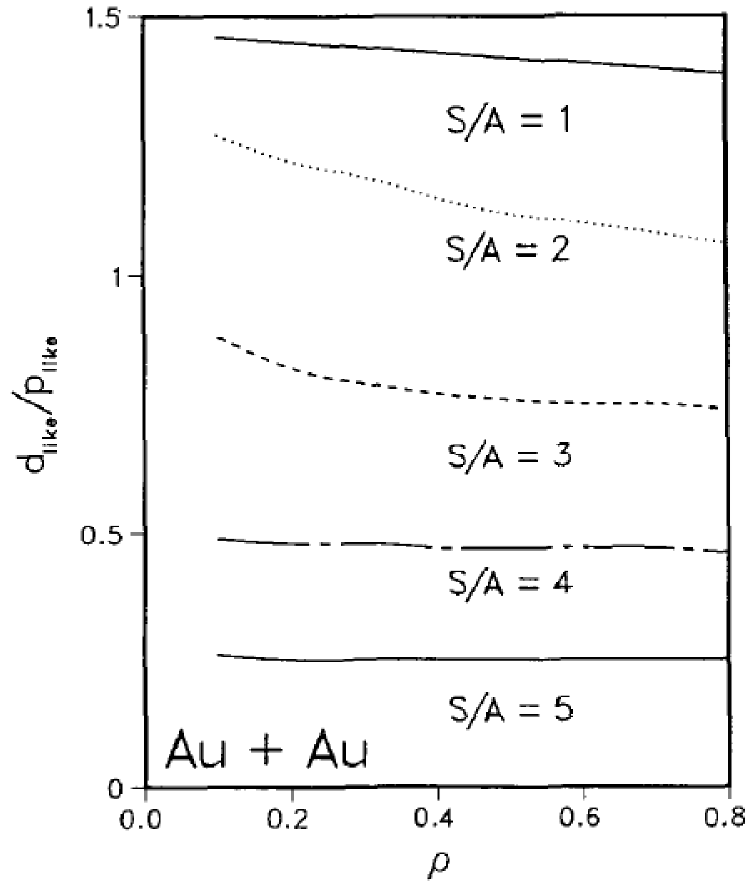


Fig. 23. Connection between d-like/p-like and the break-up density for constant entropy. The calculation has been done for the Au + Au system.

Saha equation

- ionization of a gas $X \longleftrightarrow X^+ + e^-$

$$\frac{n_e^2}{n_0} = \frac{2}{\lambda_e^3}\frac{g_1}{g_0}\exp(-\epsilon/T)$$

$n_1 = n_e$ λ_e : deBroglie

Megh Nad Saha, Phil. Mag. Series 6 40:238 (1920) 472

- equivalently: partition functions

$$\frac{Z_0}{N_0} = \frac{Z_1\, Z_e}{N_1\, N_e}$$

- equivalently: chemical potentials

$$\mu_0 = \mu_1 + \mu_e$$

Saha equation
= detailed balance
= law of mass action

Saha equation

- nuclear equivalent
- 'Nuclear Statistical Equilibrium'
- mass fraction of nucleus A:

$$p + n \longleftrightarrow d + \gamma$$

$$X_A = g_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)}$$

$$\eta = \frac{n_B}{n_\gamma} \sim 6 \cdot 10^{-10} \qquad \times \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

Kolb, Turner, The Early Universe, 1990

- this work:

$$\frac{N_A(T)}{N_p} = \frac{g_A}{g_M^{1-A}} \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{1+A}{2}}\right] A^{\frac{3}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)}$$

$$\eta_B = \frac{N_N}{N_M} \sim 0.03 \qquad \times \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)$$

$$(N_M = g_M[\zeta(3)/\pi^2]VT^3 \;,\; g_M \sim 11 - 13)$$

Heavy ion collisions

- chemical freeze-out = number of (anti-)protons/neutrons etc. constant below $T_{\rm ch}$

$$n_i^{(0)} = \frac{g}{2\pi^2}\, m_i^2 T_{\rm ch}\, {\rm K}_2(m_i/T_{\rm ch}) \;, \qquad n_i = n_i^{(0)}\, {\rm e}^{\mu_i/T}$$

- Saha equation/detailed balance: $X + A \longleftrightarrow X + \sum_i A_i$

e.g. π

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{(0)}}{\prod_i n_{A_i}^{(0)}} \;, \qquad \mu_A = \sum_i \mu_{A_i}$$

$$\mu_{^2{\rm H}} = \mu_p + \mu_n \;, \qquad \mu_{^3{\rm He}} = 2\mu_p + \mu_n \;, \qquad \ldots$$

$$N_A = V\, \frac{g_A}{2\pi^2}\, m_A^2 T\, {\rm K}_2(m_A/T)\, {\rm e}^{\mu_A/T}$$

$\mu_A(T)$? $V(T)$?

Heavy ion collisions

- isentropic expansion:

$$V/V_{\rm ch} = (T/T_{\rm ch})^{-3}$$

- non-relativistic approximation:

$$N_i(T) \simeq g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} {\rm e}^{-m_i/T}\, {\rm e}^{\mu_i/T}\, V_{\rm ch} \left(\frac{T_{\rm ch}}{T}\right)^3$$

$$\Rightarrow \mu_i \simeq 3/2\, T \ln\left(T/T_{\rm ch}\right) + m_i \left(1 - T/T_{\rm ch}\right).$$

$$\frac{N_A(T)}{N_A(T_{\rm ch})} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}(A-1)} \exp\left[B_A\left(\frac{1}{T} - \frac{1}{T_{\rm ch}}\right)\right]$$

$$\left[\frac{N_A(T)}{N_A(T_{\rm ch})}\right]_{\rm eq.} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}} \exp\left[-m_A\left(\frac{1}{T} - \frac{1}{T_{\rm ch}}\right)\right]$$

Full calculation (Thermal-FIST)

$i \in \text{stable}\ ,\ \ j \in \text{HRG}$

- particle decays:

$\langle n_i \rangle_j$ mean number of hadron i from decays of hadron j

- effective chemical potentials:

$$\tilde{\mu}_j = \sum_i \langle n_i \rangle_j \, \mu_i$$

- conservation of yields of stable hadrons:

