# A glimpse of the *H* dibaryon from a lattice QCD perspective

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THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

# Introduction — The *H* dibaryon

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**Perhaps a Stable Dihyperon***

R. L. Jaffe†

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(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of $-2$) $J^P=0^+$ dihyperon ($H$) at 2150 MeV. Another isosinglet dihyperon ($H^*$) with $J^P=1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the $H$ are discussed.

- MIT bag model predicts dihyperon state (*H*) with

$$I=0,\ S=-2,\ J^P=0^+$$

and a mass of $m_H = 2150\,\mathrm{MeV}$

- *H* dibaryon must decay weakly

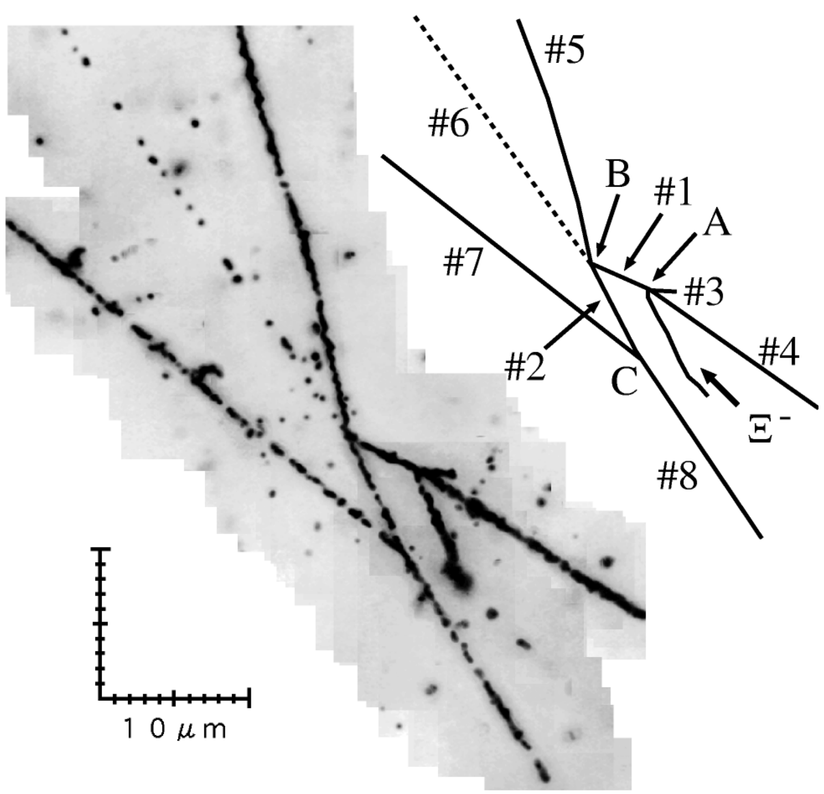
# Experimental Searches

VOLUME 87, NUMBER 21 PHYSICAL REVIEW LETTERS 19 NOVEMBER 2001

**Observation of a ${}^{6}_{\Lambda\Lambda}$He Double Hypernucleus** (E373@KEK):

A double-hyperfragment event has been found in a hybrid-emulsion experiment. It is identified uniquely as the sequential decay of ${}^{6}_{\Lambda\Lambda}$He emitted from a $\Xi^-$ hyperon nuclear capture at rest. The mass of ${}^{6}_{\Lambda\Lambda}$He and the $\Lambda$-$\Lambda$ interaction energy $\Delta B_{\Lambda\Lambda}$ have been measured for the first time devoid of the ambiguities due to the possibilities of excited states. The value of $\Delta B_{\Lambda\Lambda}$ is $1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV. This demonstrates that the $\Lambda$-$\Lambda$ interaction is weakly attractive.

**"Nagara" event**




Observation of a ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ double-hypernucleus

Binding energy:

$$B_{\Lambda\Lambda} = 7.25 \pm 0.19\,\left({}^{+0.18}_{-0.11}\right)\,\mathrm{MeV}$$

Interpreted as sequential weak decay of ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda} = 2223.7\,\mathrm{MeV}$$ @ 90% CL

# The *H* dibaryon as a dark matter candidate

- $udsuds$ bound state as dark matter candidate:
  *[G.R. Farrar, A. Strumia et al.,…]*

$m_H < 2(m_p + m_e) = 1877.6\,\mathrm{MeV}$ $\Rightarrow$ *H* dibaryon absolutely stable

$m_H > 2(m_p + \mathrm{B.E.}) = 1860\,\mathrm{MeV}$ $\Rightarrow$ Nuclei absolutely stable

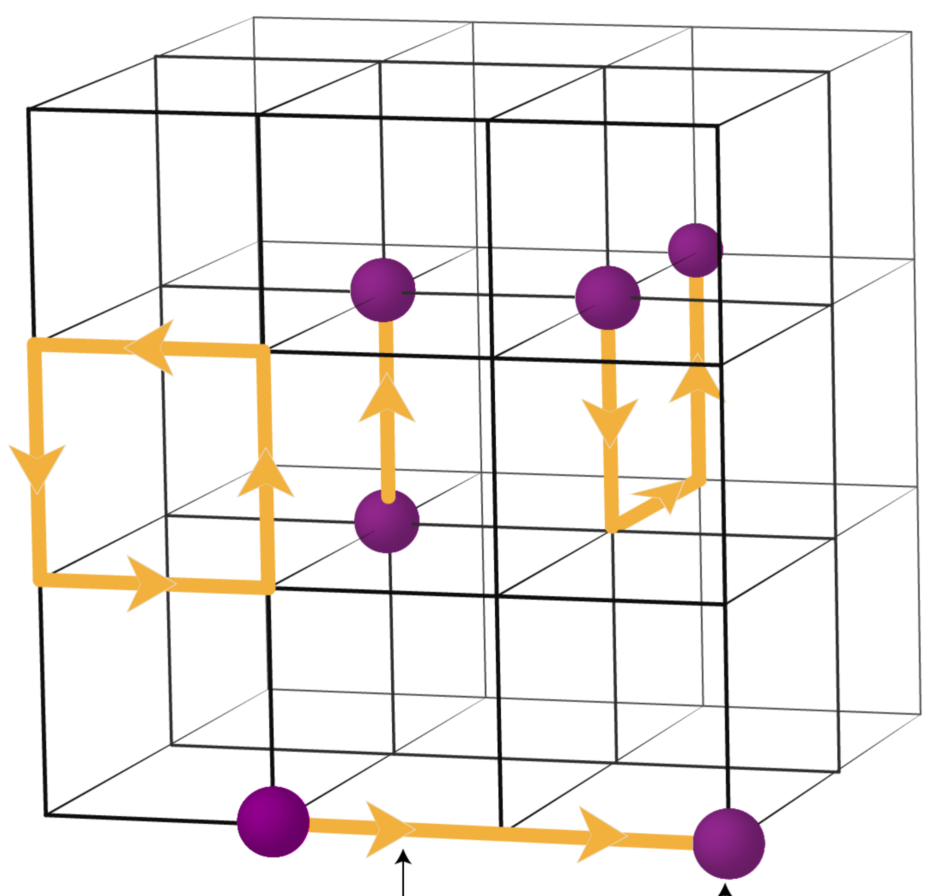
- Recall: $2m_\Lambda = 2230\,\mathrm{MeV}$

