From the Proton to the Nuclear Parton Distributions

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Heavy-Quark Hadroproduction from Collider to Astroparticle Physics

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Outline

- Experimental observations on nuclear modification of parton distributions.
- Sketch of basic mechanisms responsible for nuclear effects.
- Review the results of analysis of nuclear DIS.
- Application to W/Z boson production in $p + \mathrm{Pb}$ collisions at LHC.

References:

Alekhin, sk, Petti, PRD96 (2017) 054005
sk, ArXiv:1612.07741
Ru, sk, Petti, Zhang, PRD94 (2016) 113013
sk, ArXiV:1606.07016
sk, Petti, PRC90 (2014) 045204
sk, Petti, PRC82 (2010) 054614
sk, Petti, PRD76 (2007) 094023
sk, Petti, NPA765 (2006) 126

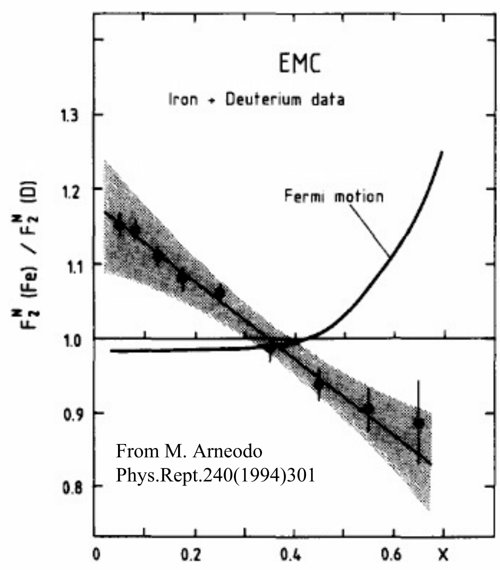
How well do we know the *nuclear PDF* if we know those of the proton and the neutron?

- A nucleus of Z protons and N neutrons; ($A = Z + N$ the total number of bound nucleons) is a loosely bound system with binding energy $E_B \ll M$ the nucleon mass.
- In DIS momentum/energy transfer $\gg$ typical momentum/energy of bound nucleons. Then we expect the incoherent sum over protons and neutrons should be a good approximation

$$p_{i/A}(x, Q^2) = Z p_{i/p}(x, Q^2) + N p_{i/n}(x, Q^2)$$

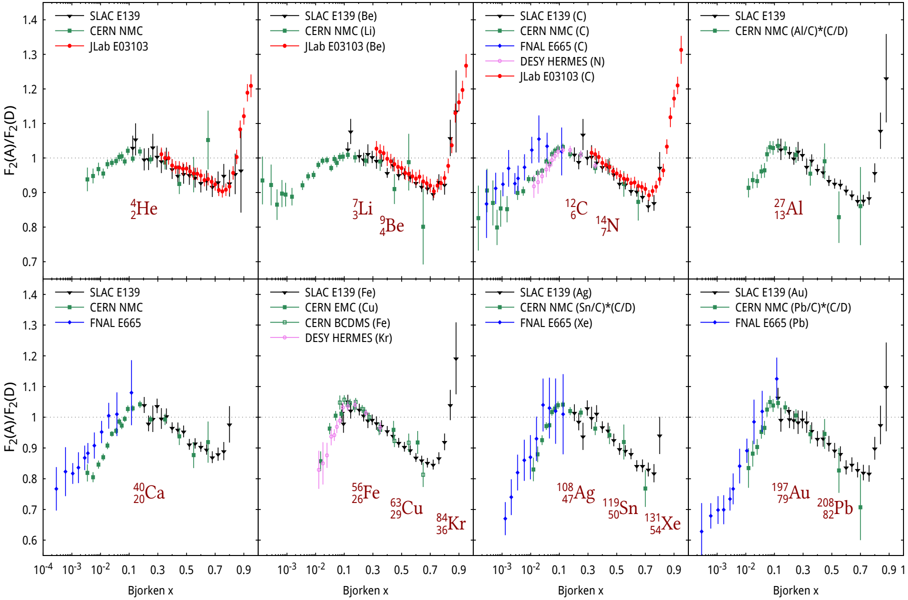
  with corrections of order $\sim p_{\text{bound nucleon}}/|\boldsymbol{q}|$ to vanish at very high energy.
- However, experimentally this hypothesis is badly violated. The measurements show corrections up to 100% depending on kinematical region.

Historic EMC measurement of nuclear effects in DIS



Direct measurement by *EMC Collaboration, 1983* indicated unexpected nuclear effects even in the DIS region. Exciting observation, although the small-x part turned out to be time dependent: initially EMC had not seen the nuclear shadowing effect but that evolved with time.

Summary on nuclear ratios data from DIS experiments



Empirical nuclear PDF

Nuclear PDF (NPDF) are phenomenologically extracted from data in a way similar to the proton analyses. Basic steps:

- Assume $p_{i/A}(x,Q^2) = Zp_{i/p}(x,Q^2) + Np_{i/n}(x,Q^2)$ with $p_{i/p}$ and $p_{i/n}$ the *bound* proton and neutron PDF, which are different from the free ones $p^0_{i/p}$ and $p^0_{i/n}$.
- Assume isospin symmetry relations $u_p = d_n$, $d_p = u_n$, $s_p = s_n$, $g_p = g_n$.
- Assume a functional form for $p_{i/A}(x)$ or for the ratio $R^A_i = p_{i/p}/p^0_{i/p}$ where $p^0_{i/p}$ is a PDF of *free* proton.
- Assume a functional form of A-dependence of the model parameters (could be potentially challenging for light nuclei).
- Fit model parameters to nuclear data.

A few analyses are available which differ by functional form (parameterization) of x and A dependencies.

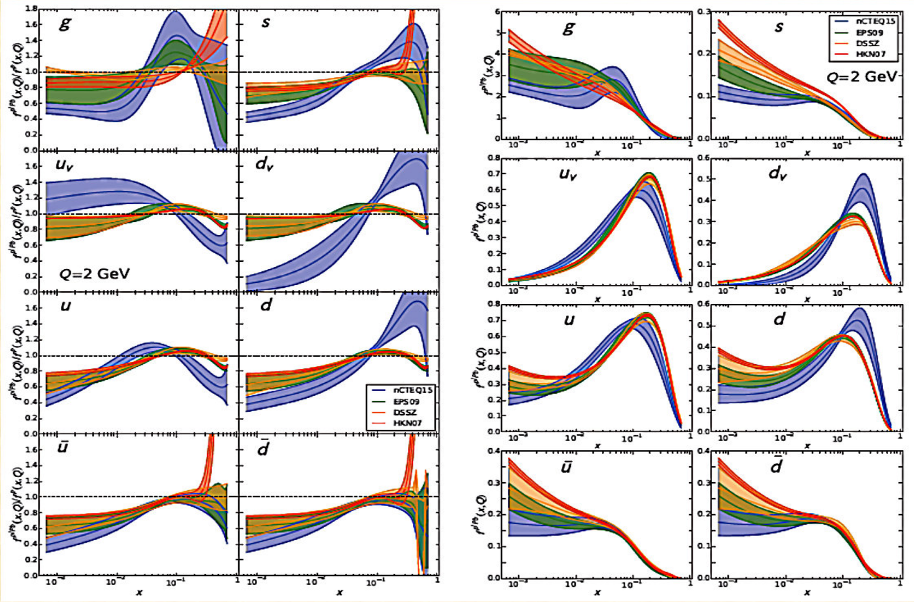
DSZS = de Florian + Sassot + Zurita + Stratmann

EPS = Eskola + Paukkunen + Salgado + ...

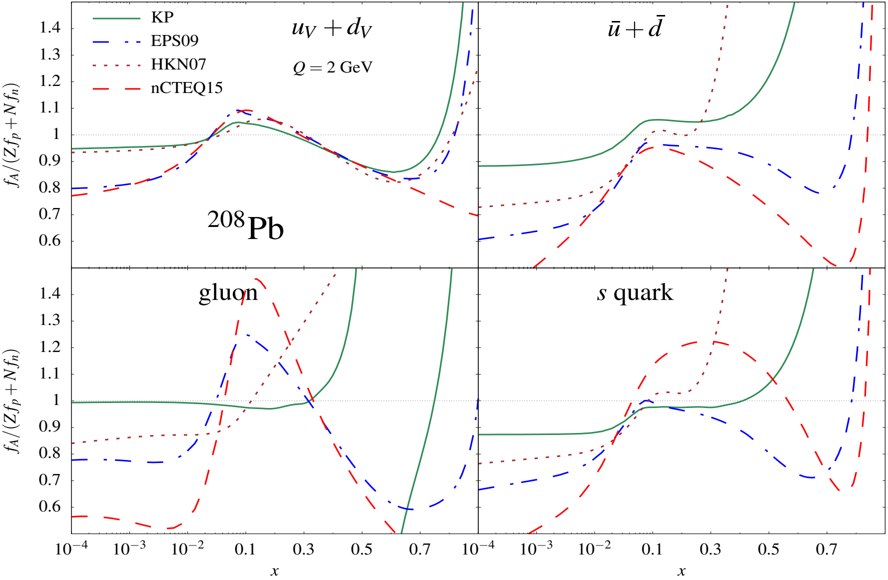
HKN = Hirai + Kumano + Nagai

nCTEQ = Kovarik + Kusina + ...

