J/ψ polarization and η_c production puzzles: a possible way out

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PLAN OF THE TALK

1. **Introduction**
2. **Background theory. The puzzle**
3. **Revisited theory. A solution**
4. **Comparison with the data**
5. **Conclusions**

Introduction

Quarkonium physics = rich area to probe the theory of strong interactions both at small and large distances

EXPERIMENTAL OBSERVABLES

– Differential cross sections for charm and bottom families
– Differential cross sections for ground states and excited states
– Direct to indirect production ratios (feed-down from χ_c and χ_b)
– P-wave production ratios $\sigma(\chi_{c1})/\sigma(\chi_{c2}),\ \ \sigma(\chi_{b1})/\sigma(\chi_{b2})$
– Polarization

THEORETICAL APPROACHES

Several approaches are competing: Color-Singlet versus Color-Octet model;
both may be extended to NLO or tree-level NNLO*;
both may be incorporated with collinear or k_T-factorization.

Background theory

Quarkonium production is a two-step process.

Production of a heavy quark pair is a hard part, the energy scale is the meson transverse mass. Perturbative QCD methods are applicable. Solid theory.

$\Rightarrow$ hard scattering matrix element $\mathcal{M}\Big(gg \to Q\bar{Q}[^{2S+1}L_J^{\rm color}]\Big)$.

Formation of a bound state is a soft part, the scale is the binding energy.

– **If the $Q\bar{Q}$ state is a color singlet**, the formation probability reduces to a single parameter, the radial wave function $|\mathcal{R}_S(0)|^2$ or $|\mathcal{R}'_P(0)|^2$ usually known from decay widths (measured) or potential models (calculated).

– **If the $Q\bar{Q}$ state is a color octet**, the emission of extra gluons is required.

$\Rightarrow$ a set of nonperturbative matrix elements for every intermediate state $|^{2S+1}L_J\rangle$ and for every final state meson, $\Big\langle \mathcal{O}^{\rm meson}[^{2S+1}L_J^{\rm color}]\Big\rangle$.

Poor theory. Nonperturbative ME's are considered as fitting parameters.

Cross section formulae (Parton Model)

$$\sigma(pp \to \eta_c + X) = \sum_{n,S,L,J} \sigma_{|nSLJ\rangle} \left\langle \mathcal{O}^{\eta_c}\left[{}^{2S+1}L_J^{\,\rm color}\right] \right\rangle$$

where

$$d\sigma_{|nSLJ\rangle} = \int \frac{2\pi}{\hat{s}\,F} \left| \mathcal{M}\Big(gg \to Q\bar{Q}\left[{}^{2S+1}L_J^{\,\rm color}\right]\Big) \right|^2$$

$$\times \mathcal{F}_g(x_1, k_{1T}^2, \mu^2)\, \mathcal{F}_g(x_2, k_{2T}^2, \mu^2)\, dk_{1T}^2\, dk_{2T}^2\, dy_\eta\, \frac{d\phi_1}{2\pi}\, \frac{d\phi_2}{2\pi}$$

k_i, k_{iT}, x_i $=$ gluon momenta, transverse momenta and longitudinal fractions;
$\hat{s} = (k_1 + k_2)^2$ $=$ total invariant energy; $F = 2\,\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$ $=$ flux factor;
$\mathcal{F}_g(x_i, k_{iT}^2, \mu^2)$ $=$ gluon distribution functions; μ $=$ factorization scale;

$\mathcal{M}\Big(gg \to Q\bar{Q}\Big)$ $=$ hard scattering matrix element;

$\left\langle \mathcal{O}^{\eta_c}\left[{}^{2S+1}L_J^{\,\rm color}\right] \right\rangle$ $=$ nonperturbative matrix elements

Restrictions from Heavy Quark Spin Symmetry

In fact, follow from a simple idea that the transition amplitudes must be identical for 'forth' and 'back' processes. The numerical prefactor comes from averaging over initial spin degrees of freedom.

$$\begin{aligned}
\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{[1]}\right]\right\rangle &= \frac{1}{3}\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{[1]}\right]\right\rangle \\
\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{[8]}\right]\right\rangle &= \frac{1}{3}\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{[8]}\right]\right\rangle \\
\left\langle \mathcal{O}^{\eta_c}\left[{}^3S_1^{[8]}\right]\right\rangle &= \left\langle \mathcal{O}^{J/\psi}\left[{}^1S_0^{[8]}\right]\right\rangle \\
\left\langle \mathcal{O}^{\eta_c}\left[{}^1P_1^{[8]}\right]\right\rangle &= 3\left\langle \mathcal{O}^{J/\psi}\left[{}^3P_0^{[8]}\right]\right\rangle \\
\left\langle \mathcal{O}^{h_c}\left[{}^1P_1^{[1]}\right]\right\rangle &= 3\left\langle \mathcal{O}^{\chi_{c0}}\left[{}^3P_0^{[1]}\right]\right\rangle \\
\left\langle \mathcal{O}^{h_c}\left[{}^1S_0^{[8]}\right]\right\rangle &= 3\left\langle \mathcal{O}^{\chi_{c0}}\left[{}^3S_1^{[8]}\right]\right\rangle
\end{aligned}$$

The η_c and J/ψ production parameters are no longer independent.

