

Coherent pion photoproduction on spin-zero nuclei

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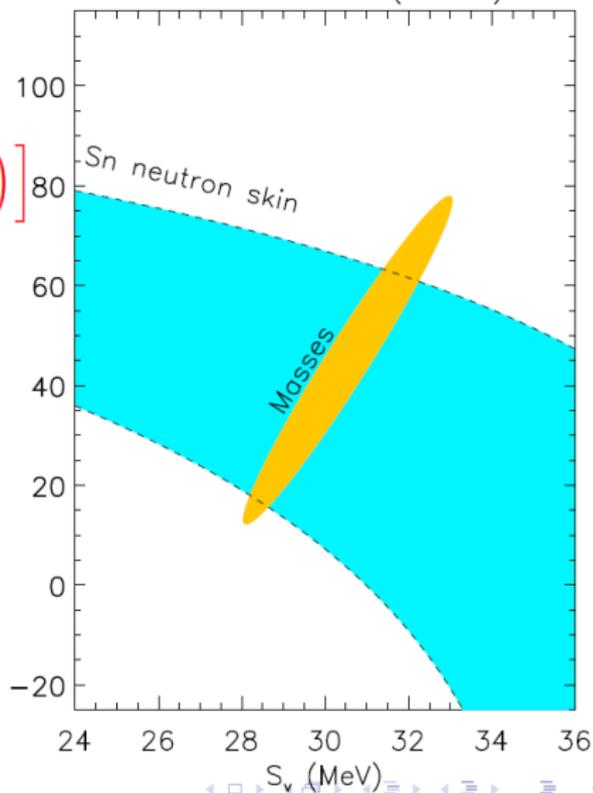
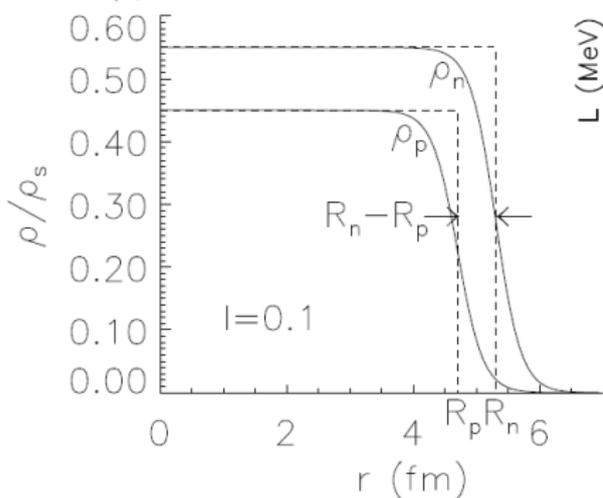
Nuclear Experimental Constraints

Tarbert et al. (2014)

Neutron Skin Thickness

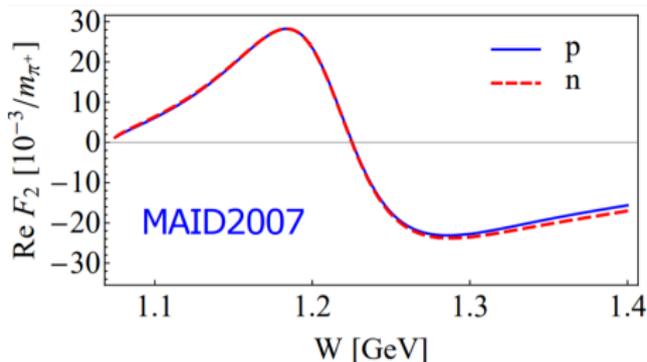
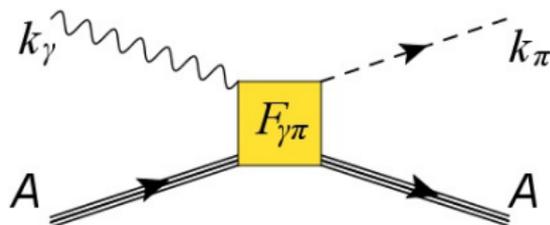
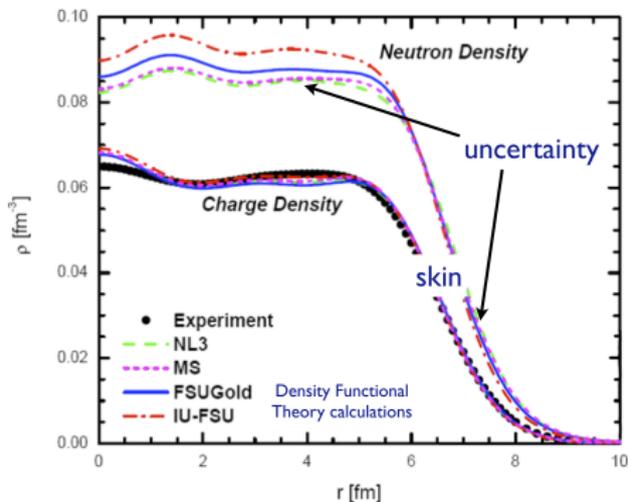
$$r_{np} = \frac{2r_0}{3S_v} \frac{1}{\sqrt{1-I^2}} (1 + S_s A^{-1/3} / S_v)^{-1} \times \sqrt{\frac{3}{5}} \left[IS_s - \frac{3Ze^2}{140r_0} \left(1 + \frac{10}{3} \frac{S_s A^{-1/3}}{S_v} \right) \right]$$