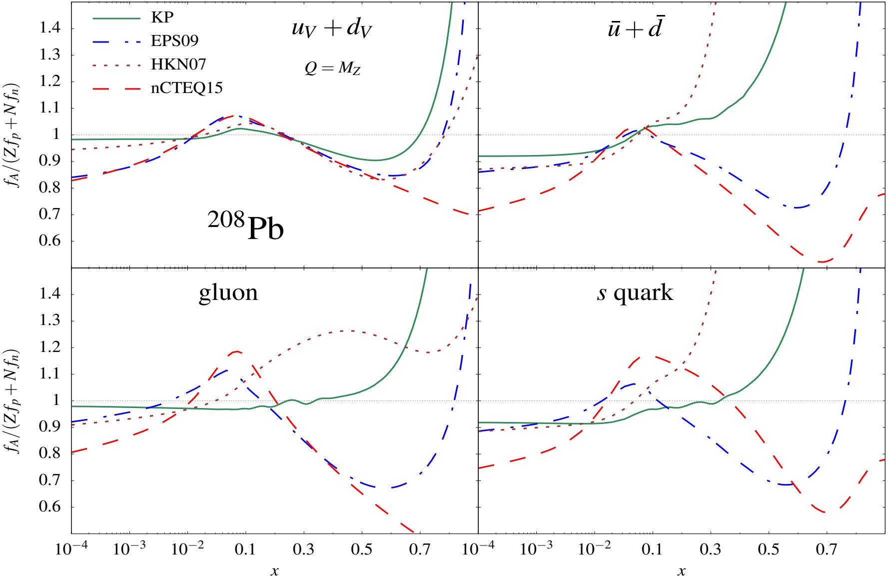
Comparison of different NPDF fits *K.Kovarik etal. arXiv:1509.00792*



Comparison of different NPDFs at $Q = 2\,\mathrm{GeV}$



Comparison of different NPDFs at $Q = M_Z$



- NPDF fits are useful in constraining the phenomenology of nuclear modification of parton distributions.
- However, it is certainly useful to understand the underlying physics mechanisms behind the observed nuclear effects.
- In this study we will discuss a few basic mechanisms of nuclear modification of parton distributions and a develop a semi-microscopic model to compute NPDFs on the base of the proton PDFs.

Why nuclear corrections survive at DIS?

Space-time scales in DIS

$$W_{\mu\nu} = \int \mathrm{d}^4x \exp(iq\cdot x)\langle p|[J_\mu(x), J_\nu(0)]|p\rangle$$

$$q\cdot x = q_0 t - |\boldsymbol{q}|z = q_0 t - \sqrt{q_0^2 + Q^2}z \simeq q_0(t-z) - \frac{Q^2}{2q_0}\,z$$

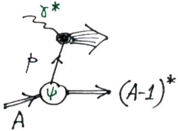
- DIS proceeds near the light cone: $|t-z| \sim 1/q_0$ and $t^2 - z^2 \sim Q^{-2}$.
- Space-time interpretation depends on the reference frame.
- In the TARGET REST FRAME the characteristic time and longitudinal distance are NOT small at all: $t \sim z \sim 2q_0/Q^2 = 1/Mx_{\mathrm{Bj}}$. DIS proceeds at the distance ~ 1 Fm at $x_{\mathrm{Bj}} \sim 0.2$ and at the distance ~ 20 Fm at $x_{\mathrm{Bj}} \sim 0.01$.
- Two different regions in nuclei from comparison of coherence length (Ioffe time) $L = 1/Mx_{\mathrm{Bj}}$ with average distance between bound nucleons r_{NN}:

$L < r_{\mathrm{NN}}$ at $x > 0.2$	Nuclear DIS $\approx$ incoherent sum of contributions from bound nucleons. Nuclear corrections $\sim EL$ and $\sim |\boldsymbol{p}|^2L^2$ where $E(p)$ typical energy (momentum) in the nuclear ground state.
$L \gg r_{\mathrm{NN}}$ at $x \ll 0.2$	Coherent effects of interactions with a few nucleons are important.

Effective theory of nuclear DIS in incoherent regime

A good starting point is approximation of incoherent scattering off bound protons and neutrons (suitable in the region of large Bjorken x). Effective T-matrix:

$$\widehat{T}_{\mu\nu} = \int \mathrm{d}^4x\mathrm{d}^4y\, e^{iqy}\, \overline{\Psi}(x)\widehat{\mathcal{T}}_{\mu\nu}(x,y)\Psi(0)$$



- $\widehat{\mathcal{T}}_{\mu\nu}(x,y)$ is effective scattering operator describing nucleon DIS.
- Ψ is the nucleon field operator.
- Matrix element over the proton (neutron) state $\langle p|\widehat{T}_{\mu\nu}|p\rangle = T^p_{\mu\nu}$ gives the proton (neutron) Compton amplitude.
- Matrix element over a nuclear state $\langle A|\widehat{T}_{\mu\nu}|A\rangle = T^A_{\mu\nu}$ gives the nuclear Compton amplitude.
- Hadronic tensor $W_{\mu\nu} = \mathrm{Im}\, T_{\mu\nu}$
- Assume $q^\mu W_{\mu\nu}(p,q) = 0$ also for the off-shell nucleon.

- Assume a nonrelativistic nuclear ground state: $|\boldsymbol{p}| \ll M,\ |\varepsilon| \ll M$ ($\varepsilon = p_0 - M$ is nonrelativistic energy.
- Examine all relevant Lorentz-Dirac structures of off-shell amplidude and make systematic expansion in $\boldsymbol{p}/M$. To order $\boldsymbol{p}^2/M^2$ we have

$$\frac{1}{M_A} W^A_{\mu\nu}(P_A, q) = \sum_{\tau=p,n} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{M+\varepsilon} \mathcal{P}^\tau(\varepsilon, \boldsymbol{p}) W^\tau_{\mu\nu}(p, q)$$

Nuclear spectral function describes bound nucleon energy-momentum distribution

$$\mathcal{P}(\varepsilon, \boldsymbol{p}) = \int \mathrm{d}t\, e^{-i\varepsilon t} \langle A|\psi^\dagger(\boldsymbol{p}, t)\psi(\boldsymbol{p}, 0)|A\rangle / \langle A|A\rangle$$

Nucleon off-shell tensor has structure similar to the on-shell one (in vicinity of the mass shell)

$$W^\tau_{\mu\nu}(p, q) = \widetilde{g}_{\mu\nu} F_1(x, Q^2, p^2) + \frac{\widetilde{p}_\mu \widetilde{p}_\nu}{p \cdot q} F_2(x, Q^2, p^2),$$

In the vicinity of the mass shell (recall, $\boldsymbol{p}/M$ effectively small), we still can describe off-shell tensor by 2 independent structure functions which have a correct on-shell limit.

Structure functions

- Apply the standard projection operators to extract the structure functions from hadronic tensor

$$F_i^A = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} K_{ij} \left(\mathcal{P}^p F_j^p + \mathcal{P}^n F_j^n \right), \quad i,j = 1,2,3$$

- Integration over the four-momentum of the bound proton (neutron) $p = (M + \varepsilon, \boldsymbol{p})$
- $\mathcal{P}^{p,n}(\varepsilon, \boldsymbol{p})$ the proton (neutron) nuclear spectral function, which describes probability to find a bound nucleon with momentum $\boldsymbol{p}$ and energy $p_0 = M + \varepsilon$. Normalized to the nucleon number $\int \mathrm{d}\varepsilon \mathrm{d}\boldsymbol{p} \mathcal{P}^p = Z$.
- The bound nucleon structure functions depend on 3 independent variables $F_2^{p,n} = F_2^{p,n}(x', p^2, Q^2)$, $x' = Q^2/2p \cdot q$ is the Bjorken variable of a nucleon with four-momentum p. Note the nucleon virtuality p^2 is additional variable for off-shell nucleon which is not present for the physical nucleon.
- The matrix of kinematical factors K_{ij}. For F_2 this matrix has only the diagonal term $K_{22} = (1 + p_z/M) \left(1 + \mathcal{O}(\boldsymbol{p}^2/|\boldsymbol{q}|^2)\right)$ (all terms are known *sk, R.Petti, 2004*).

Nuclear parton distributions

From the relations for the structure functions we obtain the relations between the nuclear and the proton/neutron PDFs in the Bjorken limit:

$$p_{i/A}(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}y\mathrm{d}p^2}{y}\, f_{\tau/A}(y,p^2)\, p_{i/\tau}(\frac{x}{y},Q^2,p^2),$$

$$f_{p,n}(y,p^2) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathcal{P}(k,\varepsilon)\left(1+\frac{k_z}{M}\right)\delta\left(y-1-\frac{\varepsilon+k_z}{M}\right)\delta(p^2-k^2)$$

Remarks:

- For any distribution function $f_{p,n}(y,p^2)$ the nuclear PDF $p_{i/A}(x,Q^2)$ automatically obeys to the DGLAP evolution equation provided that the proton/neutron PDF $p_{i/p}(x,Q^2,p^2)$ is a solution to the evolution equation.
- The nucleon distribution function $f_{p,n}(y,p^2)$ does not depend on the PDF type.

- The distribution function is normalized to the number of nucleons:

$$\int \mathrm{d}y\mathrm{d}p^2 f_{p,n}(y,p^2) = Z, N$$

- The distribution is a narrow function peaked about average light-cone momentum $y \sim 1$ (here we average over protons and neutrons)

$$\langle y \rangle = \frac{1}{A}\int \mathrm{d}y\mathrm{d}p^2\, y\, f(y,p^2) = 1 + \frac{\langle \varepsilon \rangle + \frac{2}{3}\langle T \rangle}{M}$$

$$\Delta = \langle y^2 \rangle - \langle y \rangle^2 = \frac{1}{A}\int \mathrm{d}y\mathrm{d}p^2\, (y^2 - \langle y \rangle)^2\, f(y,p^2) = \frac{2}{3}\frac{\langle T \rangle}{M}$$

where $\langle \varepsilon \rangle = \langle p_0 - M \rangle$ and $\langle T \rangle = \left\langle \boldsymbol{p}^2 \right\rangle / 2M$. Note that $\varepsilon < 0$ due to binding and $\langle \varepsilon \rangle - \langle T \rangle = \langle V \rangle$ the average potential energy of a bound nucleon.