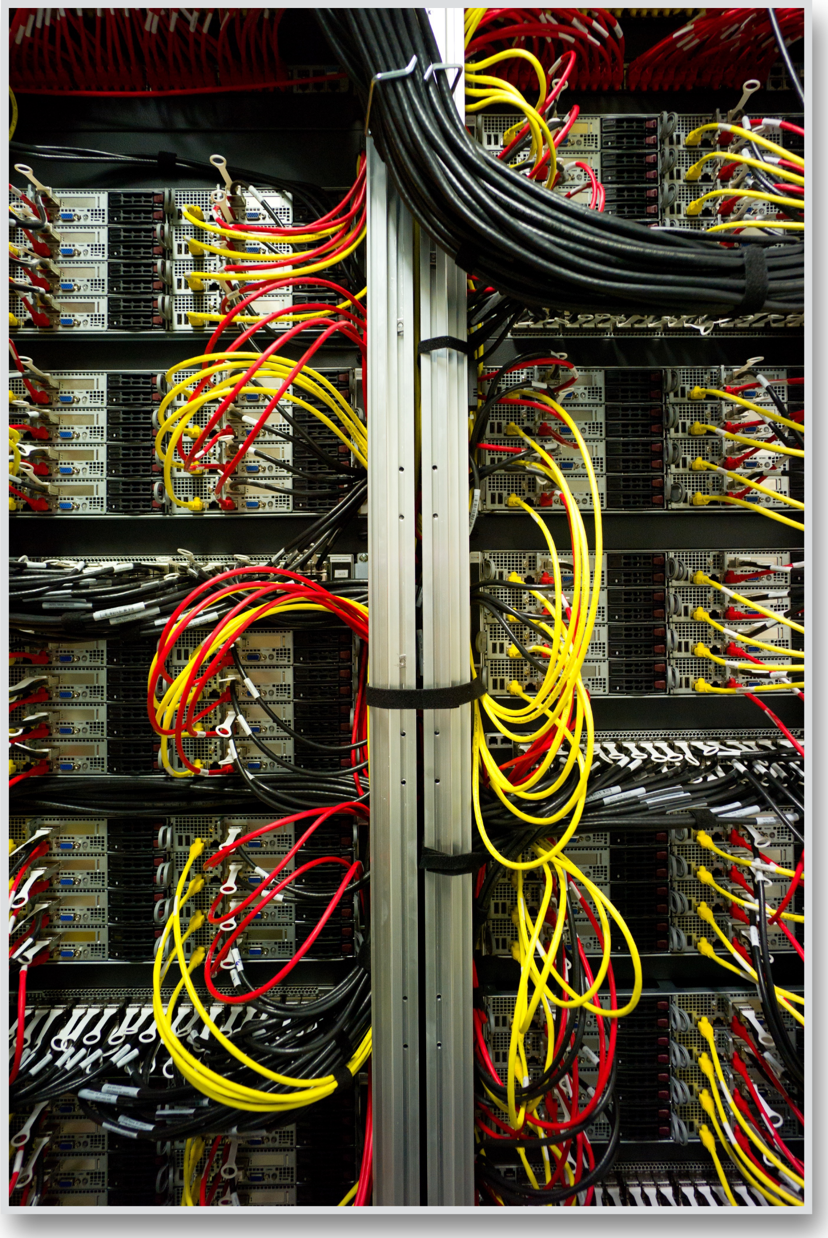
  $m_H = 2150\,\mathrm{MeV}$ (Jaffe's bag model estimate)

- Scenario requires very large binding energy of $\approx 360\,\mathrm{MeV}$

# Current Status

* *H* dibaryon not firmly established experimentally
* Is a bound *H* dibaryon a consequence of QCD?
* Try "ab initio" technique: Lattice QCD



*"Clover" @ Mainz*

# Beyond Perturbation Theory: Lattice QCD

- Non-perturbative treatment; regularised Euclidean functional integrals

  Lattice spacing: $a, \quad x_\mu = n_\mu a, \quad a^{-1} = \Lambda_{\mathrm{UV}}$

  Finite volume: $L^3 \cdot T, \quad N_s = L/a, \quad N_t = T/a$

$$\langle\Omega\rangle = \frac{1}{Z}\int \prod_{x,\mu} dU_\mu(x)\; \Omega \prod_{f=u,d,s,\ldots} \det\left(D\!\!\!/^{\,\mathrm{lat}} + m_f\right) \mathrm{e}^{-S_{\mathrm{G}}[U]}$$

- Stochastic evaluation of $\langle\Omega\rangle$ via Markov process

  Strong growth of numerical cost near physical $m_u, m_d$

- Pion mass, i.e. lightest mass in pseudoscalar channel:

$$\approx 500\,\mathrm{MeV} \longrightarrow \approx 130\,\mathrm{MeV}$$

(2001) (≳ 2015)

# Hadron spectrum in Lattice QCD

* Spectral information contained in correlation functions

$$\sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{y}-\boldsymbol{x})} \left\langle O_{\mathrm{had}}(y)\, O^{\dagger}_{\mathrm{had}}(x) \right\rangle = \sum_{n} w_n(\boldsymbol{p})\, \mathrm{e}^{-E_n(\boldsymbol{p})(y_0-x_0)}$$

$$\xrightarrow{(y_0-x_0)\to\infty} w_1(\boldsymbol{p})\, \mathrm{e}^{-E_1(\boldsymbol{p})(y_0-x_0)}$$

* $O_{\mathrm{had}}(x)$ : interpolating operator

  $\rightarrow$ projects on all states with the same quantum numbers

  Nucleon: $O_N = \epsilon_{abc} \left( u^a\, C\gamma_5\, d^b \right) u^c$

* Ground state dominates at large Euclidean times: $y_0 - x_0 \rightarrow \infty$

* Excited states are sub-leading contributions

# The *H* Dibaryon in Lattice QCD

**Flavour structure**

- *H* dibaryon lies in the $\mathbf{1}$-dimensional irrep. of $\mathrm{SU(3)}_{\mathrm{flavour}}$
- Flavour structure of two octet baryons:

$$\mathbf{8}\otimes\mathbf{8} = (\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27})_{\mathrm{S}}\oplus(\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}})_{\mathrm{A}}$$

- Upon SU(3)-symmetry breaking, $\mathbf{8}$ and $\mathbf{27}$ mix with singlet
- Singlet, octet and $\mathbf{27}$plet interpolators constructed from linear combinations of $\Lambda\Lambda$, $\Sigma\Sigma$ and $N\Xi$ operators

**Other interesting dibaryons**

- Dineutron lies in $\mathbf{27}$ irrep.
- Deuteron lies in $\overline{\mathbf{10}}$ irrep. with $J^P = 1^+$

# Interpolating operators

* Hexaquark operators (inspired by Jaffe's original bag model calculation):

$$[rstuvw] = \epsilon_{ijk}\epsilon_{lmn}\left(s^a C\gamma_5 P_+ t^b\right)\left(v^l C\gamma_5 P_+ w^m\right)\left(r^k C\gamma_5 P_+ u^n\right)$$

$$H^{(\mathbf{1})} = \frac{1}{48}\left([sudsud] - [udusds] - [dudsus]\right)$$

$$H^{(\mathbf{27})} = \frac{1}{48\sqrt{3}}\left(3[sudsud] + [udusds] - [dudsus]\right)$$

* Momentum-projected two-baryon operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk}\left(s^i C\gamma_5 P_+ t^j\right) r^k_\alpha$$

$$(BB)(\boldsymbol{P};t) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}} B_1(\boldsymbol{x},t)\,(C\gamma_5 P_+)\sum_{\boldsymbol{y}} e^{-i\boldsymbol{p}_2\cdot\boldsymbol{y}} B_2(\boldsymbol{y},t), \quad \boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$$

$\longrightarrow$ project onto $(BB)^{(1)}, (BB)^{(8)}, (BB)^{(27)}$

# Correlation matrices

* Consider set of $N_{\rm op}$ interpolating operators for a given hadron:

  Correlation matrix: $C_{ij}(\boldsymbol{P},\tau) = \left\langle O_i(\boldsymbol{P},t) O_j(\boldsymbol{P},t')^\dagger \right\rangle, \quad \tau = t - t'$

* Variational method: solve Generalised Eigenvalue Problem (GEVP):

$$\boldsymbol{C}(t_1)\, v_n(t_1,t_0) = \lambda_n(t_1,t_0)\, \boldsymbol{C}(t_0)\, v_n(t_1,t_0)$$

$$w_n^\dagger(t_1,t_0)\, \boldsymbol{C}(t_1) = \lambda_n(t_1,t_0)\, w_n^\dagger(t_1,t_0)\, \boldsymbol{C}(t_0), \quad n = 1,\ldots,N_{\rm op}$$

* Project on approximately diagonal correlator:

$$\boldsymbol{\Lambda}_{mn}(t) = w_n^\dagger\, \boldsymbol{C}(t)\, v_m$$

* Compute the effective $n^{\rm th}$ energy level:

$$E_n^{\rm eff}(t) = \frac{1}{\Delta t} \ln \frac{\Lambda_{nn}(t)}{\Lambda_{nn}(t+\Delta t)}$$

# HAL QCD Method

* Obtain baryon-baryon potential from Nambu-Bethe-Salpeter amplitude computed on the lattice

$$G_4(\boldsymbol{r}, t-t_0) = \left\langle 0 \left| (BB)^{(\alpha)}(\boldsymbol{r}, t)\, (\overline{BB})^{(\alpha)}(\boldsymbol{r}, t_0) \right| 0 \right\rangle = \phi(\boldsymbol{r}, t)\, \mathrm{e}^{-2M(t-t_0)}$$

$(BB)^{(\alpha)}(\boldsymbol{r}, t)$ : 2-baryon interpolating operator; flavour irrep. $\alpha$

