August 5-7, 2019

Lectures in Mainz, Baryogenesis / Leptogenesis

Outline

Some history

Before 1930 : no antimatter - no problem. The only known elementary particles were protons, neutrons, electrons and photons

Dirac: equations for spin $\frac{1}{2}$ particle which predicted the existence of "electron" with positive electric charge - positron

Dirac's Nobel Prize Lecture, 1933:

"If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regards it rather as an accident that the Earth (and presumably the whole solar system), contains a predominance of electrons and positive protons. It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods."

Detection of antimatter

Dirac was wrong: cosmological observations do not support the hypothesis that the distant stars and galaxies may consist of antimatter.

There are several methods for detection of antimatter:

- The search of antinuclei in cosmic rays: the probability of the process

$$pp \to antinuclei + etc$$

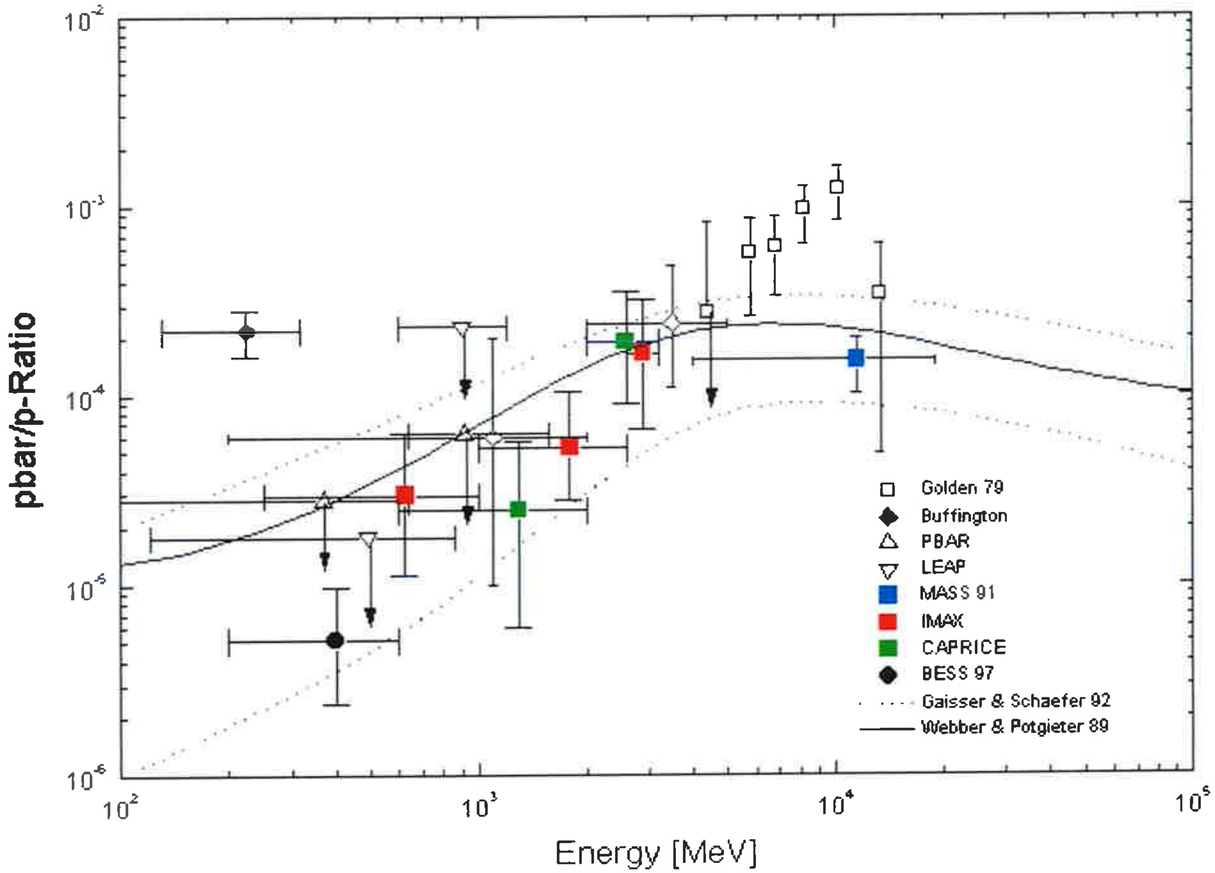
is very small! Result: no antinuclei in cosmic rays have been found. However, antiprotons have been observed

$$pp \to ppp + \bar{p} + etc$$

Detection of antimatter

(5)

The antiproton/proton ratio from AMS experiment:



Detection of antimatter

- Annihilation: particles and antiparticles annihilate:

$$
\begin{array}{ccccccccc}
 & & p\bar{p} & \rightarrow & \pi^+ & \pi^- & \pi^0 & \rightarrow & \gamma\gamma \\
 & & \nu_\mu\mu^+ & \swarrow & & \searrow & \mu^-\bar{\nu}_\mu & & \\
e^+\nu_e\bar{\nu}_\mu & \swarrow & & & & & \searrow & e^-\bar{\nu}_e\nu_\mu &
\end{array}
$$

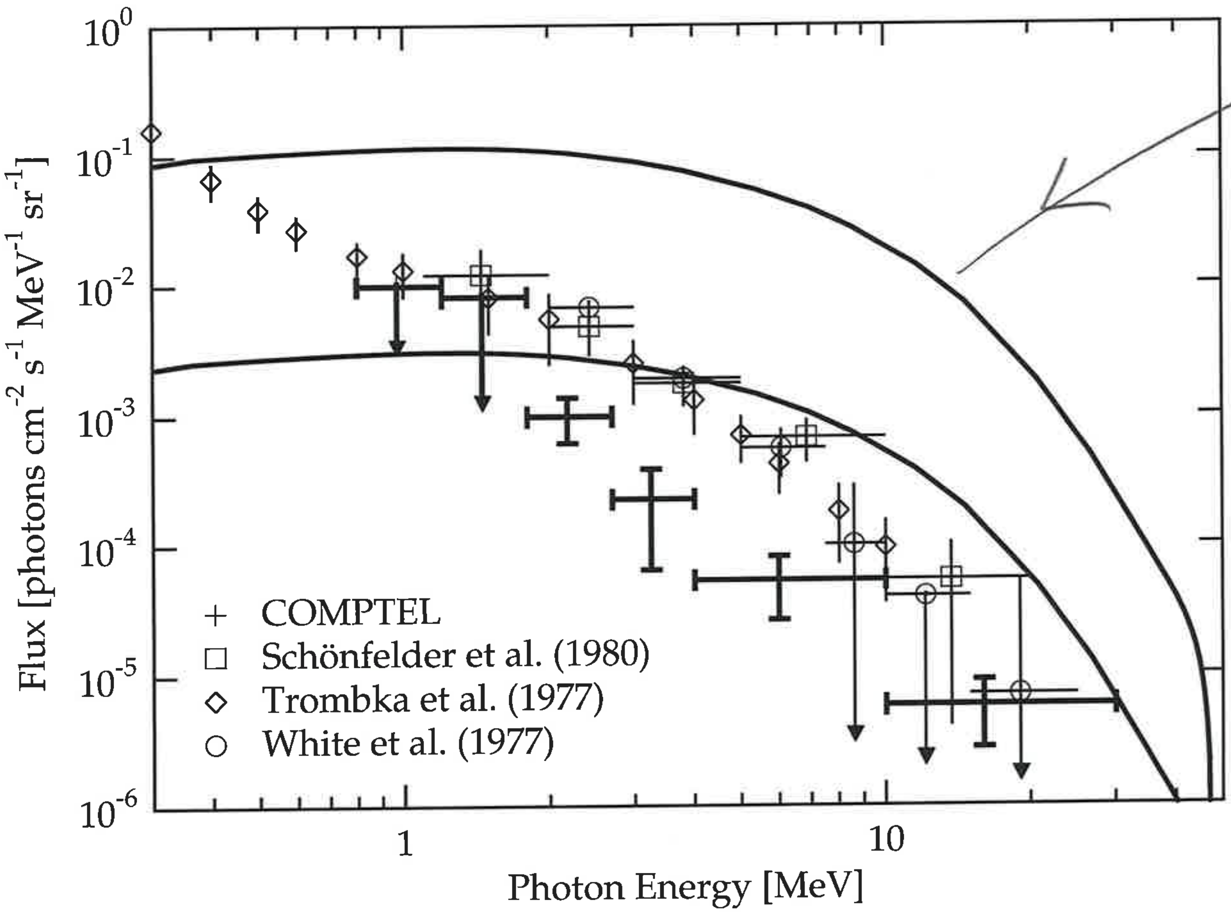
Detection of γ-rays?

$$\langle E_\gamma \rangle \sim (2GeV)/(5-6)/2 \sim 150MeV$$

However, this has not been observed!

Detection of antimatter

Data and expectations for the diffuse γ-ray spectrum (upper curve $d = 20$ Mpc, lower curve $d = 1000$ Mpc)



Cohen,
De Rújula
Glashow

Global baryon asymmetry

There are two possibilities:

- Observed universe is asymmetric and does not contain any antimatter
- The universe consists of domains of matter and antimatter separated by voids to prevent annihilation. The size of these zones should be greater than 1000 Mps, in order not to contradict observations of the diffuse γ spectrum.

The second option, however, contradicts to the large scale isotropy of the cosmic microwave background.

Thus, we are facing the question: Why the universe is globally asymmetric?

Back to particle physics:

Matter-antimatter asymmetry in particle physics

Till 1956: general believe that the nature is symmetric with respect to change of particle into antiparticle.

C - charge conjugation: $p \leftrightarrow \bar{p}, \quad n \leftrightarrow \bar{n}, \quad e^- \leftrightarrow e^+$

P - parity transformation: $\vec{x} \to -\vec{x}$, $\vec{p} \to -\vec{p}$, but $\vec{m} \to +\vec{m}$

1956: discovery of P and C breaking in weak interactions (Lee, Yang). Many manifestations, e.g. in $\pi^{\pm}$ decays. In particular, C-transformation change left-handed neutrino into left-handed antineutrino, which does not exist.

Conclusion: properties of particles and antiparticles are in fact (somewhat) different.

Matter-antimatter asymmetry in particle physics

Landau

However, the combined CP symmetry was believed to be exact:

change particles to antiparticles AND simultaneously their momenta.

Now, CP works for neutrino

$$CP: \quad \nu(\vec{v}) \longrightarrow \bar{\nu}(-\vec{v})$$

So, still no solution for the problem of baryon asymmetry of the universe!

Matter-antimatter asymmetry in particle physics

1964 (Cronin, Fitch, Christenson, Turlay): decays of K^0 mesons.

In a small fraction of cases ($\sim 10^{-3}$), long-lived K_L (a mixture of K^0 and $\bar{K}^0$ decays into pair of two pions, what is forbidden by CP-conservation.

There are other manifestations of CP breaking. For example, if CP were exact symmetry, an equal number of K^0 and $\bar{K}^0$ would produce an equal number of electrons and positrons in the reaction

$$K^0 \to \pi^- e^+ \nu_e, \quad \bar{K}^0 \to \pi^+ e^- \bar{\nu}_e,$$

Matter-antimatter asymmetry in particle physics

However, this is not the case: the number of positrons is somewhat larger ($\sim \mathbf{10^{-3}}$) than the number of electrons.

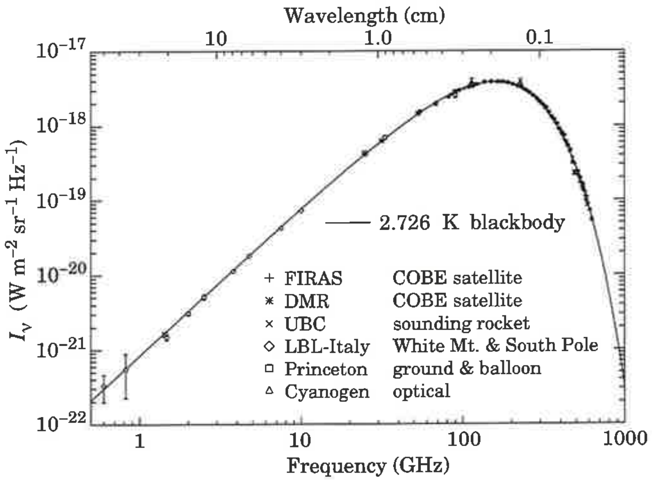
Conclusion: so, there is indeed a tiny difference between particles and antiparticles, on the level of $\mathbf{10^{-3}}$

How can this very small distinction be transformed in the 100% asymmetry of the universe we observe today?

Big Bang and baryon asymmetry

Cosmic microwave background

In 1965, Penzias and Wilson observed radio-waves in sub-millimeter range which were coming from all directions of the sky. They have a spectrum of black-body radiation with temperature $\mathbf{2.73^0}$.



Big Bang and baryon asymmetry

Big Bang theory: Friedmann, Lemaitre, Gamov



Big Bang and baryon asymmetry

Why do we care?

The existence of CMB is a proof of the Big Bang theory: universe expands and it was hot and dense in the past:

$$t = \frac{M_0}{T^2}, \quad t[sec] \sim \frac{1}{T^2[MeV]}, \quad M_0 = \frac{M_{Pl}}{1.66 N^{1/2}}$$

Let us compute the baryon asymmetry of the universe

$$\Delta(t) = \frac{n_B - \bar{n}_B}{n_B + \bar{n}_B}$$

as a function of time. We already know that $\Delta(10^{10} years) = 1$.

Big Bang and baryon asymmetry

Another interesting point: $t_0 \sim 10^{-6}$s (or $T \sim 1GeV \sim m_p$).

Thermodynamics: $n_B \sim \bar{n}_B \sim n_\gamma$

$$\Delta(t_0) = \frac{n_B - \bar{n}_B}{n_\gamma}$$

This ratio is in fact almost time-independent!

Baryon number is conserved

$$(n_B - n_{\bar{B}})R^3 = const,$$

where R is the scale factor, and the temperature of the CMB scales as $T \sim 1/R \quad \Longrightarrow \quad n_\gamma R^3 = const.$

Big Bang and baryon asymmetry

So, to find $\mathbf{\Delta}(t_0)$ just take n_B/n_γ today.

We have

- $n_\gamma \simeq$ 410 photons/cm^3 (corresponds to temperature 2.73^0K)
- $n_B \simeq$ 0.25 nucleons/m^3

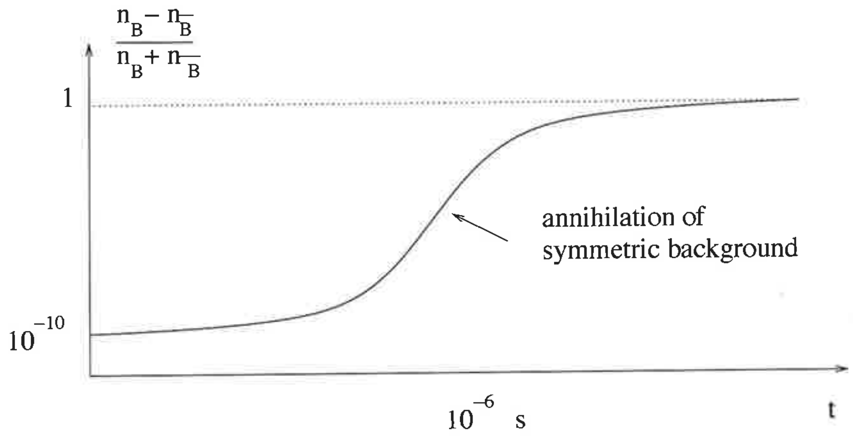
$$\Longrightarrow n_B/n_\gamma \simeq 6 \times 10^{-10}$$

Conclusion: Big Bang theory tells that the baryon asymmetry of the early universe is a very small number

$$\frac{(n_B - \bar{n}_B)}{(n_B + \bar{n}_B)} \sim 10^{-10}$$

Big Bang and baryon asymmetry

At $t \sim 10^{-6}$s after the big Bang for every 10^{10} quarks we have $(10^{10} - 1)$ antiquarks. Somewhat later the symmetric background annihilates into photons and neutrinos while the asymmetric part survives and gives rise to galaxies, stars, planets.



Dependence of baryon asymmetry on time

Big Bang and baryon asymmetry

Problems to solve:

- Why in the early universe the number of baryons is greater than the number of antibaryons
- How to compute the primordial baryon asymmetry? (from observations, $\sim \mathbf{10^{-10}}$)

Now, the comparison between of the baryon asymmetry in the early universe and the measure of the difference between particles and antiparticles in $\boldsymbol{K^0}$ decays $(\mathbf{10^{-3}})$ is much more comfortable!

(20)

Andrei Sakharov proposal, 1967:

"According to our hypothesis, the occurrence of C-asymmetry is the consequence of violation of CP-invariance in the nonstationary expansion of the hot universe during the superdense stage, as manifest in the difference between the partial probabilities of the charge-conjugate reactions."

In short: the universe is asymmetric because baryon number is not conserved in C- and CP-violating reactions which produce more baryons than antibaryons in expanding universe.

Consequence: Proton is not stable!

Sakharov model

Superheavy particles with mass $m \sim 10^{19}$ GeV = 10^{-5} grams, and lifetime 10^{-43} s

Decay modes:

$$X \to p\,p, \quad p\,e^+, \quad \bar{X} \to \bar{p}\,\bar{p}, \quad \bar{p}\,e^-$$

If C and CP are broken, X and $\bar{X}$ will produce different number of protons et antiprotons, as K^0 and $\bar{K}^0$ produce the different numbers of electrons and positrons.

It is sufficient to produce a very small asymmetry 10^{-10} which will then be converted into 100% asymmetry later on.

Sakharov model

Revolutionary solution at that time: there was a believe that the proton is absolutely stable!

Paradox: there is no antimatter in the universe since matter is unstable!

Qualitatively, universe is asymmetric due to

- baryon number non-conservation
- breaking of C and CP
- universe expansion

What kind of physics leads to

- B-violation?
- CP-violation?
- thermal nonequilibrium?

Formal derivation of Sakharov conditions:

Let $\hat{\rho}$ is the density matrix describing the Universe. Schrödinger picture:

$$i \frac{d\hat{\rho}}{dt} = [H, \hat{\rho}]$$

1. Baryon number: $B(t) = tr(\hat{B}\hat{\rho})$

If $[\hat{B}, \hat{H}] = 0$ (Baryon # conservation) and $B(t_0) = 0$, then $B(t) = 0$:

$$\frac{dB}{dt} = Tr\, \hat{B} \frac{d\hat{\rho}}{dt} = Tr\, \hat{B} \frac{1}{i} [H, \rho] =$$

$$= \frac{1}{i} Tr\, \rho [H, B] = 0$$

2. CP and C non-conservation

Take U_C as an example. If C is conserved, then $[U_C, H] = 0$. At $t = t_0$: $[\rho_0, U_C] = 0 \Rightarrow$

$[\rho(t), U_C] = 0 \Rightarrow$

$$B(t) = Tr\, \hat{B}\hat{\rho} = Tr\, \hat{B}\, U_C \hat{\rho}\, U_C^{+} =$$

$$= Tr\, U_C^{+} \hat{B}\, U_C\, \hat{\rho} = -Tr\, \hat{B}\rho = -B(t) \Rightarrow$$

$B(t) = 0.$

(3) Thermal equilibrium:

$$\rho = \exp\left[-\frac{H}{T}\right]$$

assume: no C or CP odd conserving charges.

$$\mathrm{Tr}\, B\rho = \mathrm{Tr}\, U_{CPT} B \rho U^{+}_{CPT} =$$

$$= -\mathrm{Tr}\, B\rho = 0, \text{ since } U_{CPT} H U^{+}_{CPT} = H$$

CPT theorem.

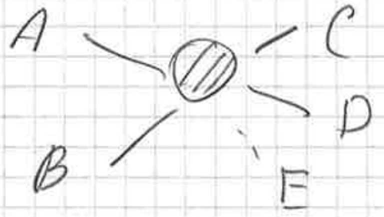
Addendum. equilibrium and non-equilibrium in expanding Universe

We never have exact equilibrium in expanding Universe: time-dependent metric.

Kinetic theory in expanding Universe.

Relatively simple case:

set of particles $A, B, C, D, \dots E$



1st collision

Second collision: 1st collision is "forgotten".

Example when this situation may not be realized:

neutrino oscillations;
after 1st collision a coherent state is formed $c_1|\nu_1\rangle + c_2|\nu_2\rangle$. The state evolves and oscillates like

$$\sin(E_1 - E_2)t \simeq \sin \frac{\Delta m^2}{E} \cdot t \equiv \sin \frac{t}{t_{osc}}$$

If the typical collision time t_{coll} is smaller than t_{osc}, $t_{coll} \lesssim t_{osc}$, Then the consequent collision „remembers" the time history.

For $t_{coll} \gg t_{osc}$ we are in the „Boltzmann" regime. The kinetic equations for particle distributions look like (homogeneous case, $n_i(p)$ are functions of momentum only):

$$\frac{\partial n_i}{\partial t} - H\vec{p}\,\frac{\partial n_i}{\partial \vec{p}} = I^i_{coll} \leftarrow \text{collision integral}$$

If $I_{coll} = 0$, then the solution of these equations gives the red shift of momenta:

$$n(\vec{p}, t) = n_0\left(\frac{\vec{p}\,R(t)}{R(t_0)}\right), \text{ where } R \text{ is}$$

the scale factor in FRW metric,

$$ds^2 = dt^2 - R^2(t)\,d\vec{x}^2$$

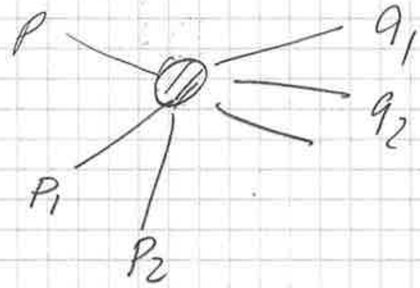
Spacially flat metric is taken for simplicity

Collision integral is written as follows: (Weinberg, 1979)

$$I_{coll} = \frac{1}{2p_0} \int D[\text{phase space}]\,(2\pi)^4 \delta^4\left(\sum p_{in} - \sum q_{out}\right)$$

$$\Big\{ |M(p \to q)|^2 n(p) n(p_1) \ldots n(p_2) (1 \pm n(q_1)) \ldots (1 \pm n(q_2)) -$$

$$|M(q \to p)|^2 n(q_1) \ldots n(q_2) (1 \pm n(p))(1 \pm n(p_1)) \ldots \Big\}$$



where + is for bosons

− is for fermions

$$D[\text{phase space}] = \frac{d^3 p_1}{(2\pi)^3 2p_1^0} \cdots$$

M is the relativistic matrix element of reaction

Important property of I_{coll}:

$$I_{coll} \Big|_{n \to \text{equilibrium functions}} = 0$$

Equation for number-densities of particles:

$$N(t) = \int \frac{d^3p}{(2\pi)^3}\, n(p)$$

$n(p)$: dimensionless, $n(p)\Big|_{eq} = \dfrac{1}{e^{\frac{\varepsilon \pm \mu}{T}} \pm 1}$

$N(t)$: dimension $1/cm^3 = GeV^3$

+ fermions

− bosons

ε - energy

μ - chemical potential

$$\frac{dN_i}{dt} + 3HN_i = \int \frac{d^3p}{(2\pi)^3} I_{coll}$$

If $I_{coll} = 0 \Rightarrow N_i \sim \dfrac{Const}{R^3(t)}$ − # of particles is conserved

For qualitative picture one can use the „relaxation time" approximation,

$$\int \frac{d^3p}{(2\pi)^3} I_{coll} = -\Gamma\,(N - N_{eq});$$

where $\Gamma = \langle \sigma n v \rangle$, σ − cross-section of reaction, or $\Gamma \sim \dfrac{1}{\tau}$ −

τ − life-time of the particle.

Approximate equation:

$$\frac{dN}{dt} + 3HN = -\Gamma(N - N_{eq})$$

Two limits :

(i) $\Gamma \gg H$ – the system is in thermal equilibrium,

and the solution is $N \approx N_{eq}$

(ii) $\Gamma \ll H$ – the system is out of thermal equilibrium

and, for stable particles,

$$N \approx \frac{Const}{R^3(t)}$$

Matching of two extremes ; at

$\Gamma \approx H$ („freeze in" or „freeze out")

$$\frac{Const}{R^3(t_0)} \approx N_{eq}(t_0)$$

Example of typical behaviour:

plasma with e^+, e^-, and γ.

