What it takes to calculate the magnetic moment of the muon in the standard model

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Uppsala University

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Bormio, January 2019

Physics beyond the standard model

- Why is there more matter in the universe than antimatter?
- What is the nature of dark matter?
- Why is strong CP violation tiny or even absent?
- What is the origin of the neutrino masses?
- How to quantize gravity?
- ...

$\hookrightarrow$ search directions for physics Beyond the Standard Model (BSM):

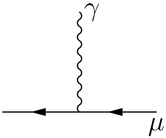
- at high(est) energies (excite BSM particles)
- with high precision (measure the forces mediated by BSM fields)
  - historical note about the pion:
    force was measured and interpreted first (Yukawa),
    afterwards particle was found with predicted mass

High-precision standard model tests

to find traces of BSM physics

- need high-precision measurements
- AND high-precision standard-model calculation

prominent example: magnetic moment of the muon ("$g-2$ of muon")



$\rightsquigarrow g = 2$

(Dirac)

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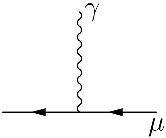

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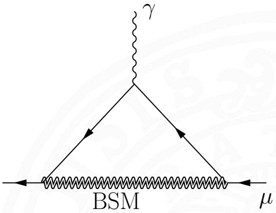
discovery potential: ($g \neq 2$)



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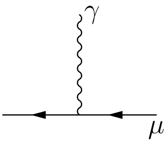

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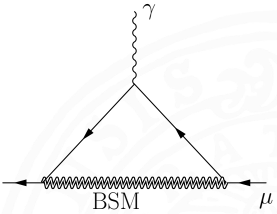
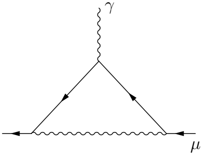
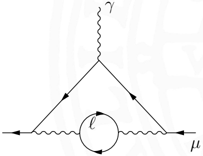


standard model corrections:



$+$



$+ \ldots$

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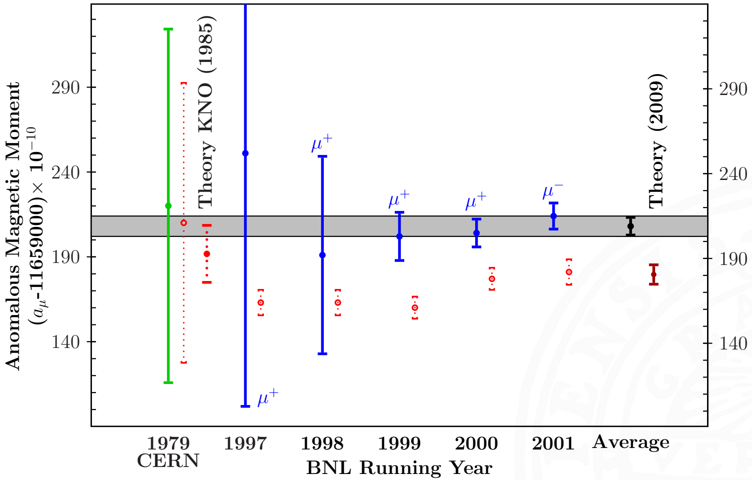
$g-2$ of the muon — status



Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

$g-2$ of the muon — experiment

- current experimental value from BNL E821

$$a_\mu = \frac{g-2}{2} = (116\,592\,089 \pm 63) \times 10^{-11}$$

- now running: Muon $g-2$ experiment at Fermilab

$\hookrightarrow$ will reduce uncertainty by factor of 4

- planned (needs approval): J-PARC E34

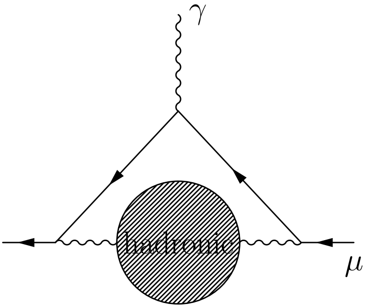
the more the merrier:

- desirable: high-quality data from several experiments

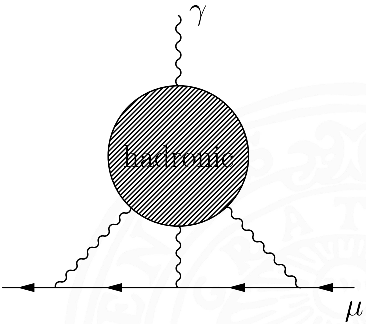
AND high-quality calculations with complementary approaches

$g-2$ of the muon — theory

Largest uncertainty of standard model: hadronic contributions



vacuum polarization
$\sim \alpha^2$



light-by-light scattering
$\sim \alpha^3$

$g-2$ of the muon — status

Standard model theory and experiment comparison [in units 10^{-11}].