$$r_{np,208} = 0.15 \pm 0.04 \text{ fm}$$



Photoproduction is a tool so study neutron distribution

⇐ CREX Workshop, March 17-19, 2013
Thomas Jefferson National Accelerator
Facility



Spin independent parts of the π^0 photoproduction amplitudes on
p and n are the same

We can measure neutron distribution!

π^0 photoproduction + proton dist. data \Rightarrow access to neutron skin

$$\langle \mathbf{k}', -\mathbf{k}' | \hat{U}_{\text{opt}}^{\text{1st}}(E) | \mathbf{k}, -\mathbf{k} \rangle = \alpha A \int d\xi d\xi_{A-1} d\xi'_{A-1} \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{k}' \cdot (\boldsymbol{\xi} - \boldsymbol{\xi}'_{A-1}/A)} e^{-i\mathbf{k} \cdot (\boldsymbol{\xi} - \boldsymbol{\xi}_{A-1}/A)} \times$$

$$e^{i(\mathbf{p}_1 - \mathbf{q}/2) \cdot (\boldsymbol{\xi} - \boldsymbol{\xi}'_{A-1})} e^{-i(\mathbf{p}_1 + \mathbf{q}/2) \cdot (\boldsymbol{\xi} - \boldsymbol{\xi}_{A-1})} \text{Tr} [\rho(\boldsymbol{\xi}'_{A-1}, \boldsymbol{\xi}_{A-1}) t(W; \mathbf{k}', \mathbf{p} - \mathbf{q}/2; \mathbf{k}, \mathbf{p} + \mathbf{q}/2)],$$

$$\hat{U} = \sum \hat{\tau}_1 + \sum_i \sum \hat{\tau}_i \hat{G} \hat{P}_0 \hat{\tau}_i + \sum \sum \sum \hat{\tau}_i \hat{G} \hat{P}_0 \hat{\tau}_i \hat{G} \hat{P}_0 \hat{\tau}_i + \dots$$

$$\langle \mathbf{k}' | \langle \Psi_0 | \hat{\tau}_1 | \Psi_0 \rangle \langle \Psi_0 | \hat{\tau}_i | \Psi_0 \rangle \dots \langle \Psi_0 | \hat{\tau}_i | \Psi_0 \rangle = \int \frac{d\mathbf{k}''}{(2\pi)^3} \text{Tr} [t(\mathbf{k}'', \mathbf{k}') \rho(\mathbf{k}'', \mathbf{k}') G(\mathbf{k}'', \mathbf{k}')] \text{Tr} [t_2(\mathbf{k}'', \mathbf{k}')]$$

$$\int \frac{d\mathbf{k}}{k_0^2 - k^2 + i0} e^{-\frac{1}{2}a^2(\mathbf{k} - \mathbf{k}')^2} \left[\frac{2\mathbf{k}'/2}{a} \left(F \left[\frac{\mathbf{k} - \mathbf{k}'}{\sqrt{2}} \right] - F \left[\frac{\mathbf{k} + \mathbf{k}'}{\sqrt{2}} \right] \right) - \frac{2\pi^2}{a^2 q} \sinh[a^2(\mathbf{k} - \mathbf{k}')^2] \right]$$