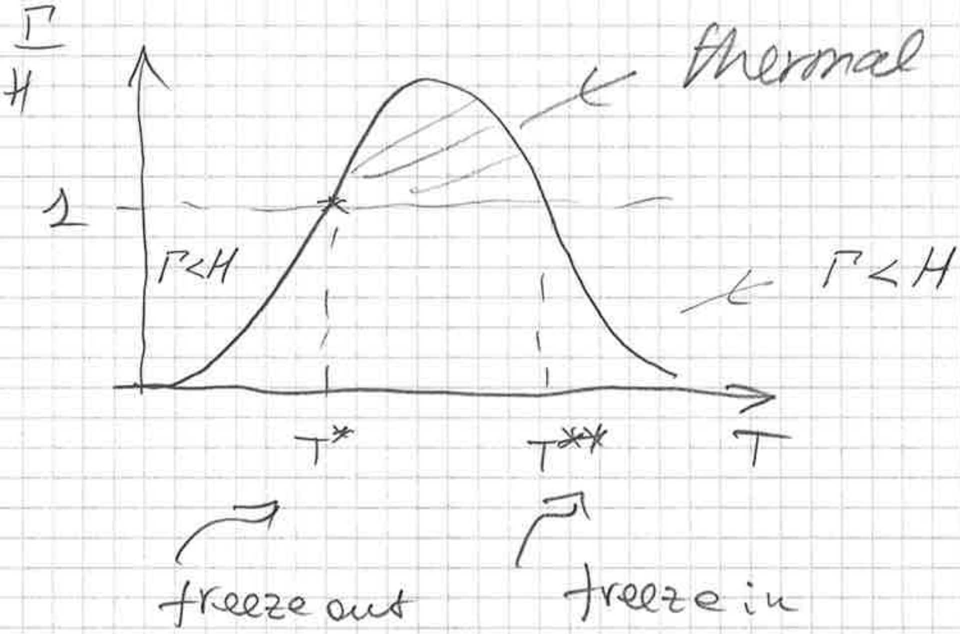
Follow concentration of electrons

Process: $e^+e^- \Leftrightarrow \gamma\gamma$

$$\sigma \sim \begin{cases} \frac{1}{v}\frac{\alpha^2}{m_e^2}, & v \ll 1 \\ \frac{\alpha^2}{\varepsilon}, & v \gg 1 \end{cases}$$

$$\Gamma \approx \sigma n v \approx \begin{cases} \frac{\alpha^2}{m_e^2} \cdot \left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}}, & T \lesssim m_e \\ \alpha^2 T, & T \gtrsim m_e \end{cases}$$

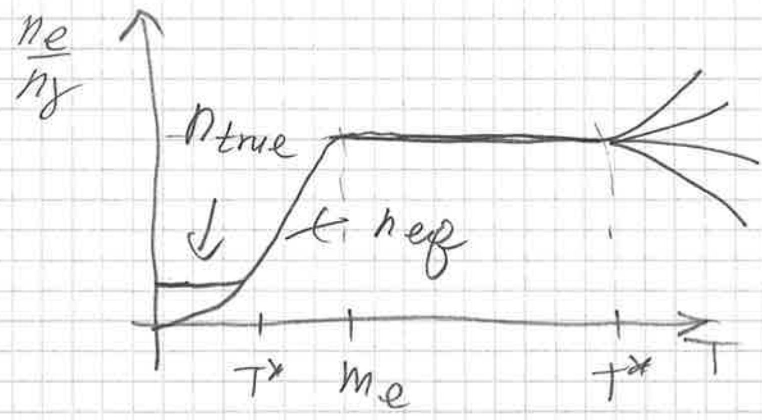
$H = \frac{T^2}{M_0}$, $M_0 \simeq \frac{M_{pl}}{1.66 N_{eff}^{1/2}}$ – Hubble rate



$T^{**} \simeq \alpha^2 M_0$

$T^* \approx \frac{m_e}{43}$

Behaviour of electron concentration

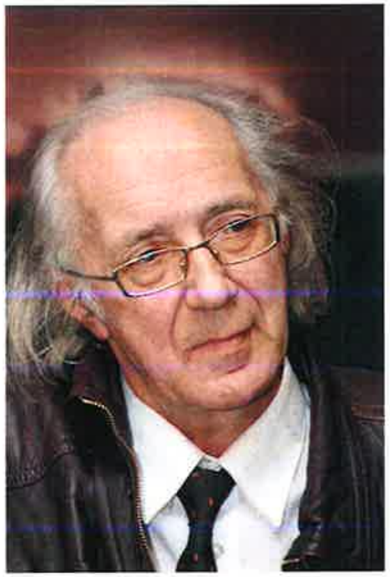


$$n_{true} \simeq \underbrace{\left(\frac{mT_*}{2\pi}\right)^{3/2} e^{-m/T^*}}_{\text{concentration at } T^*} \left(\frac{R(T^*)}{R(T)}\right)^3$$

Universe expansion

Sakharov work of 1967 “Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe ” was unnoticed for next 11 years

25



- Kuzmin, 1970: CP violation and baryon asymmetry of the universe
- change of attitude: 1978
- Ignatiev, Krasnikov, Kuzmin, Tavkhelidze, 1978: Universal CP Noninvariant Superweak Interaction and Baryon Asymmetry of the Universe
- Yoshimura, 1978: Unified Gauge Theories and the Baryon Number of the Universe

Yoshimura paper was wrong:

- He got baryon asymmetry in thermal equilibtium
- He got baryon asymmetry in minimal SU(5) GUT (not enough CP violation)

However, it largely increased an interest to this problem: everybody wrote a paper on this subject!

Literature:

- Sakharov 1967
- Kuzmin 1970
- Ignatiev et al 1977, 1978
- Yoshimura, 1978
- Weinberg, 1979: importance of inverse decays of particles, kinetic equations for GUT baryogenesis
- Kuzmin, Rubakov, MS, 1985: baryon number non-conservation in the SM at high temperatures
- Affleck-Dine baryogenesis, 1985
- Fukugita, Yanagida 1986: thermal leptogenesis
- MS, 1986: out of equilibrium condition for electroweak baryogenesis, measure of CP-violation for baryogenesis in the SM
- Cohen, Kaplan & Nelson, 1991: Electroweak baryogenesis in 1st order phase transition, because of domain walls of bubbles
- Akhmedov, Rubakov, Smirnov 1998 – low scale leptogenesis

- Asaka, MS 2005 – Kinetic theory of low scale leptogenesis
- MS 2008 – relation of leptogenesis to Dark Matter

Many technical advances in all types of baryogenesis / leptogenesis during last 10 years.

Review articles:

Leptogenesis, Davidson, Nardi & Nir, 2008

Anomalous baryon number non-conservation, Rubakov, MS, 1996

Electroweak baryogenesis, Cohen, Kaplan & Nelson, 1993; MS, 1998; Morrissey & Ramsey-Musolf, 2012

GUT, EW & Affleck-Dine Baryogenesis, Riotto, Trodden, 1999

Low scale leptogenesis, Boyarsky, Ruchayskiy, MS, 2009

Canetti, Drewes, MS 2012

Most recent review:

Garbrecht et al, 6 articles in Int. J. of Modern Physics, 2018

Anomalous fermion number non-conservation

Essential ingradient of baryogenesis:

baryon # non-conservation. The only source of B- non-conservation we are absolutely sure about is that associated with EW interactions.

Perturbative symmetries of the SM:

B, L_e, L_μ, L_τ. Only 3 of them survive if quantum effects are taken into account:

$L_e - \frac{1}{3}B$, $L_\mu - \frac{1}{3}B$ & $L_\tau - \frac{1}{3}B$.

The simplest theory which allows to explain this effect is 2D Abelian Higgs model.

Plan: - vacuum structure of Abelian theory in 2D

- fermionic level crossing
- chiral anomaly
- sphaleron rate
- Generalisation to the SM

Toy model for anomalous fermion number non-conservation

Simplest example:

Electrodynamics in $\underset{\text{space}}{1} + \underset{\text{time}}{1}$ dimensions

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu \varphi|^2 - U(\varphi)$$

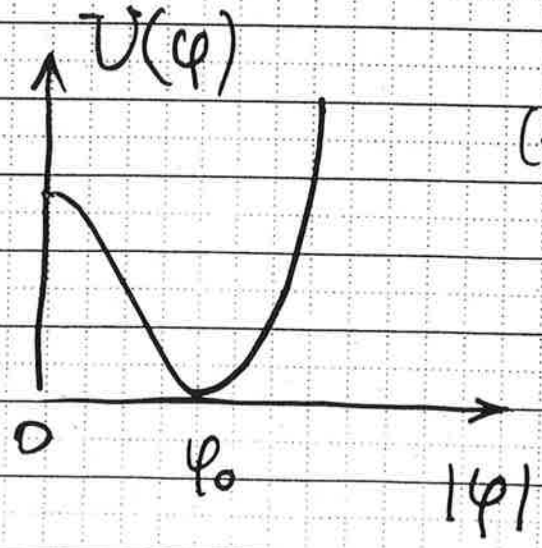
$$+ \bar{\psi}_L^{(1)} \not{D} \psi_L^{(1)} + \bar{\psi}_R^{(2)} \not{D} \psi_R^{(2)}$$

Fermionic numbers:

$F_1 = +1$; $F_2 = -1$

Fermionic current:

$$J_\mu^F = \bar{\psi}_L^{(1)} \gamma_\mu \psi_L^{(1)} - \bar{\psi}_R^{(2)} \gamma_\mu \psi_R^{(2)}$$

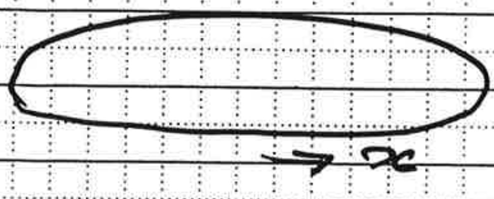


$\partial_\mu J_\mu^F = 0$?

(follows from $\mathcal{L}$)

Higgs phase

Complicated vacuum structure



Space $\equiv S_1$

Electrodynamics – gauge theory $\Rightarrow$ fix the gauge:

$A_0 = 0, \quad \partial_1 A_1 = 0$

$A_1 = \text{const}$ satisfies gauge condition

Vacuum states:

$$\varphi = \exp\left\{\frac{2\pi i n}{L} x\right\} \varphi_0$$

$$A_1 = \frac{1}{e} \cdot \frac{2\pi n}{L} \qquad n = \text{integer}$$

n – degree of mapping of space into $U(1)$ group

Quantum physics: tunnelling between differen "n" states $\Rightarrow$

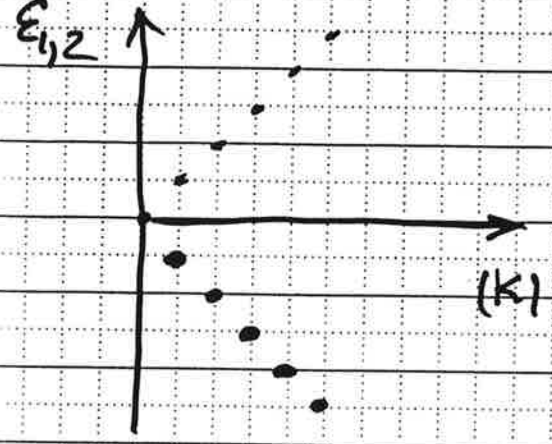
$$|Vac\rangle = \sum_n e^{in\theta} |n\rangle \qquad \text{instantons}$$

famous θ - angle

fermionic energy levels:

for $A_1 = 0$:

$\mathcal{E}_1 = \pm |k|$ $\qquad$ $\mathcal{E}_2 = \pm |k|$



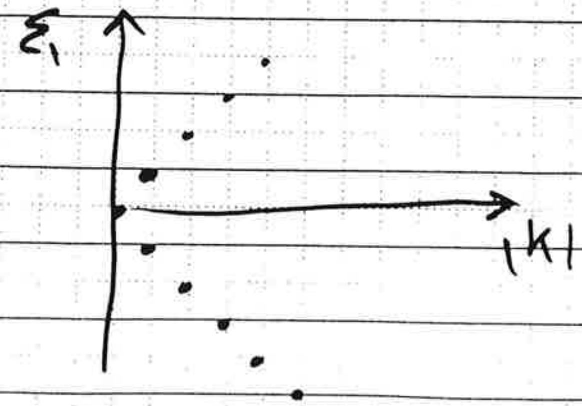
$k = \frac{2\pi l}{L}$ (periodic boundary conditions)

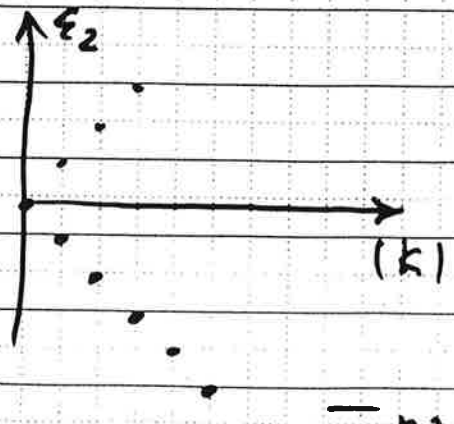
vacuum state:
all negative fermionic energy levels are occupied
(Dirac see)

for $A_1 \neq 0$:

$\mathcal{E}_1 = \pm |k| + eA_1$ $\qquad$ $\mathcal{E}_2 = \pm |k| - eA_1$

Example: A_1 changes from 0 to $n = 2$





creation of 2 $\psi_1^{(L)}$ $\qquad$ creation of 2 $\bar{\psi}_2^{(R)}$

Net fermion number:

$$\Delta F = 2 \cdot \frac{e}{2\pi} \cdot L \cdot \Delta A_1 =$$

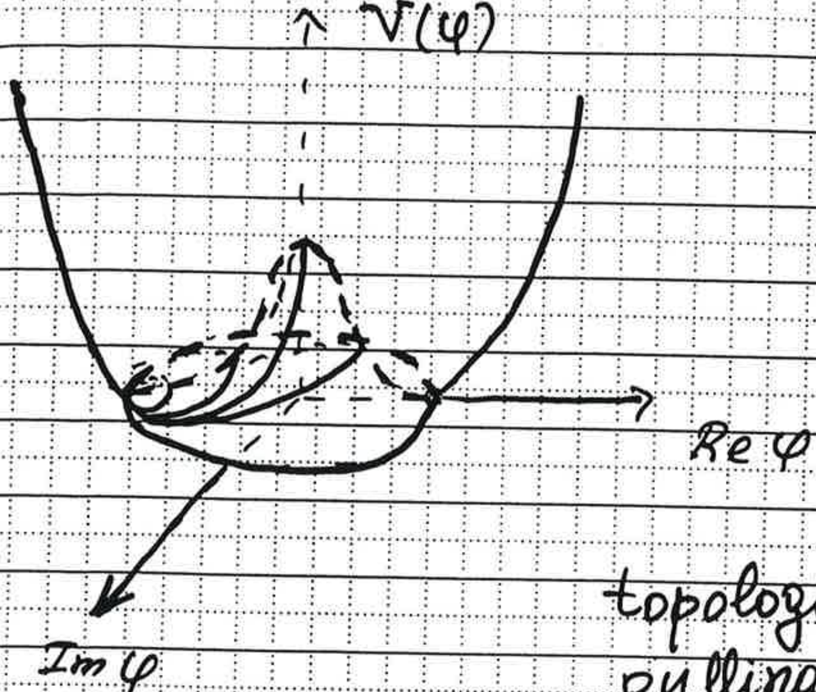
$$= 2 \cdot \underbrace{\frac{e}{2\pi} \int A_1 dx}_{\text{Chern-Simons number}}$$

The same: from anomaly equation

$$\partial_\mu j_\mu^F = 2 \cdot \underbrace{\frac{e}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu}}_{\text{density of topological number}}$$

Conclusion: fermion number is not-conserved in this theory!

Rate of $\Delta F \neq 0$?



topological transition: pulling a loop over potential hill costs energy!

tunnelling transition: instanton

exponential suppression

$$P \sim \exp(-S)$$

Non-zero temperatures

$$P \sim \exp\left(-\frac{E}{T}\right)$$

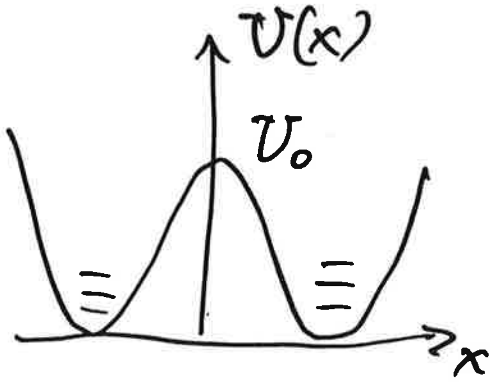
E — energy barrier to overcome

scalar + gauge field configuration corresponding to the top of barrier: sphaleron ("ready to fall")

Simplest theory:

QM with one degree of freedom

$$\mathcal{L} = \frac{1}{2}\dot{x}^2 - V(x)$$



$$V(x) = \frac{\lambda}{4}(x^2 - c^2)^2$$

$T = 0$: $\quad \mathcal{P} \sim \exp(-2S_0)$

$$S_0 = \int_{-c}^{+c} \sqrt{2V(x)}\, dx$$

$T \neq 0$: Boltzmann exponent

$$\mathcal{P} \sim \exp\left(-\frac{V_0}{T}\right) \text{ for } \frac{V_0}{T} \gg 1$$

(probability flux)

Probability flux for 1D example

$$\Gamma = \langle \delta(x)\, p\, \Theta(p) \rangle =$$

$$= \frac{\int dp\, dx\, \delta(x)\, p\, \Theta(p) \exp\left(-\beta\left(\frac{1}{2}p^2 + V(x)\right)\right)}{\int dp\, dx \exp\left(-\beta\left(\frac{1}{2}p^2 + V(x)\right)\right)} =$$

$$= \underline{\frac{\omega_0}{2\pi} e^{-\beta V_0}} \; ; \qquad \omega_0 : \; V(x) = -\frac{1}{2}\omega_0^2 x^2$$

at small x.

Sphaleron: barrier configuration:

$x = 0$.

1+1 Scalar electrodynamics:

$$\varphi_{sph}(x) = v \,\mathrm{th}\, \frac{m_H x}{2} \exp \frac{i\pi x}{L}$$

sphaleron

$$-\frac{L}{2} \leq x \leq \frac{L}{2}$$

Barrier height:

QM: $\sim \lambda c^4$

1+1 electrodynamics:

$$E_{sph} \sim \lambda v^4 \cdot m_H^{-1} =$$

$$= \mathcal{H}(\varphi_{sph})$$

Visialusation

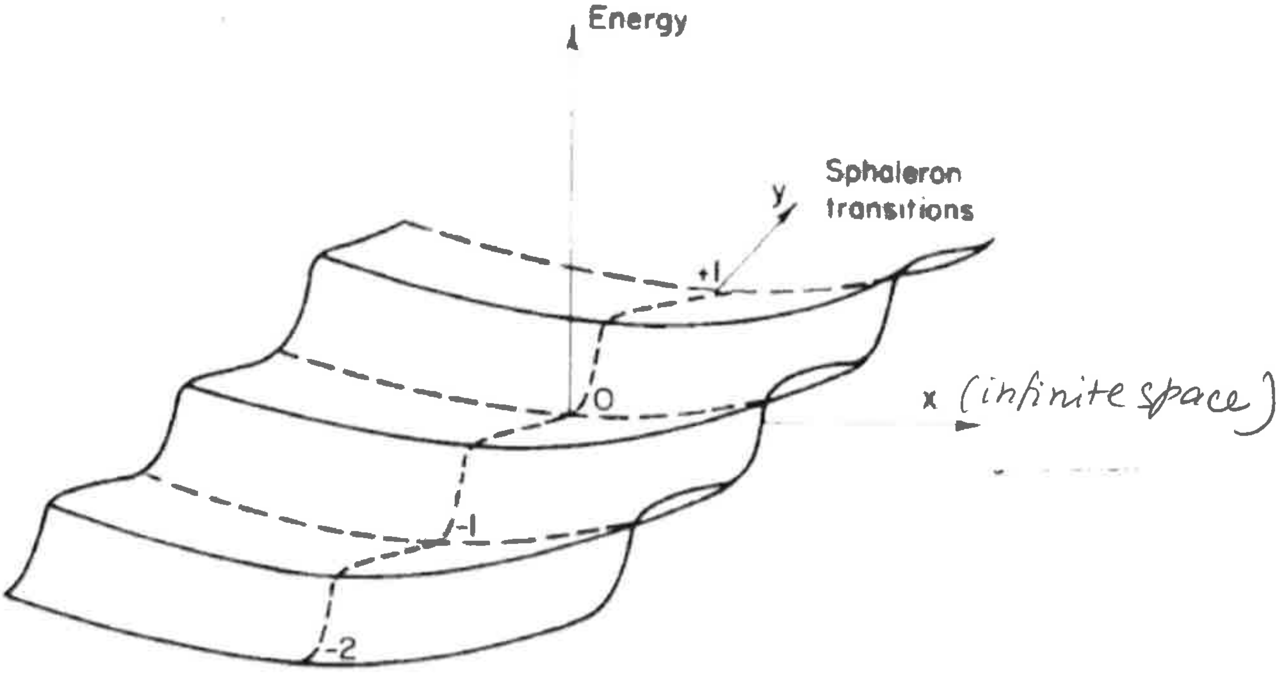


Fig. 2 Schematic dependence of the energy on the configuration of the gauge and Higgs fields. Periodicity in the direction x corresponds to the large gauge transformations.

separatrix: surface, separating different gauge sectors. $f(q)$, q – all coordinates.

Let the static energy is

$$V(q) = \int \frac{1}{4}(F_{ij})^2 + |D_i\varphi|^2 + \lambda(|\varphi|^2 - \varphi_0^2)^2$$

$$\int \left(\frac{\partial V}{\partial A}\frac{\partial f}{\partial A} + \frac{\partial V}{\partial \varphi}\frac{\partial f}{\partial \varphi} + \frac{\partial V}{\partial \varphi^*}\frac{\partial f}{\partial \varphi^*} \right) d^3x = 0$$

We need to compute the probability flux through this surface

$$\Gamma = \frac{1}{2} T \frac{\int \delta(f(q)) \left| \frac{\partial f_0}{\partial q} p \right| \exp(-\beta H)\, Dp\, Dq}{\int \exp(-\beta H)\, dp\, dq} \qquad (*)$$

Must account for

- gauge invariance [ghosts, gauge fixing, etc]
- divergencies [counter-terms]
- zero modes. For example, in 1+1d there is 1 translational zero mode: $\varphi_{sph}(x)$ and $\varphi(x-x_0)$ are solutions.

Integral (*) is divergent, integration over zero mode is to be replaced by dx_0, $\Gamma \propto V$, deviding by V gives rate per unit volume.

- Result for the sphaleron rate:

$\alpha = \frac{2M_W}{M_H}$, for $\alpha \gg 1$:

$$\Gamma = \frac{\sqrt{3}\, m_H^2}{2\pi} \sqrt{\alpha}\; 2^{\alpha - \frac{1}{4}} \left(\frac{E_{sph}}{2\pi T} \right)^{1/2} \exp\left(-\frac{E_{sph}}{T}\right)$$

Bochkarev, MS, 1987

Electroweak theory:

$SU(2) \times U(1)$ $\qquad$ $3+1$

4 conserved numbers

(will assume $m_\nu = 0$, more about $m_\nu \neq 0$ later)

B, L_e, L_μ, L_τ

Similarity with $1+1$:

topology

$$\underbrace{3d \Rightarrow SU(2) \qquad\qquad 1d \Rightarrow U(1)}_{}$$

non-trivial mapping

characterised by integer

also: anomaly, level crossing

topology:

gauge $A_0 = 0$ $\Rightarrow$

discrete set of scassical vacua,

$$A_i = \omega \partial_i \omega^{-1}$$

$$\varphi = \omega \varphi_0 \qquad \varphi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\omega = \omega(x)$$

topological number: (integer)

$$n[\omega] = -\frac{1}{24\pi^2} \int d^3x\, \varepsilon^{ijk}\, \mathrm{Tr}\,[A_i A_j A_k]$$

mapping $3d \to SU(2)$

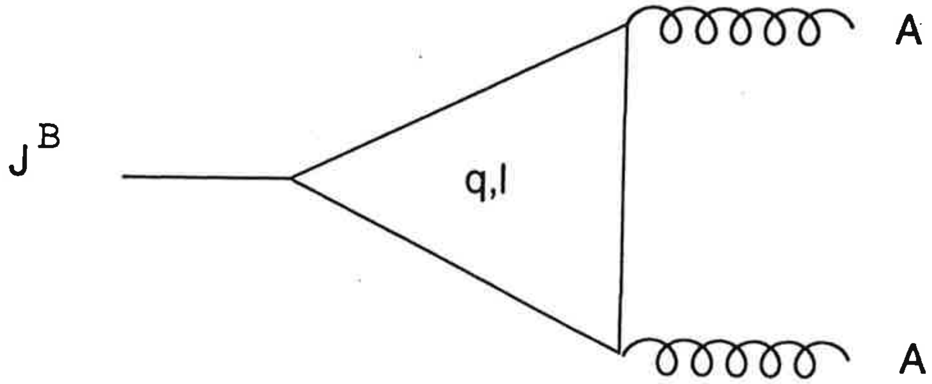
vacuum- vacuum transitions:

$$\frac{1}{32\pi^2} \int d^4x\, \mathrm{Tr}\, F_{\mu\nu} \tilde{F}_{\mu\nu} =$$

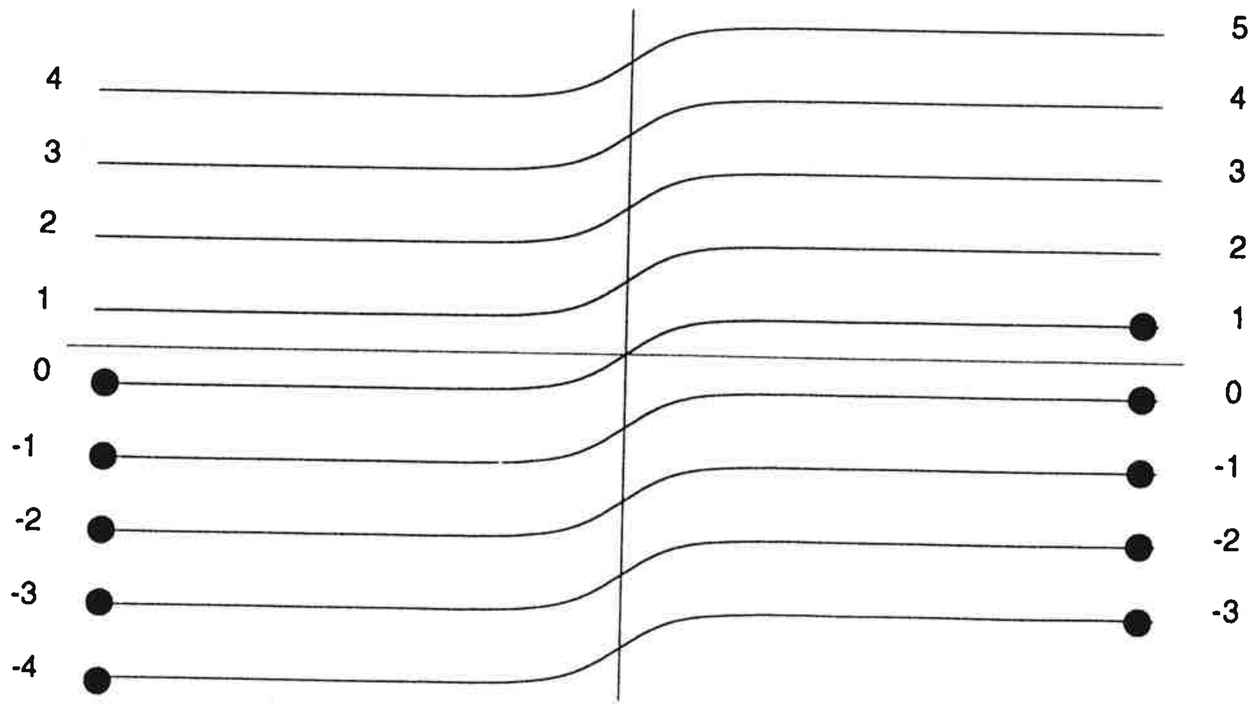
$$= n[\omega_{t=+\infty}] - n[\omega|_{t=-\infty}]$$

Quantum anomaly:

$$\partial_\mu J^B_\mu = \partial_\mu J^L_\mu = \frac{n_f}{32\pi^2}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}_{\mu\nu}\right)$$