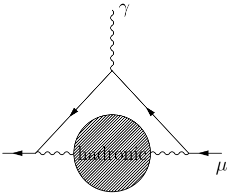
Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	−100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6
Experiment	116 592 080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3

Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

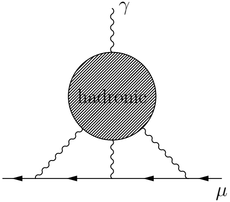
- in 2009:
  - dominant uncertainty in hadronic vacuum polarization (HVP)
  - largest relative uncertainty in hadronic light-by-light (HLbL)
- 2017 update of HVP: $(6931 \pm 34) \times 10^{-11}$

  M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C 77, 827 (2017)

Hadronic contribution to $g-2$ of the muon



how to determine size of hadronic fluctuations?



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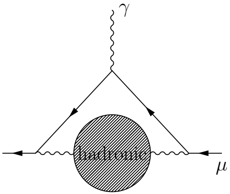

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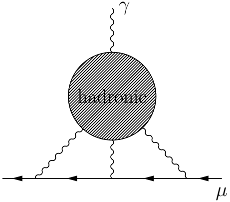

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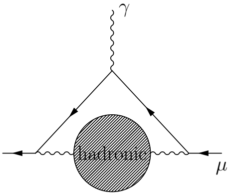

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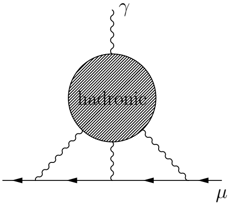
↪ does not work, contributions most sensitive to low-energy parts



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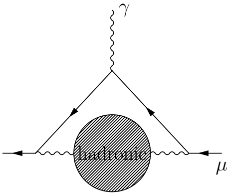

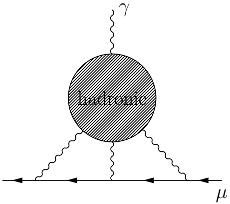

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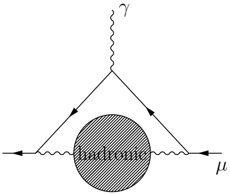

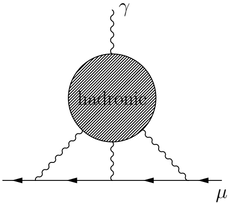

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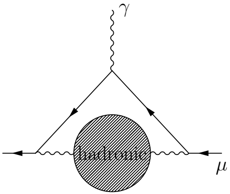

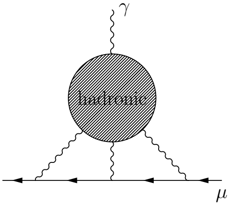

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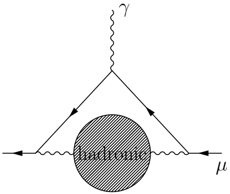

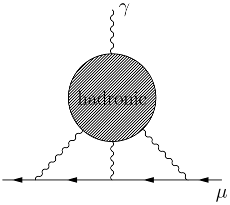

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$\hookrightarrow$ need a model independent approach

$\hookrightarrow$ lattice QCD, effective field theory, . . . , or "data"

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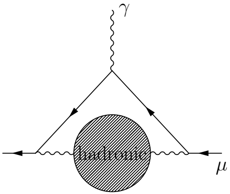

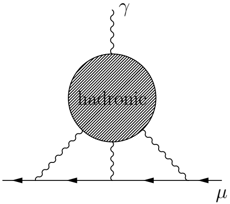

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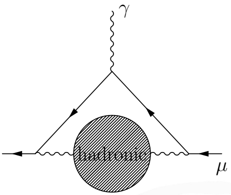
Data-driven approach

vacuum polarization (used to be dominant uncertainty)