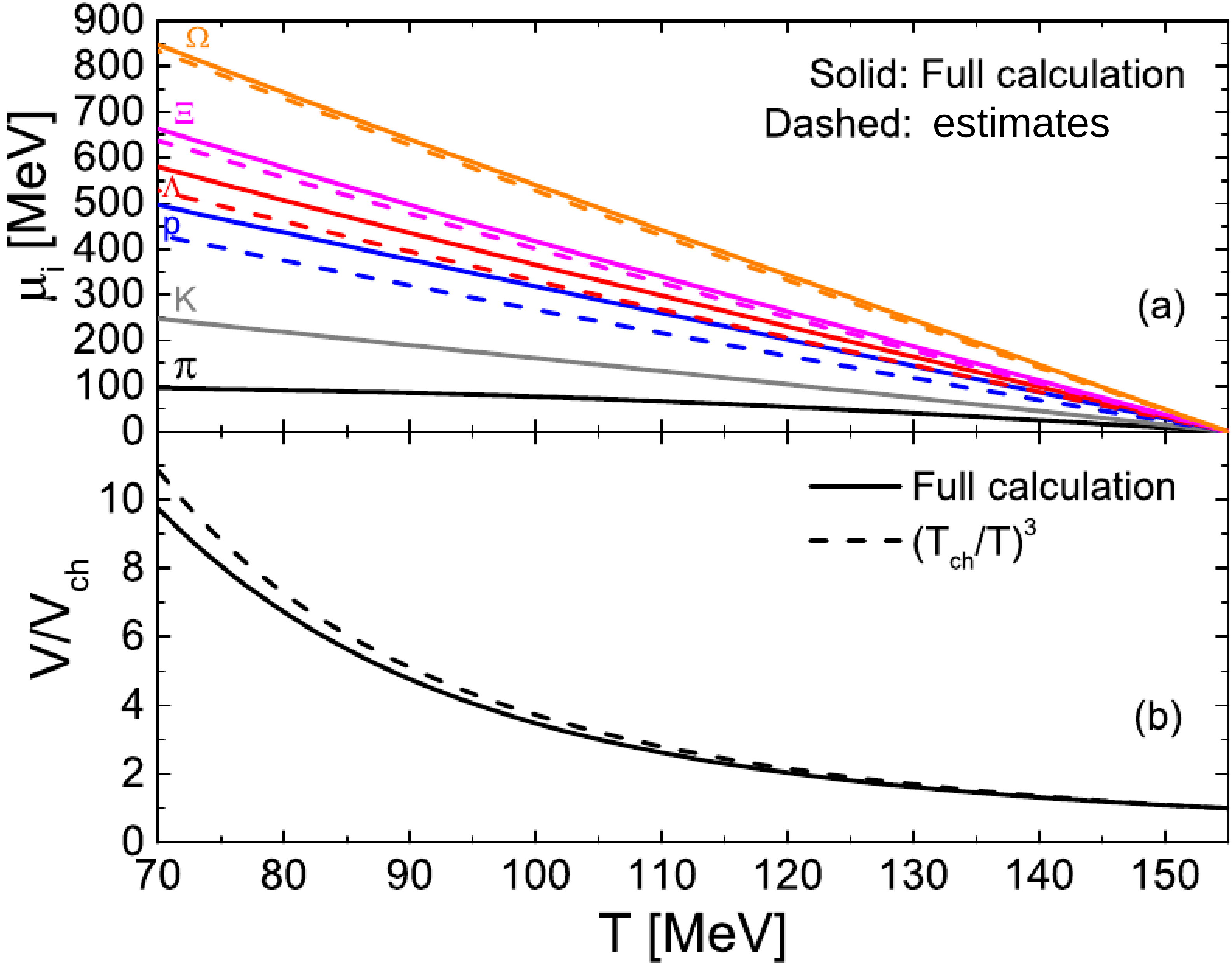
$$V \sum_i \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j) \stackrel{!}{=} N_i(T_{\text{ch}})$$

- isentropic expansion:

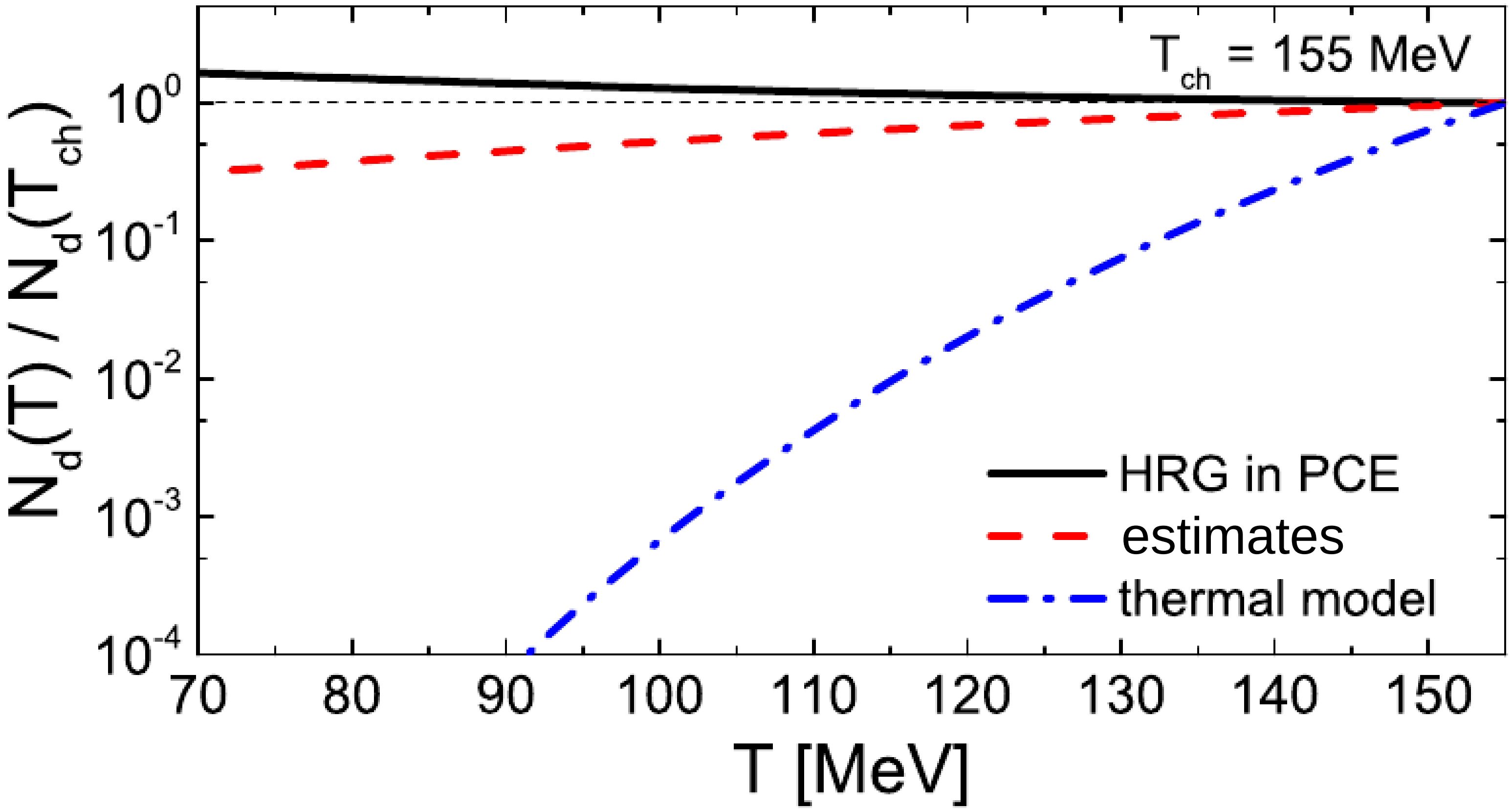
$$V \sum_i s_j(T, \tilde{\mu}_j) \stackrel{!}{=} S(T_{\text{ch}})$$