Behavior of the different contributions

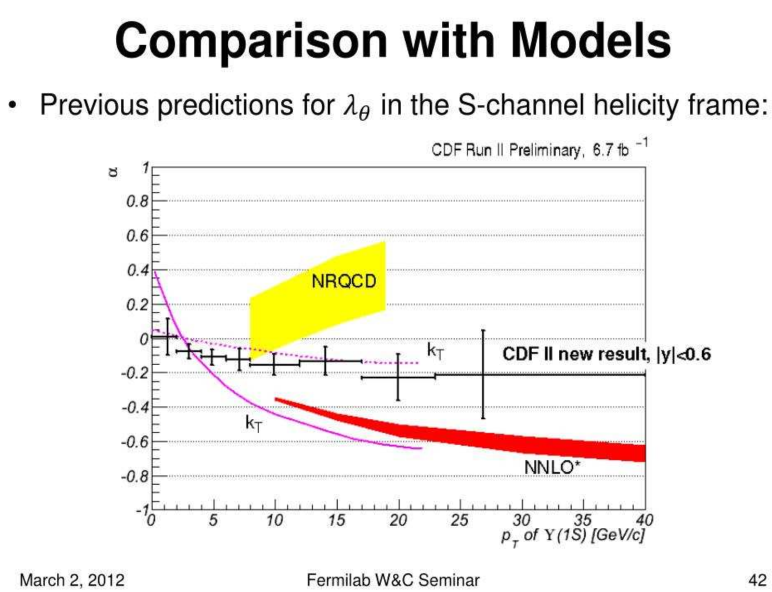
Color-Singlet Model

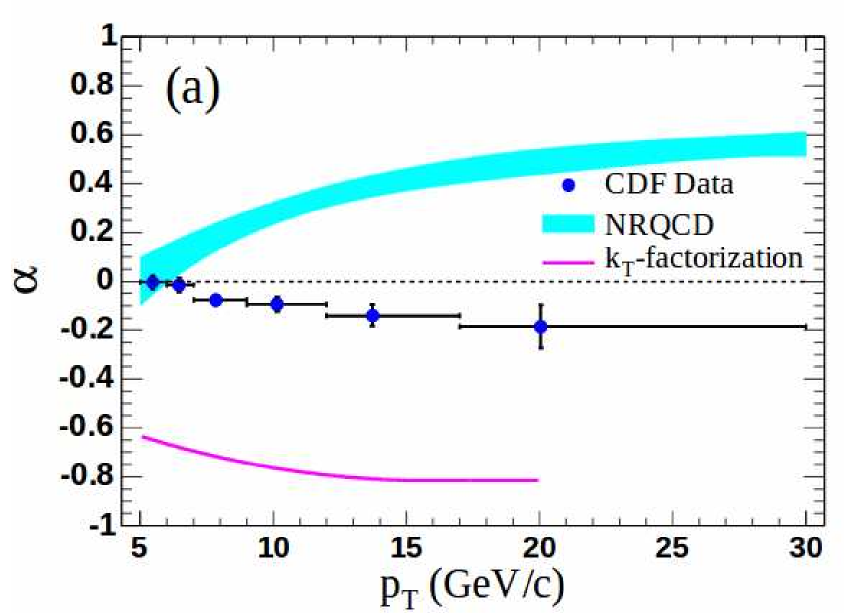
– Quarkonium production is completed already at the perturbative stage: $g+g \to J/\psi+g$. Incorrect (too steep) p_t behavior of the cross section: $d\sigma/dp_t \propto 1/p_t^8$.
– Way out: go to higher order corrections or introduce off-shell initial gluons (k_T-factorization). Then longitudinal J/ψ polarization comes from the initial gluon longitudinal polarization.

Color-Octet Model

– J/ψ production is dominated by gluon fragmentation $g+g \to g^*+g^*$ followed by $g^* \to ^3S_1^{[8]} \to J/\psi + X$ that gives correct p_t behavior $d\sigma/dp_t \propto 1/p_t^8$. At large p_t gluons are nearly massless and have strong transverse polarization further inherited by J/ψ. Polarization still increases with increasing p_t.

Great problem: J/ψ polarization...





$$\frac{d\Gamma(\psi\to\mu^+\mu^-)}{d\cos\theta\, d\phi} \propto 1 + \lambda_\theta\, \cos^2\theta + \lambda_\phi\, \sin^2\theta\, \cos 2\phi + \lambda_{\phi\theta}\, \sin 2\theta\, \cos\phi$$

$$\lambda^* = (\lambda_\theta + 3\lambda_\phi)/(1-\lambda_\phi) \qquad \alpha = \lambda_\theta$$

... or inconsistency with η_c production

To describe the polarization, one needs a large diluting contribution from ${}^1S_0^{[8]}$ spinless state. Via the HQSS relations, it makes large contribution from ${}^3S_1^{[8]}$ to η_c production. A factor of 5 above the data. The problem is called "challenging"

What is wrong in the conventional theory

– Assumes that soft gluons can change the color and other quantum numbers of a $Q\bar{Q}$ system without changing the energy-momentum. An obvious conflict with confinement that prohibits radiation of infinitely soft colored quanta. Need to consider not infinitely small energy-momentum exchange. Not only a kinematric correction!
– Long-distance matrix elements (LDMEs) for ${}^3S_1^{[8]} \to J/\psi$ transitions are treated as spin-blind numbers, a prediction of HQSS. In reality, HQSS is violated. Need to replace LDMEs with amplitudes showing well defined spin structure.