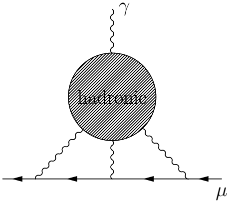
- directly related to cross sect. $e^+e^- \to$ hadrons (by dispersion relation)

$\rightsquigarrow$ measurable

$\rightsquigarrow$ ongoing improvements by international efforts

light-by-light scattering

(now dominant uncertainty)



- $\gamma^*\gamma^* \leftrightarrow$ hadron(s) not so easily accessible by experiment

$\hookrightarrow$ past: use phenom. models with conservative error estimates

$\hookrightarrow$ instead: crank dispersive machinery further

Colangelo/Hoferichter/Kubis/Procura/Stoffer, Phys.Lett. B738, 6 (2014)

Unitarity and analyticity

- constraints from local quantum field theory:
  partial-wave amplitudes for reactions/decays must be
  - unitary:

$$S\,S^\dagger = 1\,, \quad S = 1 + iT \quad \Rightarrow \quad 2\,\mathrm{Im}\,T = T\,T^\dagger$$

↪ note that this is a matrix equation:
$\mathrm{Im}\,T_{A\to B} = \sum_X T_{A\to X}\,T^\dagger_{X\to B}$

  - analytic (dispersion relations):

$$T(s) = \frac{1}{\pi}\int\limits_{-\infty}^{\infty} ds'\,\frac{\mathrm{Im}\,T(s')}{s'-s-i\epsilon}$$

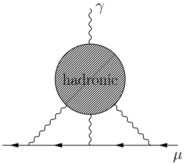
⇝ can be used to calculate whole amplitude from imaginary part

- practical limitation: too many states X at high energies

↪ in practice dispersion theory is a low-energy method ($\lesssim 1\,\mathrm{GeV}$)

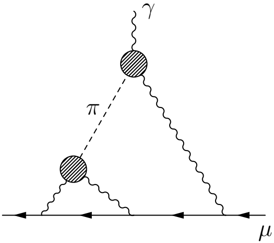
Hadronic light-by-light contribution

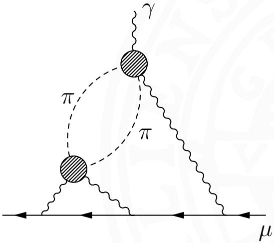
true for all hadronic contributions:



- the lighter the hadronic system, the more important
  (though high-energy contributions not unimportant for light-by-light)

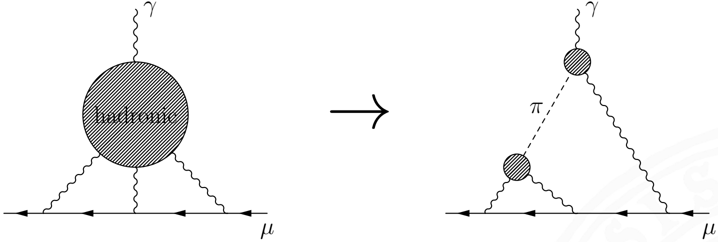
$\hookrightarrow$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow \pi^0$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow 2\pi, \ldots$



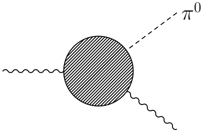


Using lowest-mass states

hadronic light-by-light (HLbL) contribution



$\rightsquigarrow$ need pion transition form factor (TFF)



$\rightarrow F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$ with γ virtualities $q_1^2,\ q_2^2$

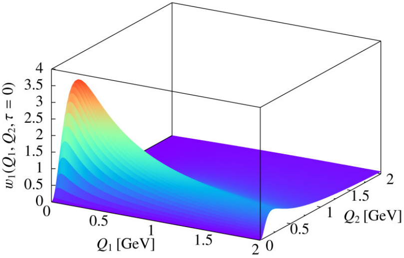
Pion-pole contribution to HLbL

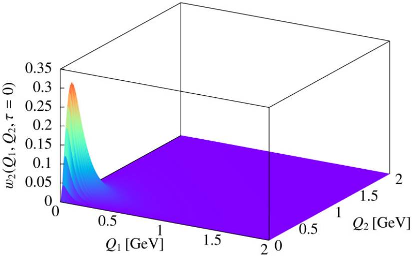
$$a_\mu^{\pi^0\text{-pole}} = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty \mathrm{d}Q_1 \int_0^\infty \mathrm{d}Q_2 \int_{-1}^1 \mathrm{d}\tau$$
$$\times \Big[w_1(Q_1, Q_2, \tau)\, F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_3^2)\, F_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0)$$
$$+ w_2(Q_1, Q_2, \tau)\, F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\, F_{\pi^0\gamma^*\gamma^*}(-Q_3^2, 0) \Big]$$

(Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009))

- $Q_{1/2}^2 = -q_{1/2}^2$
- $Q_3^2 = Q_1^2 + 2Q_1Q_2\tau + Q_2^2$
- $\tau = \cos\sphericalangle(Q_1, Q_2)$
- weight functions $w_{1/2}$ peak strongly at low Q_i's ⇝ figs.

Weight functions





- weight functions $w_{1/2}$ peak strongly at low Q_i's

$\hookrightarrow$ largest accuracy required for low-energy part of pion TFF $F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

(Hoferichter, Hoid, Kubis, Leupold, Schneider, JHEP 1810, 141 (2018))

Representation of pion transition form factor (TFF)

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = F^{\text{disp}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) + F^{\text{eff}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) + F^{\text{asym}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)$$

- $F_{\pi^0\gamma^*\gamma^*}(0,0)$ measured via $\pi^0 \to 2\gamma$ (highest precision from PrimEx)
- need low-energy part ("disp") with very high accuracy

↪ use dispersion theory and high-quality data

- need reasonable behavior at high energies ("asym")

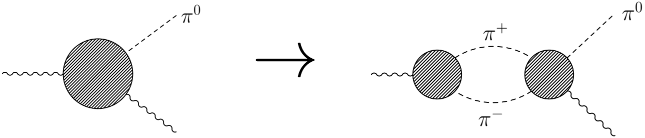
↪ guide from perturbative QCD, operator product expansion for pion distribution amplitude

- need only schematic description at intermediate energies ("eff")

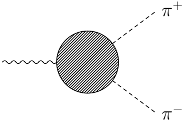
↪ use effective pole with
  - residue fitted to decay width $\pi^0 \to 2\gamma$
  - mass fitted to data on singly-virtual pion TFF $F_{\pi^0\gamma^*\gamma^*}(-Q^2,0)$

Dispersive reconstruction I

pion transition form factor

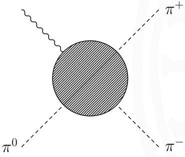


⇝ need pion vector form factor F_π^V

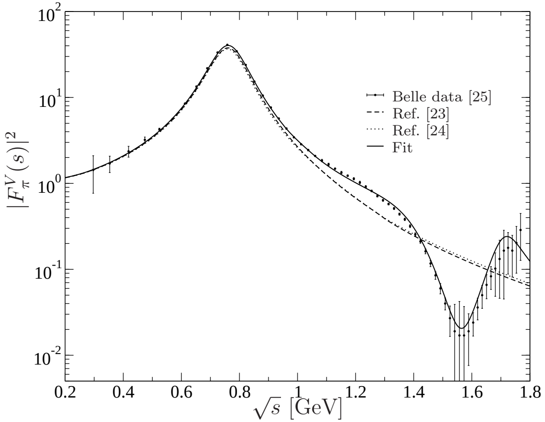


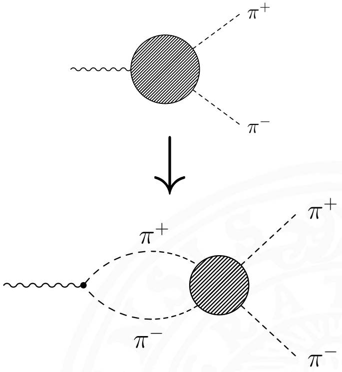
→ very well measured

and amplitude f_1 for γ^*–3-pion



Pion vector form factor



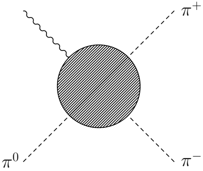


pion phase shift very well known; fits to pion vector form factor

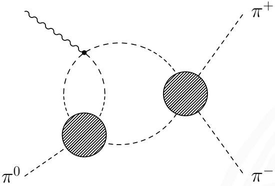
Sebastian P. Schneider, Bastian Kubis, Franz Niecknig, Phys.Rev.D86:054013,2012

