– Hadron Physics –
Lecture on Selected Topics of the Conference

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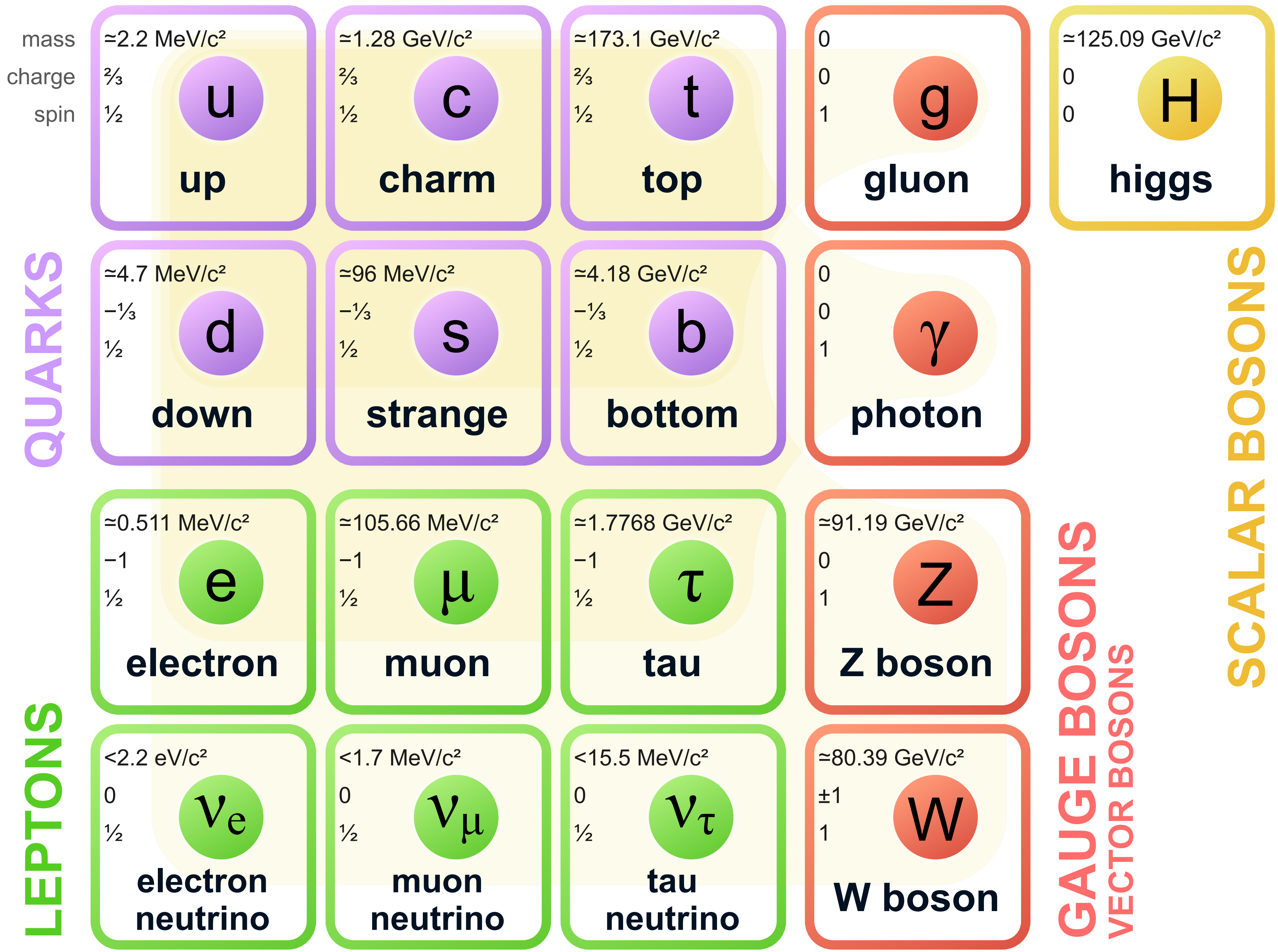
- From Hadrons to QCD $\rightarrow$ brief motivation of the fundamental theory
  - Quarks as building blocks $\rightarrow$ QCD Lagrangian
- From QCD to Hadrons $\rightarrow$ deriving expectations from QCD Lagrangian
  - e.g. Symmetries of QCD $\rightarrow$ potential models, Effective theories, Lattice
- Determination of Hadron properties
  - Methods: e^+e^- Annihilation, γ+Baryon, Hadron-Hadron Collisions, Electron Scattering
  - $\Rightarrow$ Mass, Width, Decays, Quantum Numbers,
    Wave-Function (Form-Factor, Polarizabilities, ...)
- Compare experiments with expectations: Exotics, ...

The Standard Model of Elementary Particles



Quark Model

Introduced 1964 by Gell-Mann/Zweig to clean up “particle zoo”

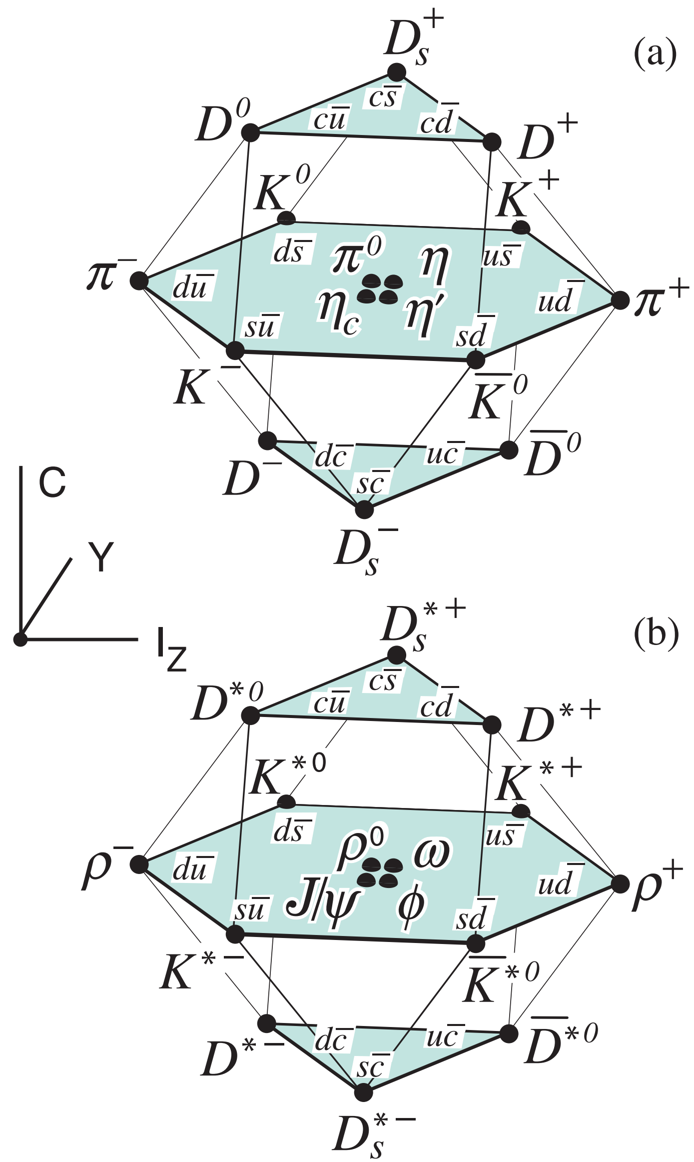
Mesons as Quark-Antiquark Pair:

Pions:

π^+	π^0	π^-	η_1
$\lvert u\bar{d}\rangle$	$\frac{1}{\sqrt{2}}\left(\lvert u\bar{u}\rangle - \lvert d\bar{d}\rangle\right)$	$\lvert d\bar{u}\rangle$	$\frac{1}{\sqrt{2}}\left(\lvert u\bar{u}\rangle + \lvert d\bar{d}\rangle\right)$

Kaons:

K^+	K^0	$\overline{K}^0$	K^-
$\lvert u\bar{s}\rangle$	$\lvert s\bar{u}\rangle$	$\lvert u\bar{s}\rangle$	$\lvert s\bar{u}\rangle$



… 6 flavours → 36 Mesons?

C: Charm, Y: Hypercharge, I_z: Isospin

Baryons

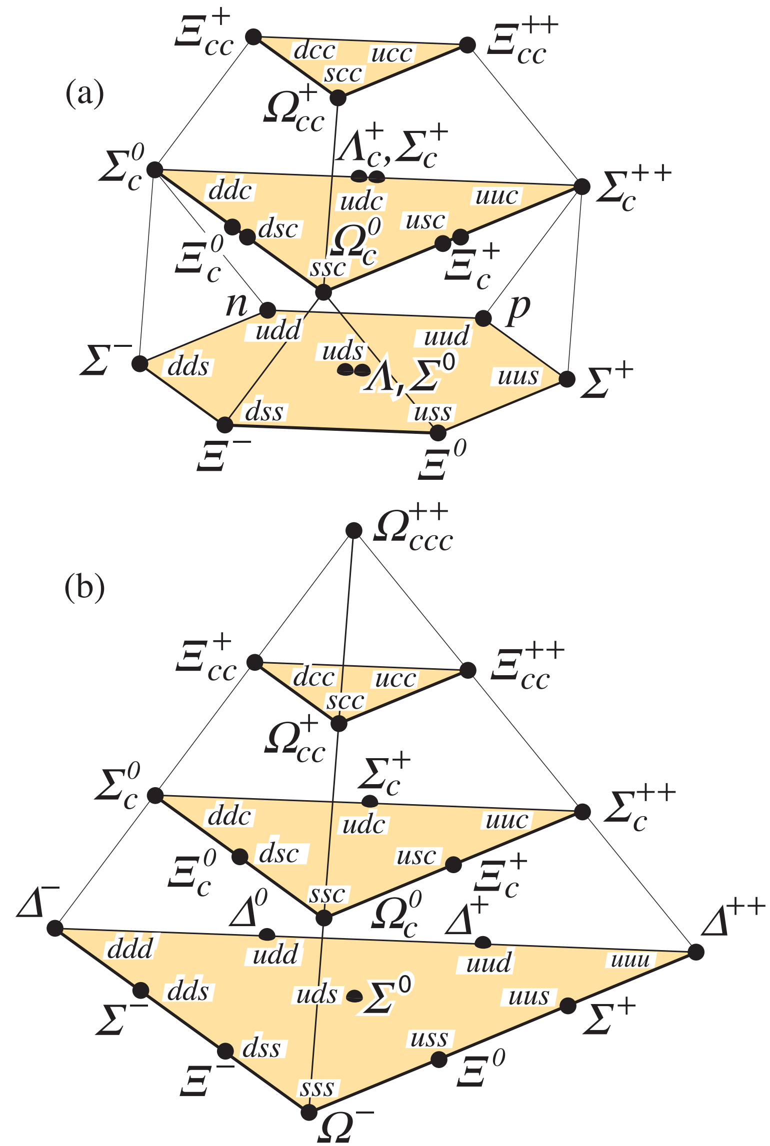
Baryons as three quark states

Examples:

$$p: \quad |u\uparrow u\downarrow d\uparrow\rangle$$
$$n: \quad |u\uparrow d\downarrow d\uparrow\rangle$$
$$\Delta(1232): \quad |u\uparrow u\uparrow d\uparrow\rangle$$
$$\Lambda: \quad |u\uparrow d\downarrow s\uparrow\rangle$$

...

Ground states are OK, excited states?



Color

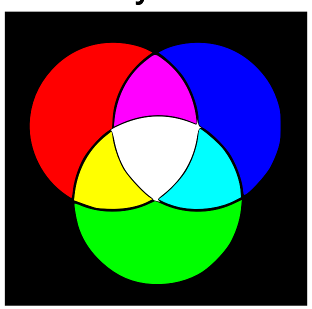
Problem: Δ^{++} with angular momentum $J = \frac{3}{2}$:

$$\Delta^{++} = \underbrace{|uuu\rangle}_{\text{flavour}} \cdot \underbrace{|\uparrow\uparrow\uparrow\rangle}_{\text{spin}} \cdot \underbrace{|l=0\rangle}_{\text{orbital } l}$$

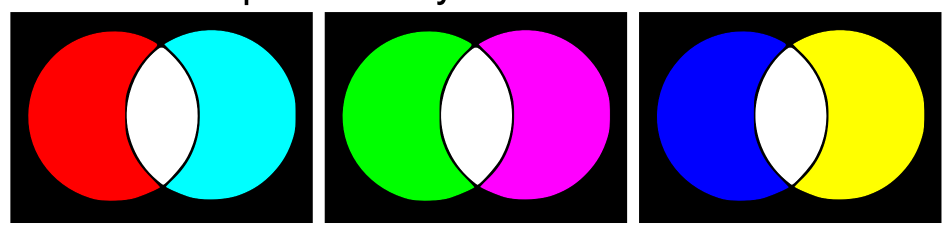
- Not possible for Fermions $\rightarrow$ additional antisymmetric charge neccessary
- Not visible for three- and two-quark states

Color Analogy:

Three colors:
Primary Colors



Two Colors:
Color – complementary Color



Physical objects are colorless (*i.e.* $SU(3)$ Color-Singulets):

Baryons: red–green–blue tripletts

$$|qqq\rangle = \sqrt{\frac{1}{6}}(|RGB\rangle - |RBG\rangle + |BRG\rangle - |BGR\rangle + |GBR\rangle - |GRB\rangle)$$

Mesons: color–anti-color pairs

$$|q\bar{q}\rangle = |R\bar{R}\rangle + |G\bar{G}\rangle + |B\bar{B}\rangle$$

$\Rightarrow SU(3)$ Symmetry of Gluons

QCD Lagrangian

Lagrangian field theory:

$L = T - V$ and Lagrange's Equation $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$

or with continuous field $\phi(x_\mu)$ $\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial x_\mu)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$

Only two ingredients for $\mathcal{L}_{\text{QCD}}$:

- Quarks are massive spin $\frac{1}{2}$ particles $\Rightarrow$ Dirac equation for free lagrangian

$$\mathcal{L}_0 = \bar{q}_j(i\gamma^\mu\partial_\mu - m)q_j$$

- Gauge invariant under $SU(3)$ color symmetry
  i.e. invariant under local phase rotation: $q(x) \to e^{i\alpha_a(x)T_a}q(x)$ with eight 3×3 matrices T_a

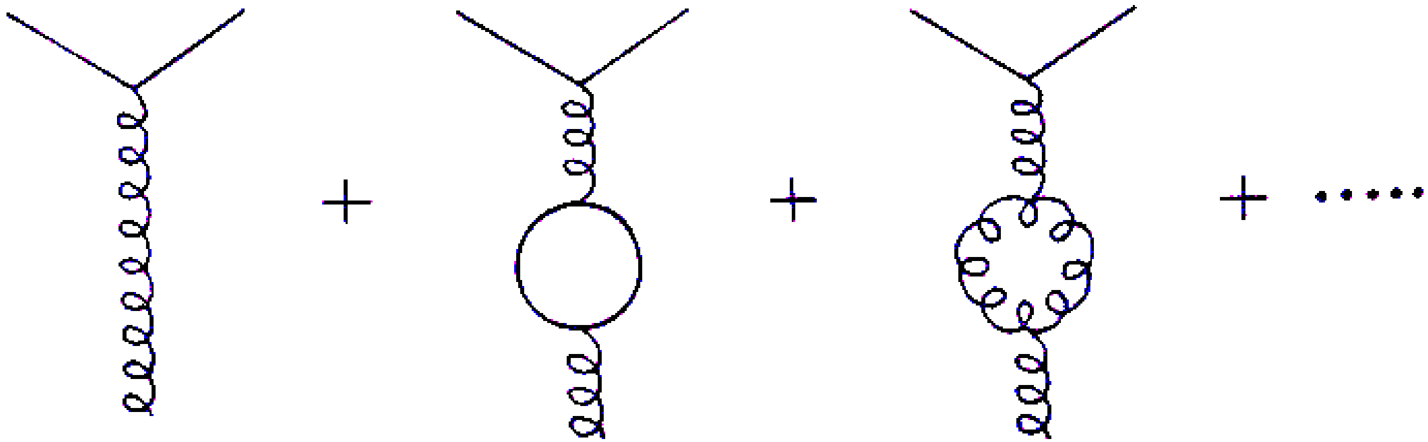
$$\mathcal{L}_{\text{QCD}} = \bar{q}(i\gamma^\mu\partial_\mu - m)q - g(\bar{q}\gamma^\mu T_a q)\, G^a_\mu - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

with 8 massless vector gauge fields transforming like $G^a_\mu \to G^a_\mu - \frac{1}{g}\partial_\mu\alpha_a - f_{abc}\alpha_b G^c_\mu$

gauge field strength tensor $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g\, f_{abc}G^b_\mu G^c_\nu$

$SU(3)$ structure constants given by $[T_a, T_b] = i\, f_{abc}T_c$ $\Rightarrow$ "non abelian"

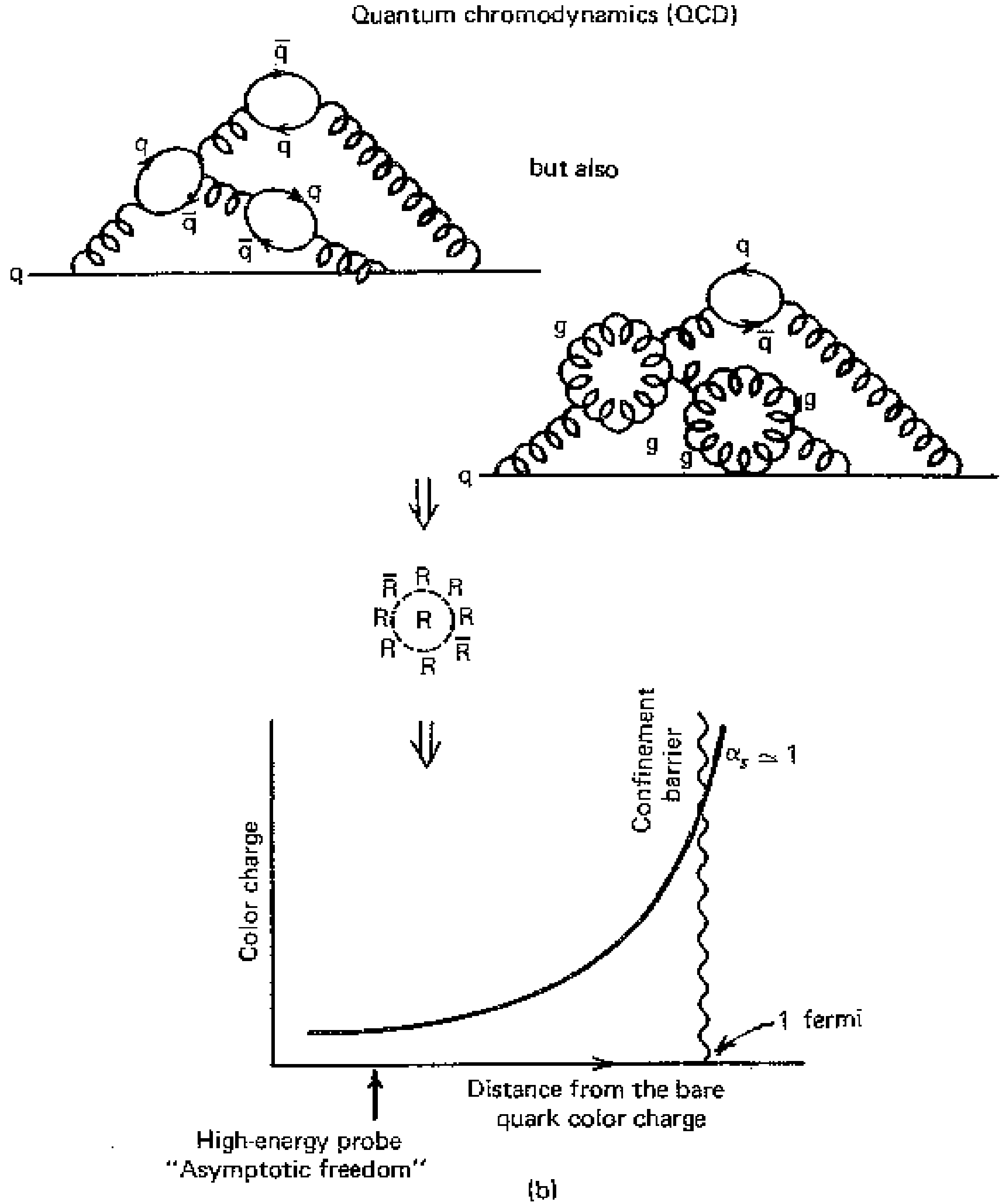
Charge Screening

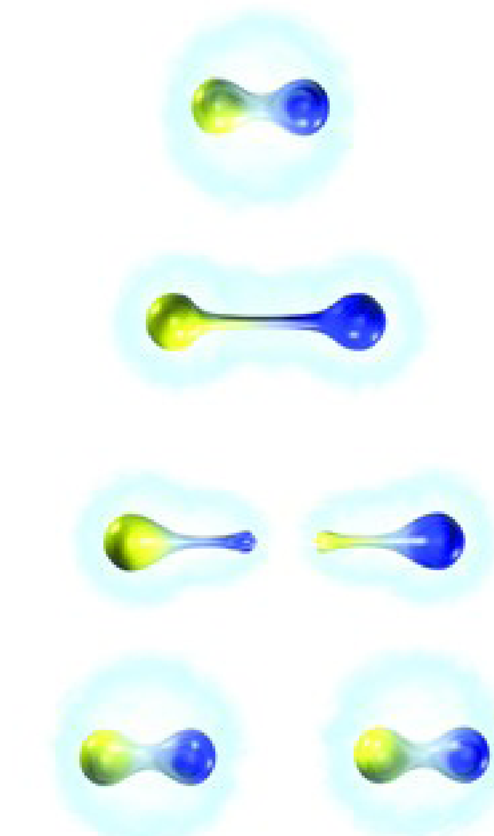


- Quark loops like lepton loops in QED
- For each flavour, large mass supressed
- Additional:
  - Gluon Loops
  - Large contribution: 8 gluons
  - opposite sign!

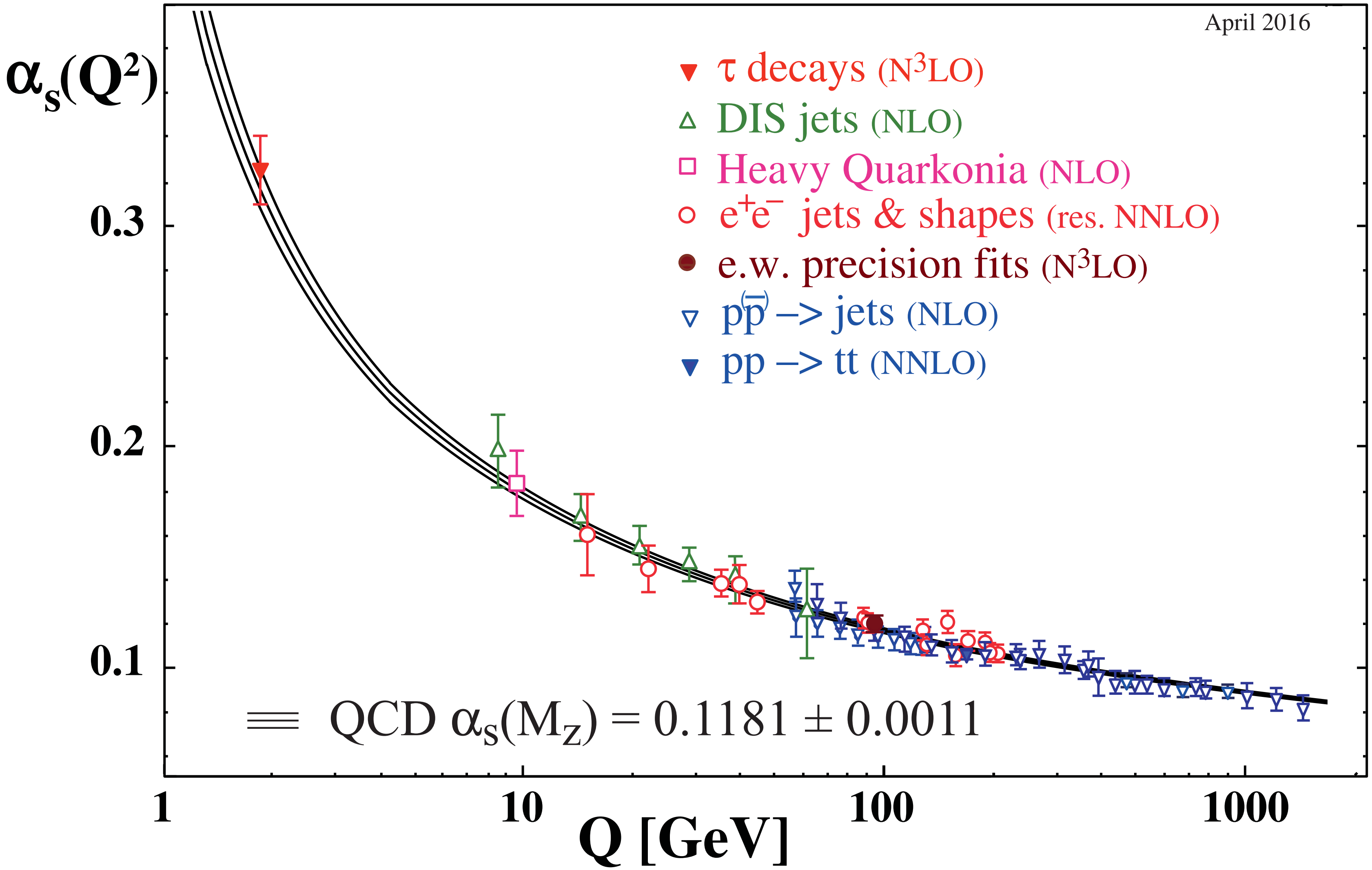
F. Halzen, A. Martin, Quarks and Leptons

Strong Coupling Constant



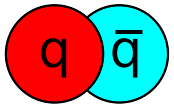


Strong Coupling Constant

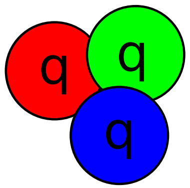


⇒ running of α_s ⇒ non-abelian structure of QCD!

