Nuclear PDFs and heavy-flavour production

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"Heavy-quark hadroproduction from collider to astroparticle physics"
Mainz Institute for Theoretical Physics
Mainz, Germany, 30/09-11/10/2019

Unrealistic Plan of this Talk

- Nuclear PDFs: Theoretical Framework
- Current status of nuclear PDFs in a nutshell
- LHC p-Pb heavy quark data and gluon shadowing
- Summary & Conclusions

Nuclear PDFs: Theoretical Framework

Theoretical Framework (pQCD formalism)

Factorization Theorems:

- Provide (field theoretical) **definitions** of the **universal** PDFs
- Make the formalism **predictive**!
- Make a statement about the **error** of the factorization formula

PDFs and predictions for **observables+uncertainties** **refer to this standard pQCD framework**

Need a solid understanding of the standard framework!

- For pp and ep collisions there a **rigorous factorization proofs**
- For pA and AA factorization is a **working assumption** to be tested phenomenologically

  There might be breaking of QCD factorization, deviations from DGLAP evolution, other nuclear matter effects to be included

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From Nucleons to Nuclei

- Starting point: global analysis framework for free nucleons
- Make sure it can be applied to the case of nuclear targets (A,Z)
  - Variable $0 < x_N < A$
  - Evolution equations
  - Sum rules
  - Observables
- Apart from validity of factorisation which is a working assumption and to be verified phenomenologically

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Formalism carries over (Backup slides)

Predictive Power

Universality: same PDFs/FFs enter different processes:

- DIS: $$F_2^A(x,Q^2) = \sum_i \left[f_i^A \otimes C_{2,i}\right](x,Q^2)$$
- DY: $$\sigma_{A+B\to\ell^+ + \ell^- + X} = \sum_{i,j} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j\to\ell^+ + \ell^- + X}$$
- A+B -> H + X: $$\sigma_{A+B\to H+X} = \sum_{i,j,k} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j\to k+X} \otimes D_k^H$$
- Predictions for unexplored kinematic regions and for your favorite new physics process

Global analysis of nuclear PDFs

Same approach as for proton PDF determinations

1. Boundary conditions:
   Parameterize x-dependence of PDFs at initial scale Q_0

$$f(x,Q_0)=A_0 x^{A_1}(1-x)^{A_2} P(x;A_3,\dots);\ f=u_v,d_v,g,\bar{u},\bar{d},s,\bar{s}$$

2. Evolve from Q_0 to Q solving the DGLAP evolution equations: $f(x,Q)$

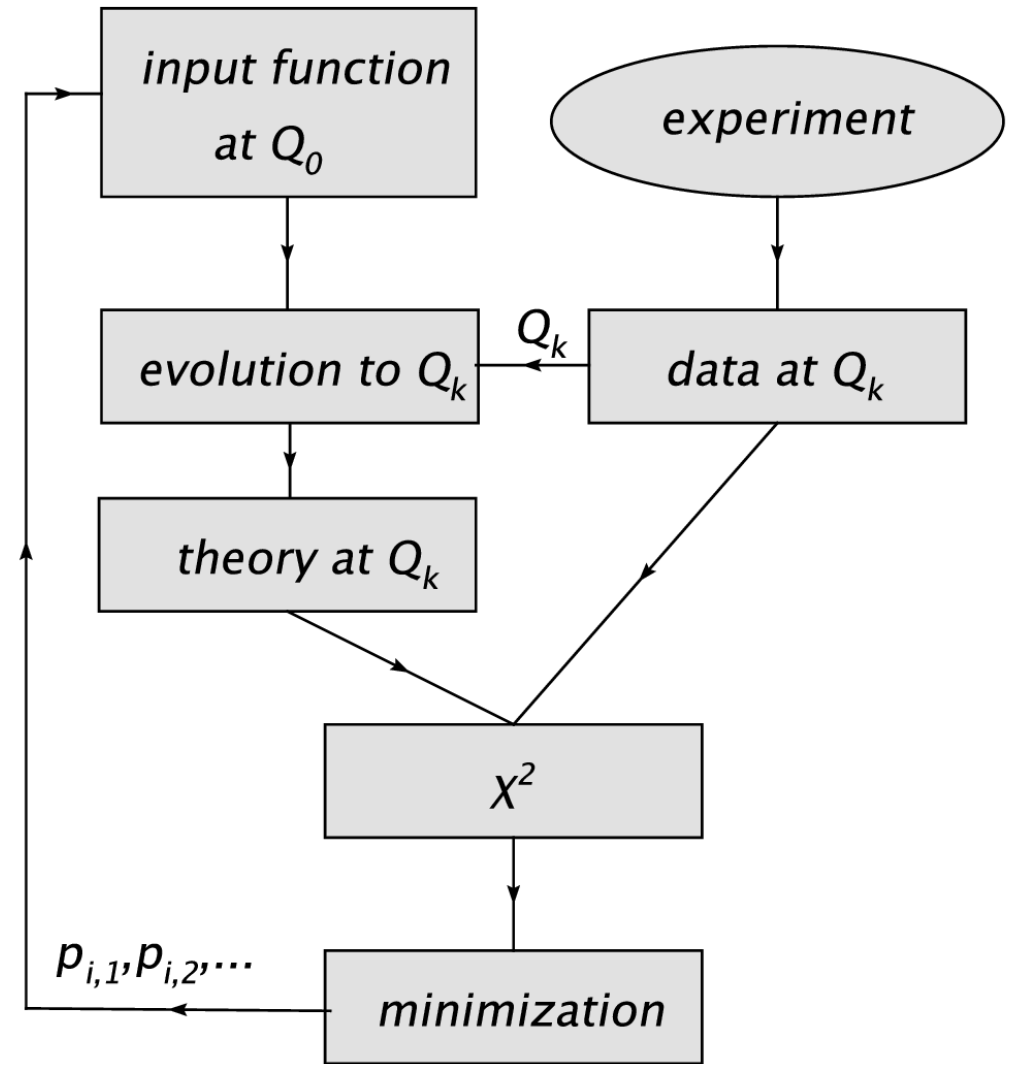
3. Define suitable χ^2 function and minimize w.r.t. fit parameters

$$\chi^2_{global}[A_i]=\sum_n w_n \chi^2_n;\ \chi^2_n=\sum_I \left(\frac{D_{nI}-T_{nI}}{\sigma_{nI}}\right)^2$$

Sum over experiments

Sum over data points

weights: default=1, allows to emphasize certain data sets



Sum rules fix 3 fit parameters (for each A)

- **Number sum rules** – connect partons to quarks from SU(3) flavour symmetry of hadrons; proton (uud), neutron (udd). For protons:

For all scales:

$$\int_0^1 dx[\underbrace{f_u(x) - f_{\bar{u}}(x)}_{u-\text{valence distr.}}] = 2 \qquad \int_0^1 dx[\underbrace{f_d(x) - f_{\bar{d}}(x)}_{d-\text{valence distr.}}] = 1$$

$$\int_0^1 dx[f_s(x) - f_{\bar{s}}(x)] = \int_0^1 dx[f_c(x) - f_{\bar{c}}(x)] = 0$$

- **Momentum sum rule** – momentum conservation connecting all flavours

For all scales:

$$\sum_{i=q,\bar{q},g} \int_0^1 dx\, x f_i(x) = 1$$

Momentum carried by up and down quarks is only around half of the total proton momentum the rest of the momentum is carried by gluons and small amount by sea quarks. In case of CT14NLO PDFs ($\mu = 1.3$ GeV):

At 1.3 GeV:

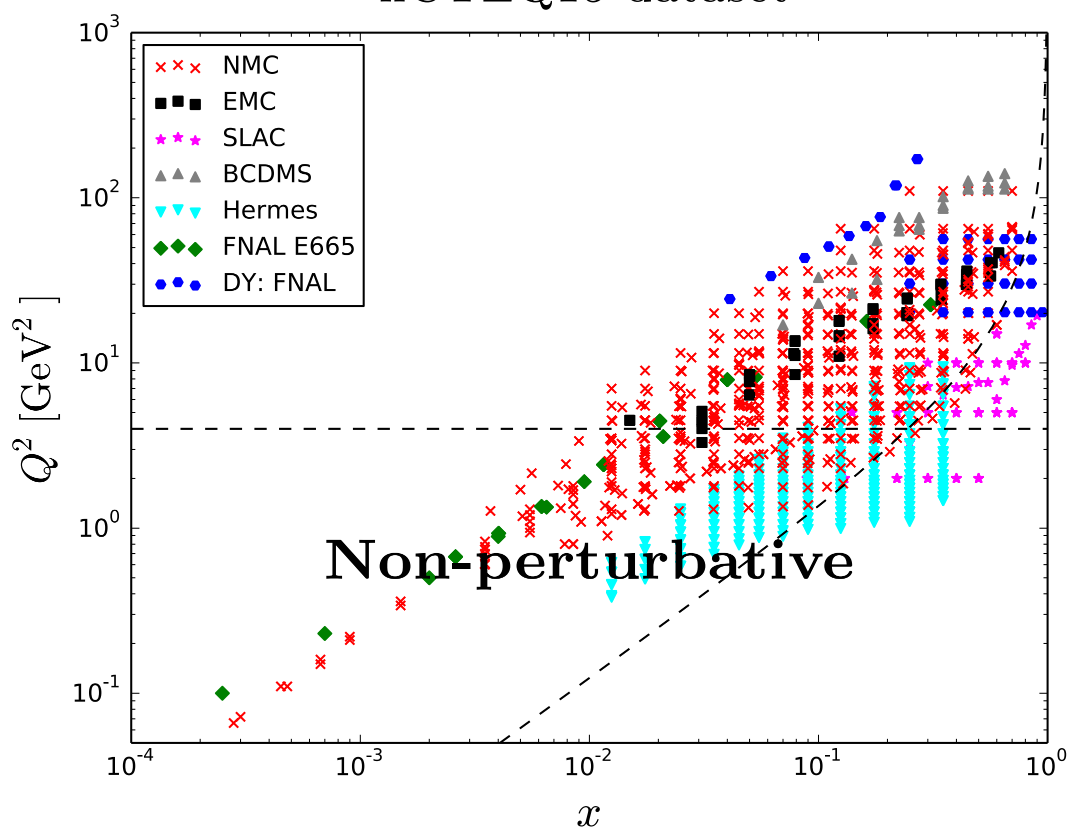
$$\int_0^1 dx\, x[f_u(x) + f_d(x)] \simeq 0.51$$

$$\int_0^1 dx\, x f_g(x) \simeq 0.40$$

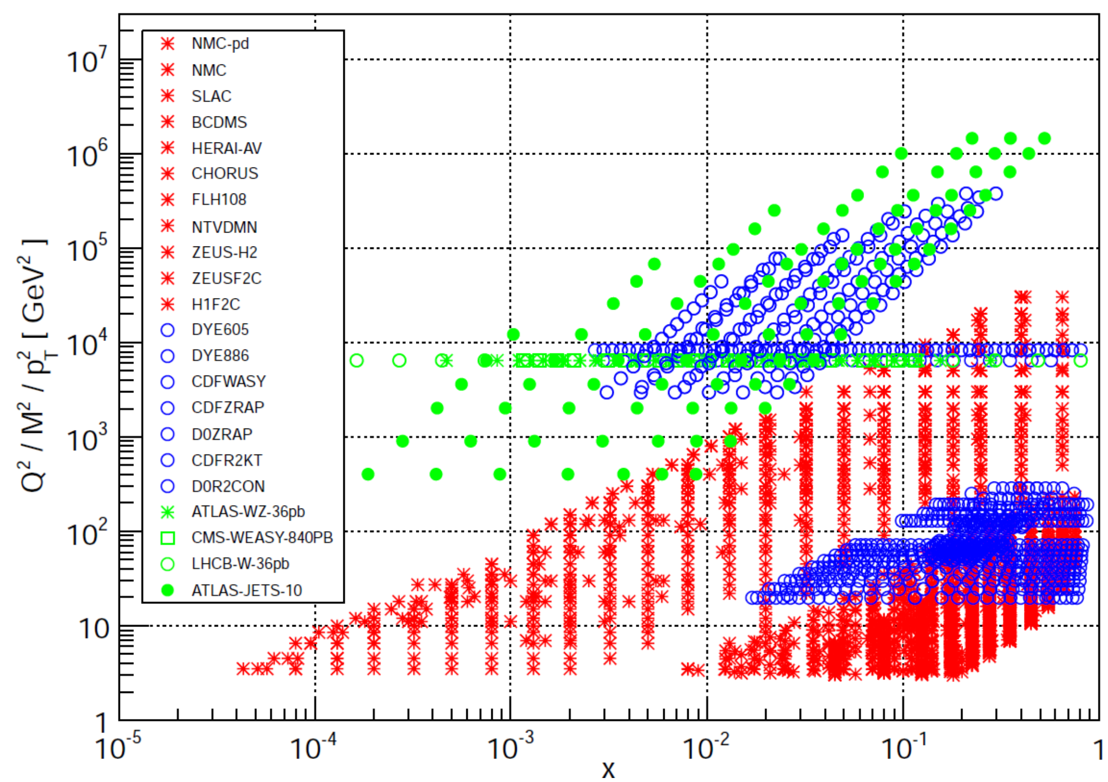
Main differences between proton and nuclear PDFs

- Theoretical status of factorization
- Parametrization: more parameters to model A-dependence
- Much less data constraints, much(!) smaller kinematic coverage

nCTEQ15 dataset



NNPDF2.3 dataset



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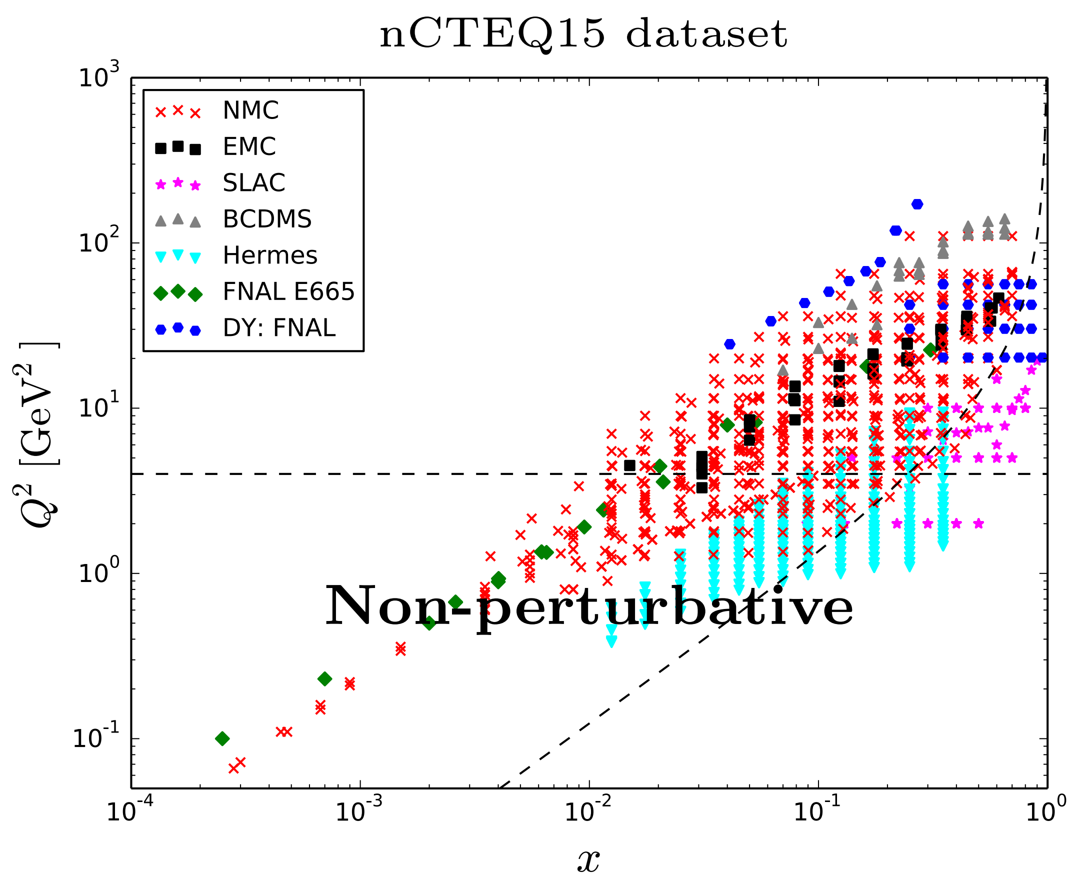
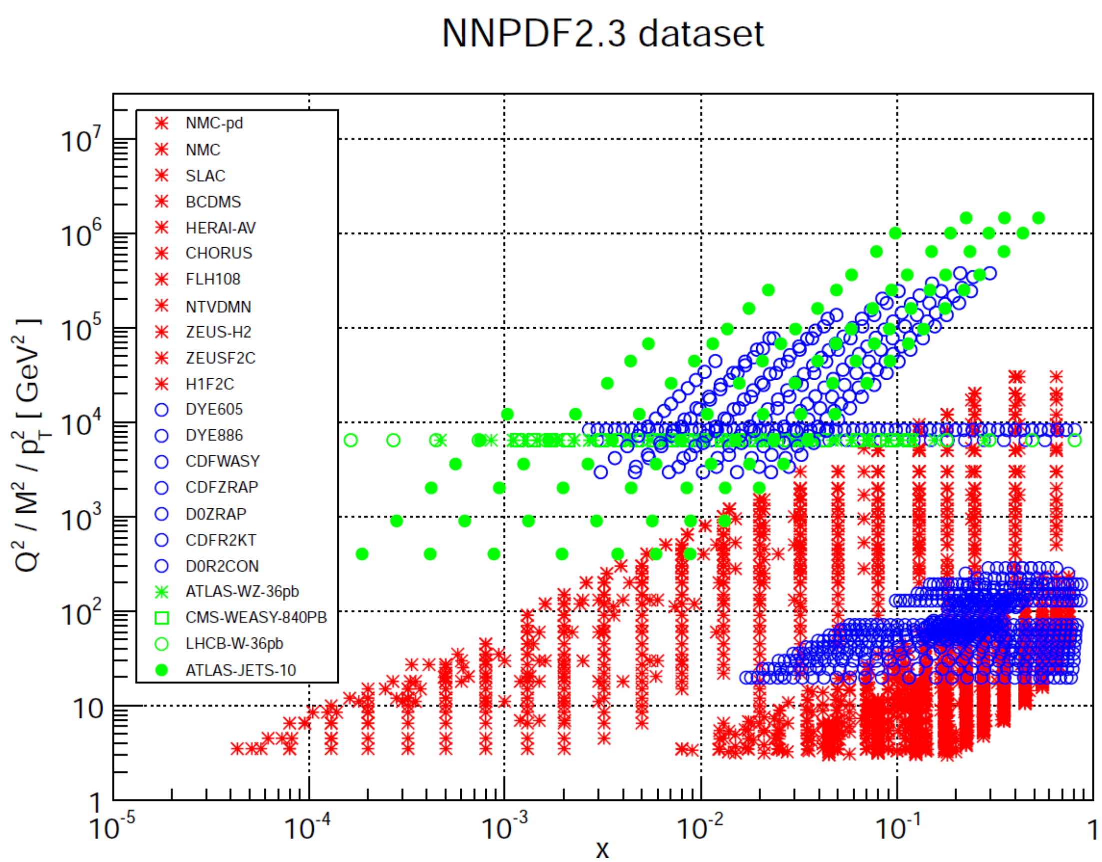