9 quarks = 3 colours × 3 generations
3 leptons = 3 generations



Dirac vacuum

Real fermion

ELECTROWEAK B NON-CONSERVATION SELECTION RULES:

TAKE ALL FERMIONIC DOUBLETS

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

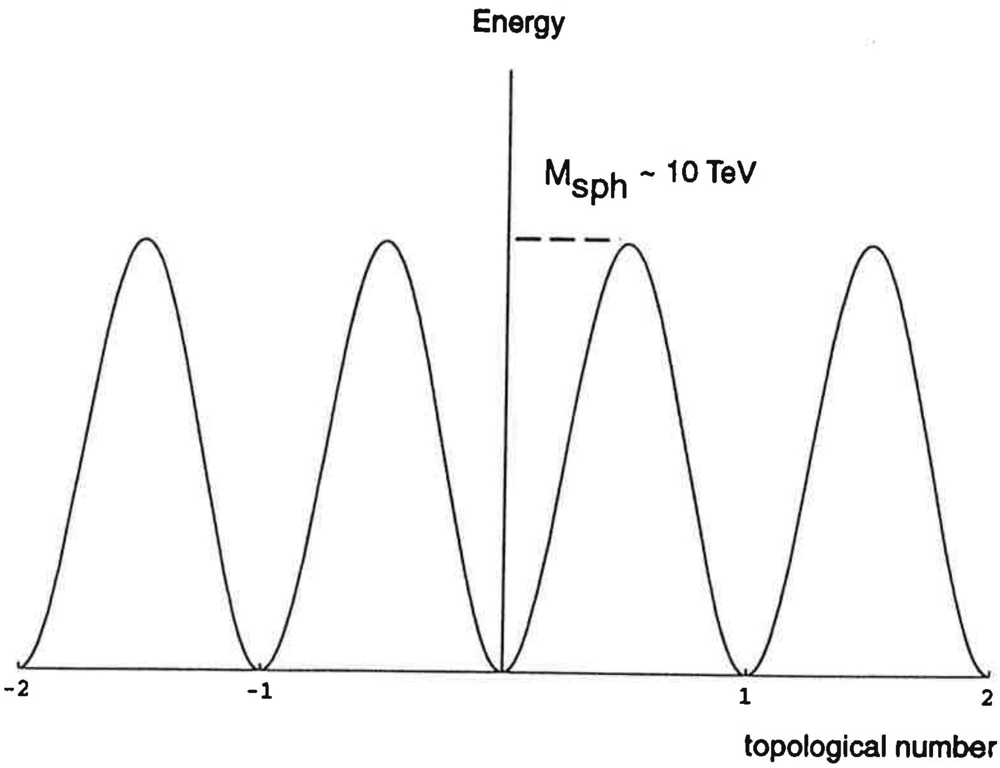
TAKE ONE $\underbrace{\text{OF}}_{}$ fermion EVERY SORT INTO REACTION

$$\Delta B = \Delta L = 3$$

proton is stable, lowest process:

$$pn \to \bar{n} e^+ \nu\nu$$

Rate of anomalous fermion number non-conservation



t'Hooft, 1976

$$\Gamma \sim \begin{cases} \exp(-\frac{4\pi}{\alpha_W}) \sim 10^{-160}, & T = 0 \\ \exp\left(-\frac{M_{sph}}{T}\right), & T < T_c \\ (\alpha_W)^{(\text{[redacted]}5)} T^4, & T > T_c \end{cases}$$

Kuzmin, Rubakov, M.S., 1985;

Arnold, McLerran, 1987

Thermal equilibrium: for

$$100GeV \sim T_c < T < (\alpha_W)^4 M_{Pl} \sim 10^{12} GeV$$

(constraint)

(i) instanton: $(T=0)$

$$A_\mu^a = \frac{1}{g}\eta_{\mu\nu a}\frac{2x^\nu}{x^2+\rho^2}$$

tunnelling (instanton action)

$$A \sim \int \frac{d\rho}{\rho^5}\exp\left(-\frac{8\pi^2}{g^2}-\pi^2 v^2\rho^2\right) \sim$$

$$\sim \exp\left(-\frac{2\pi}{\alpha_W}\right)$$

Probability:

$$P \sim A^2 \sim \exp\left(-\frac{4\pi}{\alpha_W}\right)$$

The sphaleron solution

sphaleron - from Greek, „ready to fall" - solution found by Klinkhamer and Manton in 1983

Consider theory without $U(1)$ [SM with $g'=0$]

$$\vec{A} = v \frac{f(\xi)}{\xi} \hat{\vec{r}} \times \vec{\sigma} \qquad \xi = g v r$$

$$\varphi = \frac{v}{\sqrt{2}} h(\xi) \hat{\vec{r}} \vec{\sigma} \varphi_0 \qquad \varphi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$f(0) = h(0) = 0 \,; \quad f(\infty) = h(\infty) = 1$$

f and h to be found numerically.

Important property:

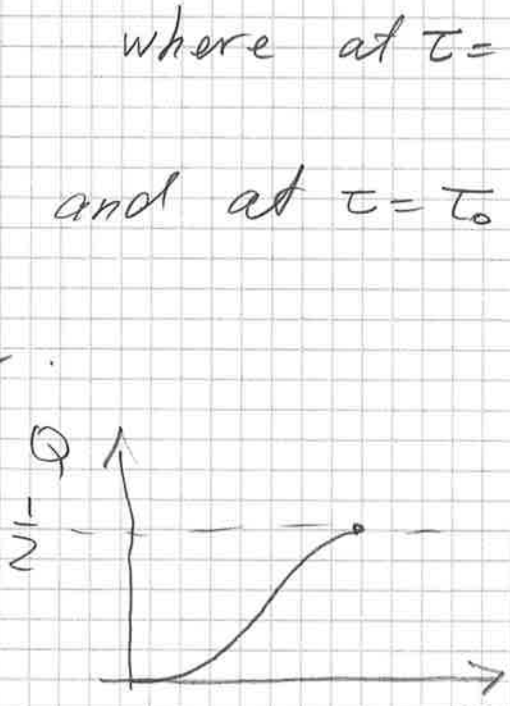
Consider

$$Q = \frac{g^2}{32\pi^2} \int d^3x \int_0^{\tau_0} F\tilde{F} d\tau,$$ where at $\tau = 0$

$\vec{A} = 0$; $\varphi = \varphi_0 \cdot v$, and at $\tau = \tau_0$

$\vec{A} = A_{sph}$; $\varphi = \varphi_{sph}$.

Then $Q = \frac{1}{2}$



Parametric dependence of energy barrier

Static energy :

$$\mathcal{E} = \int d^3x \left\{ \frac{1}{2}(\partial_i H)^2 + \frac{1}{2} m_H^2 H^2 + \frac{\lambda}{4} H^4 + \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} m_W^2 (A_i^a)^2 + \dots \right\}$$

rescaling :

$$x = \frac{1}{m_W} y \; ; \qquad H = v h ; \quad A_i \simeq v a ,$$

$$\Downarrow$$

$$\mathcal{E} = \frac{v^2}{m_W} \int dy \, \mathcal{F}(h, a) =$$

$$= \underbrace{\frac{2 M_W}{\alpha_W}}_{\text{characteristic scale}} B\left(\frac{m_H}{m_W} \right) \sim 10 \text{ TeV}$$

rate $\Gamma = \frac{T^4 \omega_-}{M_W} \left(\frac{\alpha_W}{4\pi}\right)^4 \overset{26}{N_{tr}} \overset{5.3 \cdot 10^3}{N_{rot}} \left(\frac{2 M_W}{\alpha_W T}\right)^7 e^{-\frac{\mathcal{E}}{T}}$

Sphaleron zero modes:

translations: if we have a solution $A_i^a(x)$ $\varphi(x)$, then $A_i^a(x-x_0)$, $\varphi(x-x_0)$ is also a solution. Three modes

Rotations, 3 modes.

Computing the rate: probability flux through the separatrix

Determinat - non-zero modes.

Main effect: replacement of zero T vev to temperature dependent vev.

Electroweak crossover - at $T \approx 160$ GeV (effect will be discussed later, at „electroweak Baryogenesis" part.

For $T \gtrsim 160$ GeV entirely semiclassical analysis of sphaleron rate is impossible. Qualitative picture will be described below.

(iii) $T \gg T_c$

– symmetry is "restored": symmetric phase

barrier height $\sim \dfrac{1}{g^2 \rho}$

ρ – size of configuration

Power counting estimate:

$$P \sim \exp\left[-\frac{1}{g^2 \rho T}\right] \Rightarrow$$

typical scale $\rho \sim \frac{1}{g^2 T} \Rightarrow$

by dimensional argument

rate per unit time & unit volume is

$$\Gamma \sim (\alpha_W T)^4$$ power counting estimate

Landau damping – extra power of α_W

Physics of extra suppression by α:

Challenge: understand time dependence of fluctuations with the space size $\sim l \sim \frac{1}{\alpha_w T}$

let Π : self energy of the vector $SU(2)$ field.

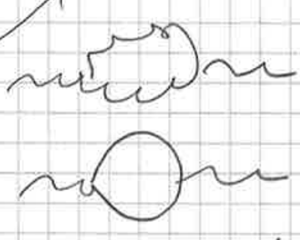
Equation for A: $\leftarrow$ different for longitudinal and transverse polarisations

$$(\omega^2 - p^2)A + \Pi(\omega)A = 0$$

Consider $\omega \leq p$

Longitudinal Π describes Debye screening

Transverse $\Pi \approx g^2 \frac{T^2\omega^2}{p^2} - i\, g^2 \frac{T^2\omega}{p}$



$\omega < p$:

scattering on particles in the plasma

Equation:

$$\left[\omega^2 - p^2 + g^2\frac{T^2\omega^2}{p^2} - i\, g^2\frac{T^2\omega}{p}\right] A = 0$$

$$\omega^2 - (g^2T)^2 + \frac{g^2T^2}{g^4T^2}\,\omega^2 - i\,\frac{g^2T^2}{g^2T}\,\omega = 0$$

$$\omega^2 - (g^2T)^2 + \frac{\omega^2}{g^2} - i\omega T \approx 0 \Rightarrow$$

Solution: $\omega \sim i g^4 T \Rightarrow$

So, the rate per unite time & unit volume is

$$\Gamma \sim \underbrace{(\alpha_w T)^3}_{\text{from size of fluctuations}} \cdot \underbrace{\alpha_w^2 T}_{\text{from time of their decay}}$$

Extra work in this Direction

- Arnold, Son, Yaffe
- Bodecker – effective field theory for sphaleron transitions

  Result: extra log in the rate

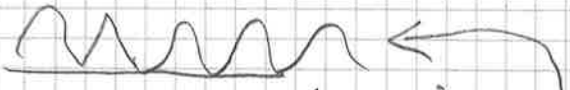
- Numerical simulations.

idea: step # 1 – create configurations with probability $\sim e^{-H/T}$

Step # 2: follow real-time evolution

Step # 3: Compute diffusion rate of topological number,

$$\langle Q^2(t) \rangle = \Gamma V t$$



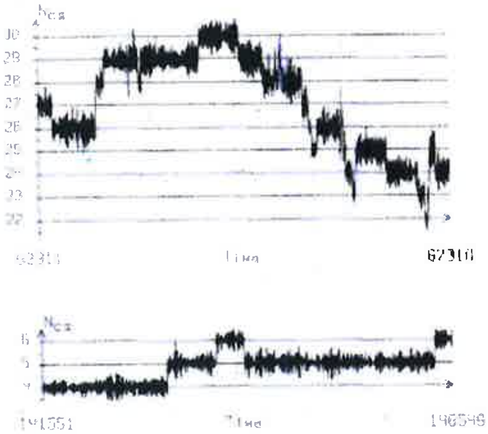
Brownian motion in potential

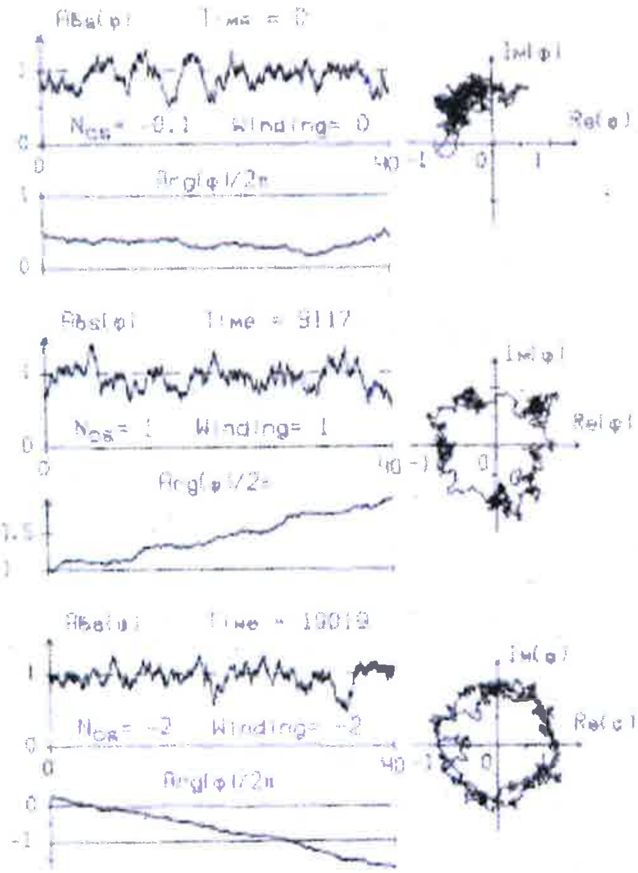
$$\Gamma = \int_0^\infty d^3x\, dt\, \frac{g^2}{32\pi^2} F\tilde{F}(t)\, F\tilde{F}(0)$$

1+1 Dimensional Simulations

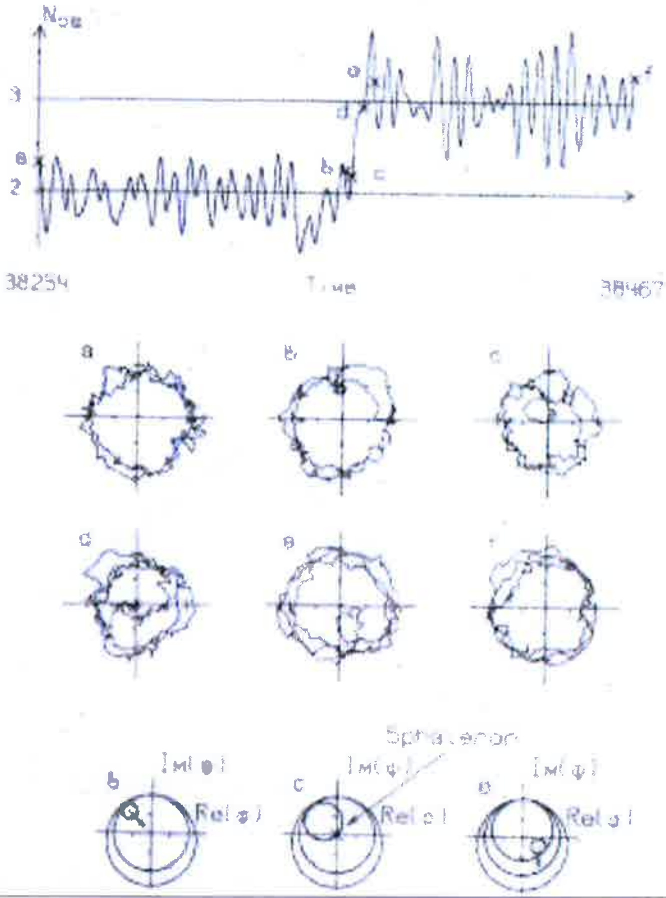
Grigoriev, Rubakov, MS

Typical time evolution of the topological number at finite temperatures derived by numerical simulations in 1+1 dimensions





see over



Final result for the SM with 125 GeV Higgs,

$$\Gamma_{symm} = (18 \pm 3)\, \alpha_w^5\, T^4 \quad , \quad T > 160 \text{ GeV}$$

$$\log \frac{\Gamma_{Higgs}}{T^4} = (0{,}83 \pm 0{,}01) \frac{T}{\text{GeV}} - (147{,}7 \pm 1{,}9)$$

$$T < 160 \text{ GeV}$$

Onofrio, Rummukainen, Tranberg, 2014

Dilution of baryon asymmetry by sphalerons.

Let us start from equilibrium question:

sphalerons are in thermal equilibrium, i.e.

$$\frac{\Gamma_{sph}}{T^3} \gg H = \frac{T^2}{M_0} \qquad H - \text{Hubble rate.}$$

What is the value of baryon asymmetry?

Suppose we know all 3 exactly conserved number of the SM,

$$\Delta_i = L_i - \frac{1}{3} B, \qquad i = e, \mu, \tau.$$

Answer: equilibrium thermodynamics

Density matrix:

$$\rho = \exp\left[-\frac{H}{T} - \sum_i \mu_i \hat{\Delta}_i - \mu_Y \hat{Y} - \mu_3 \hat{T}_3\right]$$

Y – hypercharge

T_3 : 3rd component of weak isospin

Equations for chemical potentials:

$$\Omega = \log\left[\mathrm{Tr}\,\rho\right], \quad Z - \text{statistical sum}$$

$$-\frac{\partial \Omega}{\partial \mu_i} = \frac{\mathrm{Tr}\, \Delta_i \rho}{\mathrm{Tr} \rho} \equiv \langle \Delta_i \rangle \text{ : known } \neq 0 .$$

Also,

$$\frac{\partial \Omega}{\partial \mu_Y} = \frac{\partial \Omega}{\partial \mu_3} = 0 \text{ : absense of gauge charges.}$$

These equations are universal, valid for both symmetric and Higgs phases!

The statistical sum can be represented as Euclidean path integral, see G. Moore lectures. μ_Y and μ_3 can be replaced by Euclidean fields A_4^3 [weak isospin, 4: temporal direction) and B_4 (hypercharge).

Result (Khlebnikov, MS)

$$\Omega = \frac{1}{2} m^2(T) \varphi^2 + \frac{1}{4} \lambda \varphi^4 + \frac{1}{8} \varphi^2 \overbrace{(g A_4^3 - g' B_4)^2}^{Z\text{-mass}}$$

$$+ \frac{1}{2} m_D^2 (A_4^3)^2 + \frac{1}{2} m_{D'}^2 B_4 B_4 \quad \left[\text{Debye masses} \right]$$

$$- \frac{1}{4} n_f \mu_L^2 T^2 - \frac{1}{9} n_f \mu_L^2 T^2$$

quark

$$+ i g' B_4 n_f \left(\frac{1}{3} \mu_L - \frac{1}{9} \mu_L \right) T^2$$

contributions

lepton contributions

We took for simplicity $\mu_e = \mu_\mu = \mu_\tau \equiv \mu_L$

n_f is the number of fermionic generations.

In the following n_s is the number of scalar doublets, g is SU(2) coupling, g' is U(1) coupling, λ is scalar self-coupling.

$$m_D'^2 = \left(\frac{n_s}{6} + \frac{5 n_f}{9}\right) g'^2 T^2$$

$$m_D^2 = \left(\frac{2}{3} + \frac{n_s}{6} + \frac{n_f}{3}\right) g^2 T^2$$

$$m^2(T) = -\frac{1}{2} m_H^2 + T^2 \left(\frac{1}{2}\lambda + \frac{3}{16} g^2 + \frac{1}{16} g'^2 + \frac{1}{4} g_t^2\right)$$

top quark Yukawa

Linear in chemical potential term:

minimum in φ

Equations to solve:

giving conserved charge

$$\frac{\partial \Omega}{\partial \varphi} = 0 \quad ; \quad \frac{\partial \Omega}{\partial \mu_L} = (\Delta_1 + \Delta_2 + \Delta_3)(-1)$$

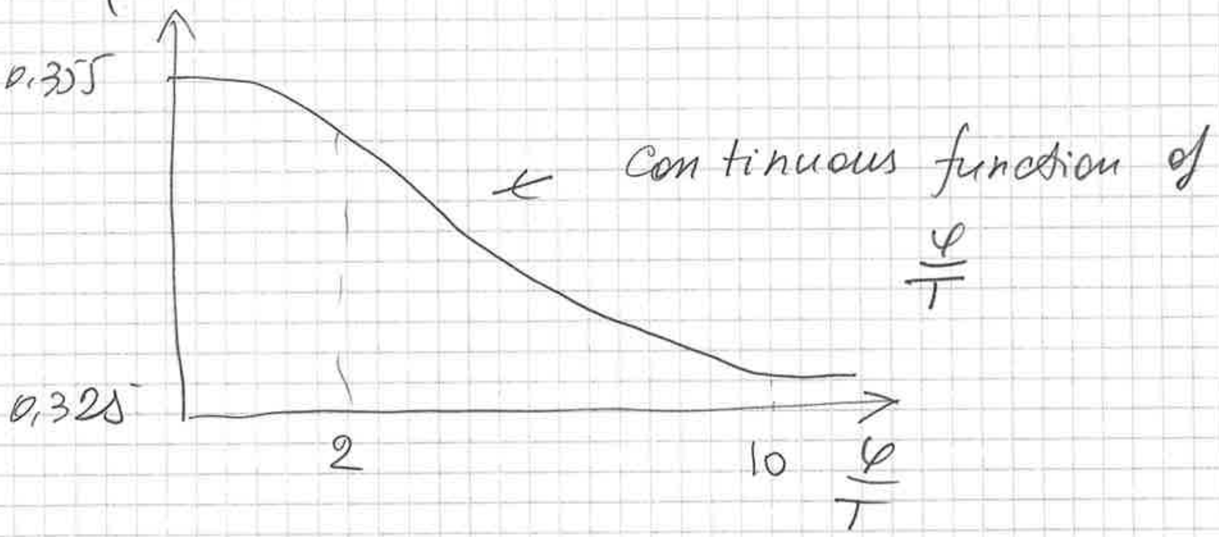
$$\frac{\partial \Omega}{\partial B_4} = 0 \quad \frac{\partial \Omega}{\partial A_3^4} = 0 \quad \text{absense of gauge charges}$$

Solution : μ_L, B_4 and A_3^4 as functions of μ_L

Use this to compute

$$\langle B \rangle = \tilde{f}(n_f, n_s, \varphi)\cdot(B-L) =$$

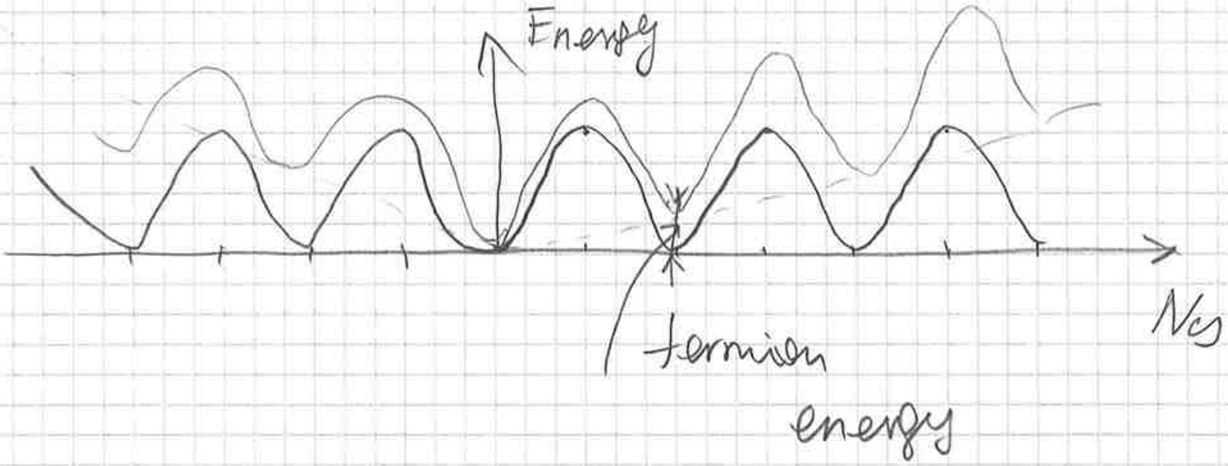
$$\begin{cases} \dfrac{4(2n_f+n_s)}{22n_f+13n_s}(B-L) & , \varphi=0 \\[2ex] \dfrac{4(2+2n_f+n_s)}{(26+24n_f+13n_s)}(B-L) & , \varphi \gg T \end{cases}$$



Continuous curve, no any jumps going from $\varphi=0$ to $\varphi\neq 0$.

Kinetic equations for Baryon number.

Adding fermions to periodic vacuum structure:

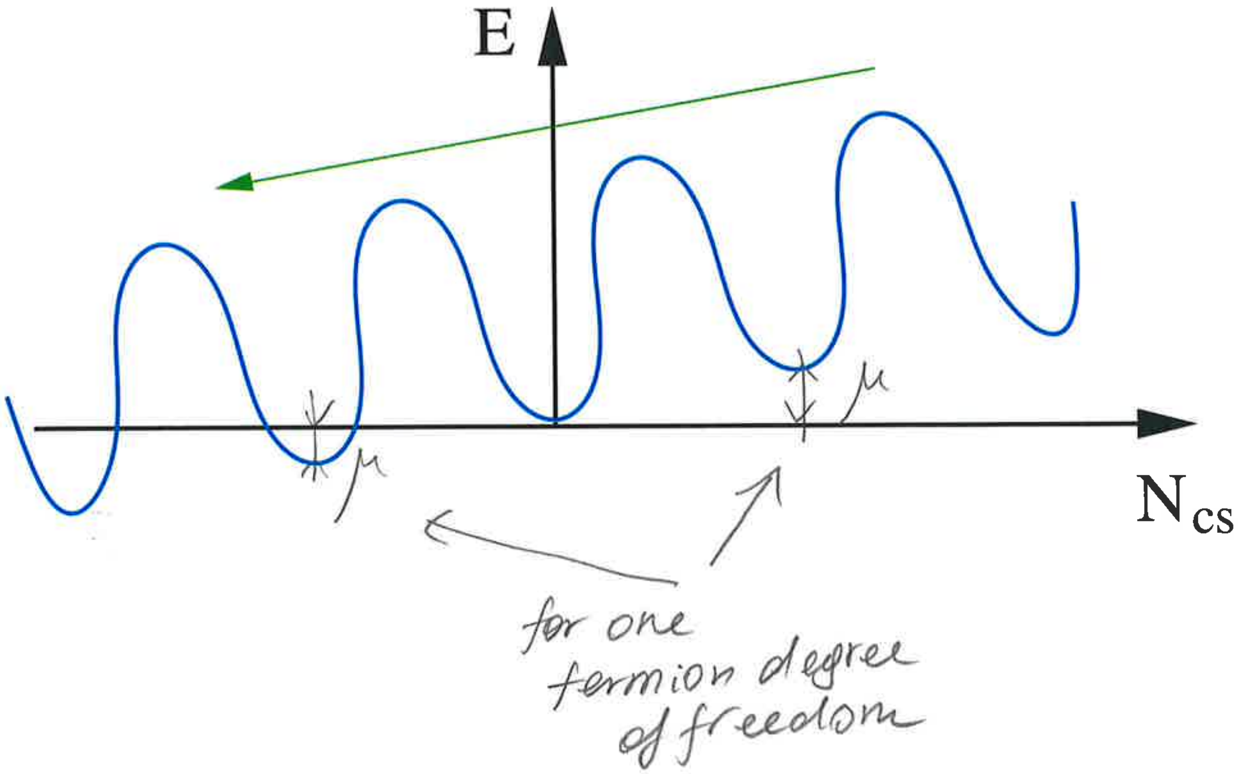


Preferred state; for $(B-L)=0$:

Baryon $\# = 0$.