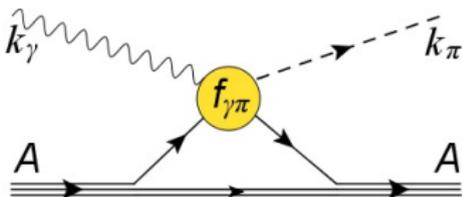
$$\xi_l = \frac{F'_l(0, \rho) + \tan \delta_l G'_l(0, \rho)}{F_l(0, \rho) + \tan \delta_l G_l(0, \rho)} \Big|_{\rho=kR_{\text{cut}}} \quad \tan \delta_l = \frac{F'_l(\eta_c, \rho) - i F_l(\eta_c, \rho)}{\xi_l G'_l(\eta_c, \rho) - i G_l(\eta_c, \rho)} \Big|_{\rho=kR_{\text{cut}}}$$

$$-2\pi i \langle \chi | \delta[G^{-1}(0)] | T |^2 | \chi \rangle = \langle \chi | \hat{T}^\dagger [G(k) - G^*(k)] \hat{T} | \chi \rangle = \langle \chi | \hat{T}^\dagger | \psi \rangle - \langle \psi | \hat{T} | \chi \rangle + \langle \chi | \hat{T} - \hat{T}^\dagger | \chi \rangle$$

$$\chi^2(p) = \sum_i^{\text{d.o.S.}} \sum_j^{n_i} \left[\frac{1}{n_i} \left(\frac{d\sigma_j^{\text{Data}} - N_i^{-1} d\sigma(p)}{\Delta d\sigma_j^{\text{Data}}} \right)^2 + \left(\frac{N_i - 1}{\Delta N_i} \right)^2 \right] + \sum_i^{\text{d.o.S.}} \sum_j^{n_i} \left(\frac{\sigma_j^{\text{Data}} - \sigma(p)}{\Delta \sigma_j^{\text{Data}}} \right)^2$$

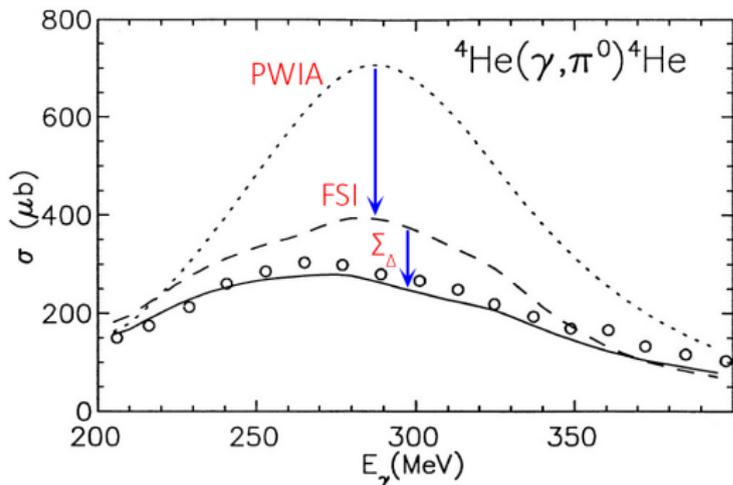
$$f(\mathbf{p}, \mathbf{p}'') = \left(\frac{A}{2\pi} \right)^3 \delta(\mathbf{p} - \mathbf{p}'')$$

Plane wave impulse approximation:



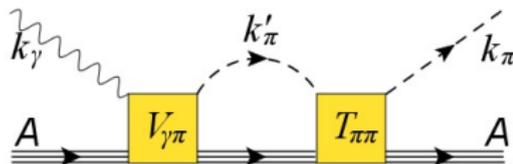
$$V_{\gamma\pi}^{\lambda} = p_A \rho(q) F_2 \left[\hat{\mathbf{k}}_{\gamma} \times \hat{\mathbf{k}}_{\pi} \right] \cdot \boldsymbol{\varepsilon}_{\lambda}$$

$\rho(q)$ - nuclear mass form factor
 F_2 - MAID2007 CGLN amplitude (spin-independent)



D. Drechsel *et al.*,
 Nuclear Physics A **660**, 423 (1999)

Final state interaction (FSI)

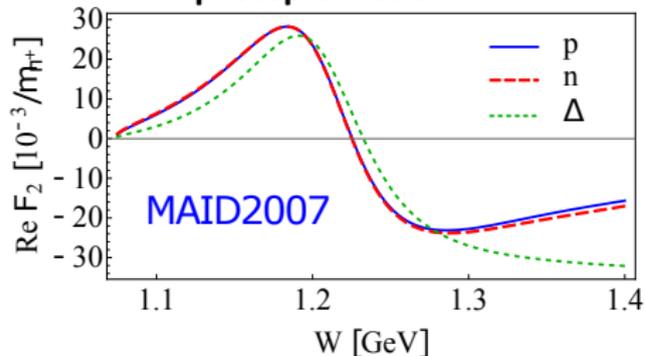


Medium effects: Δ self-energy Σ_{Δ}

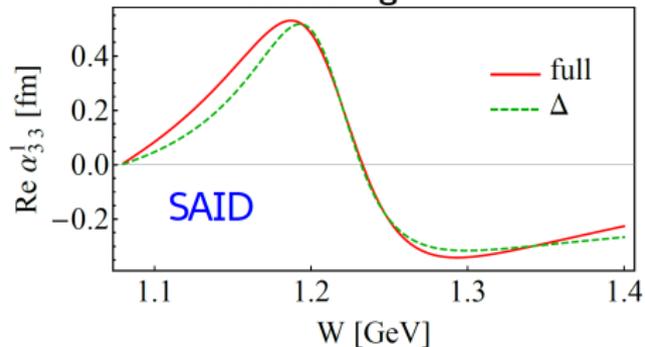
Scattering is the key to photoproduction