The key questions

Do we need intermediate color-octet states?

– **Yes.** Otherwise, we get wrong p_T dependence of the cros sections.

Can the octet-to-singlet transition be infinitely soft?

– **No.** That would be in contradiction with confinement.

$\Rightarrow$ Heavy Quark Effective Theory and Heavy Quark Spin Symmetry (HQET and HQSS) are not exact. The limit $k_g \to 0$ is unphysical.

Can we use Feynman rules for soft gluon emission?

– **No.** Long-wave gluons cannot resolve bound quarks as indiviual color charges.

$\Rightarrow$ All higher-order perturbative corrections are useless = misleading.

What is the correct theory?

– Probably, it is the classical multipole radiation formalism.

The only physical assumption is that the intermediate color-octet states are enough long-living.

Revisited soft emission theory

– **Hard**: use perturbative QCD to create a heavy quark pair $Q\bar{Q}$ in the hard gluon-gluon fusion subprocess.
– **Soft**: use multipole expansion for soft gluon radiation. Another perturbation theory where the small parameter is the relative quark velocity (or the size of the $Q\bar{Q}$ system over the gluon wavelength)

Both steps are combined into a single amplitude: $Q\bar{Q}$ spin density matrix is contracted with E1 transition amplitudes (same as for real χ_c decays).

Color-Electric Dipole transitions

$$\mathcal{A}\,(\chi_{c0}(p) \to J/\psi(p-k) + \gamma(k)) \;\propto k_\mu p^\mu \varepsilon^\nu_{(J/\psi)} \varepsilon^{(\gamma)}_\nu$$

$$\mathcal{A}\,(\chi_{c1}(p) \to J/\psi(p-k) + \gamma(k)) \;\propto \epsilon^{\mu\nu\alpha\beta}\, k_\mu\, \varepsilon^{(\chi_{c1})}_\nu\, \varepsilon^{(J/\psi)}_\alpha\, \varepsilon^{(\gamma)}_\beta$$

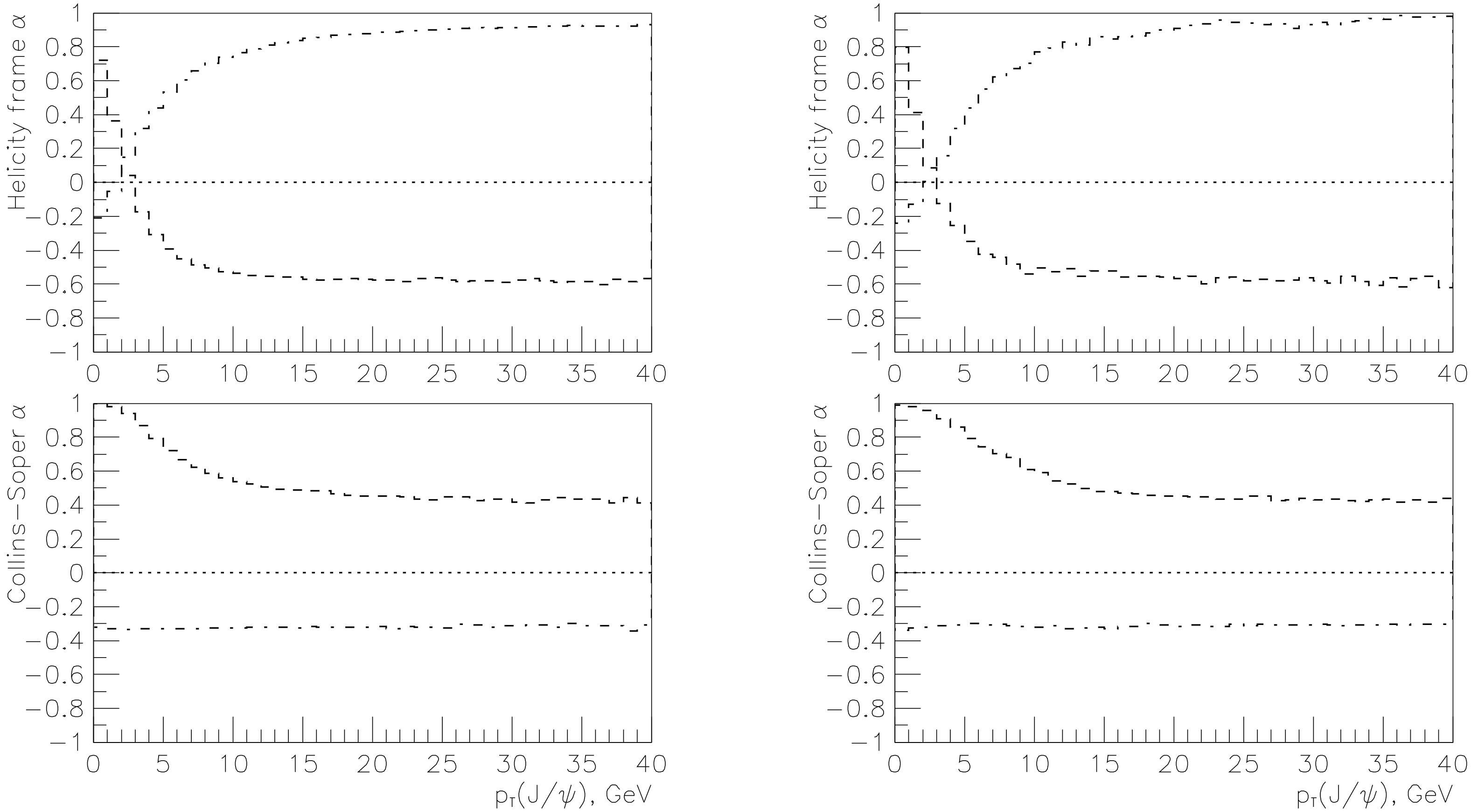
$$\mathcal{A}\,(\chi_{c2}(p) \to J/\psi(p-k) + \gamma(k)) \;\propto p^\mu\, \varepsilon^{\alpha\beta}_{(\chi_{c2})}\, \varepsilon^{(J/\psi)}_\alpha\, [k_\mu\, \varepsilon^{(\gamma)}_\beta - k_\beta\, \varepsilon^{(\gamma)}_\mu]$$