$\phi(\boldsymbol{r}, t)$ : NBS wave function

$M$ : single baryon mass

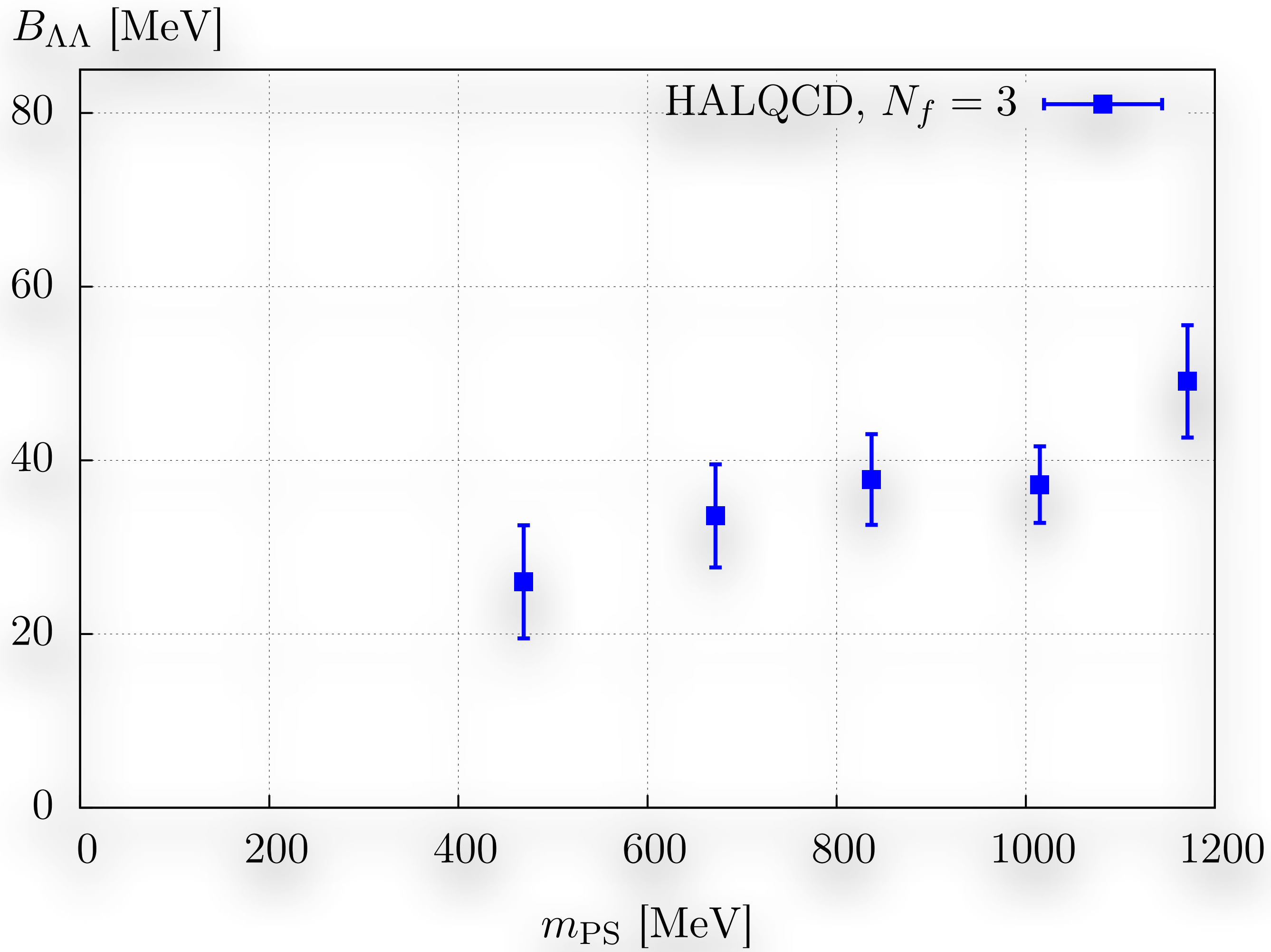
* Determine potential via $$V(r) = \frac{[-H_0 - (\partial/\partial t)]\, \phi(\boldsymbol{r}, t)}{\phi(\boldsymbol{r}, t)}$$

* Solve Schrödinger equation
  $\longrightarrow$ determine binding energies and scattering phase shifts

# HAL QCD Method

Details of the calculation: *[Inoue et al., Phys Rev Lett 106 (2011) 162002]*

- $N_f = 3,$ i.e. mass-degenerate $u, d, s$ quarks
- Single lattice spacing: $a = 0.121(2)\,\text{fm}$
- 5 pion masses in the range: $m_\pi = 469 - 1171\,\text{MeV}$




(statistical and systematic errors combined)

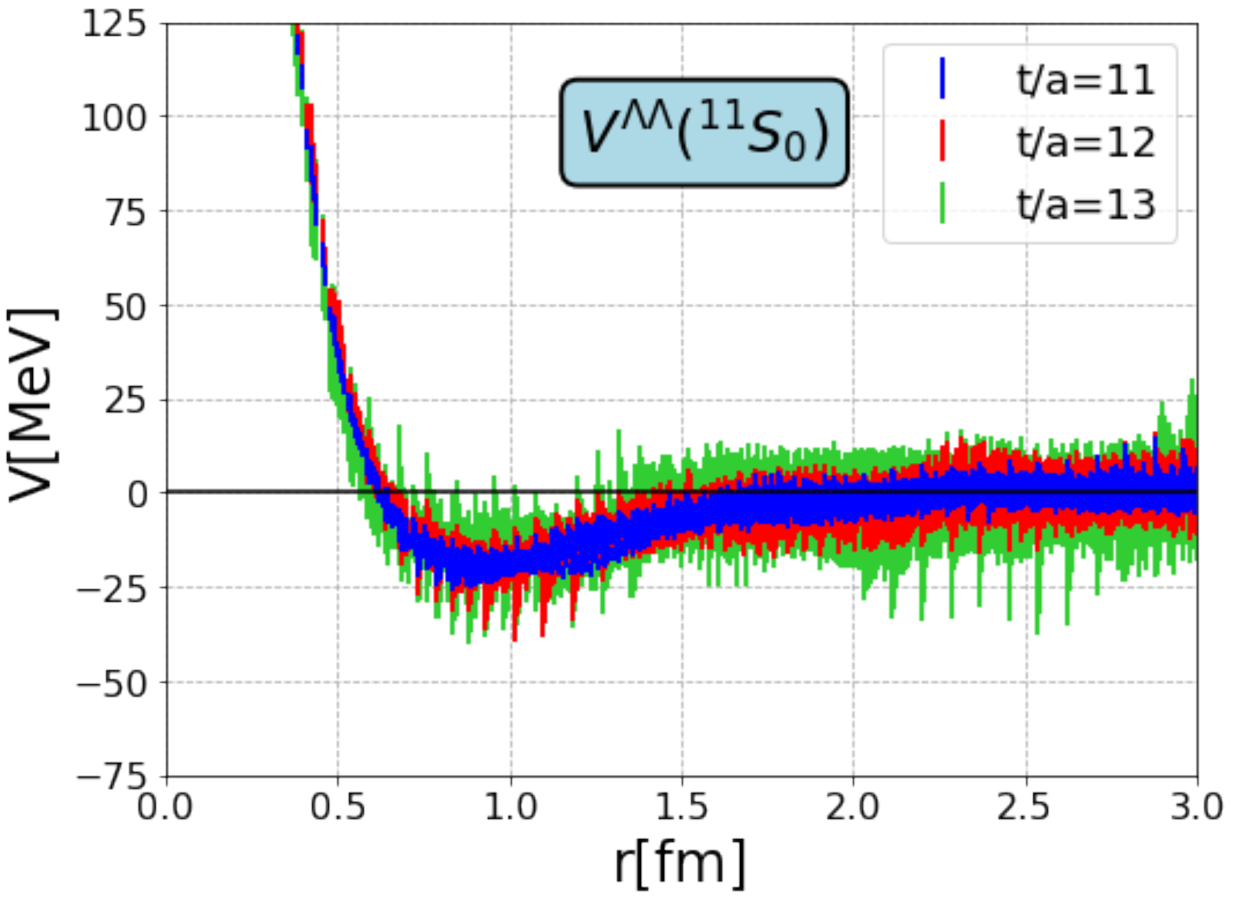
# HAL QCD Method

Details of the calculation: *[Sasaki et al., arXiv:1912.08630]*

$N_f = 2+1$, $\mathrm{O}(a)$ improved Wilson fermions

Single lattice spacing: $a = 0.0846\,\mathrm{fm}$; Volume: $L \approx 8.1\,\mathrm{fm}$

Near physical point: $m_\pi = 146\,\mathrm{MeV}$, $m_K = 525\,\mathrm{MeV}$




- * $\Lambda\Lambda$ interaction weakly attractive

$\Rightarrow$ No bound or resonant dihyperon near $\Lambda\Lambda$ threshold at the physical point

**Needs verification using a different methodology**

# The Mainz Dibaryon Project

**Collaborators:**

A. Francis, J.R. Green, A. Hanlon, P. Junnarkar, Ch. Miao, T.D. Rae, H.W.

**Gauge ensembles provided by the CLS effort:**

* $N_f = 2$ flavours of $\mathrm{O}(a)$ improved Wilson fermions; quenched strange quark

  Pion masses: $m_\pi = 450 - 1000\,\mathrm{MeV}$ (to compare with earlier studies)

