Flow Harmonic Distribution in Large and Small Systems

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Dense & Strange Hadronic Matter Group, Technical University of Munich

Based on:
C.Mordasini, A. Bilandzic, D. Karakoc, SFT, arXiv:1901.06968v2 [nucl-ex]]
SFT, arXiv: 1907.12140v1 [nucl-th]

58. International Winter Meeting on Nuclear Physics
Bormio, Italy
22 January 2020

Introduction

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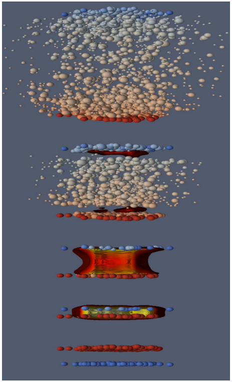
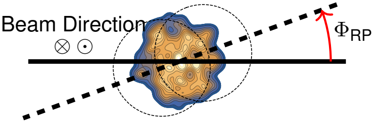
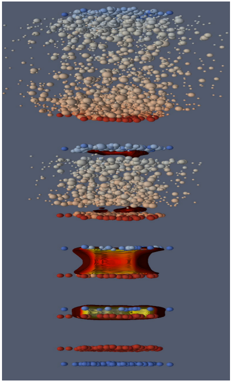


[MADAI collaboration]



Introduction

- **“Standard Model” of Heavy-Ion Collision :**
  - Pre-Equilibrium
  - Initial Condition
  - Collective Evolution (deconfined phase)
  - Hadronization
  - Collective Evolution (confined phase)
  - Freeze-Out (chemical/kinetic)
  - Free Streaming



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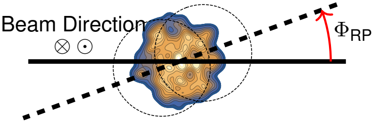

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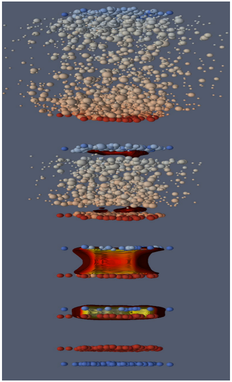
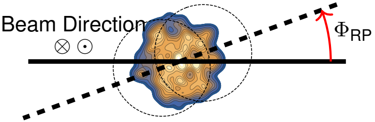


[MADAI collaboration]

- **A probe for many interesting aspects of physics**
  - Out-of-Equilibrium Physics
  - The initial state models
  - QCD Transport Coefficients, $\eta/s, \ldots$
  - QCD Phase Transition
  - QCD e.o.s.
  - Chiral Anomaly in QCD Matter (?!)
  - ⋮



What does the standard picture of the collision tell us?

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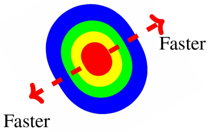
ε_2 (Ellipticity)



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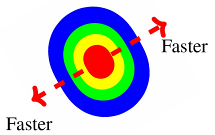

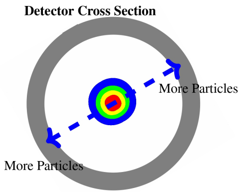

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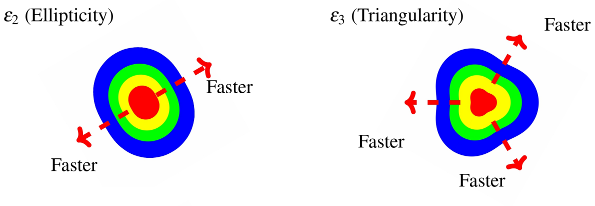

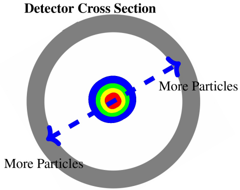

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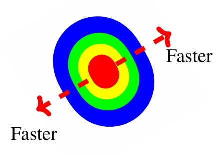
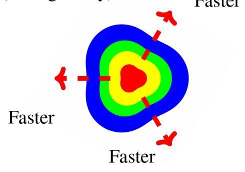
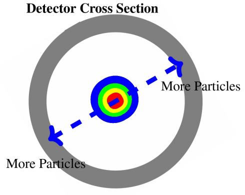
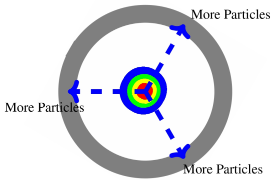