A.V.Batunin, S.R.Slabospitsky, Phys. Lett. B **188**, 269 (1987)
P.Cho, M.Wise, S.Trivedi, Phys. Rev. D **51**, R2039 (1995)

One or two subsequent transitions to convert a color octet into J/ψ:
$^3P_J^{[8]} \to J/\psi + g$ or $^3S_1^{[8]} \to {}^3P_J^{[8]} + g,\; {}^3P_J^{[8]} \to J/\psi + g,\; J = 0, 1, 2.$

J/ψ from ${}^3P_J^{[8]}$ polarization

k_T-factorization LO collinear factorization



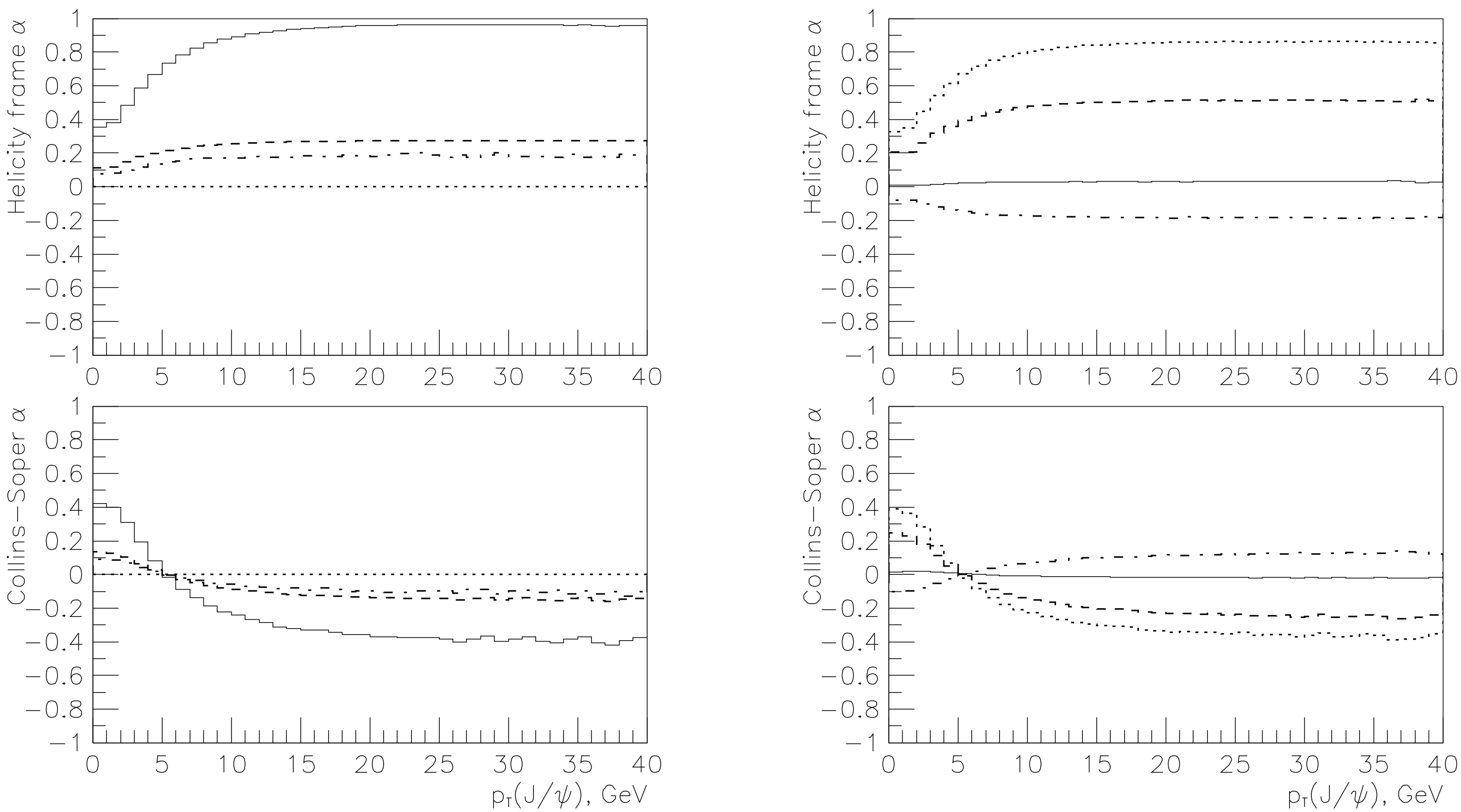
Dashed $= {}^3P_2$; dash-doted $= {}^3P_1$; doted $= {}^3P_0$