  *[Francis, Green, Junnarkar, Miao, Rae, HW, Phys Rev D99 (2019) 074505]*

* $N_f = 2+1$ flavours of $\mathrm{O}(a)$ improved Wilson fermions

  Pion masses: $m_\pi = 200 - 420\,\mathrm{MeV}$

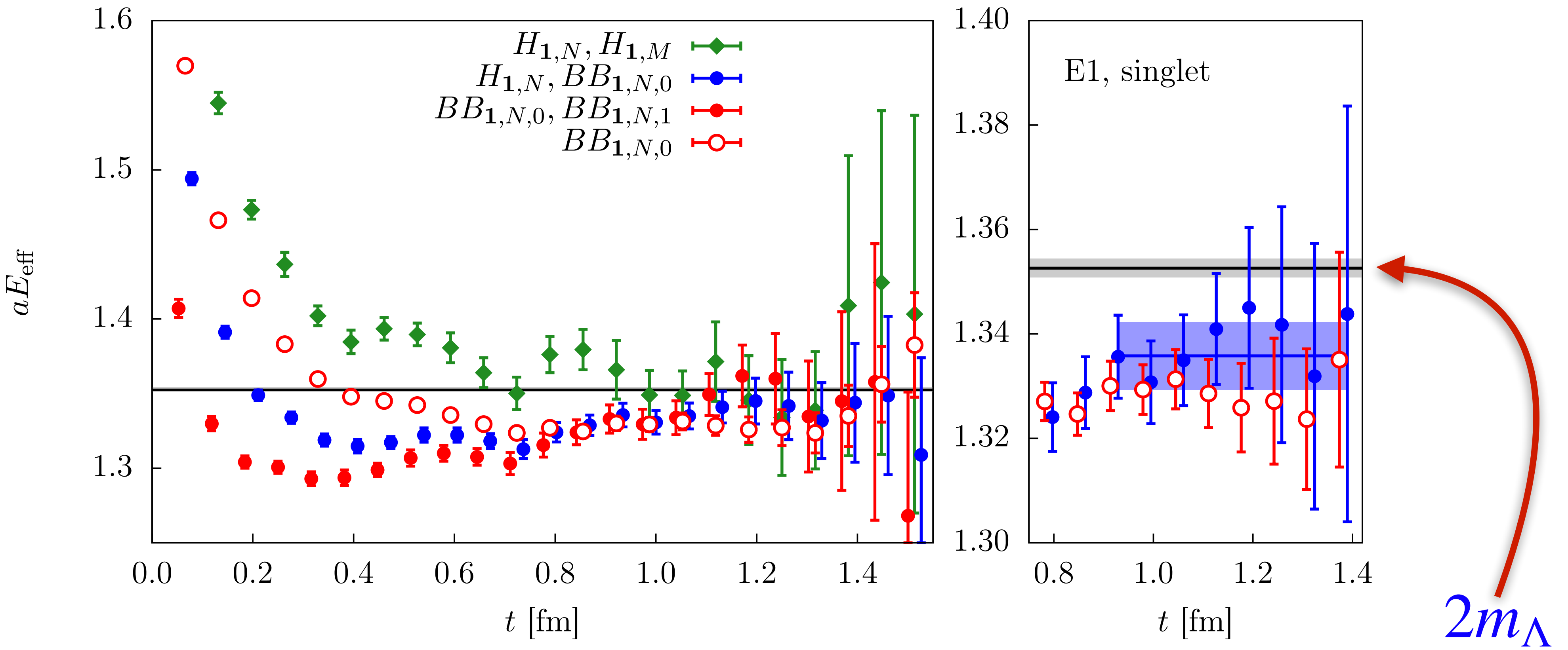
  *[Hanlon, Francis, Green, Junnarkar, HW, arXiv:1810.13282]*

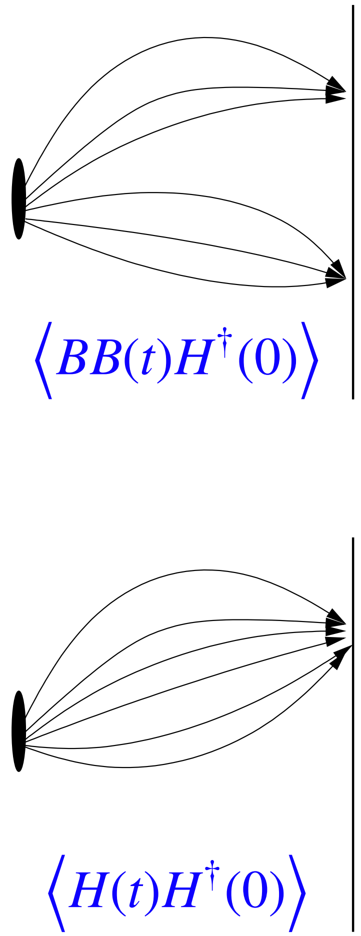
* SU(3)-symmetric and SU(3)-broken situations

* Three different lattice spacings to investigate lattice artefacts

# Finite-volume spectrum

- Ensemble E1: $N_f = 2$, SU(3)-symmetric, $m_\pi \approx 960\,\mathrm{MeV}$
- Point-to-all propagators: hexaquark operators at source, two-baryon or hexaquark operators at the sink

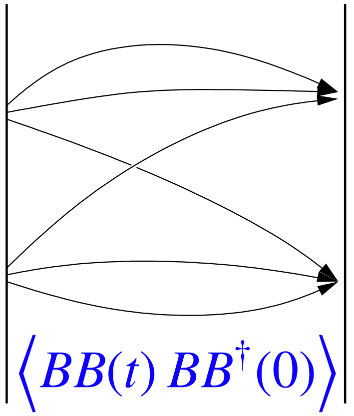



- Hexaquark operators: noisier, slower convergence towards ground state

# Finite-volume spectrum

* Compute timeslice-to-all propagators
  → **“distillation”** — Laplace-Heaviside (LapH) smearing
  *[Peardon et al., PRD 80 (2009) 054506; Morningstar et al., PRD 83 (2011) 114505]*



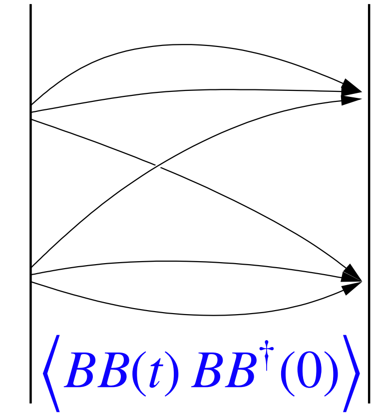

* Quark propagator with smearing matrix at source and sink:

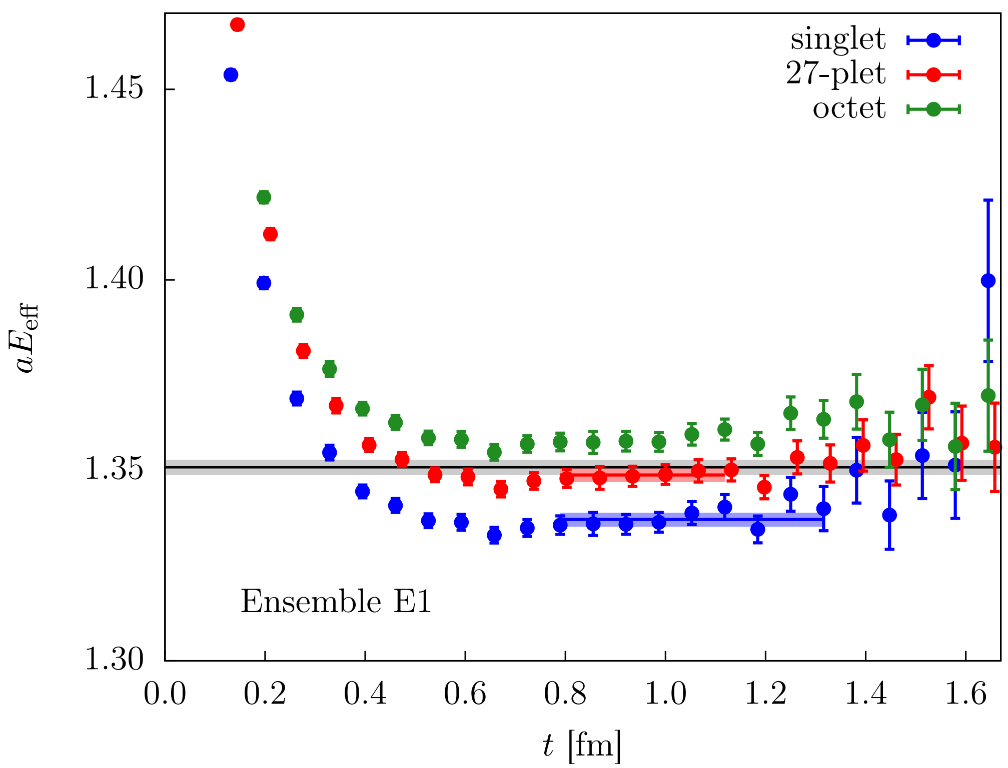
$$\mathcal{S}\, D^{-1}\, \mathcal{S}, \quad \mathcal{S}^{(t)}(\boldsymbol{x},\boldsymbol{y}) = \sum_{k=1}^{N_{\rm LapH}} V^{(k)}(\boldsymbol{x},t) \otimes V^{(k)}(\boldsymbol{y},t)^\dagger$$