Suppose initially we had some fermion number density, with chemical potential μ

Energy dependence on the CS number in the presence of non-zero fermionic density



Probability of fluctuation to the left is larger than to the right,

$$\Delta P \sim \left[1 - \exp\left(-\frac{\mu}{T}\right)\right] \simeq \frac{\mu}{T} \text{ for}$$

small $\mu \Rightarrow$ the system will be drifting to the left, and fermion number will be decreasing („eaten up" by topological number.

Result:

$$\dot{n}_B = -\Gamma_B \left(n_B - n_B^{eq} \right)$$

$$\Gamma_B = 9 \cdot \frac{869 + 333\,\Phi^2/T^2}{792 + 306\,\varphi^2/T^2} \; \frac{\Gamma_{diff}}{T^3}$$

$$\Gamma_B = H \quad @ \quad T \approx (131.7 \pm 2.3)\ \mathrm{GeV}$$

Freezing of the sphaleron transitions correspond to

$$\boxed{\frac{\varphi}{T} \approx 1.2}$$

$\varphi \approx 159\ \mathrm{GeV}$

normalisation:

$\varphi(0) \approx 246\ \mathrm{GeV}$

(61)

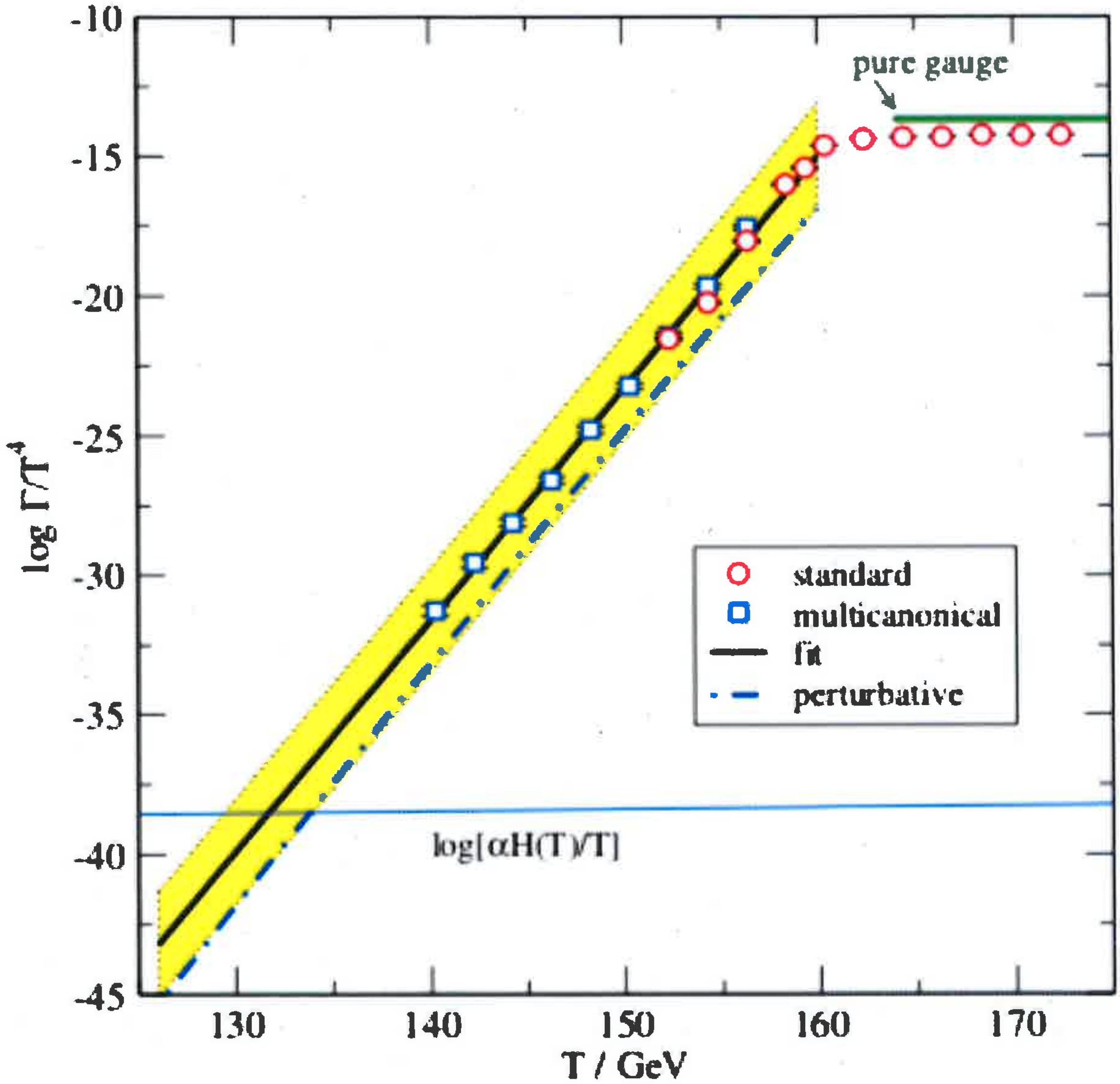


FIG. 2 (color online). The measured sphaleron rate and the fit to the broken phase rate, Eq. (7), shown with a shaded error band. The perturbative result is from Burnier *et al.* [13] with the nonperturbative correction used there removed; see main text. Pure gauge refers to the rate in hot SU(2) gauge theory [21]. The freeze-out temperature T_* is solved from the crossing of Γ and the appropriately scaled Hubble rate, shown with the almost horizontal line.

62

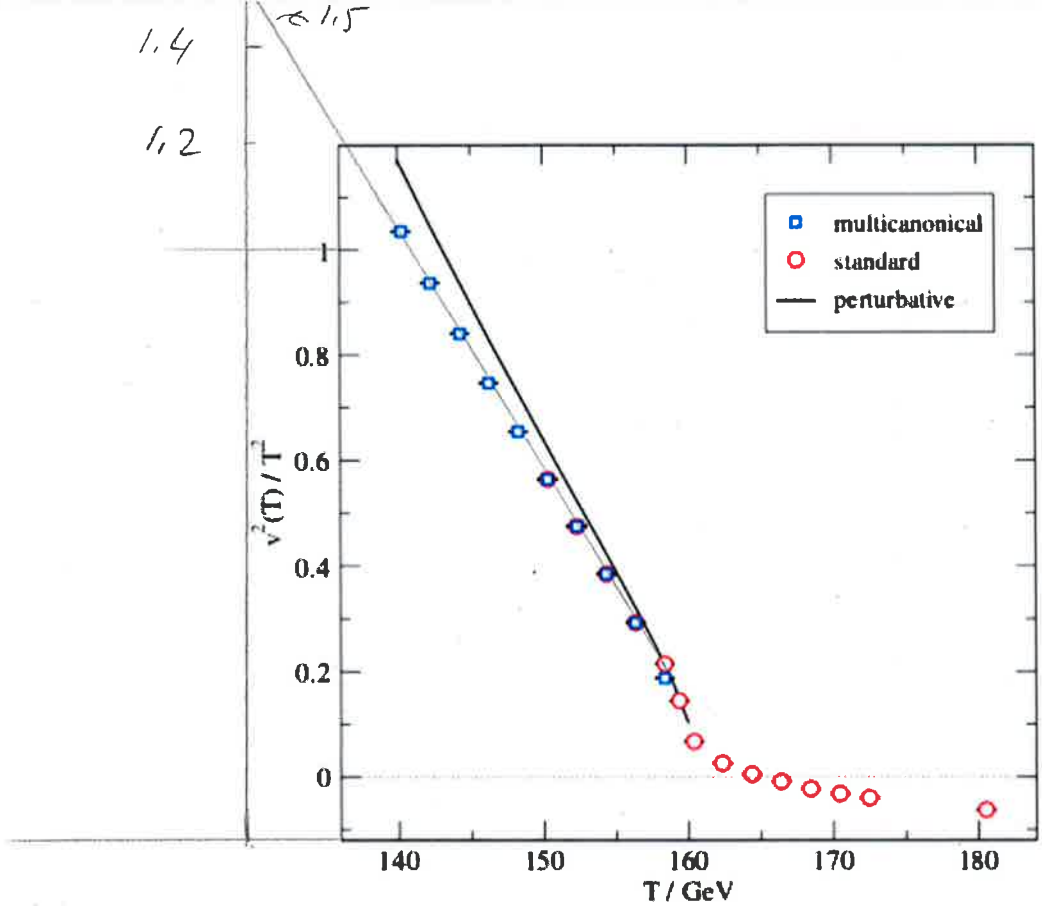


FIG. 3 (color online). The Higgs expectation value as a function of temperature, compared with the perturbative result [2].

$\frac{v^2}{T^2} = 1.5 \Rightarrow \frac{v}{T} = 1.22 \Leftarrow$ will play an importante role for electroweak baryogenesis

$v = 159\,\mathrm{GeV}$

Conclusion:

rate of B non-conseration is "ideal" for baryogenesis :

- extremely small at low T (matter is stable)

- rapid at high T (potential generation of asymmetry)

Baryon asymmetry in the standard model.

Standard Model:

- B- nonconservation: OK, $T \approx 130\,GeV$
- CP - violation: OK, KM mixing
- Departures from thermal equilibrium - OK, expansion of the Universe.

Estimate of CP- violation. Assume that the relevant scale of baryogenesis is Λ

Estimates of CP- violation as a function of Λ (MS, 1987, Farrar, MS' 1993)

Lagrangian of the SM:

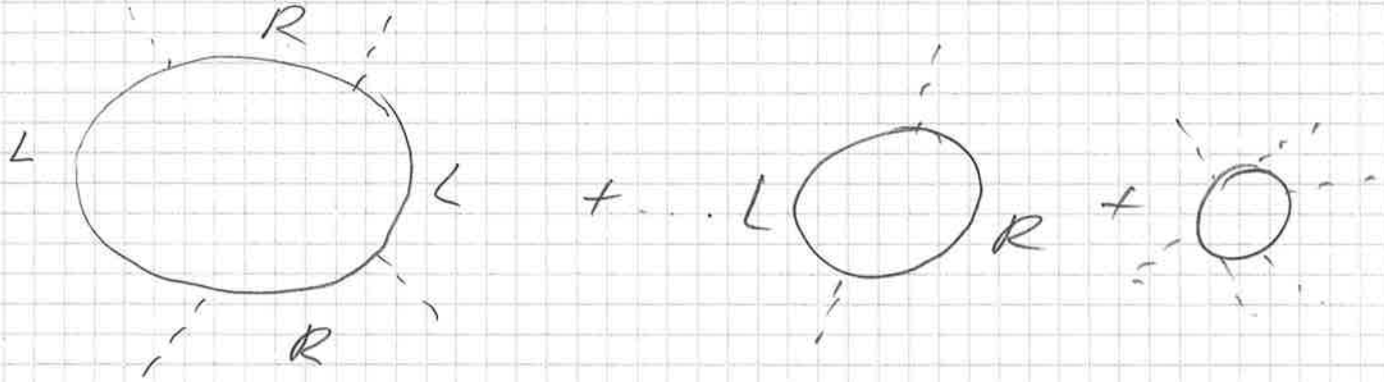
$$\mathcal{L}_Y = \bar{Q}_L K M_D D_R \varphi + \bar{Q}_L M_U U_R \varphi + h.c$$

where $M_U = \frac{g_w}{\sqrt{2} M_W} \operatorname{diag}(m_u\, m_c\, m_t)$

$$M_D = \frac{g_w}{\sqrt{2} M_W} \operatorname{diag}(m_d\, m_s\, m_b)$$

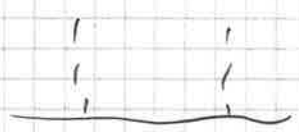
K - Kobayashi - Maskawa matrix

Suppose that $\Lambda \gtrsim m_t \Rightarrow$ all Yukawa couplings (quark masses) can be treated perturbatively. Whatever the mechanism for baryogenesis is, the result must contain the sum over all generation indexies. Symbolically:



2 building blocks:

$$= \mathcal{M}_U = M_U^2$$



$$= \mathcal{M}_D = K M_D^2 K^+$$

CP - violating result :

$$\operatorname{Im} \operatorname{Tr} \left[\text{polynomial of } \mathcal{M}_U \text{ and } \mathcal{M}_D \right]$$

The lowest order non-trivial trace:

$$\mathrm{Im}\,\mathrm{Tr}\, M_U^2 M_D^2 M_U M_D \neq 0$$

In fact, we have sum of 2 terms:

$U \Leftrightarrow D$:

and $\mathrm{Im}\,\mathrm{Tr}\left[M_U^2 M_D^2 M_U M_D + M_D^2 M_U^2 M_D M_U\right] = 0$

So, minimal possibility is

$$\mathrm{Im}\,\mathrm{Tr}\, M_U^3 M_D^2 M_U M_D =$$

$$\delta_{CP} = \left[\frac{g_W^2}{2M_W^2}\right]^7 \underbrace{s_1^2 s_2 s_3 \sin\delta}_{\text{Jarlskog determinant}, \sim 10^{-4}} m_t^6 m_b^4 m_c^2 m_s^2 = 10^{-22}$$

So, baryon asymmetry could be at most

$$\Delta_B \sim \frac{1}{N}\delta_{CP}\left(\frac{v}{\Lambda}\right)^{14} \sim 10^{-20} \text{ if } \Lambda \sim \text{sphaleron freeze-out temperature}$$

N: # of degrees of freedom

We also need deviations from thermal equilibrium: all processes, except sphalerons, are all in a very good thermal equilibrium.

: strong reactions: $q\bar{q} \to GG$,

$$\frac{\Gamma}{H} \sim 10^{-14}$$

electromagnetic reactions $ee \to \gamma\gamma$

$$\frac{\Gamma}{H} \sim 10^{-12}$$

Slowest reactions with right electron chirality flip, f_e

$$\frac{\Gamma}{H} \sim 10^{-2}$$

The only possibility to increase (*): lower the typical scale Λ down.

$m_b \lesssim \Lambda \lesssim m_t \Rightarrow$

$$\Delta_B \sim \frac{1}{N} \cdot \left(\frac{g_W^2}{2M_W^2}\right)^4 \underbrace{s_1^2 s_2 s_3 \sin\delta}\, m_b^4 m_c^2 m_s \left(\frac{v}{\Lambda}\right)^8$$

at most $\sim 10^{-10}$ for $\Lambda \sim 5$ GeV,

and larger if $\Lambda \sim 1$ GeV

If the only source for departure from thermal equilibrium is sphaleron freeze-out, then $\Lambda \simeq 130\,\text{GeV}$ and

$$\Delta_B \lesssim 10^{-20}$$ [exersize: find loopholes in 1906.04080]

Question: can we have another mechanism for thermal non-equilibrium?

Yes, 1st order phase transition!

Phase transitions in the early Universe

- What are phase transitions?
- Why we are interested in phase transitions in cosmology?
- Dynamics of first order PT
- Basics of finite temperature field theory
- Effective field theory description
- Electroweak phase transition
- Electroweak baryogenesis

What are phase transitions?

Water boils and freezes, vapor condenses, ice melts: properties of substance change abruptly with temperature.

Mathematically: PT is non-analytic behaviour of partition function with respect to temperature or other macroscopical parameters (density, external field, curvature of space-time, etc).

Boltzmann distribution:

$$P_n = \frac{1}{Z}\exp(-\beta E_n), \quad \beta = 1/T$$

E_n – energy levels of the system.

The partition function Z and the free energy F :

$$Z = \sum_n \exp(-\beta E_n) = \exp(-\beta V F) \ ,$$

V – the volume of the system.

Since $\exp(-\beta E_n)$ is an analytic function of temperature, phase transitions can occur only when infinitely many states are available, for example, if $V \to \infty$.

Terminology

- First order phase transition: F is a continuous function of T but the first derivative of F has a jump

$$\text{latent heat} = l = \Delta(T^2 \frac{\partial F}{\partial T})$$

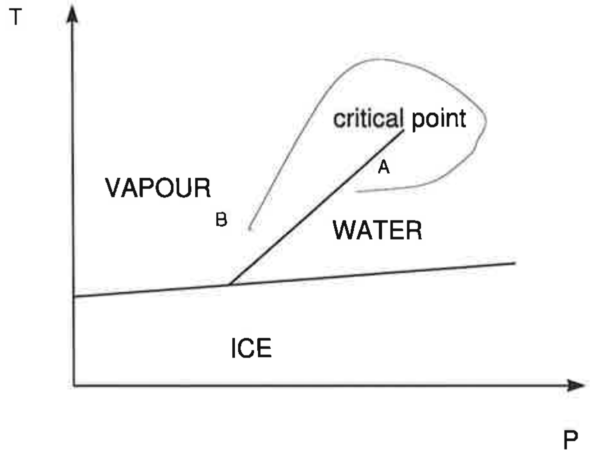
- Second order, or continuous phase transition: $l = 0$, but higher derivatives of F are singular; correlation length diverges as the temperature approaches the critical value T_c.
- (True) order parameter: an observable that is equal to zero in one phase and is non-zero in another. In other words, the value of a true order parameter unambiguously shows the phase state of the system.

Example: magnetization of a ferromagnet

$\langle \vec{M} \rangle = 0$, high T, $\langle \vec{M} \rangle \neq 0$, low T.

A system may not have a true order parameter, even if it has phase transitions

Example: a liquid–vapor phase transition



In the vicinity of the critical point many systems may behave in a similar (universal) way, which gives rise to critical phenomena.

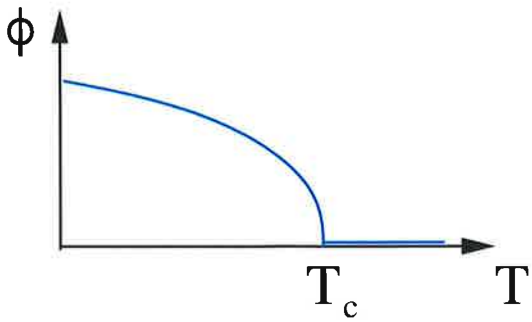
Mean field analysis:

- Second-order PT: effective potential for (true) order parameter ϕ:

$$U(\phi) = \frac{1}{2} m^2(T)\phi^2 + \frac{\lambda}{4}\phi^4\,, \quad m^2(T) = \gamma(T^2 - T_c^2)$$

The value of the order parameter ϕ is to be defined by the minimization of the potential.

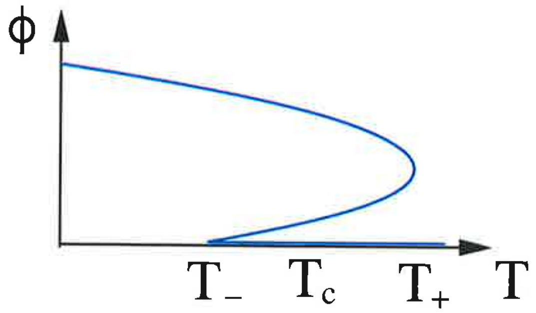
$$\phi = \begin{cases} 0, & T > T_c \\ \pm\sqrt{-\frac{m^2(T)}{\lambda}}, & T < T_c \end{cases}$$

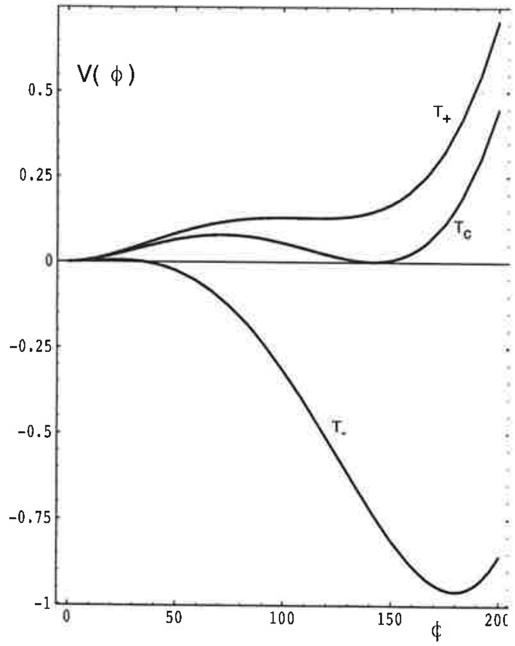


- First-order PT: effective potential

$$U(\phi) = \frac{1}{2}m^2(T)\phi^2 - \frac{1}{3}\delta T\phi^3 + \frac{\lambda}{4}\phi^4 \,,$$

$$\Delta\phi(T_c) \neq 0.$$





Why are we interested in phase transitions in cosmology?

Universe in thermal equilibrium: exactly homogeneous, does not contain any structures...

$$\Downarrow$$

Deviations from thermal equilibrium are important, they may influence a number of parameters that describe the present state of the Universe:

- primordial abundance of light elements
- baryon–photon ratio - baryon asymmetry
- density perturbations

73

Quality of thermal equilibrium:

$$q = \tau/\tau_U, \quad \tau_U = \frac{M_0}{T^2}, \quad M_0 = \frac{M_{Pl}}{(1.66\sqrt{N})}$$

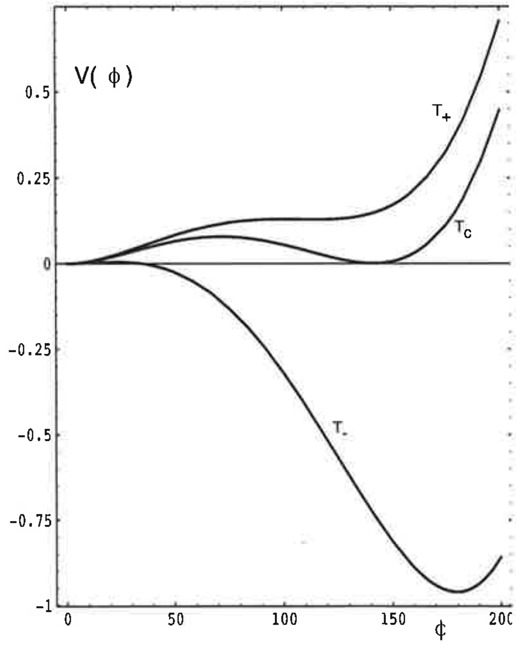
τ - typical reaction time.

Electroweak scale temperatures $T \sim 100$ GeV:

- Strong QCD reactions, e.g. $q\bar{q} \to GG$
  $q \sim T/(\alpha_s^2 M_0) \sim 10^{-14}$
- electromagnetic reactions, e.g. $e\bar{e} \to \gamma\gamma$
  $q \sim T/(\alpha^2 M_0) \sim 10^{-12}$
- slowest reaction with right electron chirality flip $e_R H \to e_L W$
  $q \sim 10^{-2}$

First order PT: source for thermal non-equilibrium in the early Universe!

- $T \gg T_c$: universe is in high-T phase
- $T_- < T < T_c$: universe is still in high-T phase
- $T = T^*$: bubble nucleation
- $T < T^*$: bubble collisions



Suppose: $T_c - T > 0$,

$(T_c - T)/T_c \ll 1$.

Probability of bubble nucleation, thin wall approximation

$\mathcal{P} = A(T)T^4 \exp(-F_c/T)$,

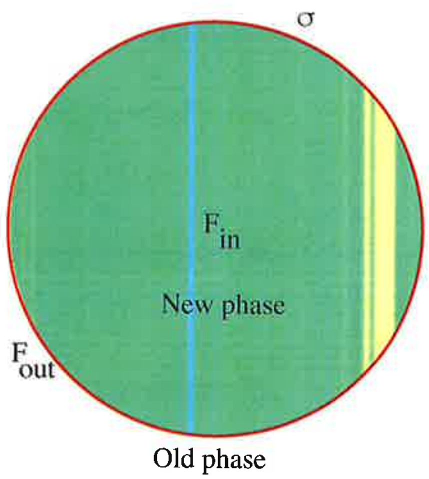
$A(T)$ - prefactor

F_c - free energy of the critical bubble, determined from:

$F(R) \; = \; -\frac{4}{3}\pi R^3 \Delta F \, + \, 4\pi R^2 \sigma$,

$\Delta F \; = \; F_{in} \, - \, F_{out} \; = \; l(1 - T/T_c)$,

l is the latent heat of the transition



Results:

The size of the critical bubble:

$R_c = 2\sigma/\Delta F$ exponential in the probability:

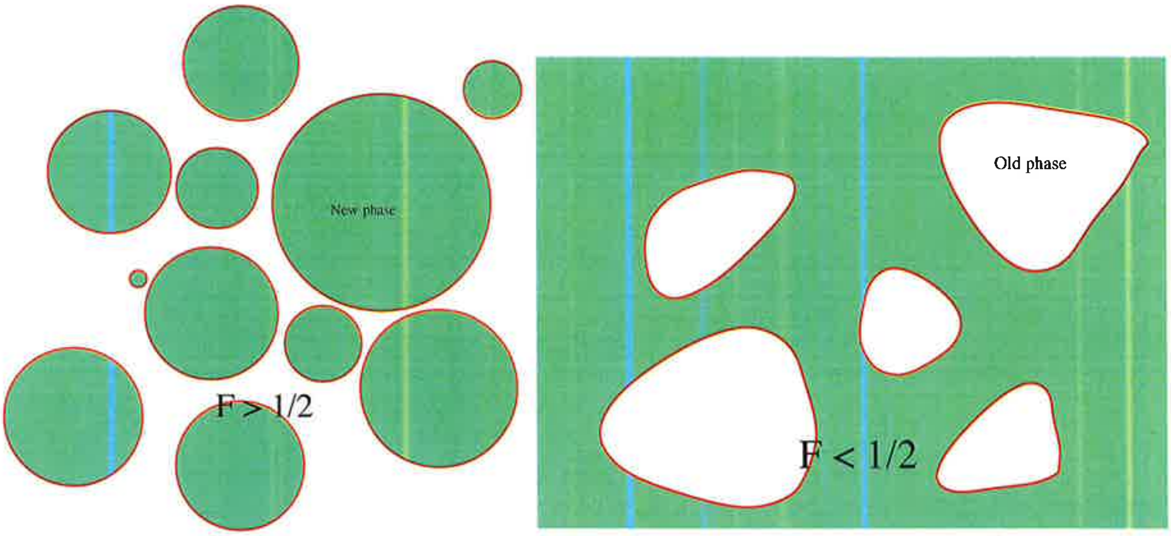
$$\frac{F_c}{T_c} = \frac{16}{3} \frac{\sigma^3 T_c^2}{l^2(T_c - T)^2} \equiv \frac{A}{x^2}, \; x = \frac{T_c - T}{T_c} \; .$$

Fraction of space $\mathcal{F}$ that is still occupied by the old phase

$$\mathcal{F} \simeq \exp\left(-\int_{t_c}^{t} dt' \mathcal{P} \frac{4}{3}\pi v^3 (t' - t_c)^3\right) ,$$

v - the velocity of the bubble walls

t_c - the time corresponding to the critical temperature.



