Heavy-quark mass effects on transverse-momentum resummed distributions

Giancarlo Ferrera

Milan University & INFN Milan

Heavy-Quark Hadroproduction from Collider to Astroparticle Physics
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Motivations

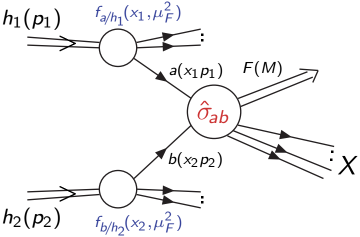
Transverse-momentum (q_T) distributions are extremely important observables at hadron colliders:

- Constraints of parton densities (PDFs).
- M_W measurement (from W q_T).
- Higgs boson characterization.
- Beyond the Standard Model searches.
- Perturbative QCD studies.

The above reasons and precise experimental data from LHC demands for accurate theoretical predictions $\Rightarrow$ computation of higher-order QCD corrections, EW corrections, heavy-quark mass effects, non-perturbative effects.

QCD factorization

$h_1 + h_2 \;\rightarrow\; F(M) + X$



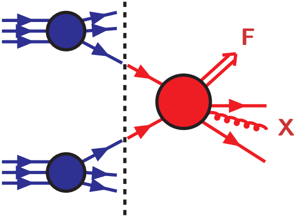
The framework: **QCD factorization formula**

$$\sigma^F(s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \hat{\sigma}^F_{ab}(x_1 x_2 s;\mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{M}\right)^p$$

- $f_{a/h}(x,\mu_F^2)$: Non perturbative universal parton densities (PDFs), $\mu_F \sim M$.
- $\hat{\sigma}_{ab}$: Hard scattering cross section. Process dependent, calculable with a perturbative expansion in the strong coupling $\alpha_S(M)$ ($M \gg \Lambda_{QCD} \sim 1\,\text{GeV}$).
- $\left(\frac{\Lambda_{QCD}}{M}\right)^p$ (with $p \geq 1$): Non perturbative power-corrections.

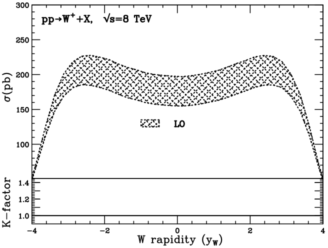
Precise predictions for σ depend on good knowledge of both $\hat{\sigma}_{ab}$ and $f_{a/h}(x,\mu_F^2)$

Higher-order calculations



Parton densities (PDFs)

Partonic cross section $\hat{\sigma}$



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

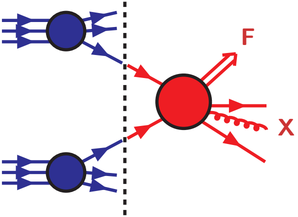
- Perturbation theory at **leading order (LO)**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)} + \alpha_S^2\, \hat{\sigma}^{(2)} + \mathcal{O}(\alpha_S^3)$$

- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO: precise prediction & **robust uncertainty**.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities**: impossible direct use of numerical techniques.

Higher-order calculations



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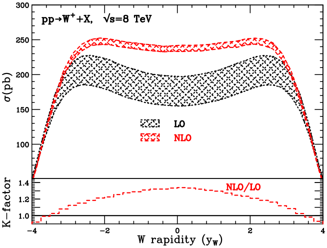

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- Perturbation theory at **next order (NLO)**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)} + \alpha_S^2\, \hat{\sigma}^{(2)} + \mathcal{O}(\alpha_S^3)$$

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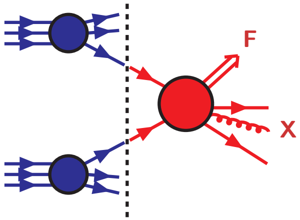