ε_3 (Triangularity)







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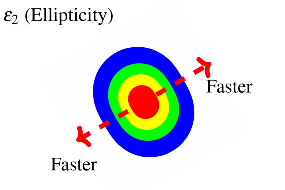

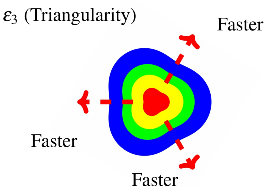

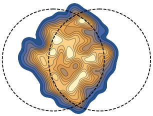

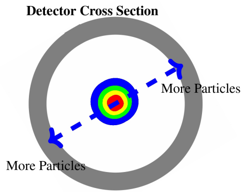

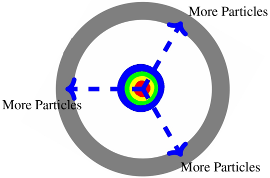

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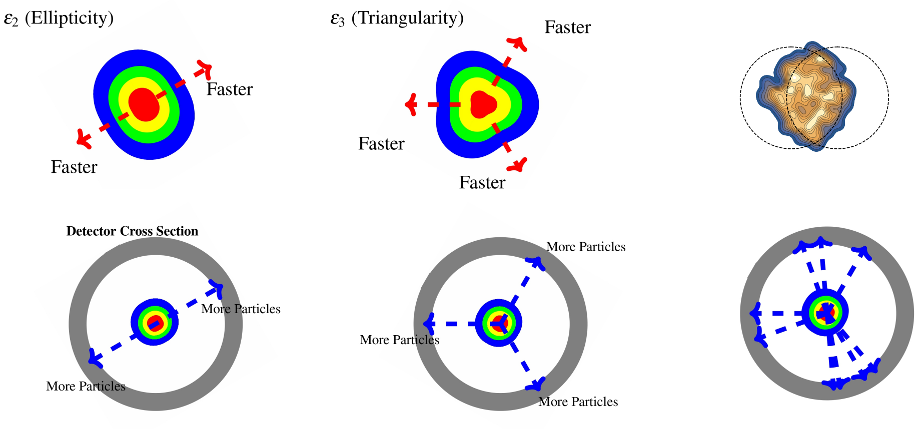
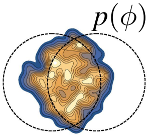
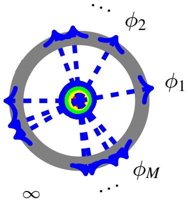


Single Particle Distribution



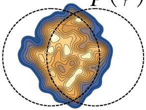


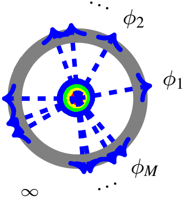
$\epsilon_2,\ \epsilon_3, \ldots$

$$p(\phi) \propto 1 + \sum_{n=1}^{\infty} 2\, v_n\, \cos\left[n(\phi - \psi_n)\right]$$

Single Particle Distribution

$p(\phi)$ ***Single Particle Distribution***





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The coefficient $v_n e^{i n \psi_n}$ *is called* n*th order* ***Flow Harmonic.***

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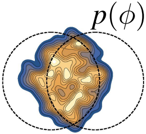

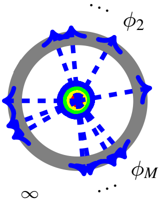

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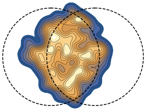
The physics is encoded in $v_n e^{i n \psi_n}$.

Event-By-Event Fluctuation



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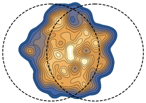
$\epsilon_n^{(1)}, \quad v_n^{(1)}$



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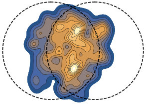

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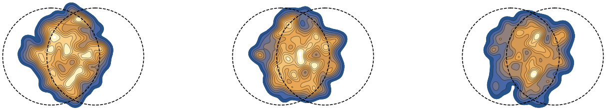

$p^{(1)}(\phi)$ $p^{(2)}(\phi)$ $p^{(3)}(\phi)$ ....