Phase transition ends at temperature (percolation)

$x \simeq \sqrt{\frac{A}{4log(M_0/T_c)}}$,

typical distance between bubble centers

$R \simeq 2vx\tau_U$.

Valid for $A/x^2 \gg 1$.

Conclusion: To describe phase transition (in thin wall approximation) one has to know

- Critical temperature
- Latent heat
- Bubble wall tension

To go beyond thin wall approximation one has to know effective action for an order parameter

Formulation of the problem

Given: Lagrangian of field theory (EW theory or its extentions, MSSM, GUT...)

Find:

- Order of phase transition (if any).
- Values of order parameters (ordinary scalar field VEVs)
- Critical temperature
- Latent heat
- Surface tension of critical bubbles
- Percolation temperature

Basics of finite temperature field theory

H - Hamiltonian of the system

Q_i - conserved charges $[Q_i, H] = 0$

$$\Downarrow$$

equilibrium state of the system is described by the density matrix ρ

$$\rho = \frac{1}{Z} \exp\left(-\frac{1}{T}(H + \sum_i \mu_i Q_i)\right),$$

μ_i – a set of chemical potentials. Statistical sum of the system Z:

$$Z = Tr\left[\exp\left(-\frac{1}{T}(H + \sum_i \mu_i Q_i)\right)\right].$$

Free energy (from now on $\mu_i = 0$):

$$Z = \exp\left(-\frac{FV}{T}\right),$$

Functional-integral representation for the statistical sum:

$$Z = \int D\phi D\Psi \exp(-S_E),$$

S_E is Euclidean action for the system, defined on a finite “time" interval $0 < \tau < \beta = \frac{1}{T}$,

$$S_E = \int_0^\beta d\tau \int d^3 x L,$$

ϕ - bosonic fields, periodic boundary conditions with respect to imaginary time, $\phi(0, x) = \phi(\beta, x)$.

Ψ - fermionic fields, antiperiodic boundary conditions, $\Psi(0, x) = -\Psi(\beta, x)$.

Gauge theories: ordinary gauge fixing and introduction of periodic ghost fields.

Imaginary time , or Matsubara Green's functions:

$$G(\tau_1, x_1, ..., \tau_n, x_n) = \frac{1}{Z} \int \phi(\tau_1, x_1)...\phi(\tau_n, x_n) D\phi D\Psi \exp(-S_E).$$

Finite temperature *equilibrium* field theory is equivalent to the Euclidean field theory defined on a finite “time" interval.

Perturbation theory at finite temperatures looks precisely like perturbation theory at $T = 0$ with substitutions of quantities associated with the zero component of 4-momentum p, as follows:

$$p_0 \to i\omega \qquad\qquad \int dp_0 \to 2\pi i T \sum_\omega$$

$$\delta(p_0) \to (2\pi i T)^{-1}\delta_{\omega,0}$$

$\omega = 2\pi n T$ for the bosons

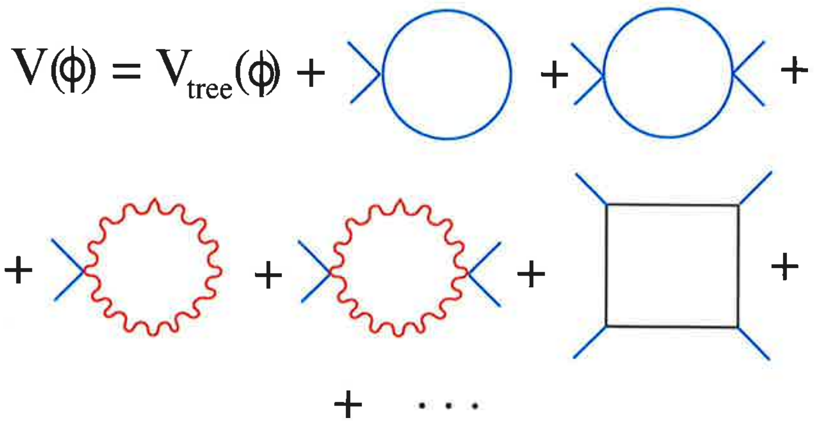
$\omega = (2n+1)\pi T$ for fermions.

The equilibrium properties of a plasma are completely defined by the statistical sum and by the set of Green's functions.

The problem of equilibrium statistics is to compute these quantities reliably.

Elementary theory of phase transitions

What can be easier – compute finite temperature effective potential for the scalar field (should be OK (?) for weakly coupled gauge theories):

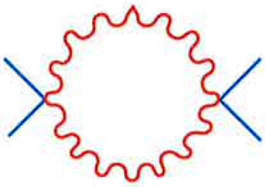


Typical one-loop expression

$$U(\phi) = \frac{1}{2}m^2(T)\phi^2 - \frac{1}{3}\delta T|\phi|^3 + \frac{\lambda}{4}\phi^4\,,$$

Non-analytic $\delta|\phi|^3$, $\delta \sim g^3$ term comes from $n = 0$ sector of finite temperature field theory:

$$T\int \frac{d^3k}{k^2 + g^2\phi^2}(g\phi)^4$$



(Wrong) conclusion from one-loop analysis:

there is always first order phase transition with

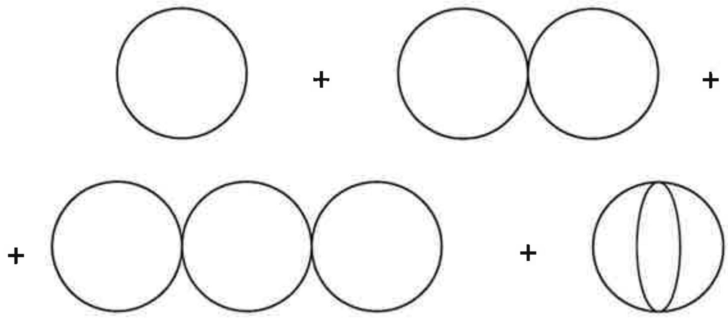
$$\Delta\phi \sim \frac{\delta T}{\lambda}$$

Infrared problem

Simplest example: scalar theory with $\lambda \ll 1$

$$L = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4$$

Statistical sum:



Two expansion parameters:

- λ - the same as at $T = 0$
- new one, $\sim \frac{\lambda T}{m}$.

Take $n = 0$ contribution:

1-loop : $T\, m^3$

2-loop : $\lambda T^2 m^2$

3-loop : $\lambda^2 T m$

4-loop : $\lambda^3 T$

Straightforward perturbation theory breaks down at $T > m/\lambda$.

For theories containing massless bosons (such as QCD) perturbation theory does not work for any temperature!

Physical reason:

$T = 0 - \lambda$ is an expansion parameter only for the processes with small number of particles.

$T \neq 0$ – number of particles, participating in collisions, may be large.

Expansion parameter $\lambda n_B(E)$:

$$n_B(E) = \frac{1}{\exp(E/T) - 1},$$

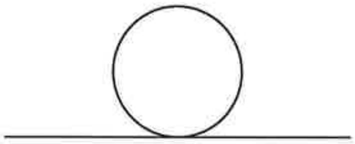
Small energies - strong coupling, $\frac{\lambda T}{E}$.

Resummation?

Idea: sum up bubble graphs for the scalar mass.

$$m^2 \to m^2_{eff}(T) = m^2 + \frac{\lambda}{4}T^2.$$

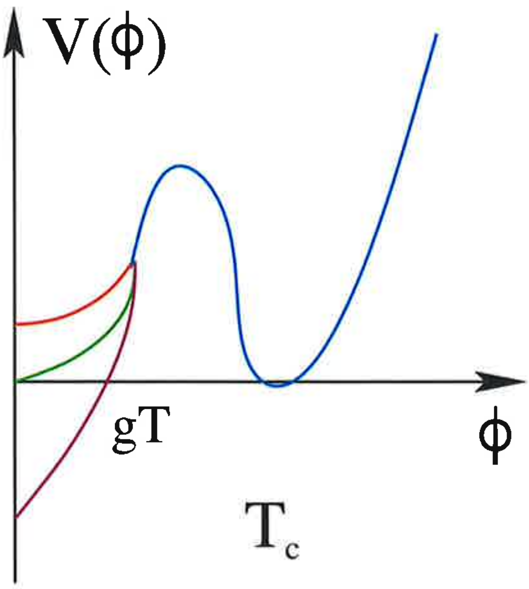
Still does not work at the vicinity of the phase transition where $m^2_{eff}(T) = 0$.



Computation of the effective potential: expansion parameter near $\phi = 0$:

$$g^2 n_B(E) \sim \frac{g^2 T}{g\phi} \simeq \frac{gT}{\phi}$$

Perturbation theory breaks down for $\phi < gT$.



Phase transitions cannot be entirely described by perturbation theory!

Lattice simulations?

Requirements:

- Lattice spacing much smaller than distances between particles, $a \ll 1/T$
- Lattice size much larger than infrared scale, $Na \gg \frac{1}{\alpha_W T}$

$$\Downarrow$$

Difficult in weakly coupled theories, $N \gg \frac{1}{\alpha_W}$

Effective field theory approach

Main idea: factorization of weakly coupled high-momentum modes, with energy $E \gg \alpha T$, and of strongly coupled infrared modes with energy $E < \alpha T$, and the construction of an effective theory for infrared modes only.

Construction of the effective field theory: perturbative

Analysis of the effective field theory: non-perturbative

- ϵ expansion
- exact renormalization group
- gap equations
- Monte Carlo simulations

Works for: QCD when $\alpha_s(T) \ll 1$, EW theory and GUTs at $T \simeq T_c$.

How to construct effective field theory?

- Expand bosonic and fermionic fields into Fourier sums,

$$\phi(x,\tau) = \sum_{n=-\infty}^{\infty} \phi_n(x) \exp(i\omega_n^b \tau)$$

$$\psi(x,\tau) = \sum_{n=-\infty}^{\infty} \psi_n(x) \exp(i\omega_n^f \tau)$$

with $\omega_n^b = 2n\pi T$ and $\omega_n^f = (2n+1)\pi T$.

- Insert these expressions into the action, and integrate over time. Result: *three-dimensional* action, containing an infinite number of fields, corresponding to different Matsubara frequencies:

$$\int d^4x L \rightarrow \sum \int d^3x L^{3d}$$

4d finite-temperature field theory is equivalent to a 3d theory with an infinite number of fields, and 3d boson and fermion masses are just the frequencies ω^b and ω^f.

Analogy to Kaluza–Klein theories with compact higher-dimensional space coordinates.

- Integrate out the weakly interacting 3d “superheavy" modes (fields with masses $\sim \pi T$) and get an effective action for light modes (bosonic fields corresponding to the zero Matsubara frequency).

$$\exp(-S_{eff}) = \int D\Psi D\phi_{n\neq 0} \exp(-S_E)$$

$$S_{eff} = cVT^3 + \int d^3x \left[L_b(T) + \sum_{n=0}^{\infty} \frac{O_n}{T^n} \right]$$

$L_b(T)$ is a super-renormalizable 3d effective bosonic Lagrangian with temperature-dependent constants,
O_n are operators of dimensionality $n+3$, suppressed by powers of temperature at $n \geq 1$,
c is a perturbatively computable number (contribution of $n \neq 0$ modes to the free energy density).

How to construct effective Lagrangian?

Matching procedure: write the most general 3d effective action, containing the light bosonic fields only, and fix its parameters (coupling constants and counter-terms) by requiring that 3d Green's functions at small spatial momenta $k \ll T$, computed with an effective Lagrangian, coincide with the initial 4d static Green's functions up to some accuracy,

$$G^{3d}(k_1, ...k_n) = G^{4d}_{\omega=0}(k_1, ...k_n)(1 + O(g^m))$$

Consistency check : the typical energy scales (masses of excitations in 3d theory m_{eff}) must be small compared with the energy scale πT that we have integrated out:

$$\left(\frac{m_{eff}}{\pi T}\right)^2 \ll 1$$

Example: pure scalar field theory

4d Lagrangian:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4$$

Effective 3d Lagrangian:

$$L_{eff} = \frac{1}{2}(\partial_i \phi_3)^2 + \frac{1}{2}m_3^2\phi_3^2 + \frac{1}{4}\lambda_3\phi_3^4 + \Delta L$$

Dimensions in 3d: ϕ_3: GeV$^{\frac{1}{2}}$; m_3^2 : GeV^2; λ_3: GeV

Tree level mapping: $\phi_3 = \phi/\sqrt{T}$, $m_3^2 = m^2$ and $\lambda_3 = \lambda T$.

Important: 3d theory is super-renormalizable:

- Z_ϕ=1
- λ_3 is finite
- m_3^2 contains linear and logarithmic divergences on the one- and two-loop levels only

In $4d$ all the couplings are scale dependent.

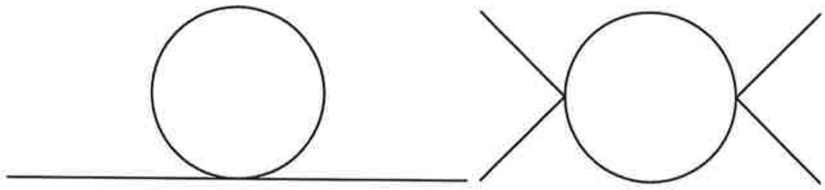
One loop mapping must have the form:

$$m_3^2 = m^2(\mu)\left[1 + \beta_m \log\frac{\mu_T}{\mu}\right] + A\lambda T^2$$

$$\lambda_3 = \lambda(\mu)T\left[1 + \beta_\lambda \log\frac{\mu_T}{\mu}\right]$$

β_m and β_λ are the β-functions corresponding to the running mass and self-coupling.

To find A and μ_T: compute diagrams



In the modified minimal subtraction scheme $\overline{\mathrm{MS}}$:

$$\mu_T = 4\pi T e^{-\gamma} \simeq 7T, \quad A = \frac{1}{4}$$

Renormalization group improvement: $\mu = \mu_T$ minimizes the corrections,

$$\lambda_3 = \lambda(\mu_T)T, \quad m_3^2 = \frac{1}{4}\lambda(\mu_T)T^2 + m^2(\mu_T)$$

Phase transition

True order parameter, related to the symmetry $\phi \to -\phi$. Symmetry restoration around the temperature where $m_3(T) = 0$:

$$T_c^2 \simeq -\frac{4m^2}{\lambda}, \quad T_c = 2v, \quad v^2 = -\frac{m^2}{\lambda}$$

The perturbative analysis of the effective theory near $T = T_c$ breaks down, since the expansion parameter $\lambda_3/|m_3|$ diverges near this point. Non-perturbative methods (such as ϵ-expansion or lattice simulations) are needed in order to clarify the nature of the system here.

The super-renormalizable theory gives the accuracy in Green's function $\frac{\Delta G}{G} \sim O(\lambda^2)$ provided we have spatial momenta $k \ll T$ and a 3d mass $|m_3^2| \ll T^2$.

Advantages of effective field theory:

4d: perturbation theory is valid at $\lambda \ll 1$ and $T^2 \ll \frac{|m^2|}{\lambda}$
4d: theory at $T^2 > \frac{m^2}{\lambda}$ contains at least two energy scales: an ultraviolet one $\sim T$ and the infrared one $\sim m_3$

The *construction* of the effective 3d field theory requires only $\lambda \ll 1$ and $|m^2| \ll (\pi T)^2$ and contains only one energy scale
3d theory is applicable near the temperature of the phase transition T_c. Far from T_c the theory is perturbatively solvable.

Electroweak theory

Universal effective theory for EW phase transition: 3d S(2)$\times$U(1) gauge theory with one Higgs doublet

$$L = \frac{1}{4} G^a_{ij} G^a_{ij} + \frac{1}{4} F_{ij} F_{ij} + (D_i \Phi)^\dagger (D_i \Phi) + \bar{m}_3^2 \Phi^\dagger \Phi + \bar{\lambda}_3 (\Phi^\dagger \Phi)^2$$

The four parameters of the 3d theory (scalar mass $\bar{m}_3^2$, scalar self-coupling constant $\bar{\lambda}_3$, and two gauge couplings $\bar{g}_3$ and $\bar{g}_3'$) are some functions of the initial parameters and temperature.

Effective description valid for MSM, MSSM and two Higgs doublet models in large part of parameter-space, etc.

Where are other scalars?

- Squarks, $m^2 \sim g_s^2 T^2$;
- Extra doublets, $m^2 \sim g_W^2 T^2$;
- Extra triplets (coming from zero components of gauge fields), $m^2 \sim g_W^2 T^2$

all have masses masses large than the infrared scale $\alpha_W T$ and can be integrated out in the vicinity of $T \simeq T_c$.

Expressions for MSM.

for couplings (tree) $g_3^2 = g^2 T, \quad g_3'^2 = g'^2 T, \quad \lambda_3 = \lambda T,$

for the scalar mass (one loop):

$$m_3^2(T) = -\frac{1}{2} m_H^2 + T^2 \left(\frac{1}{2}\lambda + \frac{3}{16} g^2 + \frac{1}{16} g'^2 + \frac{1}{4} f_t^2 \right)$$

(Potential) phase transition occurs near $T : m_3^2(T) = 0$.

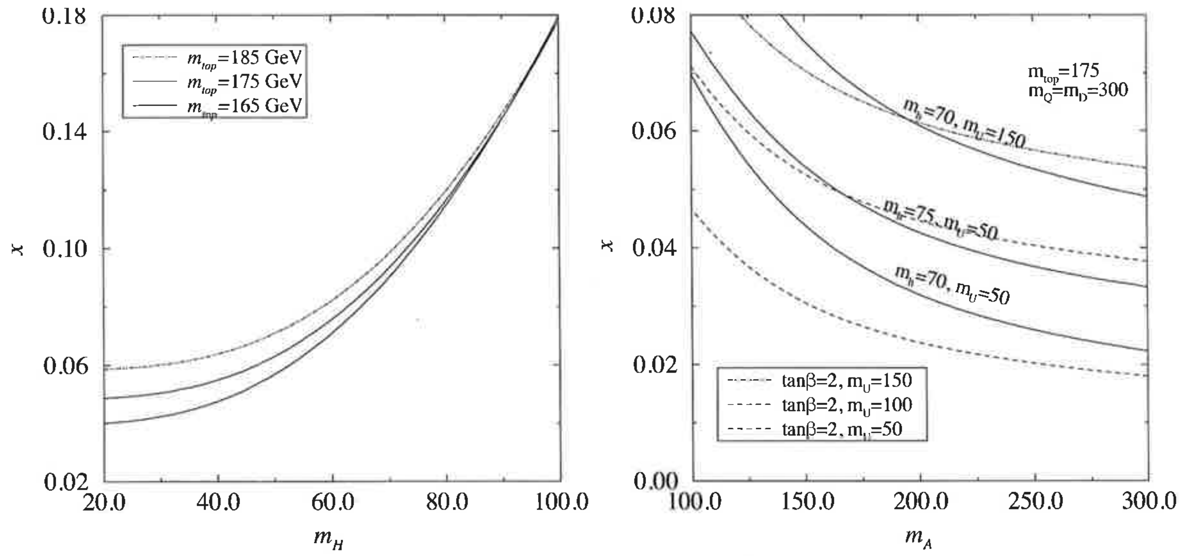
Electroweak phase transition

Relevant parameters:

- dimensionful parameter $g_3^2 \sim g^2 T$
- $z = tan\theta_W$ – known experimentally
- $y \equiv \frac{\bar{m}_3^2}{g_3^4}$ – changes with T.
- $x \equiv \frac{\bar{\lambda}_3}{g_3^2}$ – fixes properties of the EW phase transition

To solve the problem of phase transition in some version of EW theory:

- Compute perturbatively x
- Use the known non-perturbative results of the phase structure of effective 3d theory (lattice)



EW phase diagram

Naive consideration:

- $y > 0$: EW symmetry is restored, $\phi = 0$, gauge bosons are massless
- $y < 0$: EW symmetry is broken, $\phi \neq 0$, gauge bosons are massive

$$\Downarrow$$

First-order or second-order phase transition between the two phases

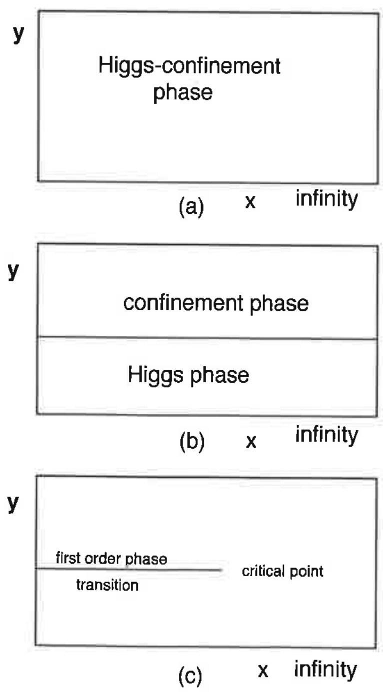
Complimentarily:

- $y > 0$: "Confinement" - bound states
  $\pi = \Phi^\dagger\Phi, \quad W_j^0 = i(\Phi^\dagger D_j\Phi - (D_j\Phi)^\dagger\Phi),$
  $W_j^+ = (W_j^-)^* = (\Phi^\dagger D_j\tilde{\Phi} - (D_j\Phi)^\dagger\tilde{\Phi})$
- $y < 0$: Massive "elementary" excitations – check that in unitary gauge "composite" fields are the same as "elementary" fields up to some irrelevant factor!

$$\Downarrow$$

no gauge-invariant local order-parameter that can distinguish between the “broken" (Higgs) and “restored" (symmetric or confinement) phases

Three possible types of phase diagram for the SU(2) gauge–Higgs system.



1-loop effective potential

$$V_1(\phi) = \frac{1}{2}m_3^2\phi^2 + \frac{1}{4}\lambda_3\phi^4 - \frac{1}{12\pi}\left(6m_T^3 + m_1^3 + 3m_2^3\right)$$

Estimate the value of the field ϕ at the *local maximum* of the effective potential.

Small λ_3: $\phi_{max} \sim \frac{g_3^3}{\pi\lambda_3}$.

Expansion parameter at this point: $\frac{g_3^2}{\pi m_T} \sim \frac{\lambda_3}{g_3^2} = x$.

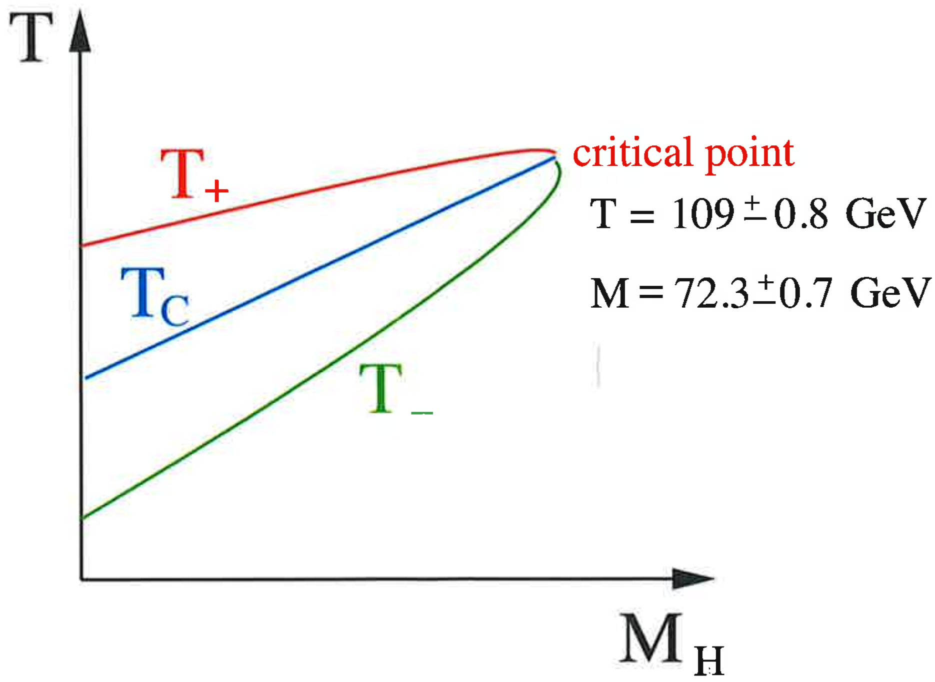
$\Downarrow$

Existence of the maximum of the effective potential is reliable at small values of x!

$\Downarrow$

First order phase transition at small x.

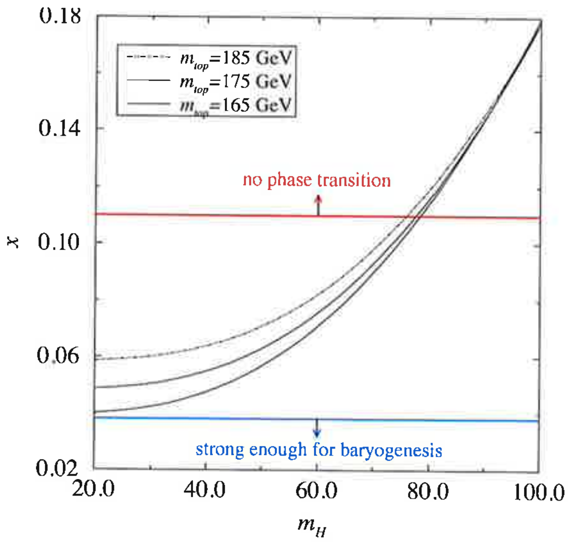
3d lattice Monte-Carlo simulations: absence of first-and second-order phase transitions at $x > 0.18$.