- Less data constraints → more **assumptions** about input PDFs
- Assumptions "hide" uncertainties!

Current status of nuclear PDFs
in a nutshell

Current nPDFs

	nNNPDF1.0 EPJC79(2019)471	EPPS16 EPJC77(2017)163	nCTEQ15 PRD93(2016)085037	KA15 PRD93(2016)014036	DSSZ12 PRD85(2012)074028	EPS09 JHEP0904(2009)065
lA DIS	✔	✔	✔	✔	✔	✔
DY in p+A		✔	✔	✔	✔	✔
RHIC π d+Au		✔	✔		✔	✔
νA DIS		✔	Standalone		✔	
DY in π+A		✔				
LHC p+Pb dijets		✔				
LHC p+Pb W,Z		✔	Soon			

Order in α_s	NNLO	NLO	NLO	NNLO	NLO	NLO
Q-cut in DIS	**1.87 GeV**	1.3 GeV	**2 GeV**	1 GeV	1 GeV	1.3 GeV
W-cut	**3.53 GeV**	-	**3.5 GeV**	-	-	-
Data points	451	1811	708	1479	1579	929
Free parameters	Neural Net	20	16	16	25	15
Error tolerance	MC replica	52	35	N.N.	30	50
Proton baseline	NNPDF3.1	CT14NLO	~CTEQ6.1	JR09	MSTW08	CTEQ6.1
Mass scheme	FONLL-B	GM-VFNS	GM-VFNS	ZM-VFNS	GM-VFNS	ZM-VFNS
Flavour sep.	-	val.+sea	valence	-	-	-

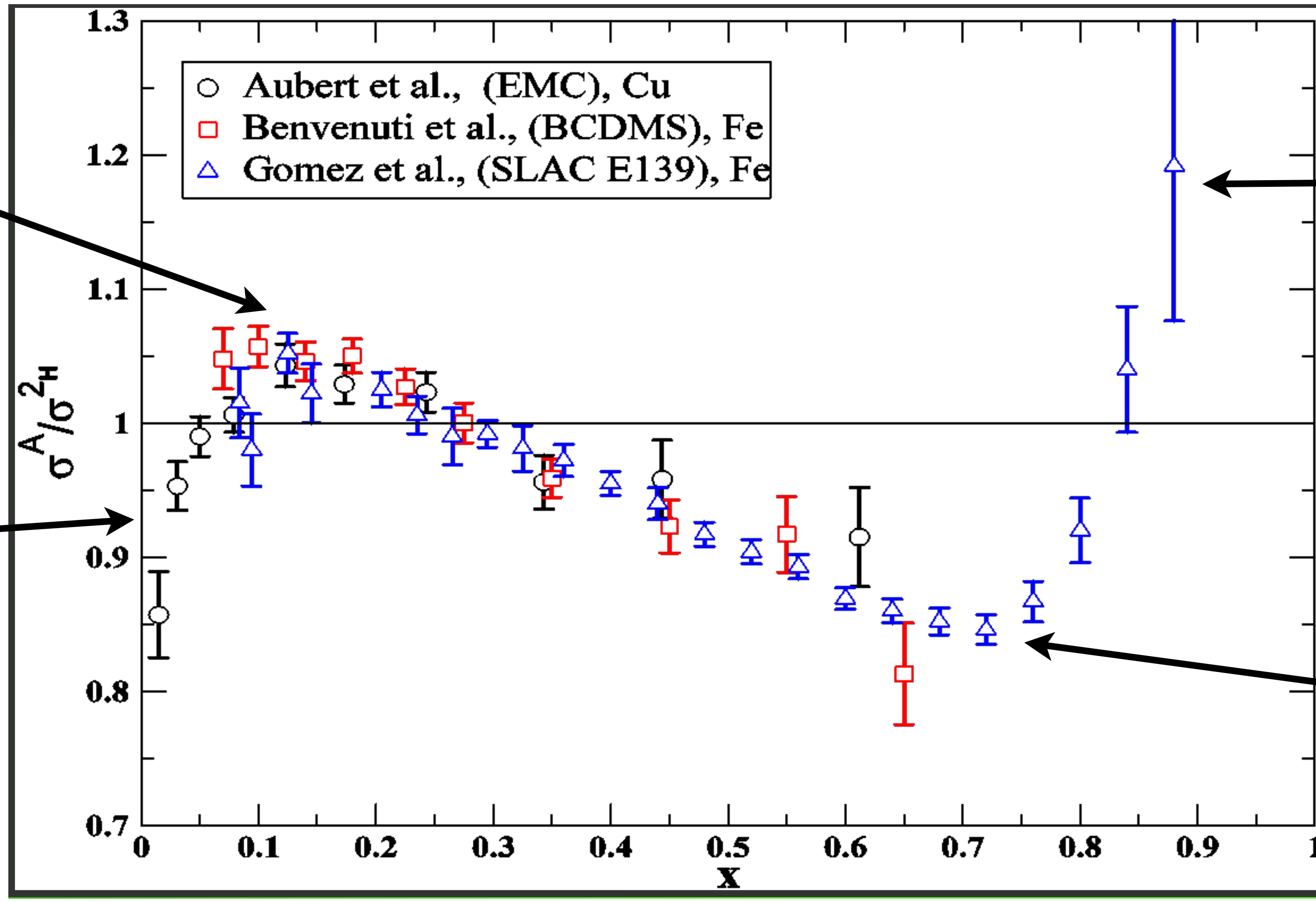
Main differences between different nPDF set

- **Processes included in the global analysis**
  - More or less conservative
- **(DIS-)cuts imposed**
  - More or less conservative
- **Parametrization**
  - Multiplicative nuclear correction factor: EPPS16, DSSZ12,…
  - Native nuclear PDFs (same x-dep. as proton PDFs): nCTEQ15
  - Neutral Network: nNNPDF1.0

Nuclear modifications of DIS structure functions

$$F_2^A(x) \neq Z F_2^p(x) + N F_2^n(x)$$

Anti-shadowing enhancement



Rise due to Fermi motion

Shadowing suppression at small x

EMC effect

Can we translate these modifications into universal nuclear PDFs?

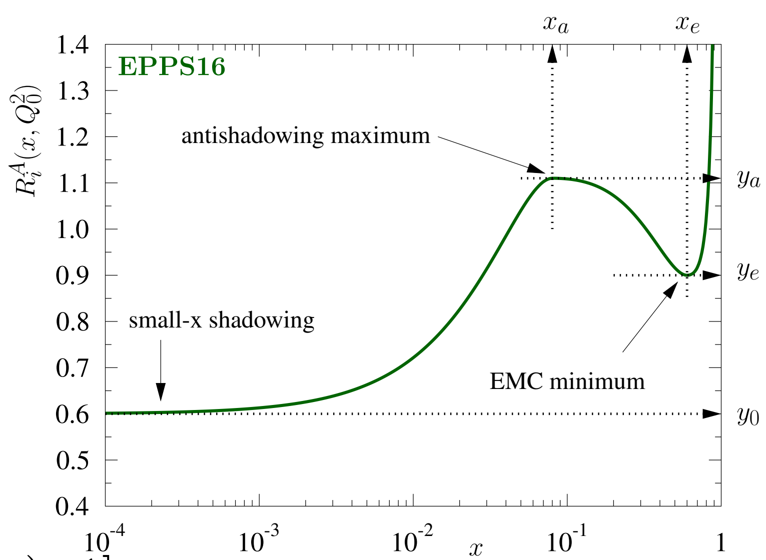
EPPS'16 framework

- NLO PDFs with errors (Hessian method, $\Delta\chi^2 = 52$)

- Parametrization (x_N<1, Q_0=1.3 GeV, i=u_v,d_v,ubar,dbar,s,g)

$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A, Z)\, f_i(x_N, \mu_0),$$

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \le x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \le x \le x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le 1 \end{cases}$$



A-dependence of fit parameters: $y_i(A) = y_i(A_{\rm ref}) \left(\frac{A}{A_{\rm ref}}\right)^{\gamma_i[y_i(A_{\rm ref})-1]}$

- CT14NLO free proton baseline, D (A=2) taken as free

- Data: lA DIS, DY, nu-A DIS, π^0@RHIC, LHC:dijets, W/Z

nCTEQ'15 framework

PRD93(2016)085037

- Functional form of the bound proton PDF same as for the free proton (`CTEQ6M`, x restricted to $0 < x < 1$)

$$xf_i^{p/A}(x, Q_0) = c_0 x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5}, \qquad i = u_v, d_v, g, \ldots$$
$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1}(1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}$$

- A-dependent fit parameters (reduces to free proton for $A = 1$)

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right), \quad k = \{1, \ldots, 5\}$$

- PDFs for nucleus (A, Z)

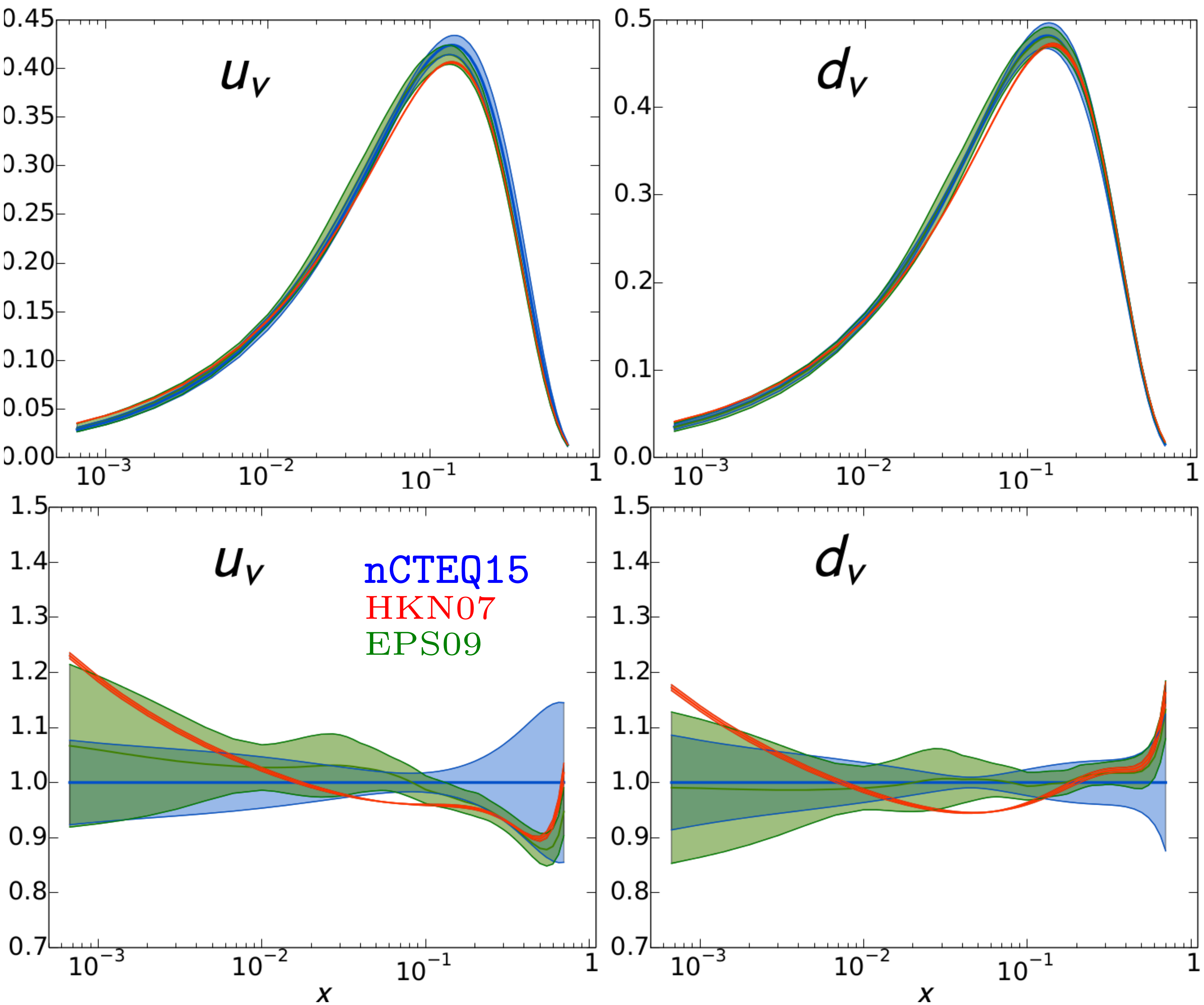
$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

(bound neutron PDF $f_i^{n/A}$ by isospin symmetry)

Valence distributions

Full lead nucleus distribution:

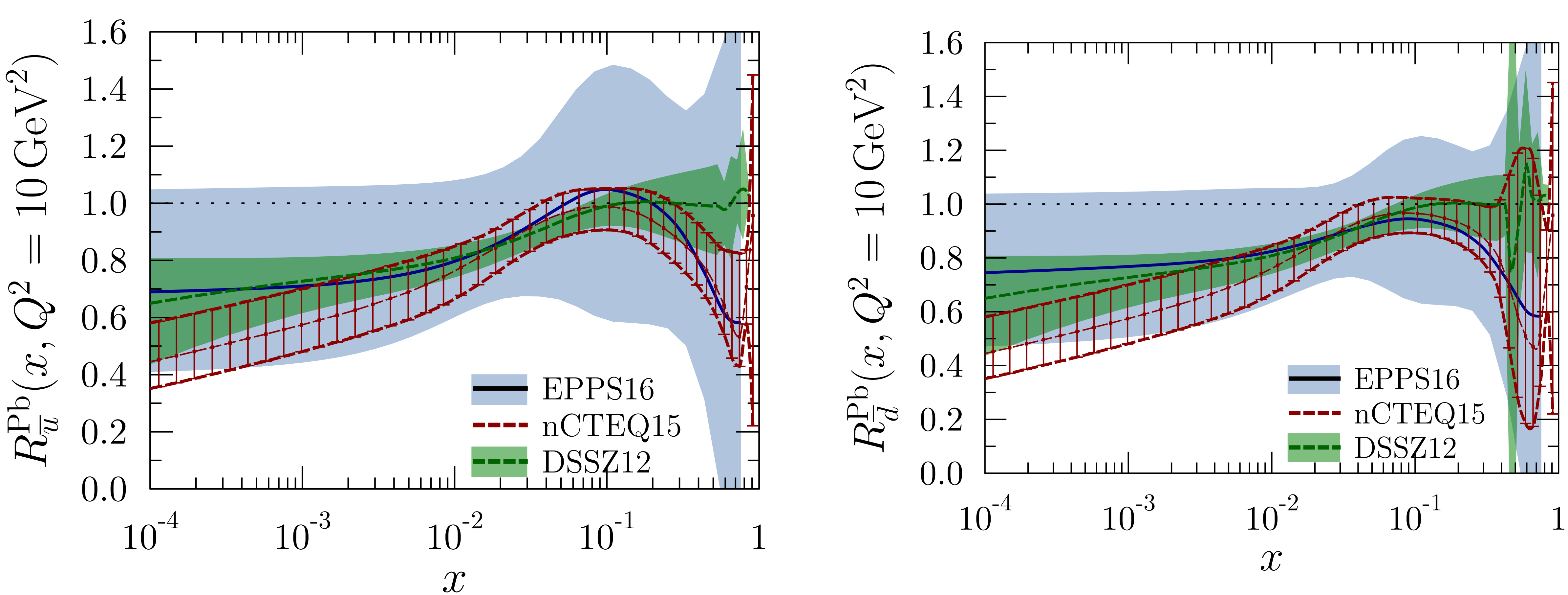
$$f^{Pb} = \frac{82}{208} f^{p/Pb} + \frac{208 - 82}{208} f^{n/Pb}$$



