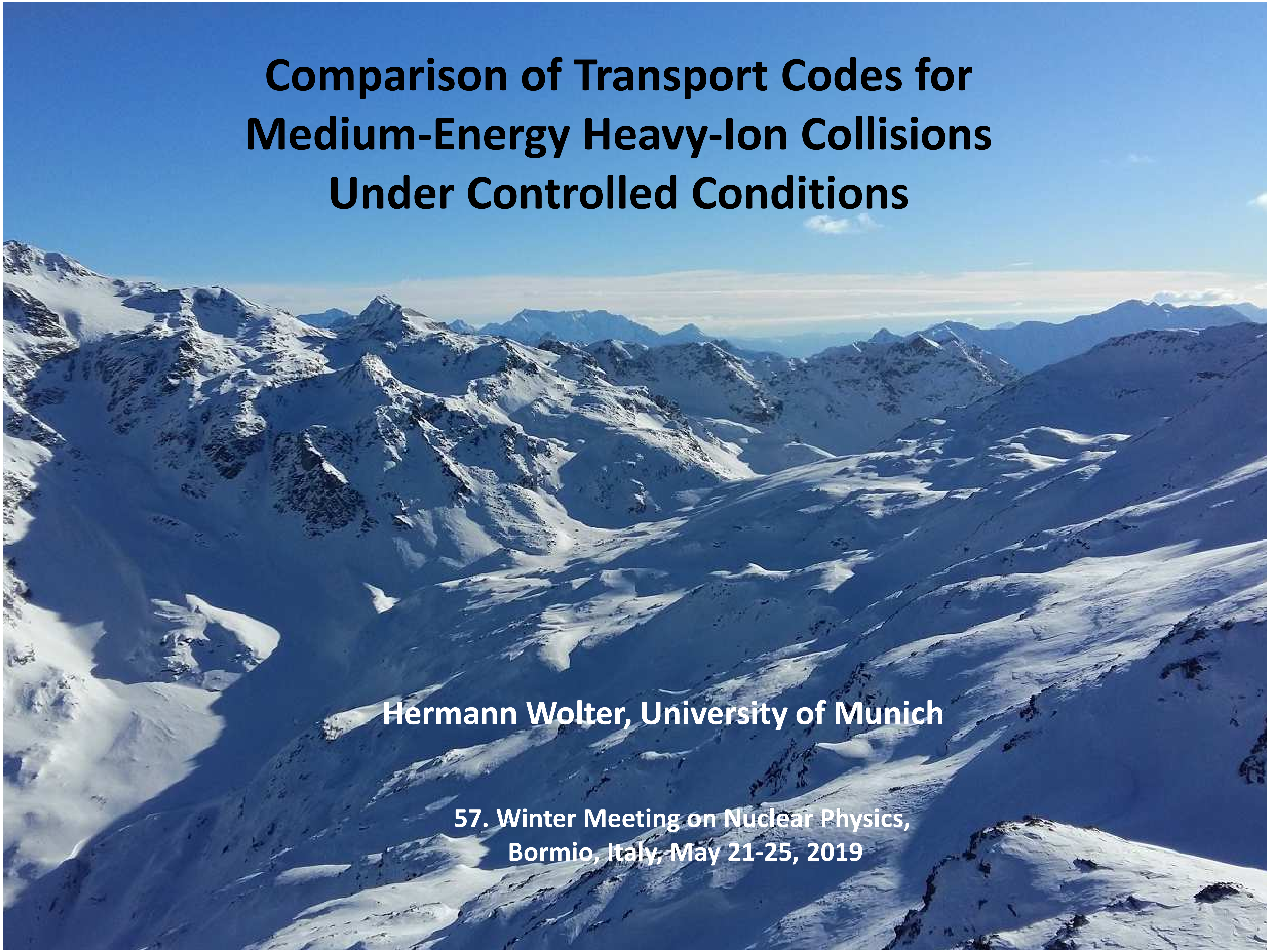
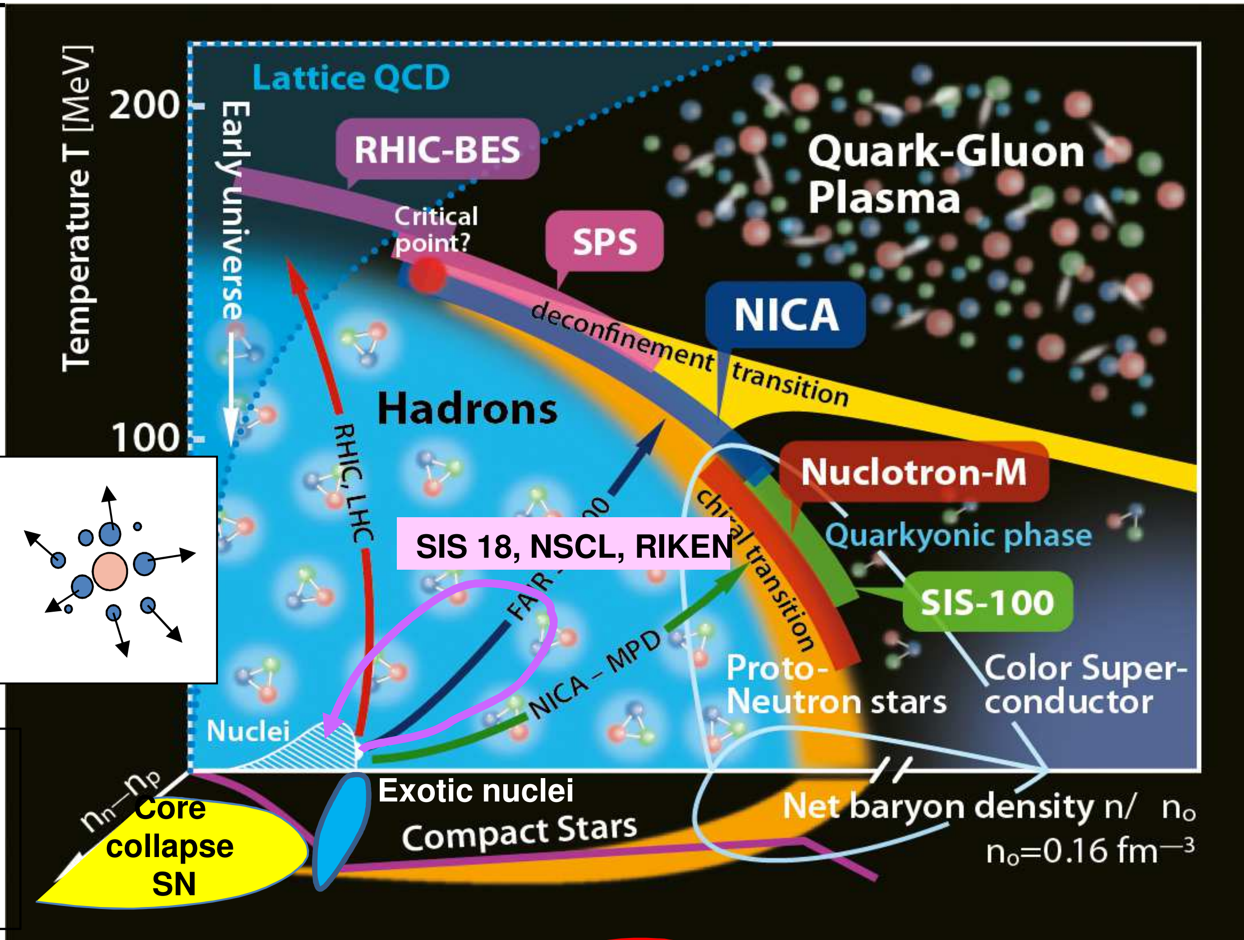
Comparison of Transport Codes for Medium-Energy Heavy-Ion Collisions Under Controlled Conditions