- Average bound nucleon virtuality $v = (p^2 - M^2)/M^2$

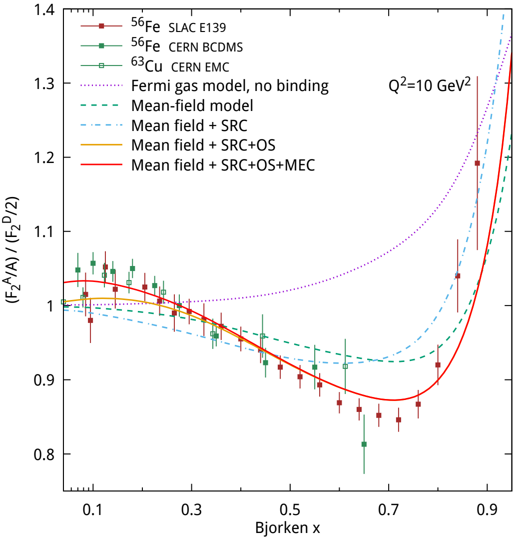
$$\langle v \rangle = \frac{1}{A}\int \mathrm{d}y\mathrm{d}p^2\, v\, f(y,p^2) = 2\frac{\langle \varepsilon \rangle - \langle T \rangle}{M}$$

Parameters of nuclear distribution for the Deuteron and Lead nuclei

Nucleus	Binding E./A (MeV)	$\langle \varepsilon \rangle$ (MeV)	$\langle T \rangle$ (MeV)	$\langle y \rangle$	Δ	$\langle v \rangle$
^{2}H	1.11	−11.56	9.33	0.994	0.0066	−0.044
^{208}Pb	7.83	−58.51	35.13	0.963	0.025	−0.197

Nuclear effects in impulse approximation

- Impulse approximation:
  $F_2(x', Q^2, p^2) = F_2(x', Q^2, M^2)$
- Momentum distribution (Fermi motion) leads to a rise at large Bjorken x *Atwood & West, 1970s*.
- Nuclear binding correction is important and results in a "dip" at $x \sim 0.6 - 0.7$
  Akulinichev, Vagradov & sk, 1984.
- However, even realistic nuclear spectral function fails to accurately explain the slope and the position of the minimum in IA. Corrections to IA are needed!



Off-shell effect

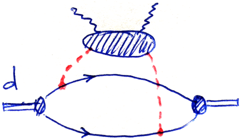
Bound nucleons are off-mass-shell ($p^2 < M^2$).
The treatment of p^2 dependence simplifies in the vicinity of the mass shell by expanding in the relative virtuality $v = (p^2 - M^2)/M^2$ *sk, Piller & Weise, 1994; sk & R.Petti, 2004*

$$q_{i/p}(x, Q^2, p^2) \approx q_{i/p}(x, Q^2) \left(1 + \delta f_i(x, Q^2)\, v\right)$$

- The function $\delta f(x, Q^2)$ describes a relative modification of the nucleon PDFs in the vicinity of the mass shell.
- Off-shell correction is closely related to modification of the nucleon size in nuclear environment.

Nuclear meson-exchange current effect (MEC)

Leptons can scatter on a meson field which mediate interaction between bound nucleons. This process generate a MEC correction to nuclear sea quark distribution *Llewellyn-Smith, Ericson, Thomas, 1983*

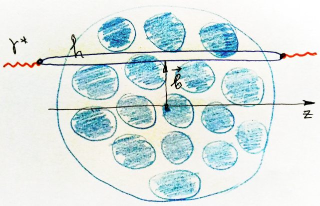


$$\delta q_{\mathsf{MEC}}(x, Q^2) = \int_x \frac{\mathrm{d}y}{y} f_{\pi/A}(y) q^{\pi}(\frac{x}{y}, Q^2)$$

- Contribution from nuclear pions (mesons) is important to balance nuclear light-cone momentum $\langle y \rangle_\pi + \langle y \rangle_N = 1$.
- The nuclear pion distribution function is localized in a region $y < p_F/M \sim 0.3$. For this reason the MEC correction to nuclear (anti)quark distributions is localized at $x < 0.3$.
- The magnitude of the correction is driven by average number of "nuclear pion excess" $n_\pi = \int \mathrm{d}y\, f_{\pi/A}(y)$ and $n_\pi/A \sim 0.1$ for a heavy nucleus like ^{56}Fe.

Nuclear shadowing

Coherent nuclear correction is due to propagation of intermediate state $\gamma^* \to h$ in nuclear environment, which can be addressed in the multiple scattering theory *Glauber, Gribov 1970s*.



$$\frac{\delta q_A^{\text{coh}}}{q_N} = \frac{\operatorname{Im}\delta\mathcal{A}}{\operatorname{Im} a}$$

$$\delta\mathcal{A} = \delta\mathcal{A}^{(2)} + \delta\mathcal{A}^{(3)} + \dots$$

$$\delta\mathcal{A}^{(2)} = ia^2 \int_{z_1<z_2} \mathrm{d}^2\boldsymbol{b}\,\mathrm{d}z_1\mathrm{d}z_2\,\rho(\boldsymbol{b}, z_1)\rho(\boldsymbol{b}, z_2)\,e^{i\frac{z_1-z_2}{L}}$$

- $\rho(\boldsymbol{r})$ is the nuclear number density, $\int \mathrm{d}^3\boldsymbol{r}\rho(\boldsymbol{r}) = A$
- $a = \frac{\sigma}{2}(i+\alpha)$ is the (effective) forward scattering amplitude of intermediate state h off the nucleon
- L is the coherence length of intermediate state which accounts finite life time of intermediate state, $1/L = Mx(1+m_h^2/Q^2)$. Its presence suppresses the coherence effect in the region of large x.

Modelling the nuclear PDFs and analysis of data

sk & R.Petti, NPA765 (2006) 126; PRC82 (2010) 054614; PRC90 (2014) 045204

$$q_{i/A} = \langle q_{i/p}(1 + v\delta f)\rangle + \langle q_{i/n}(1 + v\delta f)\rangle + \delta q_i^{\mathrm{MEC}} + \delta q_i^{\mathrm{coh}}$$

Strategy of the analysis:

- Calculate NPDF using the free proton PDF with accurate treatment of nuclear momentum distribution and energy spectrum (nuclear spectral function), MEC and coherent correction (nuclear shadowing).
- Consider the off-shell function $\delta f(x)$ and effective amplitude a as unknown and parametrize them. Study the data on the nuclear DIS in terms of the ratios $R_2(A/B) = F_2^A/F_2^B$ and determine $\delta f(x)$ and the amplitude a from data.
- Use the normalization conditions and the DIS sum rules to determine the amplitude a (responsible for nuclear shadowing) in the region of high Q^2, which is not constrained by data.
- Verify the model by comparing the calculations with data not used in analysis (new measurements at JLab, nuclear DY process, W/Z production in $p + \mathrm{Pb}$ collision at LHC).

Parameters of the model

- Off-shell structure function $\delta f(x) = C_N(x - x_1)(x - x_0)(h - x)$
  - From preliminary studies we observe that h is fully correlated with x_0, i.e. $h = 1 + x_0$.
  - C_N, x_0, x_1 are independent ajustable parameters.
- Effective amplitude

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2, \quad \bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

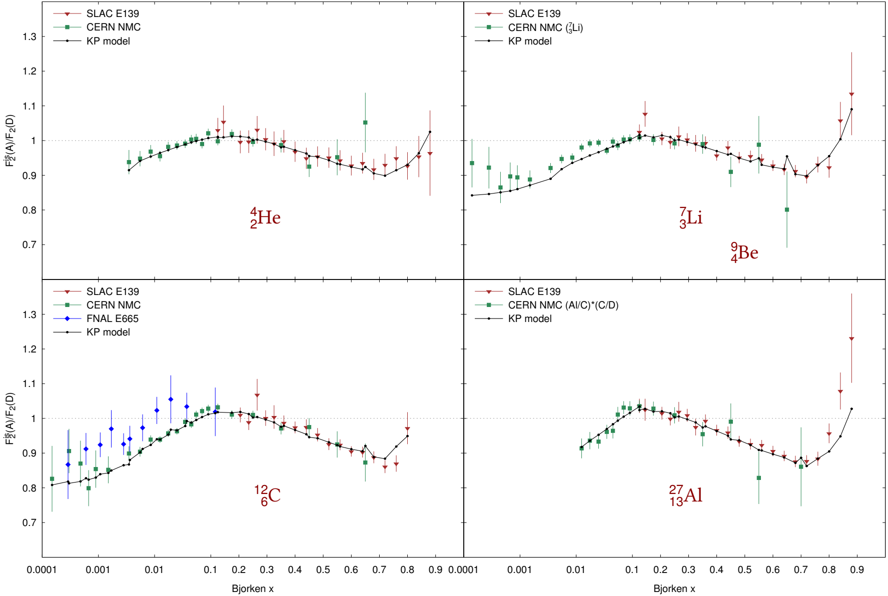
  - Parameters $\sigma_0 = 27\,\mathrm{mb}$ and $\alpha = -0.2$ were fixed in order to match the vector meson dominance model predictions at low Q.
  - Parameter $\sigma_1 = 0$ fixed (preferred by preliminary fits and fixed in the final studies).
  - Q_0^2 is adjustable scale parameter controlling transition between low and high Q regimes.