$\rightarrow \mu_i(T)\ ,\ \ V(T)$

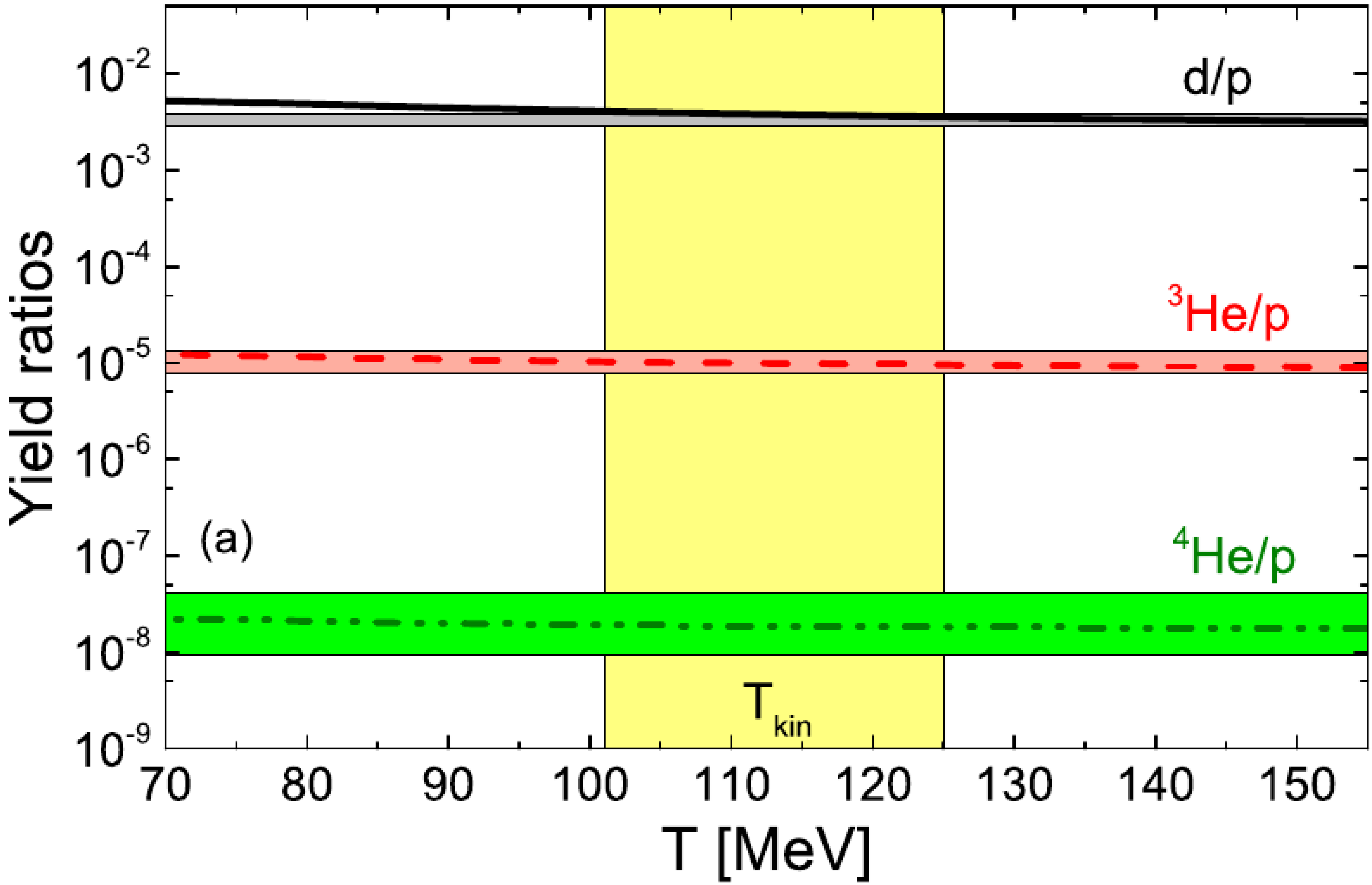
Full calculation: parameters



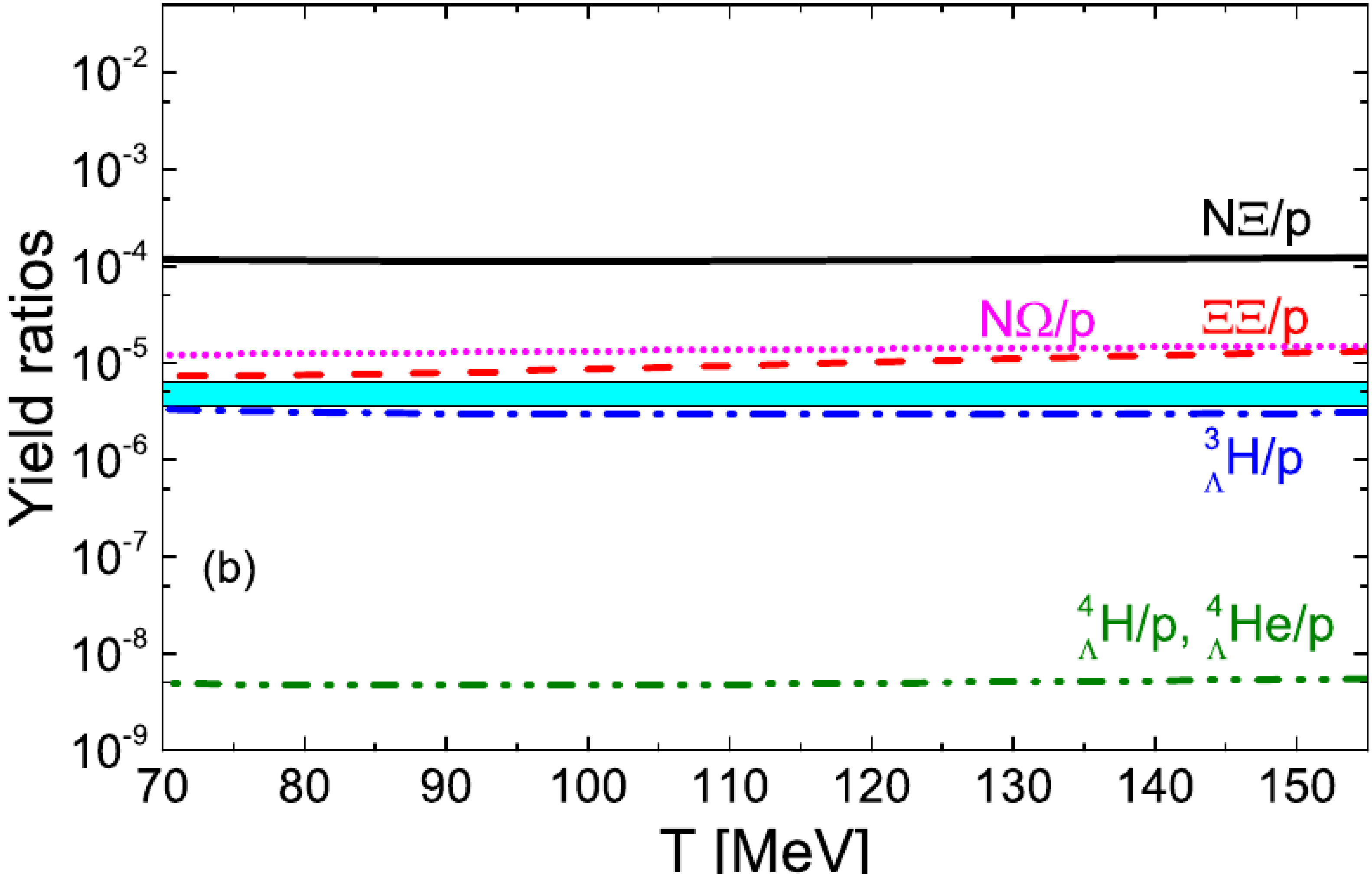
Full calculation: results for d



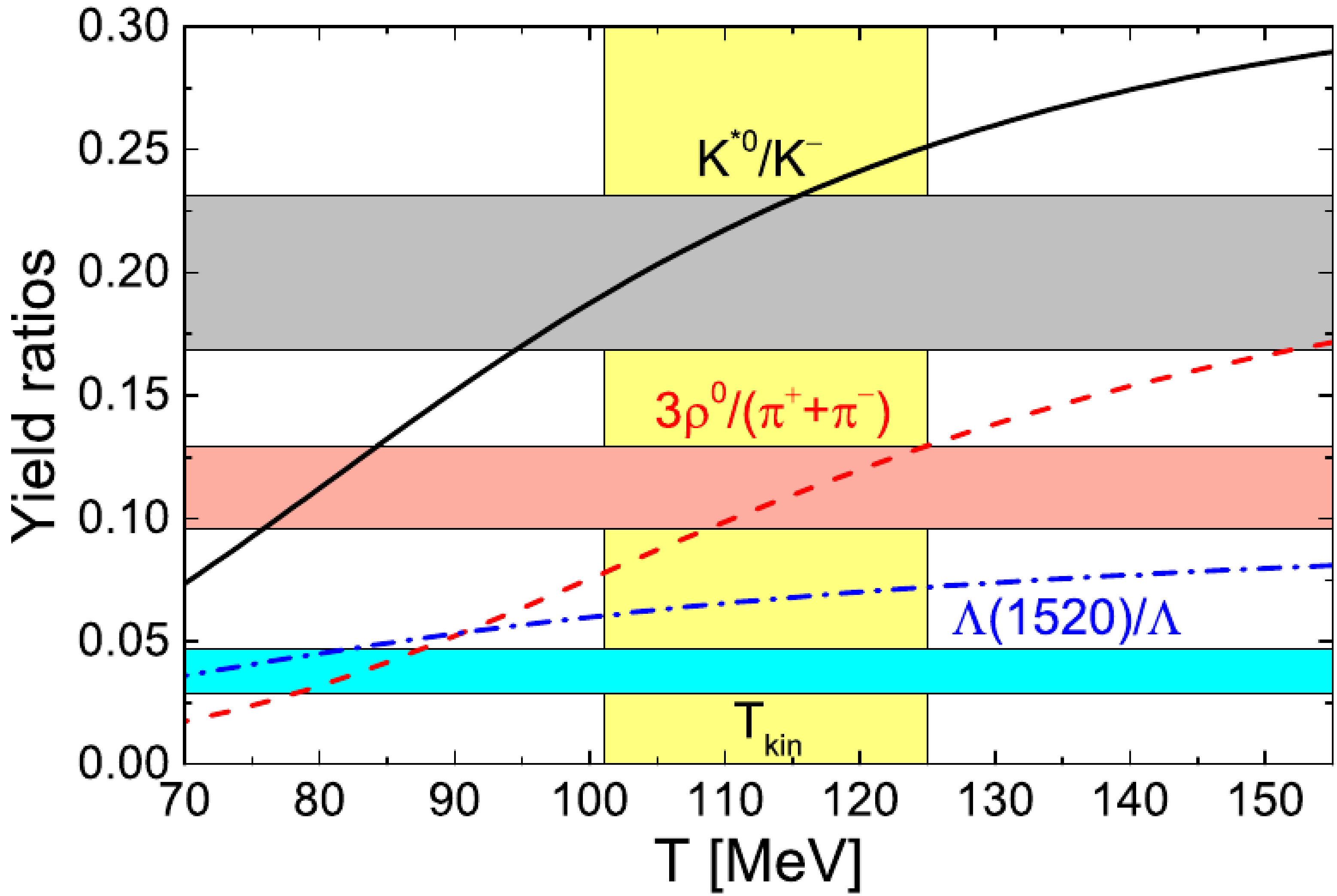
Full calculation: results (a)



Full calculation: results (b)



Full calculation: results for resonances



LHC nucleosythesis

PHYSICAL REVIEW C **99**, 044907 (2019)

Editors' Suggestion | Featured in Physics