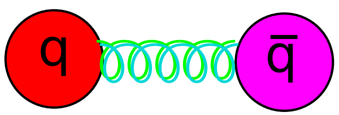
Possible Quark States



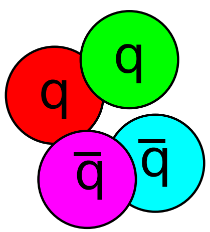
$q\bar{q}$ Meson



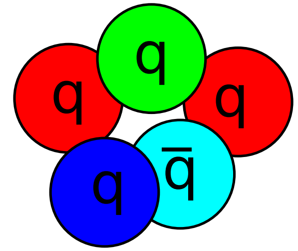
qqq Baryon



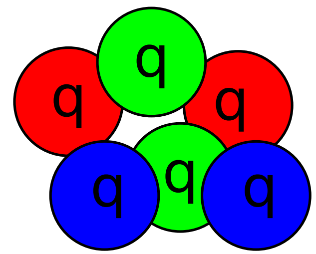
$q\bar{q}g$ Hybrid



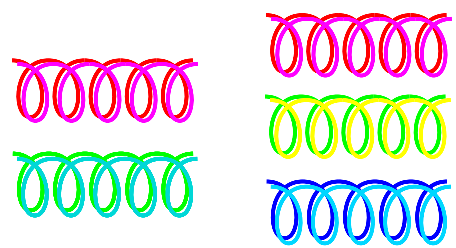
$q\bar{q}q\bar{q}$ Tetraquark



$qqqq\bar{q}$ Pentaquark



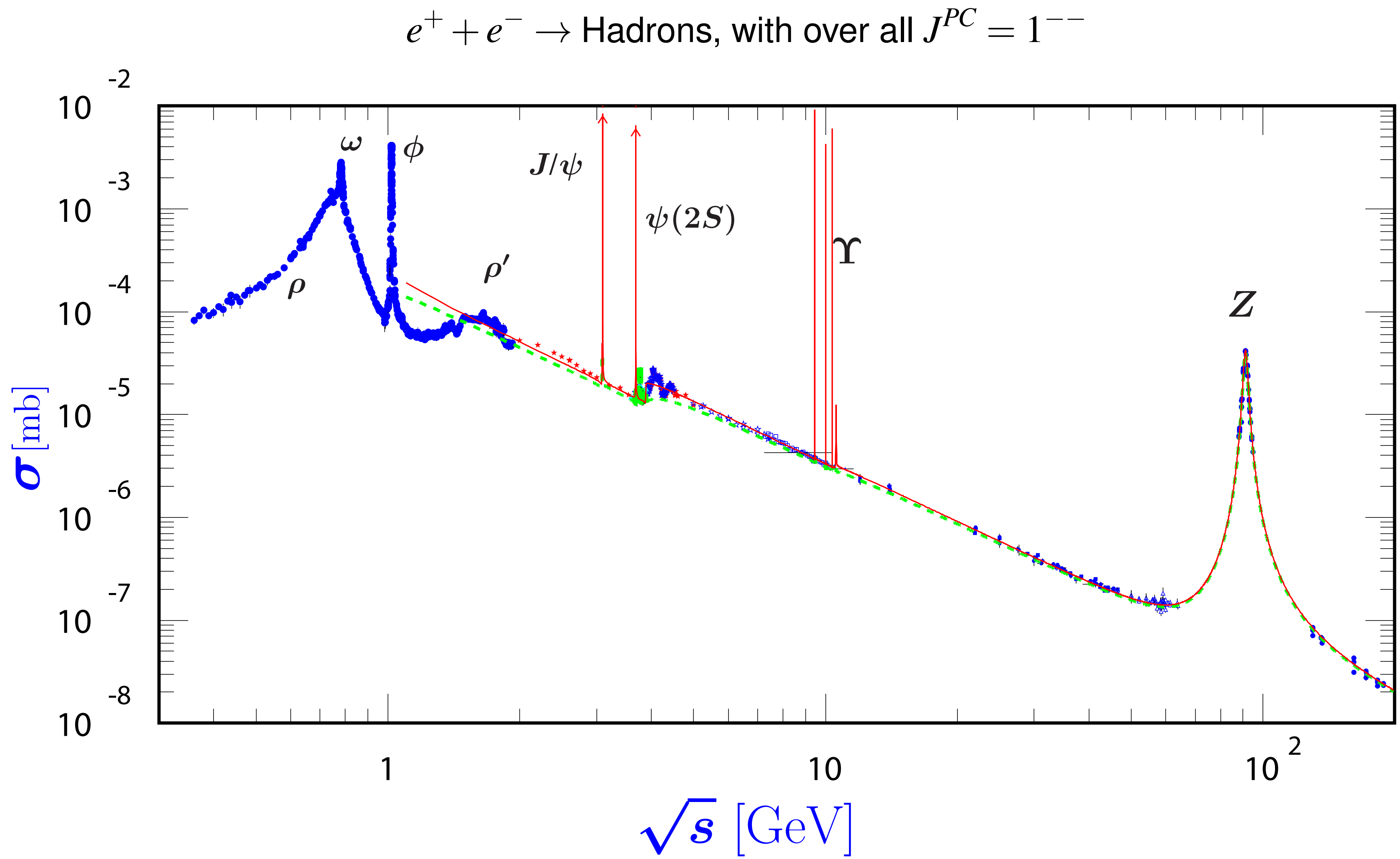
$qqqqqq$ Dibaryon
$qqq\overline{qqq}$



$g\bar{g}$, ggg Glueballs

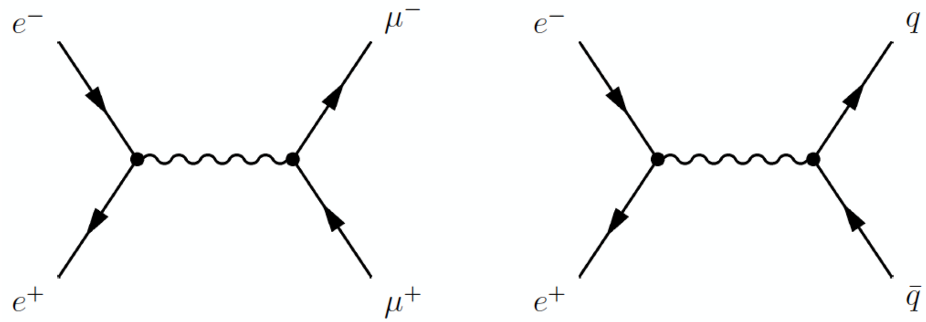
- Not only $q\bar{q}$ and qqq states $\Rightarrow$ a new zoo of “Exotics” is expected!
- Important for most of them: “Color-Singulet” does not mean “white”!
  Two singulets are always decoupled $\rightarrow$ non-trivial binding (e.g. “white” exchange) neccessary

e^+e^- Annihilation: Cross Section



e^+e^- Annihilation: general features

Idea: Relate $q\bar{q}$ cross section to known (i.e. QED) cross section (μ to be distinguishable from e):



$\mu^+\mu^-$ cross section from QED:

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

$q\bar{q}$ cross section (also only QED!):

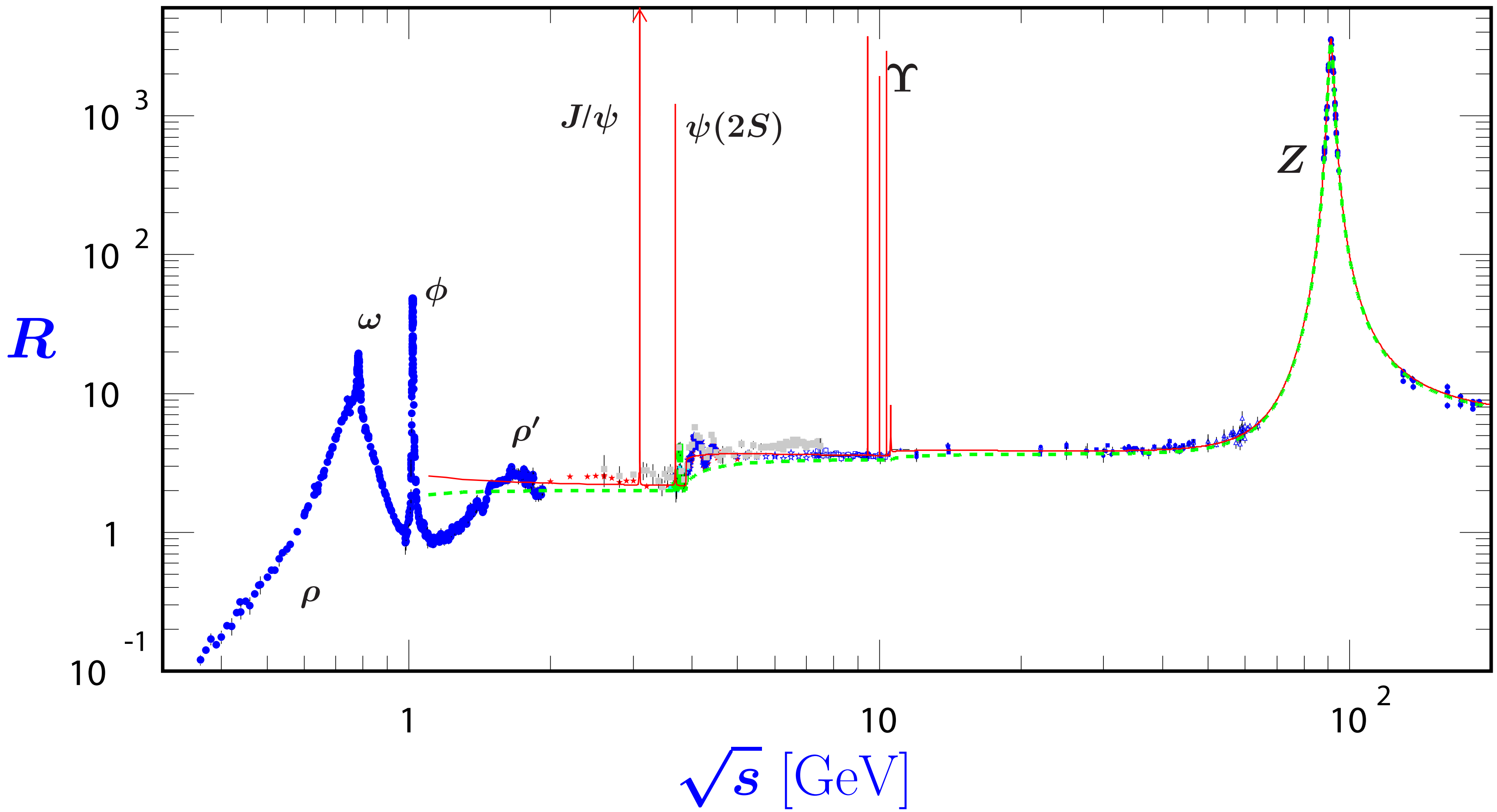
$$\sigma(e^+e^- \to q\bar{q}) = N_c e_q^2 \, \sigma(e^+e^- \to \mu^+\mu^-)$$

$$\text{with} \quad e_q = \begin{cases} -\frac{1}{3} & \text{for } q = \; d,s,b \\ +\frac{2}{3} & \qquad\quad\; u,c,t \end{cases}$$

$$\text{and} \quad N_c = 3 \quad \text{number of colors.}$$

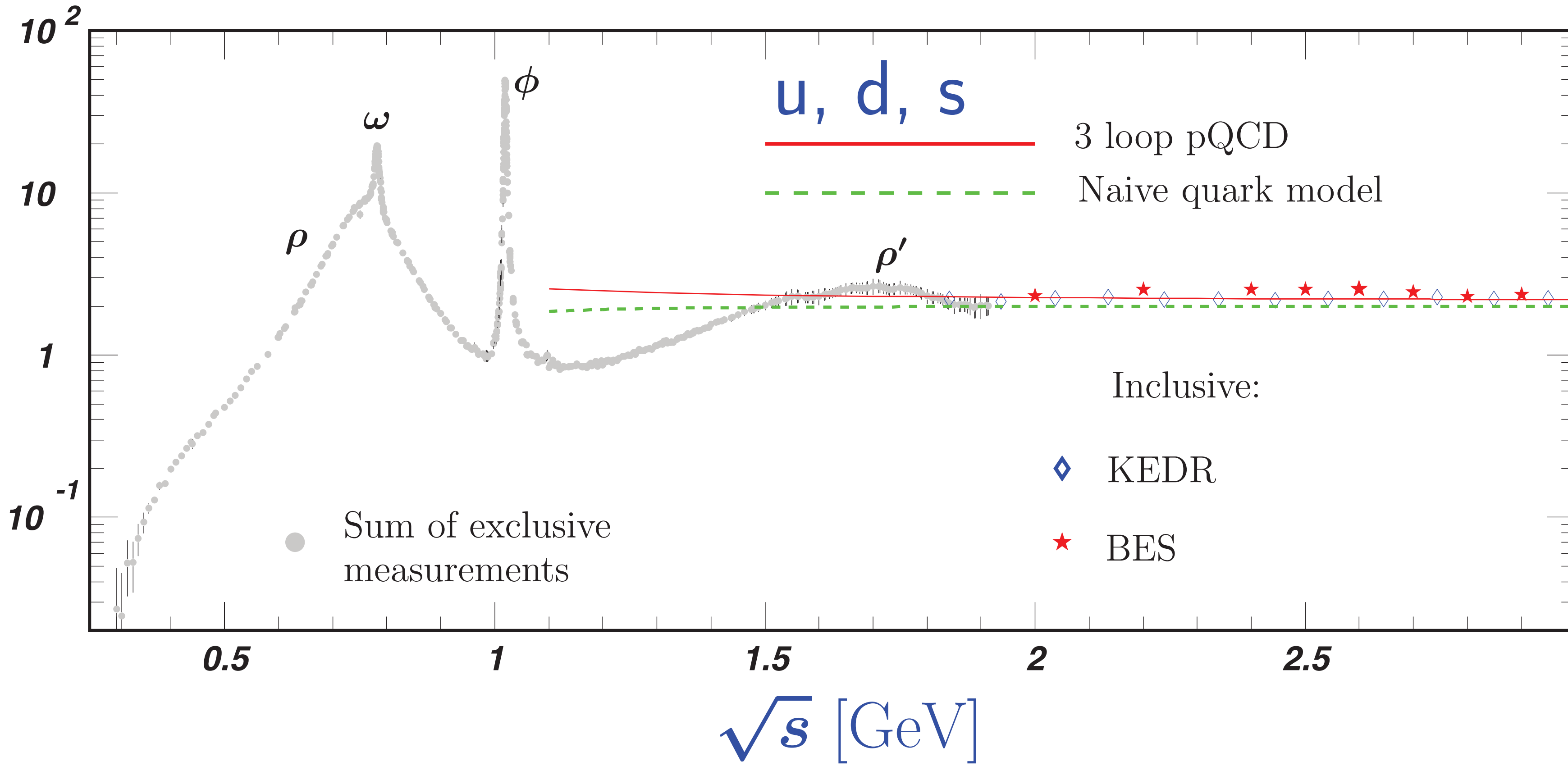
$$R = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q 3e_q^2$$

e^+e^- Annihilation



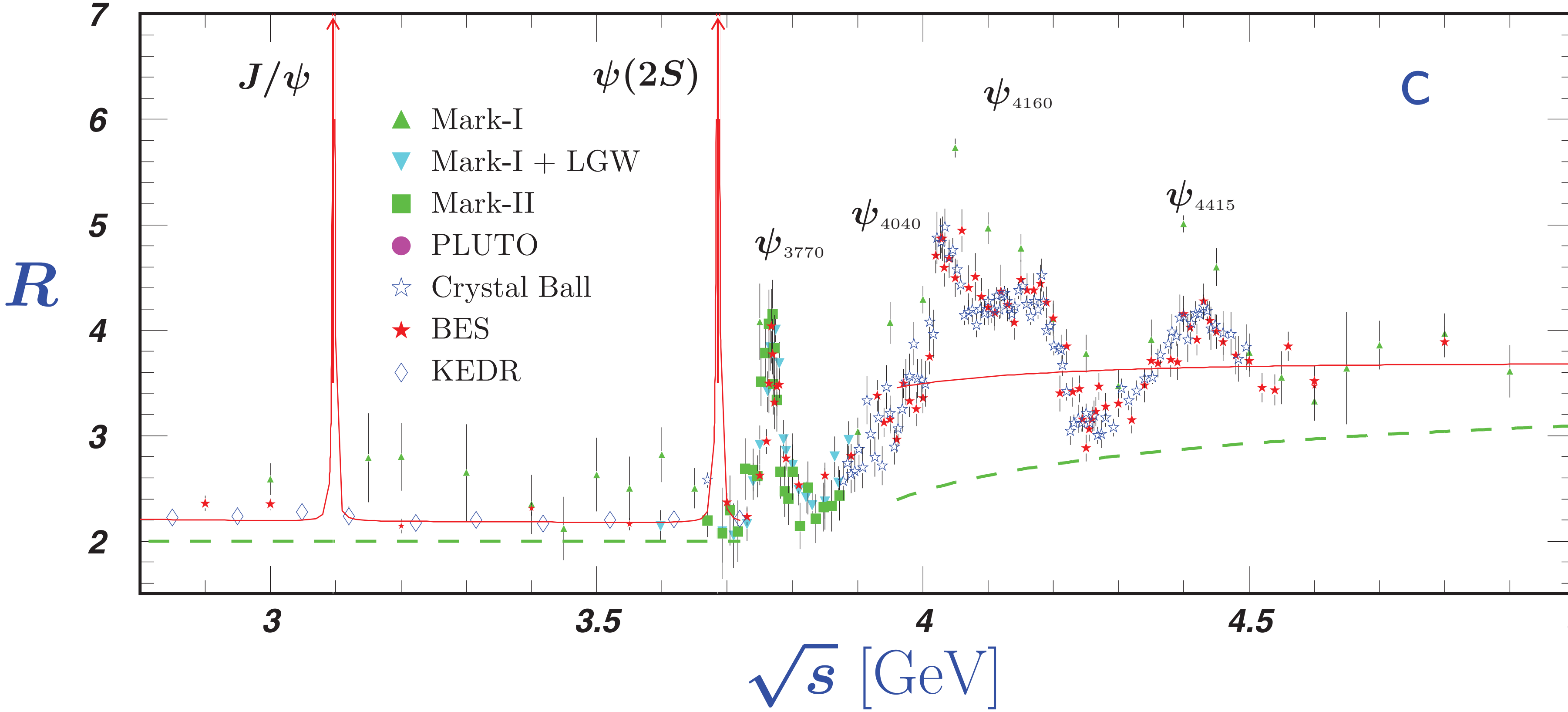
- with QCD corrections: $R = \sum_q 3e_q^2(1 + \frac{\alpha_s(Q^2)}{\pi})$
- confirms quark charge
- confirms (again) $N_c = 3$ colors

R-Ratio: u, d, s



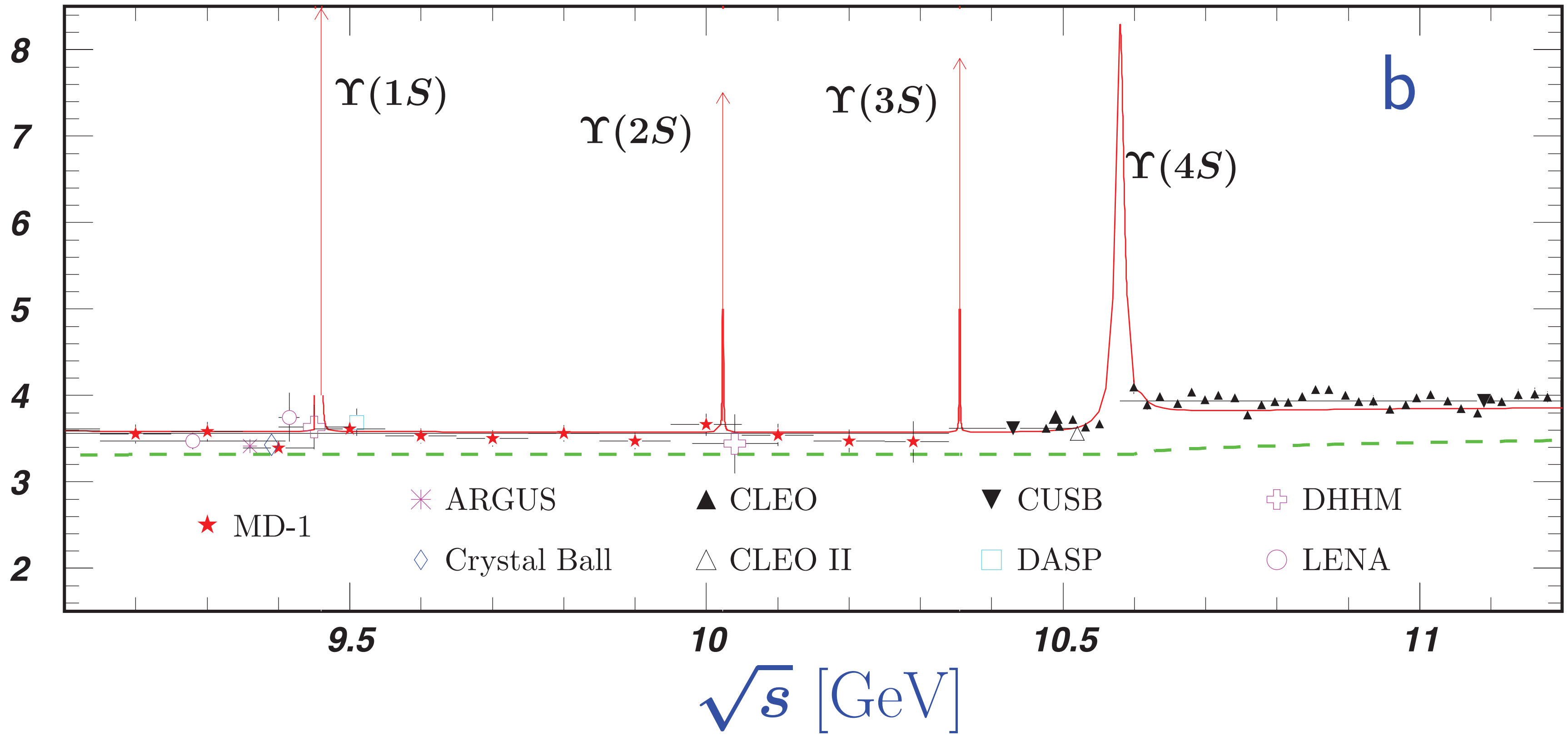
$$R = \sum_q 3e_q^2 = 3\left(\left(\frac{2}{3}\right)^2 + 1\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right) = 2$$

R-Ratio: c



$$R = \sum_q 3e_q^2 = 3\left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right) = \frac{10}{3}$$

R-Ratio: b



$$R = \sum_q 3e_q^2 = 3\left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right) = \frac{11}{3}$$

Consequences from QCD for Hadron Properties

Symmetries of the QCD Lagrangian: Parity

$\mathcal{L}_{\text{QCD}}$ is invariant under parity transformation (*i.e.* point reflection)

$$\hat{P} : (t, \vec{x}) \rightarrow (t, -\vec{x})$$

Eigenvalues:

$$\hat{P}^2(\phi(t,\vec{x})) = \hat{P}(\hat{P}(\phi(t,\vec{x})) = \hat{P}(\phi(t,-\vec{x})) = \phi(t,\vec{x})$$

$\Rightarrow \quad \hat{P}(\phi(t,\vec{x}) = P\,\phi(t,\vec{x})$ with Eigenvalues $P = \pm 1$ (actually $\pm e^{i\varphi}$, but we can redefine $\hat{P}$)

Consequences for Hadrons:

- All states can be decomposed into states with $P = +1$ or $P = -1$
  - Might be degenerated?
- System of Hadrons

  $$\hat{P}\left(\phi_1(t,\vec{x}) \otimes \phi_2(t,\vec{x}) \otimes \cdots \otimes \phi_N(t,\vec{x})\right) = P_1\left(\phi_1(t,\vec{x})\right) \times P_2\left(\phi_2(t,\vec{x})\right) \times \cdots \times P_N\left(\phi_N(t,\vec{x})\right)$$

  Parity is a “multiplicative” quantum number
- Hadrons produced via QED/QCD from a state with defined total parity have same total parity
- Additional $U(1)$ Symmetries for Baryon-Number, Charge, Lepton Number $\Rightarrow$ combined parity operators
- Define intrinsic parity $P_{\text{Proton}} = P_{\text{Neutron}} = P_{\text{Electron}} = +1$:

Symmetries of the QCD Lagrangian: Experimental determination of Parity

Example: Parity of the pion

$$^2H + \pi^- \to n + n$$

- measure angular momentum (i.e. angular distribution)
- intrinsic parity $P(p) = P(n) = 1$
- Deuteron has Spin $S_d = 1$
  Pion has Spin $S_\pi = 0$
  s-Wave $L = 0$
  n antisymmetric
  $\Rightarrow$ total orbital momentum of final state $L = 1 \Rightarrow P = (-1)^L$
- Sum

$$\underbrace{(1)}_{p\uparrow}\underbrace{(1)}_{n\uparrow}\underbrace{(P_\pi)}_{\text{Pion}} = \underbrace{(-1)}_{L=1}\underbrace{(1)}_{n\uparrow}\underbrace{(1)}_{n\uparrow}$$

$\Rightarrow$ Pion has parity $P_\pi = -1$, it is a "pseudoscalar" particle

General approach:

- calculate parity of initial state
- examine strong and electromagnetic (not weak!!!) decays, determine angular momenta
- tie to defined intrinsic parity

Symmetries of the QCD Lagrangian: Charge Conjugation

$\mathcal{L}_{\text{QCD}}$ is invariant under Charge Conjugation (*i.e.* exchange particle $\rightarrow$ antiparticle)

$$\hat{C} : |\phi\rangle \rightarrow |\overline{\phi}\rangle$$

Same properies as a parity operator

- Eigenvalues $C = \pm 1$
- Multiplicative quantum number for a system
- New: only neutral particles can be eigenstates!

Experimental determination: e.g. C-Parity of the pion from decay:

$$\pi^0 \rightarrow \gamma + \gamma$$

- C-Parity of photon $C(\gamma) = -1$ from QED
- Multiplicative $\Rightarrow C(\pi^0) = (-1)_\gamma (-1)_\gamma = 1$

Quantum numbers of the Pion: $J^{PC} = 0^{-+}$

Natural Quantum numbers

- "Natural" quantum numbers for mesons: J^{PC} with $|L-S| \leq J \leq |L+S|$

$$\hat{P}(R(r)Y_{lm}(\theta,\phi)) = Y_{lm}(\pi-\theta,\phi+\pi) = (-1)^l Y_{lm}(\theta,\phi) \quad \Rightarrow \quad \hat{P}|q\bar{q}\rangle = (-1)^{L+1}|q\bar{q}\rangle$$

- Charge Parity of a Meson as a Quark-Antiquark pair:

$$\hat{C}(|q\bar{q}\rangle) = C|q\bar{q}\rangle$$

  - Charge Conjugation corresponds to exchange of quark/antiquark
  - $L = 0,2,4,...$ symmetric, $L = 1,3,5,...$ antisymmetric $\Rightarrow$ $C \sim (-1)^L$
  - Spin $\Rightarrow$ $C \sim (-1)^{S+1}$
  - Exchange particle $\rightarrow$ antiparticle $\Rightarrow$ $C \sim (-1)$

$$\hat{C}(|q\bar{q}\rangle) = (-1)^L(-1)^{S+1}(-1)|q\bar{q}\rangle = (-1)^{L+S}|q\bar{q}\rangle$$

- Allowed: 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{-+}, 2^{++}, 3^{--}, 3^{+-}, 3^{++}, ...
  Not allowed: 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, ... $\Rightarrow$ Exotic Mesons

$^{2S+1}L_J$	S	L	J	P	C	J^{PC}	Mesons				Name
$^1S_0$	0	0	0	$-$	$+$	0^{-+}	π	η	η'	K	pseudo-scalar
$^3S_1$	1	0	0	$-$	$-$	1^{--}	ρ	ω	ϕ	K^*	vector
$^1P_1$	0	1	1	$+$	$-$	1^{+-}	b_1	h_1	h'_1	K_1	pseudo-vector
$^3P_0$	1	1	0	$+$	$+$	0^{++}	a_0	f_0	f'_0	K^*_0	scalar
$^3P_1$	1	1	1	$+$	$+$	1^{++}	a_1	f_1	f'_1	K_1	axial vector
$^3P_2$	1	1	2	$+$	$+$	2^{++}	a_2	f_2	f'_2	K^*_2	tensor

Theoretical Approaches

The “brute force” approach: Lattice QCD

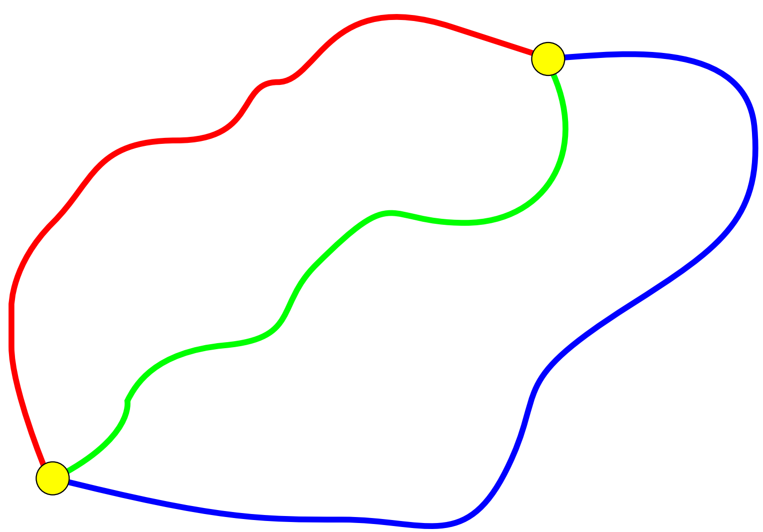
Starting point: Feynman’s Path Integral formulation of Quantum Mechanics:

$$\psi(x_2,t_2) = \frac{1}{Z}\int e^{iS}\psi(x_1,t_1)\ \mathcal{D}x$$

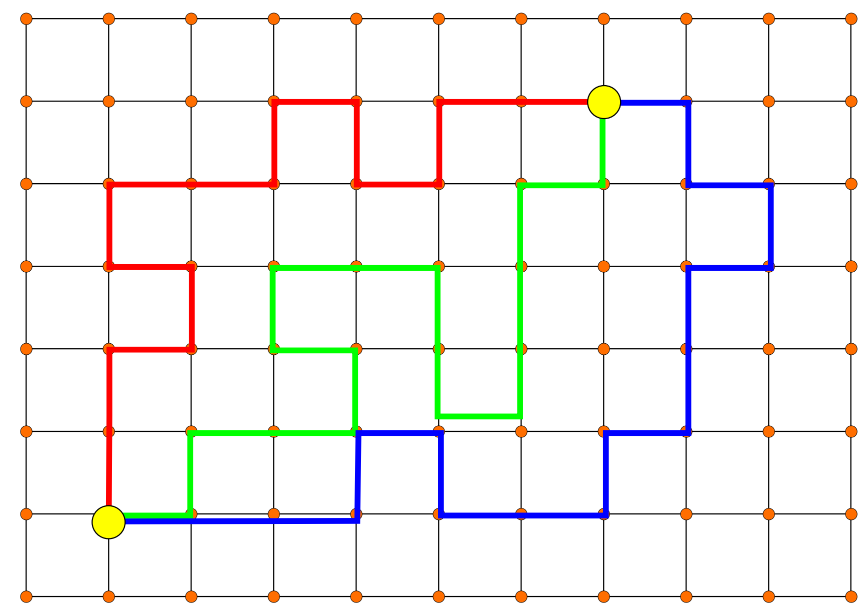
with $\int \mathcal{D}x$: Integration over *all* paths $x(t)$ with $x(0) = x_1$

and the action $S = \int_{t_1}^{t_2} L(x,\dot{x},t)\ \mathrm{d}t$

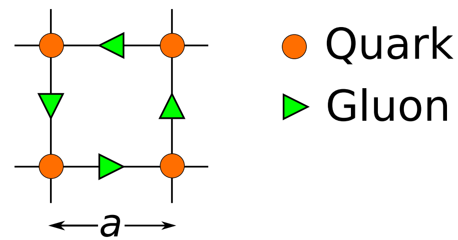
(a.k.a. Fermat’s principle, Hamilton’s principle, principle of least action)



$\Rightarrow$



Lattice QCD



- Transform to Euclidean Space (neccessary to use Monte-Carlo-Methods):

$$t \to i\tau$$
$$-(\mathrm{d}t^2) + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \to \mathrm{d}\tau^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

- Define Link Variables for gluonic field

$$U_\mu = \exp\left(iaG_\mu\left(n + \frac{\hat{\mu}}{2}\right)\right)$$

$U_{\mu\nu}(n)$: closed loop around one tile, “plaquette”