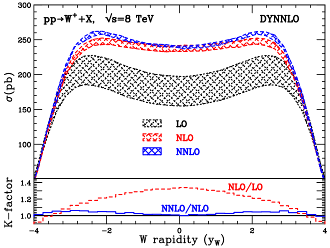

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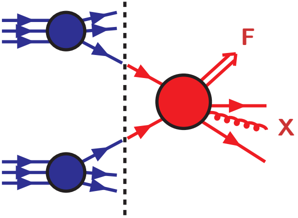
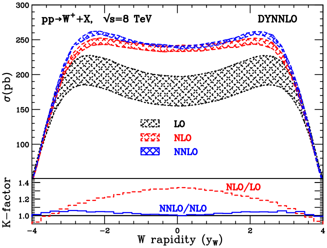


Parton densities (PDFs) | Partonic cross section $\hat{\sigma}$



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$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)} + \alpha_S^2\, \hat{\sigma}^{(2)} + \mathcal{O}(\alpha_S^3)$$

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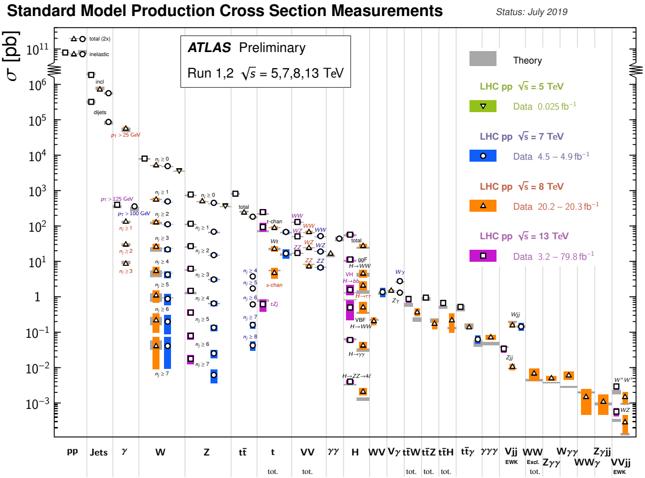
Drell-Yan process

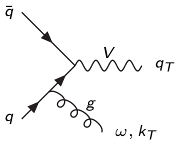
LO	**NLO**	**NNLO**
Drell,Yan (1974)	Altarelli,Ellis,Greco Martinelli,(1980-84)	Melnikov,Petriello; Catani,Cieri,deFlorian, G.F.,Grazzini (2007–09)

LHC results



- Very good agreement between experimental results and SM theoretical predictions for hard processes

All-order Sudakov resummation



Example: vector boson production at small q_T ($q_T \ll M$).
The standard fixed-order QCD perturbative expansions gives:

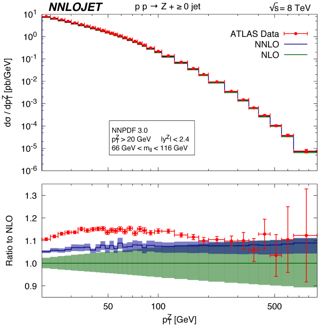
$$\int_0^{Q_T^2} dq_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim 1 + \alpha_S \left[c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right]$$

$$+\alpha_S^2 \left[c_{24} \log^4(M^2/Q_T^2) + \cdots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + \mathcal{O}(\alpha_S^3)$$

The logs are the residue of the cancellation of the real-virtual infrared singularities due to soft/collinear gluon emissions (recoiling radiation is forced to be soft/collinear).

Fixed order calculation reliable only for $q_T \sim M$

For $q_T \to 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections.



Z q_T spectrum at the LHC.