Near the critical point (the end-point of the first-order phase transition line) the 3d gauge–Higgs system admits a further simplification at large distances $\gg \frac{1}{g_3^2}$. At the critical point the phase transition is of second order, thus there is a massless scalar particle. The effective theory describing this nearly massless state is a simple scalar theory of some field χ with the Lagrangian

$$L = \frac{1}{2}(\partial_i \chi)^2 + \frac{1}{2}m^2\chi^2 + \lambda_\chi \chi^4 + h\chi.$$

Baryon asymmetry in the standard model



Baryon asymmetry is not generated

$\Downarrow$

Way out: Physics beyond the standard model

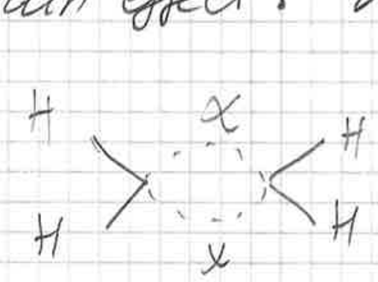
Electroweak Baryogenesis

SM does not have electroweak phase transition. If we want it, we should go beyond it. Phase transitions - scalar sector - modifications of the scalar sector. The simplest possibility: extra singlet scalar field. Adding new physics also helps with with CP-violation.

Example:

$$\Delta S = \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2} m^2 \chi^2 - \frac{\lambda_\chi}{4} \chi^4 - \frac{g}{4} H^\dagger H \chi^2$$

Main effect: decoupling of λ_3 - effective Higgs self-coupling from the Higgs mass



integrating out of scalar field χ leads to contribution to $\lambda_3 \sim -\frac{g^2 T}{m_\chi(T)}$

This decreases λ_3 and may lead to 1st order phase transition with observed Higgs.

BSM models constructed to have 1st order PT:

- complex scalar
- extra doublet
- low energy supersymmetry
- composite Higgs
- adding higher-dimensional operators

Common feature: new physics around the Fermi scale.

Main challange: hide new physics so that it does not show up at LHC and other experiments (e.g. electron dipole moment of n, anomalous magnetic moment of μ, $\mu \to e\gamma$, etc).

Suppose we have a theory with 1st order phase transition: how to create BAU?

EW baryogenesis

Cohen Kaplan & Nelson, 1991

1. Symmetric phase: $\langle\phi^\dagger\phi\rangle \simeq 0 \longrightarrow$ quarks are almost massless and B-non-conservation is rapid.

2. Higgs phase: $\langle\phi^\dagger\phi\rangle \neq 0 \longrightarrow$ quarks are massive and B-non-conservation is exponentially suppressed.

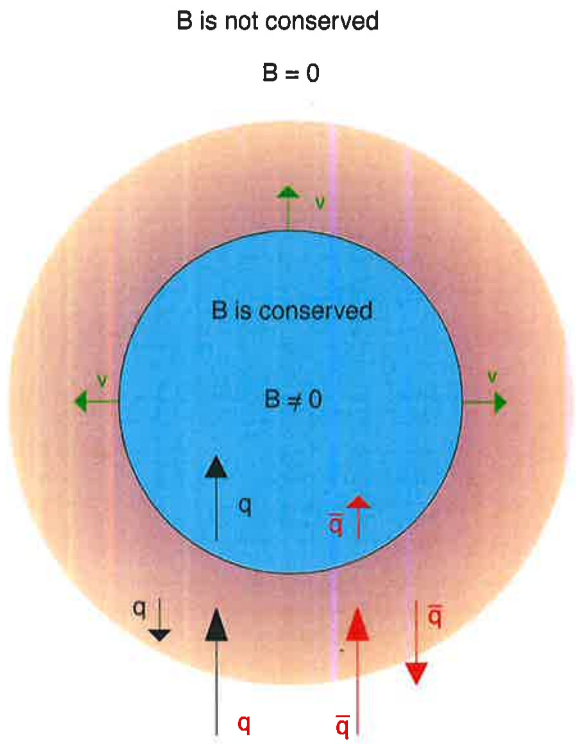
$\Downarrow$

Quarks interact in a CP-violating way (reflected and transmitted) with the surface of the bubble

$\Downarrow$

Baryon asymmetry of the Universe after EW phase transition.

Mechanism



Necessary condition: survaval of baryon asymmetry inside the bubbles $\Rightarrow$

$$\frac{\Gamma_{sph}}{H} \lesssim 1 \quad \text{at} \quad T = T_c \qquad \text{M.S., 1986}$$

in the SM this would require $\frac{v}{T} \gtrsim 1.2$

Detailed computations are complicated, should account for

- bubble nucleation rate
- bubble dynamics, their velocity
- interaction of quarks and other particles with domain walls
- interaction of quarks and other particles with the medium, transport, diffusion
- CP-violation must be included

For review see Ramsey-Musolf, 2012

most recent

Rough estimate of BAU: (and most optimistic)

$$\Delta \sim \frac{1}{N}\,(t-\bar{t})\cdot v$$

velocity of the bubble wall

transmission coefficients

assumption: all quarks in symmetric phase are „processed" by sphalerons

If not, extra suppression factor,

diffusion coefficient for baryon #

$$\sim \frac{D_B \Gamma_{sph}}{v^2}\,,\ \text{may be} \ll 1\,.$$

$$\frac{\partial n_B}{\partial t} = -D_B \nabla^2 n_B$$

Net result: may work for extentions of the SM, but is challenging to hide new physics which is needed for this mechanism.

Neutrino physics: see-saw Lagrangian

Neutrinos have masses – plenty of experimental evidence:

- atmospheric ν
- Solar ν
- reactor ν

$$\Delta m^2_{atm} = 2.4 \cdot 10^{-3}\, eV^2$$

$$\Delta m^2_{solar} = 7.5 \cdot 10^{-5}\, eV^2$$

$$\sin^2 2\theta_{13} = 0.09 \pm$$

$$\sin^2 2\theta_{12} = 0.85 \pm$$

$$\sin^2 2\theta_{23} = 1 \pm$$

Dirac phase in PMNS, 2 Majorana phases, and type of hierarchy is unknown

ν – The only high energy physics experimental evidence for BSM physics. Two others: BAU & Dark matter – come from cosmology.

Neutrinos have non-zero masses - how to incorporate this into the Standard Model? Effective field theory approach: low energy Lagrangian can contain all sorts of higher-dimensional SU(3)×SU(2)×U(1) invariant operators, suppressed by some unknown scale Λ:

$$L = L_{\rm SM} + \sum_{n=5}^{\infty} \frac{O_n}{\Lambda^{n-4}} \,.$$

Majorana neutrino mass: from five-dimensional operator

$$O_5 = A_{\alpha\beta}\left(\bar{L}_\alpha \tilde{\phi}\right)\left(\phi^\dagger L_\beta^c\right)$$

Neutrino mass matrix:

$$M_\nu \sim A_{\alpha\beta}\frac{v^2}{\Lambda}$$

Crucial questions:

- What is the physics behind non-renormalizable terms?
- What is the value of Λ ?

First try: origin of neutrino masses - unknown Planck scale physics, related to quantum gravity,

$$\Lambda = M_{Pl} \simeq 10^{19}\ \mathrm{GeV}$$

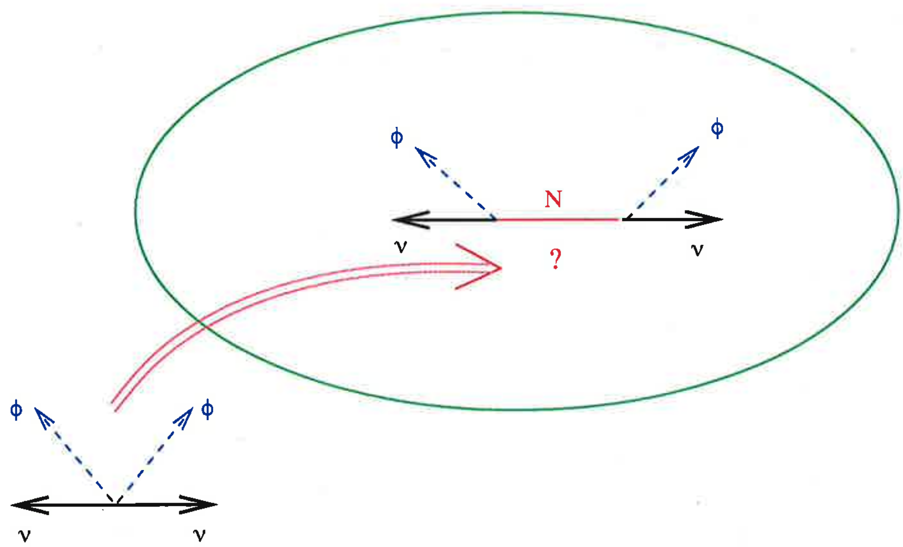
Prediction :

$$m_\nu \sim \frac{v^2}{M_{\rm Pl}} \simeq 10^{-6}\ \mathrm{eV}$$

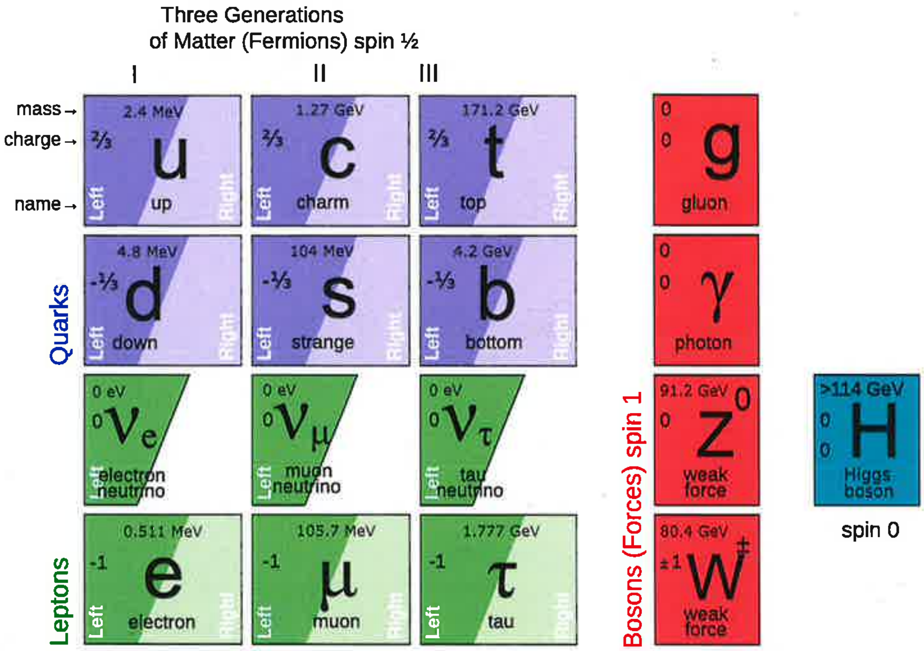
Far away from experimental observations - does not work!

Second try: origin of neutrino masses - existence of new unseen particles; complete theory is renormalisable

- Higgs triplet with hypercharge 2 - direct contribution to neutrino mass
- Singlet Majorana fermions - effective contribution to neutrino mass



the SM



The missing piece: sterile neutrinos

Most general renormalizable (see-saw) Lagrangian

$$L_{see-saw} = L_{SM} + \bar{N}_I i\partial_\mu\gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I\Phi - \frac{M_I}{2}\,\bar{N}_I^c N_I + h.c.,$$

Assumption: all Yukawa couplings with different leptonic generations are allowed.

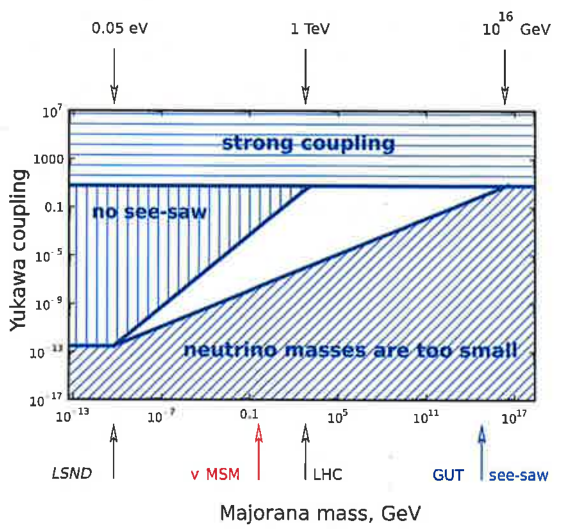
$I \leq \mathcal{N}$ - number of new particles - HNLs - cannot be fixed by the symmetries of the theory.

Let us play with $\mathcal{N}$ to see if having some number of HNLs is good for something

- $\mathcal{N} = 1$: Only one of the active neutrinos gets a mass
- $\mathcal{N} = 2$: Two of the active neutrinos get masses: all neutrino experiments, except LSND-like, can be explained. The theory contains 3 new CP-violating phases: baryon asymmetry of the Universe can be understood
  item $\mathcal{N} = 2$: Two of the active neutrinos get masses: all neutrino experiments, except LSND-like, can be explained. The theory contains 3 new CP-violating phases: baryon asymmetry of the Universe can be understood
- $\mathcal{N} = 3$: All active neutrinos get masses: all neutrino experiments, can be explained (LSND with known tensions). The theory contains 6 new CP-violating phases: baryon asymmetry of the Universe can be understood. If LSND is dropped, dark matter in the Universe can be explained. The quantisation of electric charges follows from the requirement of anomaly cancellations (1-3-3, 1-2-2, 1-1-1, 1-graviton-graviton).
- $\mathcal{N} > 3$: Now you can do many things, depending on your taste - extra relativistic degrees of freedom in cosmology, neutrino anomalies, dark matter, different scenarios for baryogenesis, and different combinations of the above.

New mass scale and Yukawas

$$Y^2 = Trace[F^\dagger F]$$



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Parameter counting:

(1) νMSM.

arbitrary complex

originaly: $F_{\alpha I}$: $3\times3\times2 = 18$ real parameters

M_{IJ} : $\frac{3\cdot 4}{2}\cdot 2 = 12$ real parameters

Symmetric complex

(i) Diagonalize $M_{IJ} \Rightarrow$ 3 real physical parameters

(ii) use 3 phases of L to remove 3 phases out of $F_{\alpha I} \Rightarrow$ 15 parameters.

$F_{\alpha I} = K \underbrace{m}_{} \tilde{K}$ 3 real angles

real diagonal, 3 parameters $\Rightarrow$

3 real angles

$15 = 3 + 6 + 6$

masses real angles phases

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

(2) ν- masses :

$(M_\nu)_{IJ}$: 12 real parameters

$-$ 3 phases $\Rightarrow$ 9 real parameters

$=$ 3 masses $+$ 3 angles $+$ $(1+2)$ phases

KM type

Majorana

HNLs - heavy neutral leptons - introduce new opportunities for baryogenesis

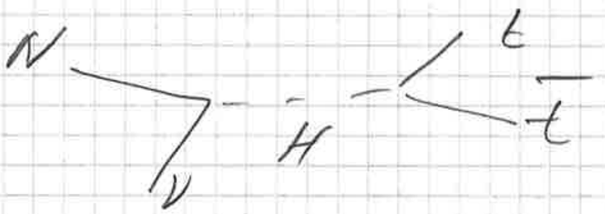
- new sources for CP-violation in comparison with SM
- new particles - new sources of lepton number violation, now we have not only sphalerons
- new particles - new sources for thermal non-equilibrium in comparison with standard model.

Rate of equilibration of new particles

Rough estimates, with the use of see-saw formula

$$m_\nu \sim \frac{f^2 v^2}{M_N} \Rightarrow f^2 \simeq \frac{m_\nu M_N}{v^2}$$

Process:



$$\Gamma \sim 10^{-2} f^2 T \cdot 4$$

$$H = \frac{T^2}{M_0}$$

$$M_0 = \frac{M_{pl}}{1.66 N^{1/2}} \approx 10^{18}\,\text{GeV}$$

$$\Gamma > H \quad \text{for} \quad 4\cdot 10^{-2} f^2 T > \frac{T^2}{M_0} ;$$

$$T < 10^{-2} f^2 M_0 \approx 10^{-2} M_0 \cdot \frac{m_\nu M_N}{v^2} \approx$$

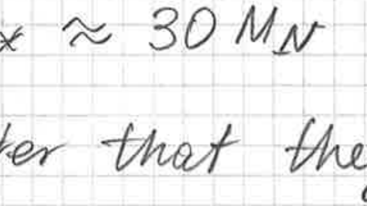
$$\approx 4\cdot 10^{-2} \cdot 10^{18} \cdot 5\cdot 10^{-11} \Big/ (250)^2 \cdot M_N \simeq 30 M_N$$

we take
$m_\nu \approx 0.05\ eV$

So, these particles enter into thermal equilibrium at the freeze-in temperature $T_{**} \approx 30\, M_N$

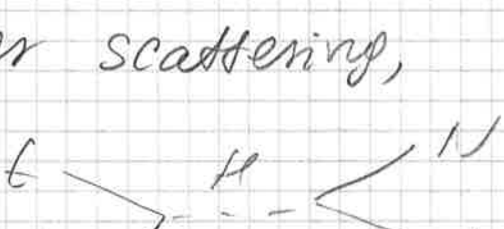
After that they go out of thermal equilibrium because at $T \lesssim M_N$ the processes which can refill their concentration are exponentially suppressed;

for example inverse decay



$\exp\left(-\frac{M_N}{T}\right)$

or scattering,

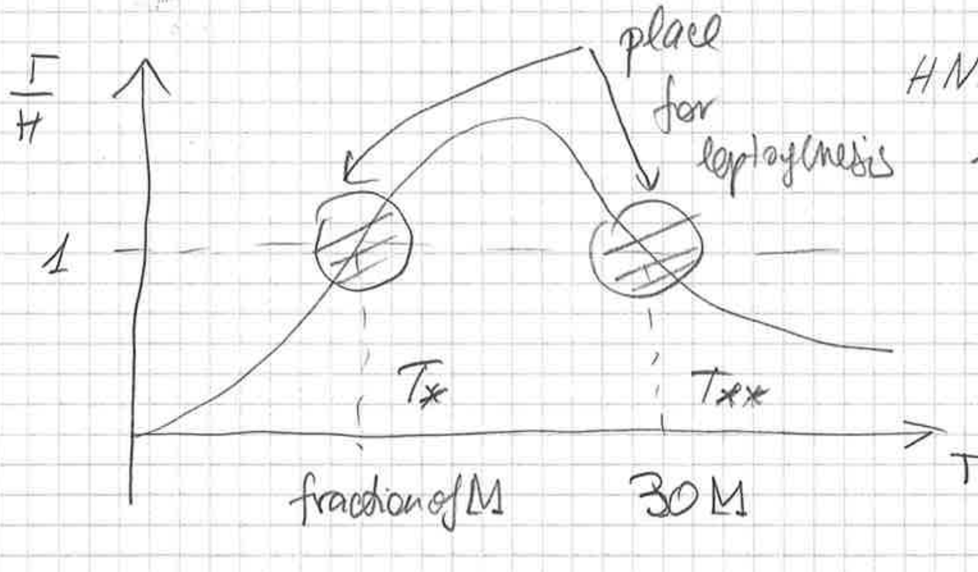


, also $\sim \exp\left(-\frac{M_N}{T}\right)$

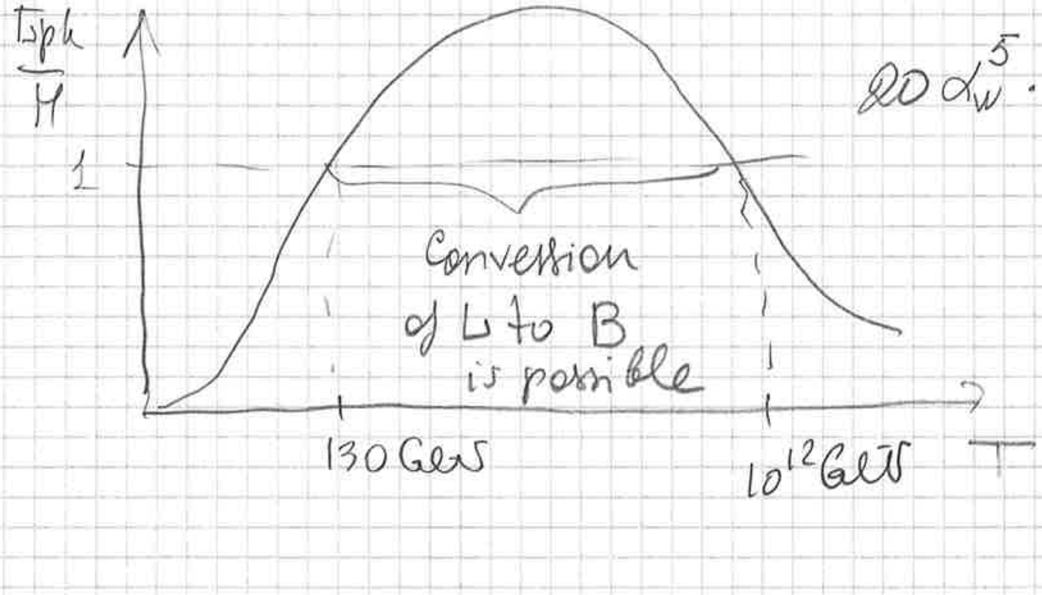
They go out of thermal equilibrium, for $M_N \gtrsim M_W$, at $T^* \sim$ fraction of M.

If $M_N \lesssim M_W$, the estimates are changing [Higgs has a mass, interaction rates are suppressed by $G_F \simeq 10^{-5}\,\mathrm{GeV}^{-2}$ (Fermi Constant), but qualitative behaviour is the same.

When we can have baryogenesis?



HNL interact with leptons $\Rightarrow$ can produce lepton asymmetry



$$20\,\alpha_W^5 \cdot T \approx \frac{T^2}{M_0}$$

$$T \approx 10^{12}\,\mathrm{GeV}$$

Two possibilities:

(i) $M_N \gtrsim T_{sph} \simeq 130$ GeV $\Longrightarrow$

N go out of equilibrium at $T \gtrsim 130$ GeV.

Lepton asymmetry is produced at $T = T_*$

"Thermal leptogenesis",

Fukugita & Yanagida 1986

(ii) $M_N \lesssim T_{sph}$,

N enter into thermal equilibrium at $T \gtrsim 130$ GeV, lepton asymmetry is produced at $T = T_{**}$

[go out at $T \lesssim T_{sph}$ – this moment is irrelevant for baryogenesis]

"low scale leptogenesis",

Akhmedov, Rubakov, Smirnov, 1998, idea

Asaka, MS, 2005, kinetic description

Thermal leptogenesis

Main idea

Stage 1: N enter in thermal equilibrium at $T^{**} \simeq 30M$, initial conditions are „forgotten".

Stage 2: N go out of thermal equilibrium at T^* and decay, producing lepton asymmetry

Stage 3: Lepton asymmetry is converted to baryon asymmetry due to sphalerons.

Stage 2 in more detail

N — Majorana particles, can decay as follows:

$$N \to h^+ l^-, \ h^- l^+, \ h^0 \bar{\nu}, \ \bar{h}^0 \nu$$

Combining $N \to \bar{h} l$, $N \to h \bar{l}$

$$\Gamma_l \qquad\qquad \bar{\Gamma}_l$$

$$CP: \Gamma_l \neq \bar{\Gamma}_l$$

Total number of leptons created in decays of N:

$$\delta = \frac{\Gamma_e - \overline{\Gamma}_e}{\Gamma_e + \overline{\Gamma}_e} \neq 0.$$

Lepton asymmetry:

$$\Delta_L \simeq \delta \cdot \frac{(N_N - N_{eq})}{s}$$

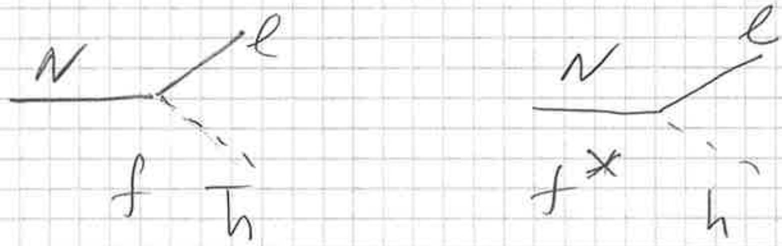
„macro" factor, accounting for kinetics

„micro" factor, accounting for CP-violation in the theory

entropy density

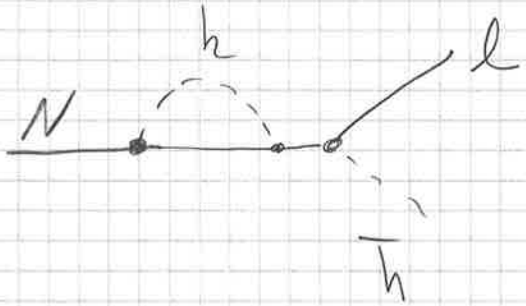
„Micro" factor δ: loop corrections to decay rates

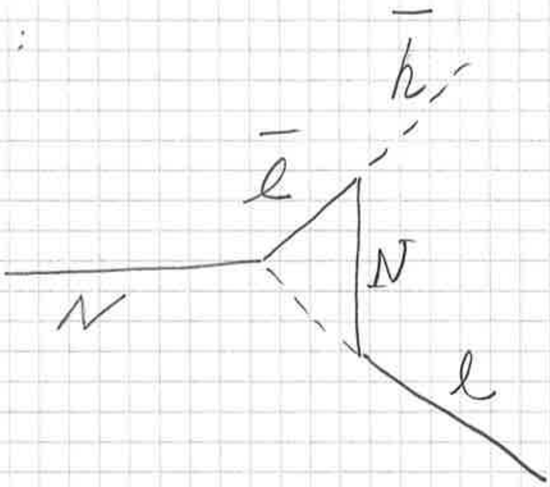
tree approximation: $\delta = 0$



$$f \cdot f^* = f^* \cdot f \Rightarrow \delta = 0$$

+ 1-loop diagrams :





$= F_1 \cdot A$

product of Yukawas

value of the loop

$= F_2 \cdot B$

product of Yukawas

value of the loop

$$\Gamma(N \to l\bar{h}) = \left| f + F_1 A + F_2 \cdot B \right|^2$$

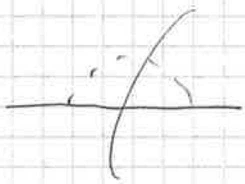
$$\Gamma(N \to \bar{l} h) = \left| f^* + F_1^* A + F_2^* B \right|^2$$

$$\Gamma(N \to l\bar{h}) \approx f f^* + F_1 f^* A + F_2 f^* B$$
$$+ F_1^* f A^* + F_2^* f B^* + \dots$$

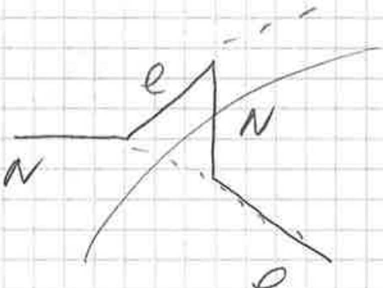
$$\Gamma(N \to \bar{l} h) \approx f^* f + F_1^* f A + F_2^* f B$$
$$+ F_1 A^* f^* + F_2 B^* f^*$$

$$\Delta\Gamma \propto \mathrm{Im}(F_1 f^*)\,\mathrm{Im}\,A + \mathrm{Im}(F_2 f^*)\,\mathrm{Im}\,B$$

This is not zero if coupling constants are complex (CP-violation)
and if diagrams have absorbtive parts:



$\neq 0$



$\neq 0$

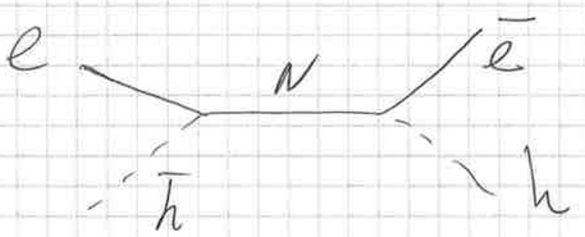
Diagrams with cuts

Physical sense:
the diagrams should „know" that there is lepton # nonconservation:
Channel with $N \to \bar{e}h$ must show up in the diagram for $N \to e\bar{h}$ decay.