Microscopic study of deuteron production in PbPb collisions at $\sqrt{s} = 2.76$ TeV via hydrodynamics and a hadronic afterburner

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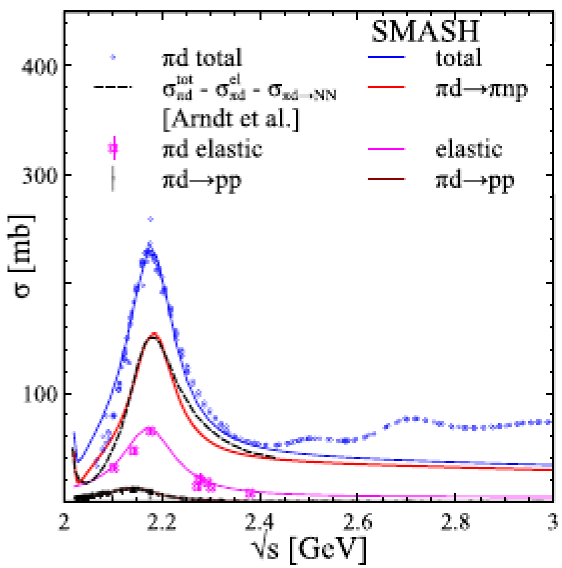


FIG. 1. Deuteron-pion interaction cross sections from SAID database [40] and partial wave analysis [41] are compared to our parametrizations (Tables II and III in the Appendix). Inelastic $d\pi \leftrightarrow$