Nuclear modifications for the light sea

$$R_i^A(x,Q^2) \equiv f_i^{\text{proton in nucleus }A}(x,Q^2)/f_i^{\text{free proton}}(x,Q^2)$$

90% CL

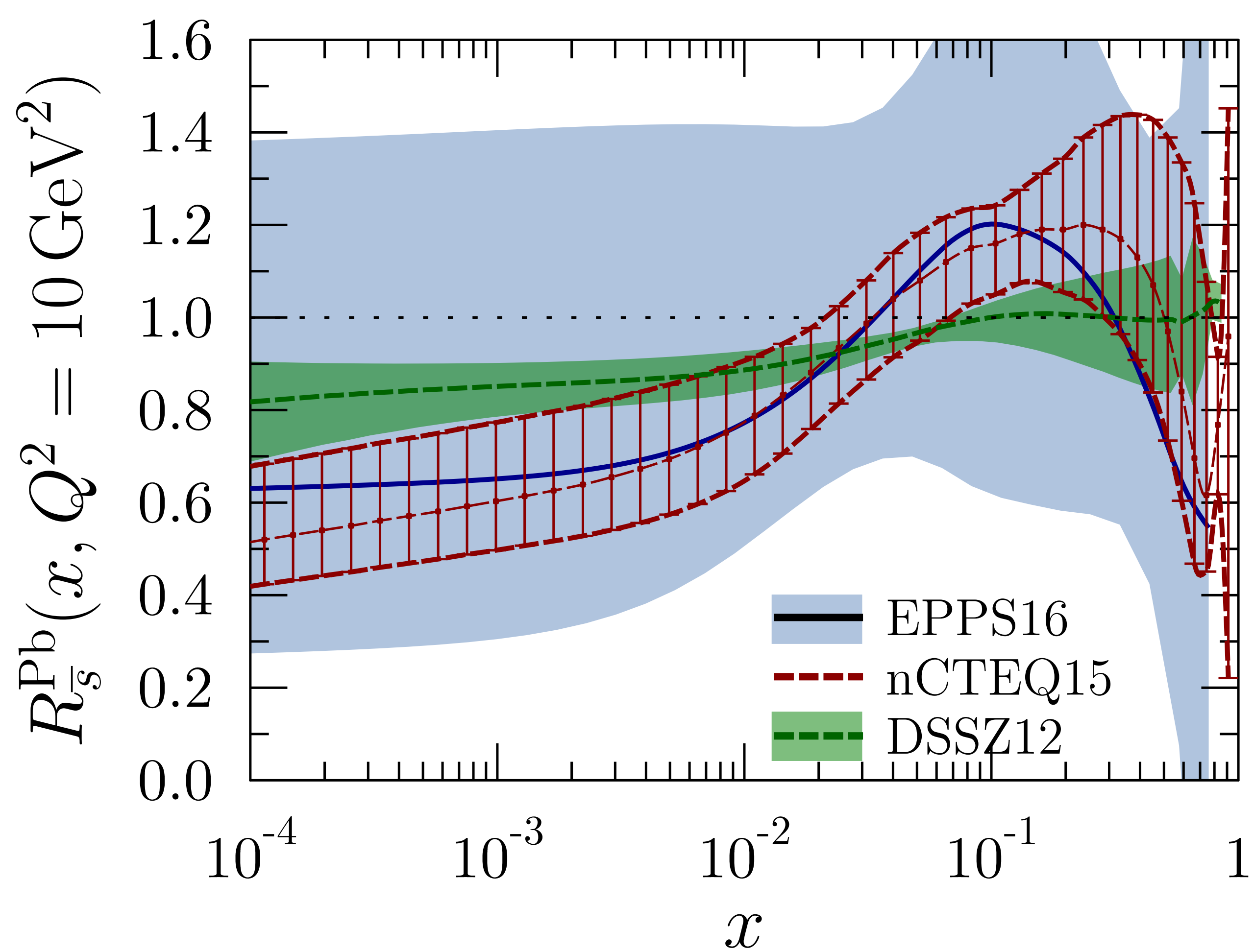


- In **EPPS16** ubar, dbar and sbar **independently** parameterized
- Therefore **EPPS16** uncertainties larger but less biased

Nuclear modifications for the strange sea

$$R_i^A(x,Q^2) \equiv f_i^{\text{proton in nucleus } A}(x,Q^2)/f_i^{\text{free proton}}(x,Q^2)$$

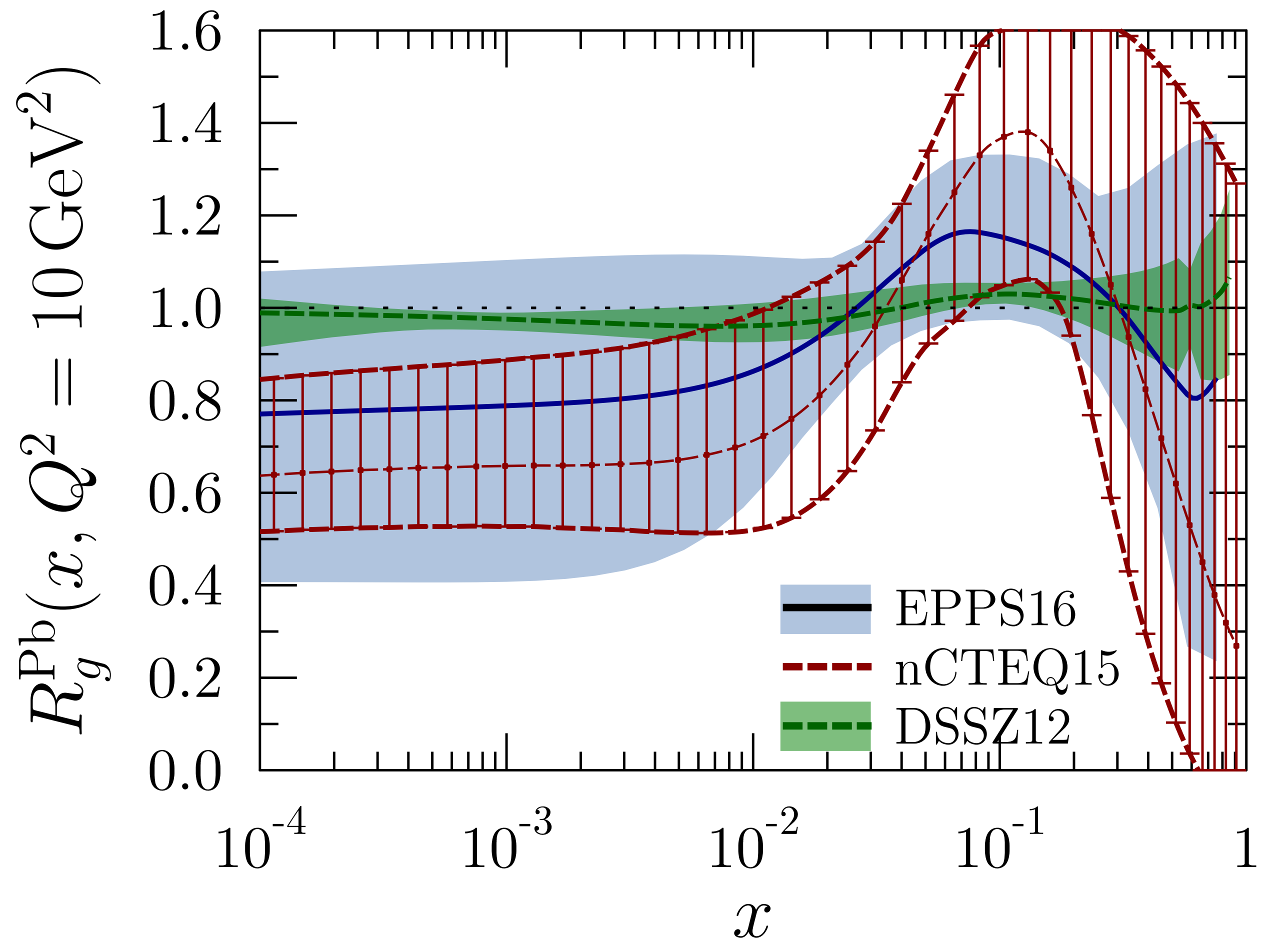
90% CL



- In **EPPS16** ubar, dbar and sbar **independently** parameterized
- In **nCTEQ15** sbar related to ubar and dbar: **sbar(x,Q$_0$)=κ(A)/2*(ubar+dbar)**
  No free parameter in the fit and hence uncertainty band **artificially** small
- **EPPS16** uncertainties large since only weak constraints from **LHC W,Z data**

Nuclear modifications to the gluon distribution

$$R_i^A(x,Q^2) \equiv f_i^{\text{proton in nucleus }A}(x,Q^2)/f_i^{\text{free proton}}(x,Q^2)$$



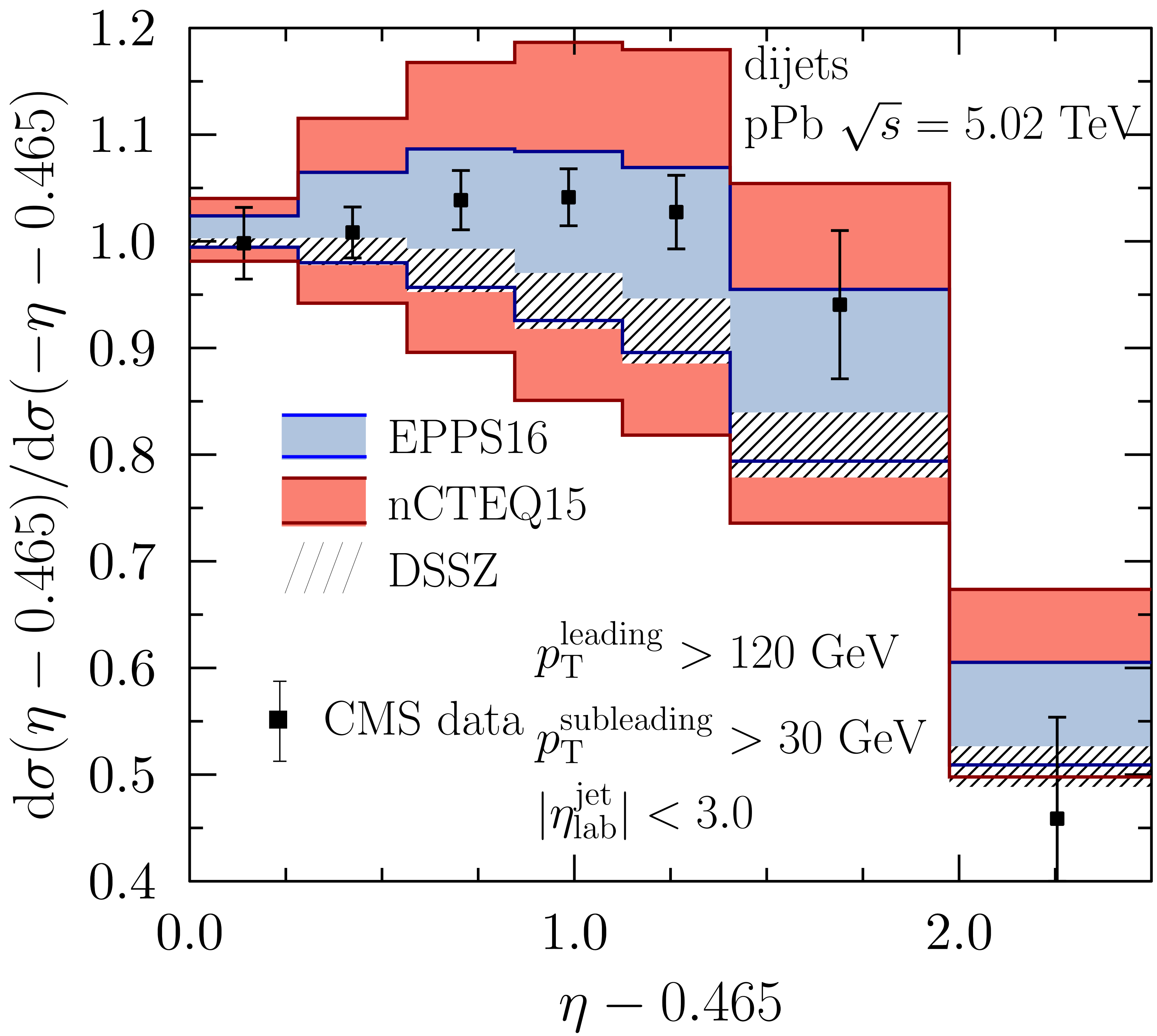
- At large-x nCTEQ15 uncertainty wider than EPPS16 (receiving constraints from dijet data)

- At small-x, true gluon uncertainty larger than from **nCTEQ15** due to parametrisation bias. This was already pointed out in arXiv:1012.1178. Happens to agree well with heavy quark data from p-Pb collisions at the LHC.

- In **DSSZ11** almost no nuclear effects in the gluon (nuclear effects were included in the FF in their analysis of RHIC pion data) and unrealistically small uncertainty band

Comparison with dijet data included in EPPS16

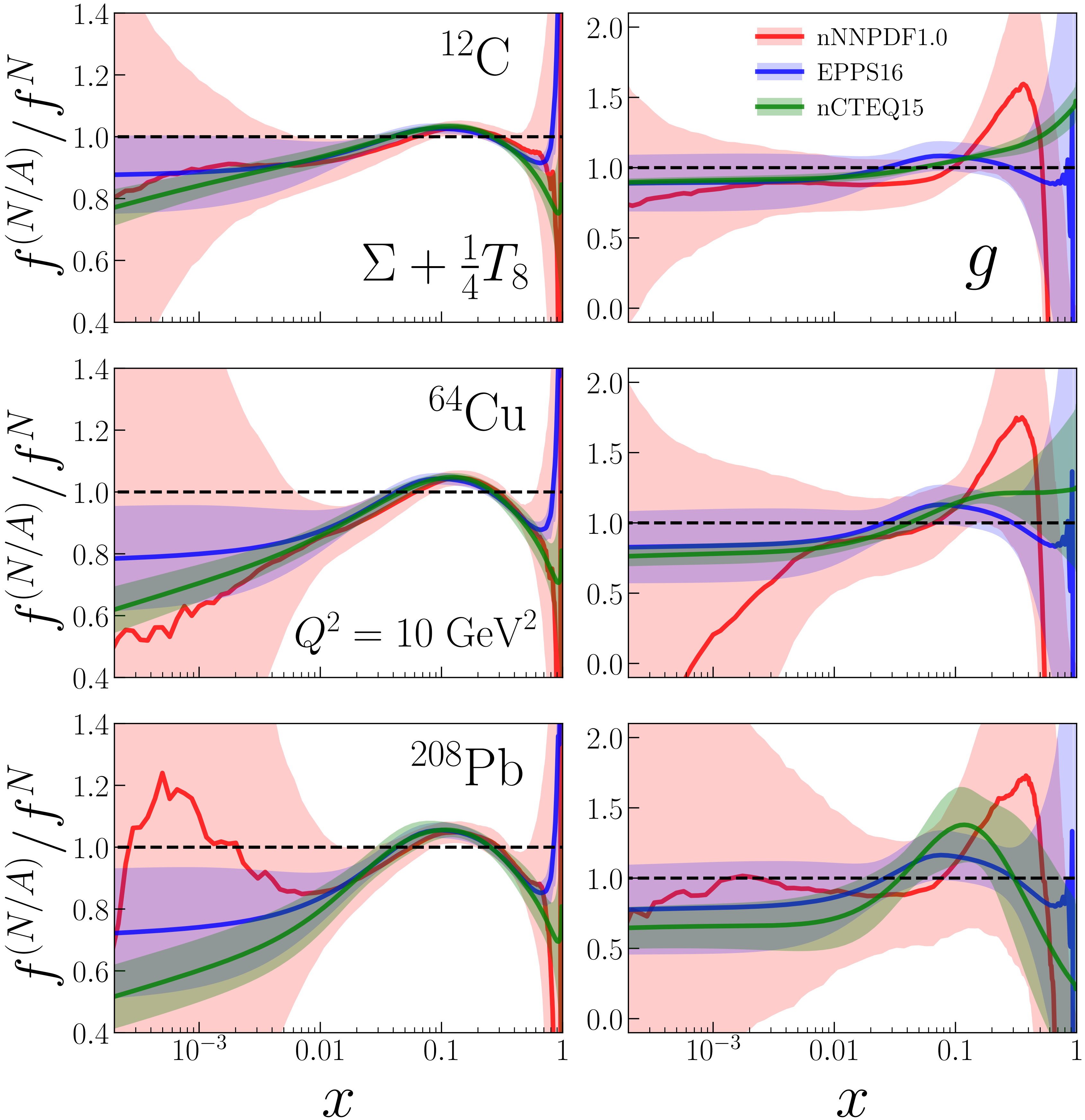
Fig. 26 in EPPS'16



- nCTEQ'15 in agreement with CMS data; including CMS dijet data in global analysis will help
- DSSZ gluon needs to be revised since not enough shadowed **OR** energy loss effects need to be included?