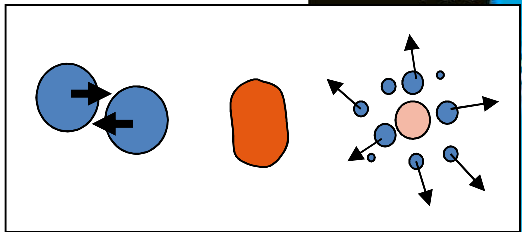


Hermann Wolter, University of Munich

57. Winter Meeting on Nuclear Physics,
Bormio, Italy, May 21-25, 2019

The Phase Diagram of Strongly Interacting Matter





Asymmetry axis
--> search for
symmetry
energy

$$E(\rho_B,\delta)/A = E_{nm}(\rho_B) + E_{sym}(\rho_B)\delta^2 + O(\delta^4) + \ldots$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Extensive efforts by:

- **Microscopic theory**
- **Neutron star observations**
- **HI experiments in the hadronic regime,**
  only way to investigate dense neutron-rich matter in the lab

Note:
HIC trajectories are non-equilibrium processes
→ transport theory is necessary
but has to check its robustness

Aim of this short contribution:

- **discussion of transport approaches to heavy-ion collisions (HIC) in the hadronic sector**
- **not interpretation of data, but robustness of description by transport approaches**
- **comparison of transport codes with identical physical input**
  i.e. controlled conditions
  with each other and with exact limits

- **highlight the role of fluctuations in the description of HIC**

On behalf of the Code Comparison Project
- of the order of 30 participants
- core group:
Maria Colonna (Catania), Akira Ono (Sendai),
Yingxun Zhang (CIAE, Beijing), Jun Xu (SINAP, Shanghai), Betty Tsang (MSU),
Pawel Danielewcz (MSU), Jongjia Wang (Houzhou), HHW (Munich)

Theoretical foundation of transport theory:
based on a chain of approximations from real-time Green functions
via Kadanoff-Baym eqs. to Boltzmann-Vlasov eq. (semi-classical , quasi-particle approx.)

In practice: two families of transport approaches

Boltzmann-Vlasov-like (BUU/BL/SMF)

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}-\vec{\nabla}U(r)\,\vec{\nabla}^{(p)}\right)f(\vec{r},\vec{p};t) = K^{<}[\sigma^{in-med}](1-f)-K^{>}[\sigma^{in-med}]\,f+\delta I_{fluct}$$

Dynamics of the 1-body phase space distribution function *f* with 2-body dissipation (collision term, gain and loss)

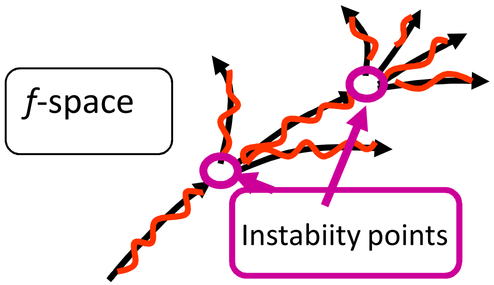
Solution with test particles, exact for $N_{TP}\rightarrow\infty$

include fluctuations around diss. solution

$$f(r,p,t)=\bar{f}(r,p,t)+\delta f(r,p,t)$$

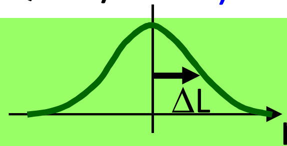
$$\boxed{\frac{df}{dt}=I_{coll}+\delta I_{fluc}}$$

Boltzmann-Langevin eq.



Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = A\prod_{i=1}^{A}\varphi(r;r_i,p_i)|0\rangle$$



$$\dot{r}_i=\{r_i,H\};\quad \dot{p}_i=\{p_i,H\};\quad H=\sum_i t_i+\sum_{i,j}V(r_i-r_j)$$

TD-Hartree(-Fock)
(or classical molecular dynamics with extended particles, Hamiltonian eq. of motion)
plus stochastic NN collisions

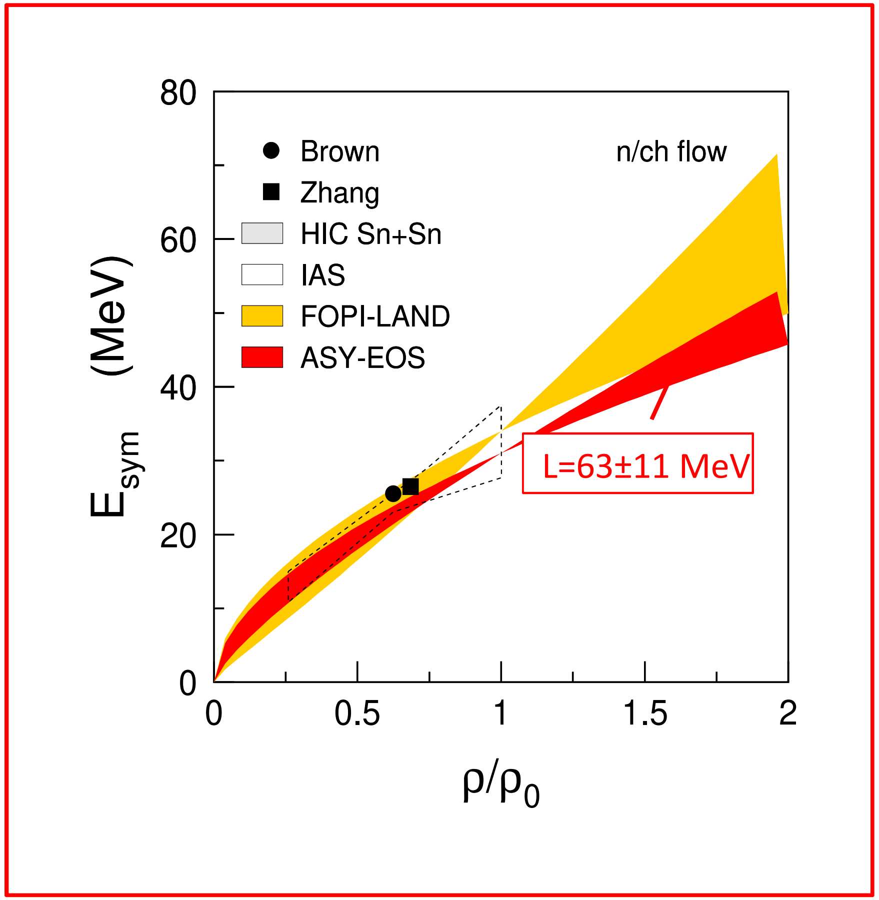
No quantum fluctuations,
but classical N-body fluctuations, damped by the smoothing.