Elementary amplitudes are building blocks

photoproduction:



scattering:



PHASE 1 **PHASE 2** **PHASE 3**

Study
scattering



Profit

$\Delta(1232)$ is driving both scattering and photoproduction

Δ propagator is modified in nuclear medium:

$$G_{\Delta} = \frac{1}{(W - m_{\Delta} + i\Gamma_{\Delta}/2)}$$

⇓

$$G_{\Delta} = \frac{1}{(W - m_{\Delta} + i\Gamma_{\Delta}/2 - \Sigma_{\Delta})}$$

Can we use the same Σ_{Δ} for both processes?

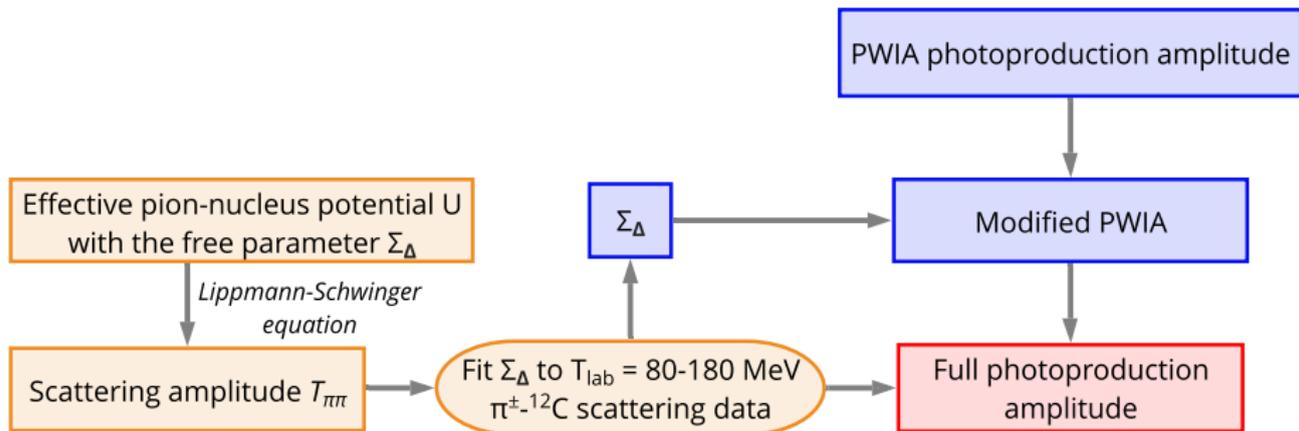
The path to the photoproduction

The Klein-Gordon equation for π :

$$(-\nabla^2 + m_\pi^2)\Phi(\mathbf{r}) + U\Phi(\mathbf{r}) = \omega^2\Phi(\mathbf{r})$$

The optical potential $U = U_{1st} + U_{2nd}$ is complex and energy dependent

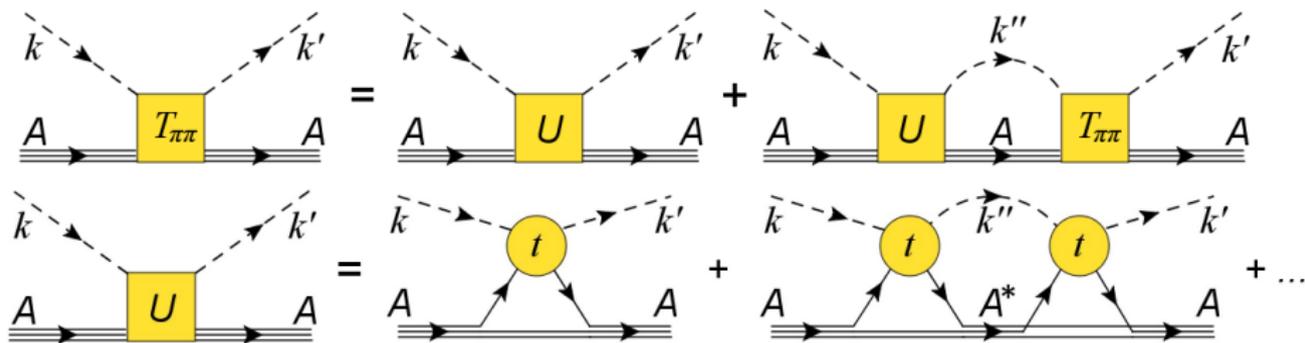
$\text{Im}[U] < 0 \implies$ pion flux is decreasing



$$\Sigma_\Delta = \text{Re } \Sigma_\Delta + i \text{Im } \Sigma_\Delta = \mathbf{const}$$