- Fermion action bei discretizing derivatives $\partial\phi_t \approx \frac{\phi(t+a)-\phi(t-a)}{2a}$

$$S = \int \overline{u}(iD_\mu\gamma_\mu + m)u\, d^4x \qquad \to \qquad D_\mu = \frac{1}{2a}\left[U_\mu(x)q(x+a\hat{\mu}) - U_\mu(x-a\hat{\mu})^\dagger q(x-a\hat{\mu})\right]$$

- Gluonic action:

$$S = -\frac{1}{2g^2}\mathrm{Tr}\int G_{\mu\nu}G^{\mu\nu}d^4x \qquad \to \qquad S_L = -\frac{1}{2g^2}\sum a^4\mathrm{Tr}\left(1 - U_{\mu\nu}(n)\right)$$

Lattice

Final Step: Numeric solution via Markov-chain Monte-Carlo:

- Choose a start-configuration C_0
- Accept a random next configuration C_{n+1} with probability

$$P = \min\left(1, \frac{W(C_{n+1})}{W(C_n)}\right)$$

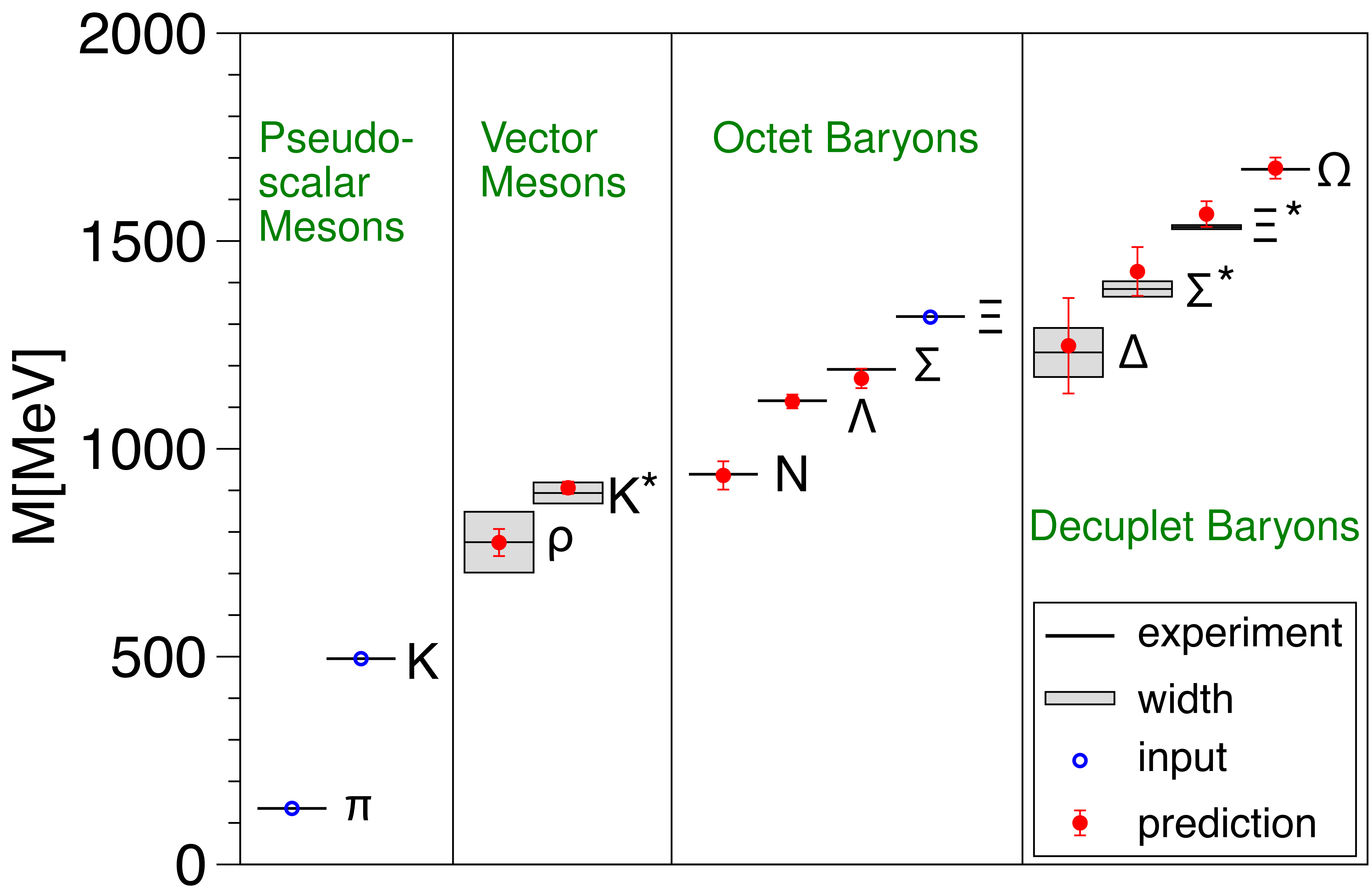
$\Rightarrow$ We don't need to know the probability density function,
we need only the *relative* weight $W(C)$,
calculated by discretized path integral!

- Repeat until "thermalization", i.e. distribution of configurations corresponds to $W(C)$
- Repeat everything with different Lattice spacing a
- Extrapolation $a \to 0$

Summary:

- Gauge invariant
- Works in the non-perturbative regime
- Finite volume, finite momentum

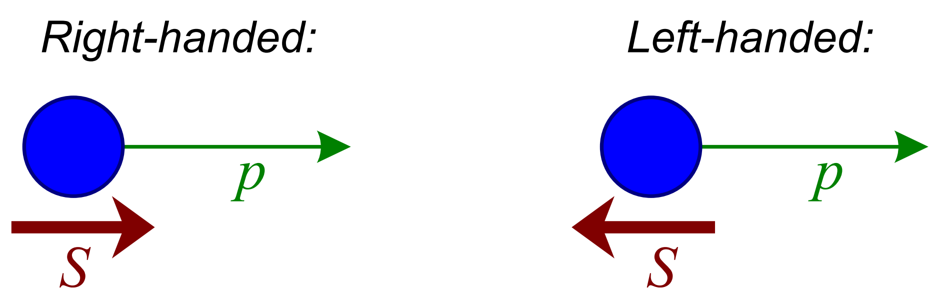
Hadron Spectrum from Lattice QCD



S. Durr, *et al.*, Science 322 (2008)

Still one symmetry of QCD not used...

Helicity: Spin projection in direction of motion



Not a good quantum number: inversion by “overtaking” reference frame!

Better: **Chirality**

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

For massless particles:

$$\gamma^5 \cdot u_+ = \gamma^5 \cdot \begin{pmatrix} \chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix} = \gamma^5 \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = +u_+ \quad \text{and} \quad \gamma^5 \cdot u_- = \gamma^5 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = -u_-$$

Eigenvalues of γ^5 are the eigenvalues of helicity for particles with $m \to 0$

Chirality $\approx$ Lorentz invariant version of Helicity

Chiral Symmetry

Projection Operator

$$\frac{1}{2}(1+\gamma^5)u = u_R \qquad \frac{1}{2}(1-\gamma^5)u = u_L$$

Consequences for *Dirac Equation* $(i\gamma^\mu p_\mu - m)u = 0$:

$$\bar{u}\gamma^\mu u = (\bar{u}_R + \bar{u}_L)\gamma^\mu(u_R + u_L) = \bar{u}_R\gamma^\mu u_R + \bar{u}_L\gamma^\mu u_L$$

for $m \to 0$: left-/right-handed particles interact only with left-/right-handed particles

Def.: Chiral Symmetry: invariant under separate rotations

$$\psi_L \to e^{\theta_L}\psi_L \quad \text{and} \quad \psi_R \to \psi_R$$

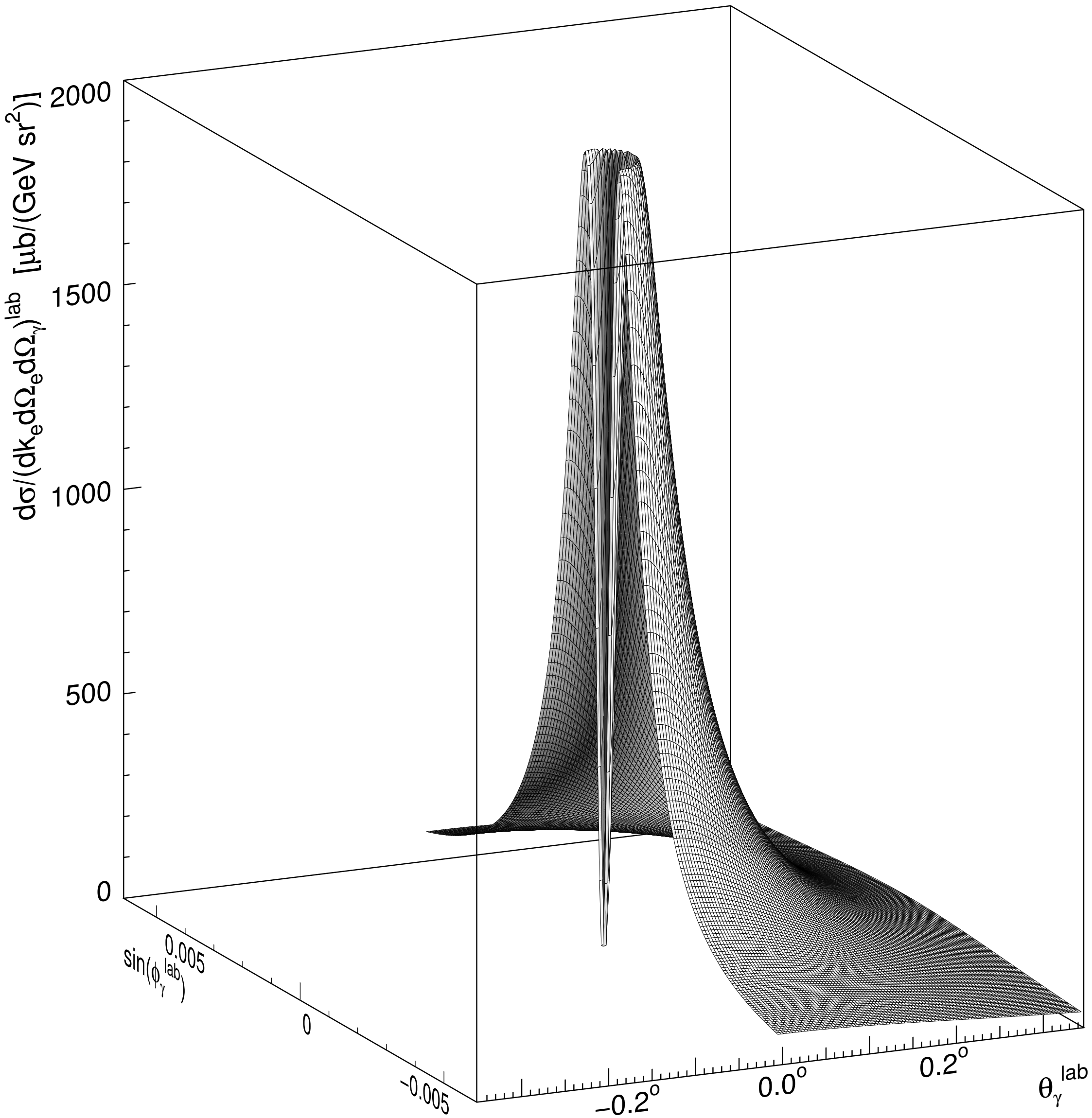
$$\text{or} \qquad \psi_R \to e^{\theta_R}\psi_R \quad \text{and} \quad \psi_L \to \psi_L$$

Chiral Symmetry in QCD: combination with Isospin rotation of $q = \begin{pmatrix} u \\ d \end{pmatrix}$:

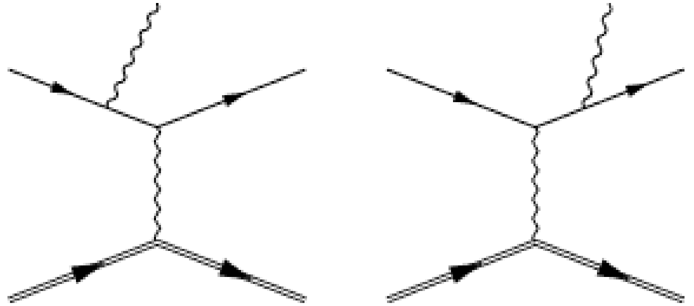
$$U(2)_L \times U(2)_R = \underbrace{SU(2)_L \times SU(2)_R}_{\text{Chiral Symmetry}} \quad \underbrace{\times U(1)_V \times U(1)_A}_{\text{Baryon number, Quan. anomaly}}$$

Chiral Symmetry: QCD invariant under separate isospin rotation for left- and right-handed quarks in the limit of massless quarks

The Power of Chiral Symmetry...



QED Example: Bremsstrahlung



- Virtual intermediate electron
- $\frac{1}{\not p - m} \to 0$ Peak in electron direction
- Exactly at $\theta_{\gamma e} = 0$:
  - Emmission of Spin 1 Photon
  - No orbital angular momentum
  - ⇒Spin Flip of electron breaks Chiral Symmetry
  - Cross section $\to 0$

⇒ Chiral symmetry is powerfull

Expectations from Chiral Symmetry for Hadron Physics

Mass of light quarks:

$$m_u = 2.2\,\mathrm{MeV} \qquad\qquad m_d = 4.7\,\mathrm{MeV}$$

$$m_q \ll m_{\text{Hadrons}}$$

Chiral symmetry $SU(2)_R \times SU(2)_L$ should be conserved at least at 1% level!

Expectations:

- Parity doublets: all light quark states have partner with oposite parity

Observation:

- No parity doubletts in baryon or meson spectrum seen! e.g. $\rho(770) < a_1(1200)$
- Three ridiculous light mesons $\pi^0,\ \pi^+,\ \pi^-$ with $m_\pi \ll \frac{2}{3}m_p$

Hypothesis:

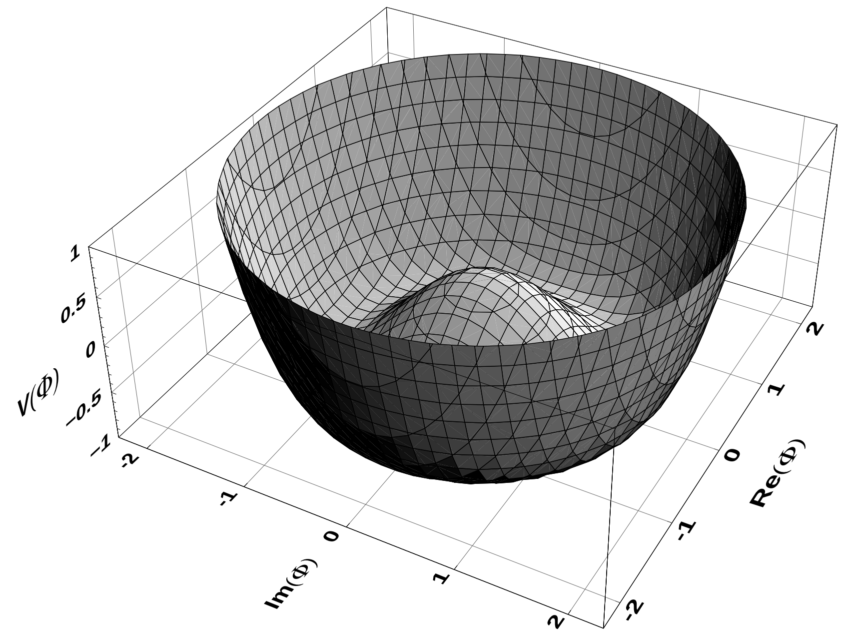
- Chiral Symmetry is spontaneously broken
- $SU(2)_L \times SU(2)_R \to SU(2)_V \times SU(2)_A$

  of standard vector $SU(2)_V$ and rest (...$SU(2)_A$ is not quite axial vector)

Spontaneous Symmetry Breaking and Goldstone-Theoreme

2-dimensional Example:

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2$$



Minimum at

$$|\phi| = k = \sqrt{-m^2/\lambda}$$

Replace complex scalar field $\phi = ke^{i\theta/k}, \qquad \theta \in \mathbb{R}$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(-ie^{-i\theta/k}\partial^\mu\theta)(ie^{i\theta/k}\partial_\mu\theta) - \frac{1}{2}m^2k^2 - \frac{\lambda}{4}k^4 = \frac{1}{2}\partial^\mu\theta\partial_\mu\theta - \underbrace{\frac{1}{2}m^2k^2 - \frac{\lambda}{4}k^4}_{\text{const. in }\theta}$$

$\Rightarrow$ Real scalar field θ ist massless!

Spontaneous Symmetry Breaking $\Rightarrow$ massless Goldstone-Bosons. $\Rightarrow$ QCD: Pions

Chiral Effective Field Theories

What are the relevant degrees of freedom? $\Rightarrow$ *e.g.* pions as Goldstone-Bosons

Ingredients for an effective field theory:

- Choose degrees of freedom: $\Rightarrow$ Pions
- Most general Lagrangian in theses DoF respecting the Symmetries of $\mathcal{L}_{QCD}$
  $\Rightarrow$ series in terms of derivatives, fields
  $\Rightarrow$ this is a perturbative theory!
- Most important: sort these terms!
  - Expansion in mass terms (explicit symmetry breaking by $-\overline{q}_f M q_f$)
  - Simultaneously expansion in p
  - Order Scheme $\rightarrow$ define what is LO, NLO, NNLO!
- Derive Feynman rules, calculate observables order by order, ...

To deal with:

- Regularization $\Rightarrow$ Low Energy Constants   Fit to experiment, limits predictive power
- Degrees of freedom: e.g. better to include resonances?
- Convergence of series
- ...

Systematic expansion, not a Model!

N.B.: Theorists' "Slang"

If a theorist uses the word

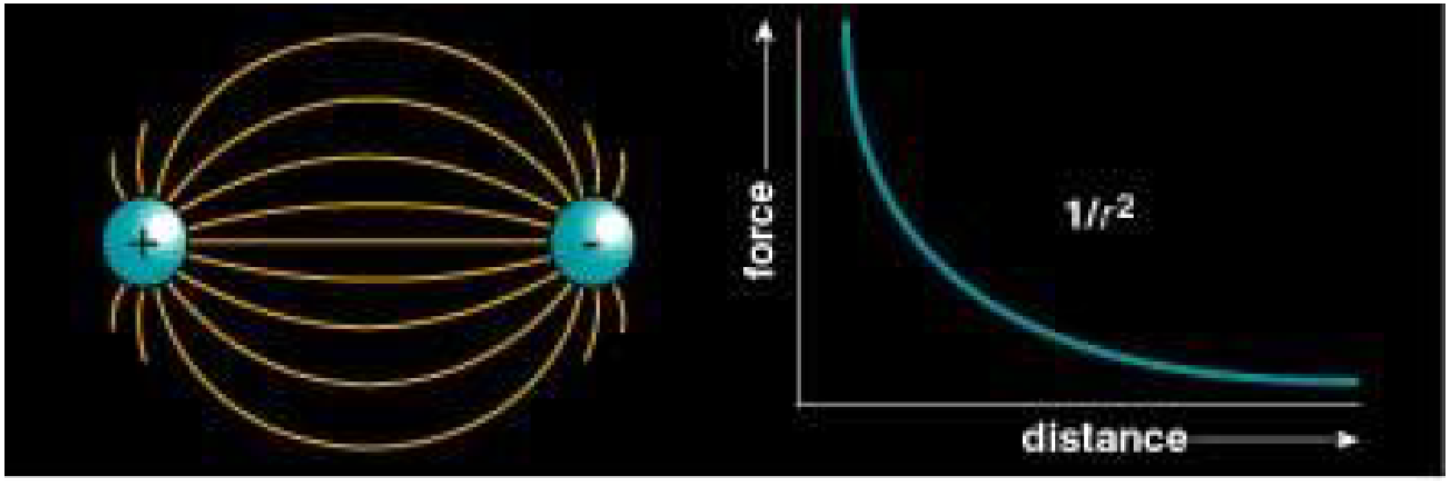
"chiral"

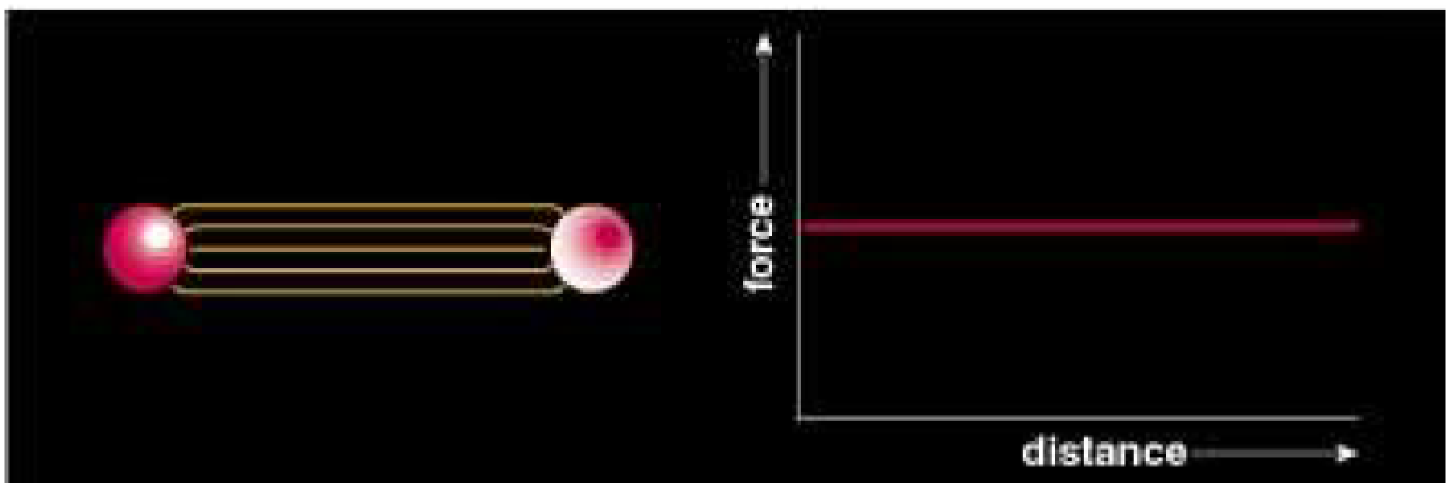
like *e.g.* in "Chiral Extrapolation of Lattice QCD" this usually means

"Using methodes from Effective Field Theories
using the Chiral Symmetry of QCD"

Potential Models

The $q\bar{q}$ Force of QCD





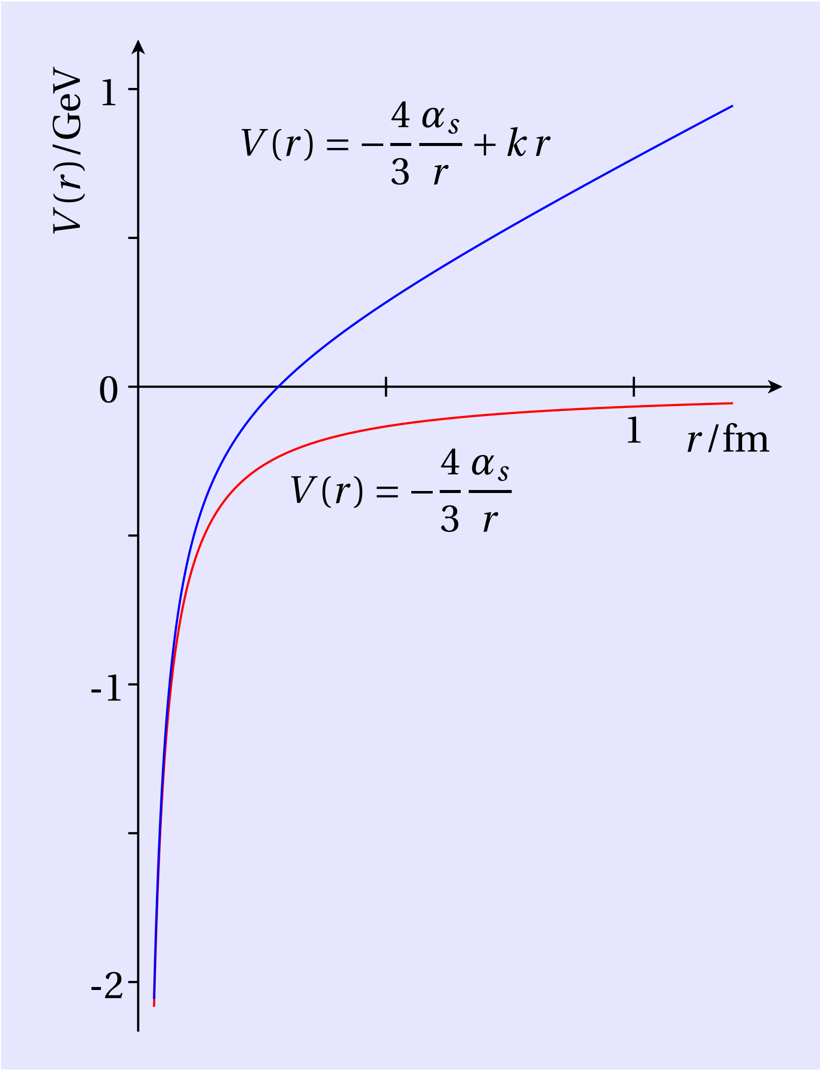
- Idea: heavy quarks $\rightarrow$ non-relativistic
- A quark in the potential of a mean field

Simple Model: Non-relativistic Potential Model

Model: quarks in the potential of the rest of the meson/baryon

- $V(r \to 0)$
  - Asymptotic freedom
  - Massless gluons
    $\to$ infinite range Coulomb like potential $\frac{1}{r}$
- $V(r \to \infty)$
  - Confinement potential $k \cdot r$
  - Running coupling constant

$$V(\vec{r}) = -\frac{4}{3}\frac{\alpha_s}{r} + k \cdot r$$



Simple Model: Non-relativistic Potential Model

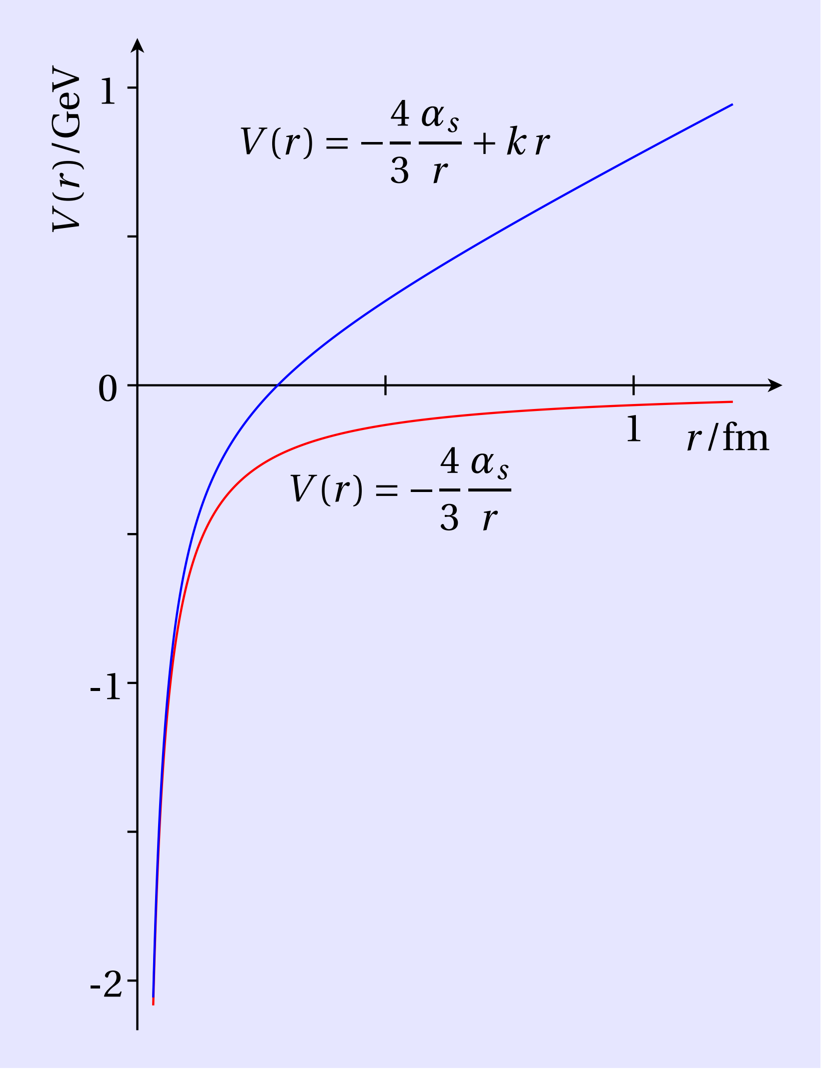
Non-relativistic $q\bar{q}$ potential:

$$V(\vec{r}) = -\frac{4}{3}\frac{\alpha_s}{r} + k \cdot r$$

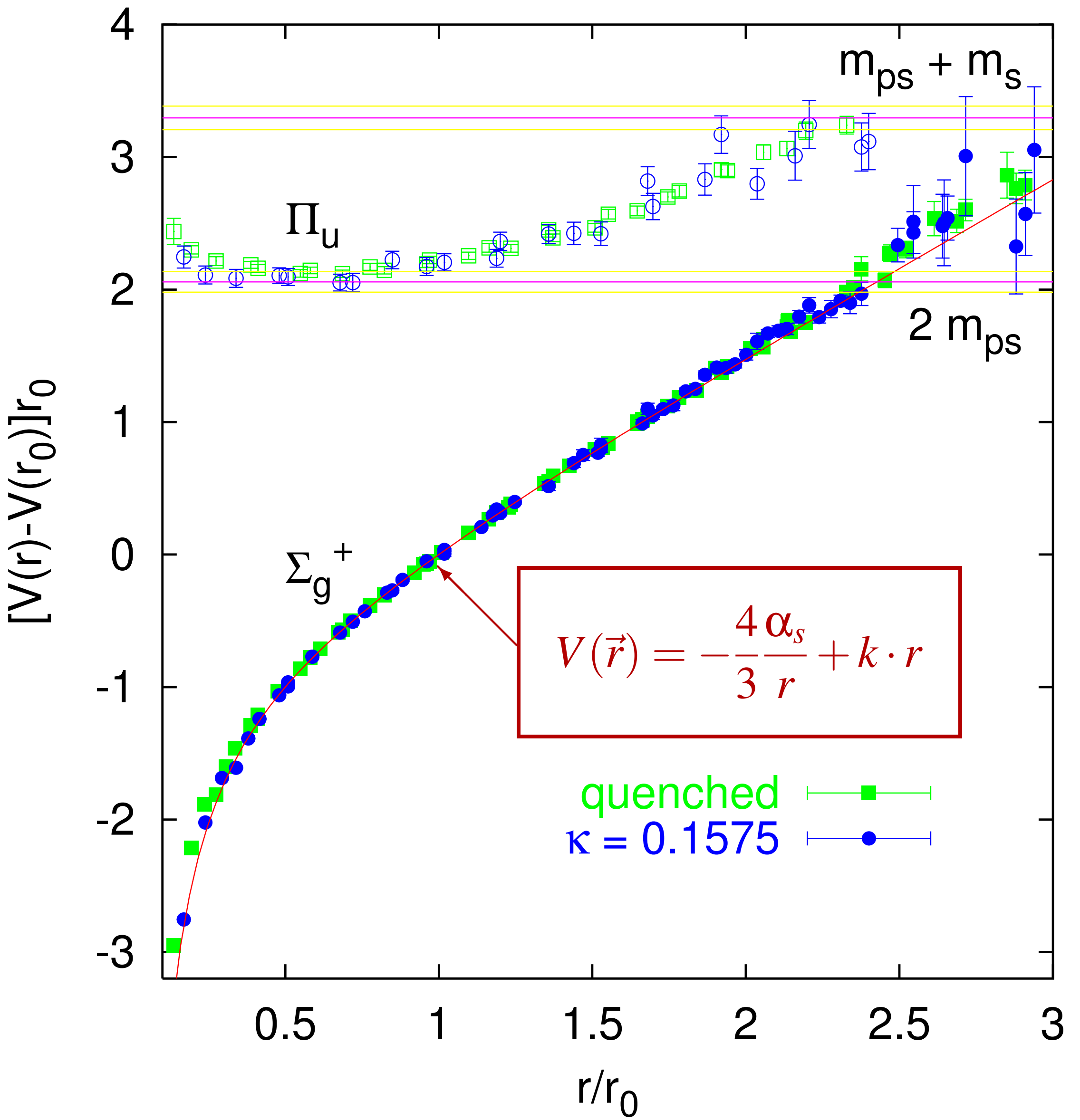
Running Coupling constant:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - n_f)\log\left(\frac{Q^2}{\Lambda^2}\right)}$$

n_f: number of flavours
$\Lambda \approx 0.2\,\text{GeV}$: QCD Scale parameter
$k \approx 1\,\frac{\text{GeV}}{\text{fm}}$: QCD String constant



$q\bar{q}$ Potential from Lattice calculation



Quenched approximation, *i.e.* no disconnected quark loops

G. S. Bali et al., PRD62 (2000)

Other usefull Ingredients: Spin Dependent Potential

Neccessary extensions of potential model:

- Spin-Orbit (fine structure)

$$V_{LS} = \frac{1}{2m_c^2 r}(\vec{L}\cdot\vec{S})\left(3\frac{\mathrm{d}V_V}{\mathrm{d}r} - \frac{\mathrm{d}V_V}{\mathrm{d}r}\right)$$

- Spin-Spin (hyperfine structure)

$$V_{SS} = \frac{2}{3m_c^2 r}(\vec{S}_1\cdot\vec{S}_2)\nabla^2 V_V(r)$$

- Tensor force

$$V_T = \frac{2}{12m_c^2}(3(\vec{S}\cdot\hat{r})(\vec{S}\cdot\hat{r}) - S^2)\left(\frac{1}{r}\frac{\mathrm{d}V_V}{\mathrm{d}r} - \frac{\mathrm{d}^2 V_V}{\mathrm{d}r^2}\right)$$

with V_V, V_S vector and scalar part of the previous potential

Finding Hadrons

$\Rightarrow$ Just looking for Bumps?