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$$p_i(\epsilon_1, \epsilon_2, \epsilon_3, \ldots, \phi_1 - \phi_2, \phi_2 - \phi_3, \ldots), \quad \xrightarrow{\text{Collective Evolution}} \quad p_f(v_1, v_2, v_3, \ldots, \psi_1 - \psi_2, \psi_2 - \psi_3, \ldots)$$

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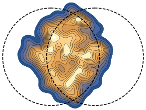

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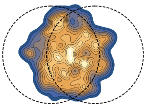

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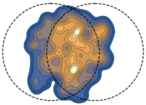

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We use cumulants to study these p.d.f.'s!

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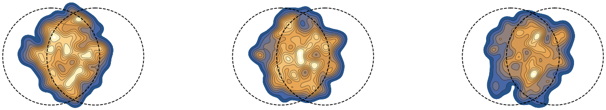

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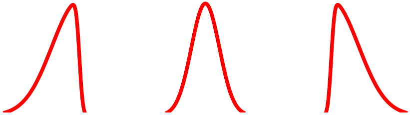
- An example: 1D ordinary p.d.f. close to Gaussian:



skewness: $\gamma_1 < 0$ $\gamma_1 = 0$ $\gamma_1 > 0$

Cumulants of Flow Fluctuations

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- Considering only one flow amplitude: $p(v_n)$
  [Borghini, Dinh, Ollitrault, '00, '01]

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- Considering two flows: $p(v_n, v_m)$
  Symmetric Cumulants:
  [Bilandzic, Christensen, Gulbrandsen, Hansen, Zhou, '13]

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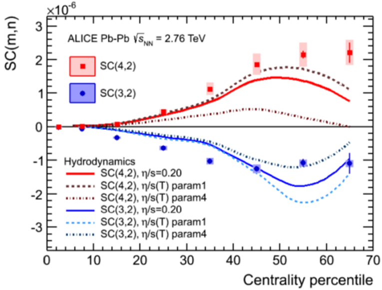

[ALICE Collab., PRL 117, 182301 (2016)]

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- Considering multi flows: $p(v_k, v_l, v_m, \ldots)$
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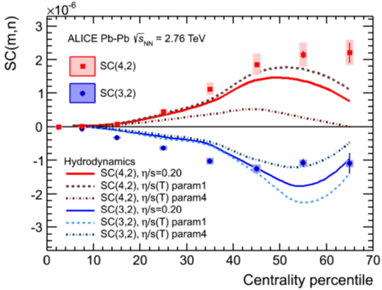
  [Mordasini, Bilandzic, Karakoc, SFT, arXiv:1901.06968v2 [nucl-ex]]



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  - Example for three harmonics:

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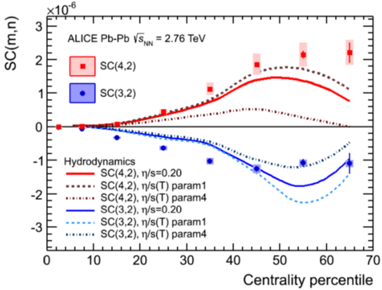
$$\mathrm{SC}_\epsilon(k,l,m) = \langle \epsilon_k^2\, \epsilon_l^2 \epsilon_m^2 \rangle - \langle \epsilon_k^2\, \epsilon_l^2 \rangle \langle \epsilon_m^2 \rangle - \langle \epsilon_k^2\, \epsilon_m^2 \rangle \langle \epsilon_l^2 \rangle - \langle \epsilon_l^2\, \epsilon_m^2 \rangle \langle \epsilon_k^2 \rangle + 2\langle \epsilon_k^2 \rangle \langle \epsilon_l^2 \rangle \langle \epsilon_m^2 \rangle$$



[ALICE Collab., PRL 117, 182301 (2016)]

Generalized Symmetric Cumulant, Realistic Monte Carlo

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- iEBE-VISHNU event generator **[Shen , Qiu, Song, Bernhard, Bass, Heinz, '14]** / PbPb collision at $\sqrt{s_{NN}} = 2.76$ TeV