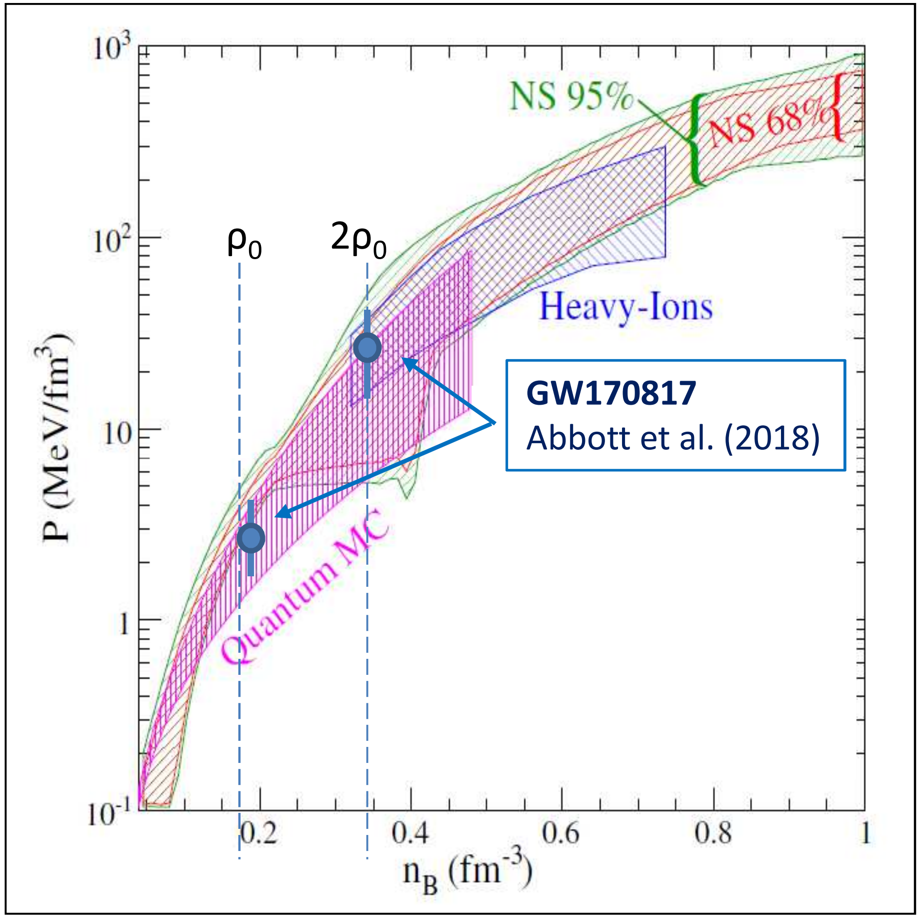
More fluctuations in QMD than in BUU, since degrees of freedom are nucleons:
→ amount controlled by width of single particle packet ΔL

Will see, that the different amount of fluctuations accounts for much the different behaviour of BUU and QMD

The Status of Symmetry Energy Research (Successes)

(taken from W. Trautmann)

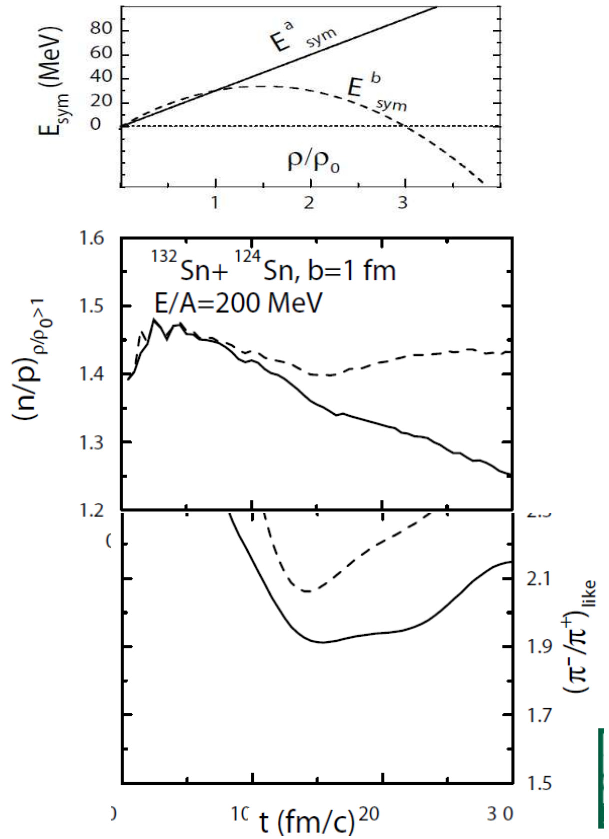




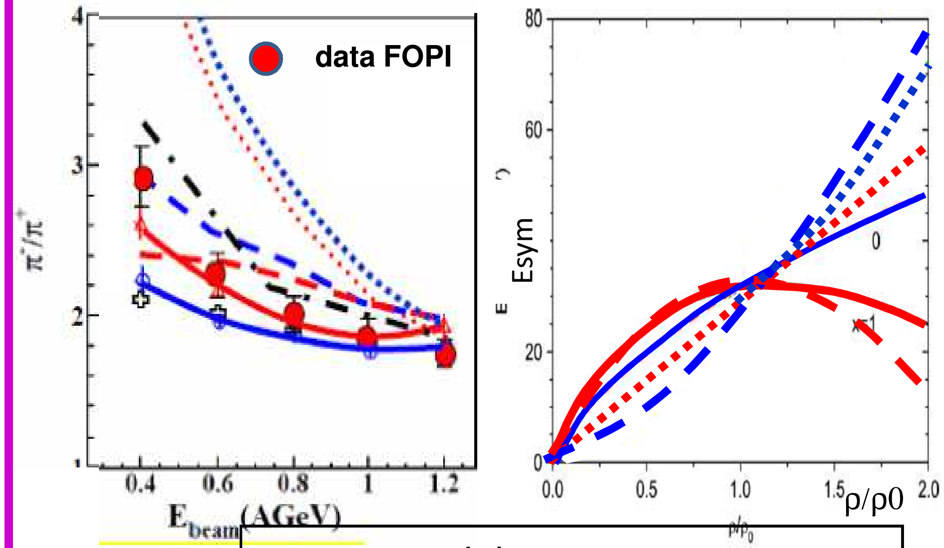
Looks encouraging, but more critical look necesary

Why Code Comparison? Failures

B.A.Li, PRL 88, 192701 (2002)



ratio of pion yields, Au+Au,0.4-1.2 GeV/A



various models
blue: stiffer symm energy
red: softer symm energy
→ no consensus, even on ordering

Reasons for differences often not clear, since calculations slightly different in the physical parameters. A need for more consistency in HI simulations: examples

→ therefore comparison of calculations with same physical input,
i.e. under controlled conditions

Code Comparison Project

History:
Workshop in Trento 2004 (1 AGeV regime, mainly particle production π,K
Workshop in Trento 2009 and Shanghai 2014 (Au+Au collisions, 100, 400 AMeV)
Workshop ICNT and NuSYM 2017, MSU 2017 (Cascade box calculations)
Conference NuSYM 2018 and Transport Workshop (Busan, Korea)
Workshop „Challenges to Transport Theory", Trento ECT*, May 20-24, 2019

Steps in Code Comparison of Transport Simulations

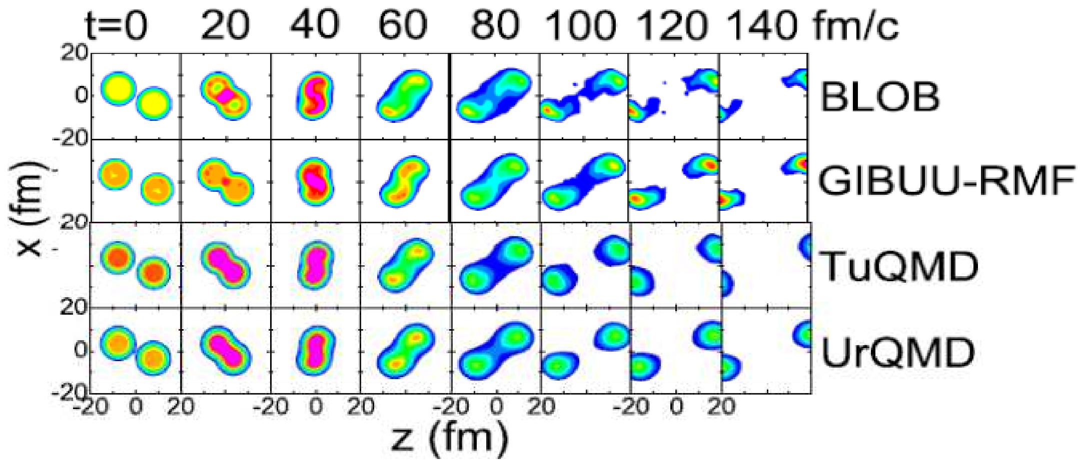
1. Full heavy ion collisions (Au+Au, 100, 400 AMeV) — done
J. Xu et al., Phys. Rev. C 93, 064609 (2016)
-> considerable discrepancies, but difficult to disentangle