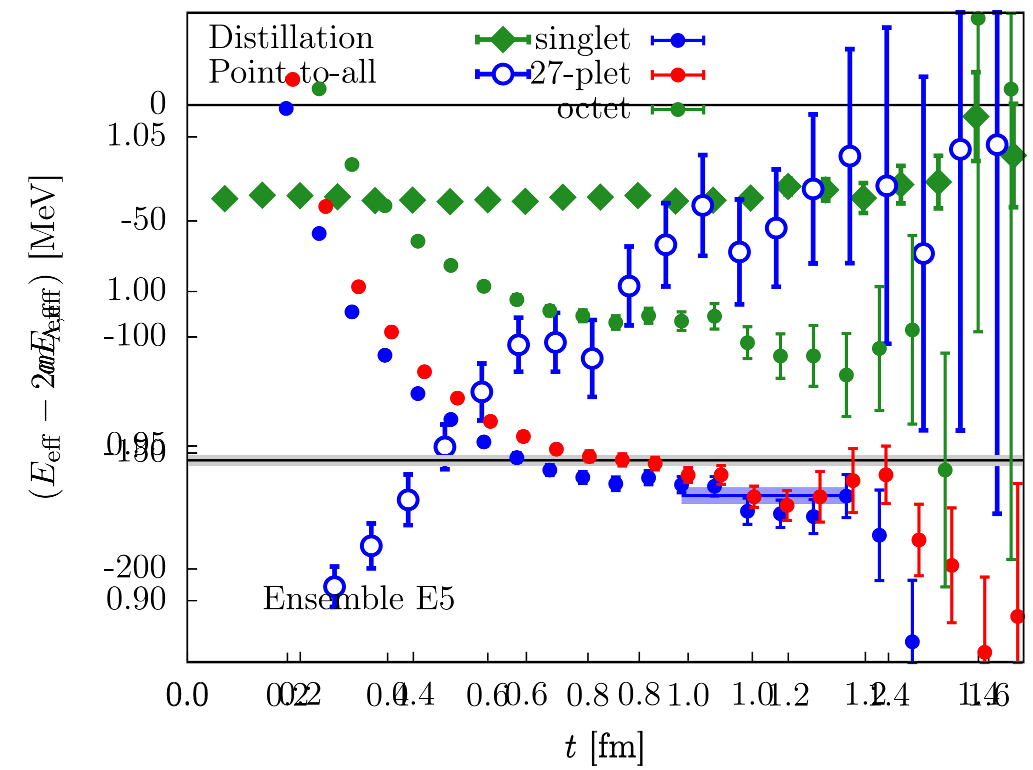
$V^{(k)}:\ k^{\rm th}$ eigenvector of Laplacian $\Delta$; has support on entire timeslice

# Finite-volume spectrum

* Compute timeslice-to-all propagators
  $\rightarrow$ **"distillation"** — Laplace-Heaviside (LapH) smearing
  *[Peardon et al., PRD 80 (2009) 054506; Morningstar et al., PRD 83 (2011) 114505]*




* Much better statistical signal

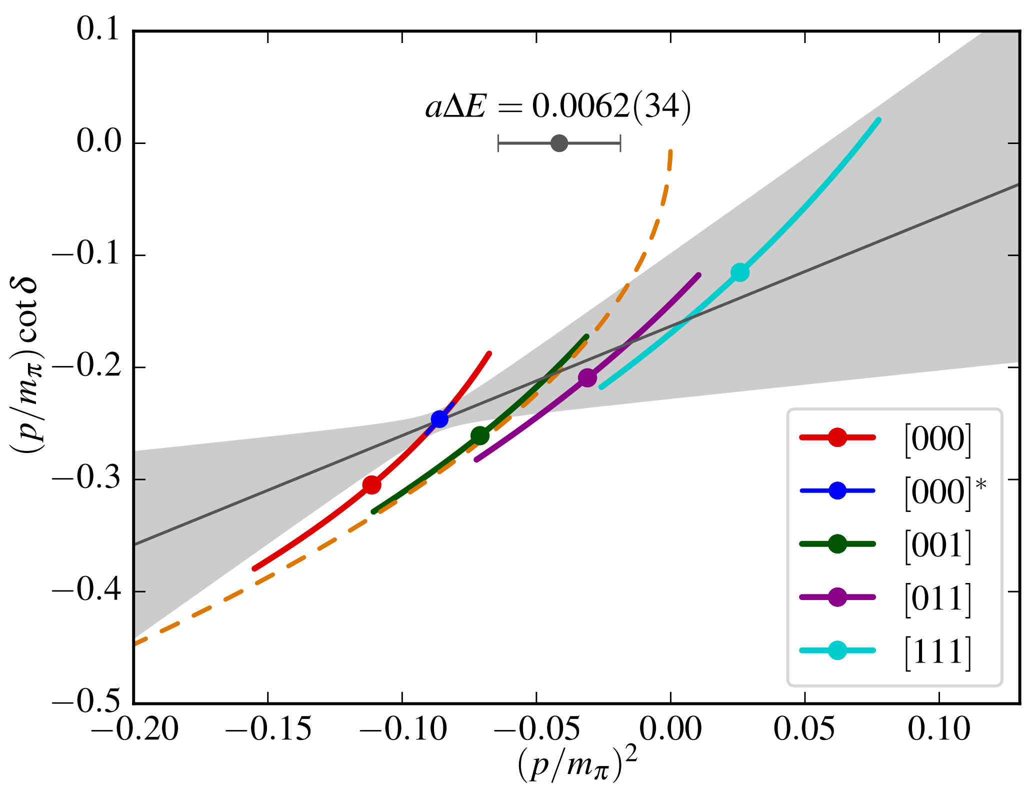
* Ensemble E5: broken SU(3)-flavour symmetry $(m_\pi = 450\,\mathrm{MeV})$

# Scattering phase shifts — Lüscher method

- Scattering momentum: $p^2 = \frac{1}{4}(E^2 - \boldsymbol{P}\cdot\boldsymbol{P}) - m_\Lambda^2$
- Scattering phase shifts: $p\cot\delta(p) = \frac{2}{\gamma L\sqrt{\pi}} Z_{00}(1,q^2), \quad q = \frac{pL}{2\pi}$

  $Z_{00}(1,q^2) = \frac{1}{\sqrt{4\pi}}\left\{\sum_{q^2\neq n^2}^{\Lambda_n}\frac{1}{q^2-n^2} - 4\pi\Lambda_n\right\}$
- Pole of the scattering amplitude: $\mathcal{A} \propto \frac{1}{p\cot\delta(p) - ip}$

  *[Lüscher 1990/91; Rummukainen & Gottlieb 1995]*
- Fit to effective range expansion: $p\cot\delta_0(p) = A + Bp^2 + \ldots \overset{!}{=} -\sqrt{-p^2}$