Results from nNNPDF1.0

EPJC79(2019)471



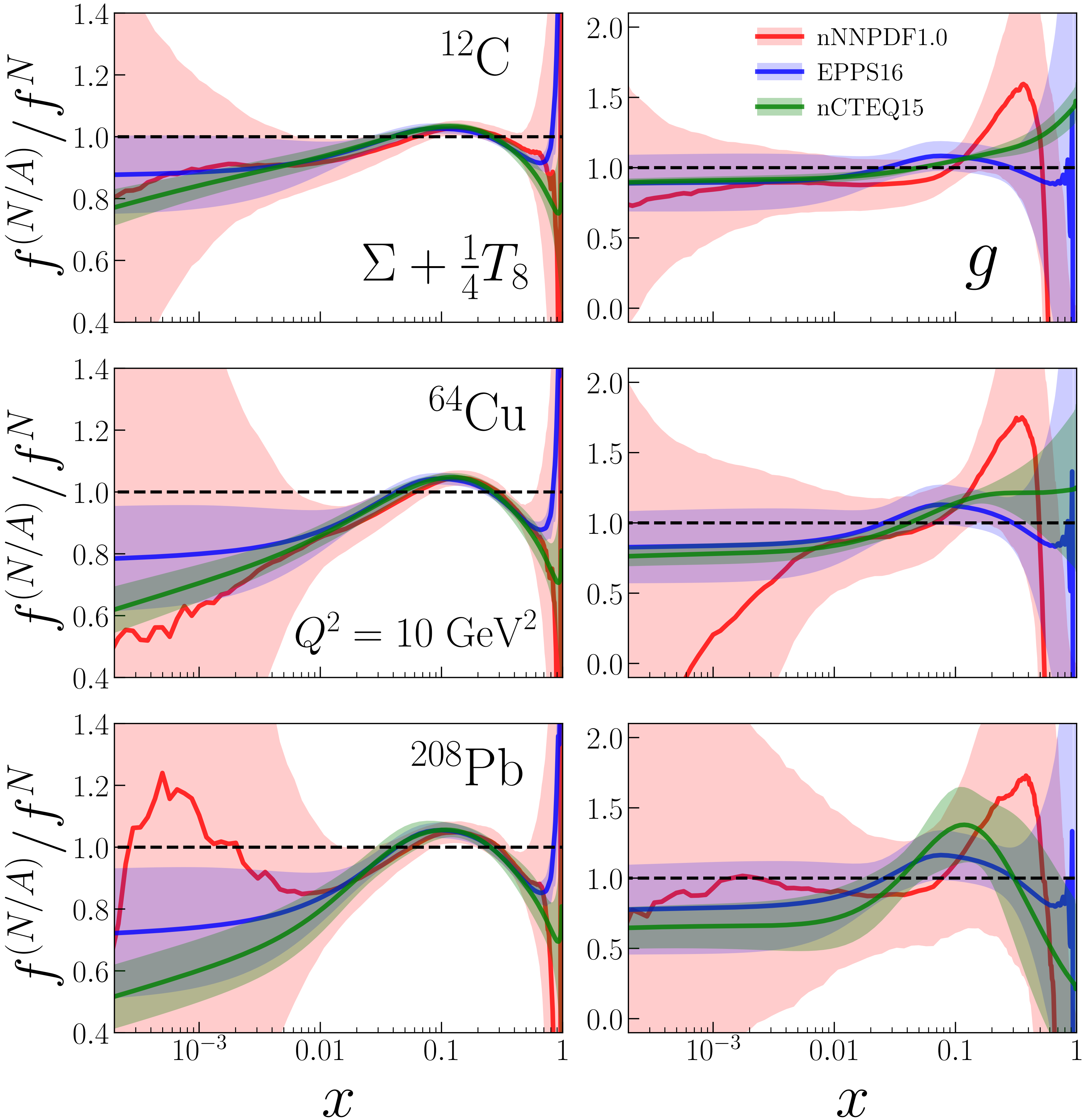
- EPPS16, nCTEQ15: 90% CL
- $\Sigma = u^+ + d^+ + s^+, T_8 = u^+ + d^+ - 2s^+$
- $F_2^{\mathrm{LO}}(x, Q^2, A) = \frac{x}{18}[4\Sigma + T_8]$

Quarks (left column)**:**

- Good agreement for this quark combination in data region x ~ [0.01, 0.65]
- Extrapolation region: Parametrisation biases in nCTEQ15, EPPS16 likely
- Reasonable to assume that small-x nuclear PDFs are not larger than the proton PDFs **BUT** it's an assumption!
- No flavour separation in this first nNNPDF1.0 analysis; only isoscalar nuclear targets
- Further constraints on quarks due to DY, neutrino DIS, lower cuts, LHC W/Z data in EPPS16 and partly in nCTEQ15

Results from nNNPDF1.0

EPJC79(2019)471



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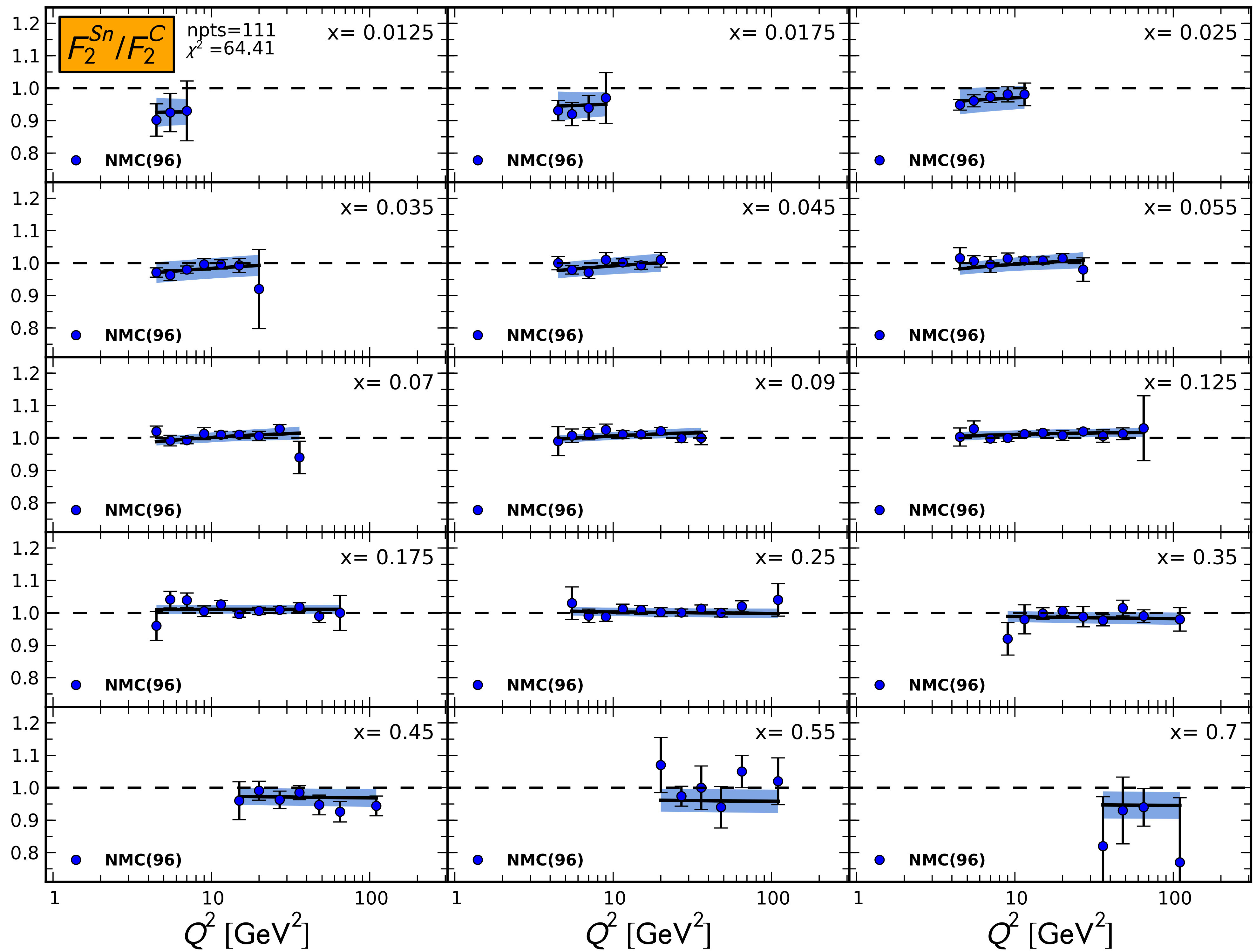
Gluons (right column):

- Note that the lA DIS data provide almost no constraint on the nuclear gluon distribution
- Therefore the nNNPDF results on the nuclear gluon are not (yet) very realistic
- The larger A/the further away from the proton boundary condition, the wider the band
- Still there is some pinch in the nNNPDF band at x~0.05 for lead.
- This may be due to the NMC data for Sn/C vs Q^2 even if the lever arm is rather short and/or possibly due to the momentum sum rule

nCTEQ15 vs NMC data

PRD93(2016)085037

Only lA-DIS set providing weak constraints on the nuclear gluon distribution



Need to include more and more collider data

- LHC dijets: gluon at medium to large x
  - Dijet data from CMS at 5 TeV already in EPPS16
  - more CMS/ATLAS data to be considered
- Inclusive W/Z production: sea quarks, strange quark
  - Run-I data already in EPPS16
  - Run-II data significantly more precise
  - Soon included in a global nCTEQ analysis!

LHC data included in EPPS16

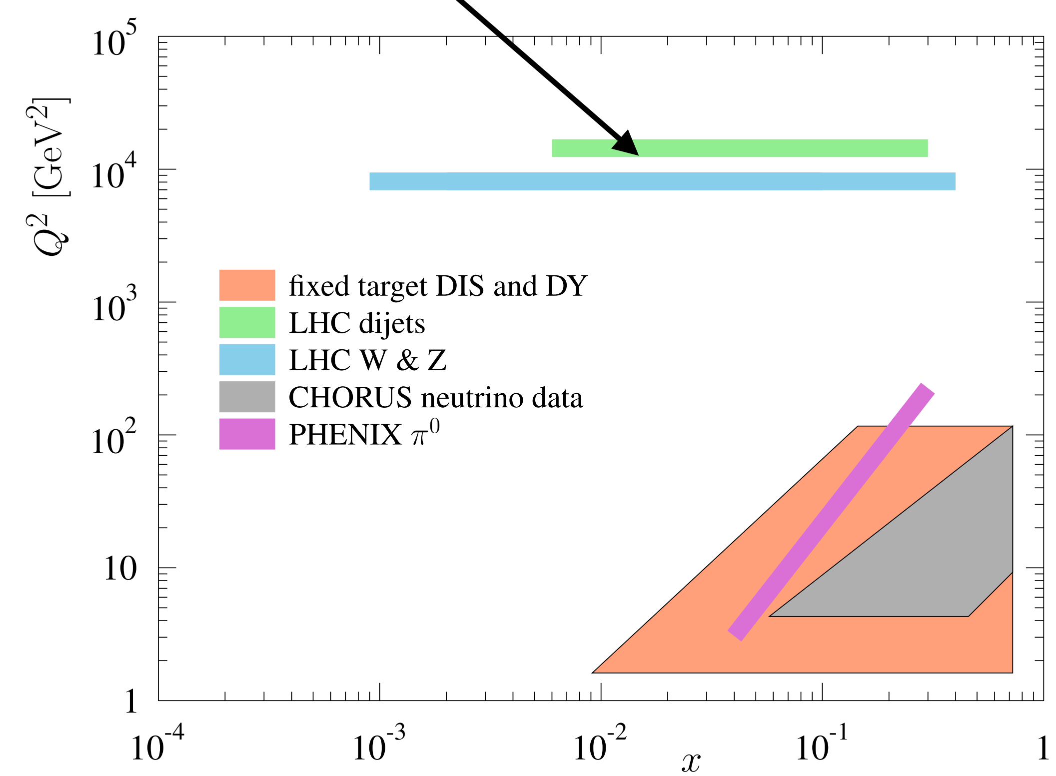


Fig. 2 The approximate regions in the (x, Q^2) plane at whic
different data in the EPPS16 fit probe the nuclear PDFs.

Collider data

- Heavy quark(-onium) data: small-x gluon
  - Works in pp case! See PROSA study for the gluon in the proton [arXiv:1503.04581]
- Inclusive prompt photons: gluon
  - Important to contrast this to gluon determinations from heavy quark data
- Low mass Drell-Yan data: sea quarks
- LHC p-Pb data on inclusive light hadrons
  - Same formalism as for inclusive π^0 production at RHIC; enter FFs

Collider data

- Heavy quark associated production Q+γ/Z/W: gluon, heavy quark PDF, strange PDF
- Top production: gluon
  - current statistics not enough to make an impact
- Data from UltraPeripheral Collisions (UPC)
  - For example dijet photoproduction
  - Heavy flavours, isolated photons

LHC p-Pb heavy quark data and gluon shadowing

Impact of LHC heavy quark data on NPDFs

A. Kusina, J.P. Lansberg, IS, H.S. Shao,
arXiv:1712.07024

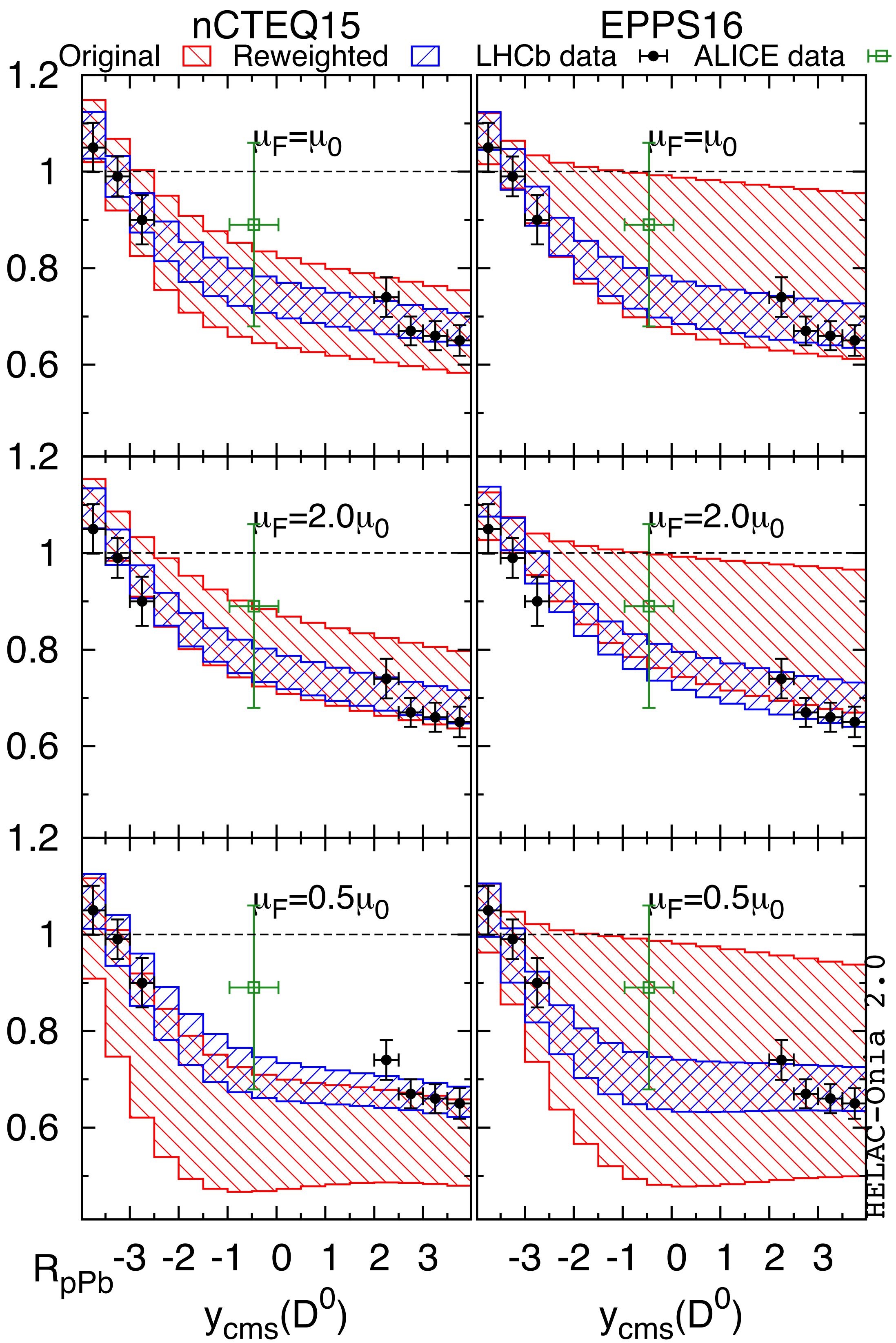
- Use data for D^0, J/Ψ, $B \rightarrow J/\Psi$, $\Upsilon(1S)$ production in p-Pb collisions at LHC at 5.02 and 8.16 TeV
- Comparison with predictions from nCTEQ15 and EPPS16
- Perform reweighting analysis of nuclear effects
- Goal: constrain small-x gluon in lead (down to $x \sim 10^{-6}$)