State of the art: q_T resummation

- Large q_T logarithms resummation in b-space [Parisi,Petronzio('79)],[Kodaira,Trentadue('82)],[Collins,Soper,Sterman('85)], [Altarelli et al.('84)],[Catani,d'Emilio,Trentadue('88)],[Catani,de Florian, Grazzini('01)],[Catani,Grazzini('10)],[Catani,Grazzini,Torre('14)]
- Various phenomenological studies [ResBos:Balasz,Yuan,Nadolsky et al.('97,'02)], [Ellis et al.('97)],[Kulesza et al.('02)],[Banfi et al.('12)], [Guzzi et al.('13)].
- Results for q_T resummation in the framework of Effective Theories and within p_T space formalisms: [Gao,Li,Liu('05)],[Idilbi,Ji,Yuan('05)],[Mantry,Petriello('10)], [Becher,Neubert('10)],[Chiu et al.('12)],[Dokshitzer,Diakonov,Troian('78)], [Frixione,Nason,Ridolfi('99)],[Erbert,Tackmann('17)],[Monni,Re,Torrielli('16)], [Bizon et al.('17,'18)].
- Studies within transverse-momentum dependent (TMD) factorization and TMD parton densities [D'Alesio,Murgia('04)],[Roger,Mulders('10)],[Collins('11)], [D'Alesio et al.('14)].
- Effective q_T-resummation can be obtained with Parton Shower algorithms. Results for NNLO predictions matched with PS obtained [Hoeche,Li,Prestel('14)], [Karlberg,Re,Zanderighi('14)], [Alioli,Bauer,Berggren,Tackmann,Walsh('14)].

Analytic resummation

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation ($L = \log(M^2/q_T^2)$).

$\alpha_S L^2$	$\alpha_S L$	$\cdots$	$\mathcal{O}(\alpha_S)$
$\alpha_S^2 L^4$	$\alpha_S^2 L^3$	$\cdots$	$\mathcal{O}(\alpha_S^2)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	$\cdots$	$\mathcal{O}(\alpha_S^n)$
dominant logs	$\cdots$	$\cdots$	$\cdots$

- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
- Ratio of two successive columns $\mathcal{O}(1/L)$: resummed expansion valid when $1/L \ll 1$.

Sudakov resummation feasible when:
dynamics AND kinematics factorize $\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions based on colour coherence (analogous of the independent multiple soft-photon emission is QED):

$$dw_n(q_1, \ldots, q_n) \simeq \frac{1}{n!} \prod_{i=1}^{n} dw_1(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform)

$$\int d^2\mathbf{q_T}\ \exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta\left(\mathbf{q_T} - \sum_{j=1}^{n}\mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

$\mathbf{q_T}$ resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int dq_T^2 \, \frac{d\hat{\sigma}^{(res)}}{dq_T^2} \overset{q_T \to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int dq_T^2 \, \frac{d\hat{\sigma}^{(fin)}}{dq_T^2} \overset{q_T \to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \, \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\{\mathcal{G}_N(\alpha_S, L)\}$$

with $\quad L \equiv \log(M^2 b^2)$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, $(\hat{\sigma}^{(0)})$; NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

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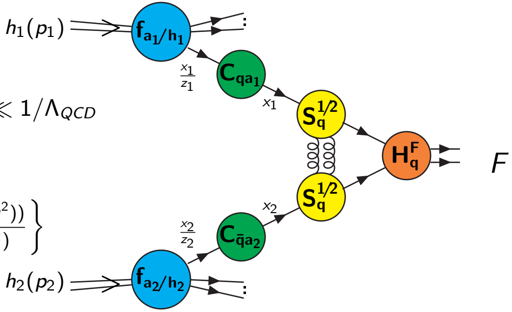
Transverse-momentum resummation formula



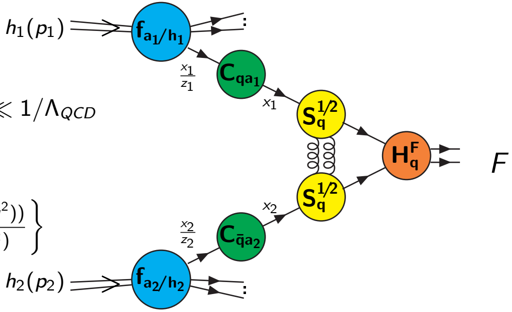
$$M \gg \Lambda_{QCD}\ ,\quad b \gg 1/M\ ,\quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times\ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln\alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\ dM^2\ dy\ d\mathbf{\Omega}} = \frac{M^2}{s}\left[d\sigma^{(0)}_{q\bar{q},F}\right]\sum_{a_1,a_2}\int\frac{d^2\mathbf{b}}{(2\pi)^2}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2}\ f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\ f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a\int_x^1\frac{dz}{z}\sqrt{S_q(M,b)}C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z,b_0^2/b^2)$$