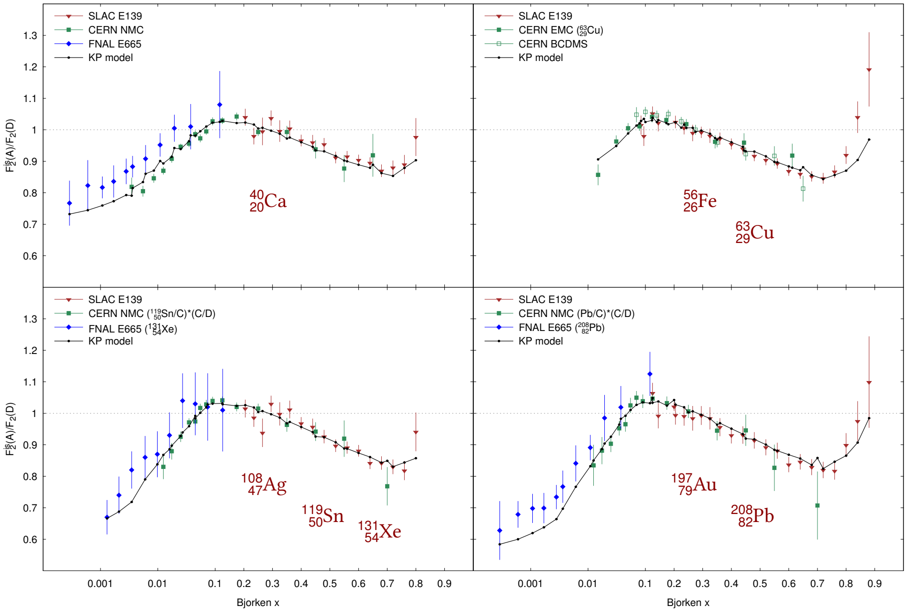
Results

- The x, Q^2 and A dependencies of the nuclear ratios are reproduced for all studied nuclei (^{4}He to ^{208}Pb) in a 4-parameter fit with $\chi^2/\text{d.o.f.} = 459/556$.
- Global fit to all data is consistent with the fits to different subsets of nuclei (light, medium, heavy nuclei).
- Parameters of the off-shell function δf and effective amplitude a_T are determined with a good accuracy.

For detailed discussion and comparison with data see *sk & R. Petti, Nucl. Phys. A765 (2006) 126*.

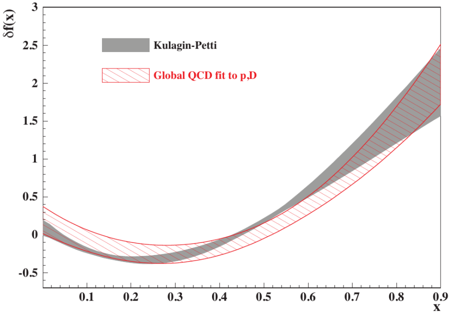
Summary of results on the nuclear ratios F_2^A/F_2^D




SLAC E139
CERN NMC
FNAL E665
KP model
$^{40}_{20}$Ca
SLAC E139
CERN EMC ($^{63}_{29}$Cu)
CERN BCDMS
KP model
$^{56}_{26}$Fe
$^{63}_{29}$Cu
SLAC E139
CERN NMC ($^{119}_{50}$Sn/C)*(C/D)
FNAL E665 ($^{131}_{54}$Xe)
KP model
$^{108}_{47}$Ag
$^{119}_{50}$Sn
$^{131}_{54}$Xe
SLAC E139
CERN NMC (Pb/C)*(C/D)
FNAL E665 ($^{208}_{82}$Pb)
KP model
$^{197}_{79}$Au
$^{208}_{82}$Pb
$F_2^{is}(A)/F_2(D)$
Bjorken x

Determination of the off-shell function $\delta f(x)$

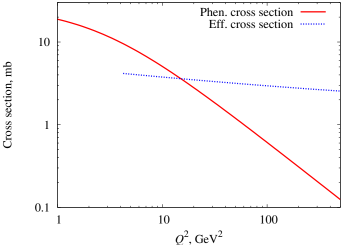
- Analysis of heavy target to deuterium ratios with a model $\delta f(x) = C_N(x-x_1)(x-x_0)(1+x_0-x)$ (gray area)
- Global QCD analysis using deuteron data and a model $\delta f(x) = Ax^2 + Bx + C$ (red) *Alekhin, sk, Petti, 2017*. Results are consistent with a heavy-target analysis of *sk & Petti, 2006*.



- The function $\delta f(x)$ provides a measure of the modification of the quark distributions in a bound nucleon.
- The slope of $\delta f(x)$ in a single-scale nucleon model is related to the change of the radius of the nucleon in the nuclear environment *sk & R.Petti, 2006*. The observed slope suggests an increase in the bound nucleon radius in the iron by about 10% and in the deuteron by about 2%.

Determination of effective cross section

- The monopole form $\sigma = \sigma_0/(1 + Q^2/Q_0^2)$ for the effective cross section of C-even $q + \bar{q}$ combination provides a good fit to data on DIS nuclear shadowing for $Q^2 < 15\ \text{GeV}^2$ with $\sigma_0 = 27\,\text{mb}$ and $Q_0^2 = 1.43 \pm 0.06 \pm 0.195\ \text{GeV}^2$. Note σ_0 is fixed from $Q^2 \to 0$ limit by the vector meson dominance model. Also we assume $\operatorname{Re} a/\operatorname{Im} a$ for C-even amplitude to be given by VMD at all energies.

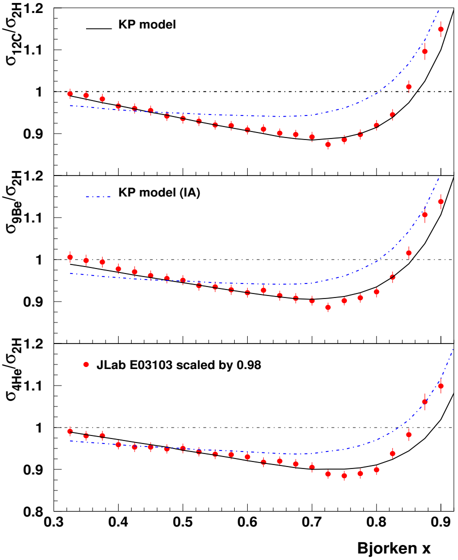


- Nuclear shadowing correction for the C-odd valence distribution $q - \bar{q}$ is also driven by same cross section σ. Note, however, important interference effect between the phases of C-even and C-odd effective amplitude.
- The cross section at high $Q^2 > 15\ \text{GeV}^2$ is not constrained by data. It is possible to evaluate σ in this region using the the normalization condition. Requiring exact cancellation between the off-shell and the shadowing correction in the normalization we have:

$$\int_0^1 \mathrm{d}x \left(\langle v \rangle\, q_{\text{val}}(x, Q^2) \delta f(x) + \delta q_{\text{val}}^{\text{coh}}(x, Q^2) \right) = 0$$

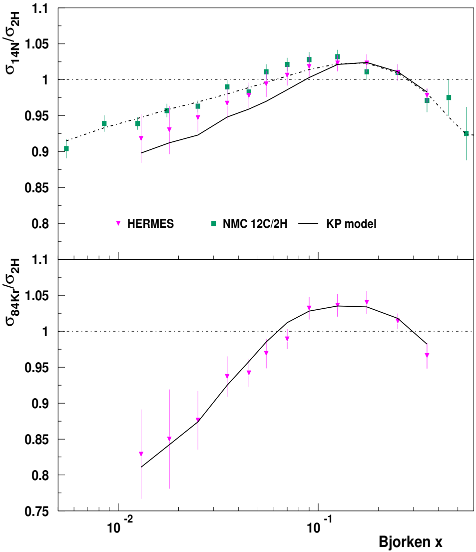
with $\langle v \rangle = \langle p^2 - M^2 \rangle / M^2$ the average nucleon virtuality. Numeric solution to this equation is shown by dotted curve.

Verification with recent JLab data (not fit)



- Very good agreement of our predictions *sk & Petti, PRC82 (2010) 054614* with JLab E03-103 for all nuclear targets: $\chi^2/d.o.f. = 26.3/60$ for $W^2 > 2$ GeV^2.
- Nuclear corrections at large x is driven by nuclear spectral function, the off-shell function $\delta f(x)$ was fixed from previous studies.
- A comparison with the Impulse Approximation (shown in blue) demonstrates that the off-shell correction is crucial to describe the data leading to both the modification of the slope and the position of the minimum of the ratios.

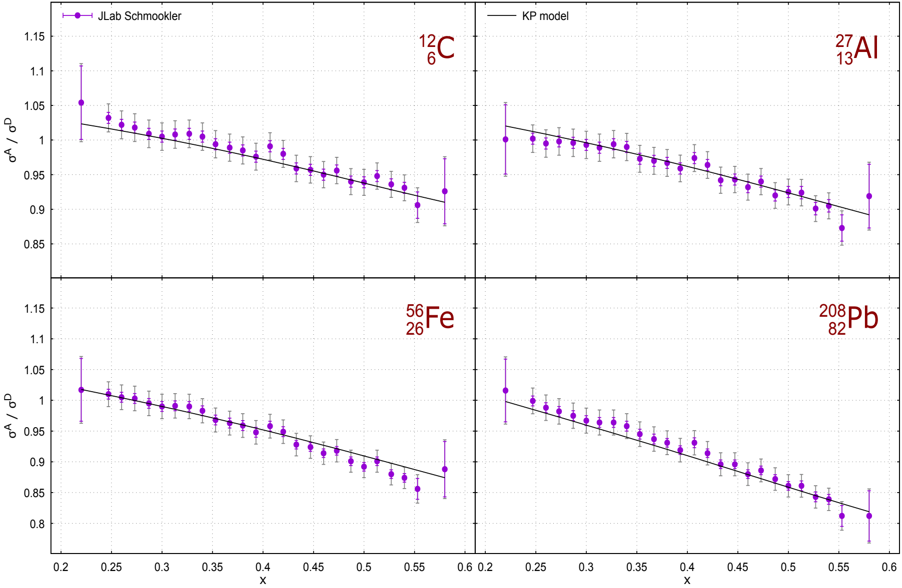
Verification with HERMES data (not fit)



- A good agreement of our predictions *sk & Petti, PRC82 (2010) 054614* with HERMES data for $^{14}\mathrm{N/D}$ and $^{84}\mathrm{Kr/D}$ with $\chi^2/d.o.f. = 14.7/24$
- A comparison with CERN NMC data for $^{12}\mathrm{C/D}$ shows a notable Q^2 dependence at small x in the shadowing region related to the Q^2 dependence of effective cross-section.