What is a Bump? The Line Shape:

- Strong Decay ⇒ Lifetime $\tau \approx 10^{-23}$ s
  ⇒ Width $\Gamma_0 \approx 100 \frac{\text{MeV}}{c}$

- Breit-Wigner Amplitude (complex mass in Dirac-propagator)

$$BW(m) = \frac{\Gamma_0/2}{m_0 - m - i\Gamma_0/2}$$

valid for $\Gamma_0 \ll m_0$
$m_0 \gg$ Threshold Energy

- Better (relativistic, orbital momentum, phase space included):

$$BW(m) = \frac{m_0\Gamma(m)}{m_0^2 - m^2 - im_0\Gamma(m)}$$

with $$\Gamma(m) = \Gamma_0 \frac{m_0}{m}\frac{p}{p_0}\frac{F_l^2(p)}{F_l^2(p_0)}$$

angular momentum barrier:

$$F_0(p) = 1$$

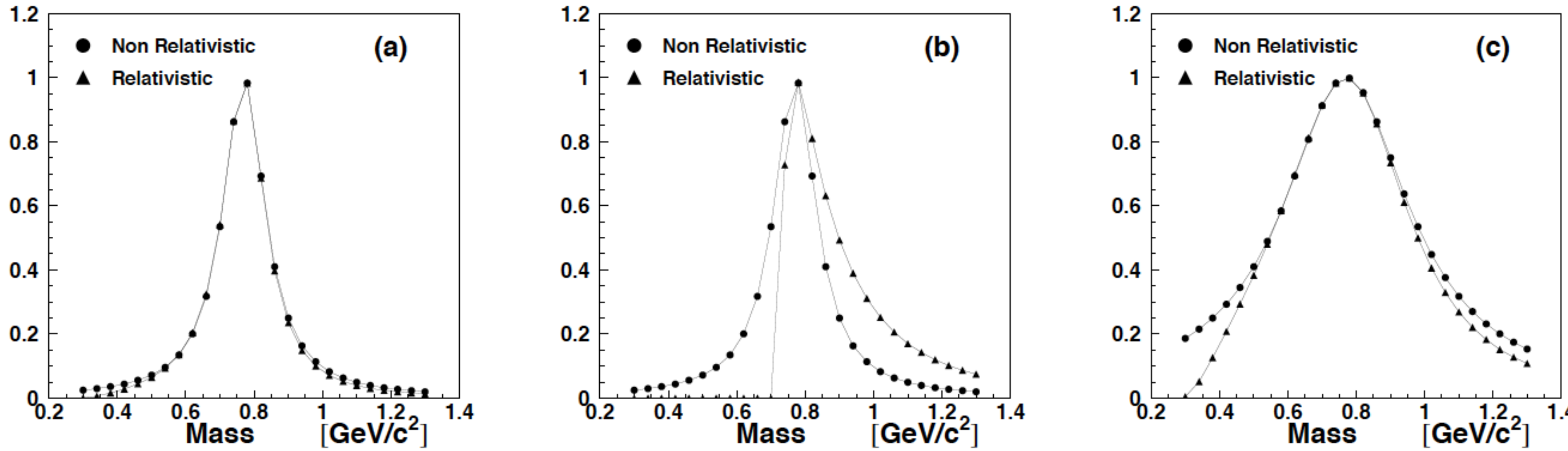
$$F_1(p) = \sqrt{2z/(z+1)}$$

with $z = (p/p_R)^2$

$$F_2(p) = \sqrt{13z^2/((z-3)^2+9z)}$$

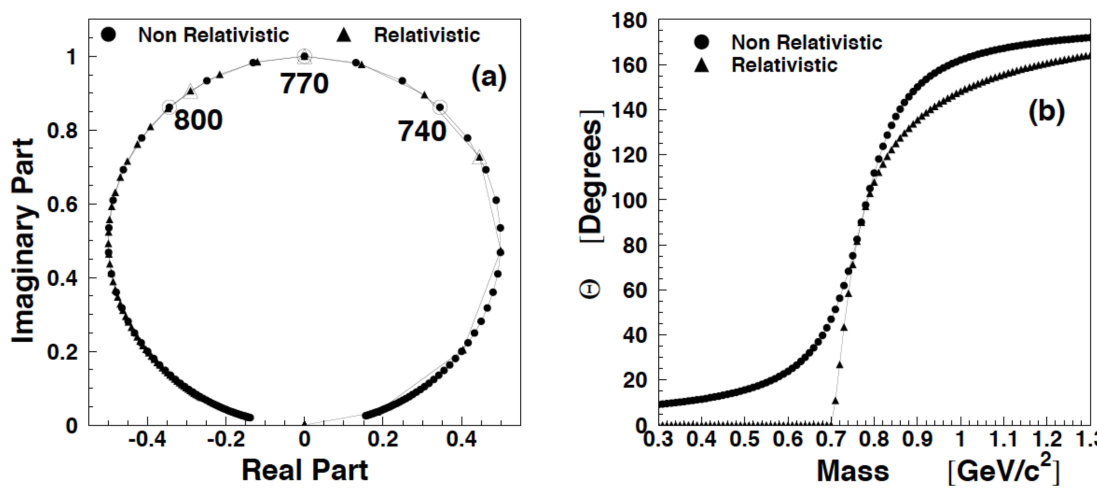
$$\ldots$$

Example $\rho(770)$

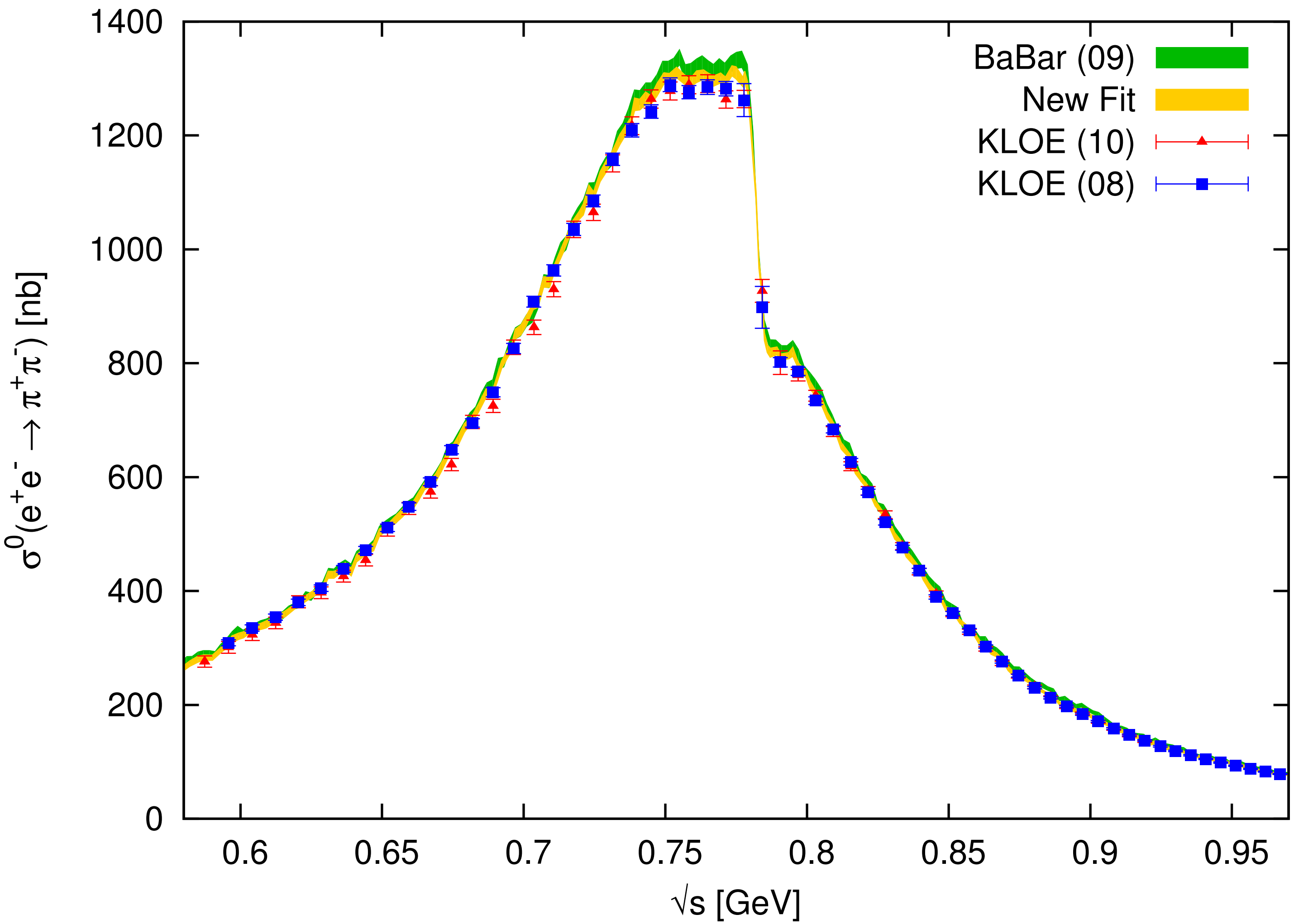


$\Gamma_0 =$	150 MeV	150 MeV	350 MeV
$m_1 = m_2 =$	140 MeV	350 MeV	140 MeV

Argand-Diagramm:



Reality Check: $\rho \to \pi^+\pi^-$



no clean Breit-Wigner $\to$ $\rho - \omega$ interference at the position of the ω mass
$\to$ amplitude and phase changed
$\Rightarrow$ all open channels have to be considered on complex amplitude level!

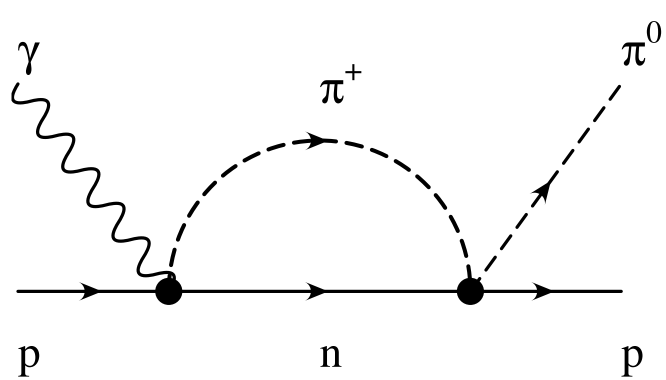
Coupled channels

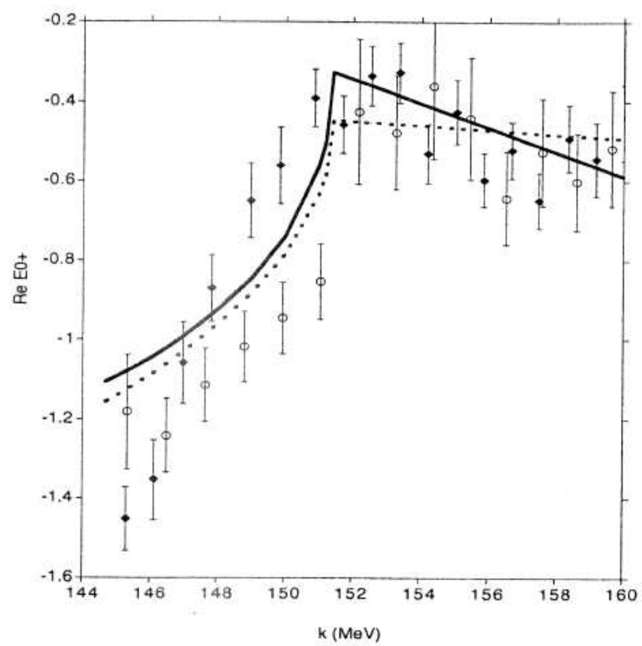
Simplest Example: proton around the pion production threshold
three open channels: $\gamma + p$, $n + \pi^+$, $p + \pi^0$

- Scattering matrix (S-Matrix) of complex transition amplitudes:

$$\begin{pmatrix} p+\gamma \\ n+\pi^+ \\ p+\pi^0 \end{pmatrix}_{\text{final}} = \begin{pmatrix} A_{\gamma\gamma} & A_{\gamma\pi} & A_{\gamma\pi} \\ A_{\gamma\pi} & A_{\pi\pi} & A_{\pi^+\pi^0} \\ A_{\gamma\pi} & A_{\pi^+\pi^0} & A_{\pi\pi} \end{pmatrix} \cdot \begin{pmatrix} p+\gamma \\ n+\pi^+ \\ p+\pi^0 \end{pmatrix}_{\text{initial}}$$

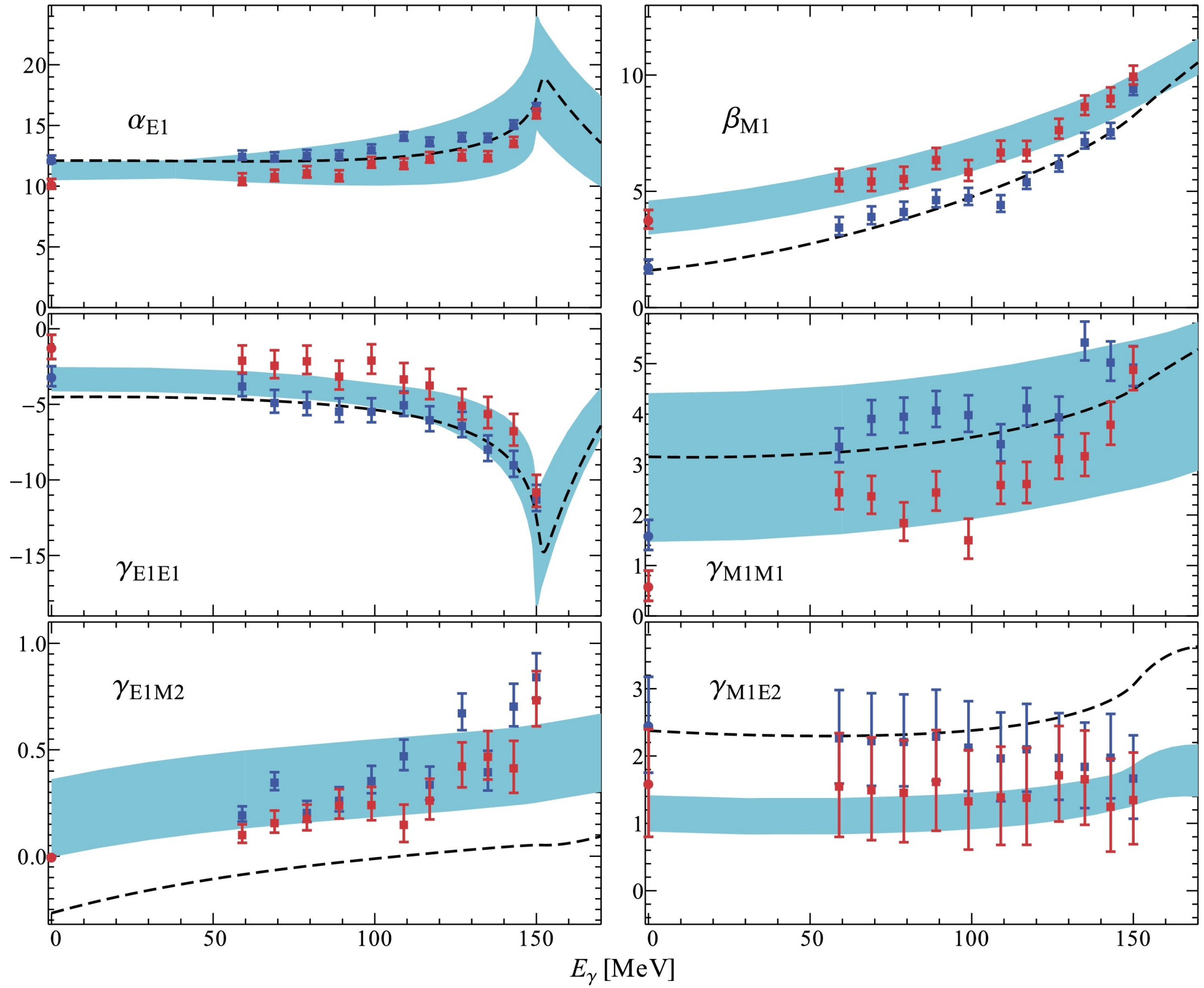
- Conservation of Probability $\Leftrightarrow$ Unitarity of S-matrix
- All channels are seen in all other channels
- $\gamma + p \rightarrow p + \pi^0$, s-wave:





Compton-Scattering

Polarizabilities in Compton Scattering (partial waves):



Is the Scattering Phase an Observable?

Quantum-mechanics: An absolute phase is not measureable!

But:

- Elastic scattering: optical Theorem

$$\sigma = \frac{4\pi}{k} \mathrm{Im}\{f(\theta = 0)\}$$

- Elastic phase is a *transition phase*
- Direct measureable at forward direction ($\theta = 0$) in

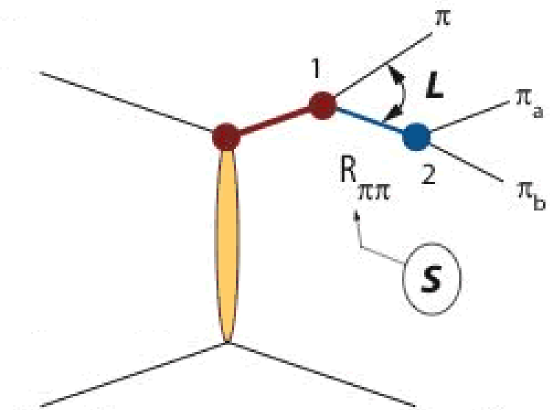
$$\gamma + p \to \gamma + p$$
$$\pi^+ + n \to \pi^+ + n$$
$$\pi^0 + p \to \pi^0 + p \qquad \text{(not measured yet...)}$$

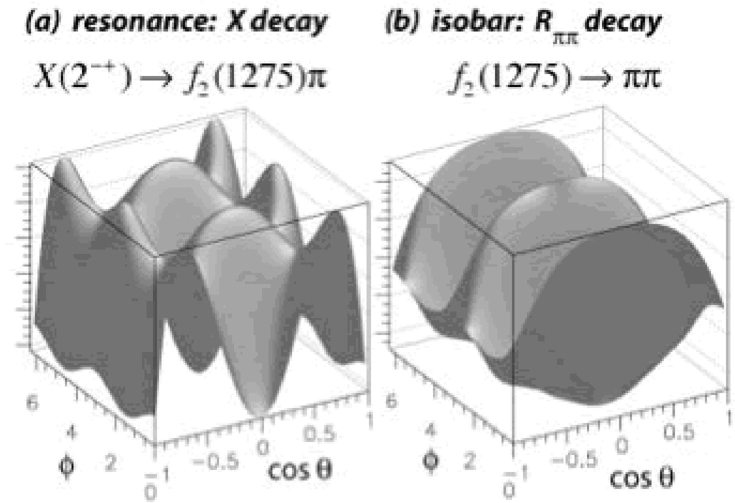
- Unitarity of S-matrix fixes phase of *all scattering amplitudes*

$\Rightarrow$ Scattering amplitudes have relative phases (inital state $\to$ final state)!

$\Rightarrow$ Production amplitudes are also **Observables** (but in reality hard to determine absolute)

The Art of Partial Wave Analysis





- Limited significance of single channels (even if this presentation is “standard” in talks...)
- All open channels have to be fitted simultanously
- Separate for every angular momentum (Partial Wave)
- Fit on *Amplitude* level (not cross section!)
- Polarization degrees of freedom
- Resonances: Breit-Wigner width (line shape, pole position), mass
- Background contributions
- Combinatorical background
- ...

Hundreds of parameters, most determined with limited significance

...choose wisely

I only believe in peaks seen

. . . in several channels

. . . by different groups,

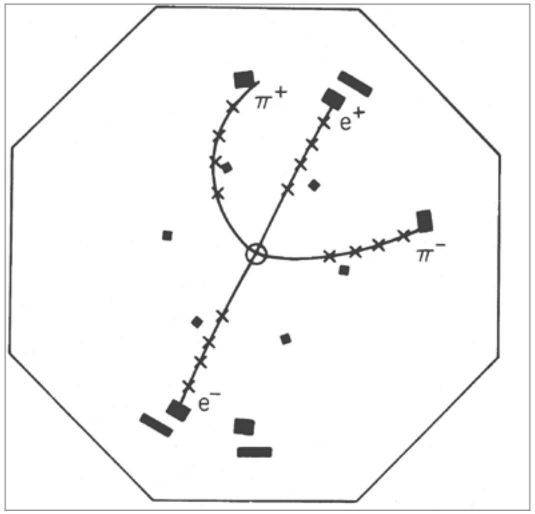
. . . measured with different apparatus,

. . . with different analysis

and still I have doubts...

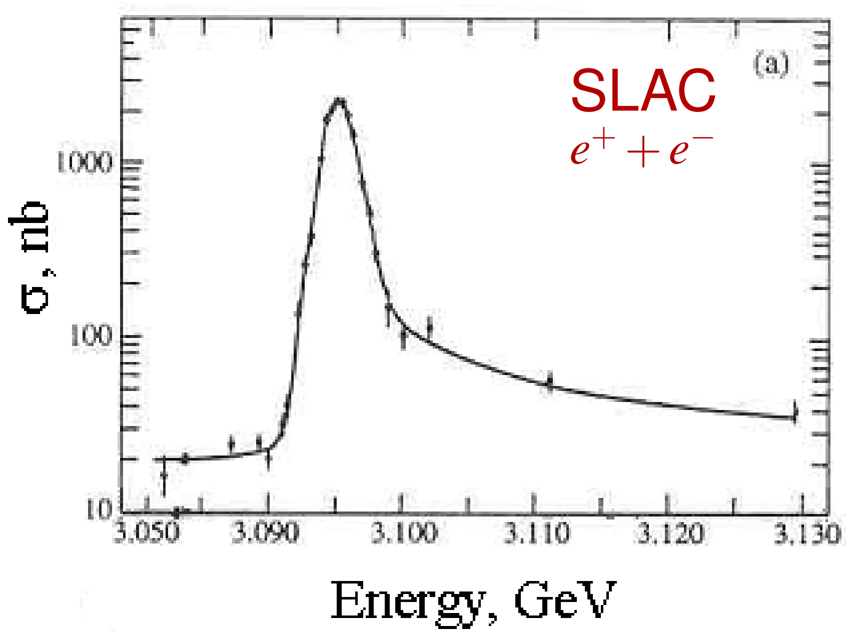
Heavy Quark Mesons

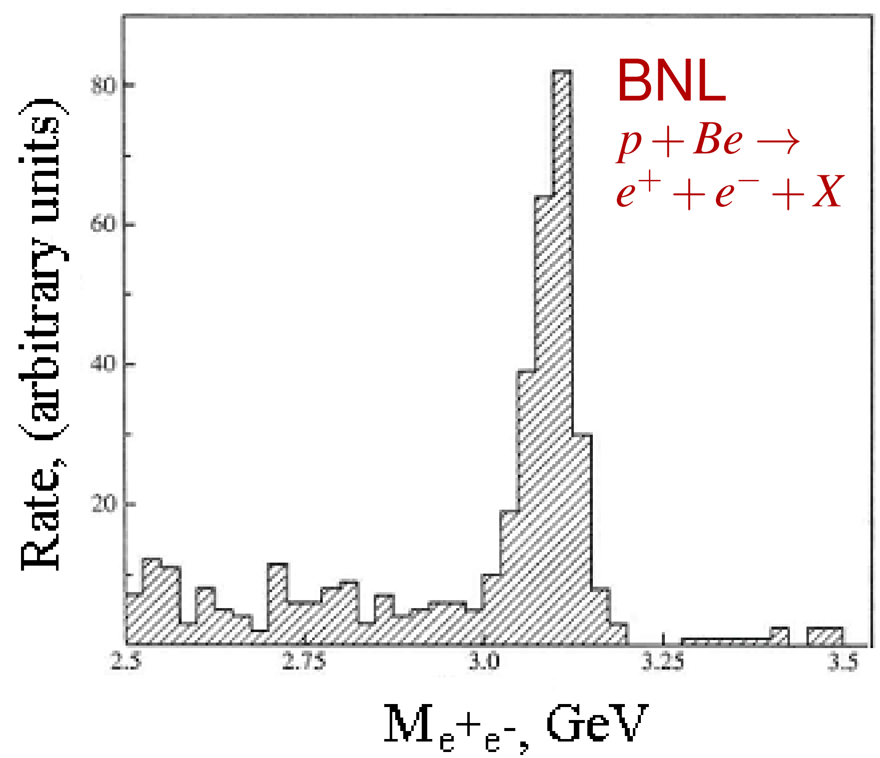
The J/ψ discovery



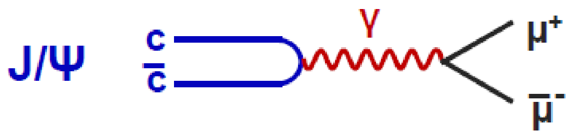
- Simultanious discovery 1974 in BNL and SLAC
- First evidence of a new quark: charm
- Confirmation of quark model (c missing partner of s)
- Bound state of $c\bar{c}$ quarks