2. Calculations of nuclear matter (box with periodic boundary conditions)
test separately ingredients in a transport approach:
a) collision term without and with blocking (Cascade) — done
Y.X. Zhang, et al., Phys. Rev. C 97, 034625 (2018)
b) mean field propagation (Vlasov) } in progress
c) π, Δ production in Cascade } in progress
d) instabilities , fragmentation } planned
e) momentum dependent fields } planned
……

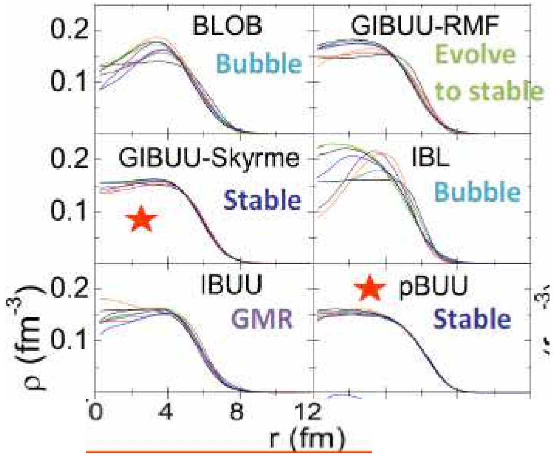
- → 19 codes of BUU- and QMD-type
- → non-rel. and relativistic codes
- → antisymmetrized QMD code: AMD, CoMD
- → BUU codes with explicit fluctuations: SMF, BLOB
- → many new Chinese codes: (I)QMD-XXX: much new activity in China, often originally closely related

Code Comparison Project (1st stage):

HIC at b=7m (midcentral)
selected contour plots;
different evolution apparent

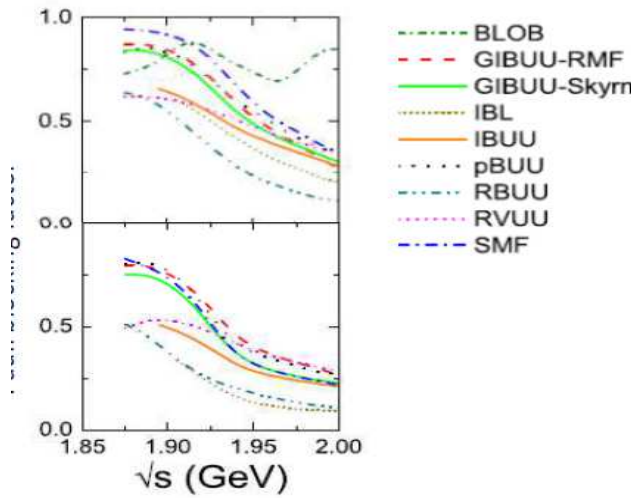


time evolution of isolated nucleus(examp)



★ Dynamical initialization (Thomas-Fermi)

blocking factor



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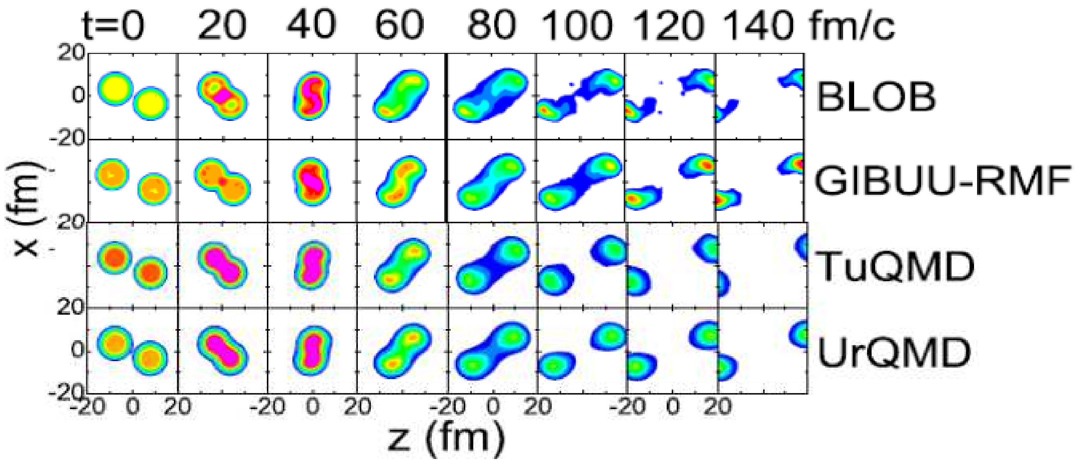
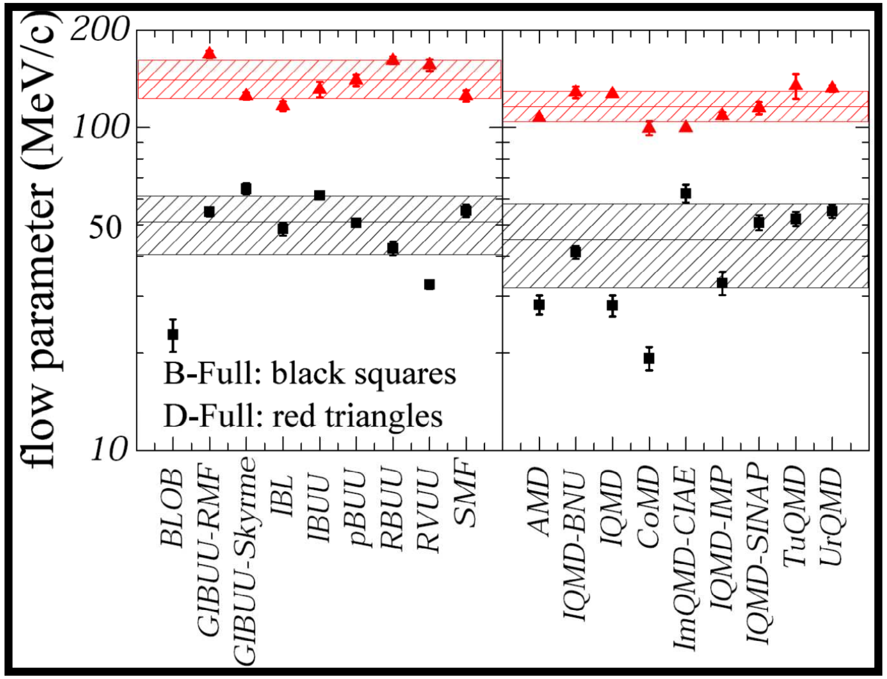


quantify spread of simulations by value of „flow“=slope at midrapidity
BUU and QMD approx. consistent