Data-driven approach

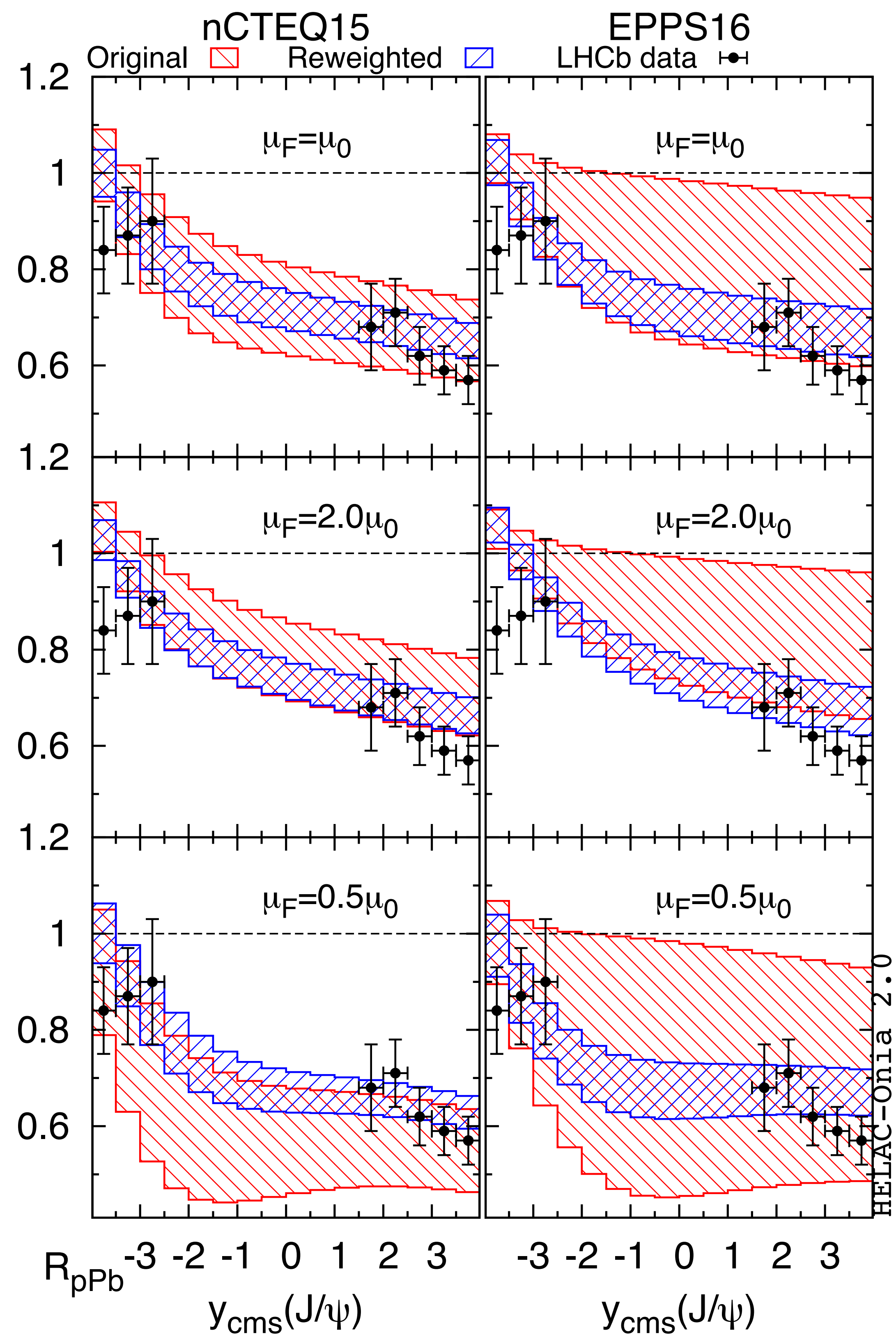
Lansberg & Shao arXiv:1610.05382

- Parameterize the squared amplitude for the partonic scattering process $g+g \rightarrow H+X$
- Convolute with modern proton PDFs
- Use data for D^0, J/Ψ, $B \rightarrow J/\Psi$, $\Upsilon(1S)$ production in pp collisions at the LHC to determine the squared amplitude
- Depends on the framework of proton PDF (scheme, order, scale choice, ...)
- Convolute squared amplitude with nuclear PDFs (same scheme, order, scale choice) to obtain predictions for p-Pb collisions