Dispersive reconstruction II

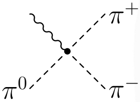
amplitude γ^*–3-pion



contains two-body correlations
(depend on s, t, u), e.g. $\rightsquigarrow$

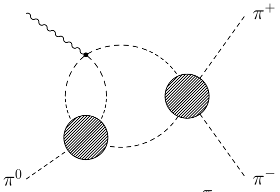


and genuine three-body correlations
(depend on $m_{3\pi}^2 = m_{\gamma^*}^2$)

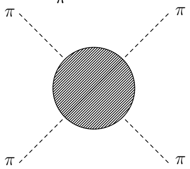


Required input

for

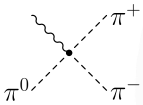


need pion phase shift



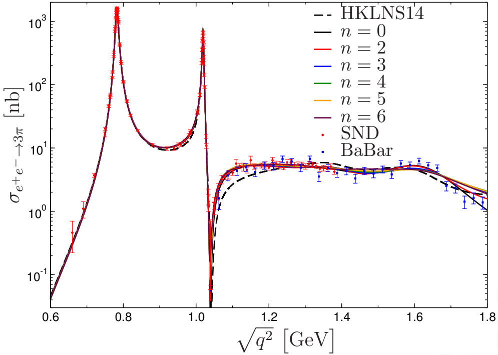
$\rightsquigarrow$ very well measured

and genuine three-body correlations
(one-parameter function!)



$\rightsquigarrow$ fit to cross section of $e^+e^- \to \pi^+\pi^-\pi^0$

Fit to $e^+e^- \to \pi^+\pi^-\pi^0$



M. Hoferichter, B. Kubis, S.L., F. Niecknig, S. P. Schneider, Eur.Phys.J. C74, 3180 (2014)

M. Hoferichter, B.-L. Hoid, B. Kubis, S.L., S. P. Schneider, JHEP 1810, 141 (2018)

$\rightsquigarrow$ fully determines p-wave projected γ^*–3-pion amplitude f_1

$\hookrightarrow$ together with pion vector form factor F_π^V this enters finally the dispersive representation of the pion transition form factor

Representation of pion transition form factor (TFF)

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = F^{\text{disp}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) + F^{\text{eff}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) + F^{\text{asym}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)$$

with double-spectral dispersive representation

$$F^{\text{disp}}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2) = \frac{1}{\pi^2}\int_{4M_\pi^2}^{s_{\text{iv}}} dx \int_{s_{\text{thr}}}^{s_{\text{is}}} dy \frac{\rho^{\text{disp}}(x,y)}{(x+Q_1^2)(y+Q_2^2)} + (Q_1 \leftrightarrow Q_2),$$

$$\rho^{\text{disp}}(x,y) = \frac{q_\pi^3(x)}{12\pi\sqrt{x}}\,\mathrm{Im}\Big[\big(F_\pi^V(x)\big)^* f_1(x,y)\Big],$$

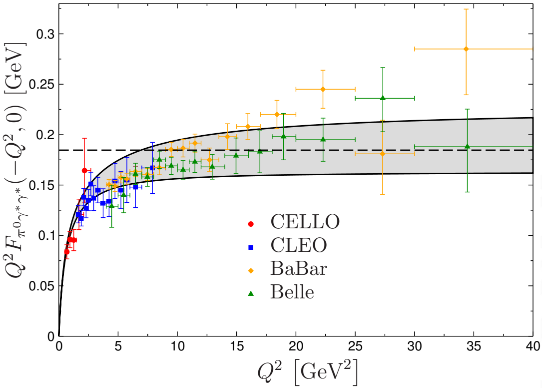
and effective pole

$$F^{\text{eff}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{g_{\text{eff}}}{4\pi^2 F_\pi}\frac{M_{\text{eff}}^4}{(M_{\text{eff}}^2-q_1^2)(M_{\text{eff}}^2-q_2^2)}$$

- g_{eff} is around 10%, i.e. comfortably small (fitted to $F_{\pi^0\gamma^*\gamma^*}(0,0)$)
- M_{eff} is around (1.5–2) GeV, i.e. in reasonable range