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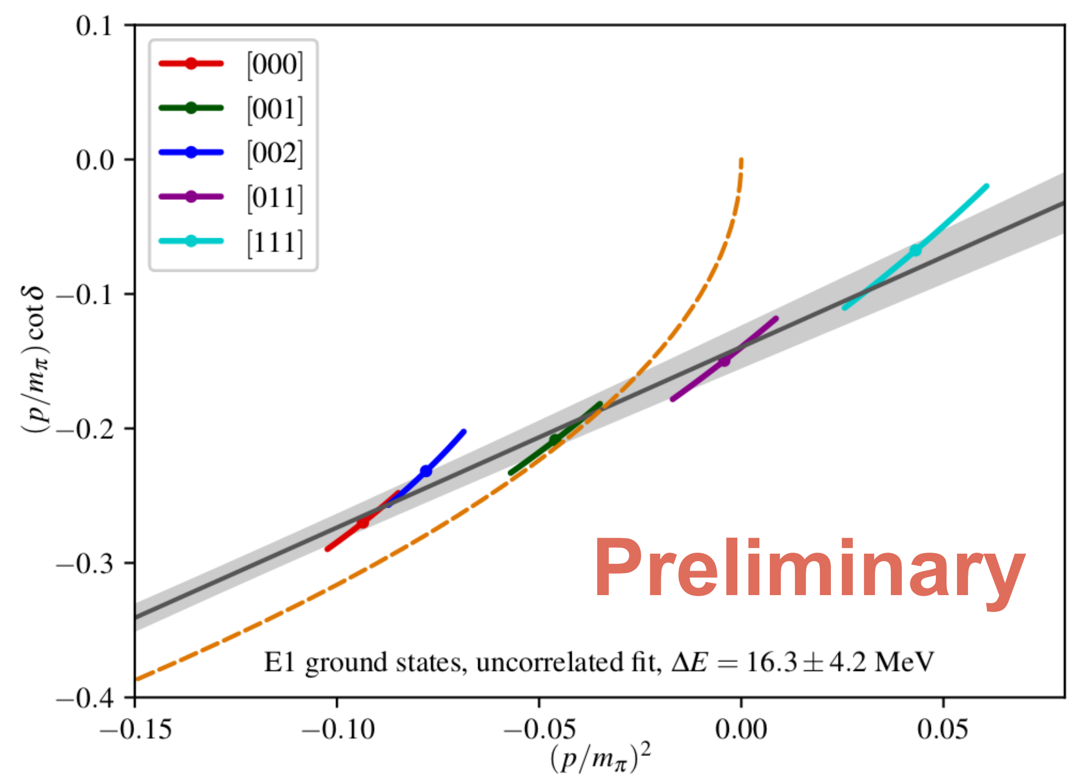
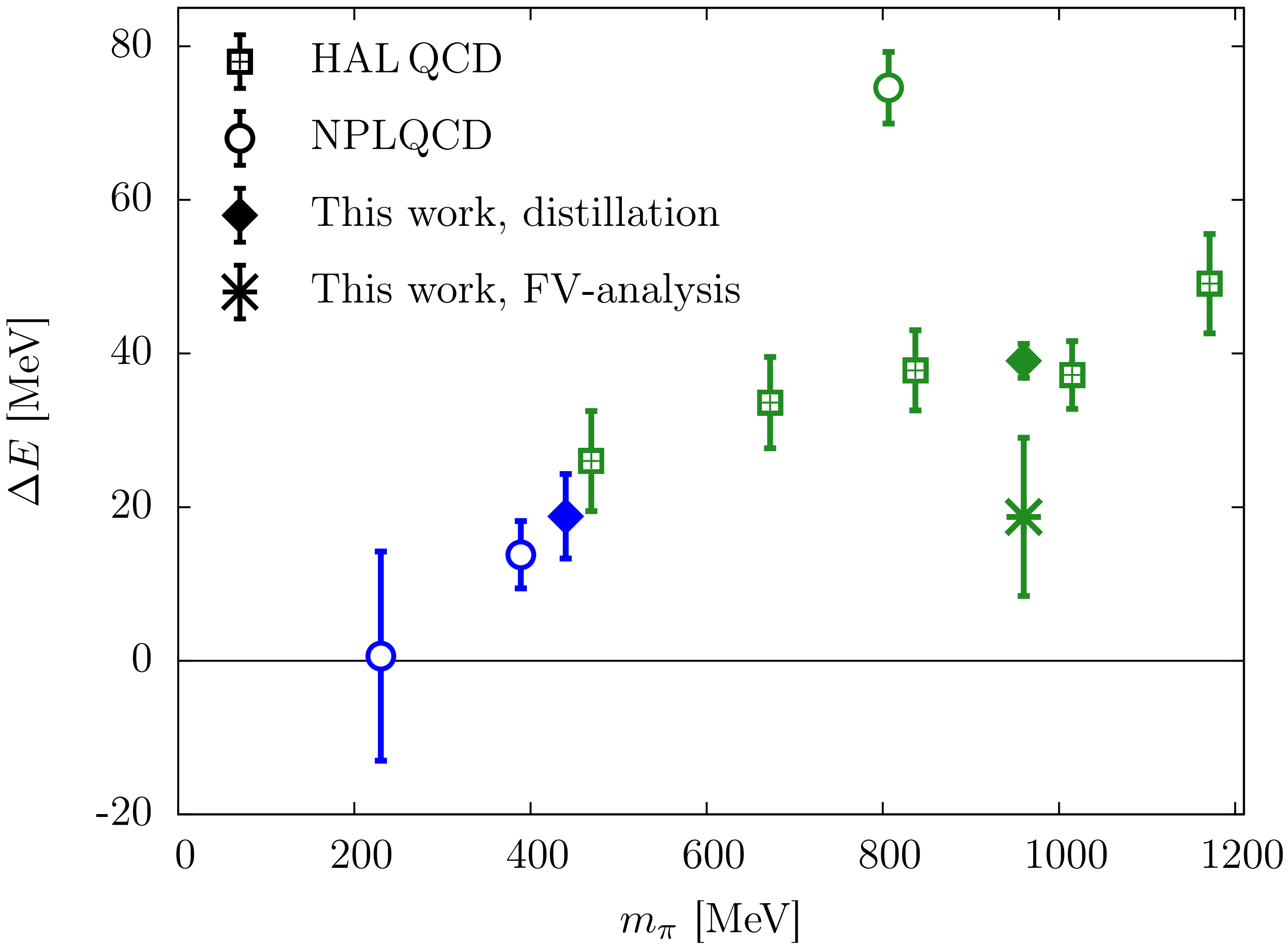


# Comparison with other calculations

- **NPLQCD Collaboration**: point-to-all propagators
- **HAL QCD Collaboration**: energy levels from lattice calculation of NBS wavefunction $(N_f = 3)$




Binding energy from FV analysis
(SU(3)-symmetric case)

$$B_{\Lambda\Lambda} = 19 \pm 10\,\text{MeV}$$

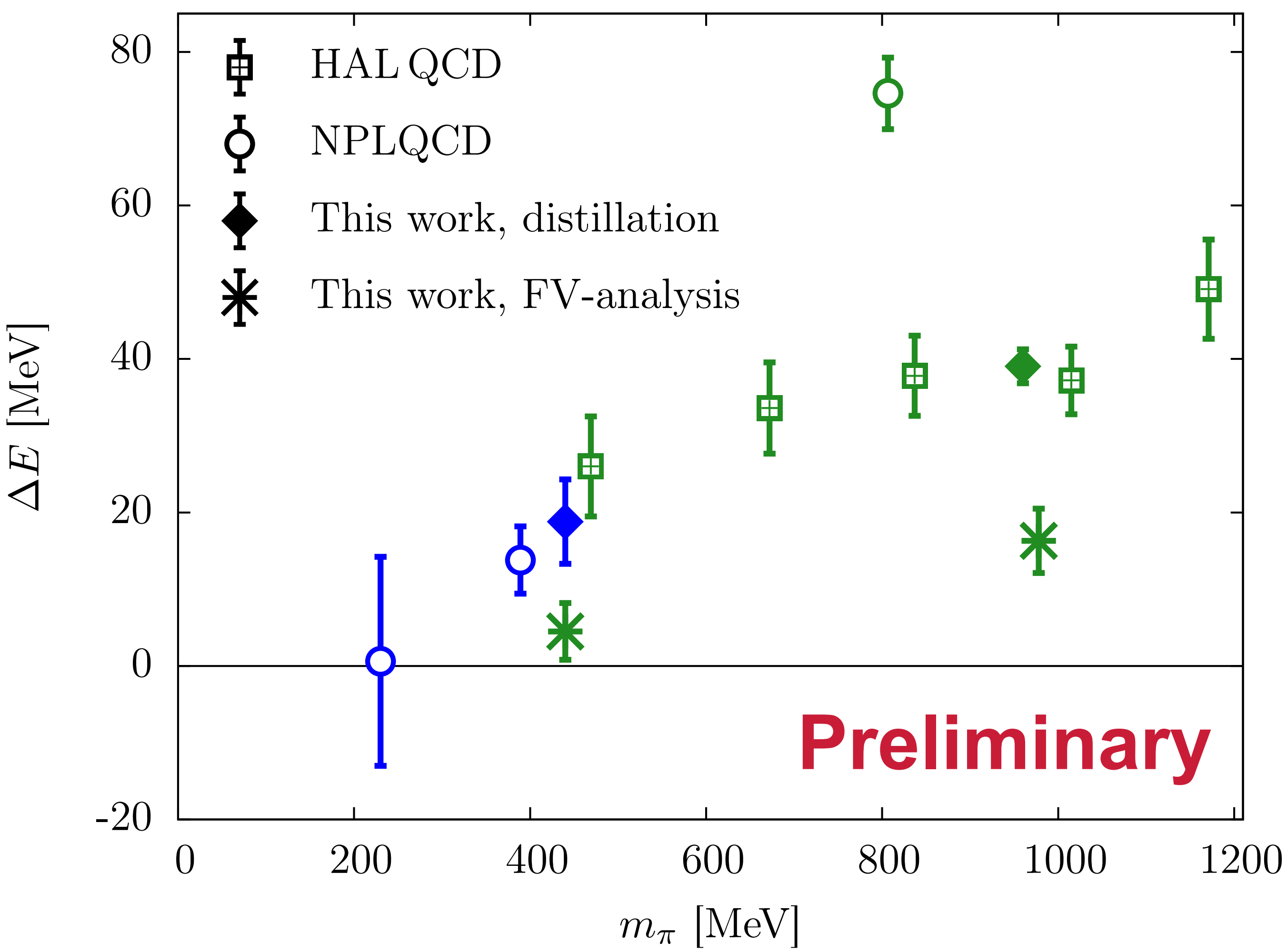
$$(m_\pi = 960\,\text{MeV})$$

(Green: SU(3)-symmetric; blue: SU(3)-broken)