The model correctly describes the observed x and Q^2 dependence.

Comparison with new JLab data by *Schmookler etal. Nature 566 (2019) 354*



W/Z boson production in $p + Pb$ collisions at LHC

The DY mechanism of W/Z production in hadron/nuclear $A + B$ collisions:

$$\frac{\mathrm{d}^2\sigma_{AB}}{\mathrm{d}Q^2\mathrm{d}y} = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b \mathsf{PDF}_{a/A}(x_a, Q^2) \mathsf{PDF}_{b/B}(x_b, Q^2) \frac{\mathrm{d}^2\widehat{\sigma}_{ab}}{\mathrm{d}Q^2\mathrm{d}y}$$

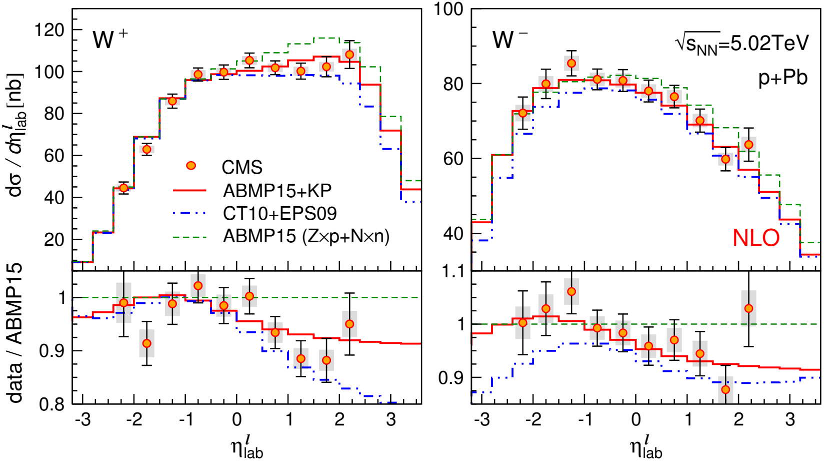
- Study rapidity distributions of produced W/Z bosons in $p + Pb$ collisions at LHC with $Q^2 \sim M_Z^2$ and $\sqrt{s} = 5.02\,\mathrm{TeV}$ using NNLO ABMP15 PDFs and DYNNLO tool.
- Compute ^{208}Pb PDFs using ABMP15 and the outlined approach.

$$x_p = \frac{M_{W,Z}}{\sqrt{s}} e^{y}, \quad x_A = \frac{M_{W,Z}}{\sqrt{s}} e^{-y}$$

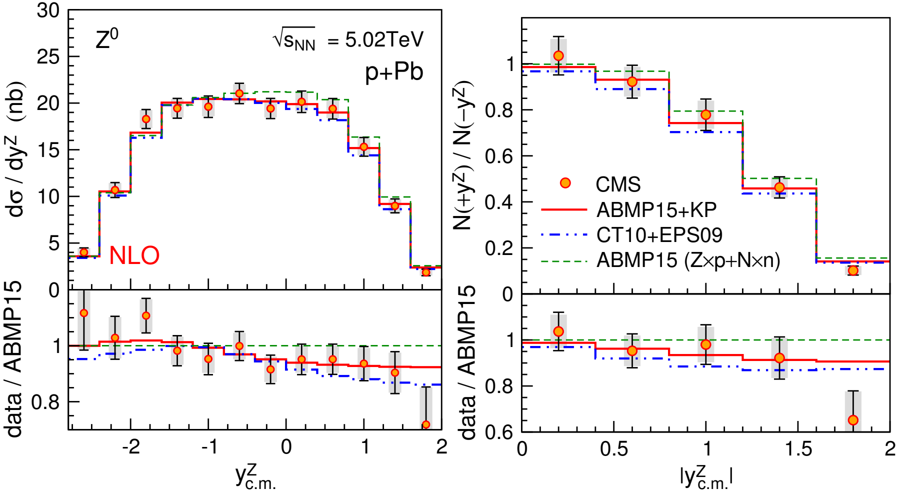
$$\frac{M_{W,Z}}{\sqrt{s}} \approx 0.016 \text{ at } \sqrt{s} = 5.02 \text{ TeV}$$

$y > 1 \implies$ small $x_A \implies$ dominated by nuclear antiquarks
$y < -1 \implies$ not so small $x_A \implies$ nuclear valence region.

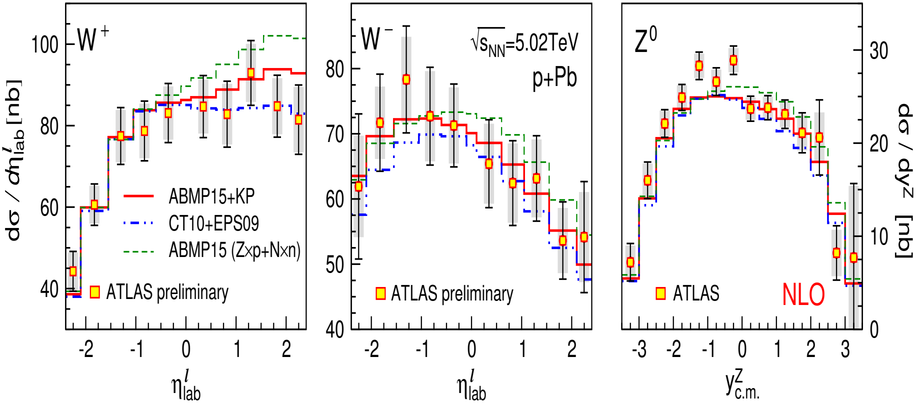
Predictions for $W^{\pm}$ in comparison with CMS data *P.Ru, sk, R.Petti, B-W.Zhang, arXiv:1608.06835*



Predictions for Z^0 in comparison with CMS data

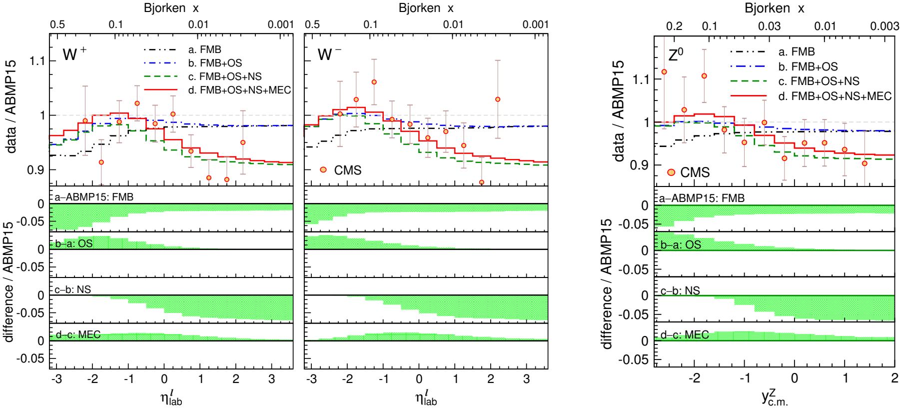


Comparison with ATLAS data on W/Z production



Separating the nuclear effects in W/Z boson production

Different nuclear effects on the production cross section of W (left) and Z boson (right) in $p + \mathrm{Pb}$ collisions at $\sqrt{s} = 5.02\,\mathrm{TeV}$ *P.Ru, sk, R.Petti, B-W.Zhang arXiv:1608.06835*.



Upper axis is Bjorken x of Pb while the lower axis is (pseudo)rapidity $(\eta)y$.

Performance of the model in terms of χ^2

Observable	N_{Data}	ABMP15 + KP	CT10 + EPS09	ABMP15 (Zp + Nn)
			CMS experiment:	
$d\sigma^+/d\eta^l$	10	1.052	1.532	3.057
$d\sigma^-/d\eta^l$	10	0.617	1.928	1.393
$N^+(+\eta^l)/N^+(-\eta^l)$	5	0.528	1.243	2.231
$N^-(+\eta^l)/N^-(-\eta^l)$	5	0.813	0.953	2.595
$(N^+ - N^-)/(N^+ + N^-)$	10	0.956	1.370	1.064
$d\sigma/dy^Z$	12	0.596	0.930	1.357
$N(+y^Z)/N(-y^Z)$	5	0.936	1.096	1.785
CMS combined	57	0.786	1.332	1.833
			ATLAS experiment:	
$d\sigma^+/d\eta^l$	10	0.586	0.348	1.631
$d\sigma^-/d\eta^l$	10	0.151	0.394	0.459
$d\sigma/dy^Z$	14	1.449	1.933	1.674
CMS+ATLAS combined	91	0.796	1.213	1.635

Summary

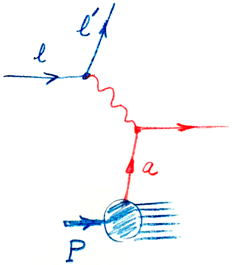
- A detailed microscopic model of nuclear PDF was presented.
  - A QCD treatment of the proton and neutron PDF and structure functions.
  - A number of nuclear effects have been addressed: Fermi motion and nuclear binding together with off-shell correction; meson-exchange currents in nuclei; coherent nuclear effects (nuclear shadowing).
  - The nuclear effects are not universal and differ for the valence and the sea-quark distributions.
- A detailed study of nuclear DIS data shows a good performance of the approach:
  - An accurate description of the ratios of nuclear structure functions F_2^A/F_2^B (nuclear EMC effect) both in the valence and the sea region.
  - A good description of the cross section of W and Z boson production in $p+\mathrm{Pb}$ collisions at LHC.
  - Not discussed today:
    - An accurate description of data on the ratio of cross sections of nuclear DY process (nuclear sea at relatively large Bjorken x).
    - A good performance in the description of (anti)neutrino differential and total cross sections from the measurements by CCFR, NuTeV, NOMAD, CHORUS *S.K.&R.Petti,PRD76(2007)094023; S.K. arXiv:1606.07016*