$\Rightarrow$ new era of heavy quark physics

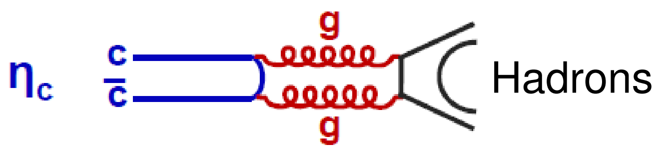




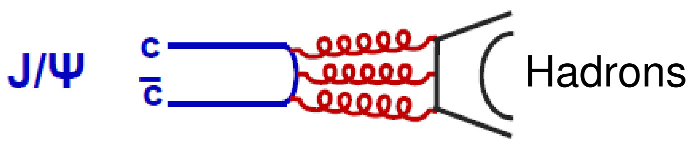
Below open charm threshold:



$J^{--} \Rightarrow$ electromagnetic decay possible



States with $C = +1$ can decay via two gluons

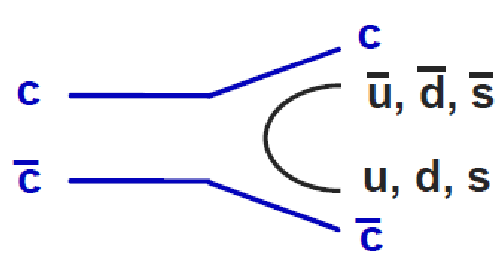


States with $C = -1$ can only decay via three gluons

$\Rightarrow$ electro-magnetic decay of same order of magnitude as strong decay

$\Rightarrow$ J/ψ is a very small resonance

Above open charm threshold:



$\Rightarrow$ broad resonances

Heavy Quark Systems

Heavy Quarks:

$$m_c = 1.3\,\text{GeV} \qquad m_b = 4.2\,\text{GeV} \qquad m_t = 170\,\text{GeV}$$

- Heavy Quark Systems are *non-relativistic*:

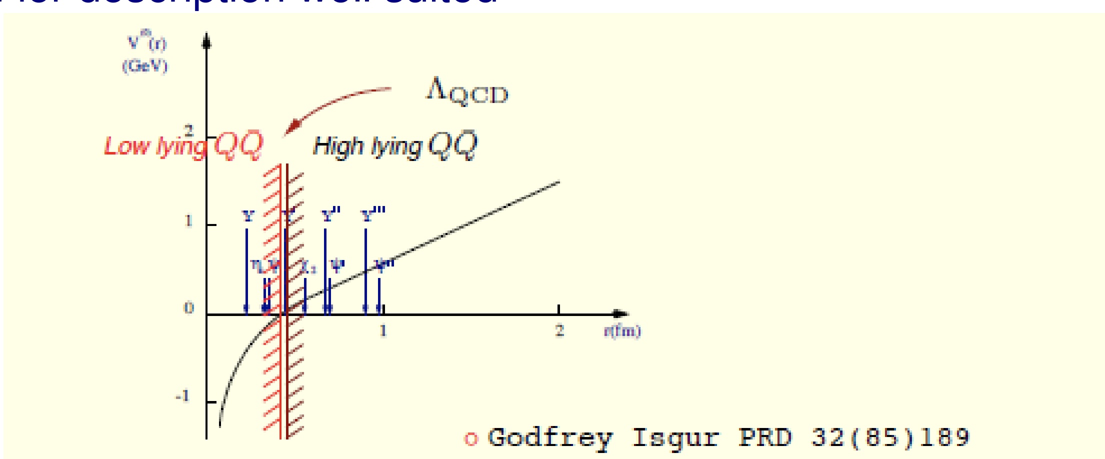
$$m_{J/\psi} = 3.1\,\text{GeV} = 2 \times m_c + 2 \times 0.25\,\text{GeV}$$

$$\Rightarrow \qquad \beta = \frac{p}{E} \approx \frac{0.25\,\text{GeV}}{1.3\,\text{GeV}} = 0.2$$

- The mass scale is *perturbative*:

$$m_Q \gg \Lambda_{QCD}$$

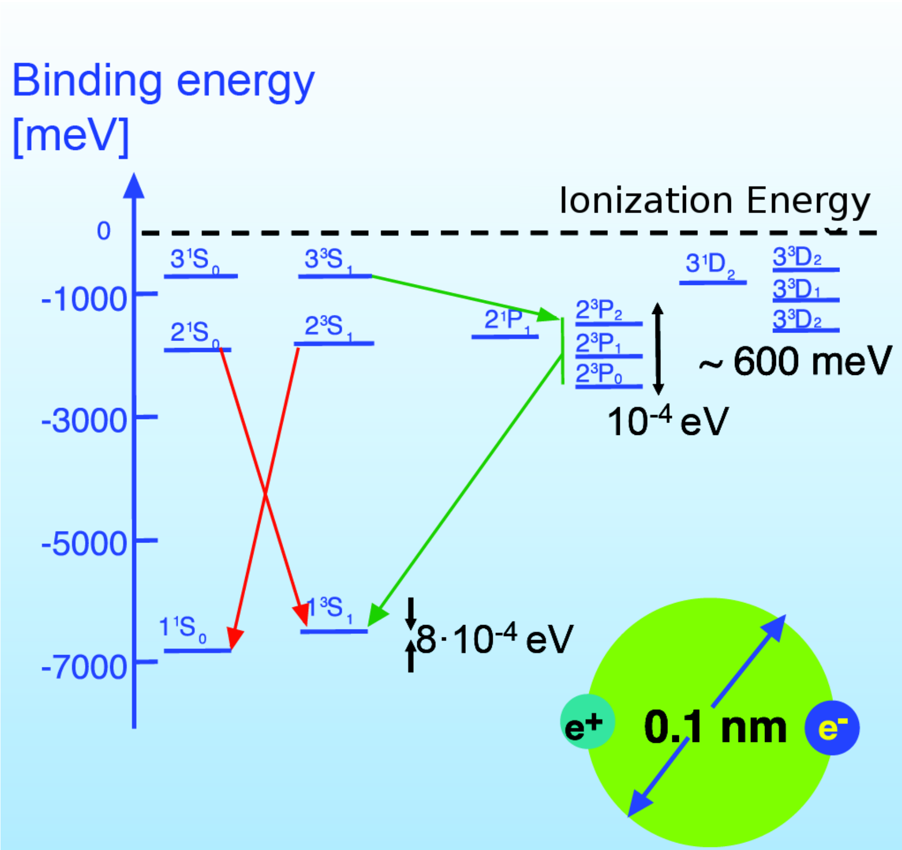
- Potential model for description well suited



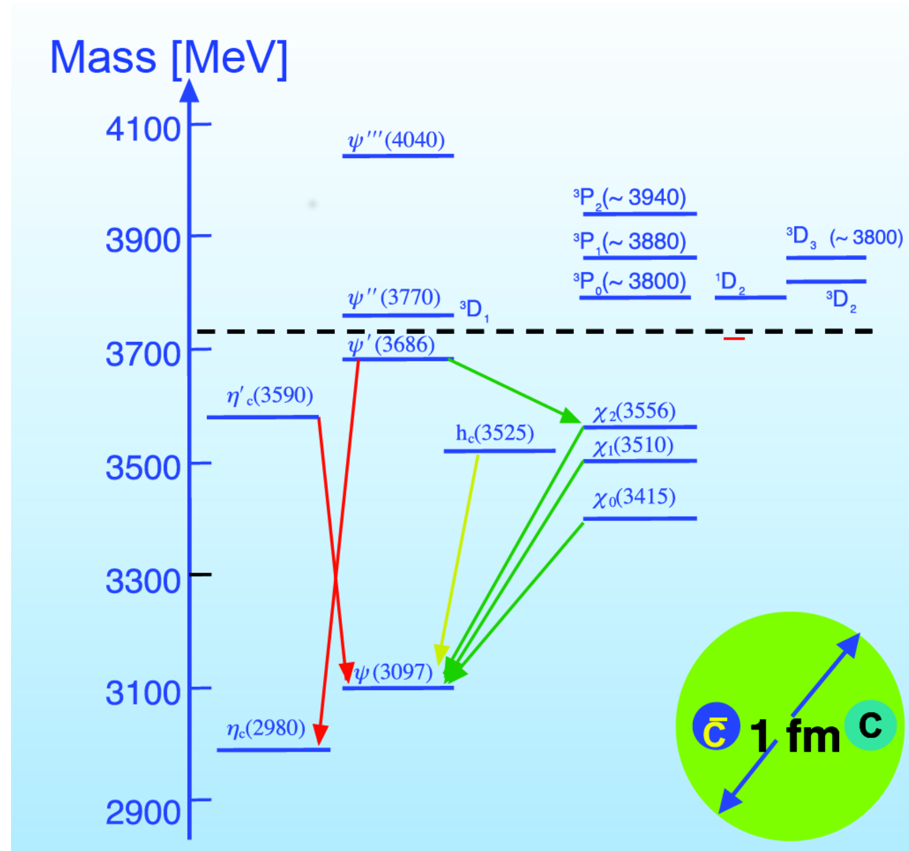
non-perturbativ – transition – perturbative regime

Positronium as Model for Quarkonium (Charmonium or Bottomonium)

Positronium

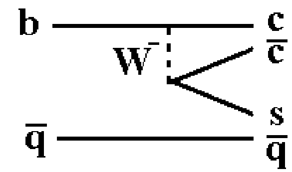


Charmonium



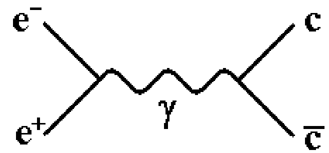
Production channels

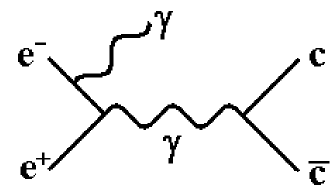
- Weak decay



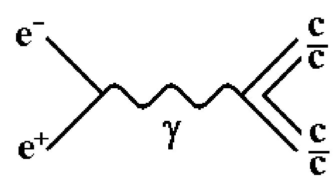
Belle

- e^+e^- annihilation and initial state radiation
  - only $J^{PC} = 1^{++}$
  - $0 <$E$<$c.m. energy



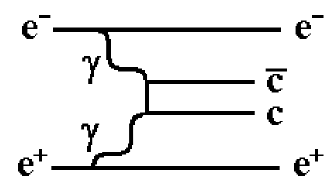


- Double Charmonium
  - $J/\psi + c\overline{c}$

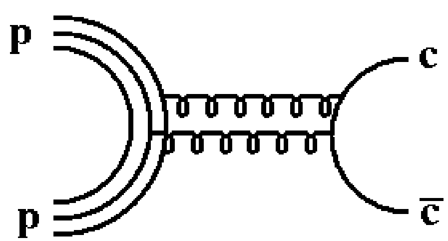


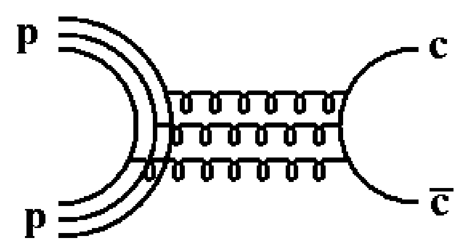
BES III

- Two-photon production
  - $C = +1$



- $p\overline{p}$ annihilation
  - 2 gluons: 0^{-+}, 0^{++}, 2^{++}
  - 3 gluons: 1^{--}, 1^{-+}

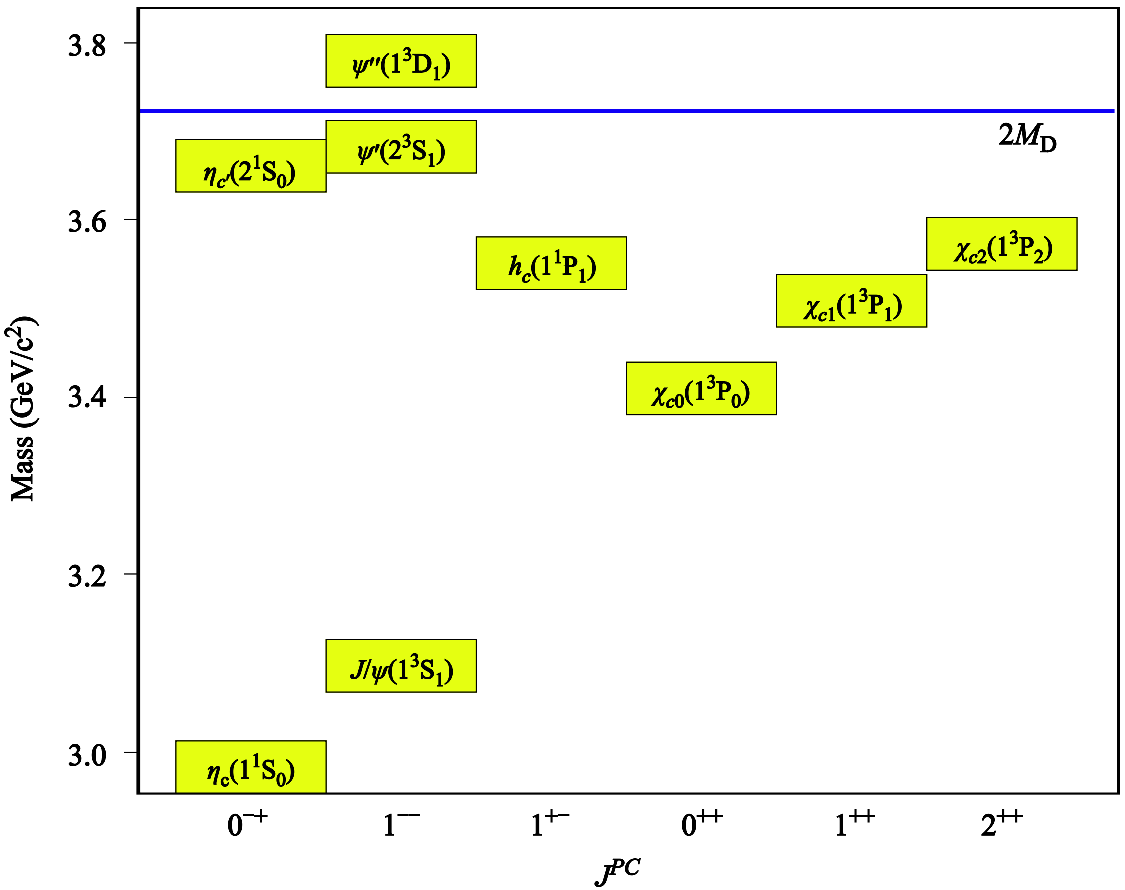




PANDA

Charmonium States below open charm threshold

Discovered Charmonium States:



- Solution of non rel. Schrödinger-Equation
- Notation:

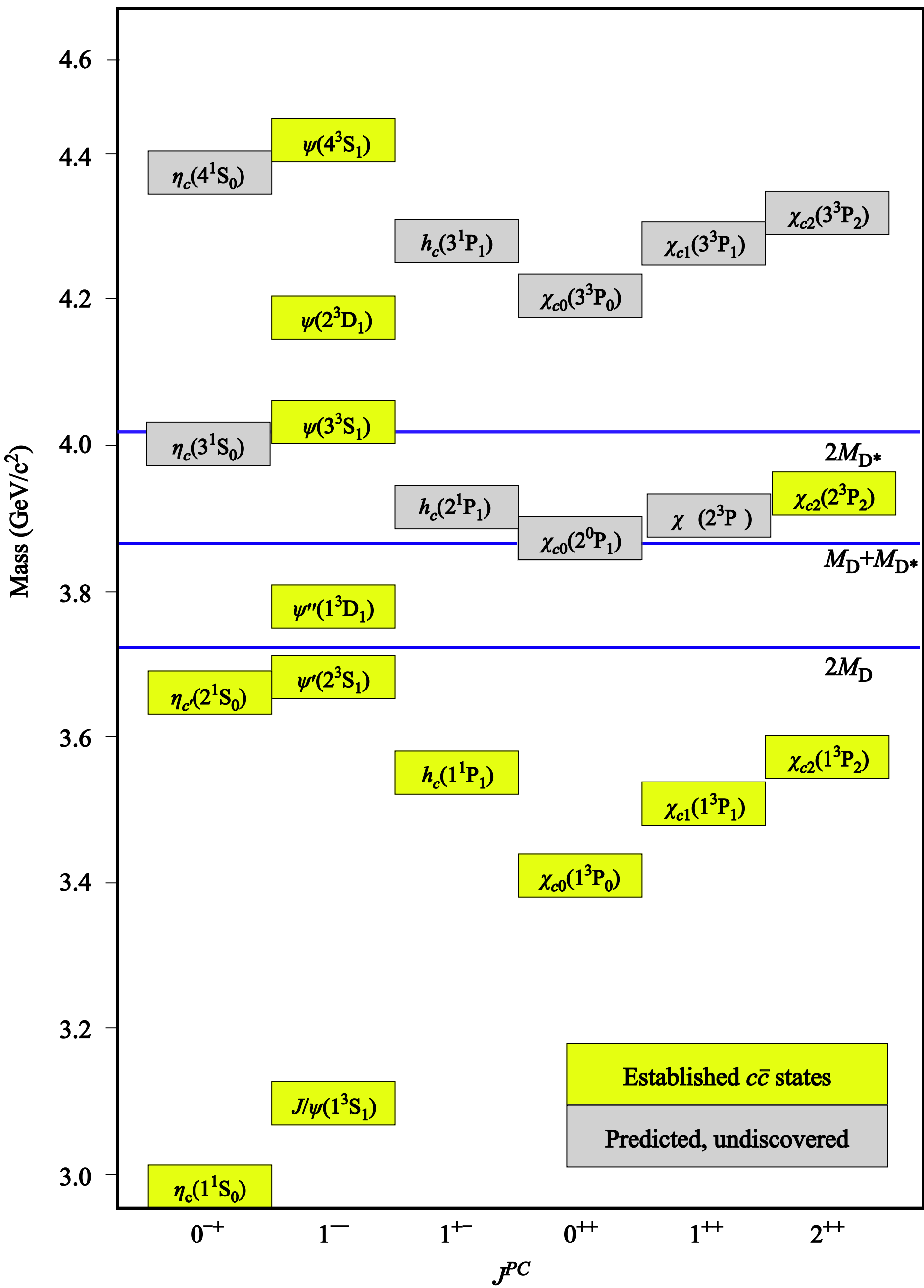
0^{-+}	1^{--}	1^{+-}	J^{++}
η_c	ψ	h_c	$\chi_{1,2,3}$

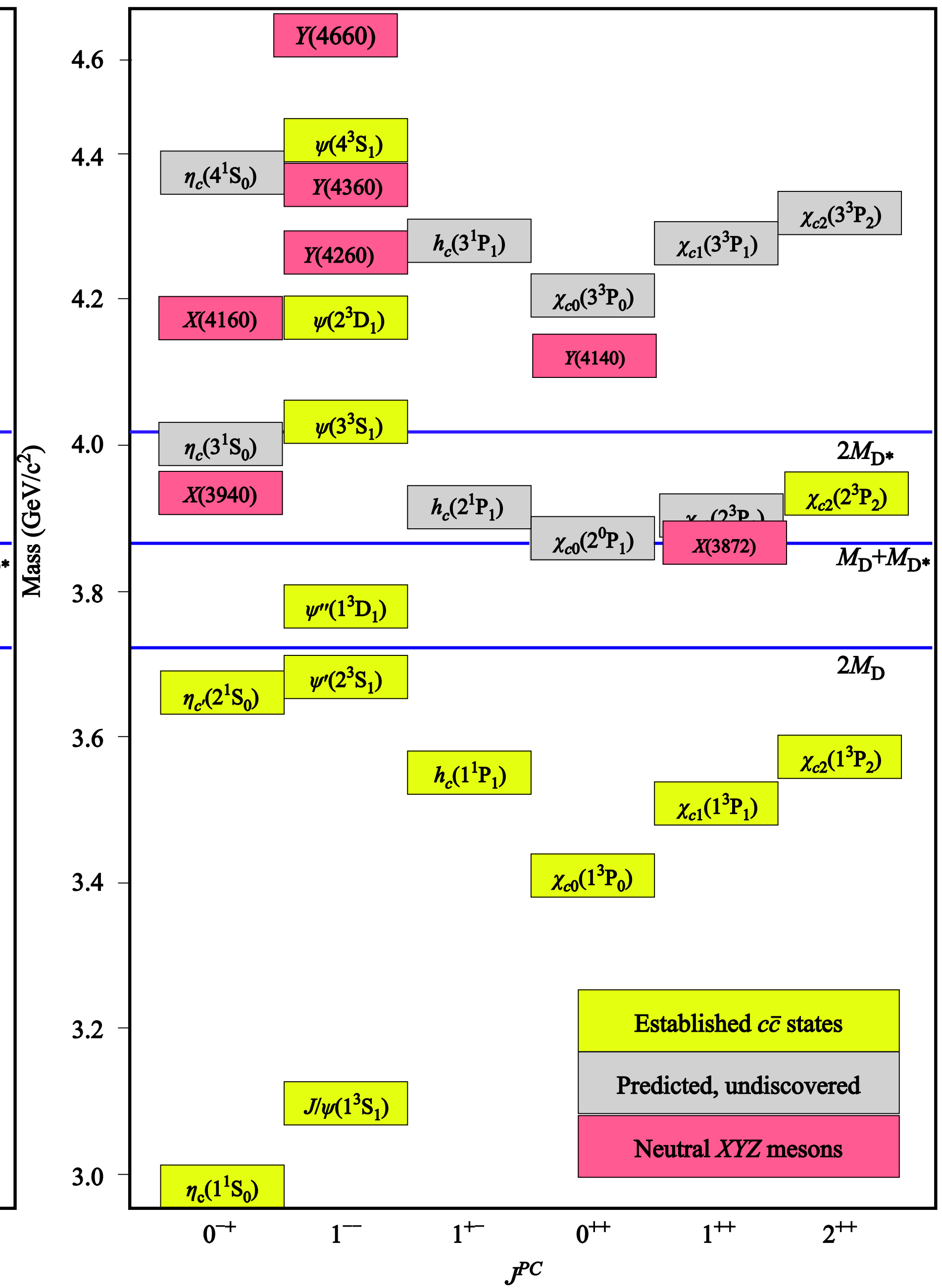
- 8 States well established
- Hyperfine splitting to adjust spin dependent potential V_{SS}

$$\Delta m_{hf}(1S) = m(J/\Psi) - m(\eta_c) = 116\,\mathrm{MeV}$$

- Look for
  - Missing States
  - Additional States

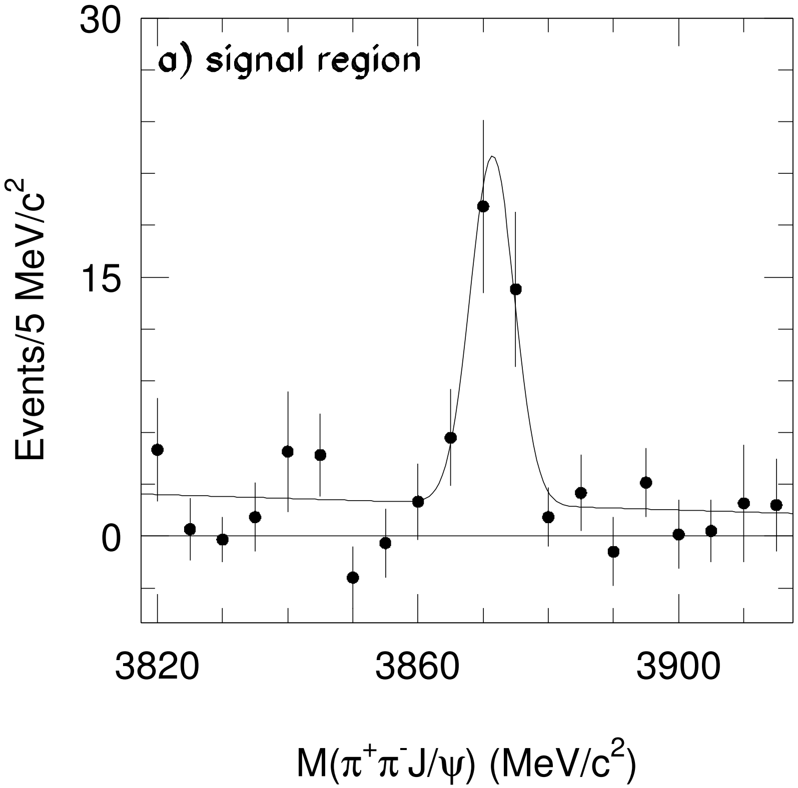
Charmonium Spectrum





The $X(3872)$ (new PDG2018 naming scheme: $\chi_{c1}(3872)$)

Belle (2013): A new state, not quite fitting into spectrum:



Discovery channel:

$$e^+e^- \to \Upsilon(4S) \to B^+B^-$$
$$B^+ \to K^+ \underbrace{\pi^+\pi^- J/\psi}_{\text{subsystem}}$$

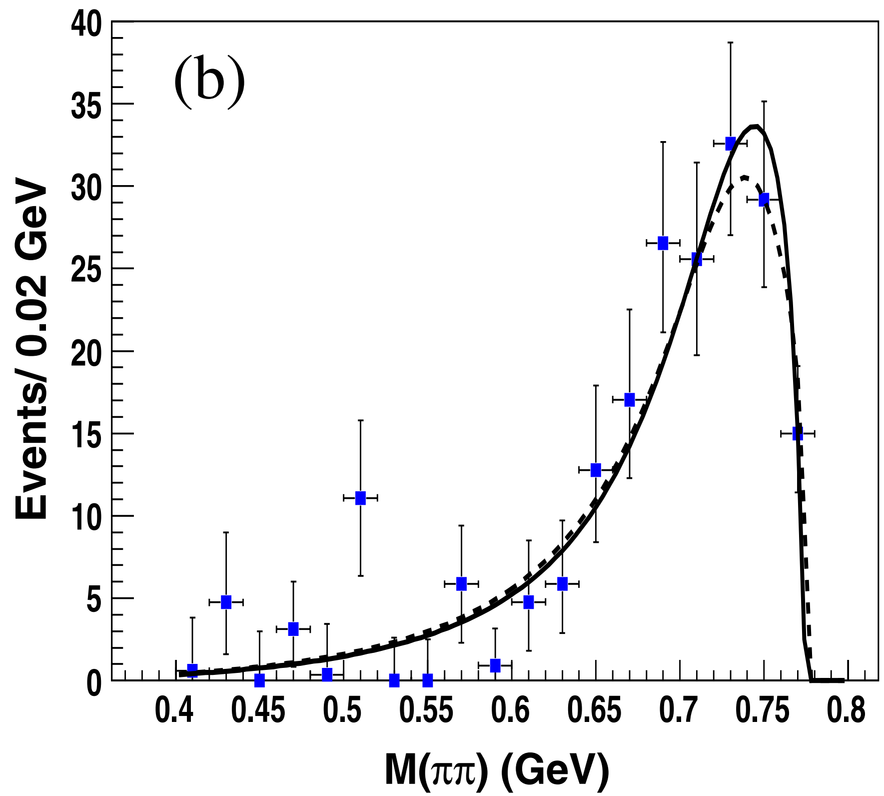
- Decay to J/ψ: $c\bar{c}$ content neccessary
- Isospin: Decay via $\rho \to \pi^+\pi^-$ or $\omega \to \pi^+\pi^-$
- ρ decay is isospin violating $\to$ suppressed
- Both channels are of same order

$\Rightarrow$ additional u and d content?

- Resonance confirmed by BaBar, BES, CDF, D0, LHCb, ...
- LHCb: Quantum Numbers $J^{PC} = 1^{++}$, $I = 0$ (these are not exotic!)

$X(3872)$ Decay to $\rho J/\psi$

$$X(3872) \to \rho + J/\psi$$
$$\rho \to \pi^+ + \pi^-$$



- Two Pion distribution described by Breit-Wigner with known $\rho(770)$ width
- Violates Isospin conservation $\Rightarrow$ at least two gluons
- Should be suppressed compared to decay via $\omega \to \pi^+\pi^-\pi^0$

Interpretations of the $X(3872)$

$X(3872)$ Properties

- Mass is very close to open charm threshold $\overline{D}_0 D_0*$
- Width is very narrow < 1.2 MeV
- small binding $\Rightarrow$ huge separation
- Decays to $\rho J/\psi$
- Decays to $\omega J/\psi$
- Decays dominant to $\overline{D}_0 D_0*$

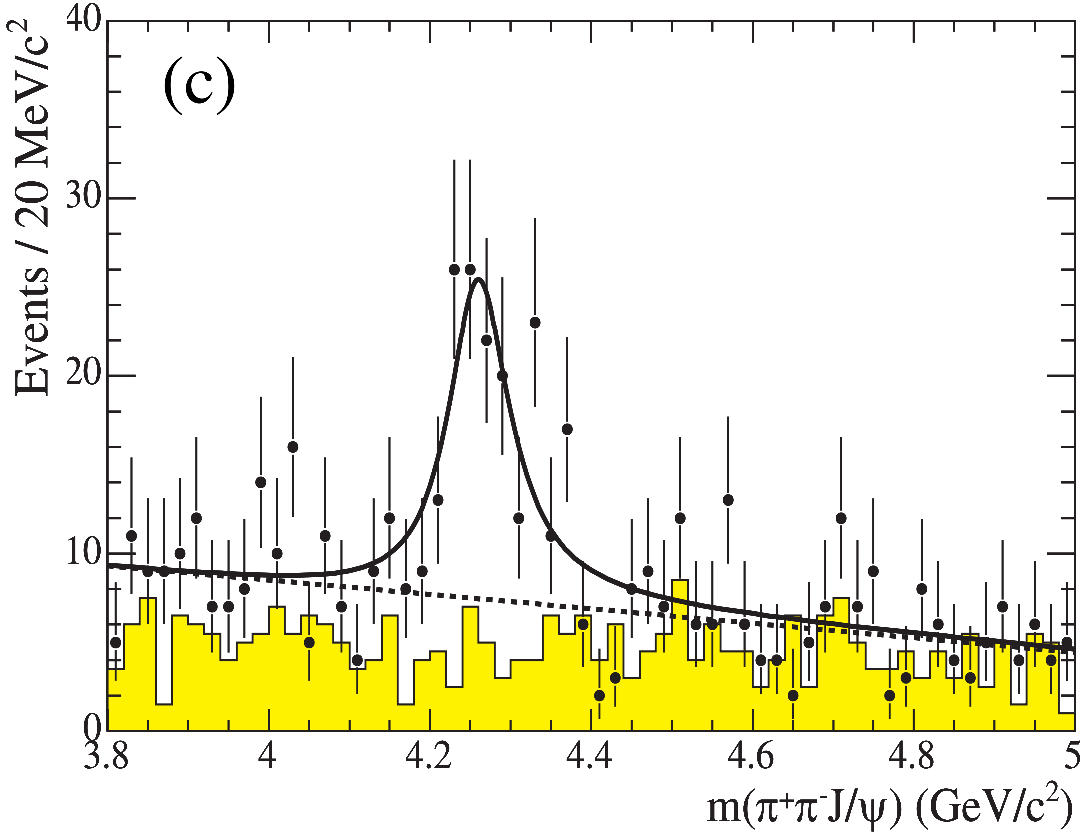
Interpretation:

- Exotic nature? Probably...
- Many interpretations on the market
- Loosely bound $\overline{D}_0 D_0*$ molecule?