Results for R_{pA} vs rapidity

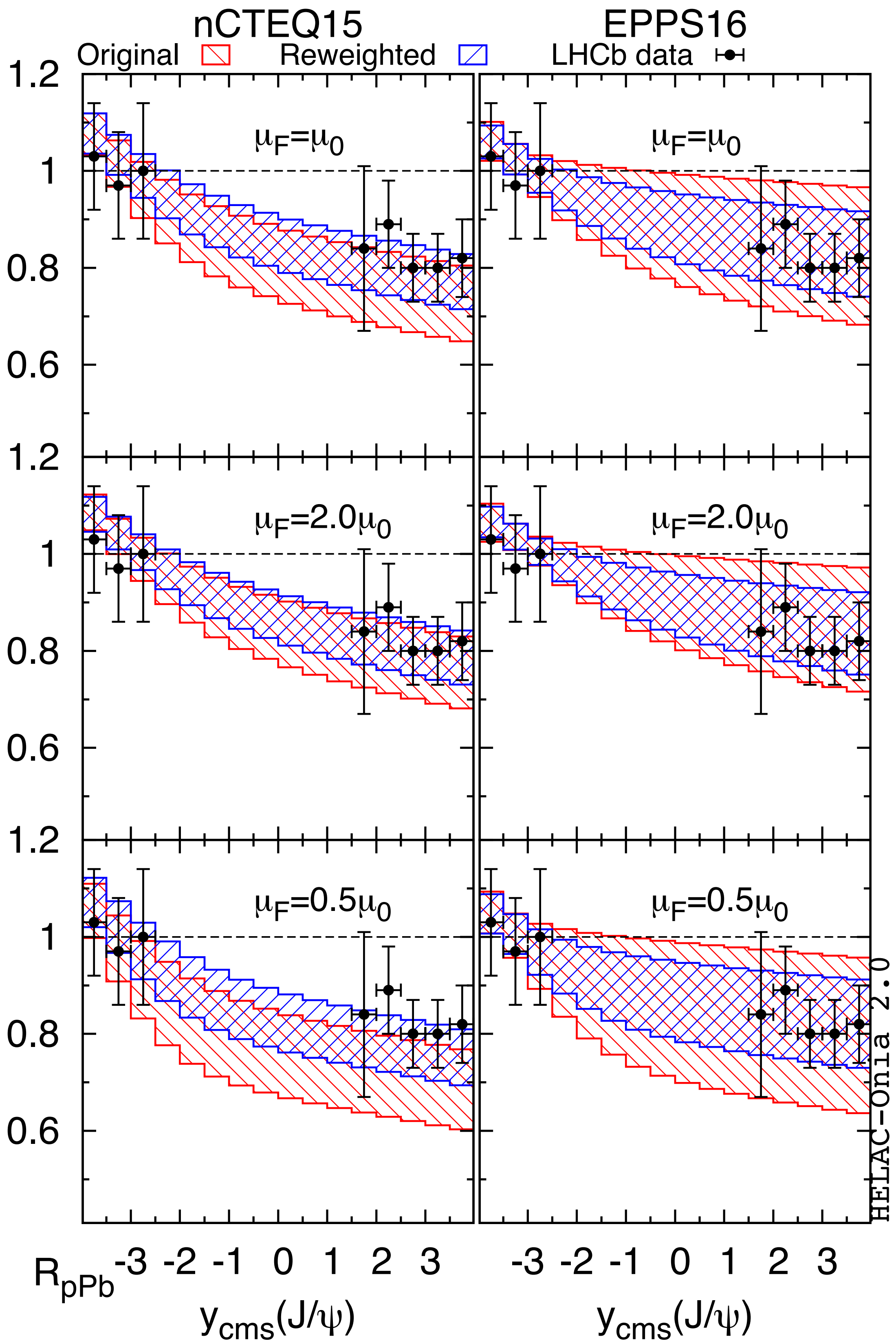


(a) Prompt D^0

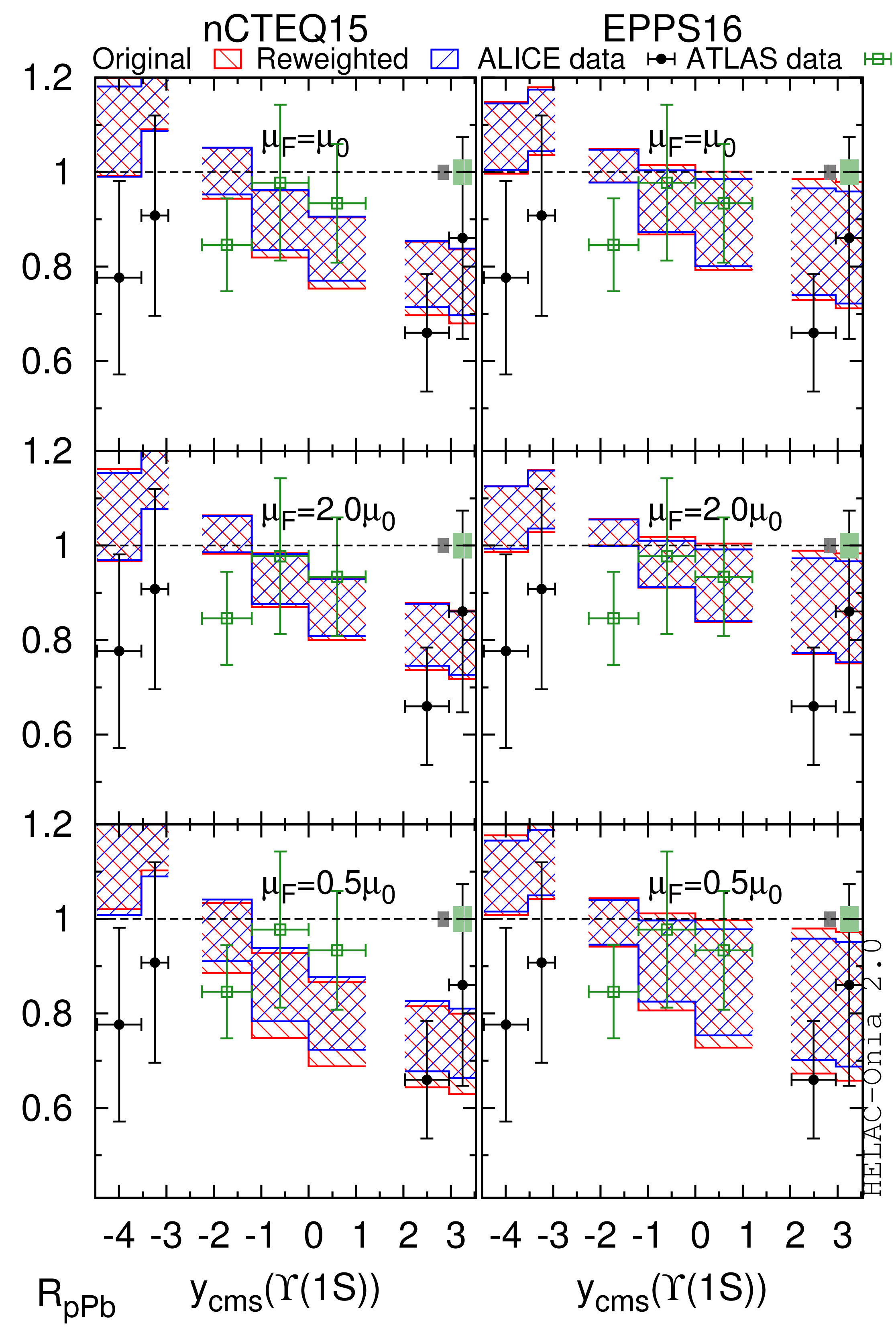


(b) Prompt J/ψ

Results for R_{pA} vs rapidity

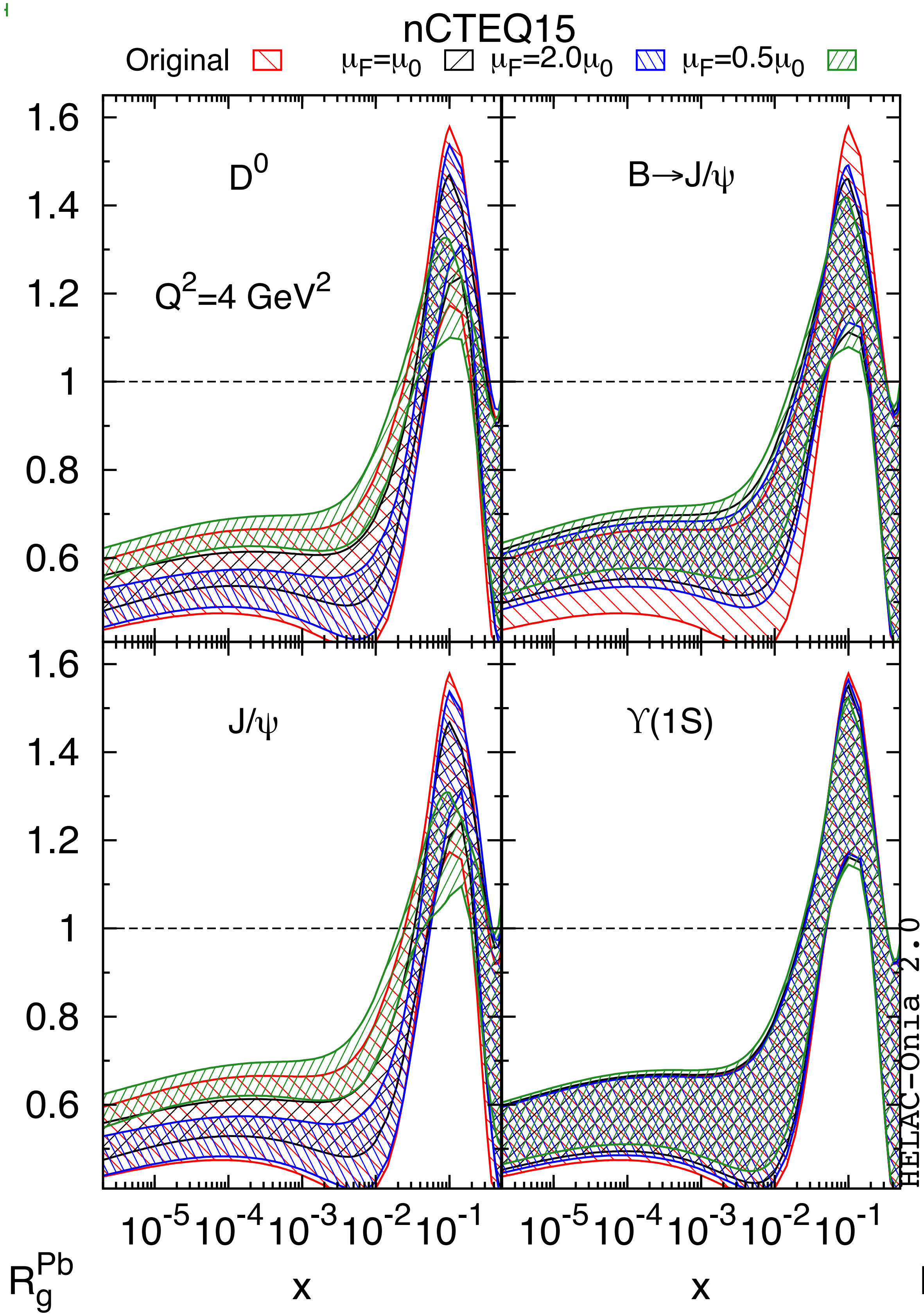


(c) $B \to J/\psi$

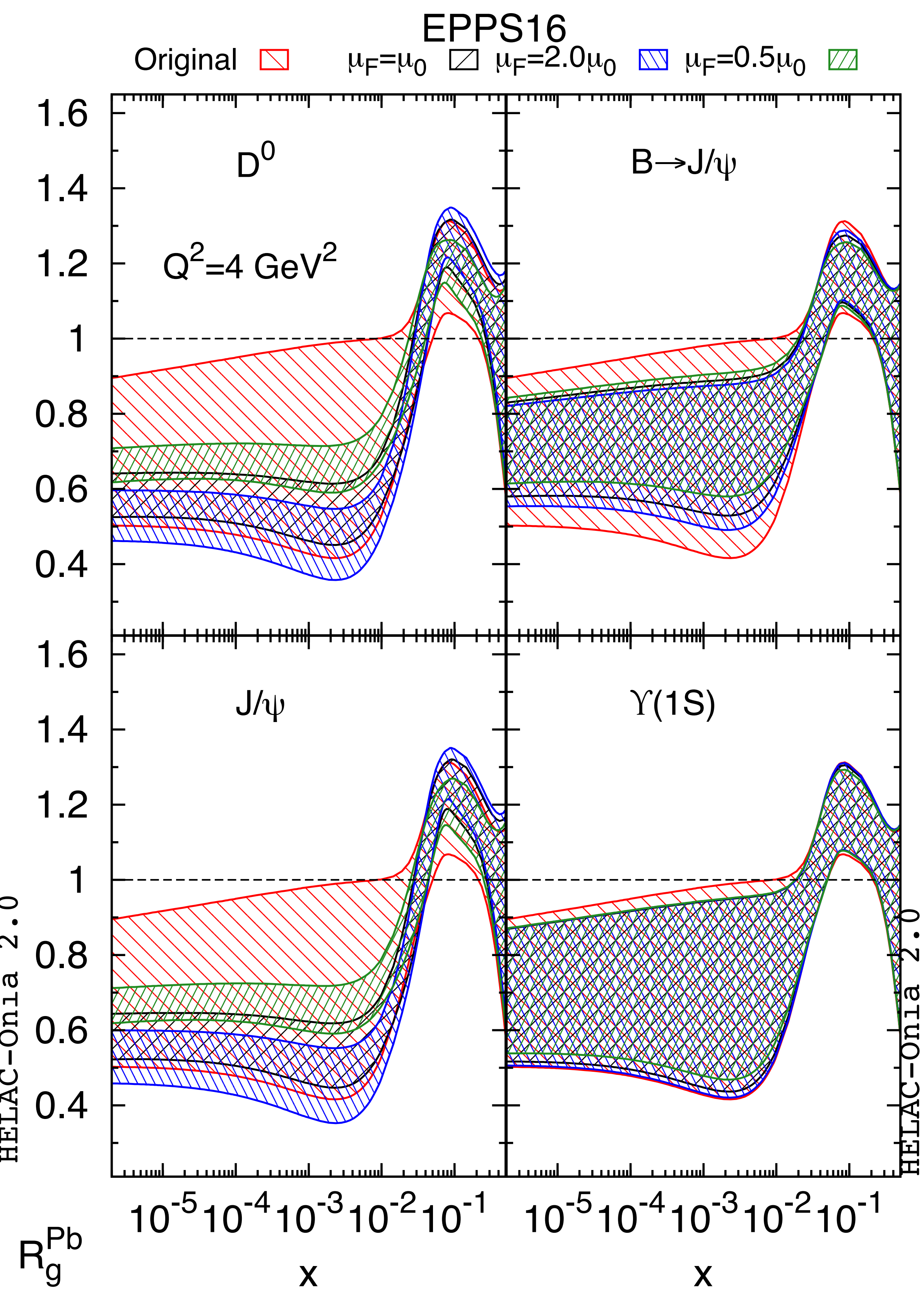


(d) $\Upsilon(1S)$

R_g^{Pb} vs x after reweighing



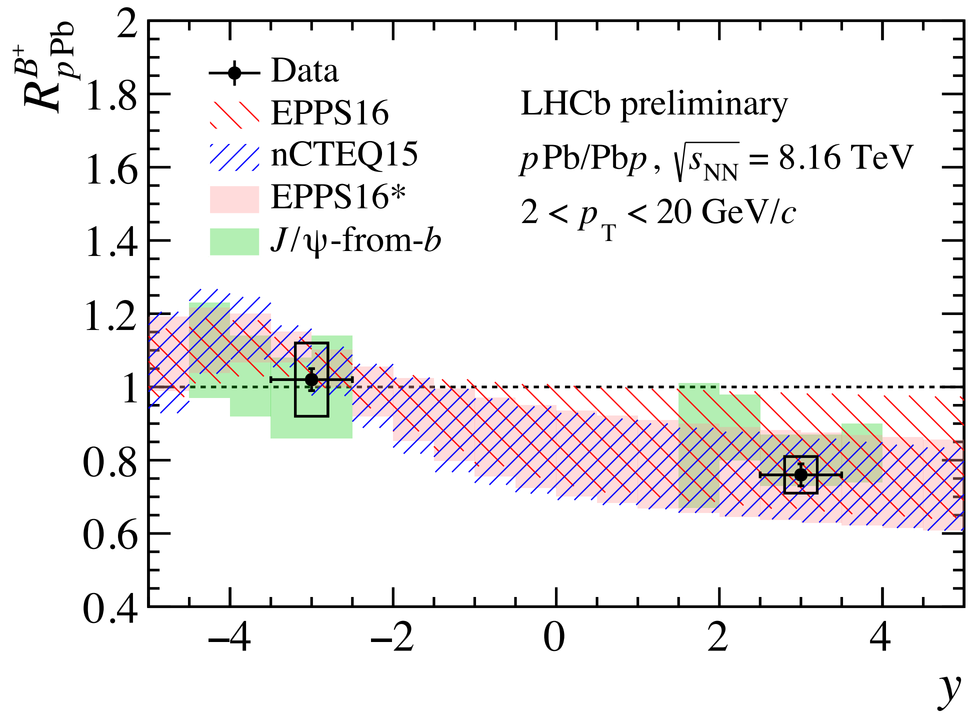
(e) nCTEQ15 nPDF



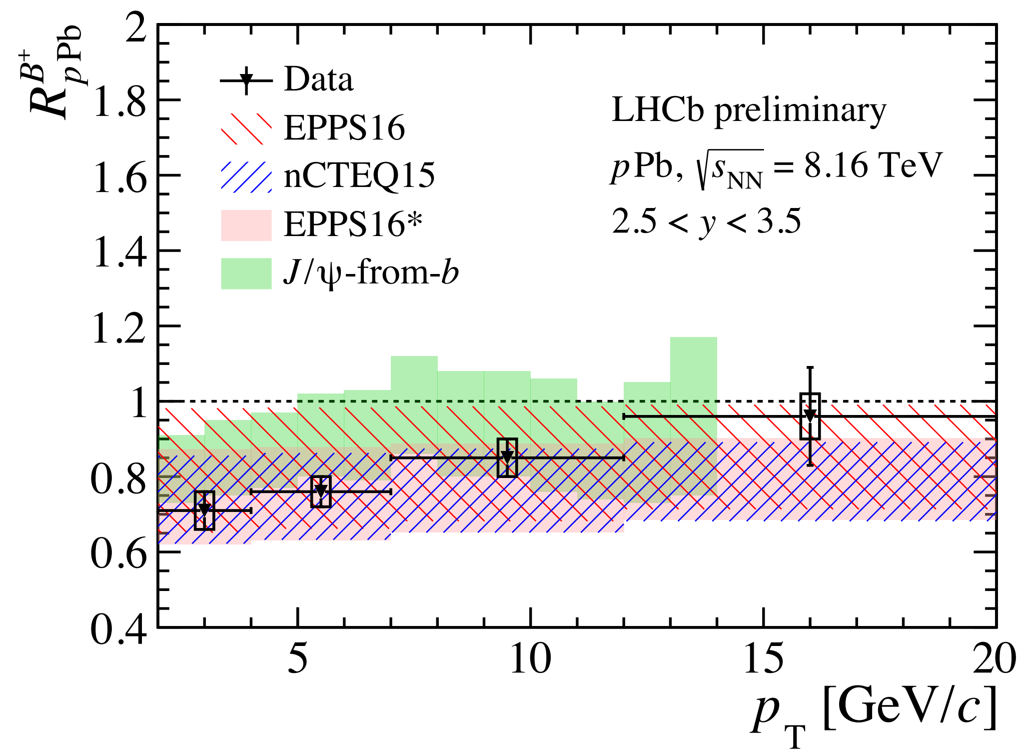
(f) EPPS16 nPDF

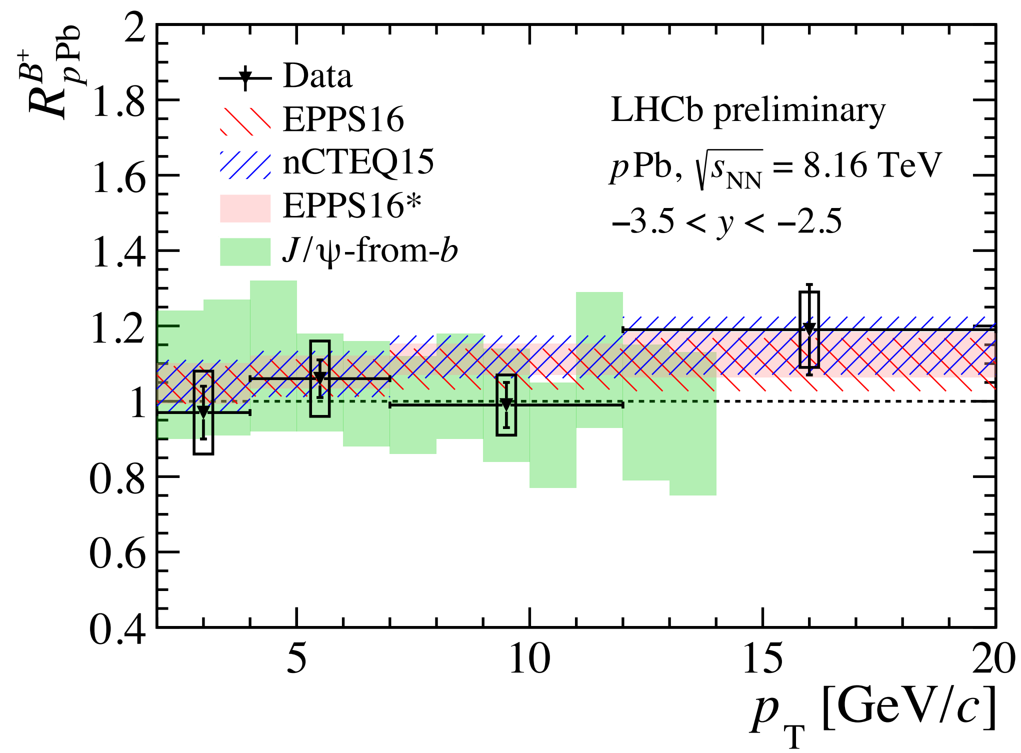
Agreement with new LHCb results

LHCb-CONF-2018-004



Reweighted nPDFs available upon request





Discussion

- A consistent description of LHC heavy quark data p-Pb data is possible in the standard pQCD framework
- Reweighting of nCTEQ15 and EPPS'16 nPDF shows unambiguosly a suppressed ('shadowed') gluon for $x \sim< 10^{-2}$
- Much reduced uncertainty band for **both** EPPS'16 and nCTEQ'15+gluons in arXiv:1012.1178
- The data-driven approach will be further compared to calculations of open heavy flavour production in the GM-VFNS

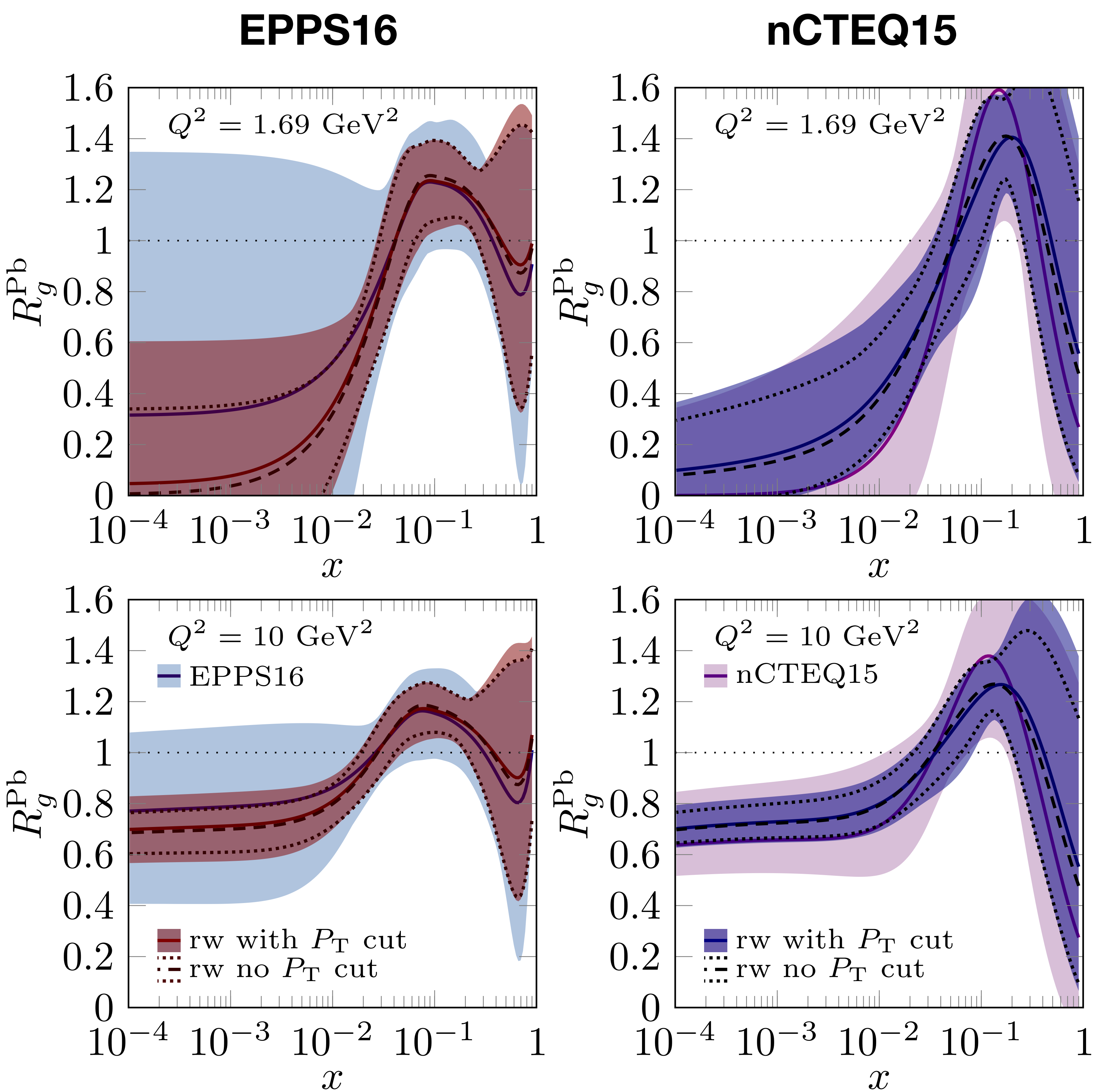
Recent study of LHCb D-meson data in p-Pb

Eskola, Helenius, Paakkinen,Paukkunen
arXiv:1906.02512

- Analysis in the GM-VFNS of LHCb D-meson data
- Similar conclusion: "compelling evidence of gluon shadowing at small-x with no signs of Parton dynamics beyond collinear factorisation"
- Perform reweighting analysis of nuclear effects