Note: of course, all N's must be considered

– other processes must be included as well, for example:

scattering



If energies of l and $\bar{h}$, $(p_e + p_{\bar{h}})^2 = m_N^2$ — resonance, similar contribution to decays.

<u>Macro factor</u> — accounts for kinetics of decays.

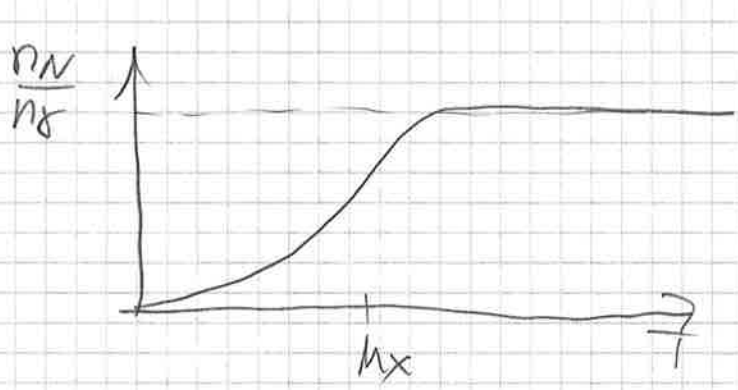
Thermal equilibrium — the rates of decays and inverse decays are the same

Rate of decays:

$$\Gamma \simeq \begin{cases} \Gamma_N \dfrac{m_N}{T}, & T \gg m_N \\ \Gamma_N, & T \ll m_N \end{cases} \quad \left(\frac{T}{M_N} \text{ - Lorentz factor}\right)$$

decay rate at rest

Equilibrium concentration



Decay temperature:

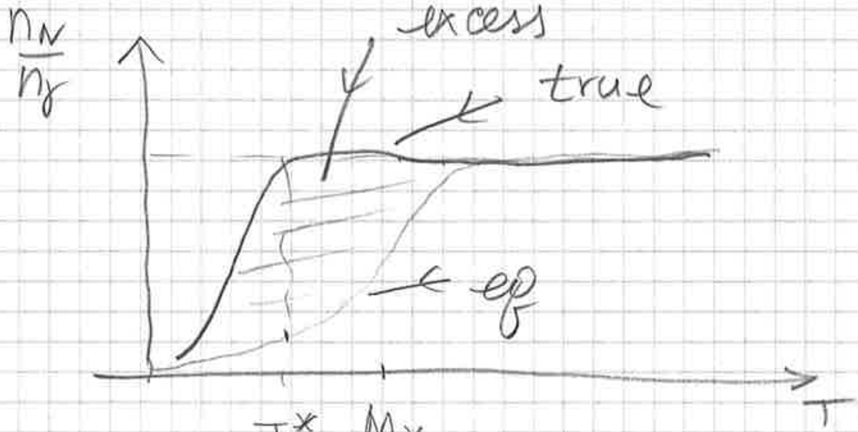
$$\Gamma = \frac{T^2}{M_0}\,, \quad T^* \approx \sqrt{\Gamma_N M_0} \quad \text{for} \quad T^* < M_N\,, \quad \frac{M_N^2}{\Gamma_N M_0} \gtrsim 1$$

$$\Gamma_N \frac{M_N}{T} = \frac{T^2}{M_0}\,; \quad T^* = (\Gamma_N M_X M_0)^{1/3}\,, \quad T^* > M_N\,, \quad \frac{M_N^2}{\Gamma_N M_0} \lesssim 1$$

Let $T^* < M_N \Longrightarrow \frac{n_N}{n_\gamma} \simeq 1$ until T^*,

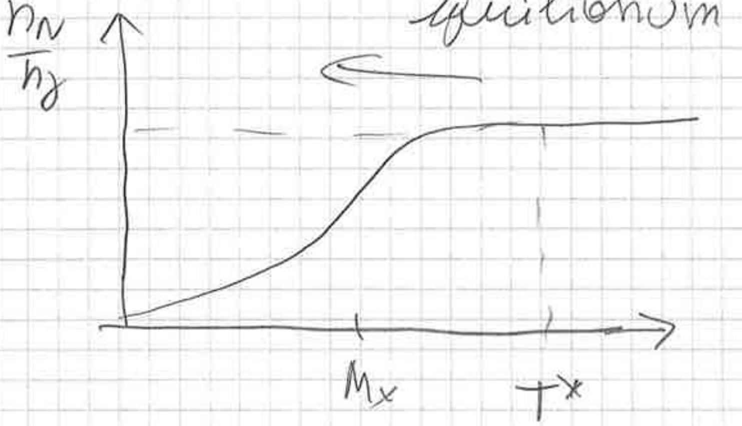
Since particles are not decaying (we also assumed that that the processes $2 \leftrightarrow 2$ like $t\bar{t} \to N\ell$ are out of thermal equilibrium)

True behaviour of n_N:



$$\Delta_L \simeq \delta \cdot \frac{1}{N_{eff}}\,, \quad \text{i.e.} \quad \frac{n_X - n_{eq}}{s} \simeq \frac{1}{N_{eff}}$$

Let now $T^* > M_N$, then n_N follows equilibrium distribution



In the first approximation $\Delta_L = 0$, in fact

$$\Delta \simeq \delta \frac{1}{N_{eff}} \cdot \underbrace{\frac{m_N^2}{\Gamma_N M_0}}_{\text{deviation from thermal equilibrium}} \quad (*)$$

Constraints on the mass : $\frac{m_N^2}{\Gamma_N M_0} \simeq \frac{m_N^2}{\frac{f^2}{4\pi} m_N M_0} \simeq \frac{4\pi v^2}{m_0 M_0} \simeq 10^{-2}$

$$\delta \sim f^4 \frac{1}{(4\pi)^2} ; \quad \Gamma \simeq \frac{f^2 M_N}{4\pi}$$

So, we can use (*)

$$\Delta \simeq f^4 \frac{1}{(4\pi)^2} \frac{1}{N_{eff}} \cdot \frac{m_N^2}{\frac{f^2 M_N}{4\pi} M_0} \gtrsim 10^{-10}$$

$$\Delta \simeq \frac{1}{4\pi N_{eff}} f^2 \frac{m_N}{M_0} \simeq \frac{1}{4\pi N_{eff}} \frac{f^2 v^2}{m_N} \frac{m_N^2}{v^2 M_0} \simeq$$

$$\simeq \frac{1}{4\pi N_{eff}} \frac{m_\nu m_N^2}{v^2 M_0} \gtrsim 10^{-10} \Rightarrow$$

$$m_N \gtrsim \left(\frac{4 \cdot 3 \cdot 100 \cdot 10^{-10} \cdot 6 \cdot 10^{4} \cdot 10^{18}}{8 \cdot 10^{-11}} \right)^{1/2} \simeq$$

$$\simeq \left(10^{11+3-10+4+18} \right)^{1/2} \simeq 10^{13}\, GeV$$

More refined bound,

for $M_2 \gg M_1$; $M_3 \gg M_1$, decays of N_1:

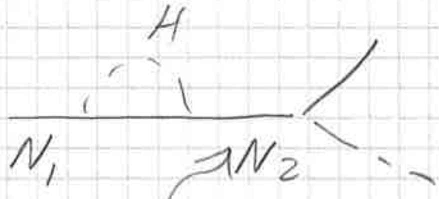
$$m_{N_1} \gtrsim 10^{9}\, GeV$$

(allows to have larger Yukawa couplings $\Rightarrow$ smaller M_N)

for more details see Davidson Nardi Nir Review.

Important: account for complicated pattern of ν mixing and masses.

In fact, the mass can be much smaller, if there is degeneracy between HNL's



Study of „resonant" leptogenesis, see Pilaftsis et al.

propagator, $\frac{1}{(M_1^2 - M_2^2)}$: enhancement factor, which may lead to $\delta \sim 1$

Important constraint: N must decay before $T_{sph} \approx 130$ GeV, otherwize lepton asymmetry cannot be transferred into baryon asymmetry:

$$\frac{M_N}{T_{sph}} \frac{f^2 M_N}{4\pi} \gtrsim \frac{T_{sph}^2}{M_0} \quad ; \text{ or}$$

$$\frac{M_N}{T_{sph}} \frac{f^2 v^2}{4\pi M_N} \frac{M_N^2}{v^2} = \frac{m_\nu M_N^2}{T_{sph} 4\pi v^2} \gtrsim \frac{T_{sph}^2}{M_0} \Rightarrow$$

$$M_N \approx \left(\frac{4\pi v^2 T_{sph}^3}{m_\nu M_0} \right)^{1/3} \approx 30 \text{ GeV}$$

In fact, the estimate of the width of N was based on the process $N \to Hl$, and requires $M_N \gtrsim M_H \simeq 130$ GeV. If N is lighter, then the width of N is suppressed by $G_F^2 \Longrightarrow$ So the more reliable estimate is $m_N \gtrsim 130$ GeV.

In overall, thermal leptogenesis works and is consistent with ν-masses and oscillations.

<u>Predictivity of thermal leptogenesis</u>

[Knowing ν-physics, can we predict the magnitude and sign of BAU?] (no)

Can we predict something for ν-physics assuming thermal leptogenesis

(no)

117

Most general renormalisable see-saw Lagrangian with Majorana neutrinos:

Minkowski; Yanagida; Gell-Mann, Ramond, Slansky; Glashow, Mohapatra, Senjanovic

$$L_{\nu MSM} = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I \Phi - \frac{M_I}{2}\,\bar{N}_I^c N_I + h.c.,$$

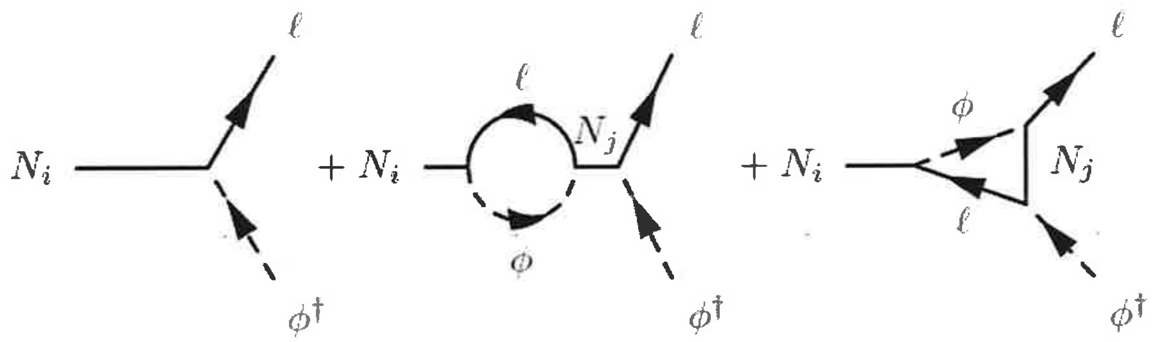
Counting “high energy” parameters, 3 HNLs:
3 Majorana masses of new neutral fermions N, 15 new Yukawa couplings in the leptonic sector (3 Dirac neutrino masses, 6 mixing angles and 6 CP-violating phases), 18 new parameters in total.

Counting “very low energy” parameters, 3 HNLs:

3 Majorana masses of active neutrinos, 3 mixing angles in PMNS matrix, 1 Dirac phase and 2 Majorana phases, 9 parameters in total, 6 of them can be measured in active neutrino oscillations

See over

The mechanism: leptogenesis with superheavy Majorana neutrinos (Fukugita, Yanagida) : HNLs go out of thermal equilibrium, decay, and produce lepton asymmetry at temperatures $T \sim 10^{10}\ GeV$. Then the lepton number is converted into baryon asymmetry by sphalerons which are active until $T \simeq 130\ GeV$. The resulting baryon asymmetry is just a numerical factor of order one smaller than the lepton asymmetry.



In general, baryon asymmetry depends on all high-energy parameters of the model. There are 18 of them. In the very best case we can only determine 9 combinations of them via the see-saw formula in low energy neutrino experiments. Therefore, neither amplitude no sign of baryon asymmetry can be predicted.

Question: Can we chose high energy parameters in such a way that we are consistent with low energy neutrino experiments and produce the necessary baryon asymmetry?

Answer: Yes, the freedom is pretty large: baryon asymmetry is just one number, and we have 9 parameters to play with!

Question: Can we get baryon asymmetry just from low energy CP-violating phases? To make sense of this question, consider Casas-Ibarra parametrisation of the matrix of Yukawa couplings:

$$F = \frac{1}{v} U_{PMNS} \sqrt{m_\nu} R \sqrt{M}$$

Here R is complex orthogonal matrix depending on 3 complex angles. Make these angles real or some of them pure imaginary to get rid of high-energy complex phases (ad-hoc choice).

Answer: Yes, the freedom is still pretty large! (Moffat, Pascoli, Petcov Turner '18)

Question: Can we get baryon asymmetry just from low energy Dirac phase (i.e. put all Majorana phases to zero)?

Answer: Yes, the freedom is still pretty large! (Moffat, Pascoli, Petcov Turner '18)

Question: Can we get baryon asymmetry if low energy CP phases are zero?

Answer: Yes, no problem!

see over

Let us decrease the number of parameters: assume that only 2 HNLs exist

Counting "high energy" parameters, 2 HNLs:
2 Majorana masses of new neutral fermions N, 9 new Yukawa couplings in the leptonic sector (2 Dirac neutrino masses, 4 mixing angles and 3 CP-violating phases), 11 new parameters in total.

Counting "very low energy" parameters, 2 HNLs:

2 Majorana masses of active neutrinos (one is almost massless), 3 mixing angles in PMNS matrix, 1 Dirac phase and 1 Majorana phases, 7 parameters in total, 6 of them can be measured in active neutrino oscillations

Still, 11>7 and therefore, neither amplitude no sign of baryon asymmetry can be predicted.

Question: Can we chose high energy parameters in such a way that we are consistent with low energy neutrino experiments and produce the necessary baryon asymmetry?

Answer: Yes, the freedom is pretty large: baryon asymmetry is just one number, and we have 4 parameters to play with!

Question: Can we get baryon asymmetry just from low energy CP-violating phases?

Answer: Yes, the freedom is still pretty large (3 parameters)! (Moffat, Pascoli, Petcov Turner '18)

Question: Can we get baryon asymmetry just from low energy Dirac phase (i.e. put all Majorana phases to zero)?

Answer: Yes, the freedom is still pretty large (2 parameters)! (Moffat, Pascoli, Petcov Turner '18)

Question: Can we get baryon asymmetry if low energy CP phases are zero?

Answer: Yes, no problem!

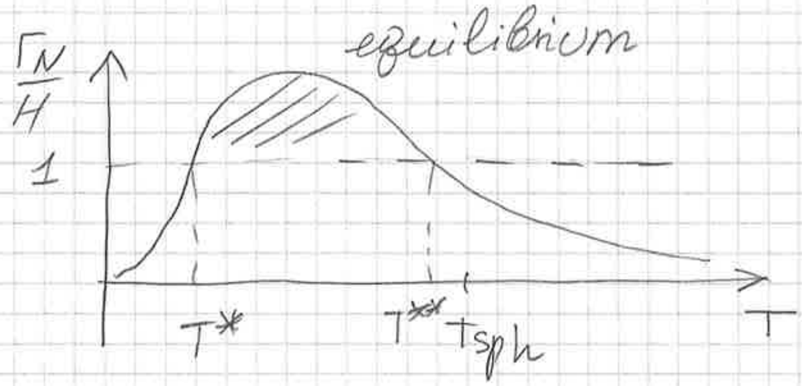
Conclusions for see-saw leptogenesis

- It is impossible to find the sign and amplitude of BAU in see-saw models, as we do not (and will not) have an access to essential information about high scales experimentally.

- BAU can be explained with low energy Dirac phase only, but there are no convincing arguments why other phases should vanish.

see over

Low scale leptogenesis and baryogenesis



$T^* \leq T_{sph}$: out-of-equilibrium below T^* cannot be used.

Domain of $T \gtrsim T_{sph}$ HNL's are out of thermal equilibrium $\Rightarrow$ production of lepton asymmetry, and processing it into baryon asymmetry due to sphalerons.

Plan

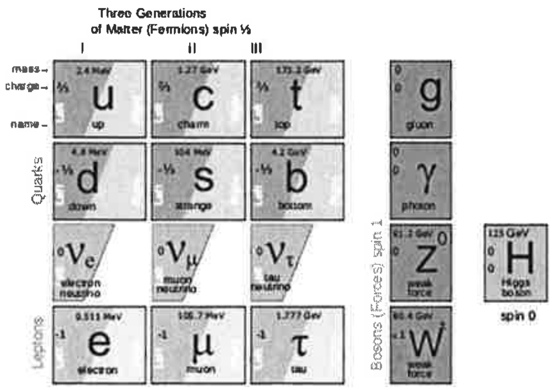
- small mass HNL's – νMSM
- kinetic description
- predictions
- testability

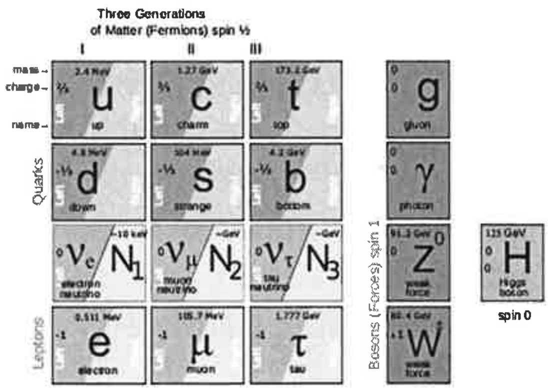
124

Low scale leptogenesis,

particle physics motivation

HNL masses are similar to SM quark and lepton masses: SM-> NuMSM





Role of the Higgs boson: break the symmetry and inflate the Universe
Role of N_1 with mass in keV region: dark matter.
Role of N_2, N_3 with mass in 100 MeV – GeV region: "give" masses to neutrinos and produce baryon asymmetry of the Universe

125

Yukawa couplings:

- KeV scale DM sterile neutrino N1: $F \sim 10^{-13}$ to have sufficiently large lifetime
- GeV scale N2 and N3: $F \sim 10^{-6}$ to explain neutrino masses

Note: the SM does not provide any explanation of the origin and magnitude of Yukawa couplings of quarks and charged leptons, they are all taken from experiment and scatter from $f_t \sim 1$ for the top quark to $f_e \sim 10^{-5}$ for electron.

Leptogenesis with GeV HNLs

Creation of baryon asymmetry is a complicated process involving creation of HNLs in the early universe and their coherent CP-violating oscillations, interaction of HNLs with SM fermions, sphaleron processes with lepton and baryon number non-conservation. One need to deal with resummations, hard thermal loops, Landau-Pomeranchuk-Migdal effect, etc.

Initial idea: Akhmedov, Rubakov, Smirnov '98

Formulation of kinetic theory and demonstration that NuMSM can explain simultaneously neutrino masses, dark matter, and baryon asymmetry of the Universe: Asaka, M.S. '05

Analysis of baryon asymmetry generation in the NuMSM: Asaka, M.S., Canetti, Drewes, Frossard; Abada, Arcadia, Domcke, Lucente; Hernández, Kekic, J. López-Pavón, Racker, J. Salvado; Drewes, Garbrech, Guetera, Klariç; Hambye, Teresi; Eijima, Timiryasov; Ghiglieri, Laine,...

Since Yukawa couplings are small, large lepton (and thus - Baryon) asymmetry can only appear if we have resonance enhancement $\Rightarrow$ degeneracy. Kinetic description a'la Boltzmann is not possible, one has to take into account oscillations of HNL.

Elements of formalism: Sigl, Raffelt, 1993; Eijima, MS, 2018

We have two Majorana leptons N_2, N_3 with masses $M_2 \approx M_3$. (We keep N_1 for Dark Matter). Let us unify them into one Dirac fermion Ψ. Lagrangian can be written as:

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{\Psi} i \gamma^\mu \partial_\mu \Psi - M \bar{\Psi} \Psi + \mathcal{L}_{int}$$

$$\mathcal{L}_{int} = -\frac{\Delta M}{2} \left(\bar{\Psi} \Psi^c + \bar{\Psi}^c \Psi \right)$$

$$- h_{\alpha 2} \langle \Phi \rangle \bar{\nu}_{L\alpha} \Psi + h_{\alpha 3} \langle \Phi \rangle \bar{\nu}_{L\alpha} \Psi^c + h.c.$$

We neglect interaction with true Higgs excitation for simplicity.

Let us fix momentum of leptons, p.

Particle number density operators:

$$a_+^+ a_+ \; ; \; a_-^+ a_-$$

$$b_+^+ b_+ \; ; \; b_-^+ b_-$$

$+$ and $-$ are projections of spin on momentum,

$$- : \; \overrightarrow{}^{\,p} \; \overrightarrow{}_{\,\sigma} \qquad + : \; \overrightarrow{}^{\,p} \; \overleftarrow{}_{\,\sigma}$$

The operator
Density matrix for HNL's contains non-diagonal operators, such as:

$$a_+^+ a_- \; ; \; a_+^+ b_+ ; \; a_+^+ b_- , \text{ etc.}$$

and contains 16 elements in total

Symbolic notation of all these operators: Q_i^0, and the density matrix is

$$q_i^0 = Tr[Q_i^0 \rho]$$

Let us use Heisenberg representation.

We can write

$$i \dot{q}_i^0 = Tr\, [H, Q_i^0]\rho$$

Hamiltonian can be written as $H = H_0 + H_{int}$, with H_{int} corresponding to $\mathcal{L}_{int}$, and H_0 contains everything else.

Note that $[H_0, Q_i^0] = 0$, since all fermionic states $a_\pm^+ |0\rangle$ and $b_\pm^+ |0\rangle$ are degenerate. Commutator of $[Q_i^0, H_{int}]$ contains the terms of the structure

$a_\sigma^+ b_\nu$, $a_\sigma^+ b_\nu^+$, etc, where b_ν are the operators of neutrinos. To get a self-contain system of equations, commute these operators with H, and use the idea of separation of time scales:

- slow reactions, with rates $\sim h\nu$, are kept in the kinetic equations
- rapid reactions [e.g. all SM processes] are „integrated out". The final system of equations i obtained after truncation of the infinite chain of commutators [3 commutation are necessary]

and is written for 2 density matrices, for „particles" HNL, and „antiparticles" HNL, for chemical potentials to now almost conserved quantum numbers $\Delta_i = (L_i - \frac{1}{3}B)$, and to baryon number B.

For temperatures above the sphaleron freeze out B can be excluded.

The density matrices for HNL's are:

$$\left\langle \begin{matrix} a_+^+ a_+ & a_+^+ b_- \\ b_-^+ a_+ & b_-^+ b_- \end{matrix} \right\rangle \equiv \rho_N$$

$$\left\langle \begin{matrix} a_-^+ a_- & a_-^+ b_+ \\ b_+^+ a_- & b_+^+ b_+ \end{matrix} \right\rangle \equiv \rho_{\bar{N}} ,$$

all other ellements happen do disappear from kinetic equation.

$\omega_{\alpha\beta}$: susceptibility matrix, Higgs phase:

$$\frac{1}{207}\begin{pmatrix} 78 & 10 & 10 \\ 10 & 78 & 10 \\ 10 & 10 & 78 \end{pmatrix}, \quad \Gamma \text{ – different rates,}$$

equilibration of lepton #

$$i\frac{d\Delta_\alpha}{dt} = -i\left[\frac{12}{T^3}\int\frac{d^3k}{(2\pi)^3}\Gamma_{\nu_\alpha} f_f(1-f_f)\right]\omega_{\alpha\beta}\Delta_\beta$$
$$+\, i\int\frac{d^3k}{(2\pi)^3}\left[\mathrm{Tr}[\tilde{\Gamma}_{\nu_\alpha}\rho_{\bar{N}}] - \mathrm{Tr}[\tilde{\Gamma}^*_{\nu_\alpha}\rho_N]\right].$$

$\nu \Leftrightarrow N$ transfer

$$i\frac{d\rho_N}{dt} = [H_N, \rho_N] - \frac{i}{2}\{\Gamma_N, \rho_N\}$$
$$-\,\frac{i}{2}\sum_\alpha \tilde{\Gamma}^\alpha_N\,\omega_{\alpha\beta}\left[\frac{12\Delta_\beta}{T^3} f_f(1-f_f)\right].$$

$$i\frac{d\rho_{\bar{N}}}{dt} = [H^*_N, \rho_{\bar{N}}] - \frac{i}{2}\{\Gamma^*_N, \rho_{\bar{N}}\}$$

equilibration HNL's

$$+\,\frac{i}{2}\sum_\alpha (\tilde{\Gamma}^\alpha_N)^*\,\omega_{\alpha\beta}\left[\frac{12\Delta_\beta}{T^3} f_f(1-f_f)\right].$$

oscillation of HNL's

$\nu \Leftrightarrow N$ transfer

There are several analytic formulas for Δ_i, following from perturbative solution of these equations, valid when $\Gamma t \lesssim 1$

Otherwize, one should use numerics.