$Y(4260)$

BaBar (2005) via Initial State Radiation

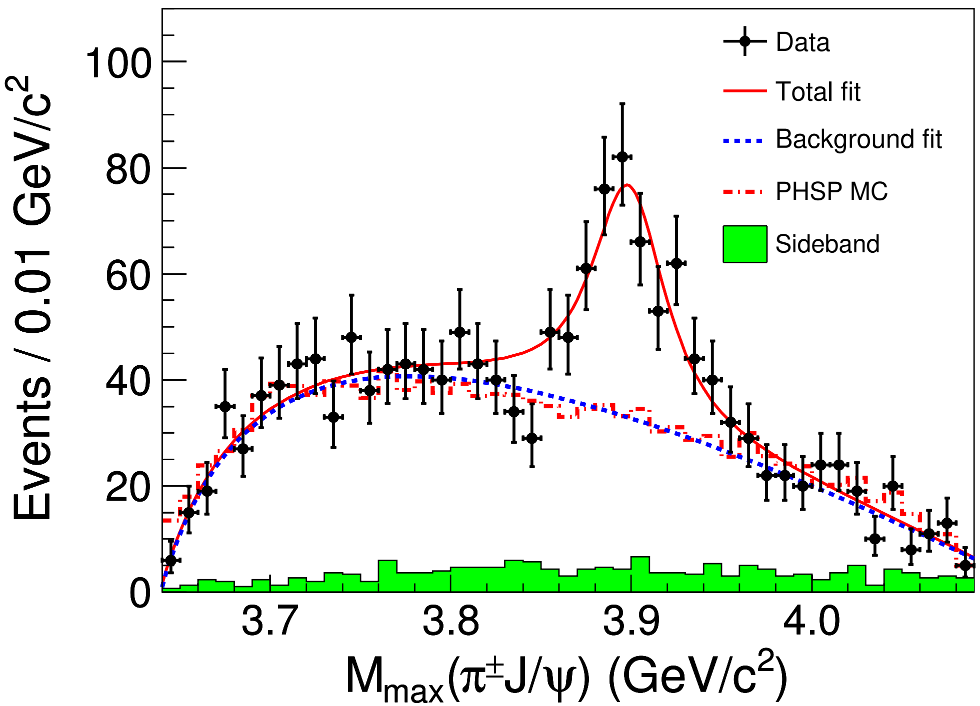
$$e^+ e^- \rightarrow \gamma_{ISR}\, \pi^+ \pi^- J/\psi$$



- Quantum numbers are now $J^{PC} = 1^{--}$
- Confirmed by CLEAO, CLEOIII, Belle, BESIII
- Weak coupling consistent with hybrid meson

$Z_c^+(3900)$

BES III (2013)



$$e^+e^- \to \pi^- \underbrace{\pi^+ J/\psi}_{\text{subsystem}}$$

- Decay to J/ψ:
  $\Rightarrow c\bar{c}$ content neccessary
- Charged!!!!!!
  $\Rightarrow$ at least $c\bar{c}u\bar{d}$

Status:

- Confirmed by several experiments
- Several states
- also Z_b^+ states seen
- PDG 2018 naming scheme:

X	now χ	Isospin 0
Y	now ψ	
Z		Isospin 1

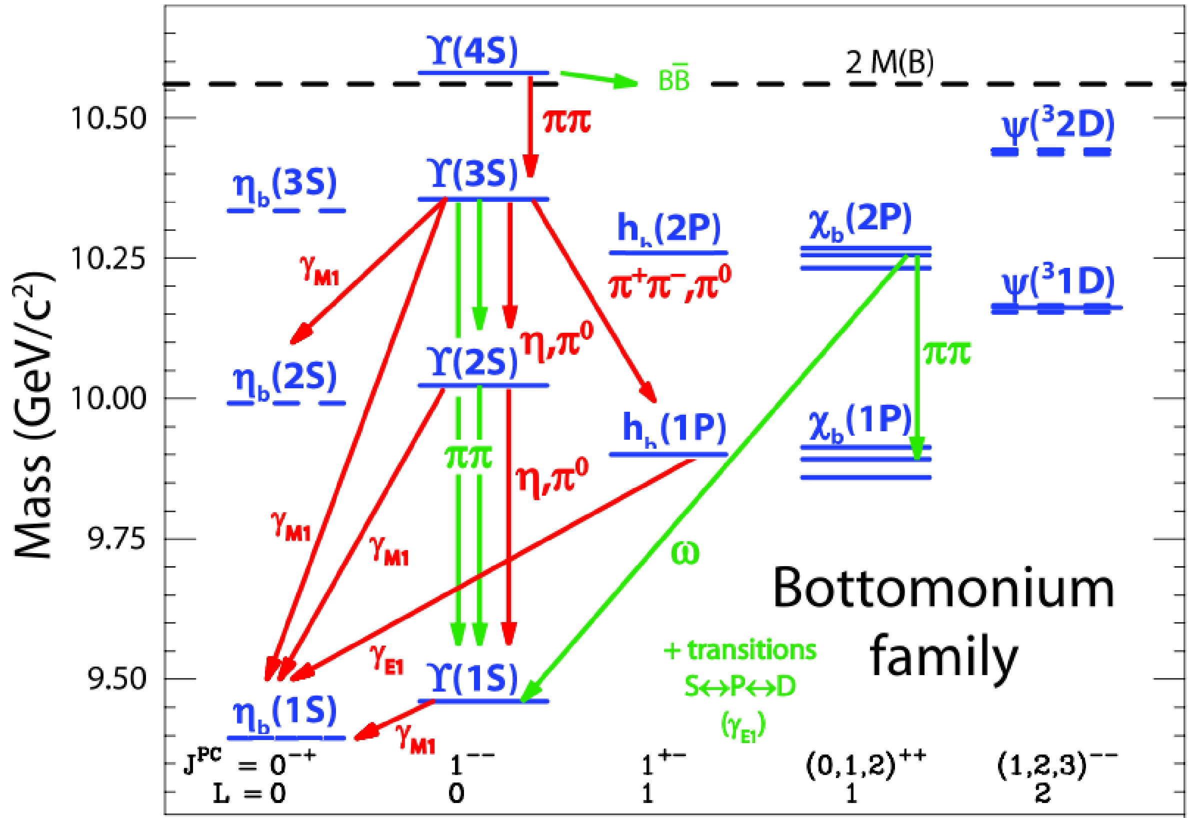
Growing number of states...

Particle Data Group (2018): States near open $c\bar{c}$ or $b\bar{b}$ threshold

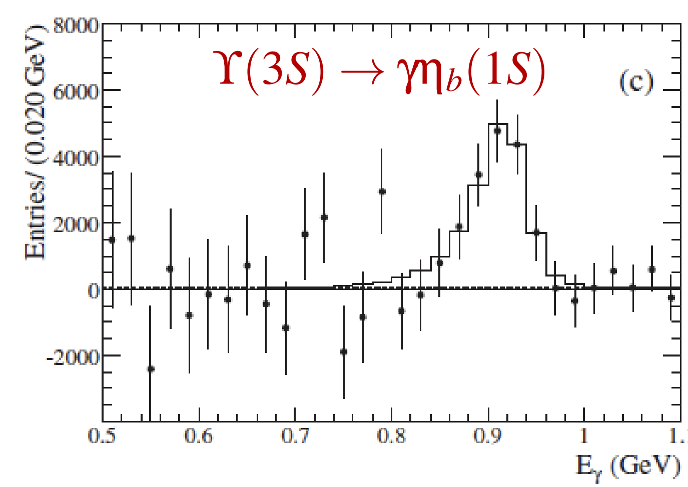
PDG Name	Former/Common Name(s)	m (MeV)	Γ (MeV)	$I^G(J^{PC})$	Production	Decay	Discovery Year	Summary Table
$\chi_{c1}(3872)$	$X(3872)$	3871.69±0.17	< 1.2	$0^+(1^{++})$	$B \to KX$ $p\bar{p} \to X...$ $pp \to X...$ $e^+e^- \to \gamma X$	$\pi^+\pi^- J/\psi$ $3\pi J/\psi$ $D^{*0}\overline{D}^0$ $\gamma J/\psi$ $\gamma\psi(2S)$	2003	YES
$Z_c(3900)$		3886.6 ± 2.4	28.2 ± 2.6	$1^+(1^{+-})$	$\psi(4260) \to \pi^- X$ $\psi(4260) \to \pi^0 X$	$\pi^+ J/\psi$ $\pi^0 J/\psi$ $(D\bar{D}^*)^+$ $(D\bar{D}^*)^0$	2013	YES
$X(4020)$	$Z_c(4020)$	4024.1 ± 1.9	13 ± 5	$1^+(?^{?-})$	$\psi(4260, 4360) \to \pi^- X$ $\psi(4260, 4360) \to \pi^0 X$	$\pi^+ h_c$ $\pi^0 h_c$ $(D^*\bar{D}^*)^+$ $(D^*\bar{D}^*)^0$	2013	YES
$Z_b(10610)$		10607.2 ± 2.0	18.4 ± 2.4	$1^+(1^{+-})$	$\Upsilon(10860) \to \pi^- X$ $\Upsilon(10860) \to \pi^0 X$	$\pi^+\Upsilon(1S, 2S, 3S)$ $\pi^0\Upsilon(1S, 2S, 3S)$ $\pi^+ h_b(1P, 2P)$ $(B\bar{B}^*)^+$	2011	YES
$Z_b(10650)$		10652.2 ± 1.5	11.5 ± 2.2	$1^+(1^{+-})$	$\Upsilon(10860) \to \pi^- X$	$\pi^+\Upsilon(1S, 2S, 3S)$ $\pi^+ h_b(1P, 2P)$ $(B^*\bar{B}^*)^+$	2011	YES

...and ≈ 25 more unassigned states above threshold

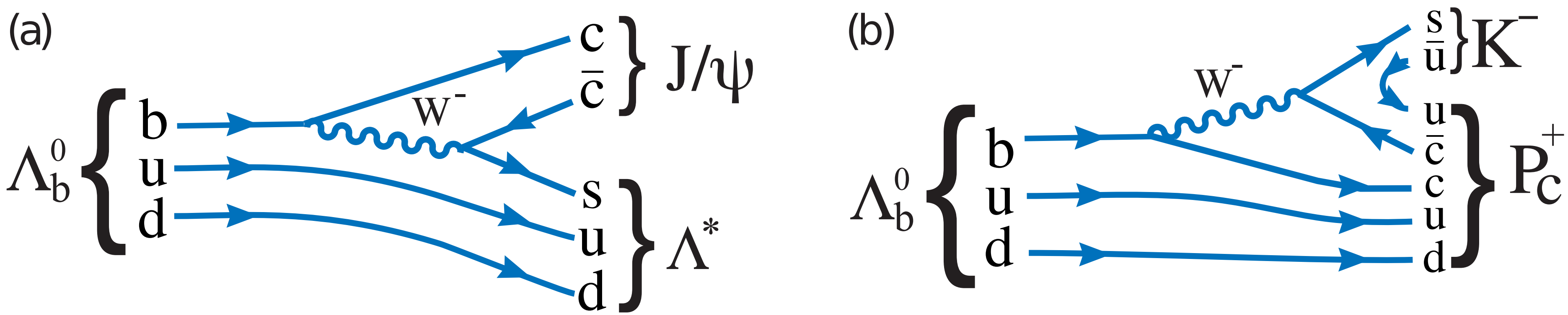
Bottomonium

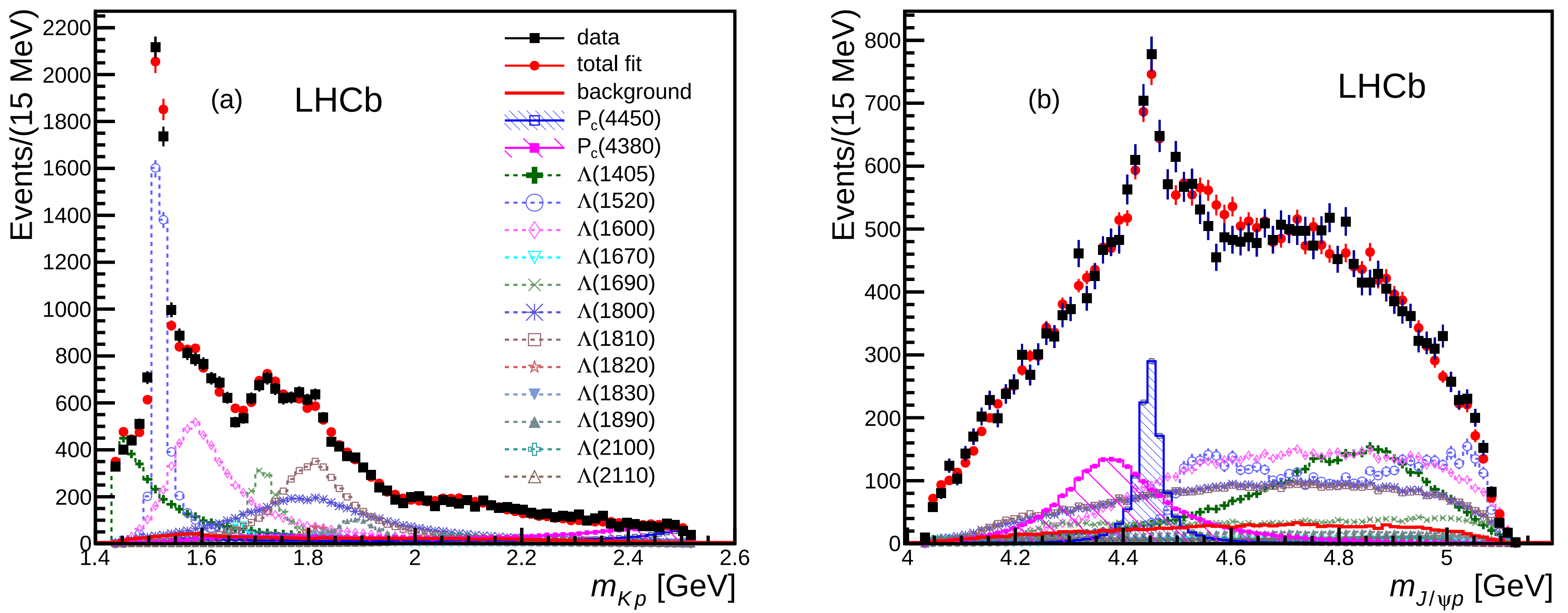


- higher b-quark mass
- lower coupling $\alpha_s(Q^2)$
- dominated by Coulomb term of the potential
- better description by potential models
- ground state $\eta_b(1S)$ discovered 2008



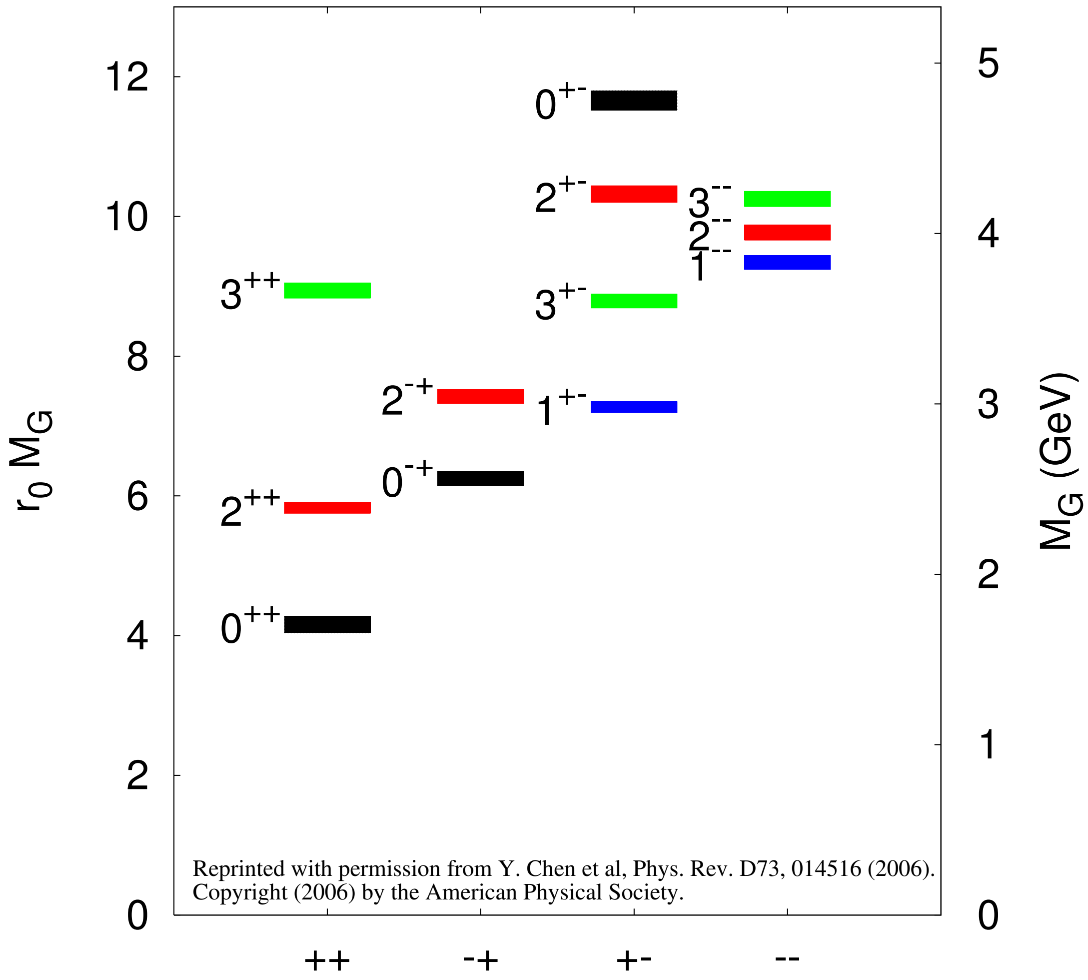
Pentaquark (LHCb 2015)





$$\Lambda_b^0 \to J/\psi + K^- + p$$

Glueballs



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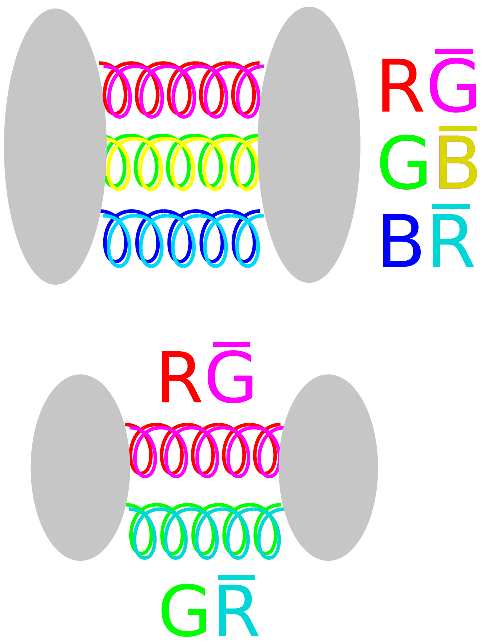


- Calculable in Lattice QCD
- Predictions:

$$J^{PC} = 0^{++}, 2^{++}$$

- Mixing with scalar mesons $f_0(1370)$
- Candidates $f_0(1500)$, $f_0(1710)$, ...
- No clear signature yet

Testing the wave-function: Form-Factors

Form-Factor of the Nucleon

Elastic Cross Section (Rosenbluth-Formula):

$$\frac{d\sigma}{d\Omega_e} = \left(\frac{d\sigma}{d\Omega_e}\right)_{\text{Mott}} \frac{1}{(1+\tau)} \left[G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)\right]$$

with
$$\tau = \frac{Q^2}{4m_p^2}$$
$$\varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

$G_E(Q^2)$: Electric Form-Factor $\rightarrow$ related to charge distribution
$G_M(Q^2)$: Magnetic Form-Factor $\rightarrow$ related to distribution of magnetic moments

Normalization:

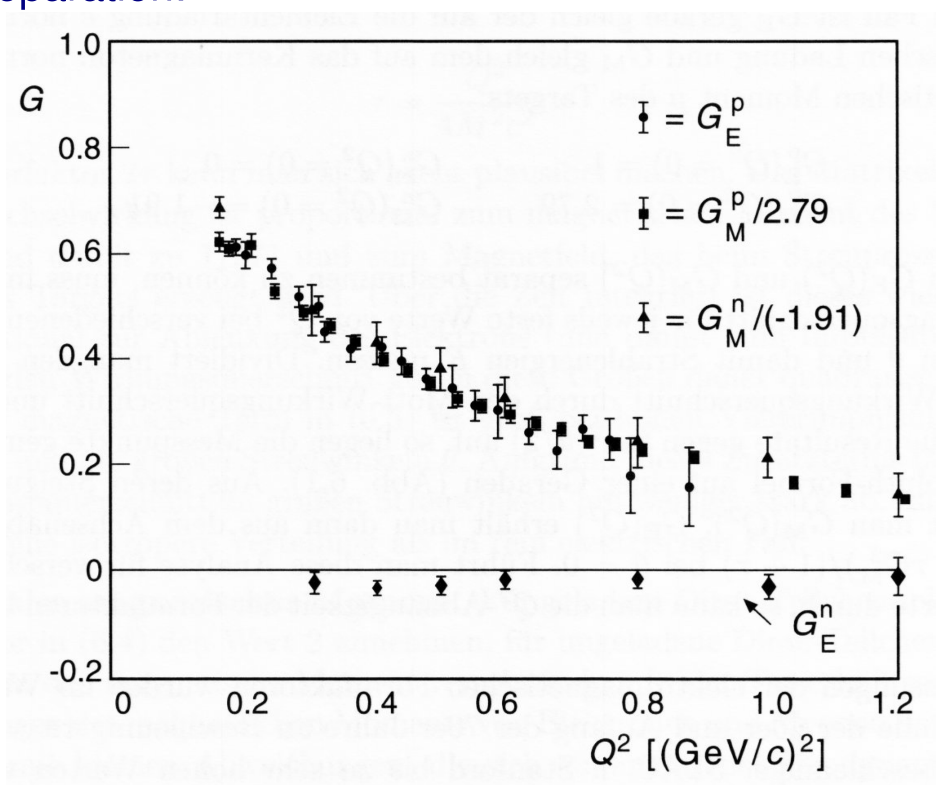
$$G_E^p(Q^2=0) = 1 \qquad G_E^n(Q^2=0) = 0$$
$$G_M^p(Q^2=0) = 2.79 \qquad G_M^n(Q^2=0) = -1.91$$

Root-Mean-Square Radius:

$$\langle r_E^2 \rangle = -6 \frac{d}{dQ^2} G_E(Q^2)\Big|_{Q^2=0} \qquad \langle r_M^2 \rangle = -\frac{6}{\mu_p} \frac{d}{dQ^2} G_M(Q^2)\Big|_{Q^2=0}$$

Gross features of the Nucleon Form-Factors: Dipole formula

Results of Rosenbluth separation:



Empirical dipole fit:

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{2.79} = \frac{G_M^n(Q^2)}{-1.91} = G^{Dipole}(Q^2)$$

$$G_{Dipole}(Q^2) = \left(1 + \frac{Q^2}{0.71 GeV^2}\right)^{-2}$$

Polarization Transfer

Polarization transfer reaction:

$$\vec{e} + n \to e + \vec{n}$$

Longitudinal and transverse polarization:

$$P_l = \frac{E+E'}{mI_0}\sqrt{\tau(1+\tau)}G_M^2 \tan^2\frac{\theta}{2}$$

$$P_t = -\frac{2}{I_0}\sqrt{\tau(1+\tau)}G_E G_M \tan\frac{\theta}{2}$$

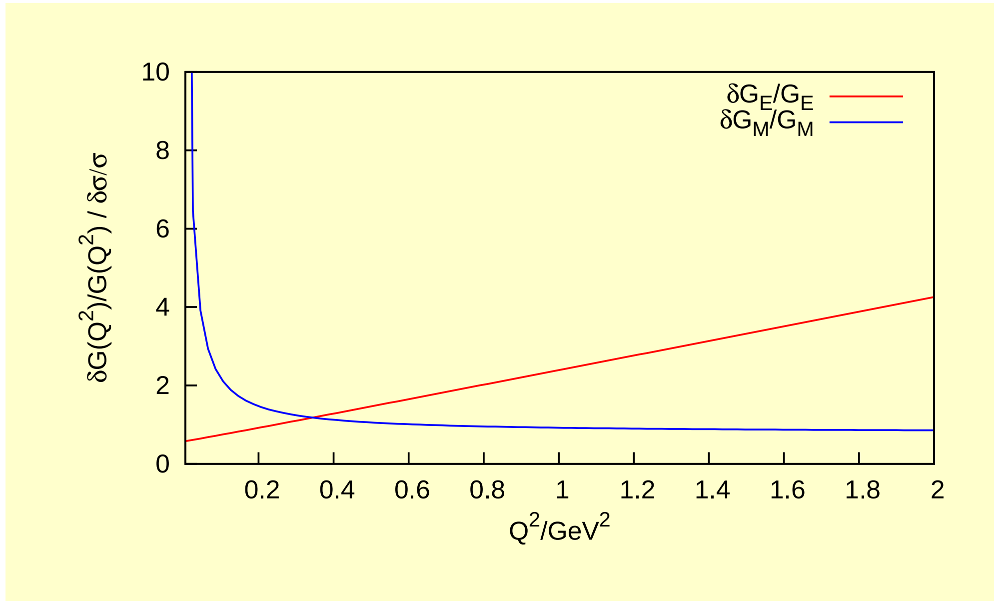
$$I_0 = G_E^2 + \tau\left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]G_M^2$$

$$\Rightarrow \quad \frac{G_E}{G_M} = -\frac{P_t}{P_l}\frac{(E+E')}{2m}\tan\frac{\theta}{2}$$

- Signal proportional to $G_E(Q^2)$
- Systematic errors cancel out
- Requires measurement of recoil polarization

Rosenbluth vs. Polarization Transfer

Error propagation for Rosenbluth-separation (example: $\varepsilon = 0.2, 0.9$):

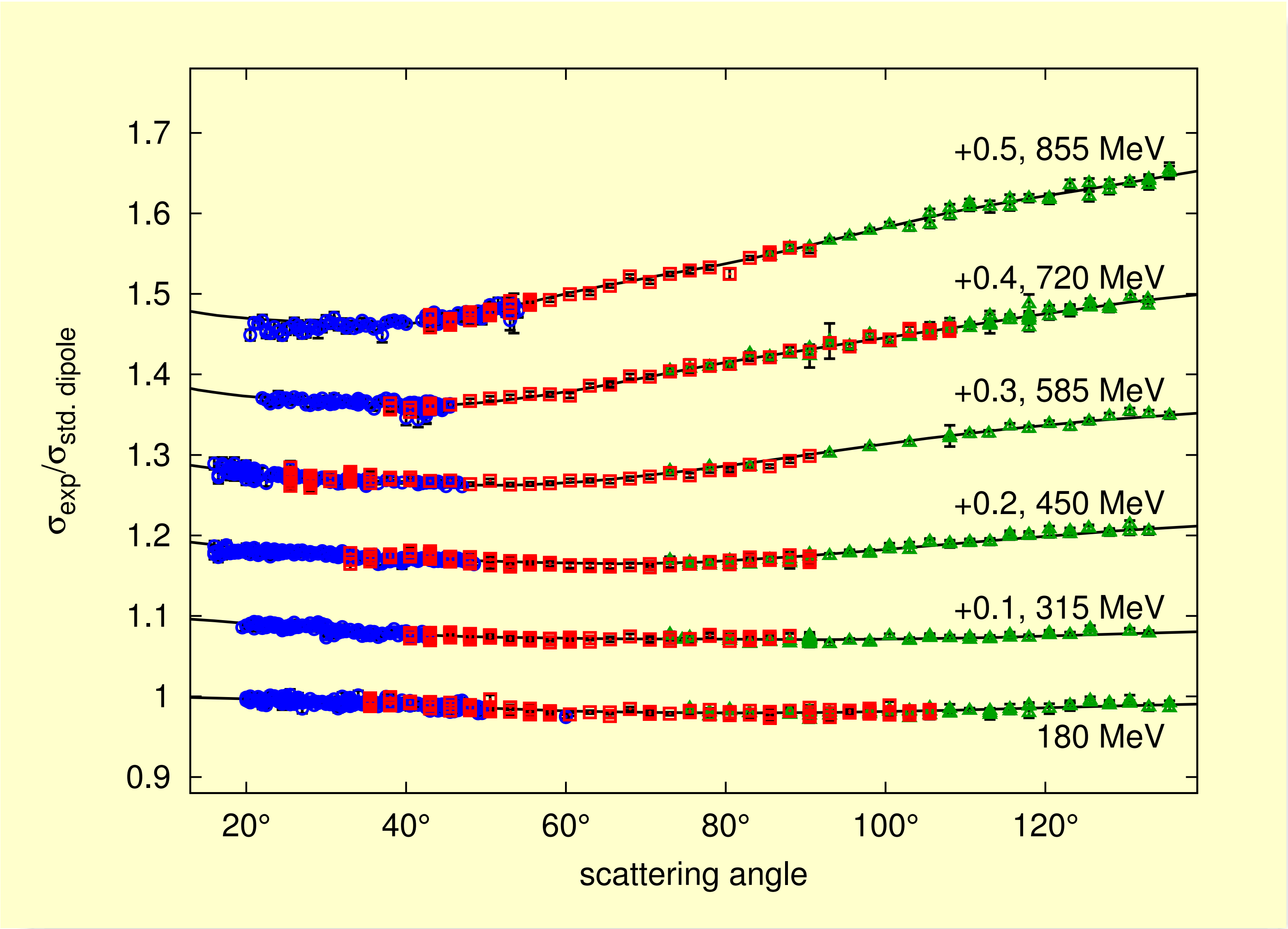


$$\frac{d\sigma}{d\Omega_e} = \left(\frac{d\sigma}{d\Omega_e}\right)_{\text{Mott}} \frac{1}{(1+\tau)} \left[G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)\right]$$