Approximate cancellation between ${}^3P_1^{[8]}$ and ${}^3P_2^{[8]}$ channels.

J/ψ from ${}^3S_1^{[8]}$ polarization

Pure ${}^3P_J^{[8]}$ states

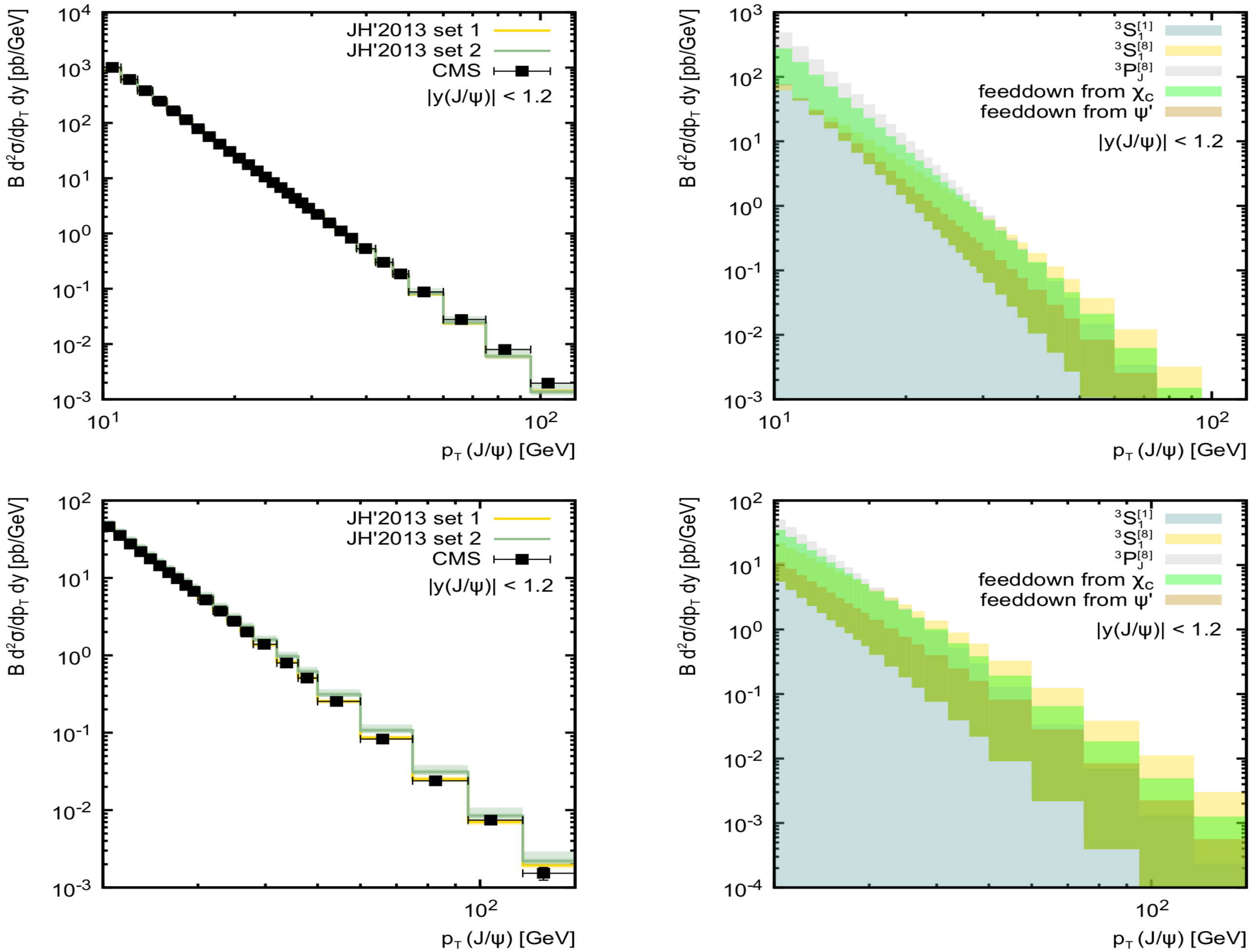
Interfering channels



Dash=${}^3P_2$; dash-dot=${}^3P_1$; dot=${}^3P_0$;
Solid $= {}^3S_1^{[8]}$ spin preserved.