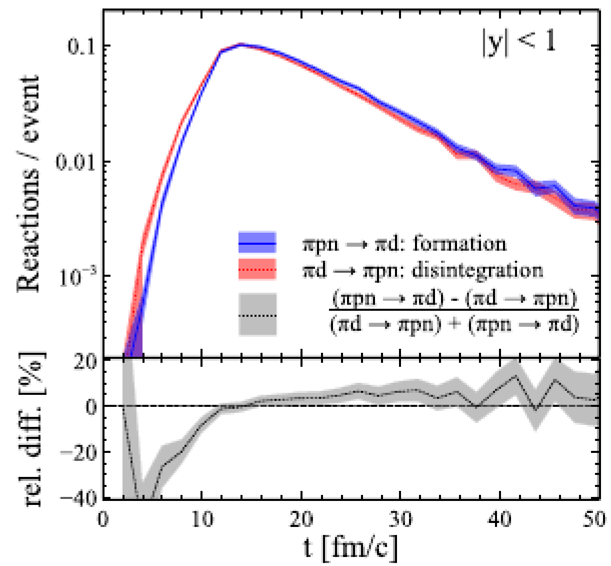
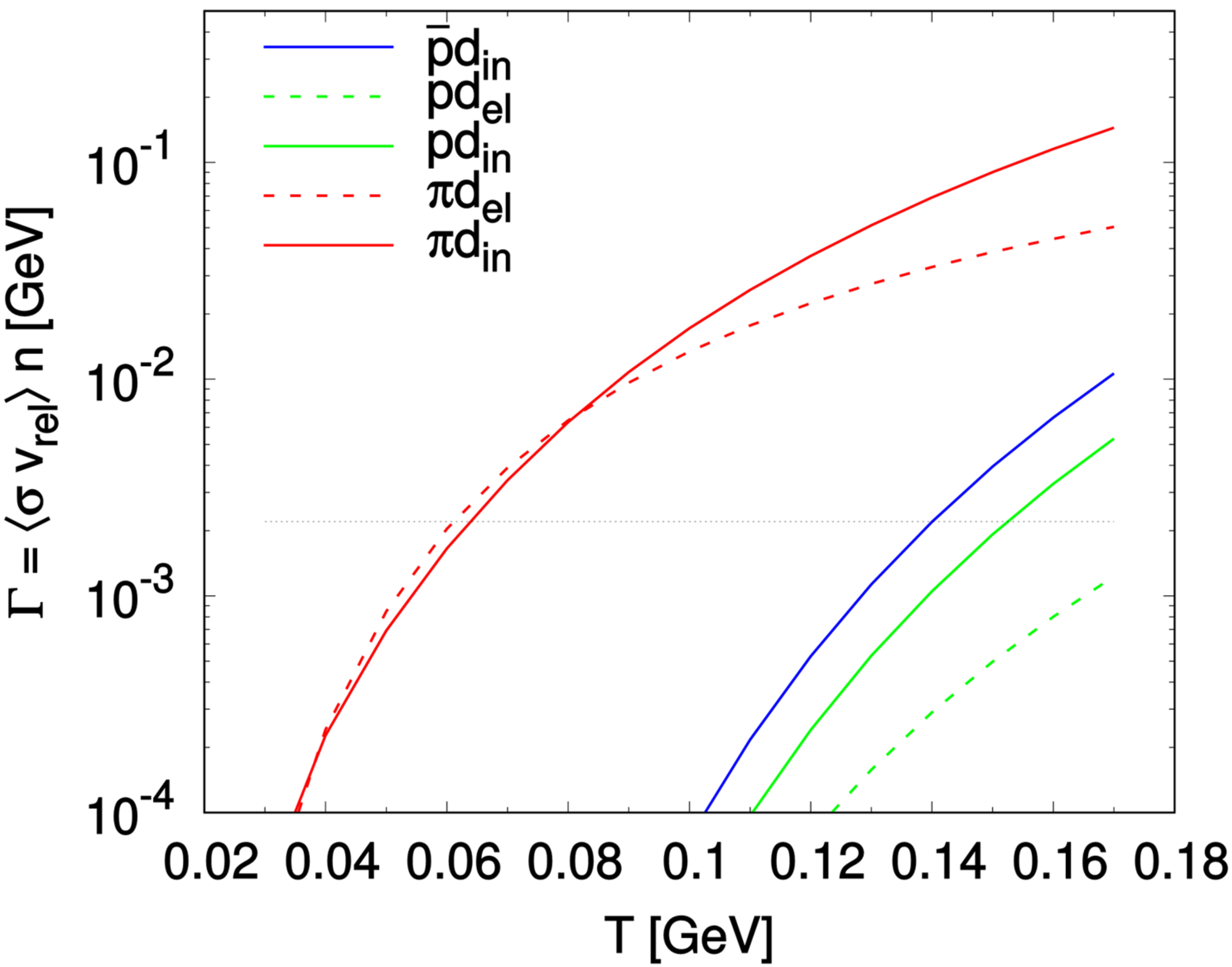


FIG. 5. Reaction rates of the most important $\pi d \leftrightarrow \pi pn$ reaction in forward and reverse direction.

- law of mass action at work

Rate



$$\Delta E \underset{Heisenberg}{\simeq} \Gamma$$

$$\leftrightarrow \quad \Delta E_{\mathrm{bound}}$$

$$\leftrightarrow \quad \Gamma_{\mathrm{expansion}}$$

("kinetic freezeout")

Conclusions

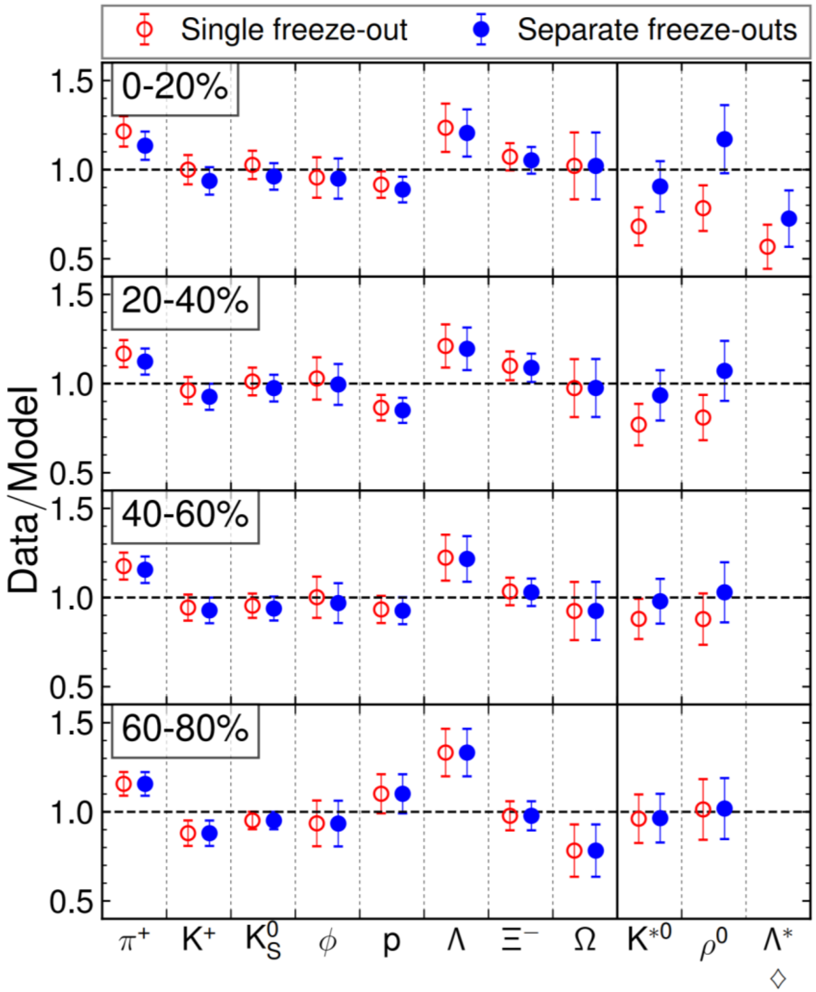
- Saha equation gives natural explanation of agreement of thermal model predictions and experimental observations
- light nuclei may be formed at any $T < T_{ch}$!
- who can give the answer?
- building of (pre-)clusters (Hagedorn states)
- coalescence
- rate equations
- transport simulations (cf. D.Oliinychenko et al.)
- …