Recent study of LHCb D-meson data in p-Pb

Before and after reweighing



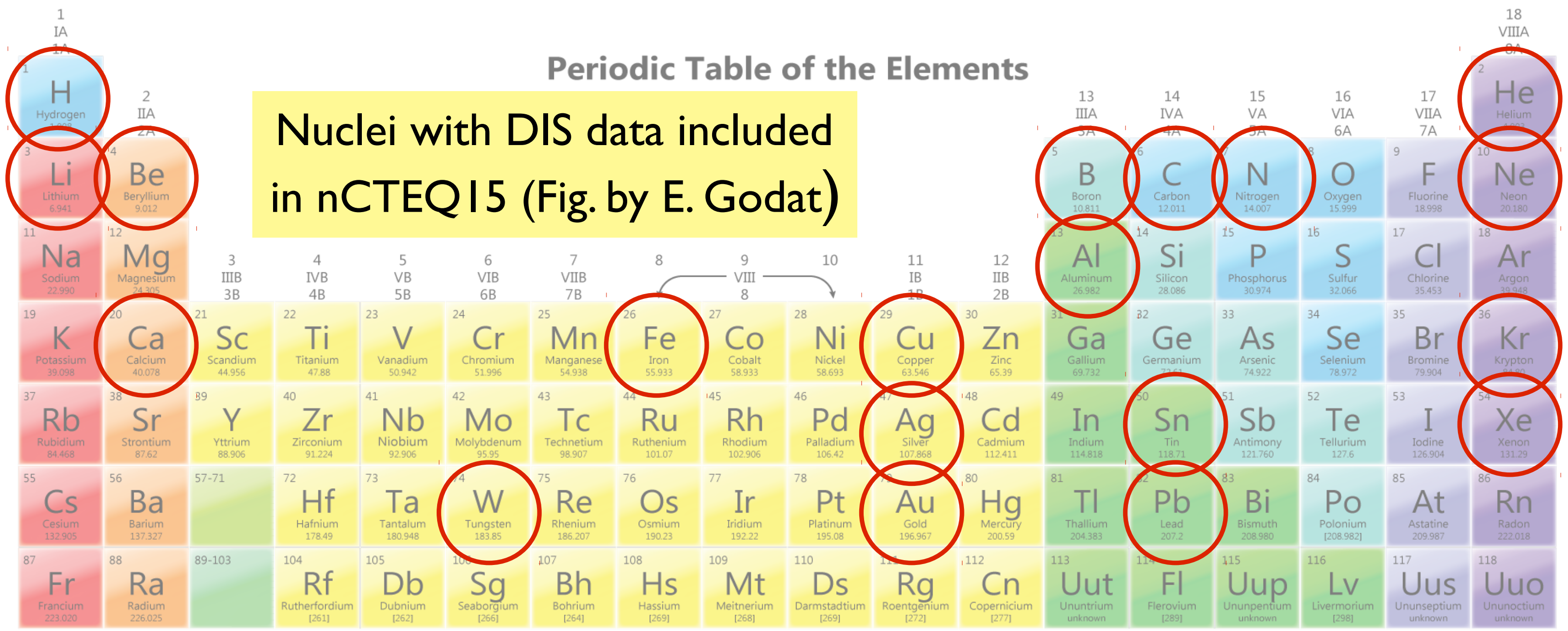
Summary & Conclusions

Conclusions

- Much activity with hope for future improvements!
  - EPPS16 first analysis including LHC data
  - nCTEQ: soon LHC W/Z data
  - First nNNPDF analysis of lA-DIS data
- Heavy quark data from p-Pb collisions point to strongly shadowed gluon at small-x

  More work needed to unambiguously disentangle from other mechanisms (saturation, coherent energy loss)
- More and more collider and data to be included in the next few years
- Neutrino data, Hi-x data,…
- Comparisons between improved nPDFs from nCTEQ, EPPS, nNNPDF will advance the field

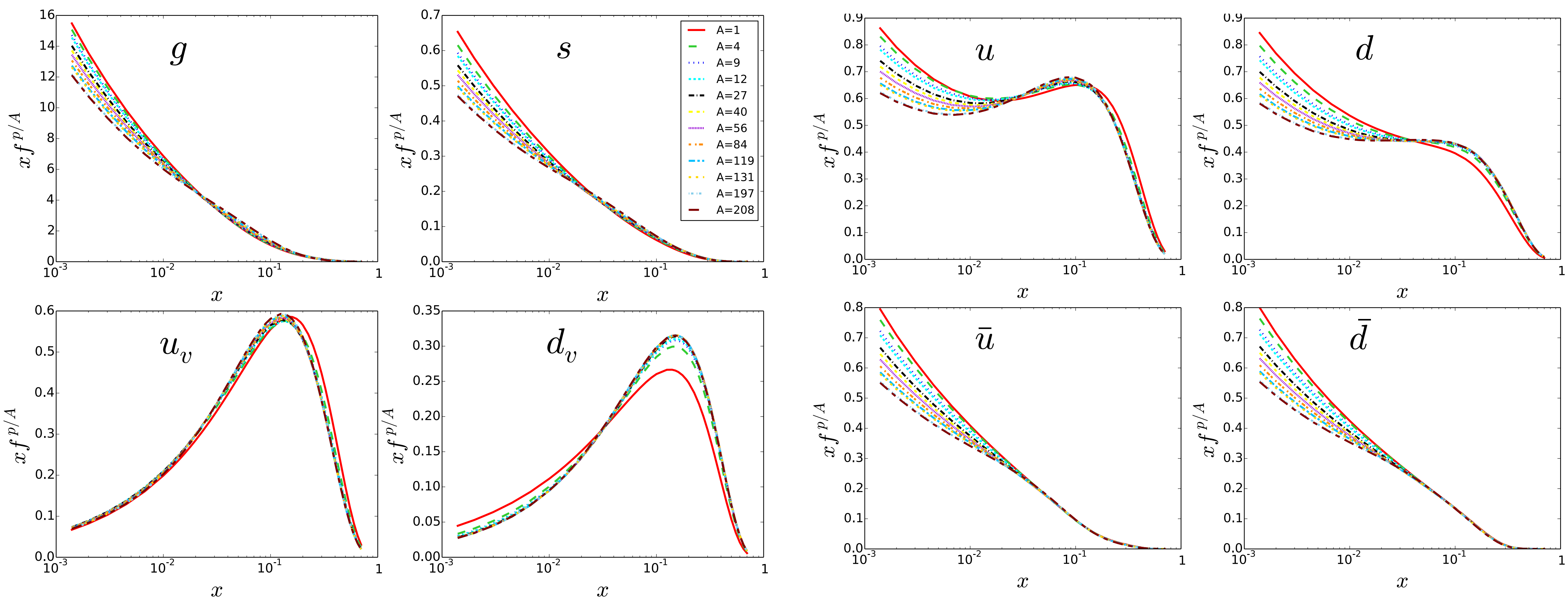
A-dependence of the partonic structure



- Fundamental quest
- New data from LHC, EIC, will allow a refined parametrization; zoom in on high-x region
- Ultimately, fits to lead only (or other targets); no need to combine different A in one analysis

nCTEQ15, arXiv:1509.00792

$$x f_i^{p/A}(x, Q_0) = x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4} x)^{c_5} \qquad c_k(A) = c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right)$$

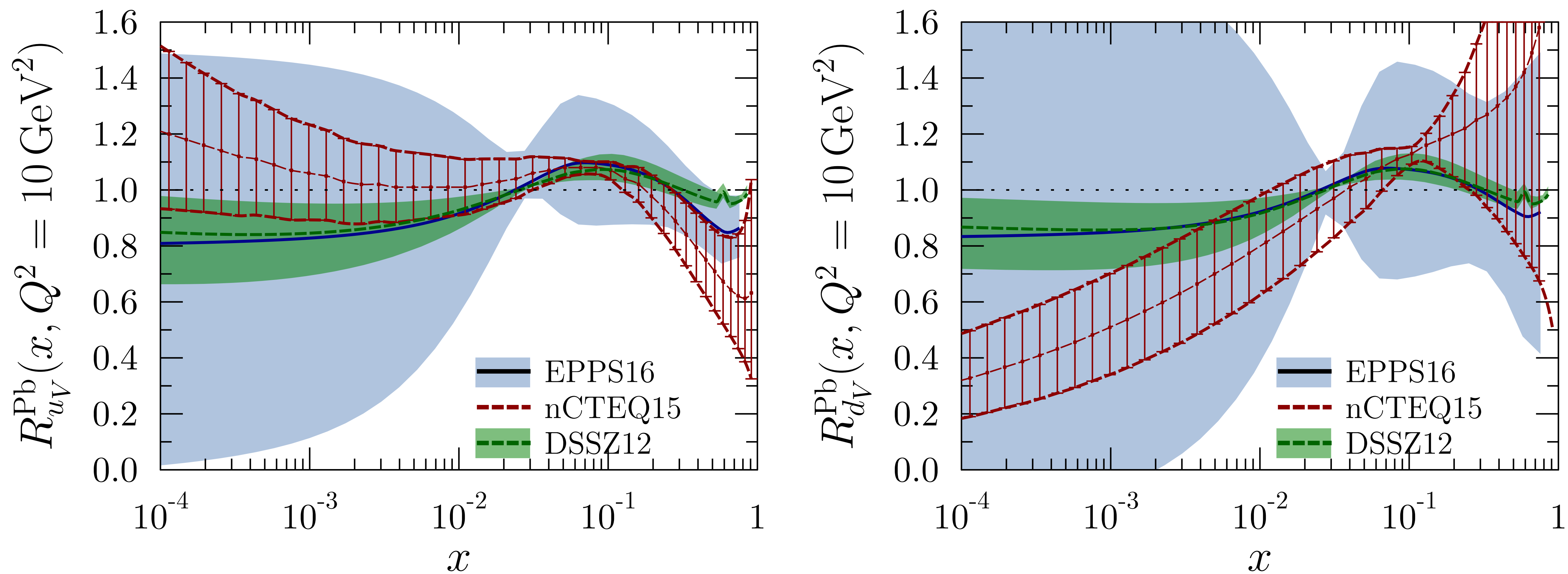


Backup slides

Nuclear modifications for u_v and d_v

$$R_i^A(x,Q^2) \equiv f_i^{\text{proton in nucleus } A}(x,Q^2)/f_i^{\text{free proton}}(x,Q^2)$$

90% CL



- In **EPPS16** and **nCTEQ15** u_v and d_v **independently** parameterized

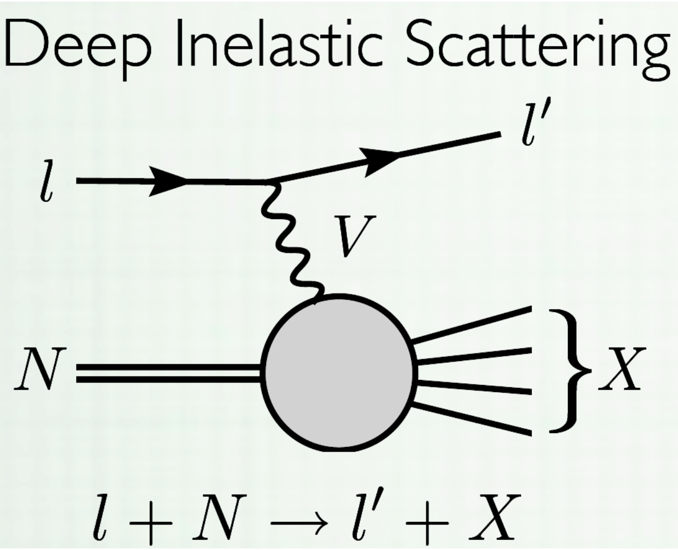
- Note: $\mathbf{R_i^A}$ as defined above **<u>not</u>** the best quantity!
  Much better agreement for full distributions $\mathbf{f^{Pb}=82\ f^{p/Pb}+(208-82)f^{n/Pb}}$ **, ($f=u_v$, d_v)**

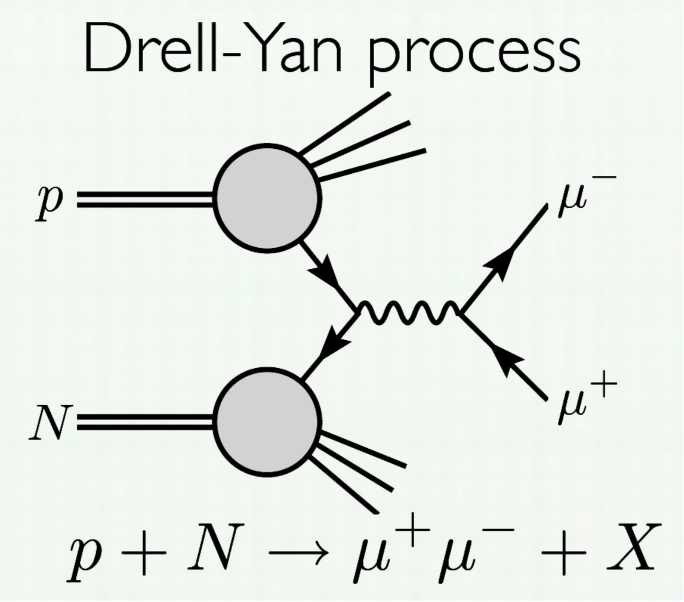
- Note: valence distributions very small at $x \sim 10^{-3}$; large uncertainties not relevant there.

nCTEQ'15 framework: Data sets

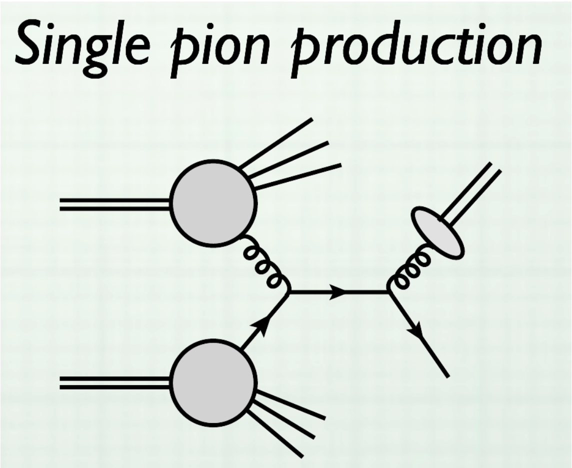
- NC DIS & DY

  CERN BCDMS & EMC & NMC
  N = (D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W)
  FNAL E-665
  N = (D, C, Ca, Pb, Xe)
  DESY Hermes
  N = (D, He, N, Kr)
  SLAC E-139 & E-049
  N = (D, Ag, Al, Au, Be,C, Ca, Fe, He)
  FNAL E-772 & E-886
  N = (D, C, Ca, Fe,W)





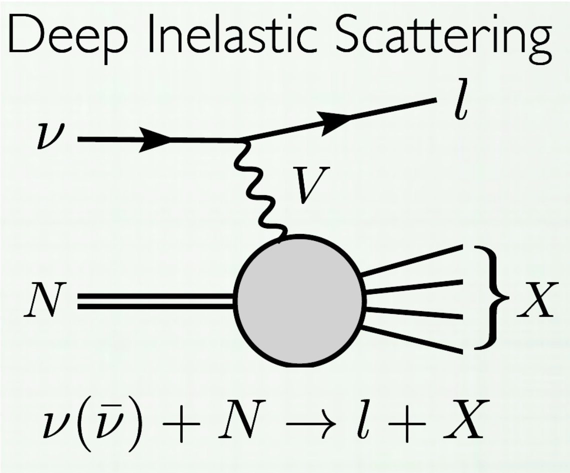
- Single pion production (new)



RHIC - PHENIX & STAR

N = Au

- Neutrino (to be included later)



CHORUS CCFR & NuTeV

N = Pb N = Fe

Fit details

PRD93(2016)085037

Fit properties:

- fit @NLO
- $Q_0 = 1.3\text{GeV}$
- using ACOT heavy quark scheme
- kinematic cuts:
  $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
  $p_T > 1.7$ GeV
- 708 (DIS & DY) + 32 (single π^0)
  = 740 data points after cuts
- 16+2 free parameters
  - 7 gluon
  - 7 valence
  - 2 sea
  - 2 pion data normalizations
- $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$

Error analysis:

- use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$$

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)
- eigenvalues span 10 orders of magnitude $\rightarrow$ require numerical precision
- use noise reducing derivatives