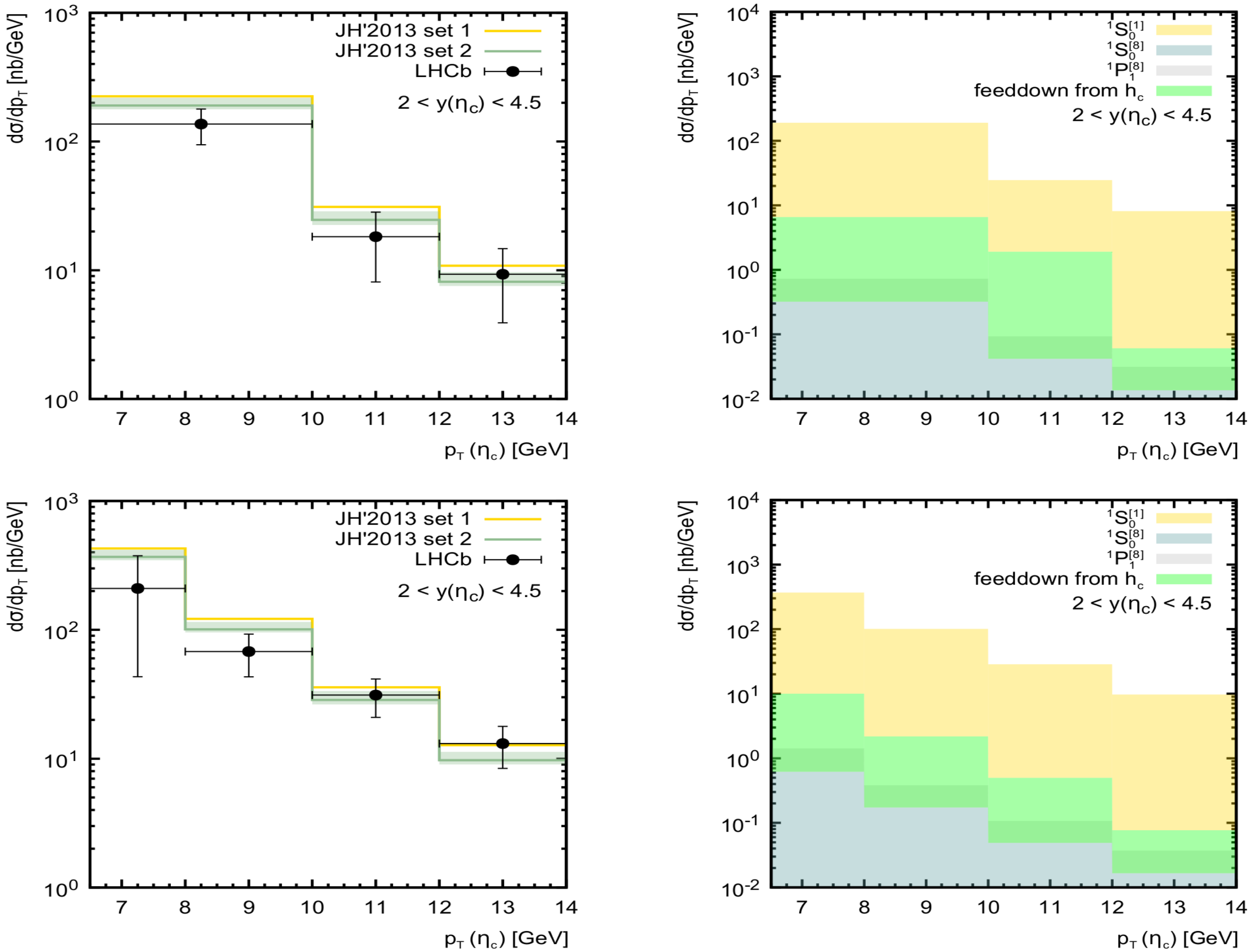
Solid=$\left|{}^3P_2^{[8]} +{}^3 P_2^{[8]} +{}^3 P_2^{[8]}\right|^2$
normalized to $\sqrt{2J+1}$