*[Francis et al., PRD 99 (2019) 074505; Green et al., in prep.]*

# Comparison with other calculations

* **NPLQCD Collaboration**: point-to-all propagators
* **HAL QCD Collaboration**: energy levels from lattice calculation of NBS wavefunction ($N_f = 3$)




(Green: SU(3)-symmetric; blue: SU(3)-broken)

Binding energy from FV analysis (SU(3)-symmetric case)

$$B_{\Lambda\Lambda} = 16.3 \pm 4.2\,\text{MeV}$$
$$(m_\pi = 960\,\text{MeV})$$

$$B_{\Lambda\Lambda} = 4.5 \pm 3.7\,\text{MeV}$$
$$(m_\pi = 450\,\text{MeV})$$

*[Francis et al., PRD 99 (2019) 074505; Green et al., in prep.]*

# Higher spin states

* So far: focus on $J^P = 0^+$ and $S = -2$
* Extend calculation to higher spins $\rightarrow$ include additional irreps.
* Spin-1 interpolating operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk}\left(s^i C\gamma_5 P_+ t^j\right) r_\alpha^k$$

$$(BB)_i(\boldsymbol{p}_1, \boldsymbol{p}_2) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}} B_1(\boldsymbol{x}, t)\,(C\gamma_i P_+) \sum_{\boldsymbol{y}} e^{-i\boldsymbol{p}_2\cdot\boldsymbol{y}} B_2(\boldsymbol{y}, t)$$

$\Rightarrow$ Deuteron: $$(BB)^{(n)}_{i;T_1^+} = \frac{1}{N} \sum_{\boldsymbol{p};\, p^2=n} (BB)_i(-\boldsymbol{p}, \boldsymbol{p})$$

* Study *H* dibaryon and additional states in QCD with $N_f = 2+1$
* Move toward physical pion mass

# Gauge ensembles with $N_f = 2+1$

* $O(a)$ improved Wilson fermions — CLS effort
* SU(3)-symmetric point: $m_\pi = m_K \approx 420\,\text{MeV}$

| Label | $L^3 \times T$ | $a$ [fm] | $m_\pi$ [MeV] | |
|---|---|---|---|---|
| U103 | $24^3 \times 128$ | 0.0865 | 420 | SU(3)-symmetric |
| H101 | $32^3 \times 96$ | 0.0865 | 420 | SU(3)-symmetric |
| B450 | $32^3 \times 64$ | 0.0765 | 415 | SU(3)-symmetric |
| U102 | $24^3 \times 128$ | 0.0865 | 350 | SU(3)-broken |
| D200 | $64^3 \times 128$ | 0.0644 | 200 | SU(3)-broken |

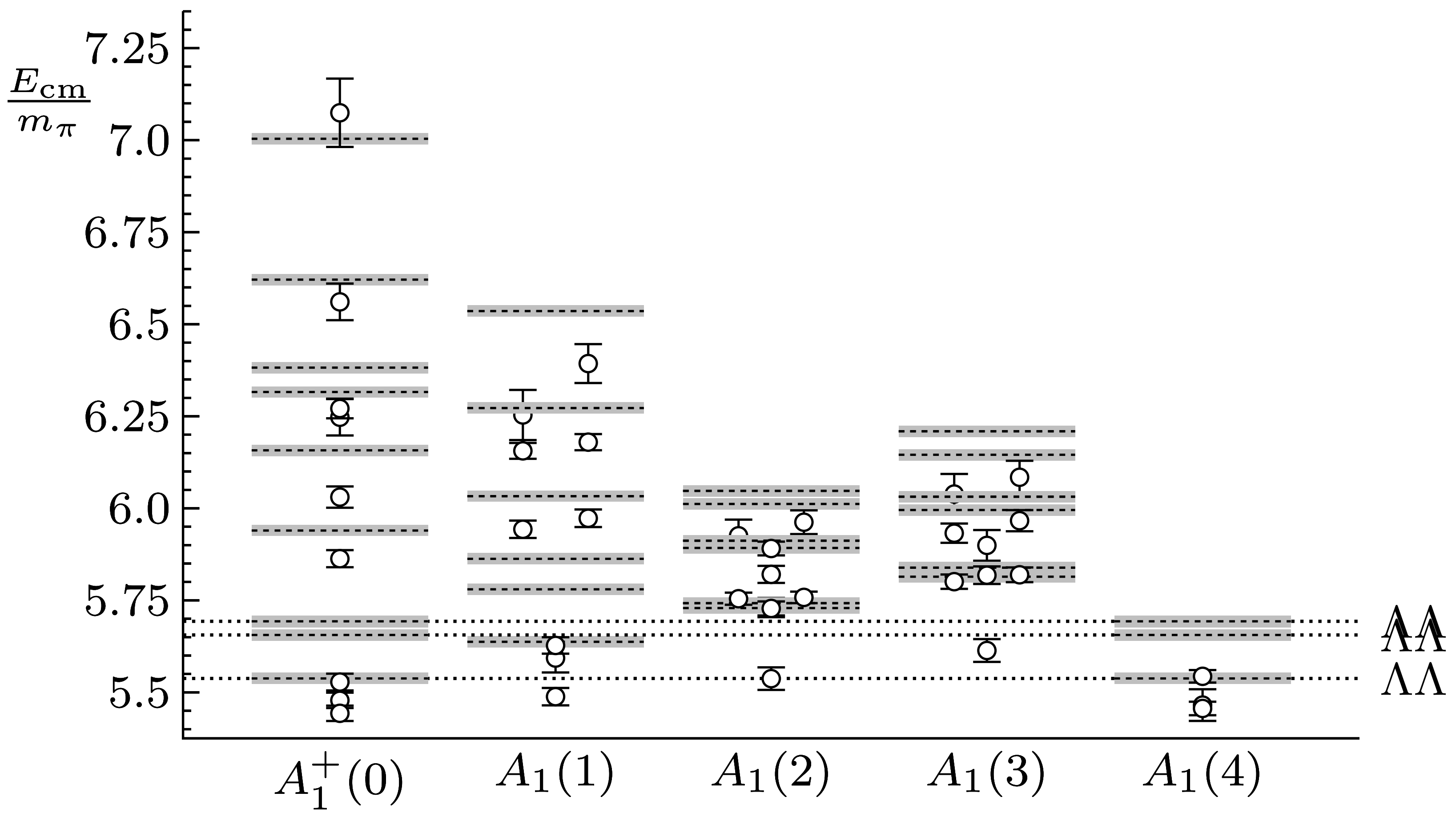
* Approach physical point along chiral trajectory defined by

$$\mathrm{Tr}\, M_q = \text{const.} \quad \Leftrightarrow \quad \tfrac{1}{2} m_\pi^2 + m_K^2 \approx \text{const.}$$

* Compute spectrum using distillation and GEVP

# Preliminary results for $N_f = 2+1$

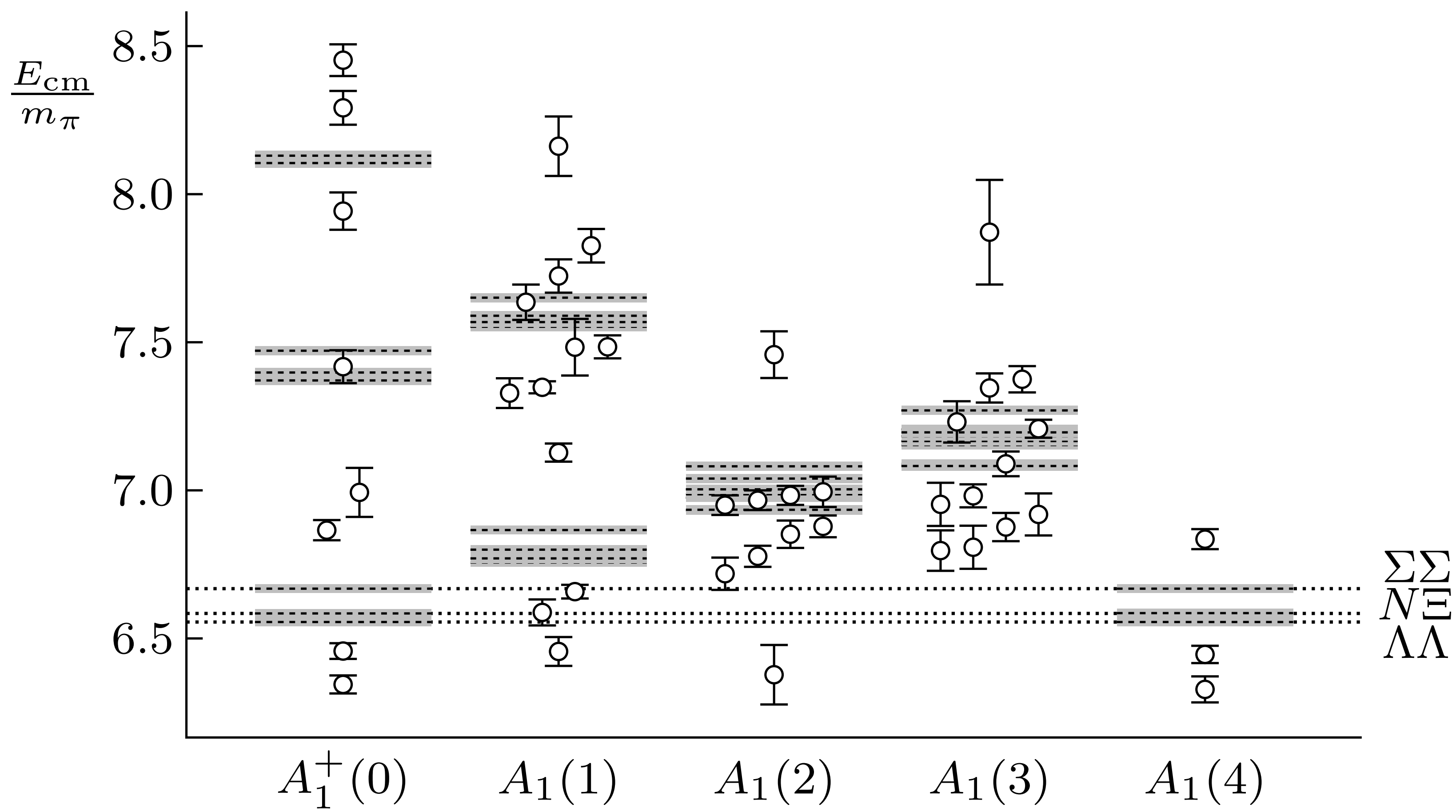
- Singlet channel, spin-0, SU(3)-symmetric
- Finite-volume energy levels in $A_1$ irrep. in different frames