uncertainity

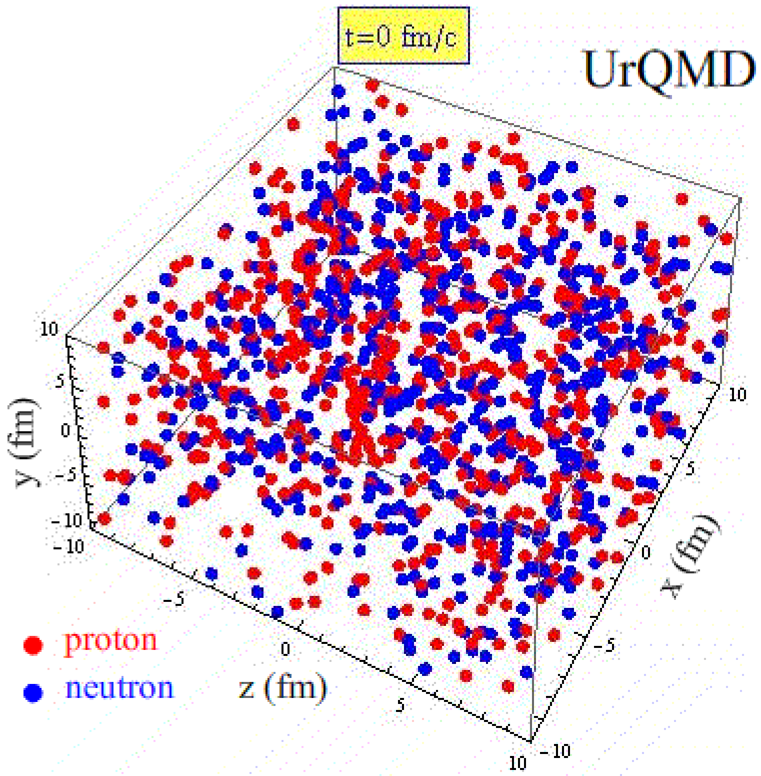
100 AMeV: ~30%
400 AMeV: ~13%



Difficult to disentangle origin of discrepancies

2. Box calculation comparison

simulation of the static system of infinite nuclear matter,
→ solve transport equation in a periodic box



Useful for many reasons:

- **check consistency of calculation**
  e.g. thermodynamical consistency
- **check consistency of simulation:**
  collision numbers, blocking
  (exact limits from kinetic theory)
- **check aspects of simulation separately**
  Cascade: only collisions
  without/with blocking
  Vlasov: only mean field propagation
- **check ingredients of particle production**
  e.g. pion production

PHYSICAL REVIEW C **97**, 034625 (2018)

Comparison of heavy-ion transport simulations: Collision integral in a box

Ying-Xun Zhang,[1,2,*] Yong-Jia Wang,[3,†] Maria Colonna,[4,‡] Pawel Danielewicz,[5,§] Akira Ono,[6,‖] Manyee Betty Tsang,[5,¶] Hermann Wolter,[7,#] Jun Xu,[8,**] Lie-Wen Chen,[9] Dan Cozma,[10] Zhao-Qing Feng,[11] Subal Das Gupta,[12] Natsumi Ikeno,[13] Che-Ming Ko,[14] Bao-An Li,[15] Qing-Feng Li,[3,11] Zhu-Xia Li,[1] Swagata Mallik,[16] Yasushi Nara,[17] Tatsuhiko Ogawa,[18] Akira Ohnishi,[19] Dmytro Oliinychenko,[20] Massimo Papa,[4] Hannah Petersen,[20,21,22] Jun Su,[23] Taesoo Song,[20,21] Janus Weil,[20] Ning Wang,[24] Feng-Shou Zhang,[25,26] and Zhen Zhang[14]

Collision term in box calculations

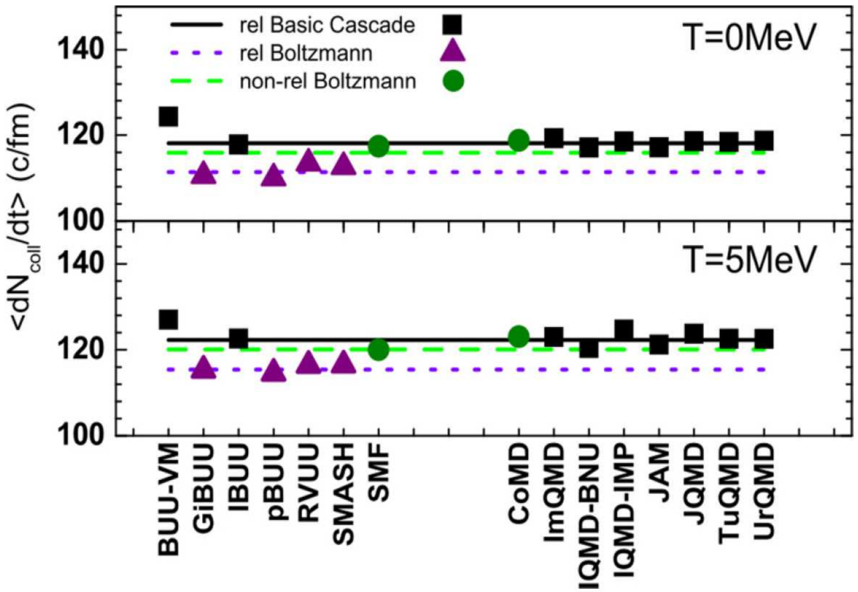
Collision rates in a cascade box calculation (w/o mean field, T=0 and 5 MeV)

collision probability

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'} \, v_{21} \, \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} (1-f_1)(1-f_2) - f_1 f_2 (1-f_{1'})(1-f_{2'}) \right]$$

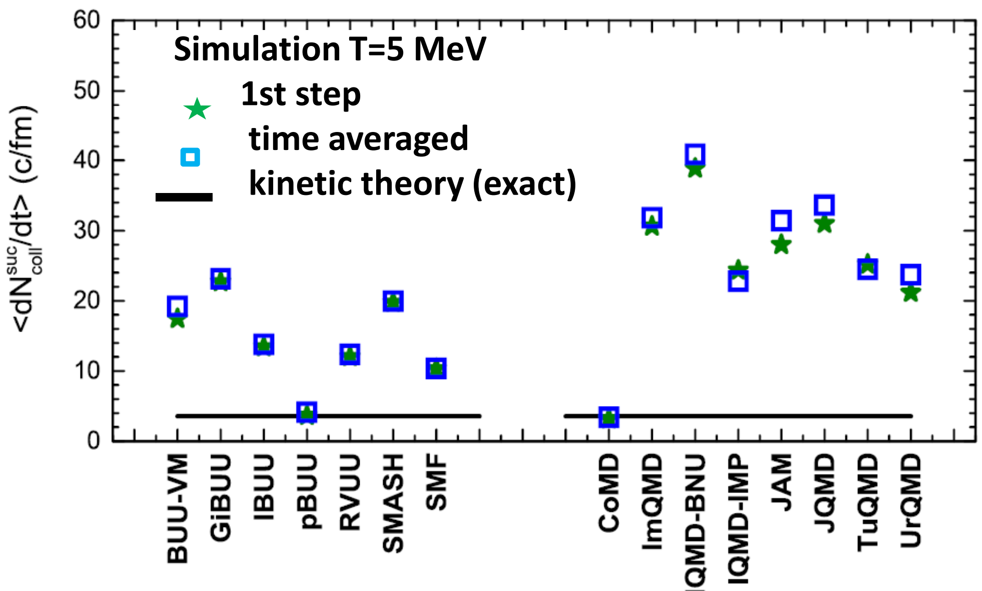
without blocking: comparison to exact limit

$$\frac{dN_{\rm coll}}{dt} = \frac{A}{2\rho} g^2 \int \frac{d^3p\, d^3p_1}{(2\pi\hbar)^6} v_{\rm rel}\, \sigma^{\rm med} f(p) f(p_1) = \frac{1}{2} A \rho \langle v_{\rm rel}\, \sigma^{\rm med} \rangle .$$