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- $$\text{NSC}(k,l,m) = \frac{\text{SC}(k,l,m)}{\langle v_k^2\rangle\langle v_l^2\rangle\langle v_m^2\rangle},$$
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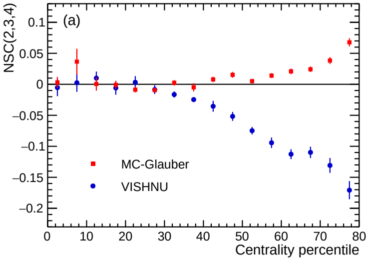

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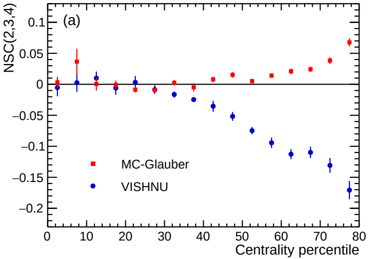

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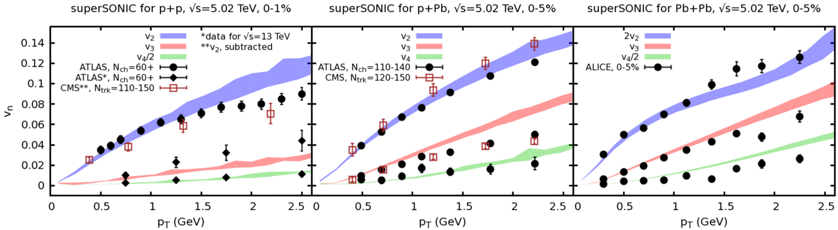
The observable is sensitive to the hydrodynamic evolution.

QGP droplet in pp Collision?

Four-Particle Correlation Sign Puzzle in pp Collision

One fluid may or may not rule them all

$$v_n\{2\} \equiv (c_n\{2\})^{1/2}$$



[Wellerland, Romatschke,'17]

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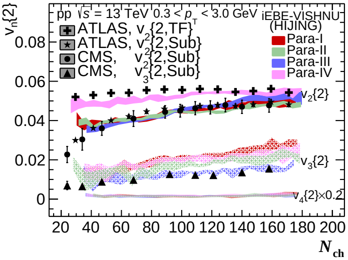

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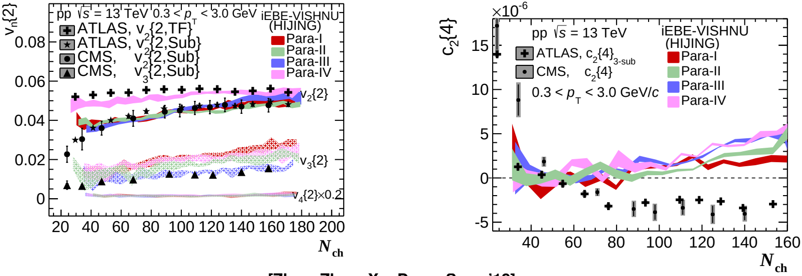

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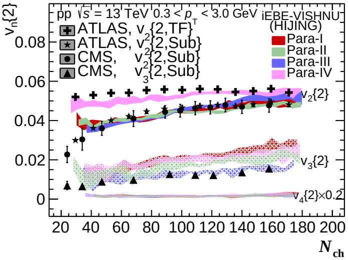

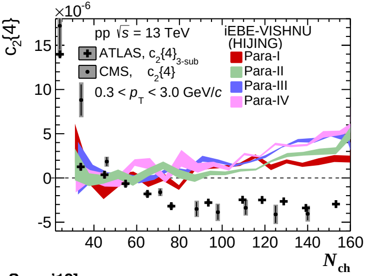

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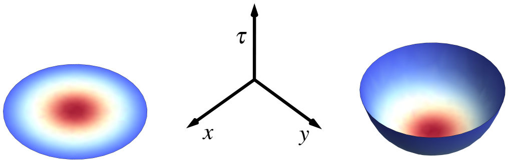

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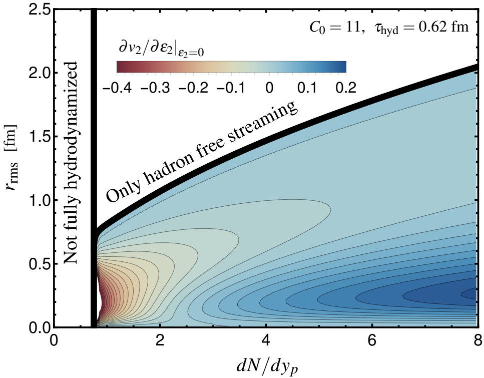