- High Q^2: $G_M^2 \approx 2.79^2 \times G_E^2$
  G_E^2 suppressed by $\tau >> 1$
- Low Q^2: G_M^2 suppressed by $\tau = \frac{Q^2}{4M^2} \to 0$

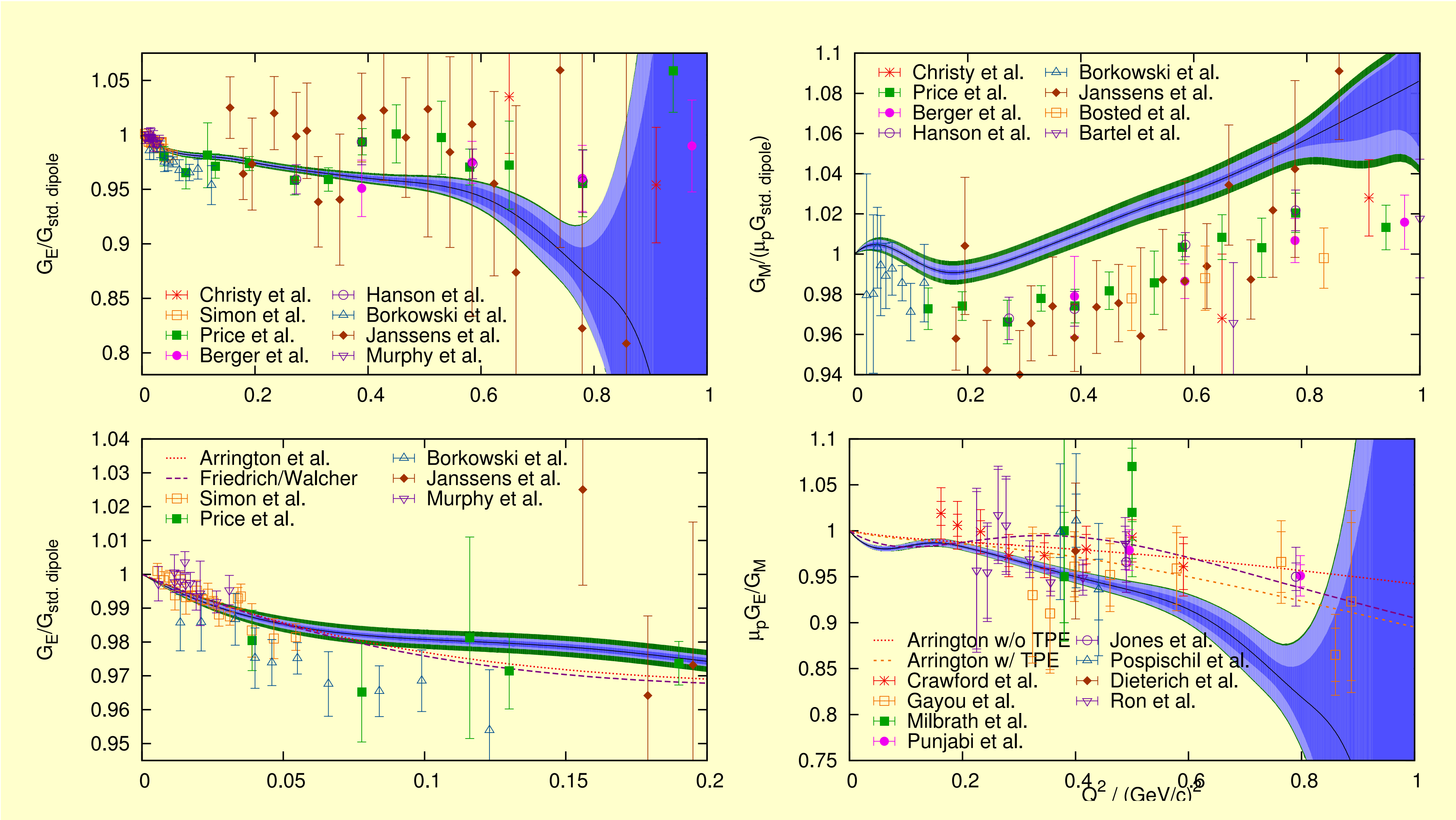
  - BUT: Recoil polarization difficult below $Q^2 \approx 0.2\,\text{GeV}^2/c^2$
  - Utilize knowledge at $Q^2 = 0 \quad \Rightarrow \quad \mu G_E = G_M$

Rosenbluth-Separation by Fit



Data spectrometer A, B, C, error bars by spread of data → 0.2% – 0.4% (stat.: 0.1% – 0.3%)
— Spline fit

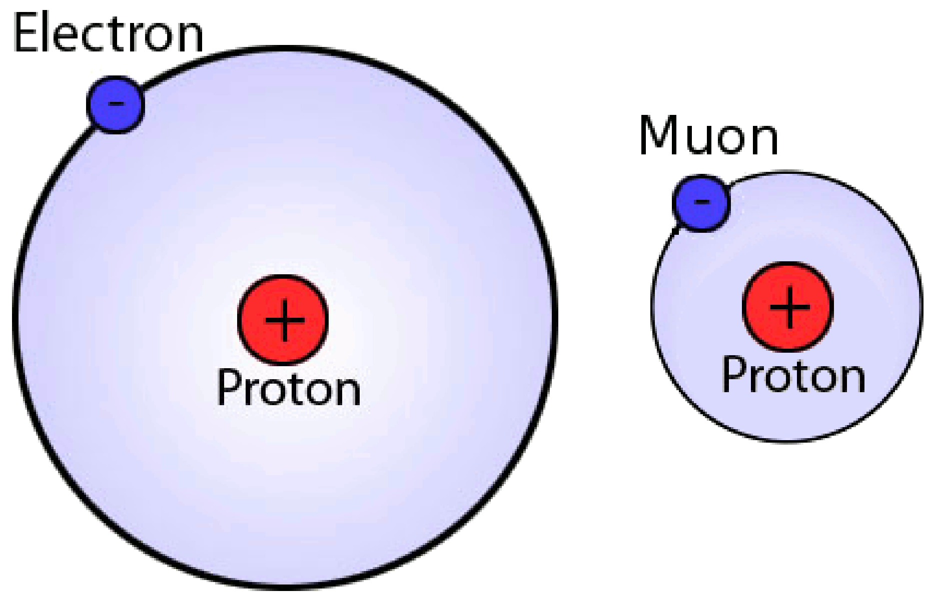
Form-Factor results



$$r_E = 0.879 \pm 0.005_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.002_{\text{model}} \pm 0.004_{\text{group}}\ \text{fm},$$
$$r_M = 0.777 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.005_{\text{model}} \pm 0.002_{\text{group}}\ \text{fm}.$$

Jan C. Bernauer *et al.*, Phys. Rev. Lett. 105, 242001 (2010), Phys. Rev. C 90, 015206 (2014)

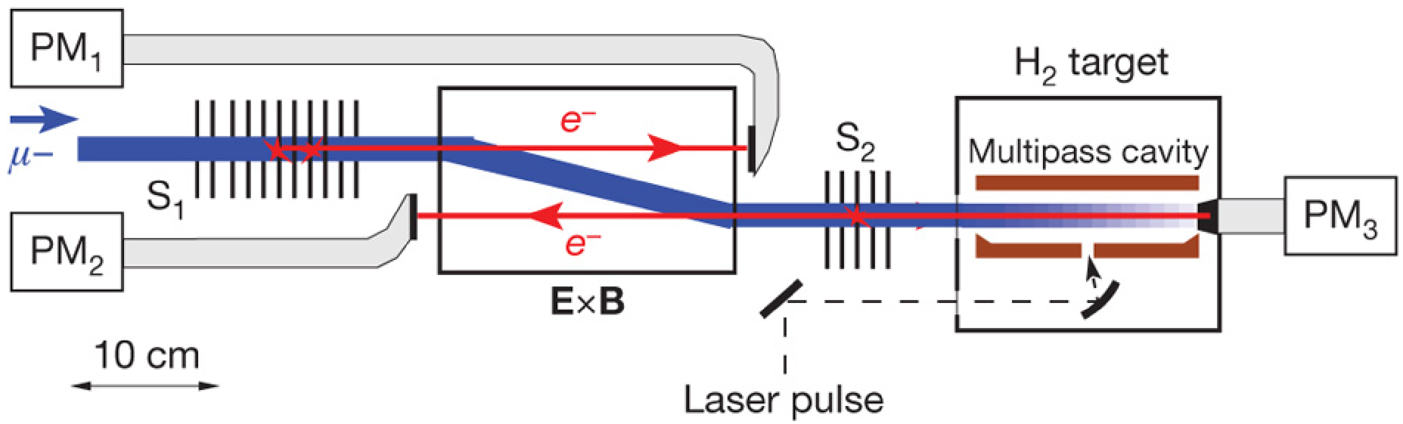
Muonic Hydrogen

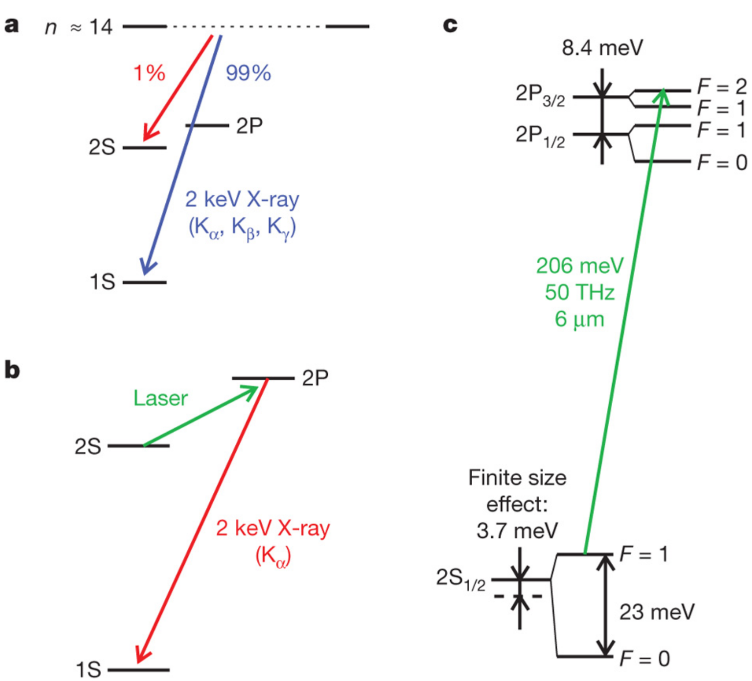


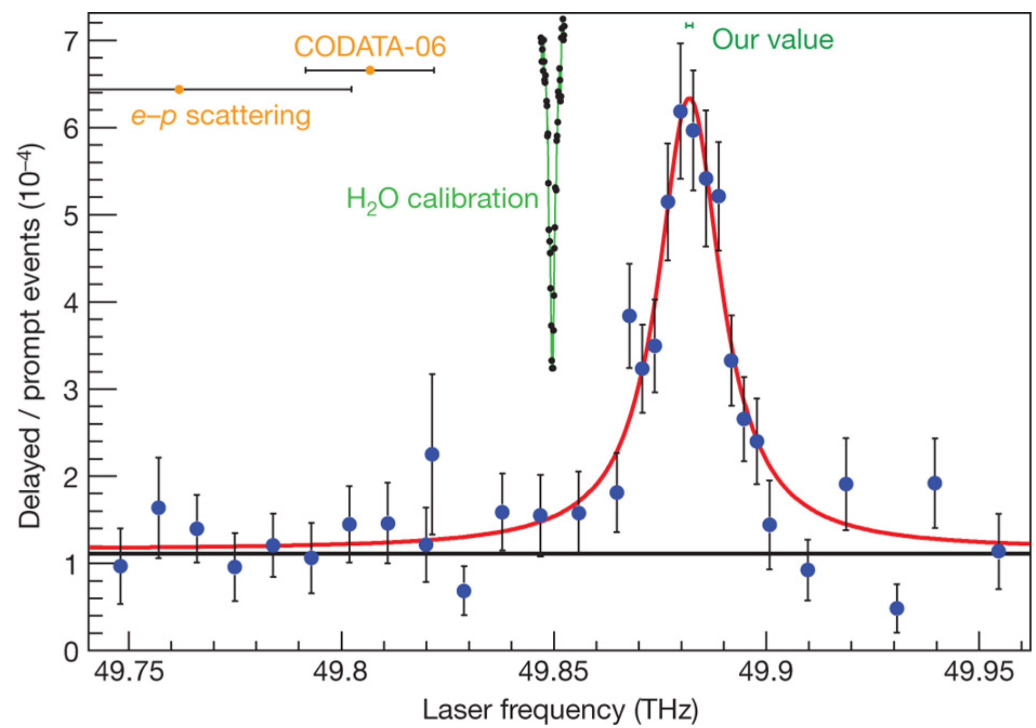
- $\frac{m_\mu}{m_e} \approx 200 \Rightarrow$ Muon spends more time at center
- Increased overlap wave-function with nucleus (at least s-wave...)
- Incresed sensitivity to charge distribution
- First order: electric radius of the proton

The Radius Puzzle: Lamb shift in μH-Atom

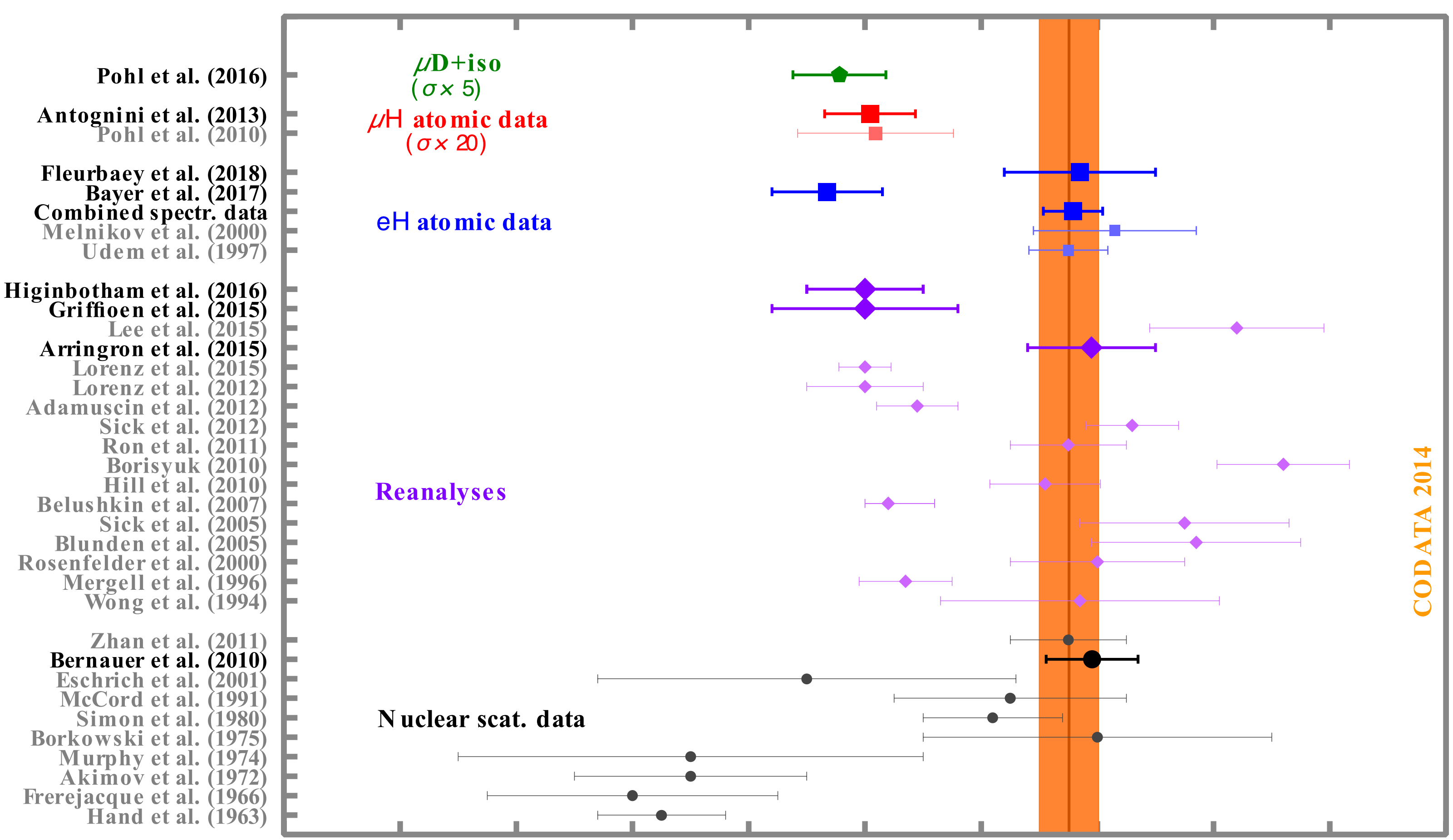
Nature **466**, 213-216 (8 July 2010)







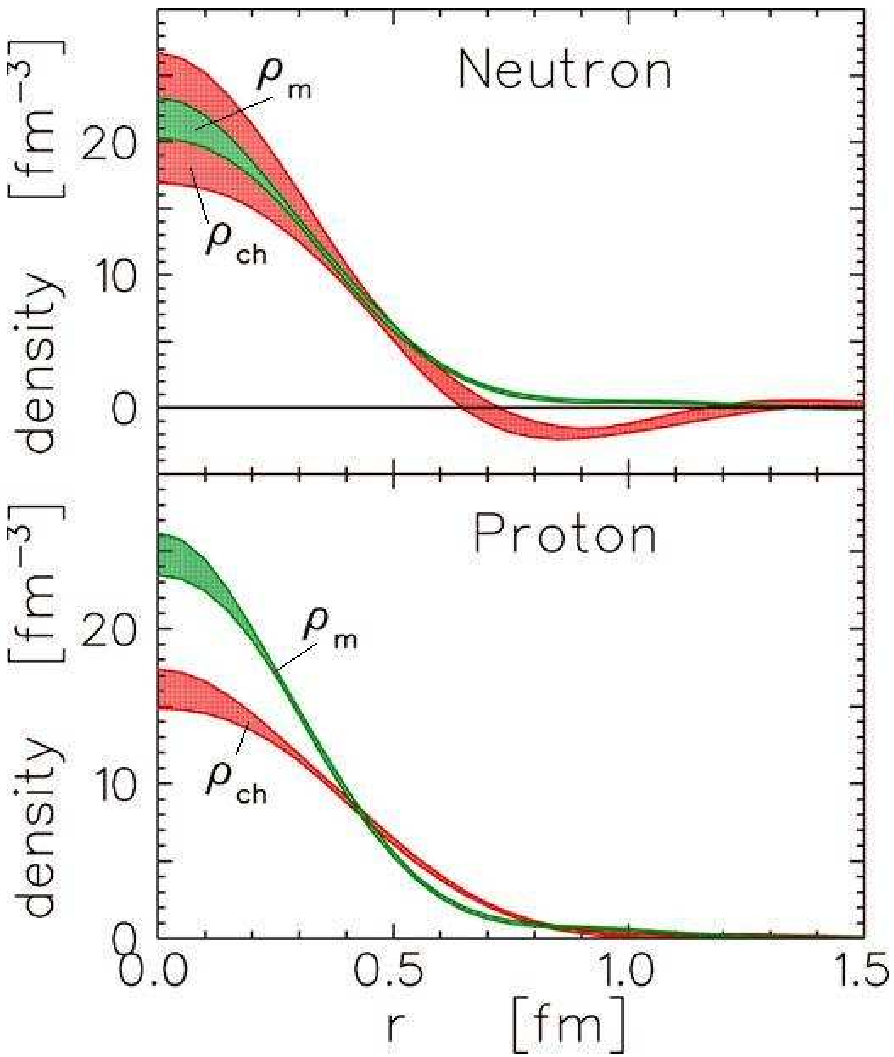
Radius of the Proton



- 5σ Discrepancy between atomic physics and electron scattering
- Situation still unclear
- Serious problem far beyond nuclear science: *e.g.* Rydberg Constant
- Trigger experimental program in atomic and nuclear physics

Models for Form-Factors

Charge Distribution



J. J. Kelly, Phys. Rev. C 66 (2002)

- Non-Relativistic charge distribution: Fourier transform of form factor $F(k)$

$$F(k) = \int_0^\infty r^2 j_0(kr)\rho(r)dr$$
$$\rho(r) = \int_0^\infty k^2 j_0(kr)F(k)dr$$

with $k = |\vec{q}|$

- Lorentz boost

$$k^2 \to Q^2/(1+\tau)$$

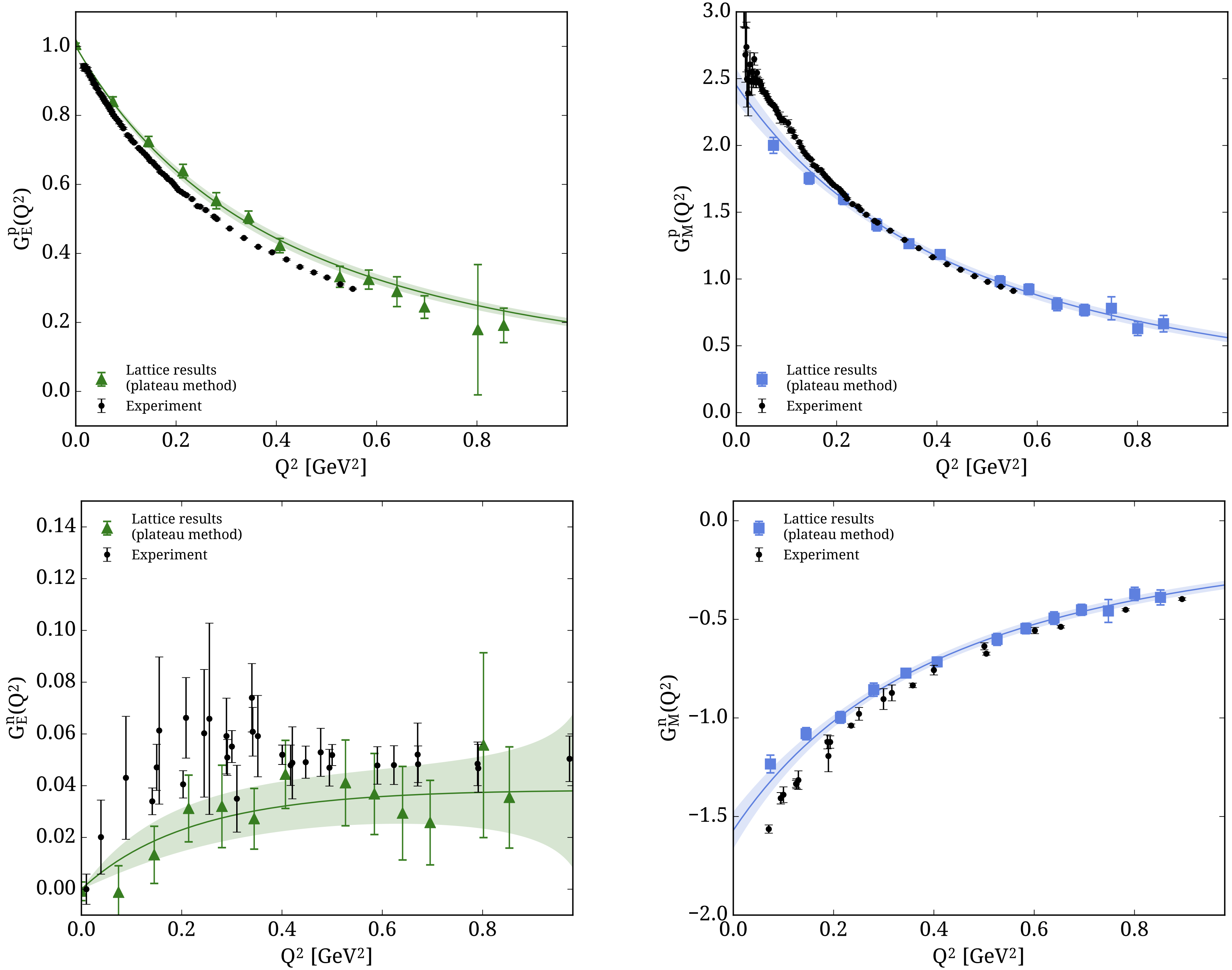
- Relativistic prescription (not unique)

$$F(k) = (1+\tau)^2 G(Q^2)$$

- Limit for k (position uncertainties!)

$$k_{max} = 2M$$

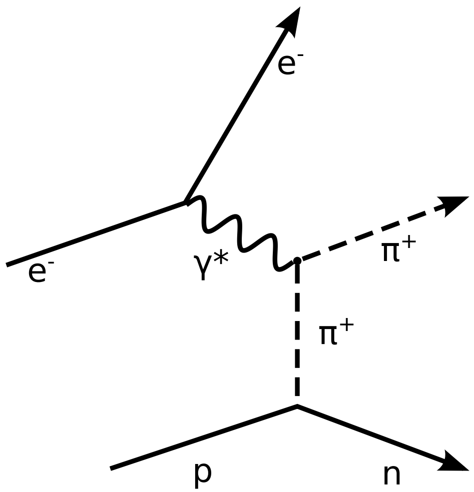
Nucleon Form-Factor in Lattice QCD



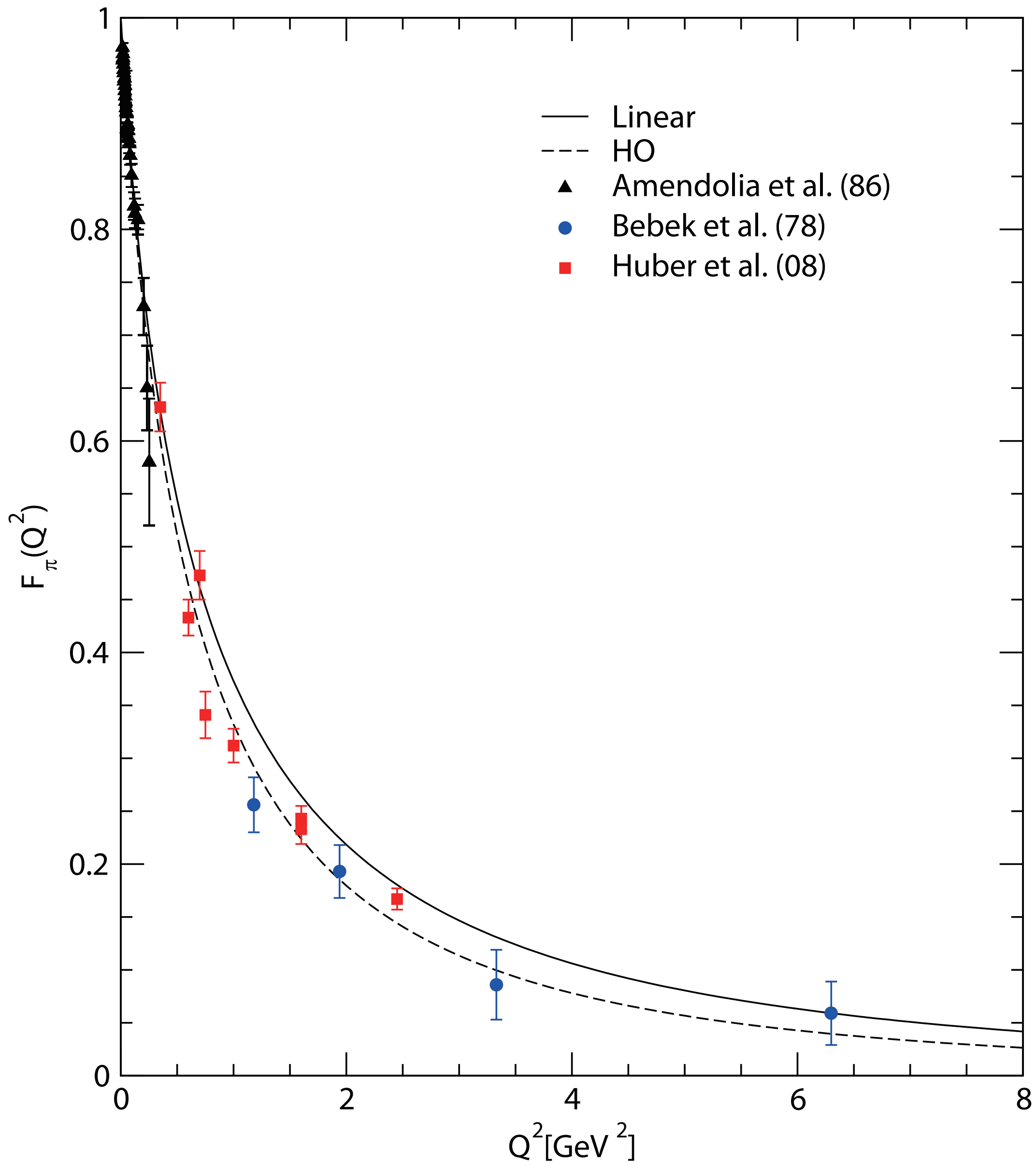
C. Alexandrou et al., PRD96 (2017)

