Meson-Baryon interaction in the Fock-Tani Formalism

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Abstract

The Fock-Tani formalism is a first principle method to obtain effective interactions from microscopic Hamiltonians. The idea consist in a change of representation such that the operators associated with composite particles could be rewritten in operators who satisfy the canonical anticomutation relations. Starting from Fock space and using creation and annihilation operators to the constituents particles, we consider a sistem contending quarks and antiquarks which could form bonded-states. In this new representation, the meson/baryon states can be constructed from meson/baryon creation operators. Originally derived for meson-meson or baryon-baryon scatttering, we will present the correponding equations for meson-baryon scattering.

The Fock-Tani Formalism

The Fock-Tani formalism uses a unitary operator U to rewrite the particle operators, redefining meson and baryon states as ideals elementary hadron states that satisfy the canonical commutation relations [1].

$$|\Omega\rangle \longrightarrow |\Omega) = U^{-1}|\Omega\rangle \qquad , \qquad O \longrightarrow O_{\rm FT} = U^{-1} O U \, .$$

Since the operator U is unitary, the scalar product and the matrix elements are preserved under the change of representation:

$$\langle\Omega|\Omega\rangle = (\Omega|\Omega) \qquad , \qquad \langle\Omega|\, O \,|\Omega\rangle = (\Omega|O_{\rm FT}|\Omega) \, .$$

If $|\alpha\rangle$ is a meson state, it could be rewriten by an "ideal" meson under the following transformation

$$|\alpha\rangle \longrightarrow U^{-1}|\alpha\rangle \equiv |\alpha) = m_\alpha^\dagger|0) \, ,$$

with the meson "ideal" operators, $m_\alpha^\dagger$ and m_α, satisfy the canonical comutation relation

$$[m_\alpha, m_\beta] = 0 \qquad , \qquad \left[m_\alpha, m_\beta^\dagger\right] = \delta_{\alpha\beta} \, ,$$

and are kinetically independent of the quark and antiquarks operators

$$[q_\mu, m_\alpha] = \left[q_\mu, m_\alpha^\dagger\right] = [\bar{q}_\nu, m_\alpha] = \left[\bar{q}_\nu, m_\alpha^\dagger\right] = 0 \, .$$

The change of Fock to Fock-Tani representation from meson ands baryon can be seen in the following table, where the representation of the composed nature and structure of mesons and baryons are $D_{\alpha\beta}$ and $\Delta_{\alpha\beta}$ respectively.

Fock	Fock-Tani
$\|\alpha\rangle = M_\alpha^\dagger\|0\rangle$	$\|\alpha) = m_\alpha^\dagger\|0)$
$[M_\alpha, M_\beta] = 0$	$[m_\alpha, m_\beta] = 0$
$[M_\alpha, M_\beta^\dagger] = \delta_{\alpha\beta} - D_{\alpha\beta}$	$[m_\alpha, m_\beta^\dagger] = \delta_{\alpha\beta}$
$[q_\mu, M_\alpha] = [\bar{q}_\nu, M_\alpha] = 0$	$[q_\mu, m_\alpha] = [\bar{q}_\nu, m_\alpha] = 0$
$[q_\mu, M_\alpha^\dagger] = \Phi_\alpha^{\mu\nu}\bar{q}_\nu^\dagger$	$[q_\mu, m_\alpha^\dagger] = 0$
$[\bar{q}_\nu, M_\alpha^\dagger] = -\Phi_\alpha^{\mu\nu} q_\mu^\dagger$	$[\bar{q}_\nu, m_\alpha^\dagger] = 0$
$\|\alpha\rangle = B_\alpha^\dagger\|0\rangle$	$\|\alpha) = b_\alpha^\dagger\|0)$
$\{B_\alpha, B_\beta\} = 0$	$\{b_\alpha, b_\beta\} = 0$
$\{B_\alpha, B_\beta^\dagger\} = \delta_{\alpha\beta} - \Delta_{\alpha\beta}$	$\{b_\alpha, b_\beta^\dagger\} = \delta_{\alpha\beta}$
$\{q_\mu, B_\alpha\} = 0$	$\{q_\mu, b_\alpha\} = 0$
$\{q_\mu, B_\alpha^\dagger\} = \sqrt{\frac{3}{2}}\Psi_\alpha^{\mu\mu_2\mu_3} q_{\mu_2}^\dagger q_{\mu_3}^\dagger$	$\{q_\mu, b_\alpha^\dagger\} = 0$