Singly-virtual pion transition form factor

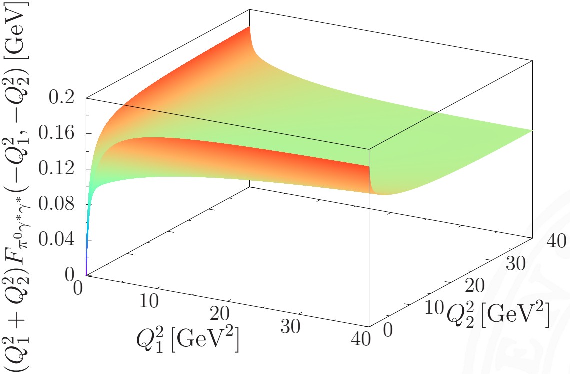
M_{eff} is determined by fit to singly-virtual pion TFF



M. Hoferichter, B.-L. Hoid, B. Kubis, S.L., S. P. Schneider,

Phys. Rev. Lett. 121, 112002 (2018) and JHEP 1810, 141 (2018)

Sketch of pion transition form factor



M. Hoferichter, B.-L. Hoid, B. Kubis, S.L., S. P. Schneider,

Phys. Rev. Lett. 121, 112002 (2018) and JHEP 1810, 141 (2018)

Final result and discussion of uncertainties

pion-pole part of hadronic light-by-light contribution to $a_\mu = (g-2)/2$:

$$a_\mu^{\pi^0\text{-pole}} = 62.6(1.7)_{F_{\pi\gamma\gamma}}(1.1)_{\text{disp}}\binom{2.2}{1.4}_{\text{BL}}(0.5)_{\text{asym}} \times 10^{-11}$$
$$= \left(62.6^{+3.0}_{-2.5}\right) \times 10^{-11}$$

with uncertainties from

- decay width $\pi^0 \to 2\gamma$ ("$F_{\pi\gamma\gamma}$")
- pion vector form factor, pion phase shift, fit to $e^+e^- \to 3\pi$, range of low-energy representation, ... ("disp")
- fit to singly-virtual pion TFF (Brodsky-Lepage, "BL")
- onset of asymptotic region ("asym")

$\hookrightarrow$ dominant uncertainties can be reduced by improved data on $F_{\pi^0\gamma^*\gamma^*}(0,0)$ (PrimEx-II) and $F_{\pi^0\gamma^*\gamma^*}(-Q^2,0)$ (Belle II)

Summary and Outlook

- traces of physics beyond the standard model might be found in low-energy observables by contrasting
  - high-precision measurements
  - high-precision standard-model calculation

$\hookrightarrow$ promising observable: magnetic moment of the muon

$\hookrightarrow$ at present: $> 3\sigma$ deviation

$\hookrightarrow$ experiments will improve accuracy ($\delta a_\mu \times 10^{11} \approx 63 \to 16$)

$\hookrightarrow$ calls for improvement of standard-model calculation

- previously: hadronic light-by-light contribution to $a_\mu = (g-2)/2$

$$a_\mu^{\text{complete HLbL}} = (116 \pm 39) \times 10^{-11} \quad \text{(model dependent!)}$$

Summary and Outlook

- have presented data driven approach (dispersive method) for hadronic light-by-light contribution to $a_\mu = (g-2)/2$

↪ achievement: model independent determination of pion-pole part with small uncertainty

$$a_\mu^{\mathrm{HLbL,pion\,pole}} = \left(62.6^{+3.0}_{-2.5}\right) \times 10^{-11}$$

- note: this is not the complete HLbL contribution, but its largest fraction

Summary and Outlook

complementary activities within data-driven approach:

- pion box

$$a_\mu^{\mathrm{HLbL,pion\,box}} = (-15.9 \pm 0.2) \times 10^{-11}$$

Colangelo, Hoferichter, Procura, Stoffer, Phys. Rev. Lett. 118, 232001 (2017)

- rest of two-pion contribution
  (includes "f_0- and f_2-pole contributions")

↪ work in progress (Colangelo, Hoferichter, Procura, Stoffer)

- η- and η'-pole contributions

↪ work in progress (Hanhart, Kubis, Kupsc, Wirzba, . . .)