Extra slides

- PDFs are light-cone momentum distributions of partons (quarks, antiquarks, gluons) in a target. PDFs drive cross sections of different hard processes

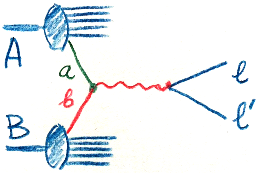
Lepton DIS

$$F_2^\gamma = (\tfrac{2}{3})^2 x(u+\bar u) + (\tfrac{1}{3})^2 x(d+\bar d) + \dots$$
$$F_2^{W^+} = x(d+\bar u + \dots)$$
$$F_2^Z = (\tfrac{1}{3} - \tfrac{8}{9}\sin^2\theta_W)x(u+\bar u) + (\tfrac{1}{6} - \tfrac{2}{9}\sin^2\theta_W)x(d+\bar d) + \dots$$



Hadron hard collisions

$$\sigma_{AB} = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b\, \mathsf{PDF}_{a/A}(x_a)\mathsf{PDF}_{b/B}(x_b)\widehat{\sigma}_{ab}$$



- PDFs are hard to calculate from first principles in QCD since it requires solving QCD in a strong coupling regime.
- PDFs scale dependence is governed by perturbative quark-gluon interaction with the coupling $\alpha_S(Q^2)$ (DGLAP evolution equation *Dokshitser-Gribov-Lipatov-Altareli-Parisi, 1970s*).
- At practice the proton PDF are obtained from global fits to high-energy data (charged-lepton DIS, Drell-Yan process, W-boson production) assuming some functional form of PDFs at a fixed scale $Q^2 = Q_0^2$

$$p_i(x, Q_0^2) = A_i x^{a_i} (1-x)^{b_i} (1 + c_i x + \cdots) \quad i = u_V,\ d_V,\ \bar{u},\ \bar{d},\ \bar{s},\ g$$

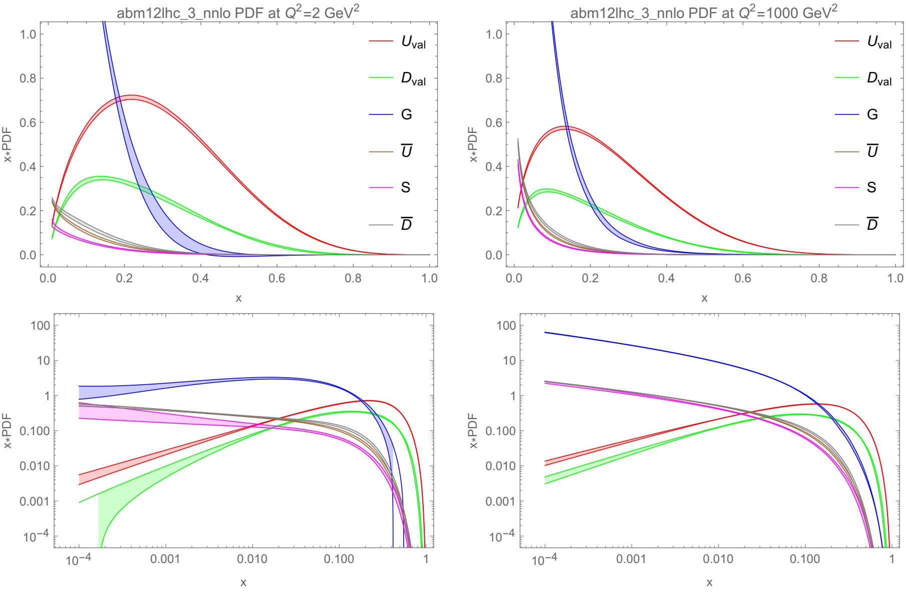
- A few global analyses are available which are regularly updated. To name a few:

ABM = Alekhin + Blümlein + Moch + ...

CTEQ = Coordinated Theoretical-Experimental project on QCD

HERAPDF = H1 and ZEUS Collaborations from HERA

MRST = Martin + Stirling + Thorn + ...


abm12lhc_3_nnlo PDF at Q^2=2 GeV^2
abm12lhc_3_nnlo PDF at Q^2=1000 GeV^2
U_val
D_val
G
Ū
S
D̄
x*PDF
x

Targets	χ^2/DOF						
	NMC	EMC	E139	E140	BCDMS	E665	HERMES
^{4}He/^{2}H	10.8/17		6.2/21				
^{7}Li/^{2}H	28.6/17						
^{9}Be/^{2}H			12.3/21				
^{12}C/^{2}H	14.6/17		13.0/17				
^{9}Be/^{12}C	5.3/15						
^{12}C/^{7}Li	41.0/24						
^{14}N/^{2}H							9.8/12
^{27}Al/^{2}H			14.8/21				
^{27}Al/^{12}C	5.7/15						
^{40}Ca/^{2}H	27.2/16		14.3/17				
^{40}Ca/^{7}Li	35.6/24						
^{40}Ca/^{12}C	31.8/24					1.0/5	
^{56}Fe/^{2}H			18.4/23	4.5/8	14.8/10		
^{56}Fe/^{12}C	10.3/15						
^{63}Cu/^{2}H		7.8/10					
^{84}Kr/^{2}H							4.9/12
^{108}Ag/^{2}H			14.9/17				
^{119}Sn/^{12}C	94.9/161						
^{197}Au/^{2}H			18.2/21	2.4/1			
^{207}Pb/^{2}H						5.0/5	
^{207}Pb/^{12}C	6.1/15					0.2/5	

Values of χ^2/DOF between different data sets with $Q^2 \geq 1$ GeV^2 and the model predictions *NPA765(2006)126; PRC82(2010)054614*.

Structure functions not only PDFs

If Q^2 is large compared the nucleon mass, the operator product expansion in QCD produces power series:

$$F_2(x,Q^2) = F_2^{LT,TMC}(x,Q^2) + \frac{H_2(x,Q)}{Q^2} + \cdots$$

The leading term is given in terms of PDFs convoluted with coefficient functions:

$$\begin{aligned} F_2^{LT} &= \left[1 + \frac{\alpha_S}{2\pi} C_q^{(1)}\right] \otimes x \sum_q e_q^2 (q + \bar{q}) \\ &+ \frac{\alpha_S}{2\pi} C_g^{(1)} \otimes xg + \mathcal{O}(\alpha_S^2) \end{aligned}$$

The HT terms involve interaction between quarks and gluons and lack simple probabilistic interpretation. In the region of high Bjorken x and/or low Q^2 (small W^2) one has to account for the target mass correction *Georgi & Politzer, 1976*

$$F_2^{LT,TMC}(x,Q^2) = \frac{x^2}{\xi^2\gamma^2} F_2^{LT}(\xi,Q^2) + \frac{6x^3M^2}{Q^2\gamma^4} \int_\xi^1 \frac{\mathrm{d}z}{z^2} F_2^{LT}(z,Q^2) + \mathcal{O}(Q^{-4})$$

$\xi = 2x/(1+\gamma)$ is the Nachtmann variable and $\gamma^2 = 1 + 4x^2M^2/Q^2$. In this work we use the results of the PDF global analysis performed to QCD NNLO approximation (i.e. to order α_S^2) and which includes the proton (and deuteron) data sets from DIS, DY and collider data. Kinematical range $0.8 < Q^2 < 10^5$ GeV^2 and $10^{-6} < x < 1$ with the cut $W > 1.8$ GeV *Alekhin, Melnikov, Petriello, 2007*; *Alekhin, S.K., Petti, 2007*

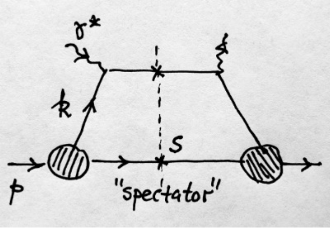
Off-shell effect and the bound nucleon radius

The valence quark distribution in a (off-shell) nucleon

S.K., Piller & Weise, 1994; *S.K. & Petti, 2004*

$$q_{\text{val}}(x,p^2) = \int^{k^2_{\text{max}}} \mathrm{d}k^2 \Phi(k^2,p^2)$$

$$k^2_{\text{max}} = x\left(p^2 - s/(1-x)\right)$$



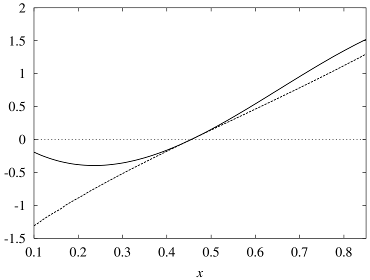
- A one-scale model of quark k^2 distribution: $\Phi(k^2) = C\phi(k^2/\Lambda^2)/\Lambda^2$, where C and ϕ are dimensionless and Λ is the scale.
- Off-shell: $C \to C(p^2),\ \Lambda \to \Lambda(p^2)$
- The derivatives $\partial_x q_{\text{val}}$ and $\partial_{p^2} q_{\text{val}}$ are related in this model

$$\delta f(x) = \frac{\partial \ln q_{\text{val}}}{\partial \ln p^2} = c + \frac{\mathrm{d}q_{\text{val}}(x)}{\mathrm{d}x} x(1-x)h(x)$$

$$h(x) = \frac{(1-\lambda)(1-x) + \lambda s/M^2}{(1-x)^2 - s/M^2}$$

$$c = \frac{\partial \ln C}{\partial \ln p^2},\quad \lambda = \frac{\partial \ln \Lambda^2}{\partial \ln p^2}$$

- A simple pole model $\phi(y) = (1-y)^{-n}$ (note that $y < 0$ so we do not run into singularity) provides a resonable description of the nucleon valence distribution for $x > 0.2$ and large Q^2 ($s = 2.1$ GeV^2, $\Lambda^2 = 1.2$ GeV^2, $n = 4.4$ at $Q^2 = 15 \div 30$ GeV^2).
- The size of the valence quark confinement region $R_c \sim \Lambda^{-1}$ (nucleon core radius).
- Fix c and λ to reproduce $\delta f(x_0) = 0$ and the slope $\delta f'(x_0)$. We obtain $\lambda \approx 1$ and $c \approx -2.3$. The positive parameter λ suggests the decreasing scale Λ in nuclear environment (or "swelling" of a bound nucleon).