Multi-energy fit to $\pi^\pm - ^{12}\text{C}$ total, reaction and differential elastic cross sections

Second-order pion-nucleus optical potential



$$U_{1st}(\mathbf{k}', \mathbf{k}) = t_0(\mathbf{k}', \mathbf{k})\rho(\mathbf{q})$$

$$U_{2nd}(\mathbf{k}', \mathbf{k}) = - \int \frac{d\mathbf{k}''}{(2\pi)^3} G_0(\mathbf{k}'') [t_0(\mathbf{k}', \mathbf{k}'')t_0(\mathbf{k}'', \mathbf{k})C(\mathbf{k}' - \mathbf{k}'', \mathbf{k}'' - \mathbf{k}) + 2t_1(\mathbf{k}', \mathbf{k}'')t_1(\mathbf{k}'', \mathbf{k})D(\mathbf{k}' - \mathbf{k}'', \mathbf{k}'' - \mathbf{k})]$$

Correlation functions in the momentum space:

$$D(\mathbf{q}_1, \mathbf{q}_2) = \int d\mathbf{r}_1 d\mathbf{r}_2 e^{-i(\mathbf{q}_1 \cdot \mathbf{r}_1 + \mathbf{q}_2 \cdot \mathbf{r}_2)} \rho_{ex}(\mathbf{r}_1, \mathbf{r}_2);$$

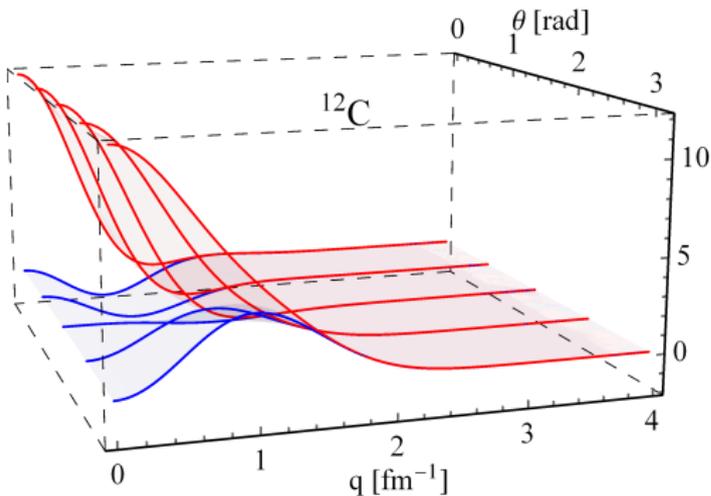
$$C(\mathbf{q}_1, \mathbf{q}_2) = \int d\mathbf{r}_1 d\mathbf{r}_2 e^{-i(\mathbf{q}_1 \cdot \mathbf{r}_1 + \mathbf{q}_2 \cdot \mathbf{r}_2)} C(\mathbf{r}_1, \mathbf{r}_2),$$

$$\rho_{ex}(\mathbf{r}_1, \mathbf{r}_2) = \rho_2(\mathbf{r}_1, \mathbf{r}_2) - \rho(\mathbf{r}_1)\rho(\mathbf{r}_2) \quad C(\mathbf{r}_1, \mathbf{r}_2) = \rho_{ex}(\mathbf{r}_1, \mathbf{r}_2) - \frac{1}{A}\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)$$

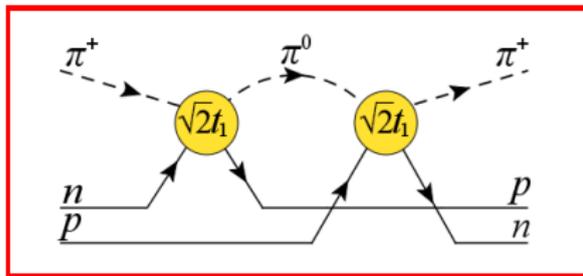
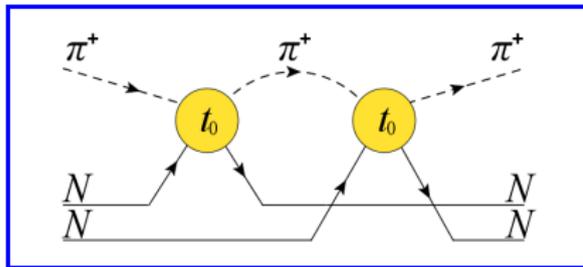
The second order part restores Pauli blocking