prospects: will finally beat previous result in accuracy

Collaborators, publications

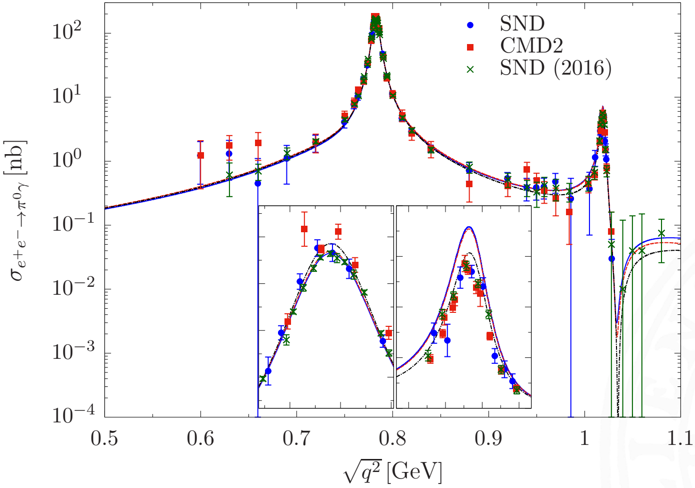
- Martin Hoferichter (Seattle)
- Bai-Long Hoid (Bonn)
- Bastian Kubis (Bonn)
- Franz Niecknig (Bonn, now industry)
- Sebastian Schneider (Bonn, now industry)

Hoferichter, Hoid, Kubis, Leupold, Schneider, Phys. Rev. Lett. 121, 112002 (2018)
Hoferichter, Hoid, Kubis, Leupold, Schneider, JHEP 1810, 141 (2018)
Leupold, Hoferichter, Kubis, Niecknig, Schneider, EPJ Web Conf. 166, 00013 (2018)
Hoferichter, Kubis, Leupold, Niecknig, Schneider, Eur. Phys. J. C 74, 3180 (2014)

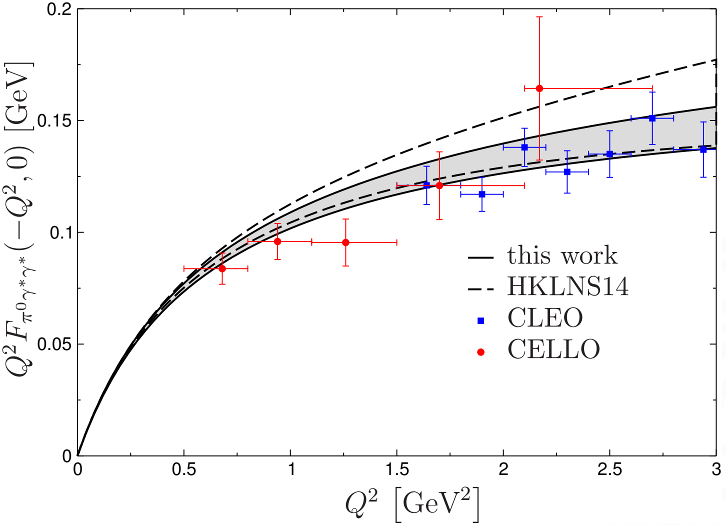
related previous work of the Bonn group:
F. Niecknig, B. Kubis and S. P. Schneider, Eur. Phys. J. C 72, 2014 (2012)
S. P. Schneider, B. Kubis and F. Niecknig, Phys. Rev. D 86, 054013 (2012)
M. Hoferichter, B. Kubis and D. Sakkas, Phys. Rev. D 86, 116009 (2012)

backup slides

Postdictions and predictions

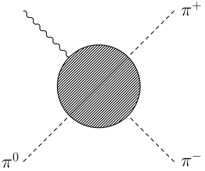


Postdictions and predictions

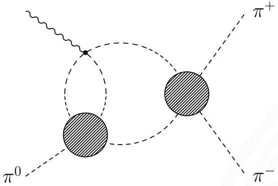


Dispersive reconstruction II

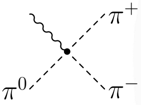
amplitude γ^*–3-pion



contains two-body correlations
(depend on s, t, u), e.g. $\rightsquigarrow$

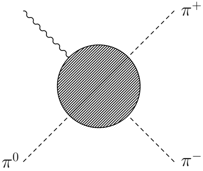


and genuine three-body correlations
(depend on $m_{3\pi}^2 = m_{\gamma^*}^2$)



Dispersive reconstruction II

amplitude γ^*–3-pion



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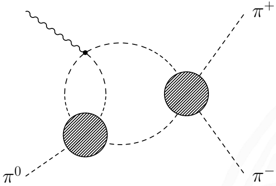

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