Main challenge: computation of rates Γ, most complete study due to

Ghiglieri & Laine.

Main Questions: – what are parameters of the model, leading to production of BAU, and consistent with ν experiments.

– can we find experimentally these particles?

– Most complet scan of parameter space:

Eijima, Timiryasov, MS, 2018

Leptogenesis with GeV HNLs

Creation of baryon asymmetry is a complicated process involving creation of HNLs in the early universe and their coherent CP-violating oscillations, interaction of HNLs with SM fermions, sphaleron processes with lepton and baryon number non-conservation. One need to deal with resummations, hard thermal loops, Landau-Pomeranchuk-Migdal effect, etc.

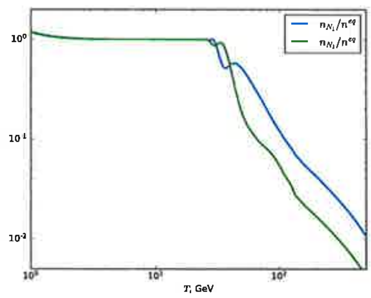
Initial idea: Akhmedov, Rubakov, Smirnov '98

Formulation of kinetic theory and demonstration that NuMSM can explain simultaneously neutrino masses, dark matter, and baryon asymmetry of the Universe: Asaka, M.S. '05

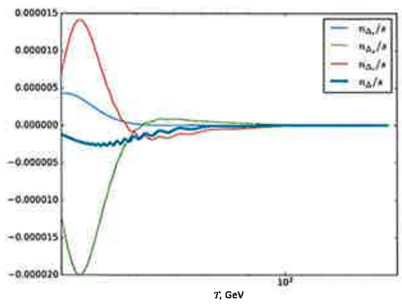
Analysis of baryon asymmetry generation in the NuMSM: Asaka, M.S., Canetti, Drewes, Frossard; Abada, Arcadia, Domcke, Lucente; Hernández, Kekic, J. López-Pavón, Racker, J. Salvado; Drewes, Garbrech, Guetera, Klariç; Hambye, Teresi; Eijima, Timiryasov; Ghiglieri, Laine,...

Time evolution

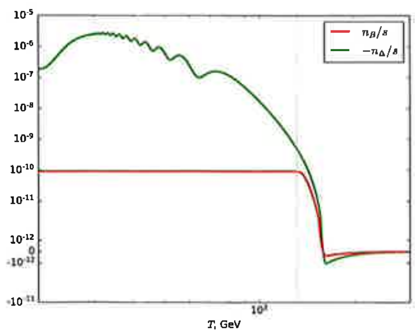
HNL densities



Lepton asymmetries



Baryon asymmetry



Experimental challenges of HNL searches:

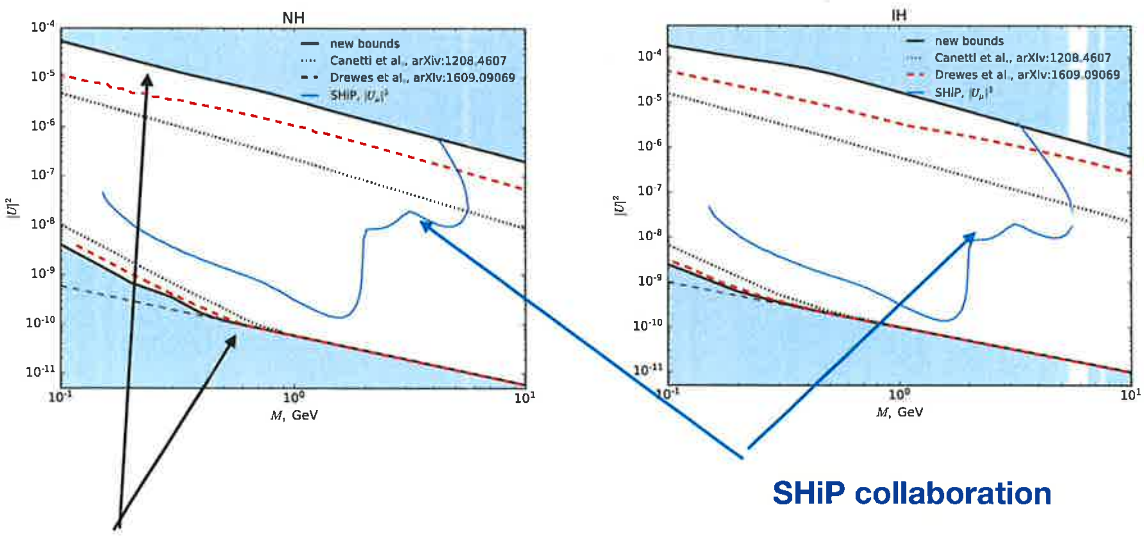
HNL production and decays are highly suppressed – dedicated experiments are needed:

- Mass below ~ 5 GeV - Intensity frontier, CERN SPS: NA62 in beam dump mode, SHiP
- Mass below ~ 5 GeV - Energy frontier, LHC: MATHUSLA
- Mass above ~ 5 GeV - FCC in e+e– mode in Z-peak, LHC

Generic purpose experiments to search for all sorts of relatively light dark sector particles (dark photons, hidden scalars, etc).

p+ target $\to$ mesons + ...
e.g. $D_s \to \mu N$, then, N-decays, $N \to \pi \mu$

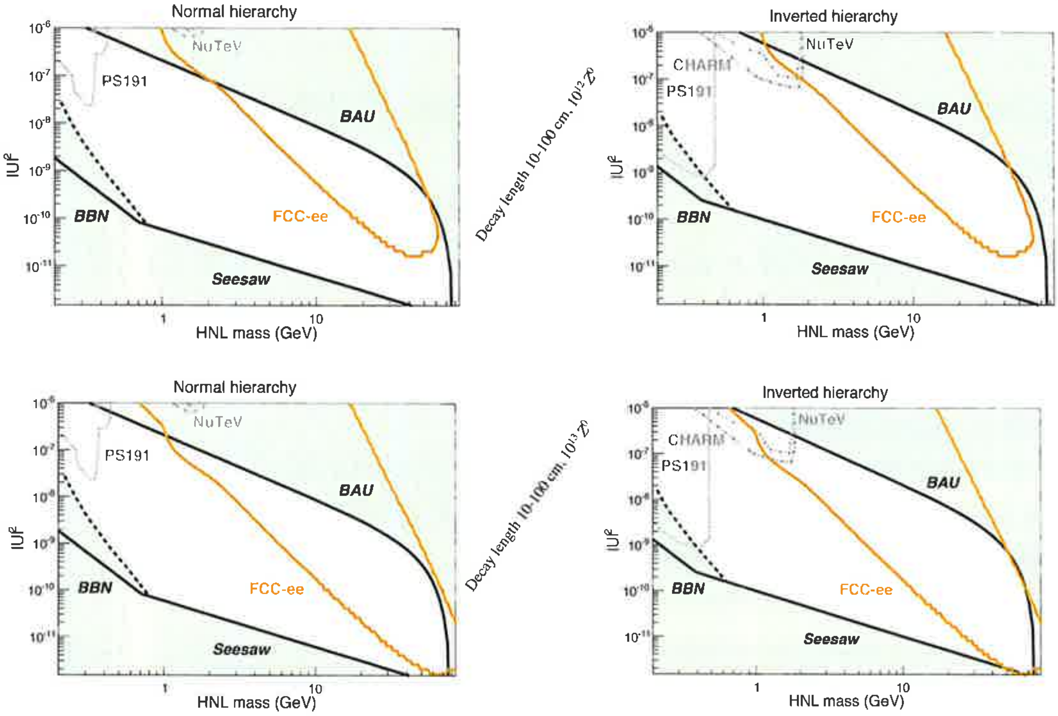
SHiP



SHiP collaboration

Eijima, M.S., Timiryasov

see over

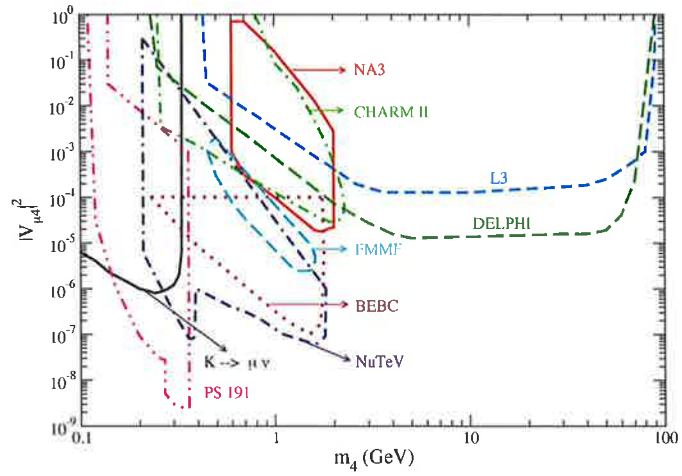


Blondel, Graverini, Sera, M.S.

process: $e^+e^- \to Z \to N \nu$

$\hookrightarrow$ decay of N

Survey of constraints



From arXiv:0901.3589, Atre et al

Suppose, HNL's are discovered.

What will we learn for ν-physics & cosmology?

Dark Matter HNL N1 decouples from see-saw formula and leptogenesis: Yukawa are too small

Counting “low energy” parameters, 2 HNLs:
2 Majorana masses of new neutral fermions N, 9 new Yukawa couplings in the leptonic sector (2 Dirac neutrino masses, 4 mixing angles and 3 CP-violating phases), 11 new parameters in total.

Counting “very low energy” parameters, 2 HNLs:

2 Majorana masses of active neutrinos (one is almost massless), 3 mixing angles in PMNS matrix, 1 Dirac phase and 1 Majorana phases, 7 parameters in total, 6 of them can be measured in active neutrino oscillations

- Dream scenario. Both HNLs N2 and N3 are discovered, their masses and decay branching ratios to electron, muon and tau flavours are found, and CP-violation in their decays is observed. 3 phases must be determined (at least 1 in HNL decays, 2 others can come from “very low energy” neutrino data). This determines all NuMSM parameters. The amplitude and sign of baryon asymmetry is predicted, and all “very low energy parameters” are fixed. The model is tested by the comparison with “very low energy” neutrino data.

More realistic scenario. From baryogenesis: masses of HNLs N2 and N3 are close to each other

$$\Delta M/M \sim 10^{-1} - 10^{-13}$$

and thus their mass splitting may not be resolved at experiments. They will look like a single particle. Then only part of the NuMSM parameters which can be determined experimentally (mass and decay branching ratios to electron, muon and tau flavours). If CP-violating effects are tiny, they also are not seen experimentally. So, we can determine only 1+3=4 "low energy" parameters. One can show that 2 combinations of these 4 parameters have no influence on "very low energy" neutrino parameters.

The amplitude and sign of baryon asymmetry cannot be predicted as it depends essentially on HNL mass difference and "low energy" CP-violating phase.

For active neutrino masses, the theory is equivalent to NuMSM with degenerate N2 and N3 and is characterised by 9 instead of 11 parameters ($\Delta M = M_2 - M_3$ and one CP-phase are out). 7 of them propagate to "very low energies" (only 2 combinations of them determined experimentally).

Suppose that all 7 "very low energy" parameters are fixed by experiments (neutrino oscillations and neutrino less double beta decay). So, we get 7 equations for 5 unknowns, meaning that we have 2 consistency relations which must be satisfied in the NuMSM.

Forget about Dark Matter and use all 3 HNLs for baryon asymmetry and neutrino mass generation

Counting "low energy" parameters, 3 HNLs:
3 Majorana masses of new neutral fermions N, 15 new Yukawa couplings in the leptonic sector (3 Dirac neutrino masses, 6 mixing angles and 6 CP-violating phases), 18 new parameters in total.

Counting "very low energy" parameters, 3 HNLs:

3 Majorana masses of active neutrinos, 3 mixing angles in PMNS matrix, 1 Dirac phase and 2 Majorana phases, 9 parameters in total, 6 of them can be measured in active neutrino oscillations

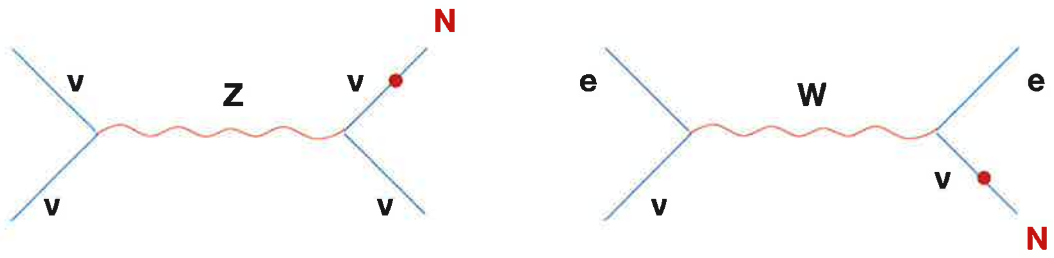
- Dream scenario. All three HNLs are discovered, and their decay branching ratios to electron, muon and tau flavours are found. Several CP-violating effects are found (6), fixing all the phases. All "low energy" parameters are found, the amplitude and sign of baryon asymmetry is predicted. We also get 9 consistency relations with "very low energy" neutrino data.

- More realistic scenario. Some of "low energy" parameters are not determined (such as CP-violation phases). The amplitude and sign of baryon asymmetry can not be predicted. However, we get several consistency relations with "very low energy" neutrino data.

Dark matter candidate: long lived ($\tau_N > t_{Universe}$), but unstable, sterile neutrino N1 with the mass in keV range

Dodelson, Widrow; Shi, Fuller; Abazajian, Fuller, Patel; ... Asaka, Laine, MS;...

Production of Dark matter in the early Universe.

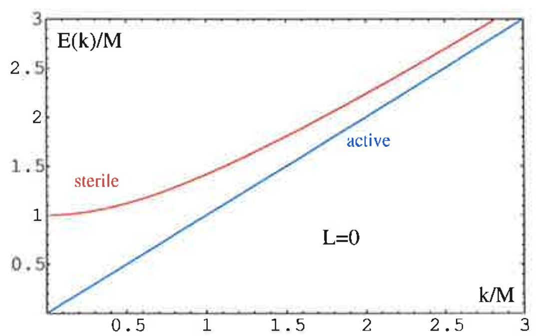


The temperature of production of DM sterile neutrinos:

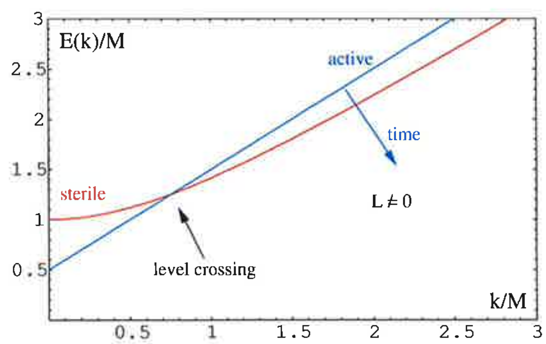
$$T \sim 130 \left(\frac{M_1}{1\ \mathbf{keV}} \right)^{1/3} \mathbf{MeV}$$

Non-resonant and resonant production

Dispersion relations for active and sterile neutrinos



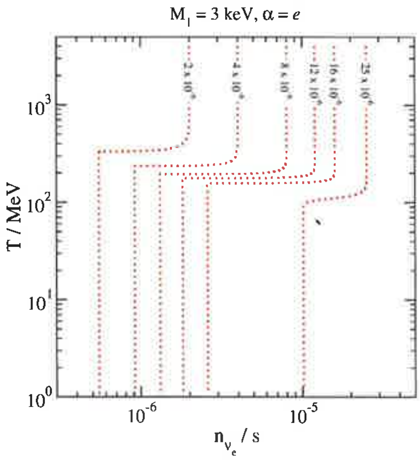
Non-resonant transitions, zero lepton asymmetry



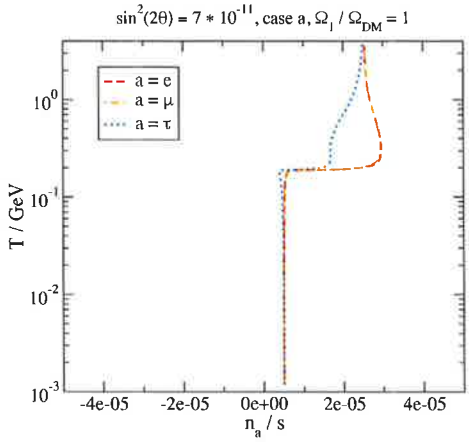
Resonant transitions in the presence of lepton asymmetry

140

Large fraction of lepton asymmetry is transferred to DM

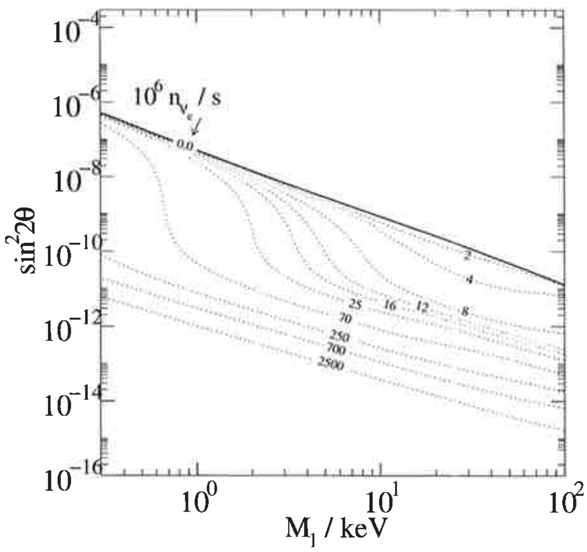


Laine, MS



Ghiglieri, Laine, 7 keV

Sterile neutrino Dark Matter abundance



Mixing angle, lepton asymmetry and the N1 mass leading to observed DM abundance

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Master formular for DM Production

Equation for density matrix:

$$i\frac{d\rho}{dt} = [\hat{H}, \hat{\rho}]$$

$$\hat{H} = \hat{H}_{SM} + \hat{H}_S + \hat{H}_{int}$$

kinetic term for sterile ν

Yukawa interaction

initial condition - no sterile ν:

$$\rho = \hat{\rho}_{SM} \times \underbrace{|0\rangle\langle 0|}_{\text{vacuum for sterile } \nu}$$

„free" Hamiltonian: $H_0 = H_{SM} + H_S$

interaction picture:

$$\hat{\rho}_I = e^{i\hat{H}_0 t}\,\hat{\rho}\,e^{-i\hat{H}_0 t}$$

140.2

Equation in interaction picture:

$$i \frac{d\rho_I}{dt} = \left[\hat{H}_I(t), \hat{\rho}_I(t)\right]$$

Perturbation theory with respect to H_I:

$$\hat{\rho}_I(t) = \hat{\rho}_0 - i\int_0^t dt' \left[H_I(t'), \rho_0\right] + (-i)^2 \int_0^t dt' \int_0^{t'} dt'' \times$$

$$\left[H_I(t'), \left[H_I(t''), \rho_0\right]\right] + \dots$$

Distribution function of sterile ν:

$$\frac{d\hat{N}_I}{d^3x\, d^3q} = \frac{1}{V} \sum a_I^+ a_I \Leftarrow \text{\# operator}$$

$$\{a_I\, a_I^+\} = \delta^3(p-q)\, \underset{\nearrow \text{ type}}{\delta_{IJ}}\, \underset{\nearrow \text{ spin}}{\delta_{st}}$$

$$\frac{dN_I(x,q)}{d^3x\, d^3q} = Tr\left[\frac{d\hat{N}_I}{d^3x\, d^3q} \cdot \hat{\rho}_I(t)\right]$$

140.3

$$\hat{H}_I = \int d^3x \left[M_{D\alpha I} \bar{\nu}_\alpha a_L N_I + h.c. \right]$$

$$a_L = \frac{1-\gamma_5}{2}$$

$$N_I = \int \frac{d^3p}{\sqrt{(2\pi)^3 2p_0}} \left[a\, e^{-ipx} + a^+ e^{ipx} \right]$$

1st order — does not give contribution

2nd order $\neq 0$:

$$\frac{d}{dt} \frac{dN_I}{d^3x\, d^3q} = \frac{1}{(2\pi)^3} (M_D^*)_{\alpha I} (M_D)_{\beta I}\, \mathcal{F}(q) \Bigg|_{q^0 = \sqrt{m_N^2 + \vec{q}^2}}$$

$$\mathcal{F}(q) = \mathrm{Tr} \int e^{iQx}\, \cancel{Q} \underbrace{\langle \nu_\alpha(0) \nu_\beta(x) + \nu_\beta(0) \nu_\alpha(x) \rangle}_{\text{active neutrino propagator}}$$

$$\vec{q} = \vec{Q}$$

Valid, if sterile ν are out of thermal equilibrium typical structure of ν propagator:

real part: gives contribution to dispersional relation

imaginary part: absorption of ν

$$F(q) \sim \frac{1}{\left(1 + \frac{2p}{M_N^2}\left(b(p,T) \pm c\right)\right)^2 + \theta^2}$$

$$\theta = \frac{M_D}{M_N}$$

$$b = \frac{16 G_F^2}{\pi \alpha_W} p \left(2 \cos^2\theta\right) \frac{7\pi^2 T^4}{360}$$

$$C(T) = 3\sqrt{2}\, G_F \left(1 + \sin^2\theta_W\right) \underbrace{\left(n_{\nu_e} - n_{\bar{\nu}_e}\right)}_{\text{Lepton asymmetry}}$$

Resonance:

$$M_N^2 + 2p\left[b \pm c\right] = 0$$

Effective θ-angle:

$$\theta^2 \to \theta^2(T) = \theta^2 \cdot f(q)$$

(140.5)

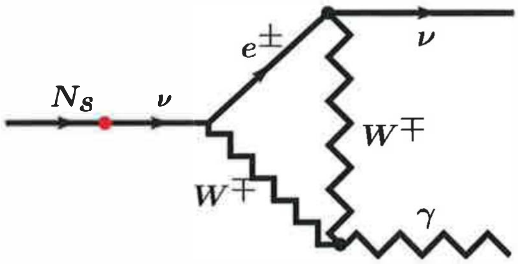
large T:

$\Theta(T) \sim \frac{\Theta}{T^6}$ huge suppression

$T^6: \sim b \cdot p \sim p^2 T^4 \sim T^6$

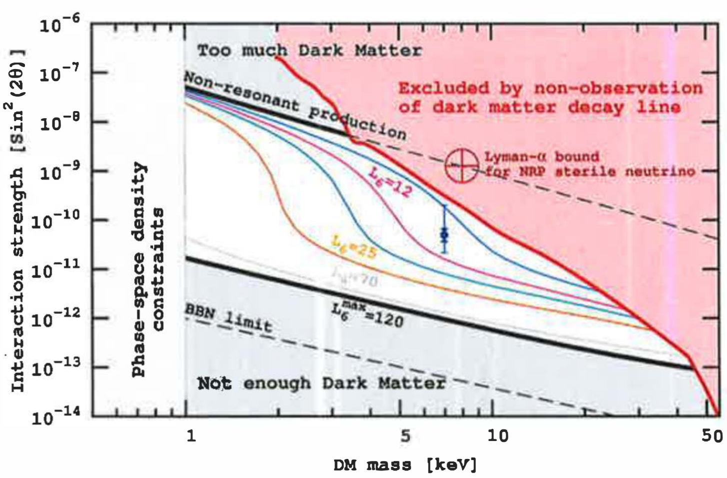
Experimental searches

DM particle is not stable. Main decay mode $N_1 \to 3\nu$ is not observable. Subdominant radiative decay channel: $N_1 \to \nu\gamma$



The photon line with energy $E_\gamma = \frac{M}{2}$ can be searched for with X-ray telescopes

3.5 keV line: E. Bulbul et al, Boyarsky et al



Conclusions:

- Baryon asymmetry of the Universe is a signal for physics beyond the SM
- What is this physics exactly, we cannot find theoretically, and need an experimental input
- Main ingredients of BAU explanations are
  - baryon # non-conservation
  - CP-breaking
  - departure from thermal equilibrium

Different proposals deal with these issues in a different way.

- Back in 2004, I counted a number of different mechanisms were invented since Sakharov paper in 1967. I found 42 proposals (now this number must be larger!)

Main reason: one number $\Rightarrow$ many explanations

$$\# \text{ of explanations} = \frac{\text{const}}{\# \text{ of things to explain}}$$

Progress over last 25 years

(Since 1967)

Today we know exactly 42 different ways to create baryons in the Universe!

The mechanisms I discussed are underlined

How to create baryons

1. GUT baryogenesis ← same type of physics as thermal leptogenesis
2. GUT baryogenesis after preheating
3. Baryogenesis from primordial black holes
4. String scale baryogenesis
5. Affleck-Dine (AD) baryogenesis
6. Hybridized AD baryogenesis
7. No-scale AD baryogenesis
8. Single field baryogenesis
9. Electroweak (EW) baryogenesis
10. Local EW baryogenesis
11. Non-local EW baryogenesis
12. EW baryogenesis at preheating

144

How to create baryons

13. SUSY EW baryogenesis
14. String mediated EW baryogenesis
15. Baryogenesis via leptogenesis
16. Inflationary baryogenesis
17. Resonant baryogenesis
18. Spontaneous baryogenesis
19. Coherent baryogenesis
20. Gravitational baryogenesis
21. Defect mediated baryogenesis
22. Baryogenesis from long cosmic strings
23. Baryogenesis from short cosmic strings
24. Baryogenesis from collapsing loops

How to create baryons

25. Baryogenesis through collapse of vortons
26. Baryogenesis through axion domain walls
27. Baryogenesis through QCD domain walls
28. Baryogenesis through unstable domain walls
29. Baryogenesis from classical force
30. Baryogenesis from electrogenesis
31. B-ball baryogenesis
32. Baryogenesis from CPT breaking
33. Baryogenesis through quantum gravity
34. Baryogenesis via neutrino oscillations
35. Monopole baryogenesis
36. Axino induced baryogenesis

37. Gravitino induced baryogenesis
38. Radion induced baryogenesis
39. Baryogenesis in large extra dimensions
40. Baryogenesis by brane collision
41. Baryogenesis via density fluctuations
42. Baryogenesis from hadronic jets

My personal preference: νMSM, which expains simultaneously ν masses and oscillations, BAU, DM, inflation through non-minimal coupling of the Higgs field to Ricci scalar, $\xi h^2 R$. The model is ad least partially testable at future experiments.