Prompt J/ψ production. Theory versus data



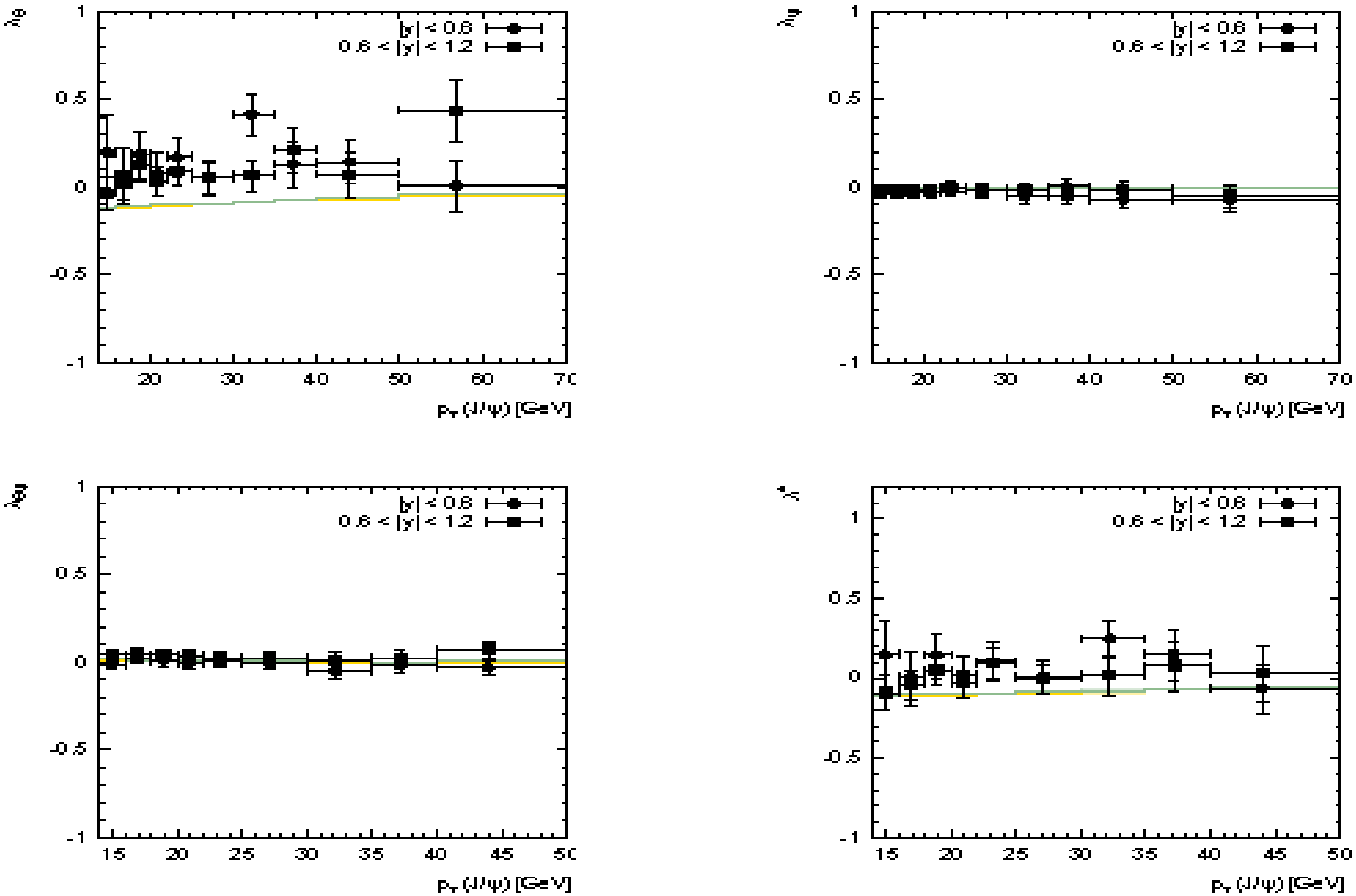
Data from CMS Collab., Phys. Rev. Lett. **114**, 191802 (2015); Phys. Lett. B **780**, 251 (2018)