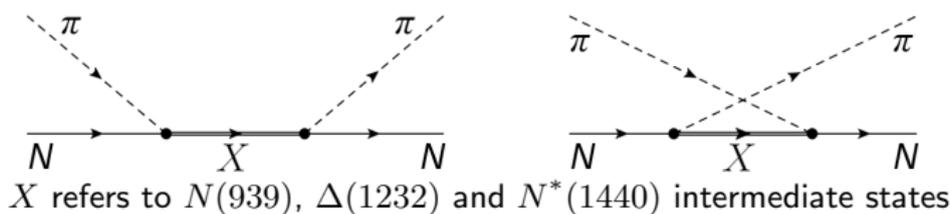
$$U_{2\text{nd}}(\mathbf{k}', \mathbf{k}) = - \int \frac{d\mathbf{k}''}{(2\pi)^3} G_0(\mathbf{k}'') \left[t_0(\mathbf{k}', \mathbf{k}'') t_0(\mathbf{k}'', \mathbf{k}) C(\mathbf{k}' - \mathbf{k}'', \mathbf{k}'' - \mathbf{k}) \right. \\ \left. + 2t_1(\mathbf{k}', \mathbf{k}'') t_1(\mathbf{k}'', \mathbf{k}) D(\mathbf{k}' - \mathbf{k}'', \mathbf{k}'' - \mathbf{k}) \right]$$

Harmonic oscillator shell model for D and C :

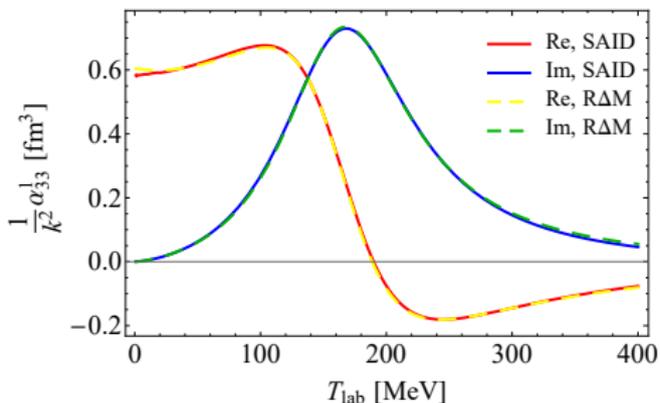


Diagonal part of $D(q, q, \theta)$ and $C(q, q, \theta)$.





E. Oset, H. Toki, W. Weise, Phys. Reports **83**, 281 (1982)

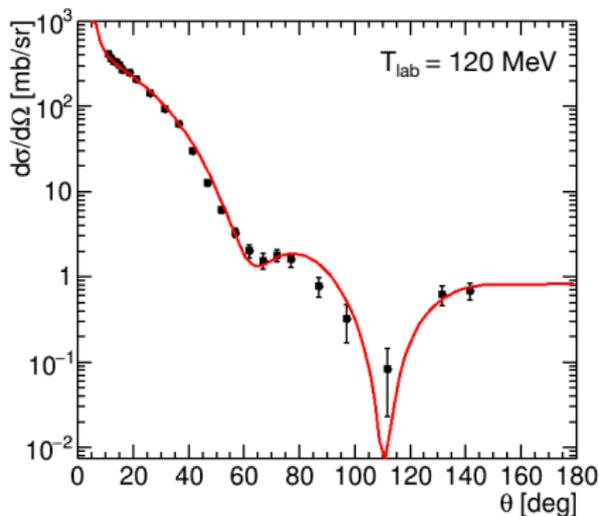
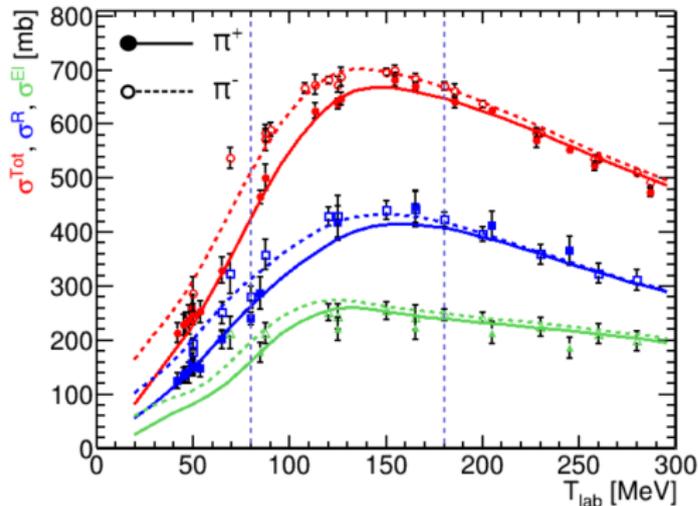