Transverse-momentum resummation formula



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$$\times \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{q\bar{q};a_1a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

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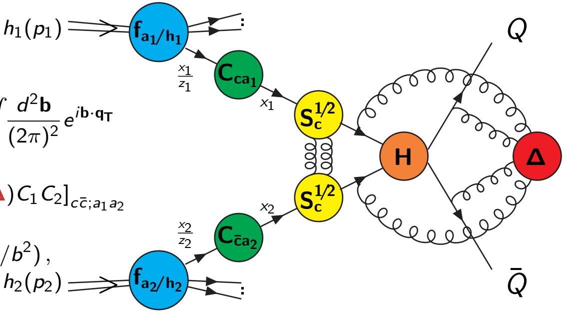
$\mathbf{q_T}$ resummation for heavy-quark hadroproduction

[Catani,Grazzini,Torre,Sargsyan('14,'18)]

$$\frac{d\sigma^{(res)}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c}}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}$$

$$\times\; S_c(M,b)\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\,[(\mathbf{H}\boldsymbol{\Delta})C_1C_2]_{c\bar{c};a_1a_2}$$

$$\times\; f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\,,$$



- Main difference with colourless case: soft factor (colour matrix) $\boldsymbol{\Delta}(\mathbf{b},M;\mathbf{\Omega})$ which embodies soft (wide-angle) emissions from $Q\bar{Q}$ and from initial/final-state interferences (no collinear emission from heavy-quarks).
- Soft radiation produce colour-dependent azimuthal correlations at small-q_T entangled with the azimuthal dependence due to gluonic collinear radiation.
- Soft-factor $\boldsymbol{\Delta}(\mathbf{b},M;\mathbf{\Omega})$ consistent with breakdown (in weak form) of TMD factorization (additional process-dependent non-perturbative factor needed) [Collins,Qiu('07)].

The q_T resummation formalism [Catani et al.('01,'03,'06))]

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2b^2) \to \ln(Q^2b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2b^2) \to \widetilde{L} \equiv \ln(Q^2b^2+1) \quad \Rightarrow \quad \exp\{\alpha_S^n \widetilde{L}^k\}\big|_{b=0} = 1 \Rightarrow \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right) = \hat{\sigma}^{(tot)};$$

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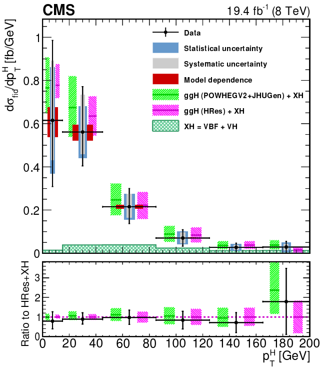
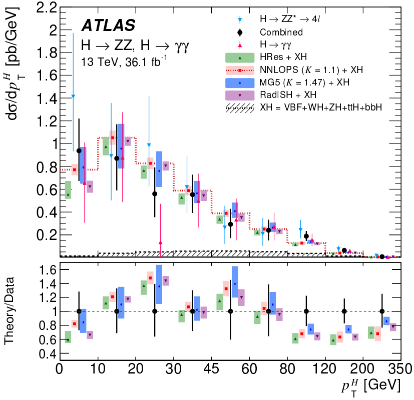
Higgs results: q_T-resummation with H boson decay

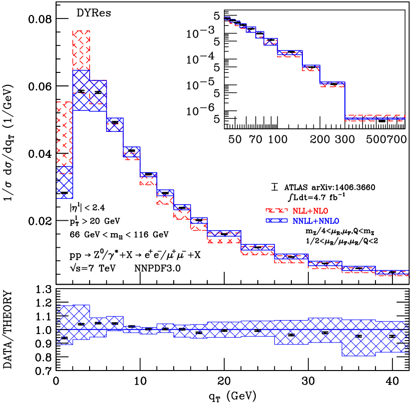


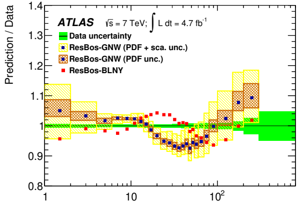
H q_T spectrum ($H \to WW$): theory predictions (HRes [de Florian, G.F., Grazzini, Tommasini ('12)]) compared with CMS data (from [CMS Coll. ('16)]).
Lower panel: ratio to theory (HRes).