Prompt η_c production. Theory versus data



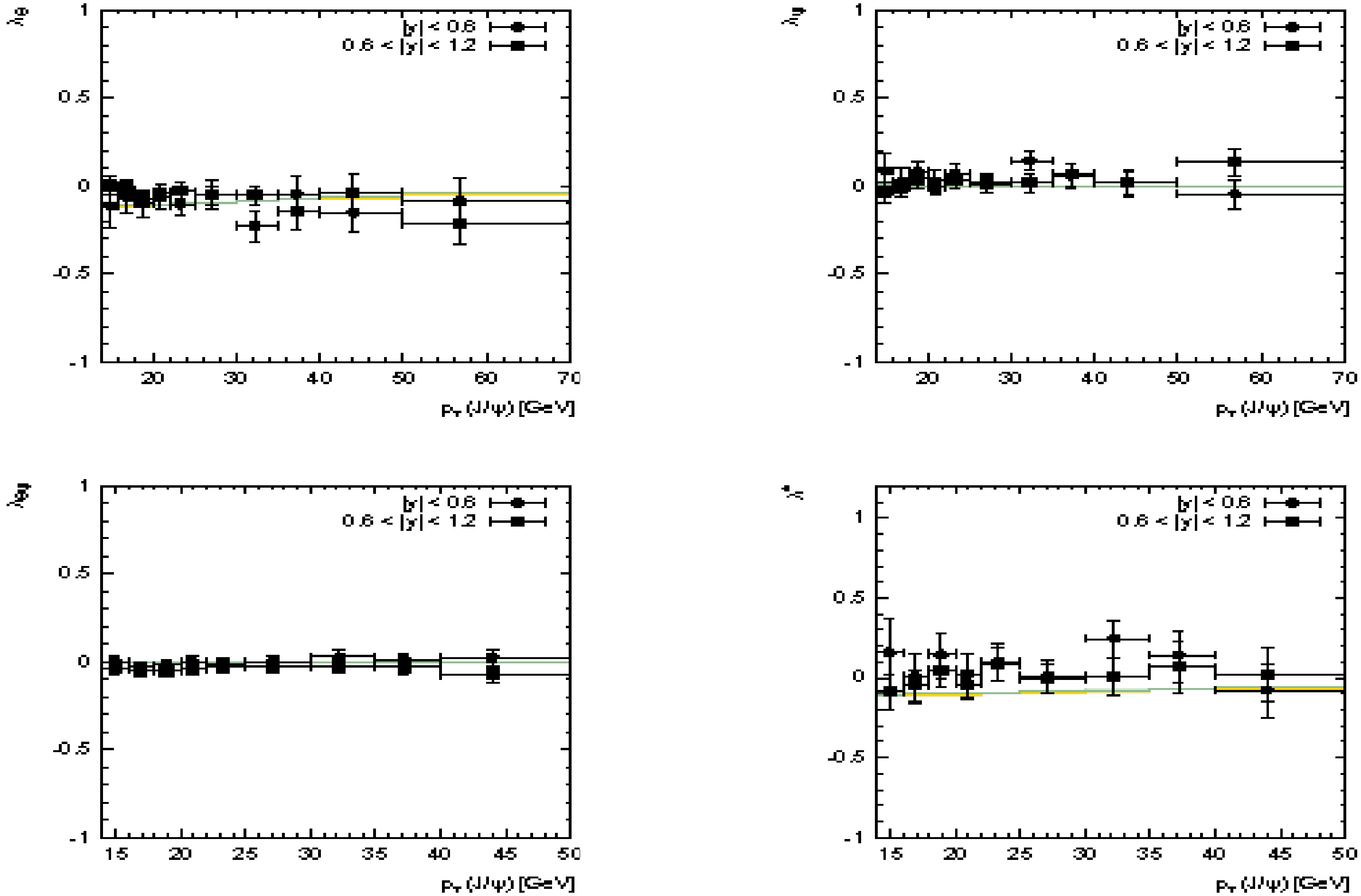
Data from LHCb Collab., Eur. Phys. J. C **75**, 311 (2015)

J/ψ polarization: λ_θ, λ_ϕ, $\lambda_{\theta\phi}$, λ^*. Helicity frame



Data from CMS Collab., Phys. Lett. B **727**, 381 (2013)

J/ψ polarization: λ_θ, λ_ϕ, $\lambda_{\theta\phi}$, λ^*. Collins-Soper frame



Data from CMS Collab., Phys. Lett. B **727**, 381 (2013)

Nonperturbative matrix elements for J/ψ and η_c production

	JH set 1	JH set 2	Kniehl *et al.* [1]	Gong *et al.* [2]
$\langle \mathcal{O}^{J/\psi}[^3S_1^{[1]}]\rangle$ /GeV^3	1.16	1.16	1.32	1.16
$\langle \mathcal{O}^{J/\psi}[^1S_0^{[8]}]\rangle$ /GeV^3	0.0	0.0	0.304	0.097
$\langle \mathcal{O}^{J/\psi}[^3S_1^{[8]}]\rangle$ /GeV^3	4.2 $\cdot 10^{-4}$	1.6$\cdot 10^{-3}$	1.7$\cdot 10^{-3}$	-4.6$\cdot 10^{-3}$
$\langle \mathcal{O}^{J/\psi}[^3P_0^{[8]}]\rangle$ /GeV^5	0.023	0.024	-0.0091	-0.0214
$\langle \mathcal{O}^{\eta_c}[^1S_0^{[1]}]\rangle$ /GeV^3	0.39	0.39	0.44	0.39
$\langle \mathcal{O}^{\eta_c}[^3S_1^{[8]}]\rangle$ /GeV^3	0.0	0.0	0.304	0.097
$\langle \mathcal{O}^{\eta_c}[^1S_0^{[8]}]\rangle$ /GeV^3	1.4 $\cdot 10^{-4}$	5.3 $\cdot 10^{-4}$	5.6$\cdot 10^{-4}$	-1.5$\cdot 10^{-3}$
$\langle \mathcal{O}^{\eta_c}[^1P_1^{[8]}]\rangle$ /GeV^5	0.069	0.072	-0.027	-0.064

[1] M. Butenshoen, B.A. Kniehl, Phys. Rev. D **84**, 051501(R) (2011)
[2] B.Gong, L.-P.Wan, J.-X.Wang, H.-F.Zhang, Phys. Rev. Lett. **110**, 042002 (2013)

Conclusions

An adequately formulated soft gluon radiation solves all problems:

– J/ψ and η_c differential cross sections agree with the data;
– J/ψ polarization agrees with the data;
– Heavy Quark Symmetry relations are strictly observed

$\Rightarrow$ A fully consistent theoretical approach is found and presented

Thank you!