$$\alpha_{33}^1(k) = \frac{K_{33}^1}{1 - ikK_{33}^1}$$

$$K_{33}^1 = \frac{1}{3} \frac{k^2}{4\pi m_\pi^2} \frac{m_N}{\sqrt{s}} \left[\frac{8f_N^2 m_N}{m_N^2 - \bar{u}} + \frac{8f_{N^*}^2 m_{N^*}}{m_{N^*}^2 - \bar{u}} \right] + \left(\frac{2f_\Delta^2 m_\Delta}{m_\Delta^2 - s} + \frac{1}{9} \frac{2f_\Delta^2 m_\Delta}{m_\Delta^2 - \bar{u}} \right)$$

P_{33} wave : **in nuclear medium** $m_\Delta \longrightarrow m_\Delta + \Sigma_\Delta$.
 S_{11} , S_{31} , P_{11} , P_{31} , P_{13} partial waves from **SAID**

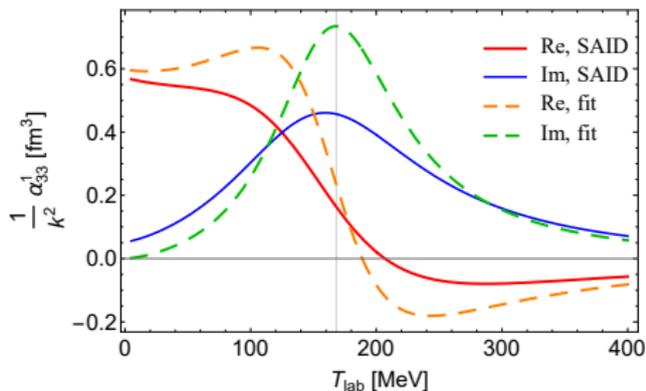
Fit to $T_{\text{lab}} = 80 - 180$ MeV $\pi^{\pm-12}\text{C}$ scattering data



$$\chi^2/\text{ndf} = 2.05$$

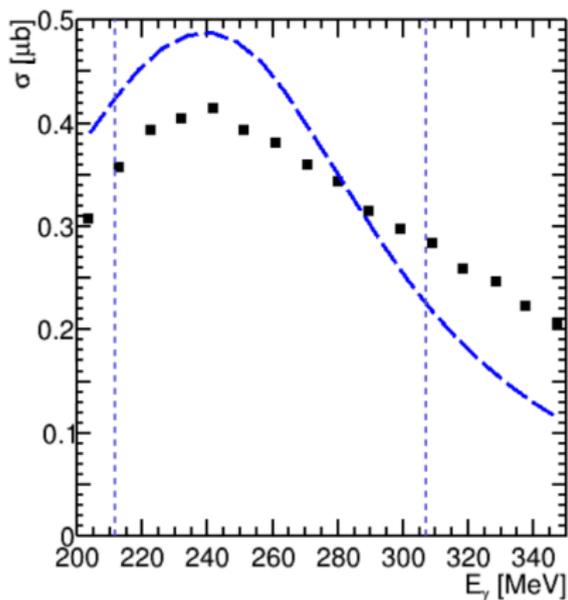
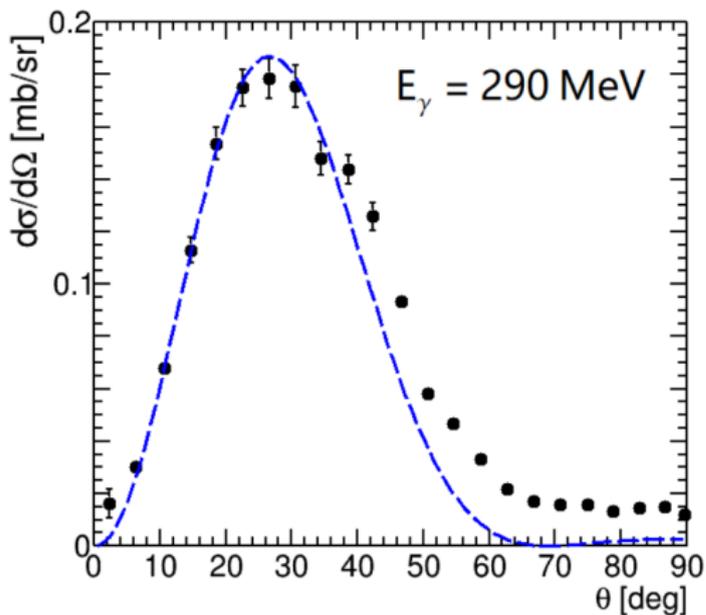
$$\text{Re } \Sigma_{\Delta} = 9.3 \pm 0.9 \text{ MeV}$$