**B450 / H101 ensemble:** $m_\pi = m_K = 420 / 415$ MeV, $L = 2.45 / 2.77$ fm, $a = 0.0765 / 0.0865$ fm

# Preliminary results for $N_f = 2+1$

* Singlet channel, spin-0, SU(3)-**broken**
* Finite-volume energy levels in $A_1$ irrep. in different frames



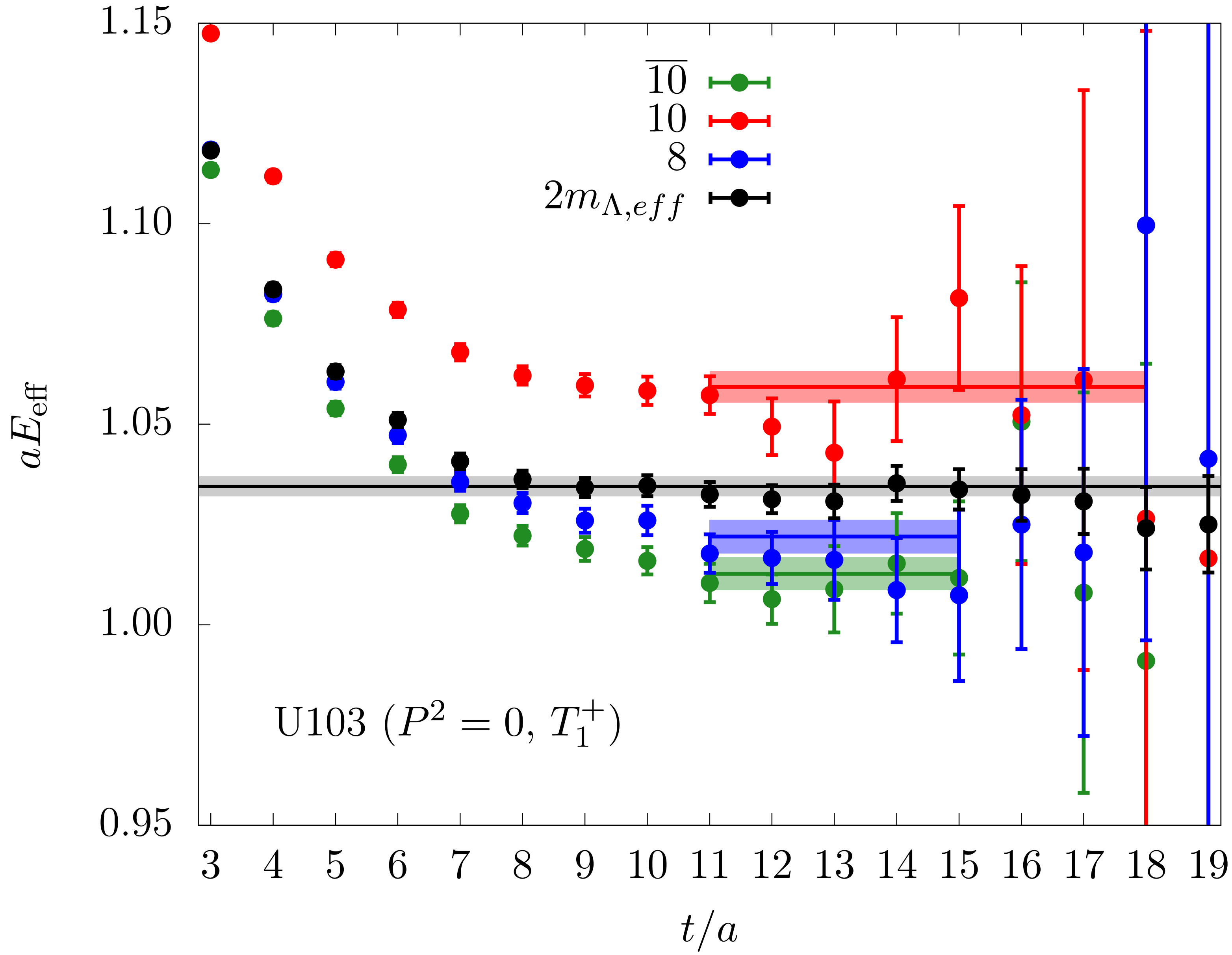

* Excellent resolution of excitation spectrum

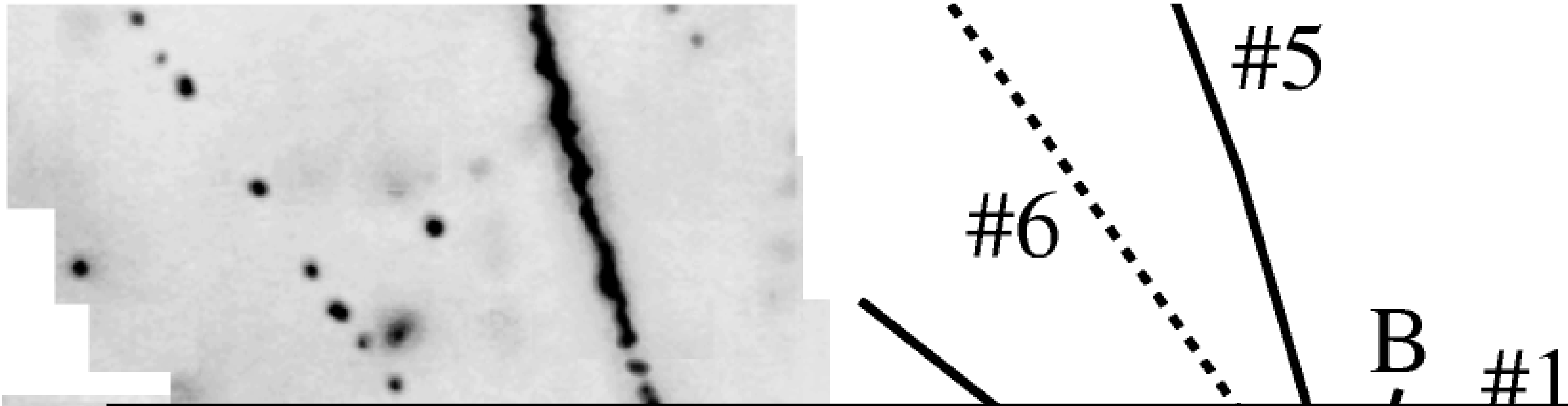
U102 ensemble: $m_\pi = 350$ MeV, $m_K = 450$ MeV, $L = 2.08$ fm, $a = 0.0865$ fm

* Interpretation of the energy levels very involved

# Preliminary results for $N_f = 2+1$

* Is the deuteron bound at $m_\pi = m_K \approx 420\,\mathrm{MeV}$ ?






# Summary

- **Distillation** & **GEVP**: powerful method to determine energy levels in a finite volume for a wide range of dibaryon channels
- Lüscher's finite-volume quantisation condition: rigorous formalism to study hadron-hadron interactions on the lattice
- ***H* dibaryon**: Binding energies extracted from Lüscher formalism significantly lower than the naïve energy difference $E_{\Lambda\Lambda} - 2m_\Lambda$
- SU(3)-symmetric point: $B_{\Lambda\Lambda} = 5 - 20\,\text{MeV}, \qquad m_\pi \geq 450\,\text{MeV}$

  $\Rightarrow$ significantly lower than Jaffe's bag model estimate

**Next steps:**

- Investigate SU(3)-breaking
- Compute binding energies as the quark masses are tuned towards the physical situation