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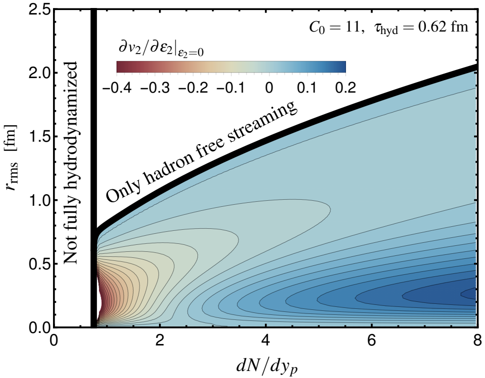

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- Translate total transverse energy into multiplicity:

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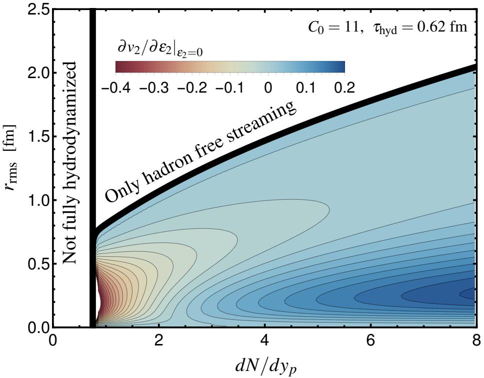

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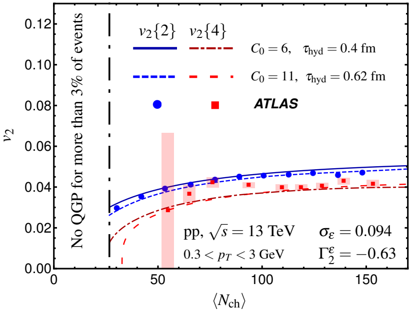

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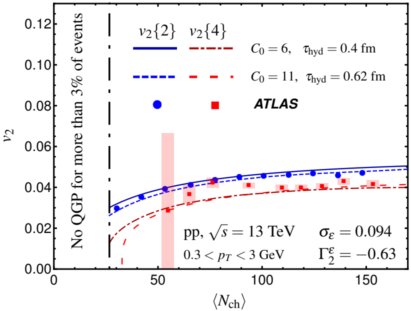

The model can explain $-v_2\{4\}^4 \equiv c_2\{4\} < 0$ with correct sign!

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Grazie!

Backup

Generalized Symmetric Cumulant, Realistic Monte Carlo

- iEBE-VISHNU **[Shen , Qiu, Song, Bernhard, Bass, Heinz, ’14]**
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$$NSC(k,l,m) = \frac{SC(k,l,m)}{\langle v_k^2\rangle\langle v_l^2\rangle\langle v_m^2\rangle}, \qquad NSC(k,l,m) = \frac{SC(k,l,m)}{\langle \epsilon_k^2\rangle\langle \epsilon_l^2\rangle\langle \epsilon_m^2\rangle}$$

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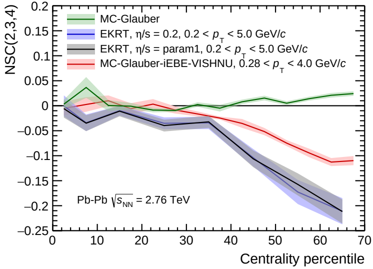

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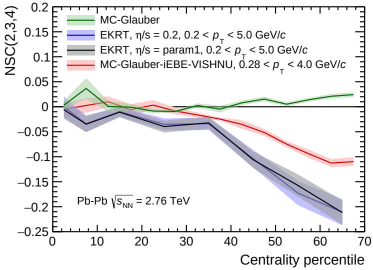

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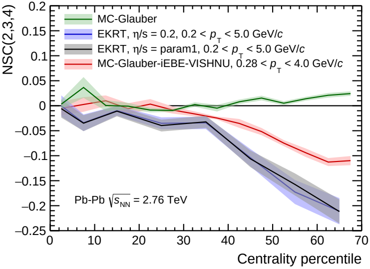