The Fock-Tani Hamiltonian

Once a microscopic interaction Hamiltonian H is defined at the quark level, a new transformed Hamiltonian can be obtained. The transformed Fock-Tani Hamiltonian is a result of the the application of the unitary transformation on the microscopic Hamiltonian

$$H_{\rm FT} = U_B^{-1} U_M^{-1} H \, U_M \, U_B .$$

The transformed Hamiltonian $H_{\rm FT}$ describes all possible processes involving mesons, baryons and quarks.

$$\mathcal{H} = \mathcal{H}_q + \mathcal{H}_m + \mathcal{H}_b + \mathcal{H}_{mq} + \mathcal{H}_{bq} + \mathcal{H}_{mb} \, .$$

After apply the Fock-Tani transformation in the microscopic quark-antiquark Hamiltonian

$$\begin{aligned} H_{2q} \;=\; & T(\mu)\, q_\mu^\dagger q_\mu + T(\nu)\, \bar{q}_\nu^\dagger \bar{q}_\nu + V_{q\bar{q}}(\mu\nu;\sigma\rho) q_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho q_\sigma \\ & + \frac{1}{2} V_{qq}(\mu\nu;\sigma\rho) q_\mu^\dagger q_\nu^\dagger q_\rho q_\sigma + \frac{1}{2} V_{\bar{q}\bar{q}}(\mu\nu;\sigma\rho) \bar{q}_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho \bar{q}_\sigma , \end{aligned}$$

we obtain the following meson-baryon potential with quark and gluon exchange [2]

$$V_{\rm mb}(\alpha\beta;\delta\gamma) = \sum_{i=1}^{4} V_i(\alpha\beta;\delta\gamma)\, m_\alpha^\dagger\, b_\beta^\dagger\, m_\gamma\, b_\delta$$

where

$$\begin{aligned} V_1(\alpha\beta;\delta\gamma) &= -3V_{qq}(\mu\nu;\sigma\rho)\, \Phi_\alpha^{*\mu\nu_2} \Psi_\beta^{*\nu\mu_2\mu_3} \Phi_\gamma^{\rho\nu_2} \Psi_\delta^{\sigma\mu_2\mu_3} \\ V_2(\alpha\beta;\delta\gamma) &= -3V_{q\bar{q}}(\mu\nu;\sigma\rho)\, \Phi_\alpha^{*\mu_1\nu} \Psi_\beta^{*\mu\mu_2\mu_3} \Phi_\gamma^{\sigma\rho} \Psi_\delta^{\mu_1\mu_2\mu_3} \\ V_3(\alpha\beta;\delta\gamma) &= -3V_{qq}(\mu\nu;\sigma\rho)\, \Phi_\alpha^{*\mu\nu_2} \Psi_\beta^{*\mu_1\nu\mu_3} \Phi_\gamma^{\mu_1\nu_2} \Psi_\delta^{\sigma\rho\mu_3} \\ V_4(\alpha\beta;\delta\gamma) &= -6V_{q\bar{q}}(\mu\nu;\sigma\rho)\, \Phi_\alpha^{*\nu_1\nu} \Psi_\beta^{*\mu_1\mu\mu_3} \Phi_\gamma^{\mu_1\rho} \Psi_\delta^{\nu_1\sigma\mu_3} \, . \end{aligned}$$

Results and Perspectives

We use the meson-baryon potential to calculate the low energy cross section interactions, as the charm system J/ψ-Nucleon, DN and kaon-nucleon system K^+N system for which we also calculate the annihilation potential [3]. As the kaon-nucleon system have resonances for energies greater than 300 MeV [4], we use the Adhikari method [5] to calculate the cross section from the T-matrix and we are studying elastic channels behavior as $\bar{K}^0 + p$, $\eta + p$ and $D^+ + p$.

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