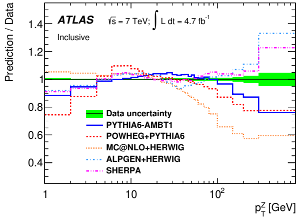


H q_T spectrum ($H \to \gamma\gamma$): various theory predictions (HRes [de Florian, G.F., Grazzini, Tommasini ('12)]) compared with ATLAS data (from [ATLAS Coll. ('18)]).
Lower panel: ratio to theory (HRes).

q_T spectrum of Z boson: theory vs ATLAS data

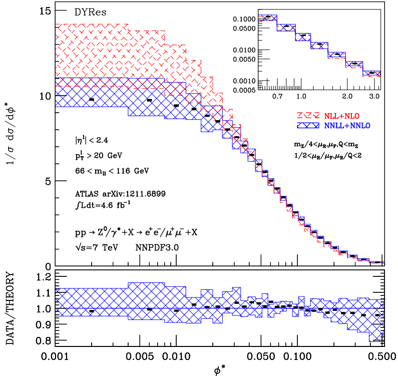




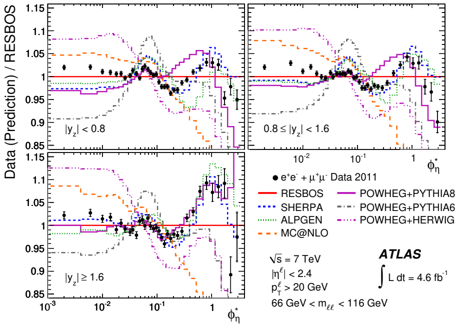


Left: NLL+NLO and NNLL+NNLO bands for Z/γ^* q_T spectrum compared with and ATLAS data (7 TeV).
Right Top: Ratios between ResBos predictions and ATLAS data.
Right Bottom: Ratios between various MC generators results and ATLAS data.

ϕ^* spectrum of Z boson: theory vs ATLAS data

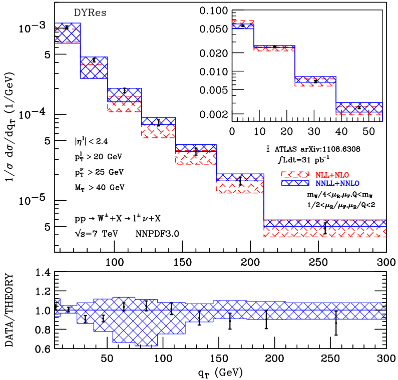


NLL+NLO and NNLL+NNLO bands for Z/γ^* ϕ^* spectrum compared with ATLAS data.

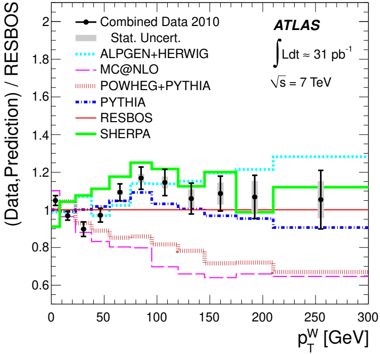


MC generators results and ATLAS data ratio to ResBos.

q_T spectrum of W: theory vs ATLAS data



NLL+NLO and NNLL+NNLO bands for $W^{\pm}$ q_T spectrum compared with ATLAS data.