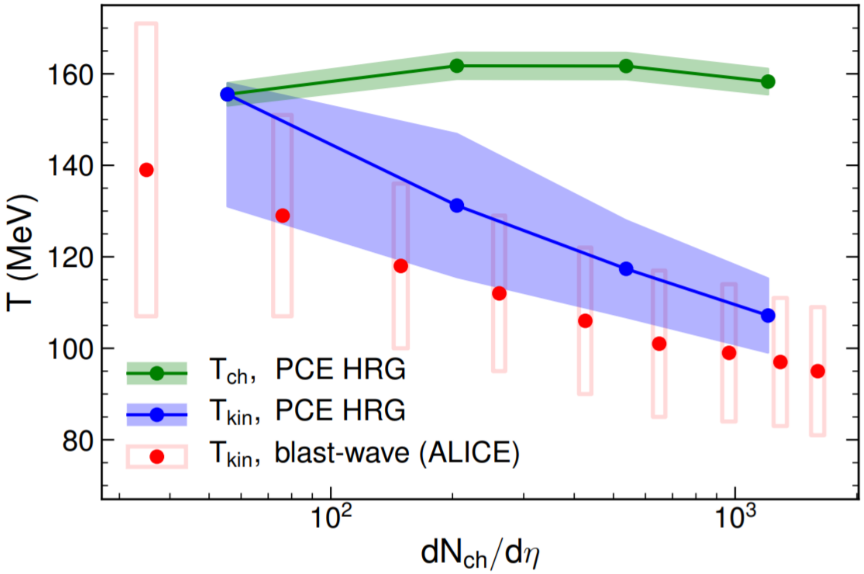
- quantum mechanical treatment of creation/decreation and decoherence of bound systems in medium ("open quantum systems") needed

Backups

Kinetic freeze-out temperature from resonance yields

- Fit of K* and ρ^0 extracts the kinetic freeze-out temperature

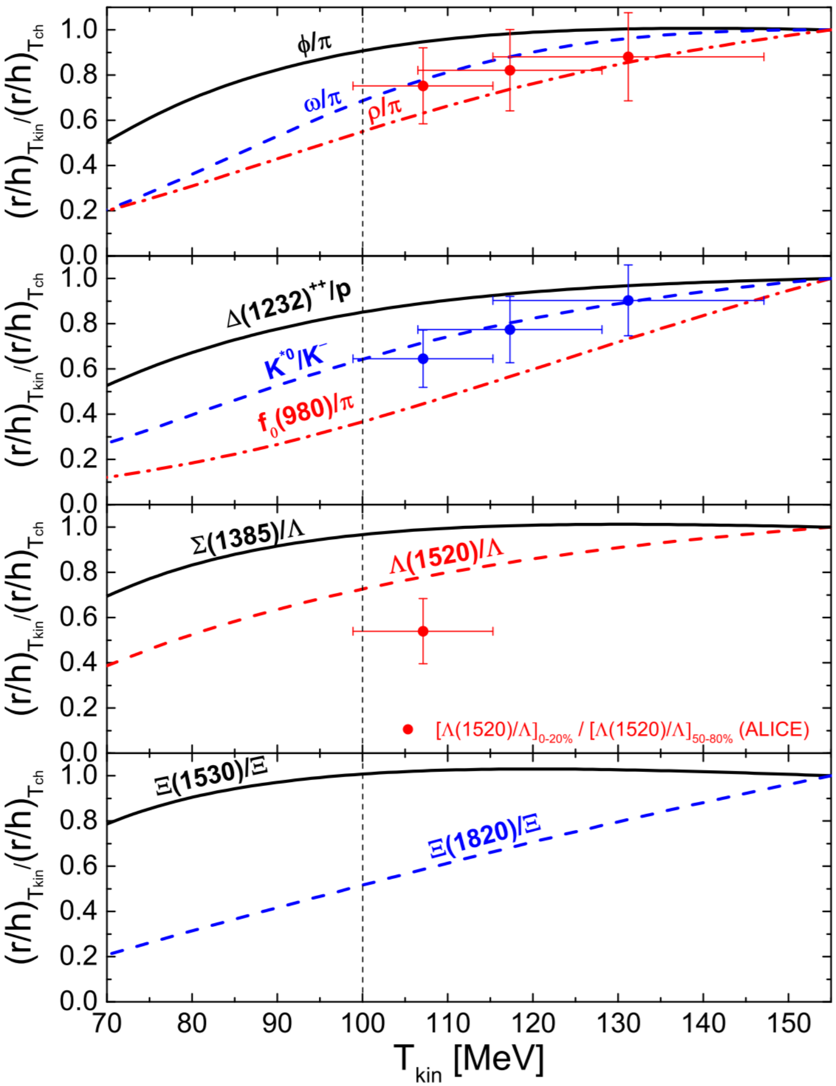




Centrality	T_{ch} (MeV)	T_{kin} (MeV)	χ^2/dof
0-20%	160.2 ± 3.1	–	23.6/8
	158.3 ± 2.8	107.1 ± 8.2	10.5/7
20-40%	162.9 ± 3.1	–	19.5/8
	161.7 ± 2.9	117.3 ± 10.8	12.8/7
40-60%	162.3 ± 3.0	–	12.5/8
	161.8 ± 2.9	131.2 ± 15.9	10.6/7
60-80%	155.5 ± 2.5	–	19.1/8
	155.5 ± 2.5	$155.5^{+2.5}_{-24.5}$	19.1/7

A. Motornenko, V. Vovchenko, CG, H. Stoecker, 1908.11730

Predictions for other resonances



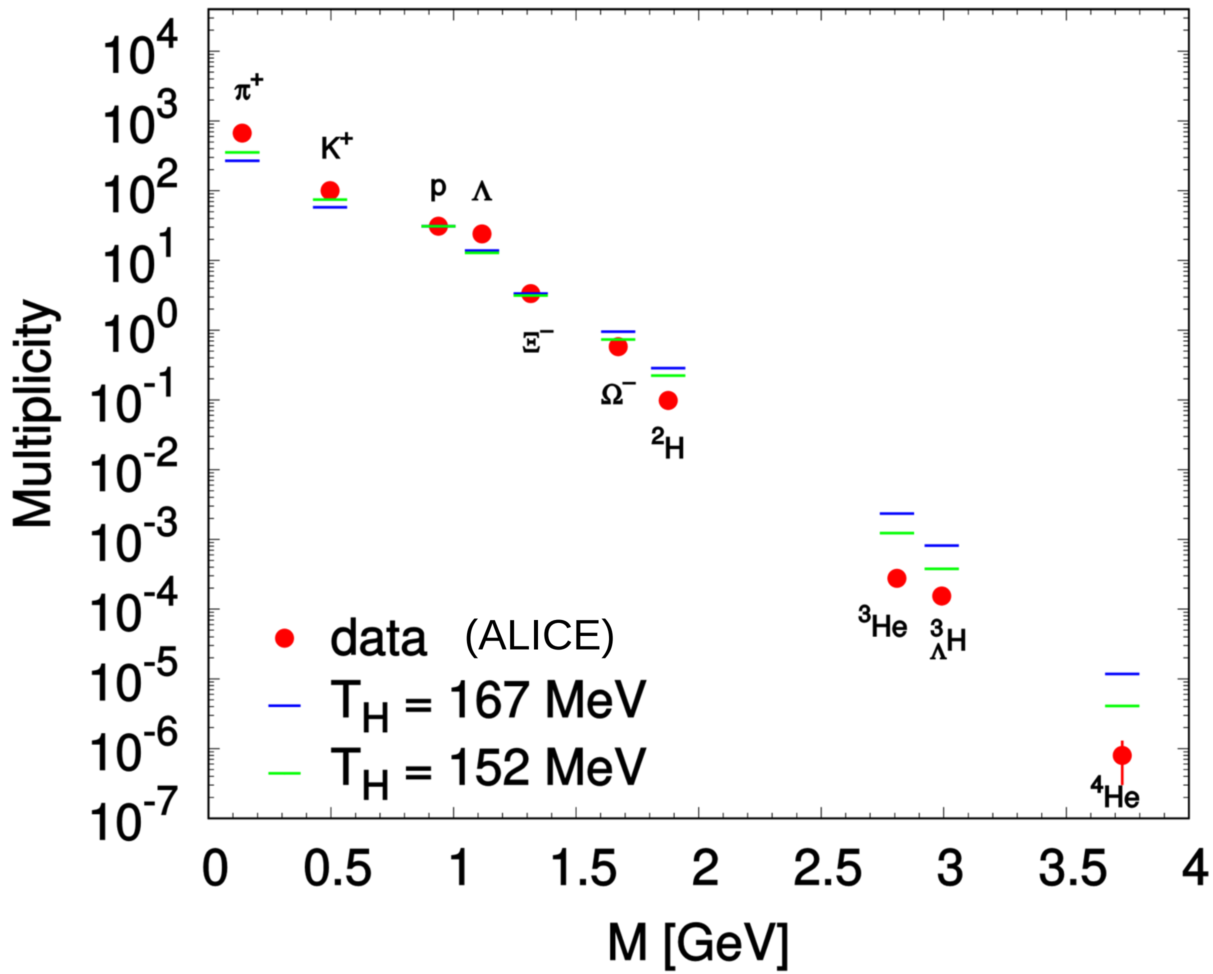
- Short-lived $\Delta(1232)$ and $\Sigma(1385)$ not suppressed