good agreement with corresponding exact result; collision probability ok

blocking:
Compare to exact solution of kinetic equation

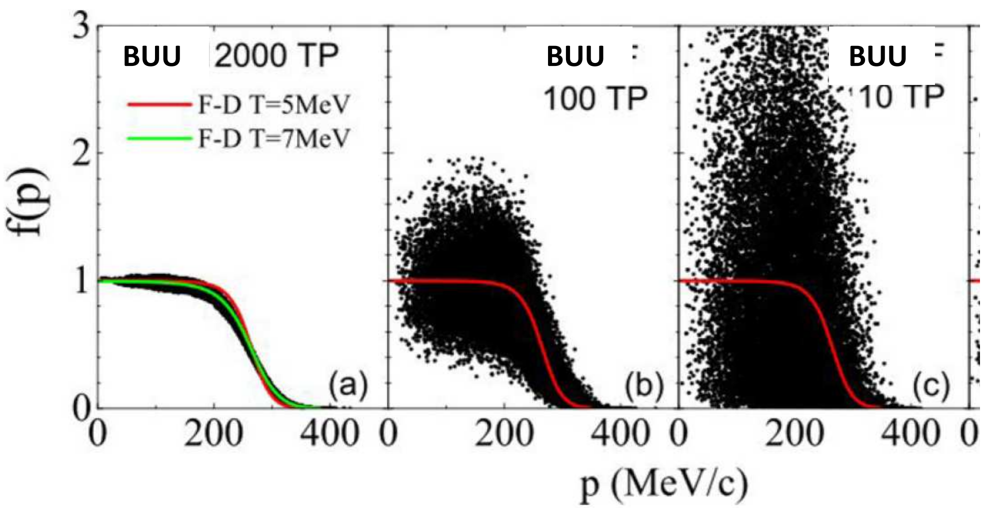


- almost all codes have too little blocking, i.e. allow too many collisions,
- QMD codes more
- can be connected to amount of fluctuations

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'}\, v_{21}\, \sigma_{12}^{in-med}(\Omega)\, (2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} (1-f_1)(1-f_2) - f_1 f_2 (1-f_{1'})(1-f_{2'}) \right]$$

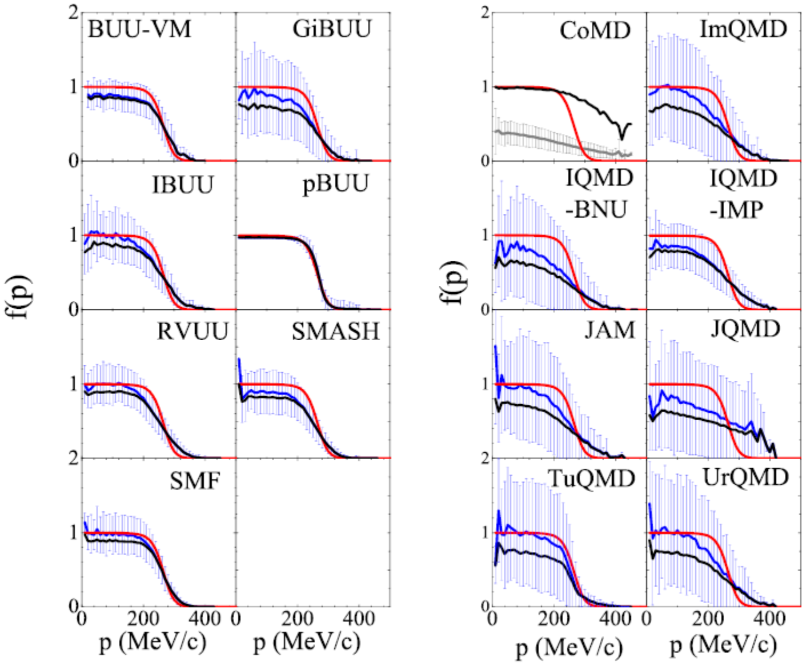
with blocking

Understand differences in collision probabioiies :
Sampling of occupation prob.
in comparison to prescribed FD distribution (red)



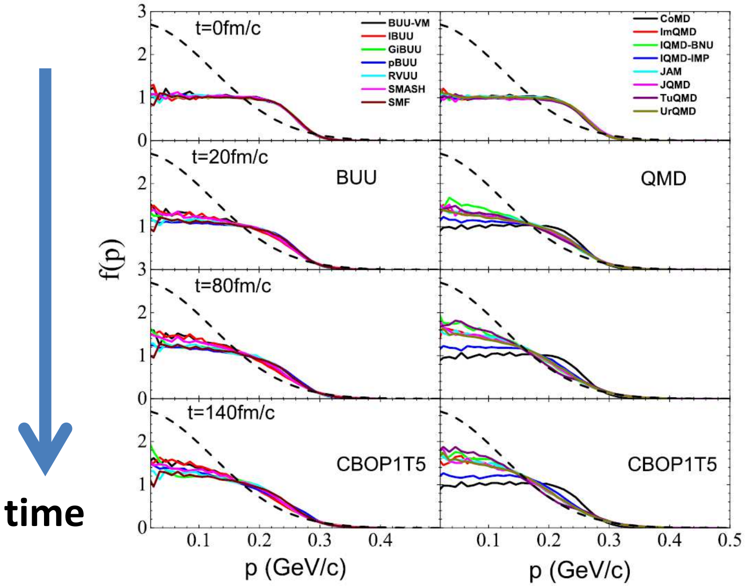
Fluctuations very different in transport models
- **fluctuation in BUU controlled by TP number, can be made arbitrarily small**
- **fluctuation in QMD given by width of wave packet**

Collision rates with blocking



width and averages of calculated occupation numbers in different codes

- prescribed occupation (red line)
- average calculated occupation (blue line)
- average of f<1 occupation (used for the blocking) (black line)



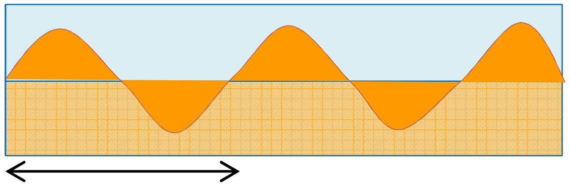
- the momentum distribution moves away from the stable Fermi-Dirac distribution towards the classical Maxwell-Boltzmann distribution (dotted line),
- depending on collision rates

Fluctuations influence dynamics of transport calculations.
However the proper treatment of fluctuations in transport is under debate.
--> implement Boltzmann-Langevin eq.

Box simulations: test of m.f. dynamics (in progress)

Symmetric matter   only mean-field potential

- Initialize standing wave and follow the time evolution of ρ(z)

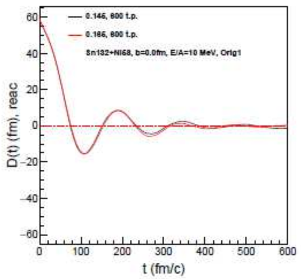


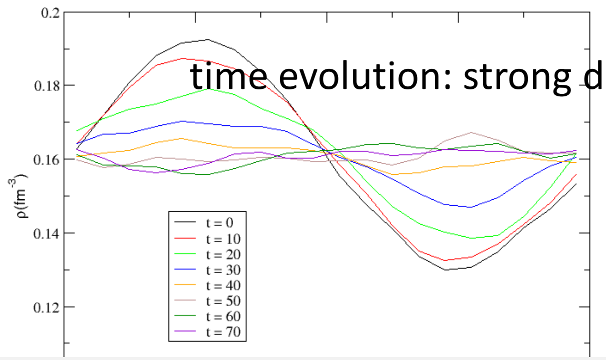
$$\rho(z,t=t_0)= \rho_0 + a_\rho \sin(k_i z)$$

$$k_i=n_i\, 2\pi/L,\ a_0=2\rho_0$$

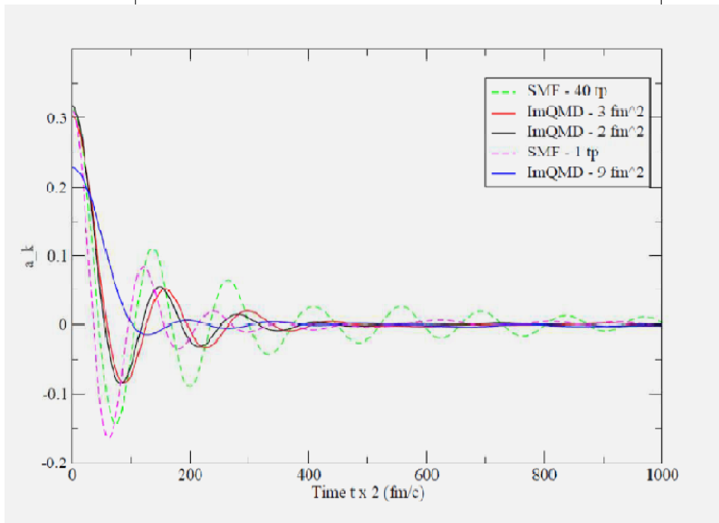
1. Extract the Fourier transform in space

$$\rho_k(t) = \int dz\, \sin(kz)\, \rho(z,t)$$





time evolution: strong damping



Generally: strong damping

- SMF (BUU-like, dashed curves)

smaller no of TP: more damping, larger frequency

- ImQMD (solid curves)

increasing width Δx of wave packet:

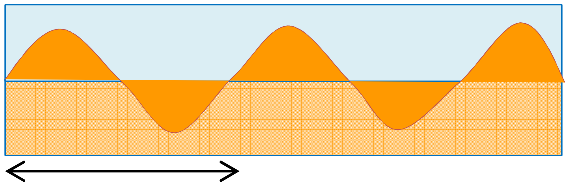
larger fluctuations in QMD $\rightarrow$ stronger damping

smaller effective forces in QMD $\rightarrow$ larger frequencies

Box simulations: test of m.f. dynamics (in progress)

Symmetric matter only mean-field potential

- Initialize standing wave and follow the time evolution of $\rho(z)$



$\rho(z,t=t_0)= \rho_0 + a_\rho \sin(k_i z)$

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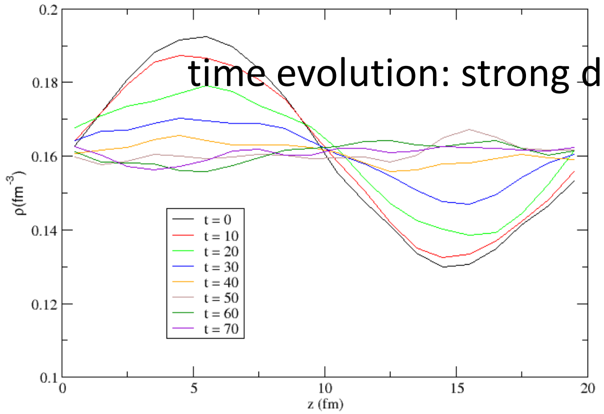

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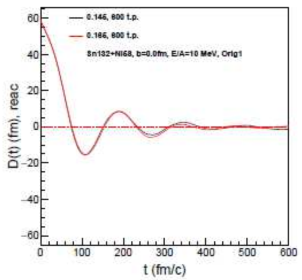
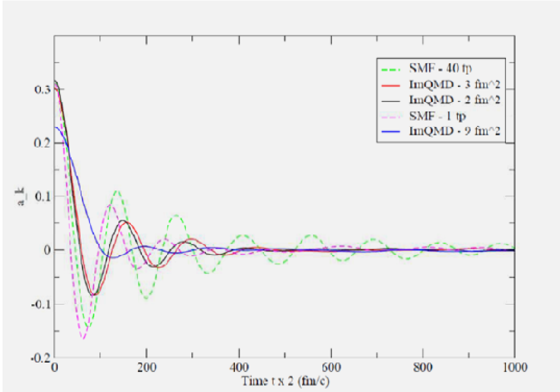
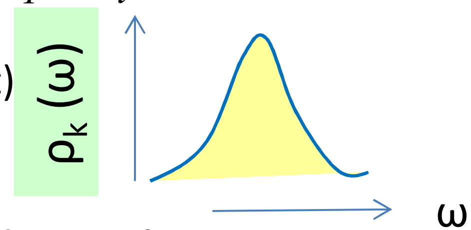
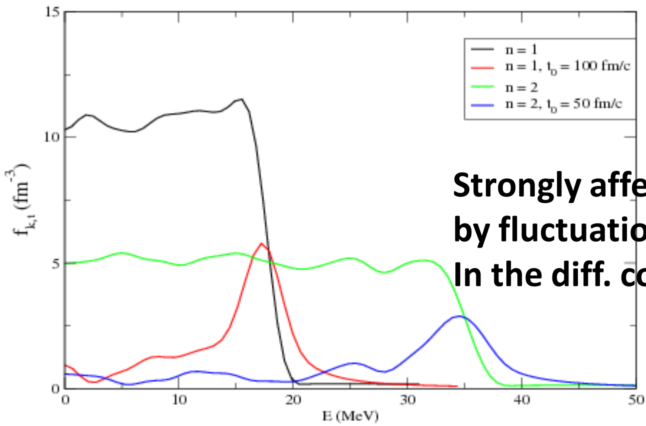




2. Fourier transform in time:
extract the oscillation frequency

$\rho_k(\omega) = \int dt \cos(\omega t)\, \rho_k(t)$





Strongly affected by fluctuations In the diff. codes.

Compare to dispersion relation of interaction

π,Δ production in box cascade calculation:

(in progress, preliminary, code names blanked out)

energy dep cross sect.

$$\sigma(NN \to N\Delta) = \frac{(\sqrt{s} - 2M_N - M_\pi)^2}{(0.015\ \mathrm{GeV}^2) + (\sqrt{s} - 2M_N - M_\pi)^2} \times 20\ \mathrm{mb}$$

Δ spectral function

$$A(m) = \frac{4 M_\Delta^{0\,2} \Gamma_\Delta}{(m^2 - M_\Delta^{0\,2})^2 + M_\Delta^{0\,2} \Gamma_\Delta^2}$$

N,Δ, no pions

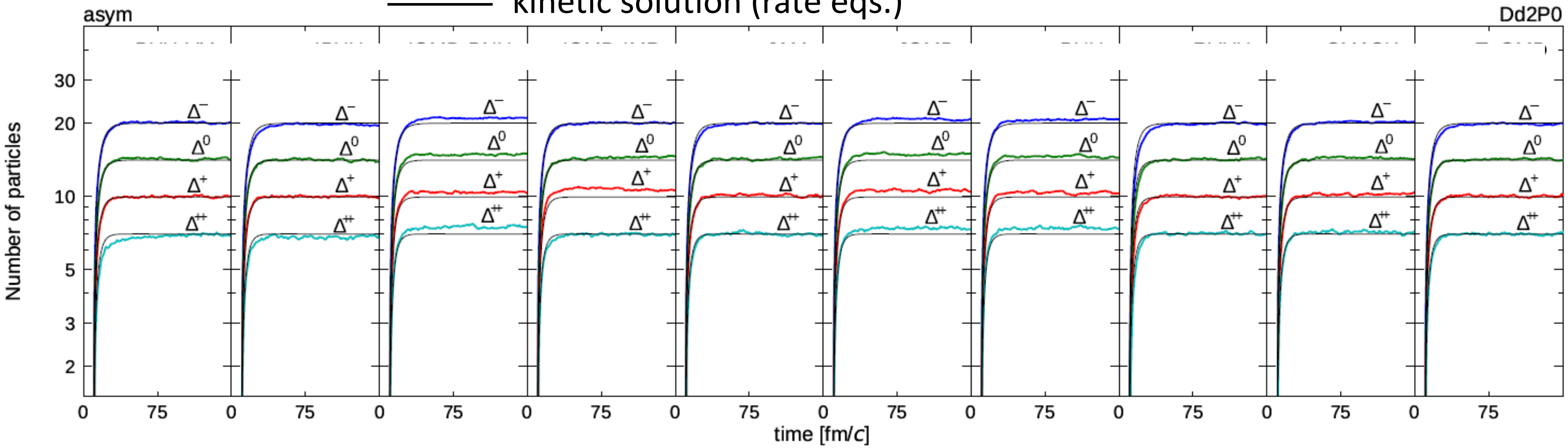
—— kinetic solution (rate eqs.)

two- ways

$$NN \leftrightarrow N\Delta$$

Time dependence of no. of Δ's in different codes (names blanked)

—— kinetic solution (rate eqs.)