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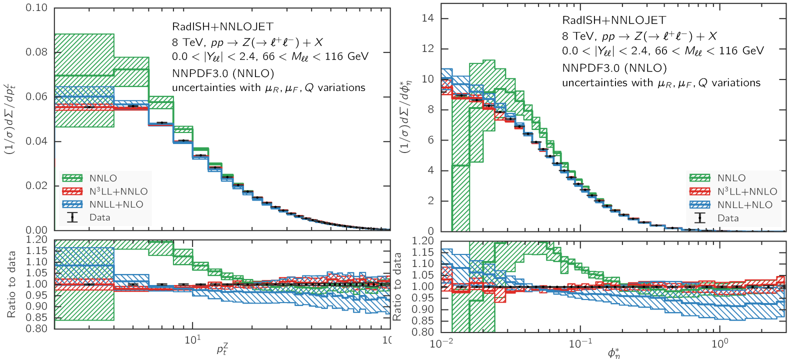
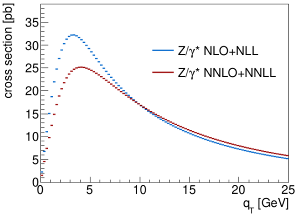


NNLO,NLL+NNLO and N3LL+NNLO bands for Z/γ^* q_T and ϕ^* spectrum compared with and ATLAS data (7 TeV). Matching with $\mathcal{O}(\alpha_S^3)$ at large q_T, from [Bizon et al.('18])

Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott (in preparation)]

Example calculation



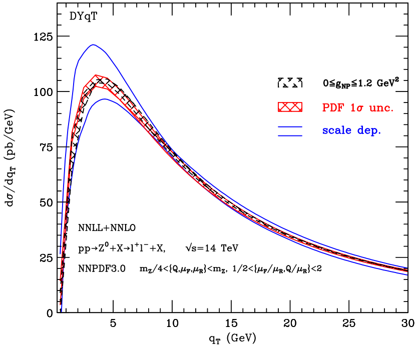
- Example calculation for Z p_T spectrum at 13 TeV
  - No cuts on the leptons
  - Full rapidity range
  - 100 p_T bins
  - 20 parallel threads

Time required	RES	CT	V+jet
NLO+NLL	6 s	0.2 s	4 min
NNLO+NNLL	10 s	0.7 s	3.4 h

- The most demanding calculation is V+jet

  → can use APPLgrid/FASTnlo for this term

PDF uncertainties and NP effects



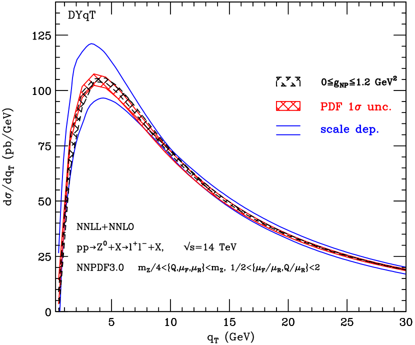
NNLL+NNLO result for Z q_T spectrum at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\,GeV^2$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \to \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

- NP effects increase the hardness of the q_T spectrum at small values of q_T.
- NNLL+NNLO result with NP effects very close to perturbative result except for $q_T < 3GeV$ (i.e. below the peak).

PDF uncertainties and NP effects



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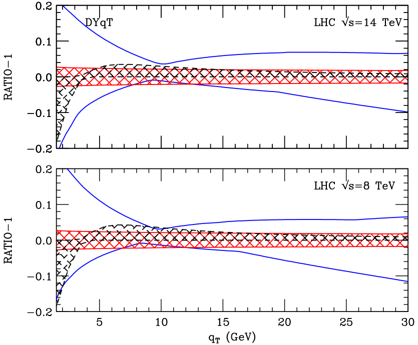

NNLL+NNLO result for Z q_T spectrum at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects normalized to central NNLL+NNLO prediction.

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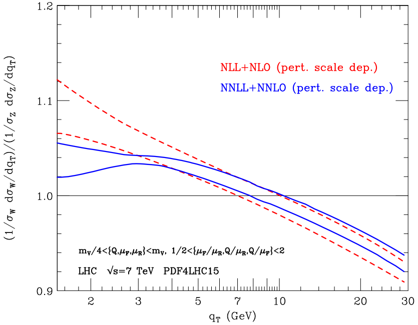
W/Z ratio: the q_T spectrum



Ratio of NNLL+NNLO and NLL+NLO results for W/Z q_T spectra at the LHC. Perturbative scale dependence.