Charged Pion Form-Factor

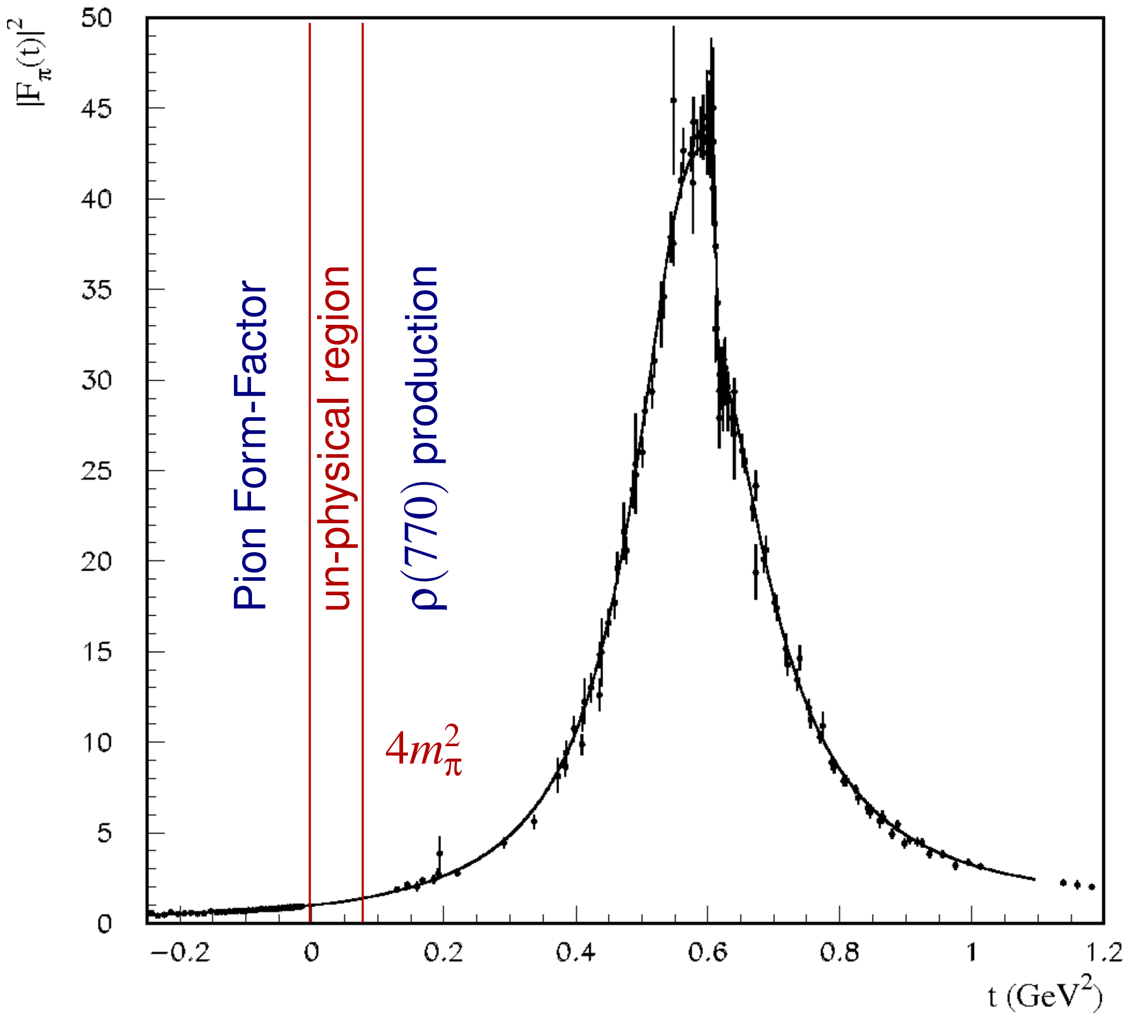
Measurement:



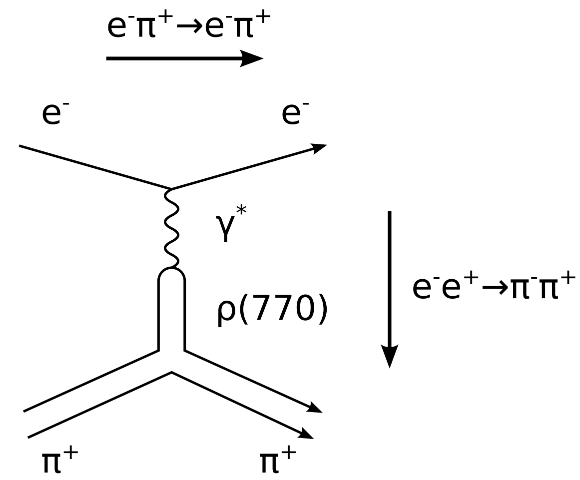
- Pion electroproduction
- Virtual pion in initial state
- Measurement at different virtualities
- Extrapolation to viruality zero



Pion Form-Factor



Interpretation:

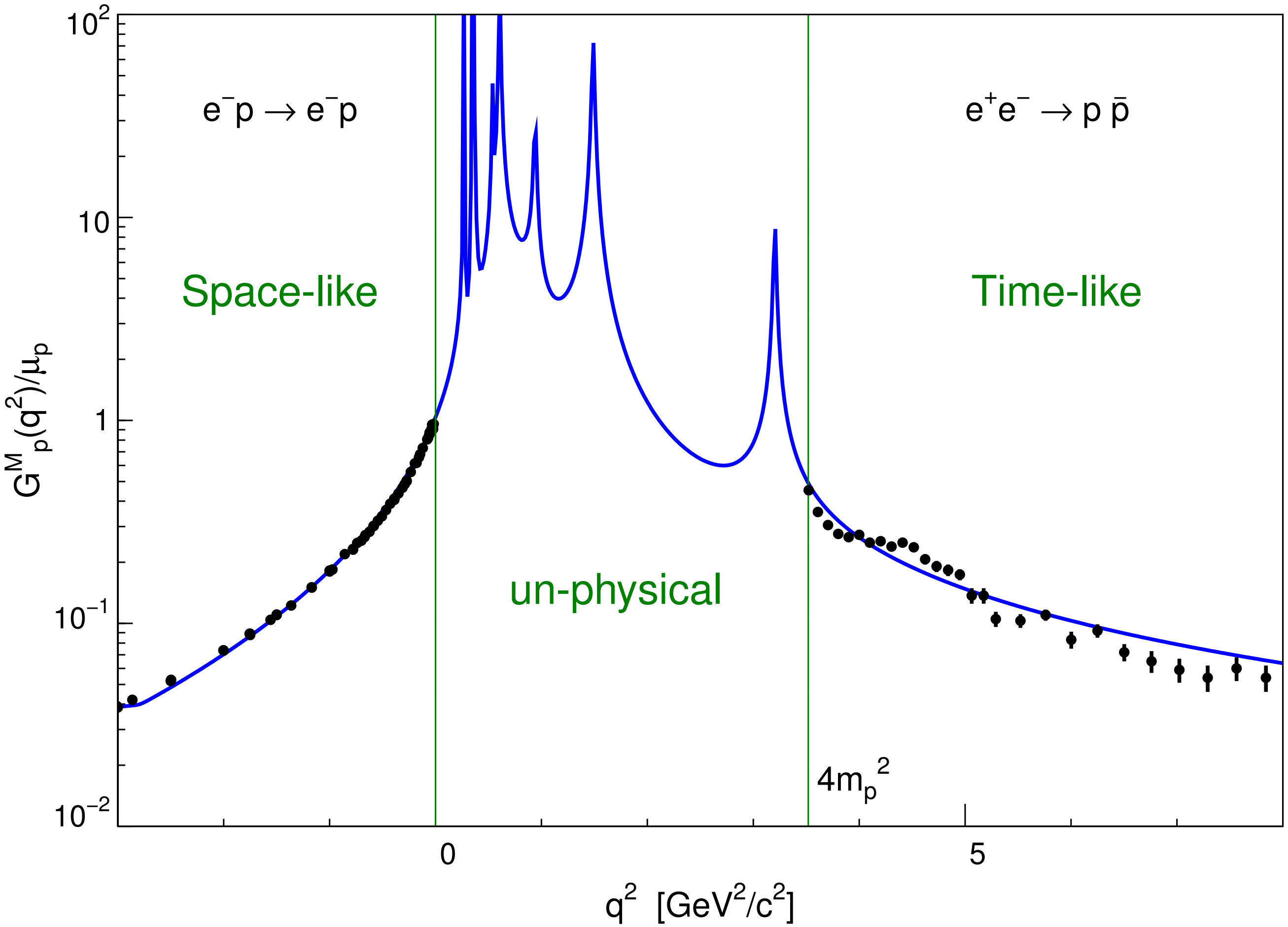


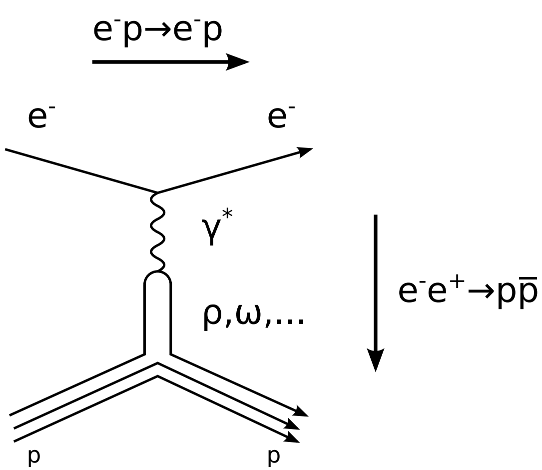
- Crossing symmetry
- Photon is virtual vector meson

$\Rightarrow$ "Vector Meson Dominance"

Vector Meson Dominance

Also the Proton Form-Factor can be described by mesons:



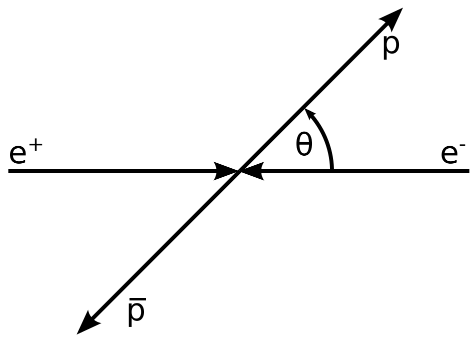


Space-like: ∃ reference frame where virtual photon is emmitted and absorbed at the same time
Time-like: ∃ reference frame where virtual photon is emmitted and absorbed at the same position

NB: Blue line from 1974 (G. Höhler), data are much later!

Time-like Form-Factors

Cross section of $e^+e^- \to p\overline{p}$:



$$\sigma = \frac{4\pi}{3}\frac{\alpha^2\beta}{q^2}C\left(\left|G_M(q^2)\right|^2 + \frac{2m^2}{q^2}\left|G_E(q^2)\right|^2\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s}C\left(\left|G_M(q^2)\right|^2(1+\cos^2\theta) + \frac{4m^2}{q^2}\left|G_E(q^2)\right|^2\sin^2\theta\right)$$

with Coulomb correction factor $C = \frac{1}{1-e^-y}$ and $y = 2\pi\frac{\alpha m}{\beta\sqrt{q^2}}$

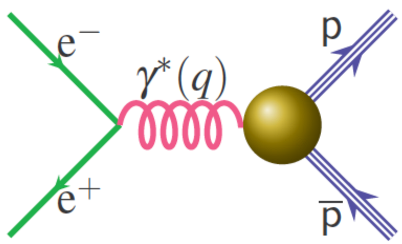
- Separation of $G_E(q^2)$ and $G_M(q^2)$ via angular structure
- $q^2 \to 4m^2$ at threshold:

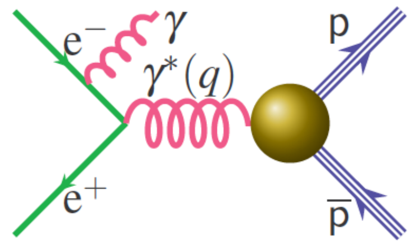
$$G_E(4m^2) = G_M(4m^2)$$

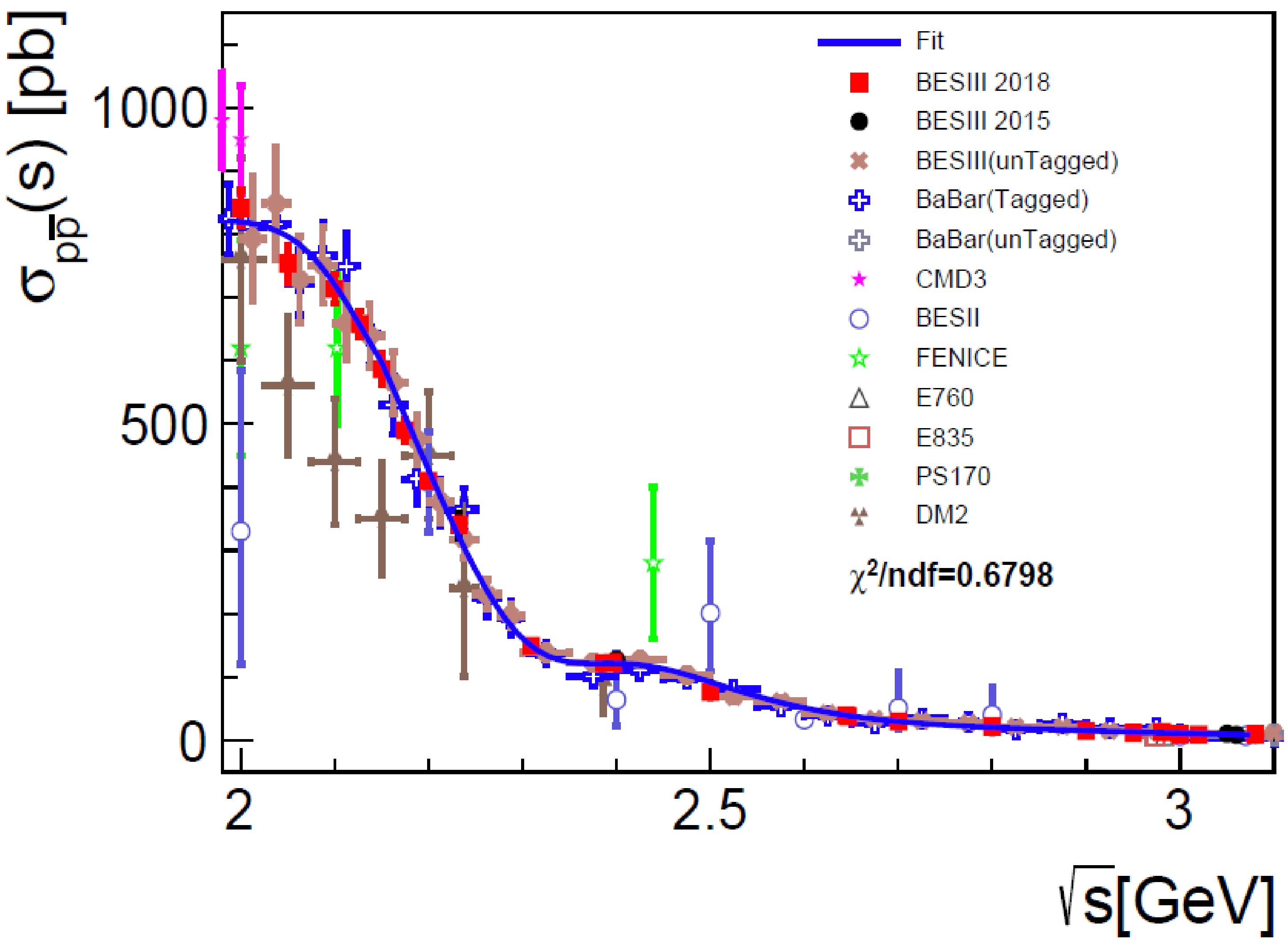
- $q^2 \to \infty$ limits from perturbative QCD:

$$F_1(q^2) \sim \frac{\alpha^2(q^2)}{q^4} \qquad F_1(q^2) \sim \frac{\alpha^2(q^2)}{q^4} \qquad \left|\frac{G_M^n(q^2)}{G_M^p(q^2)}\right|^2 \approx \left(\frac{q_d}{q_u}\right)^2 = \frac{1}{4}$$

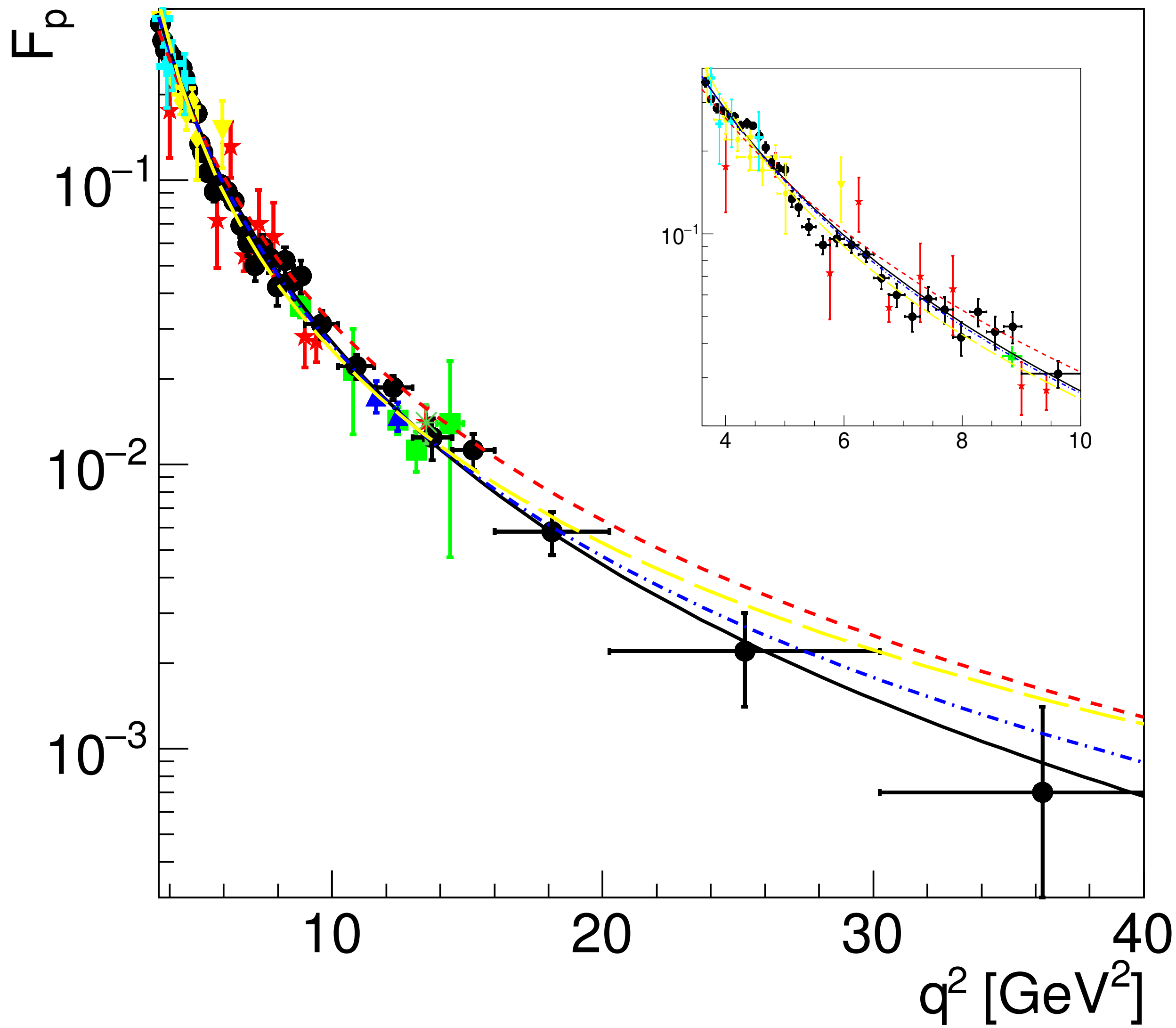
Time-like Form-Factors (BESIII)







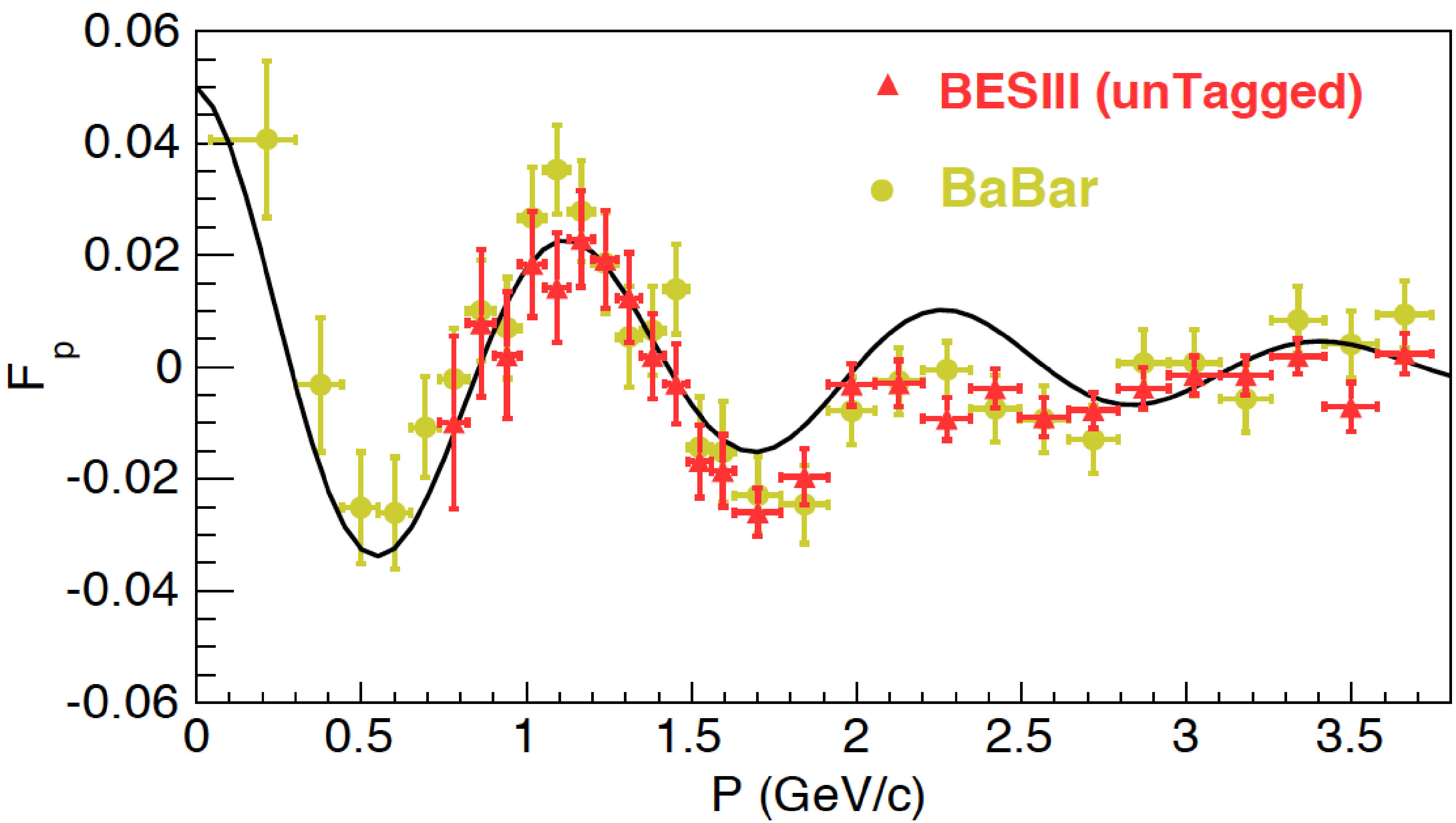
Proton Timelike Form-Factor (BaBar)



Efficive Form-Factor with assumption $G_E = G_M$: $\sigma = \frac{2\pi\alpha^2}{3\beta q^2}(2 + \frac{4m^2}{q^2})|F_p|^2$

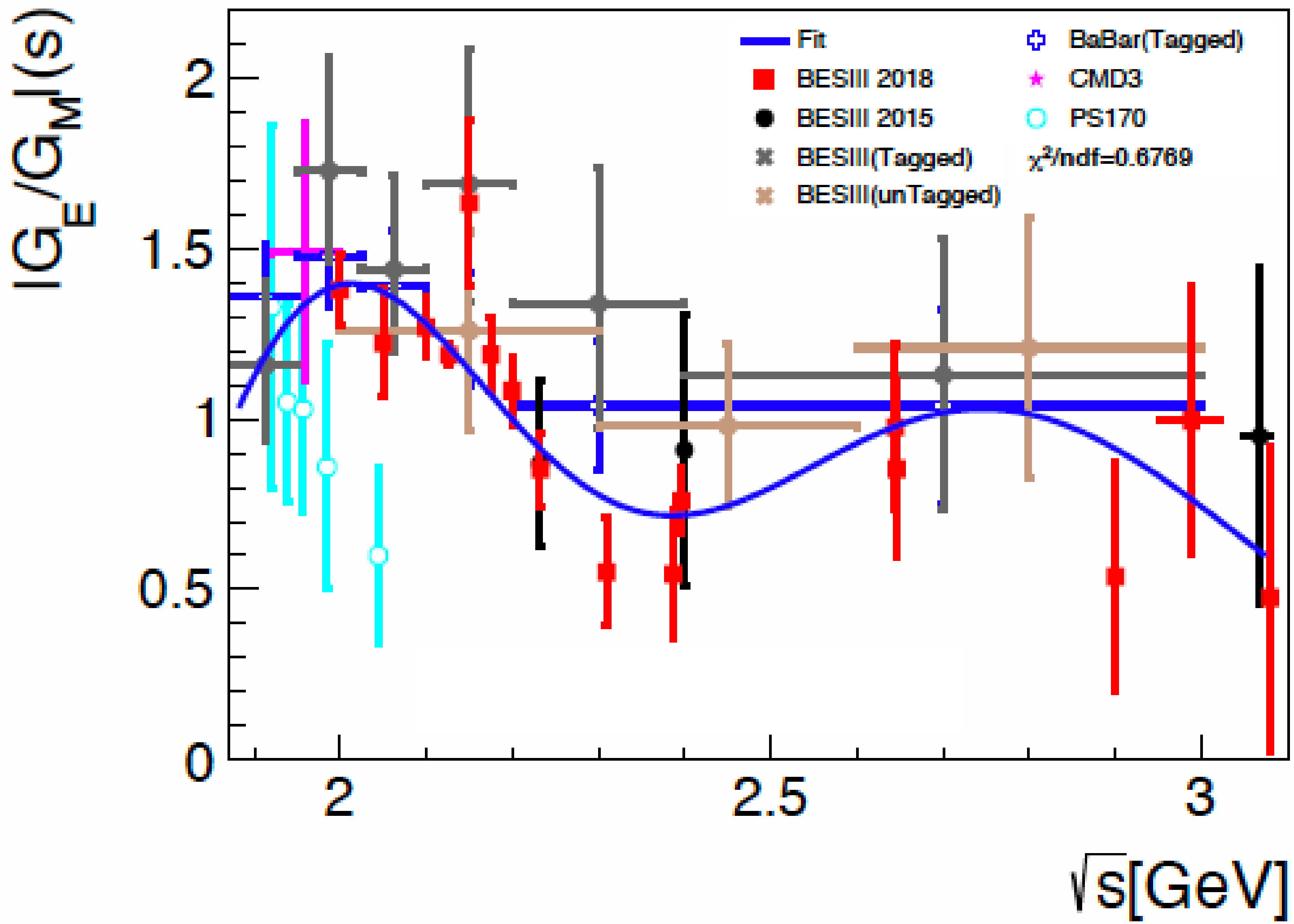
Low q^2 region

"Dipole-like" over-all fit subtracted:



- Interference structure
- Interpretation: Rescattering in $p\overline{p}$ final state, e.g. π, ρ, ω exchange

Separation $G_E - G_M$



- Separation via angular structure: $\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s} C \left(\left|G_M(q^2)\right|^2 (1+\cos^2\theta) + \frac{4m^2}{q^2} \left|G_E(q^2)\right|^2 \sin^2\theta \right)$

Conclusions

Hadron Physics

- An invaluable tool for a deep understanding of strong interaction and QCD
- Exciting experimental Results
  - New discoveries $\approx$ 1/year
  - XYZ and clear signatures of Exotic States
- Continuing Progress in Theory
  - Lattice QCD
  - Modelling of exotic states
- Running and new Facilities for Spectroscopy
  - LHC, e^+e^- Colliders
  - JLab 12
  - PANDA at FAIR
- Precission Physics
  - Determination of the Wave Function
  - Connection to Atomic Physics
- And still a lot to do ...