- $f_0(980)$ suppressed if it interacts in the hadronic phase
  → if confirmed experimentally will imply short $f_0(980)$ lifetime

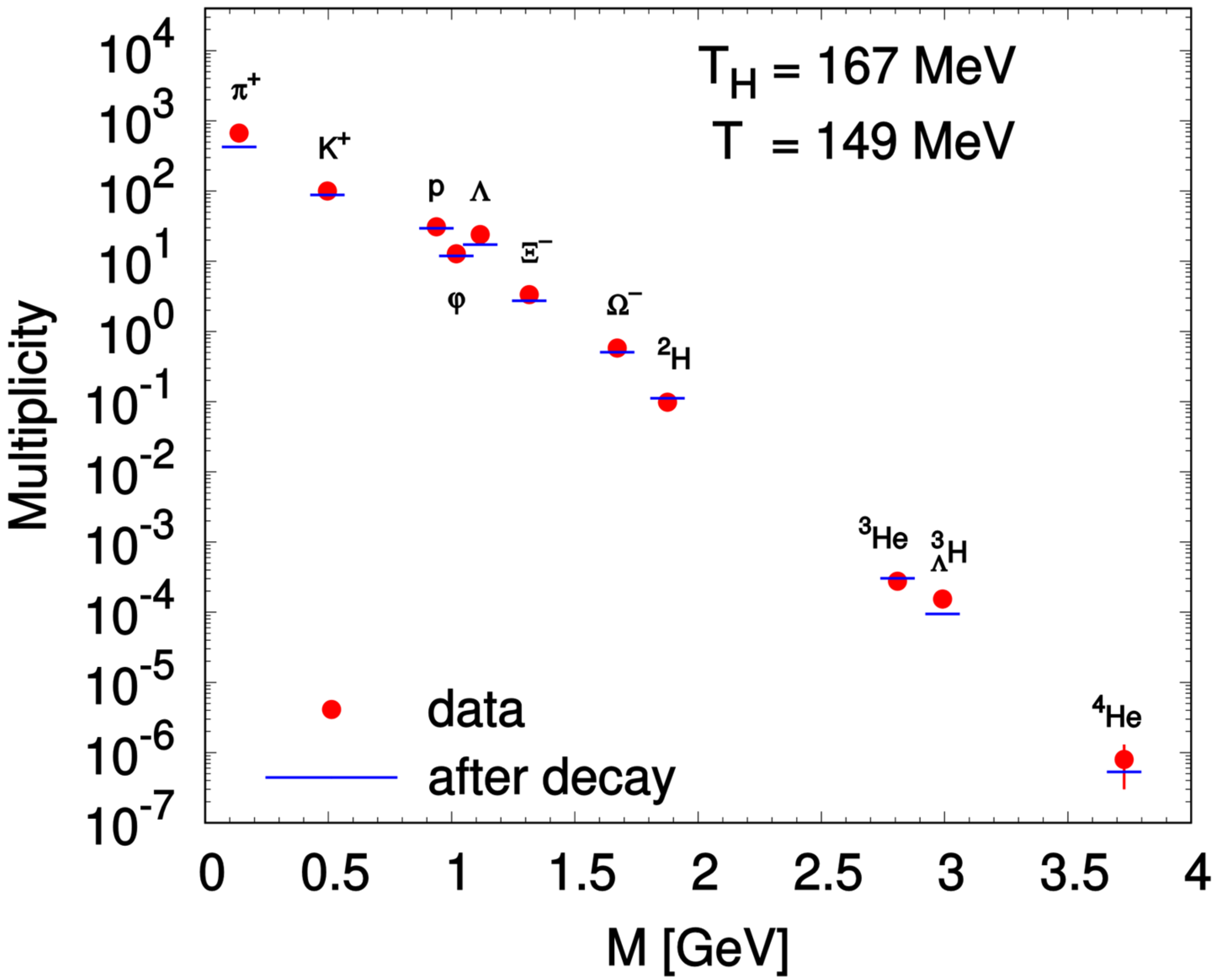
A. Motornenko, V. Vovchenko, CG, H. Stoecker, 1908.11730

single Hagedorn decay chain

- *B*,*S*,*Q*: averaged according $\tau_{BSQ}(m_H)$
- m_H=10 GeV (${m_H}\gtrsim$4GeV: branchings nearly independent of m_H)