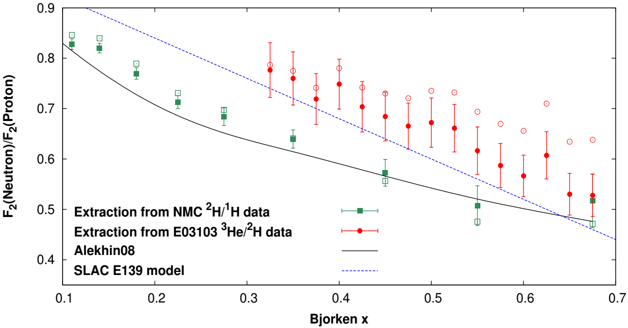


$$\frac{\delta R_c}{R_c} \sim -\frac{1}{2}\frac{\delta\Lambda^2}{\Lambda^2} = -\frac{1}{2}\lambda\frac{\langle p^2 - M^2\rangle}{M^2}$$

^{56}Fe : $\delta R_c/R_c \sim 0.09$

^{2}H : $\delta R_c/R_c \sim 0.02$

Using F_2^n/F_2^p as a consistency test of nuclear data

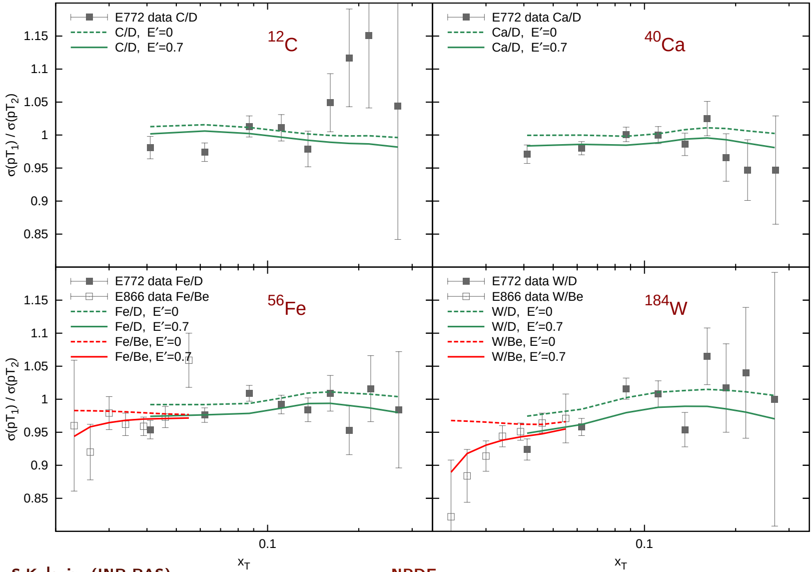


Extraction of F_2^n/F_2^p from F_2^p/F_2^D (NMC) and $F_2^{^3\mathrm{He}}/F_2^D$ (JLab) with account of nuclear effect (full symbols) and also with no nuclear effects (open symbols).

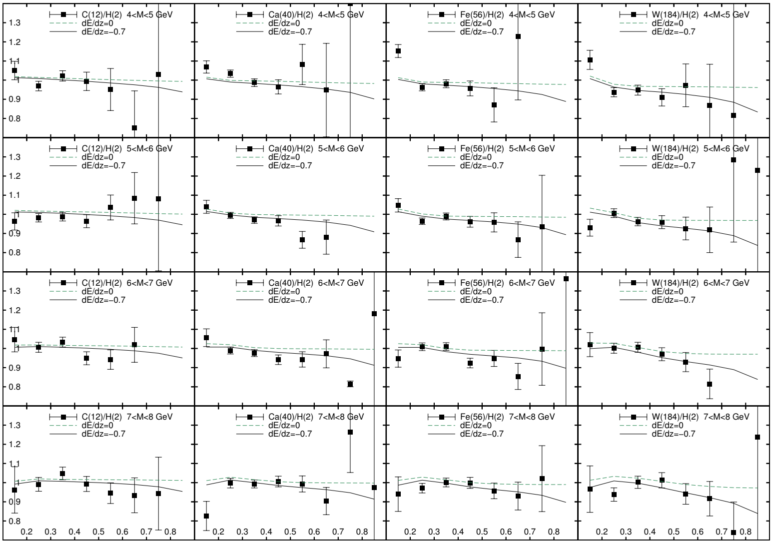
- Mismatch in F_2^n/F_2^p extracted from different experiments. At $x \sim 0.35$, where nuclear corrections are negligible, the ratio F_2^n/F_2^p from JLab E03-103 is 15% bigger than that from NMC.
- Normalization of F_2^n/F_2^p is directly related to the normalization of $^3\mathrm{He/D}$. Requiring F_2^n/F_2^p from JLab to match NMC, we obtain a renormalization factor of $1.03^{+0.006}_{-0.008}$ for the central values of JLab $^3\mathrm{He/D}$ measurement.

Comparison with E772 & E866 measurements

S.K. & R.Petti, PRC90(2014)045204



Detailed comparison with E772 by dimuon mass bin

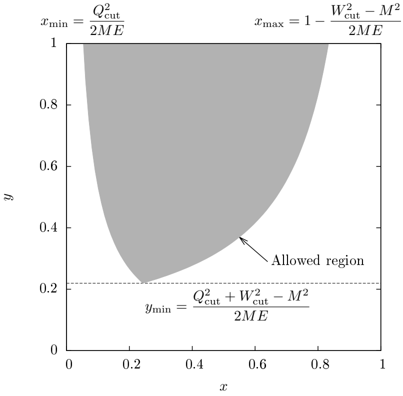


Neutrino cross sections

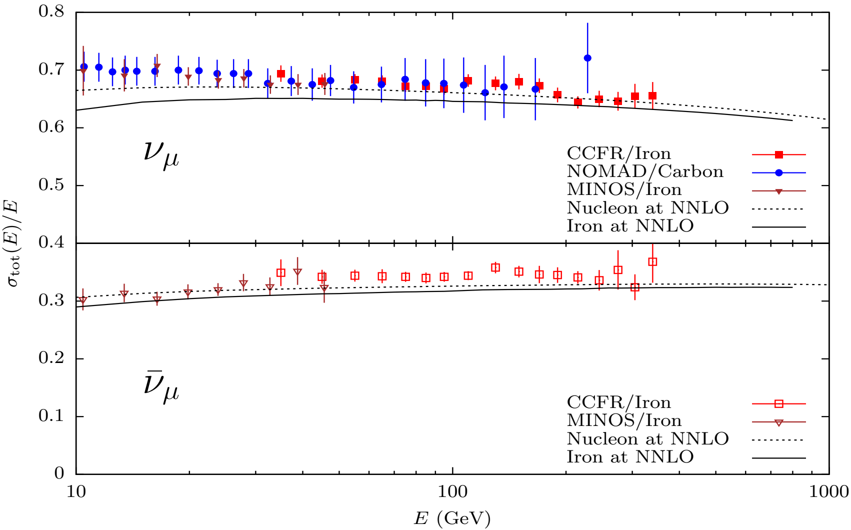
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = \int \mathrm{d}y \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y}\theta(Q^2 - Q^2_{cut})\theta(W^2 - W^2_{cut})$$

$$\sigma_{\text{tot}} = \int \mathrm{d}x\mathrm{d}y \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y}\theta(Q^2 - Q^2_{cut})\theta(W^2 - W^2_{cut})$$

In general the variables x and y are bound to $0 < x < 1$, $0 < y < 1$. The cuts on Q^2 and W^2 lead to further restrictions on the integration region.



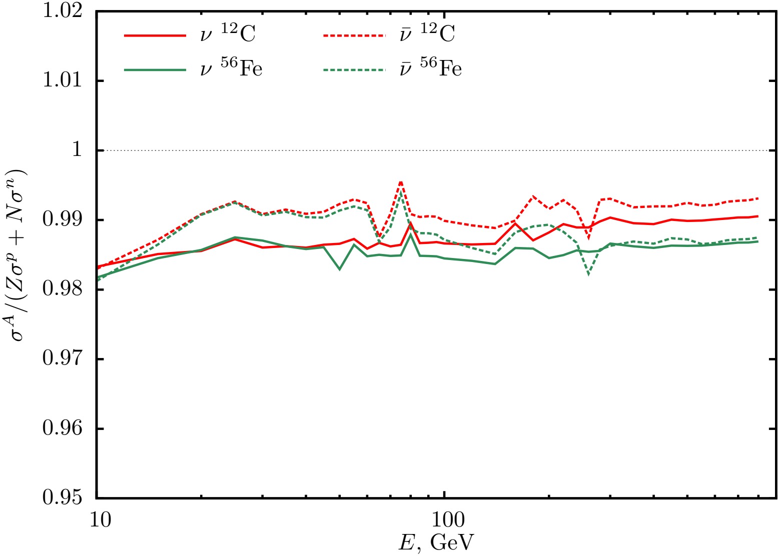
Neutrino total cross sections



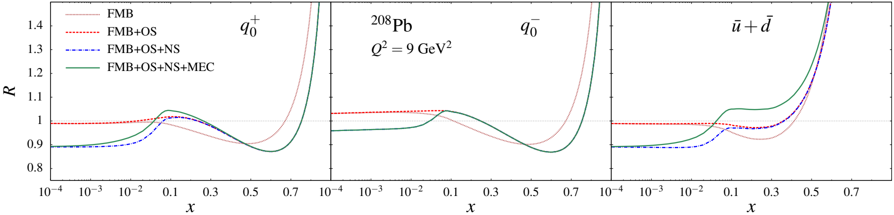
Note that theory includes DIS contribution only.