Looks reasonably ok!

Now switch on pions

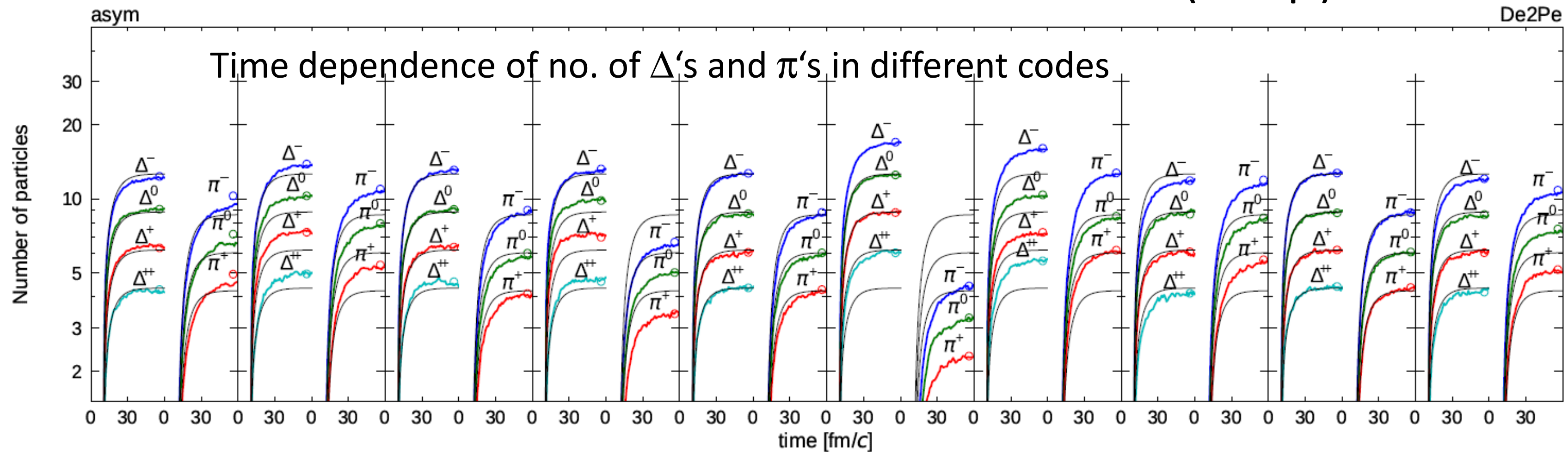
π,Δ production in box cascade calculation:
(in progress, preliminary, codes names blanked)

now including pions

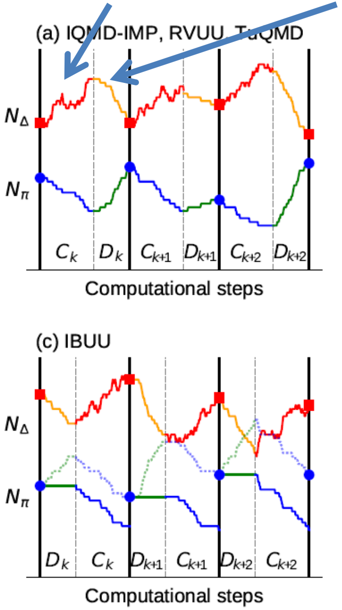
$$NN \leftrightarrow N\Delta, \quad \Delta \leftrightarrow N\pi$$

kinetic solution (rate eqs.)

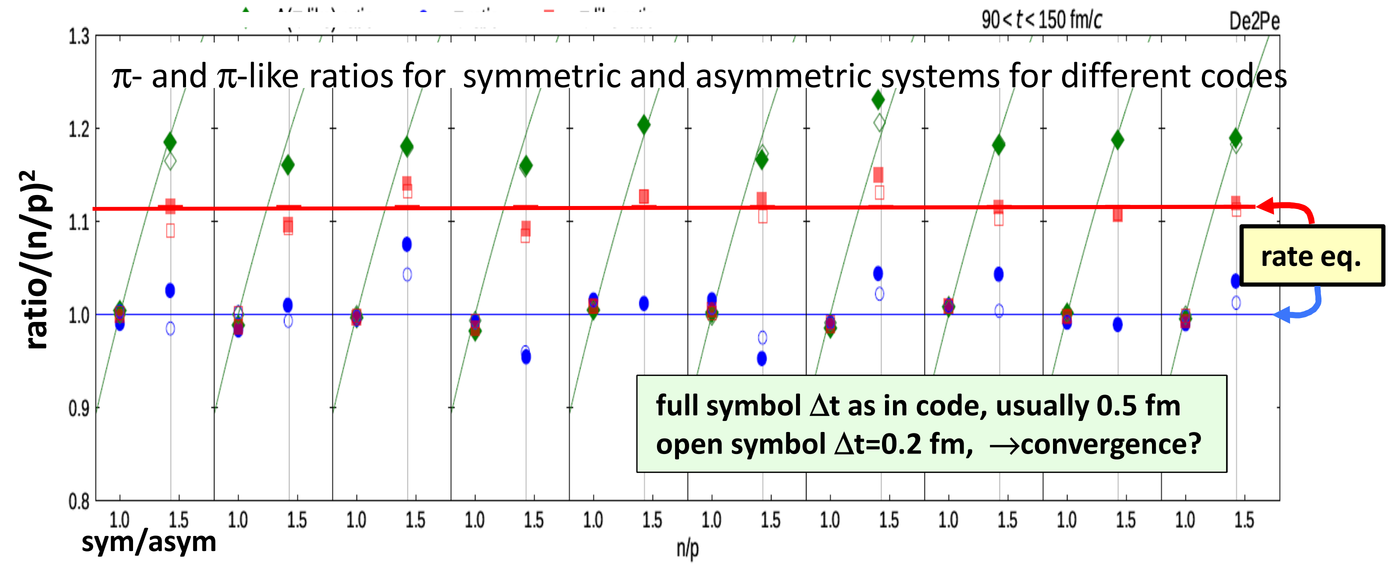
large differences between models and exact result



(partly) due to sequence of handling collisions (C_k) and decays (D_k)



ratios π ratio $= \pi^-/\pi^+$ $\quad$ π-like ratio $= \dfrac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+}$



→ towards a better understanding of the differences in the pion ratios

Summary

-Transport approaches necessary to extract physics information from complex non-equilibrium processes, as e.g. heavy ion collisions.

However, there are open problems in the application of transport theories:
- physical (degrees of freedom, fields and in-medium cross-sections, fluctuations, correlations, short range)
- questions of implementation: simulation, rather than solution of the transport equations
- involves strategies not strictly given by the equations, such as
  representation of the phase space, coarse graining, criteria for collisions and Pauli blocking

- these may affect the deduction on physical properties from collisions and lead to a kind of systematical theoretical error
- here attempt to understand, quantify and hopefully reduce these uncertainities in a Transport Code Comparison under Controlled Conditions

Results:
- Comparison of full HIC makes evident the discrepancies (initializations, collision term), but difficult to disentangle
- Box calculations to study the different ingredients of transport
  (collisions, blocking, mf evolution, particle production)
- Important influence of fluctuations on the simulations
- Fluctuations (and correlations) go beyond the one-body description. Implementions differ in BUU (explicit fluctuation term) and QMD (classical correlations + smoothing by wave packet)
- particle production: strategy of treatment of inelastic collisions and of decay has an influence

- continue in the future, e.g. in fragmentation in instable regime, pion production in full HIC, off-shell effects …

Thank you for the attention