$$\text{Im } \Sigma_{\Delta} = -32.0 \pm 0.7 \text{ MeV}$$



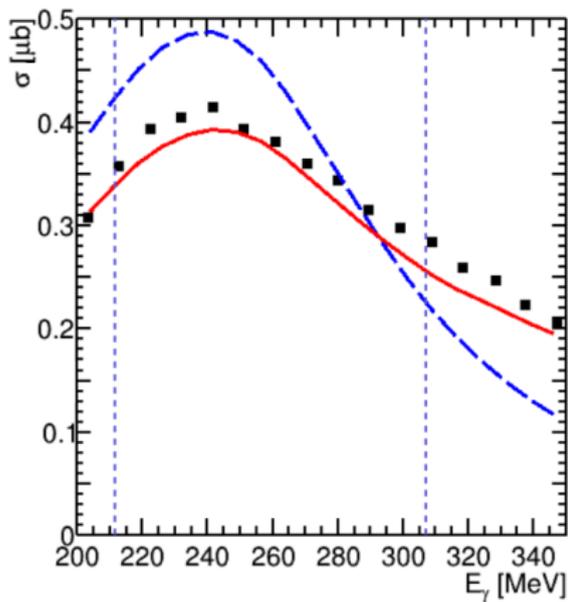
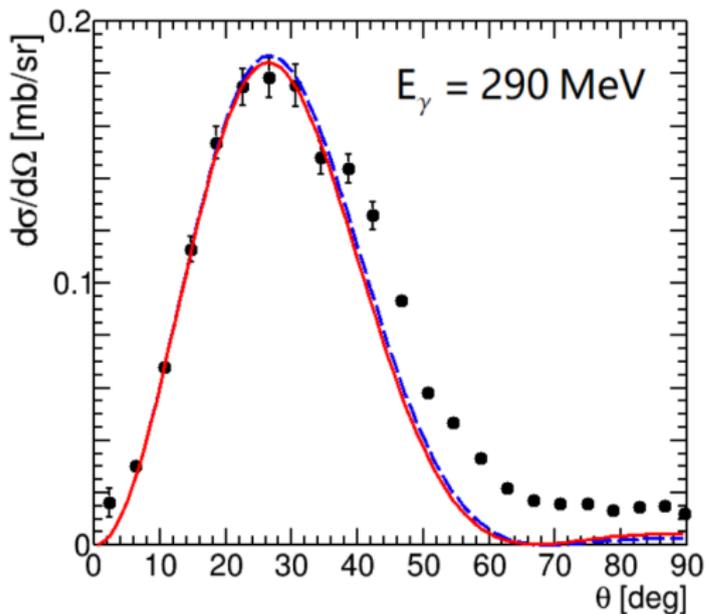
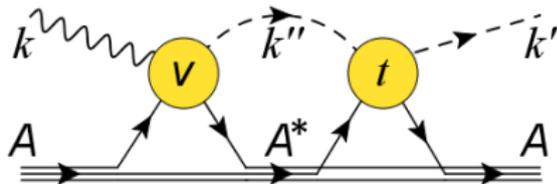
$$F_2 = F_2^{(B)} + 2M_{1+}^{(\Delta)}$$

$$M_{1+}^{(\Delta)} = F_{\gamma N \Delta} \frac{\Gamma_{\Delta} m_{\Delta} e^{i\phi}}{m_{\Delta}^2 - W^2 - im_{\Delta} \Gamma_{\Delta}} F_{\pi N \Delta}$$



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$$M_{1+}^{(\Delta)} = F_{\gamma N \Delta} \frac{\Gamma_{\Delta} m_{\Delta} e^{i\phi}}{m_{\Delta}^2 - W^2 - im_{\Delta} \Gamma_{\Delta}} F_{\pi N \Delta}$$



- Medium effects in π^\pm scattering and π^0 photoproduction described by introducing phenomenological Δ self-energy Σ_Δ
- Derived optical potential provides adequate fits for $T_{lab} = 80 - 180$ MeV scattering
- The full photoproduction amplitude modified by Σ_Δ (from the scattering fit) is consistent with the data
- production of charged pion followed by charge exchange on a second nucleon causes a significant shift in the cross section

- **Exploration:** sensitivities of the model
theoretical error estimate
- **Extension:** application to heavy nuclei, e.g. ^{40}Ca , ^{48}Ca , ^{208}Pb

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Thank you for your attention!