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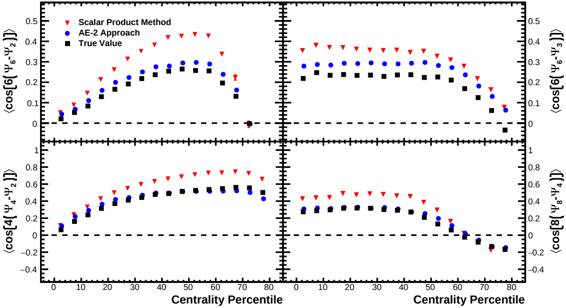
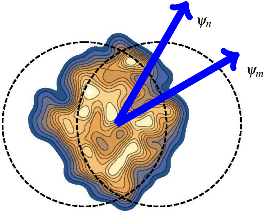
- $SC(2,3) < 0 \quad \rightarrow \quad v_2$ and v_3 are anti-correlated.

Symmetry-Plane Correlation

[M. Lesch, A. Bilandzic, SFT, in progress]

$$\langle\cos\left[k(\psi_m-\psi_n)\right]\rangle_{SP}=\frac{\langle v_m^{am}\,v_n^{an}\,\cos[k(\psi_m-\psi_n)]\rangle}{\sqrt{\langle v_m^{2am}\rangle\langle v_n^{2an}\rangle}},\qquad \langle\cos\left[k(\psi_m-\psi_n)\right]\rangle_{AE}=\frac{\langle v_m^{am}\,v_n^{an}\,\cos[k(\psi_m-\psi_n)]\rangle}{\sqrt{\langle v_m^{2am}\,v_n^{2an}\rangle}}$$





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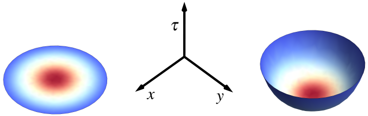

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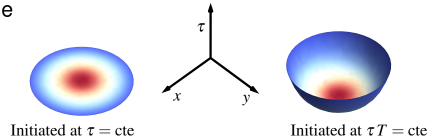

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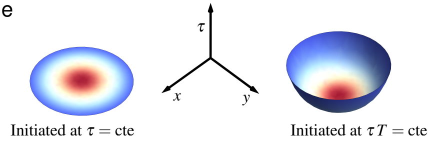

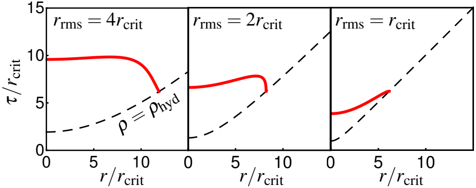

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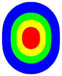
- *Assumption 2*: The average ellipticity is equal to zero, $\bar{\epsilon}_2 \equiv \Big\langle$  $\Big\rangle_{\text{many events}} = 0$

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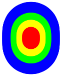
- *Assumption 2*: The average ellipticity is equal to zero, $\bar{\epsilon}_2 \equiv \Big\langle$  $\Big\rangle_{\text{many events}} = 0$

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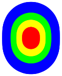
- *Assumption 2*: The average ellipticity is equal to zero, $\bar{\epsilon}_2 \equiv \Big\langle$  $\Big\rangle_{\text{many events}} = 0$
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$$p(r_{\text{rms}}) = \frac{r_{\text{rms}}}{\sigma_r^2} \exp\left[-\frac{r_{\text{rms}}^2}{2\sigma_r^2}\right].$$

The width of the distribution: $\langle r_{\text{rms}}^2 \rangle = \pi\sigma_r^2/2 = 2\sigma_{\text{NN}}^{\text{inel}}(\sqrt{s})/14.3$ **[Heinz, Moreland,'11]**.
The value of $\sigma_{\text{NN}}^{\text{inel}}(\sqrt{s})$ has been measured by **[TOTEM Collaboration, '17]**.

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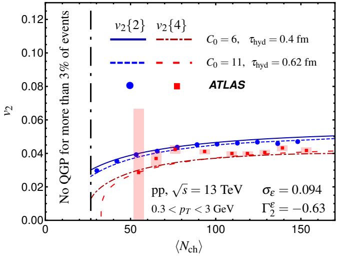

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