thermal Hagedorn gas

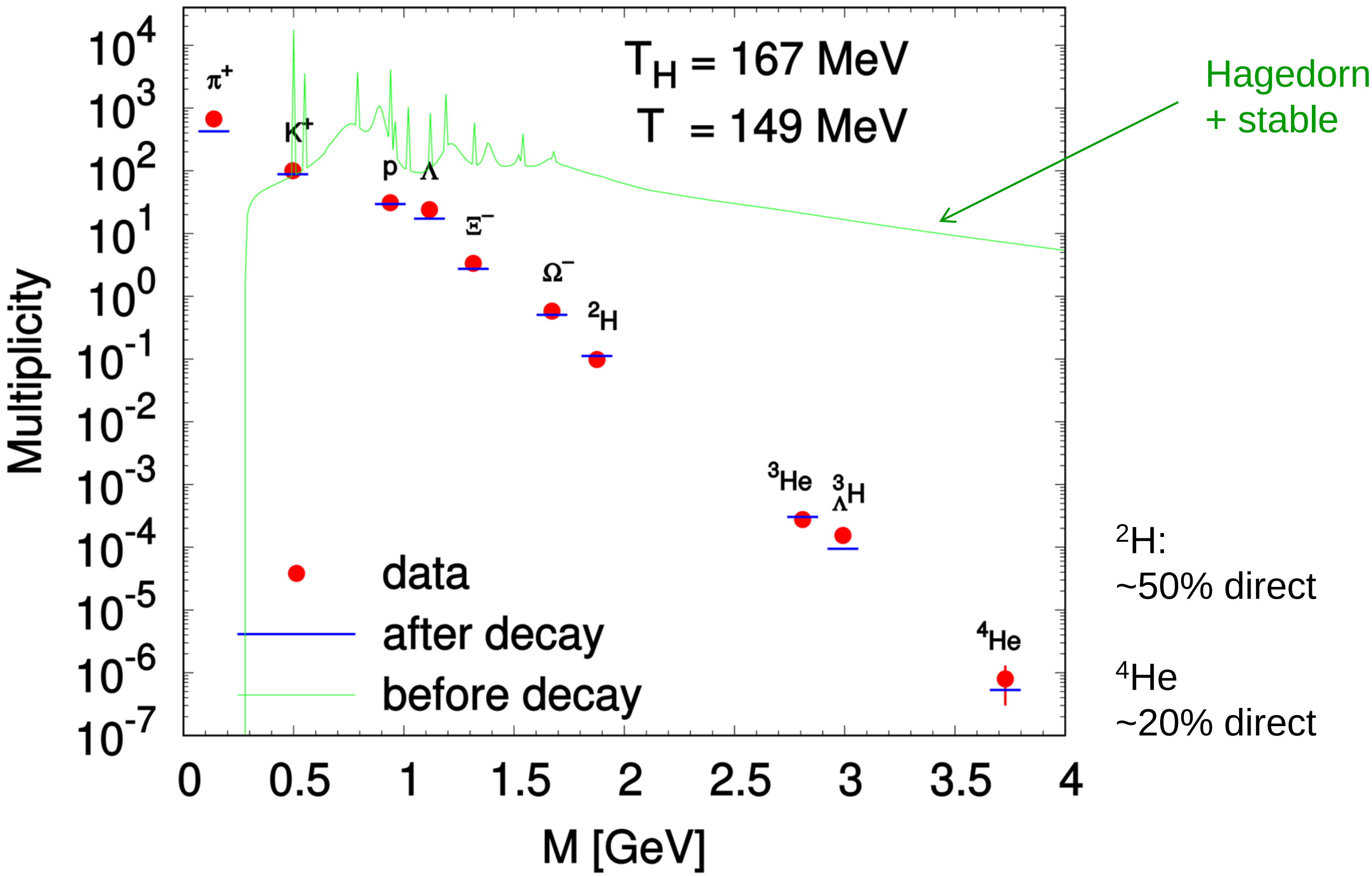


equivalent:
T_H = 152 MeV
T = 144 MeV

^{2}H:
~50% direct

^{4}He
~20% direct

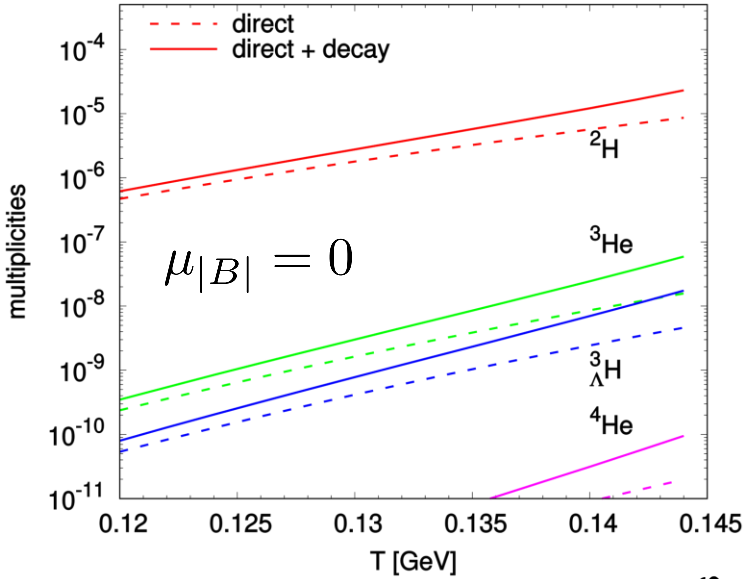
thermal Hagedorn gas



^2H:
~50% direct

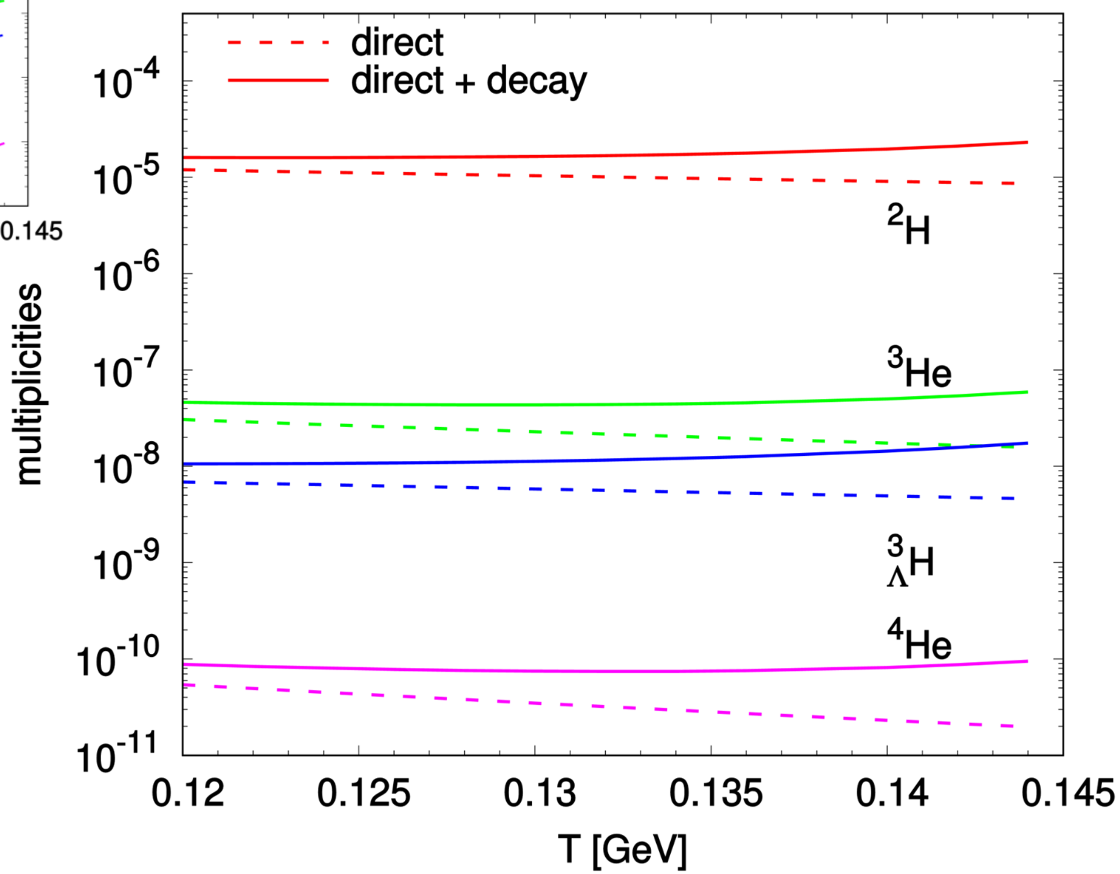
^4He
~20% direct

Hagedorn and Saha



$$\mu_{|B|} = \mu_{|B|}(T)$$

(fixed before decay)



yields do not depend on temperature