Fit details

PRD93(2016)085037

Kinematic cuts

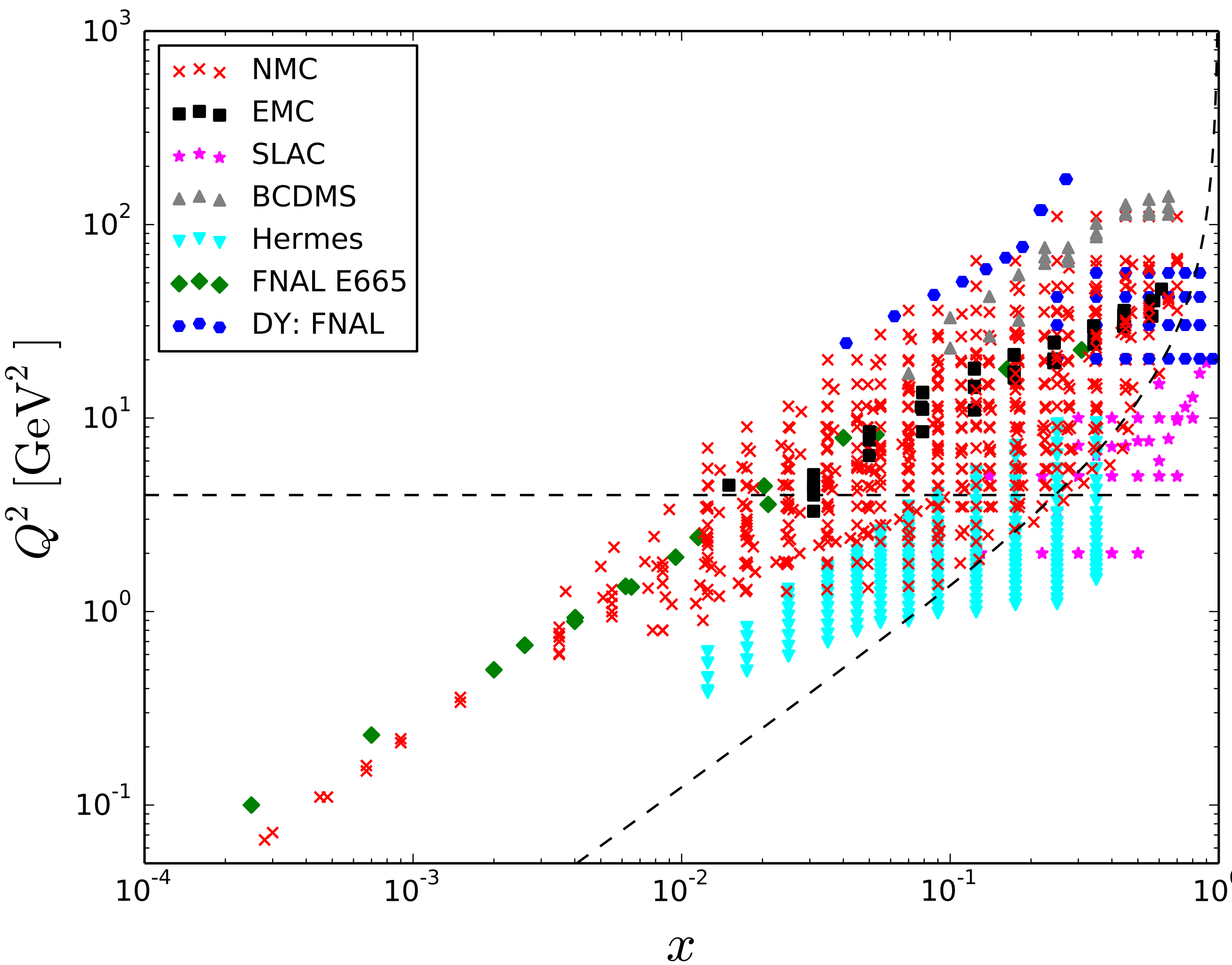
nCTEQ:

$$\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$$

EPS: $Q > 1.3$ GeV

HKN: $Q > 1$ GeV

DSSZ: $Q > 1$ GeV



nCTEQ: 740 data points

EPS09: 929 data points

Fit details

PRD93(2016)085037

Fit properties:

- fit @NLO

Fit quality

- $\chi^2/dof = 0.81$

sc

$Q > 2\text{GeV},\ W > 3.5\text{GeV}$
$p_T > 1.7$ GeV

- 708 (DIS & DY) + 32 (singl
$= 740$ data points after cuts
- 16+2 free parameters
  - 7 gluon
  - 7 valence
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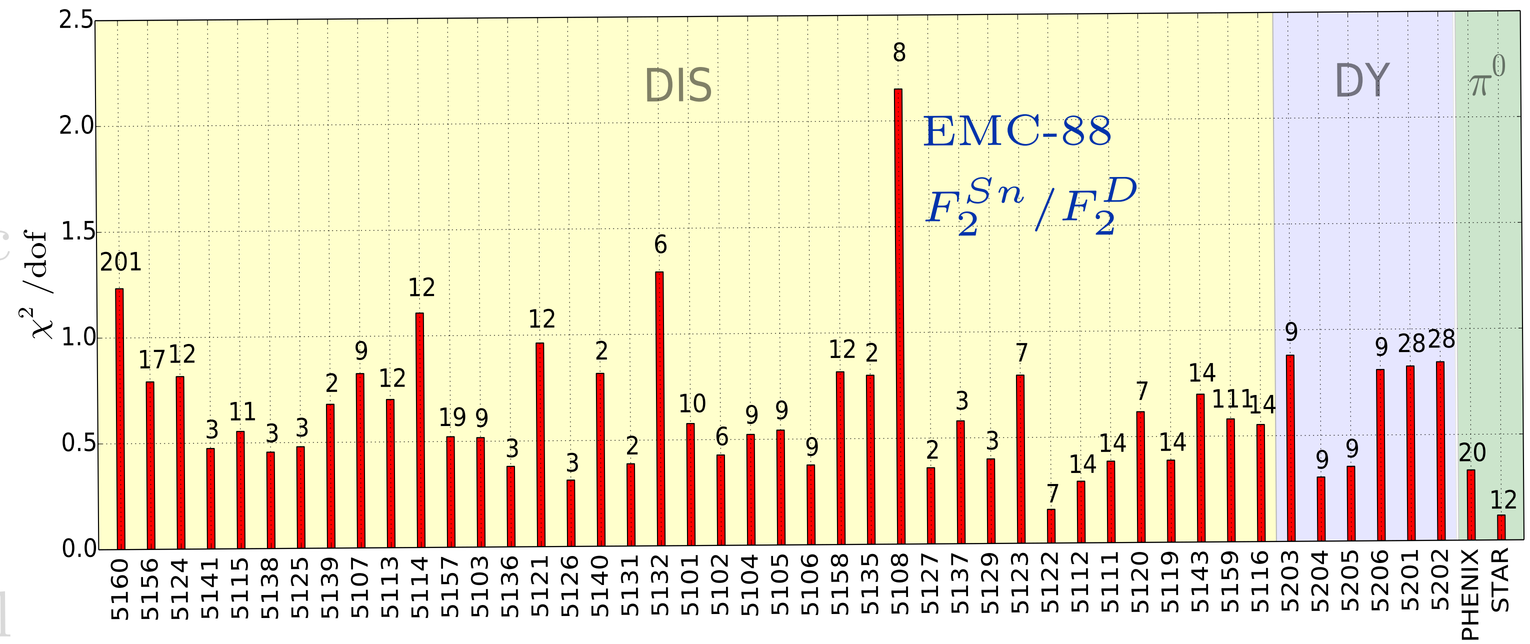
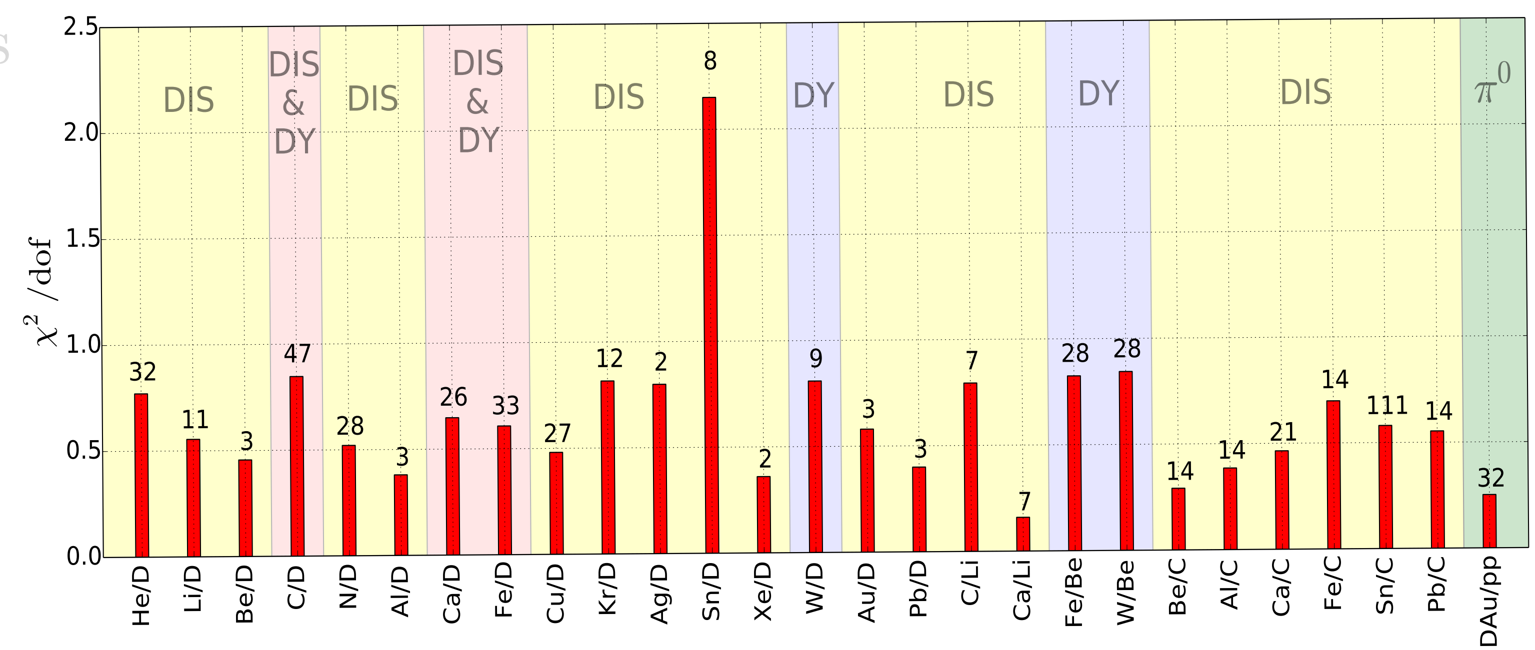
Error analysis:





From nucleons to nuclei

From Protons to Nuclei

- Starting point: global analysis framework for free nucleons
- Make sure it can be applied to the case of nuclear targets (A,Z)
  - Variable $0 < x_N < A$
  - Evolution equations
  - Sum rules
  - Observables
- Apart from validity of factorisation which is a working assumption and to be verified phenomenologically

DIS on nuclear targets

Consider deep inelastic lepton–nucleon collisions: $l(k) + A(p_A) \rightarrow l'(k') + X$

Introduce the usual DIS variables: $q \equiv k - k'$, $\quad Q^2 \equiv -q^2$, $\quad x_A \equiv \frac{Q^2}{2p_A \cdot q}$

Hadronic tensor: $W_{\mu\nu}^A \propto \langle A(p_A)|\ J_\mu J_\nu^\dagger\ |A(p_A)\rangle = \sum_i\ a_{\mu\nu}^{(i)}\ \tilde{F}_i^A(x_A, Q^2)$,
where $a_{\mu\nu}^{(i)}$ are Lorentz-tensors composed out of the 4-vectors q and p_A and the metric $g_{\mu\nu}$

Express structure functions in the QCD improved parton model in terms of NPDFs

$$\tilde{\mathcal{F}}_k^A(x_A, Q^2) = \int_{x_A}^1 \frac{\mathrm{d}y_A}{y_A}\ \tilde{f}_i^A(y_A, Q^2) C_{k,i}(x_A/y_A) + \tilde{\mathcal{F}}_k^{A,\tau\geq 4}(x_A, Q^2)$$

NPDFs: Fourier transforms of matrix elements of twist-two operators composed out of the quark and gluon fields:

$$\tilde{f}_i^A(x_A, Q^2) \propto \langle A(p_A)|\ O_i\ |A(p_A)\rangle$$

Definitions of $\tilde{F}_i^A(x_A, Q^2)$, $\tilde{f}_i^A(x_A, Q^2)$, and the varibale $0 < x_A < 1$ carry over one-to-one from the well-known free nucleon case

Evolution Equations and Sum Rules

DGLAP as usual:

$$\frac{\mathrm{d}\tilde{f}_i^A(x_A, Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_A}^1 \frac{\mathrm{d}y_A}{y_A} P_{ij}(y_A)\, \tilde{f}_j^A(x_A/y_A, Q^2) \,, \\ = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_A}^1 \frac{\mathrm{d}y_A}{y_A} P_{ij}(x_A/y_A)\, \tilde{f}_j^A(y_A, Q^2) \,,$$

Sum rules:

$$\int_0^1 \mathrm{d}x_A\, \tilde{u}_v^A(x_A, Q^2) = 2Z + N \,, \\ \int_0^1 \mathrm{d}x_A\, \tilde{d}_v^A(x_A, Q^2) = Z + 2N \,,$$

and the momentum sum rule

$$\int_0^1 \mathrm{d}x_A\, x_A \left[\tilde{\Sigma}^A(x_A, Q^2) + \tilde{g}^A(x_A, Q^2)\right] = 1 \,,$$

where $N = A - Z$ and $\tilde{\Sigma}^A(x_A) = \sum_i (\tilde{q}_i^A(x_A) + \tilde{\bar{q}}_i^A(x_A))$ is the quark singlet combination

Rescaled definitions!

Problem: average momentum fraction carried by a parton $\propto A^{-1}$
since there are 'A-times more partons' which have to share the momentum

- Different nuclei (A, Z) not directly comparable
- Functional form for x-shape would change drastically with A
- Need to rescale!

PDFs are number densities: $\tilde{f}_i^A(x_A)\,\mathrm{d}x_A$ is the number of partons carrying a momentum fraction in the interval $[x_A, x_A + \mathrm{d}x_A]$

Defi ne rescaled NPDFs $f_i^A(x_N)$ with $0 < x_N := Ax_A < A$:

$$\boxed{f_i^A(x_N)\,\mathrm{d}x_N := \tilde{f}_i^A(x_A)\,\mathrm{d}x_A}$$

The variable x_N can be interpreted as parton momentum fraction w.r.t. the **average** nucleon momentum $\bar{p}_N := p_A/A$

Rescaled evolution equations and sum rules

Evolution:

$$\frac{\mathrm{d}f_i^A(x_N,Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_{x_N/A}^{1}\frac{\mathrm{d}y_A}{y_A}\,P(y_A)\,f_i^A(x_N/y_A,Q^2)\,,$$
$$= \frac{\alpha_s(Q^2)}{2\pi}\int_{x_N}^{A}\frac{\mathrm{d}y_N}{y_N}\,P(x_N/y_N)\,f_i^A(y_N,Q^2)\,.$$

Assume that $f_i^A(x_N) = 0$ for $x_N > 1$, then **original, symmetrical** form recovered:

$$\frac{\mathrm{d}f_i^A(x_N,Q^2)}{\mathrm{d}\ln Q^2} = \begin{cases} \frac{\alpha_s(Q^2)}{2\pi}\int_{x_N}^{1}\frac{\mathrm{d}y_N}{y_N}\,P(y_N)\,f_i^A(x_N/y_N,Q^2) & : 0 < x_N \le 1 \\ 0 & : 1 < x_N < A, \end{cases}$$

Sum rules for the rescaled PDFs:

$$\int_0^A \mathrm{d}x_N\,u_v^A(x_N) = 2Z+N\,,$$
$$\int_0^A \mathrm{d}x_N\,d_v^A(x_N) = Z+2N\,,$$

and

$$\int_0^A \mathrm{d}x_N\,x_N\,\left[\Sigma^A(x_N)+g^A(x_N)\right] = A\,,$$

Rescaled structure functions

The rescaled structure functions can be defi ned as

$$x_N \mathcal{F}_i^A(x_N) := x_A \tilde{\mathcal{F}}_i^A(x_A) \ ,$$

with $\mathcal{F}_{1,2,3}(x) = \{F_1(x), F_2(x)/x, F_3(x)\}$.

More explicitly:

$$\begin{aligned} F_2^A(x_N) &:= \tilde{F}_2^A(x_A) \ , \\ x_N F_1^A(x_N) &:= x_A \tilde{F}_1^A(x_A) \ , \\ x_N F_3^A(x_N) &:= x_A \tilde{F}_3^A(x_A) \ . \end{aligned}$$

This leads to consistent results in the parton model using the rescaled PDFs.

Effective PDFs of bound nucleons

Further decompose the NPDFs $f_i^A(x_N)$ in terms of effective parton densities for **bound** protons, $f_i^{p/A}(x_N)$, and neutrons, $f_i^{n/A}(x_N)$, inside a nucleus A:

$$f_i^A(x_N, Q^2) = Z\, f_i^{p/A}(x_N, Q^2) + N\, f_i^{n/A}(x_N, Q^2)$$

- The bound proton PDFs have the **same** evolution equations and sum rules as the free proton PDFs **provided** we neglect any contributions from the region $x_N > 1$
- Neglecting the region $x_N > 1$, is consistent with the DGLAP evolution
- The region $x_N > 1$ is expected to have a minor influence on the sum rules of less than one or two percent (see also [PRC73(2006)045206])
- Isospin symmetry: $u^{n/A}(x_N) = d^{p/A}(x_N)$, $d^{n/A}(x_N) = u^{p/A}(x_N)$

An observable $\mathcal{O}^A$ is then given by:

$$\mathcal{O}^A = Z\, \mathcal{O}^{p/A} + N\, \mathcal{O}^{n/A}$$

In conclusion: the free proton framework can be used to analyse nuclear data