Nuclear corrections on total neutrino cross sections

Ratios $\sigma_A/(Z\sigma_p + N\sigma_n)$ of total cross sections as calculated in our model.

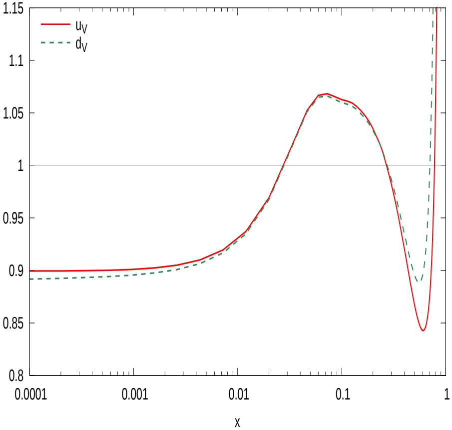


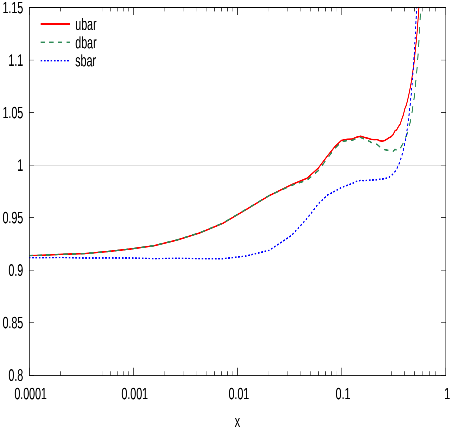
Splitting nuclear effects on different PDFs



Nuclear effects on valence quarks vs. antiquarks

The ratios $R_a = q_{a/A}/(Zq_{a/p} + Nq_{n/A})$ computed for the valence u and d (left) and the corresponding antiquarks (right) *S.K. & R.Petti, PRC90(2014)045204*.





Nuclear spectral function

The nuclear spectral function describes probability to find a bound nucleon with momentum $\boldsymbol{p}$ and energy $p_0 = M + \varepsilon$:

$$
\begin{aligned}
\mathcal{P}(\varepsilon, \boldsymbol{p}) &= \int \mathrm{d}t\, e^{-i\varepsilon t} \langle \psi^\dagger(\boldsymbol{p}, t) \psi(\boldsymbol{p}, 0) \rangle \\
&= \sum_i \left| \langle (A-1)_i, -\boldsymbol{p} | \psi(0) | A \rangle \right|^2 2\pi\delta \left(\varepsilon + E_i^{A-1}(\boldsymbol{p}) - E_0^A \right)
\end{aligned}
$$

- The sum runs over all possible states of the spectrum of $A-1$ residual system.
- The nuclear spectral function determines the rate of nucleon removal reactions such as $(e, e'p)$. For low separation energies and momenta, $|\varepsilon| < 50$ MeV, $p < 250$ MeV/c, the observed spectrum is dominated by bound states $A-1$ similar to those predicted by the mean-field model.
- High-energy and high-momentum components of nuclear spectrum is not described by the mean-field model and driven by correlation effects in nuclear ground state (short-range correlations, or SRC). We combine the mean-field together with SRC contributions and consider a two-component model $\mathcal{P} = \mathcal{P}_{\mathrm{MF}} + \mathcal{P}_{\mathrm{cor}}$ *Ciofi degli Atti & Simula, 1995*; *S.K. & Sidorov, 2000*; *S.K. & Petti, 2004*

Sketch of the mean-field picture

In the the mean-field model the bound states of $A-1$ nucleus are described by the one-particle wave functions ϕ_λ of the energy levels λ. The spectral function is given by the sum over the occupied levels with the occupied number n_λ:

$$\mathcal{P}_{\mathrm{MF}}(\varepsilon, \boldsymbol{p}) = \sum_{\lambda<\lambda_F} n_\lambda |\phi_\lambda(\boldsymbol{p})|^2 \delta(\varepsilon - \varepsilon_\lambda)$$

- Due to interaction effects the δ-peaks corresponding to the single-particle levels acquire a finite width (fragmentation of deep-hole states).
- High-energy and high-momentum components of nuclear spectrum can not be described in the mean-field model and driven by short-range nucleon-nucleon correlation effects in the nuclear ground state as witnessed by numerous studies.

High-momentum part

- As nuclear excitation energy becomes higher the mean-field model becomes less accurate. High-energy and high-momentum components of nuclear spectrum can not be described in the mean-field model and driven by correlation effects in nuclear ground state as witnessed by numerous studies.
- The corresponding contribution to the spectral function is driven by $(A-1)^*$ excited states with one or more nucleons in the continuum. Assuming the dominance of configurations with a correlated nucleon-nucleon pair and remaining $A{-}2$ nucleons moving with low center-of-mass momentum we have

$$|A{-}1, -\boldsymbol{p}\rangle \approx \psi^{\dagger}(\boldsymbol{p}_1)|(A{-}2)^*, \boldsymbol{p}_2\rangle\delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}).$$

The matrix element can thus be given in terms of the wave function of the nucleon-nucleon pair embeded into nuclear environment. We assume factorization into relative and center-of-mass motion of the pair

$$\langle (A{-}2)^*, \boldsymbol{p}_2 \,|\psi(\boldsymbol{p}_1)\psi(\boldsymbol{p})|\, A\rangle \approx C_2 \psi_{\rm rel}(\boldsymbol{k}) \psi_{\rm CM}^{A-2}(\boldsymbol{p}_{\rm CM}) \delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}),$$

where $\psi_{\rm rel}$ is the wave function of the relative motion in the nucleon-nucleon pair with relative momentum $\boldsymbol{k} = (\boldsymbol{p} - \boldsymbol{p}_1)/2$ and $\psi_{\rm CM}$ is the wave function of center-of-mass (CM) motion of the pair in the field of $A{-}2$ nucleons, $\boldsymbol{p}_{\rm CM} = \boldsymbol{p}_1 + \boldsymbol{p}$. The factor C_2 describes the weight of the two-nucleon correlated part in the full spectral function.

$$\mathcal{P}_{\rm cor}(\varepsilon, \boldsymbol{p}) \approx n_{\rm cor}(\boldsymbol{p}) \left\langle \delta\left(\varepsilon + \frac{(\boldsymbol{p} + \boldsymbol{p}_{A-2})^2}{2M} + E_{A-2} - E_A \right) \right\rangle_{A-2}$$

Average separation and kinetic energies

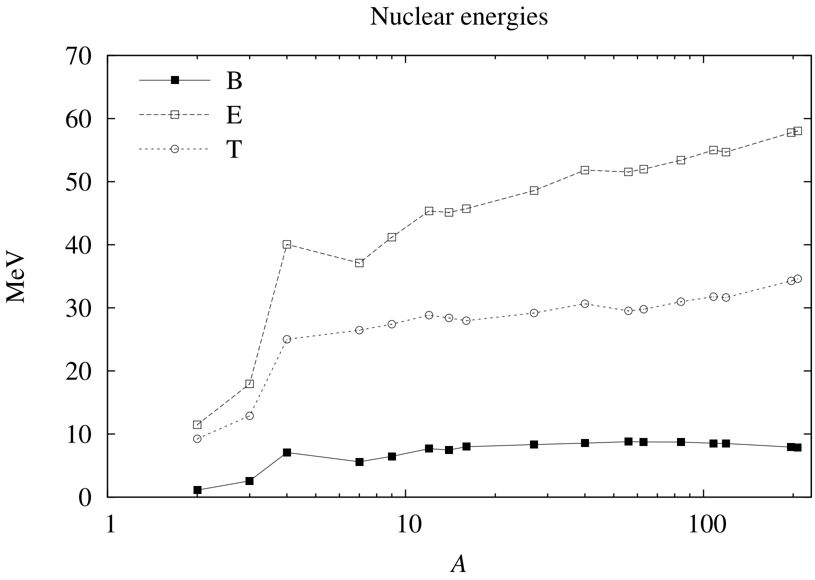
Average separation $\langle\varepsilon\rangle$ and kinetic $\langle T\rangle$ energies are related by the Koltun sum rule (exact relation for nonrelativistic system with two-body forces)

$$\langle\varepsilon\rangle + \langle T\rangle = 2\varepsilon_B,$$

where $\varepsilon_B = E_0^A/A$ is nuclear binding energy per bound nucleon

$$\langle\varepsilon\rangle = A^{-1}\int[\mathrm{d}p]\mathcal{P}(\varepsilon,\boldsymbol{p})\varepsilon,$$
$$\langle T\rangle = A^{-1}\int[\mathrm{d}p]\mathcal{P}(\varepsilon,\boldsymbol{p})\frac{\boldsymbol{p}^2}{2M}.$$

Nuclear binding, separation and kinetic energies



The two-component model of the spectral function

In what follows we combine the mean-field together with SRC contributions and consider a two-component model *Ciofi degli Atti & Simula, 1995 S.K. & Sidorov, 2000 S.K. & Petti, 2004*

$$\mathcal{P} = \mathcal{P}_{\mathrm{MF}} + \mathcal{P}_{\mathrm{cor}}$$

We assume that the normalization is shared between the MF and the correlated parts as 0.8 to 0.2 for the nuclei $A \geq 4$ [for $^{208}\mathrm{Pb}$ 0.75 to 0.25] following the observations on occupation of deeply-bound proton levels *NIKHEF 1990s, 2001*.