- Ratio of W/Z observables substantially reduces both the experimental and theoretical systematic uncertainties [Giele,Keller('97)].
- *Correlated* ($\mu^W/M_W = \mu^Z/M_Z$) scale variations by factor 2 (avoiding ratios larger than 2) gives reasonable estimate of pert. uncertainty (nice overlap of scale variation bands for $q_T > 3\,GeV$).
- PDF uncertainty dominates at very small (q_T $q_T \lesssim 5\,GeV$).
- Non trivial interplay of perturbative and NP effects.

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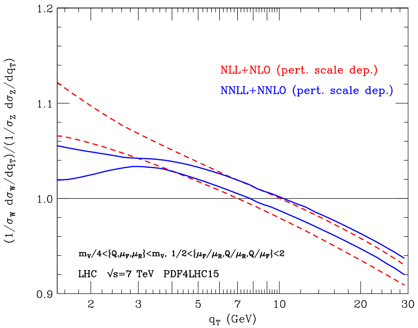

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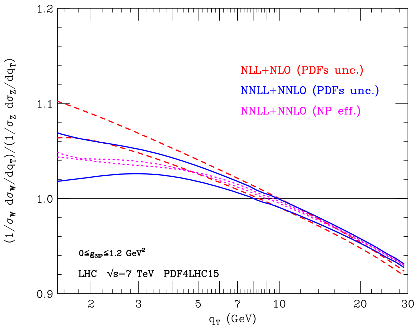

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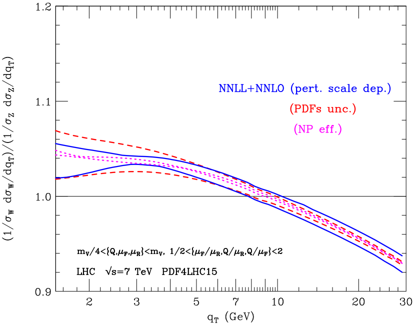

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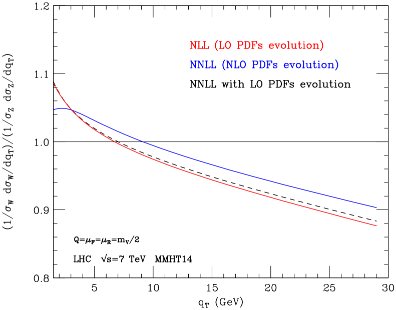
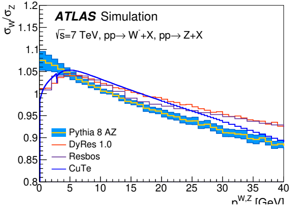
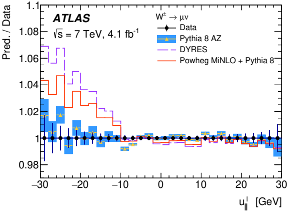


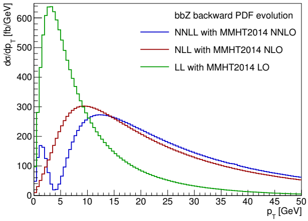




Left: Ratio of NNLL results for W/Z q_T spectra at the LHC, effect of PDFs evolution.
Right: Ratios of the normalised W/Z q_T spectra predicted by Pythia 8 and several resummation programs for W^+ and W^-.

PDF evolution and heavy-quark thresholds

$bb \to Z$ in Dyres



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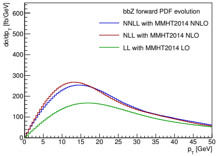
$bb \to Z$ in Dyres with forward evolution

A possible improvement is using VFN scheme forward evolution, from Q_0 to $b0/b \sim p_T$, using Pegasus, and matching exactly the order of the evolution with the selected PDF.



The p_T distribution of the $bb \to Z$ process becomes more physical, and better converging at LL, NLL, and NNLL.

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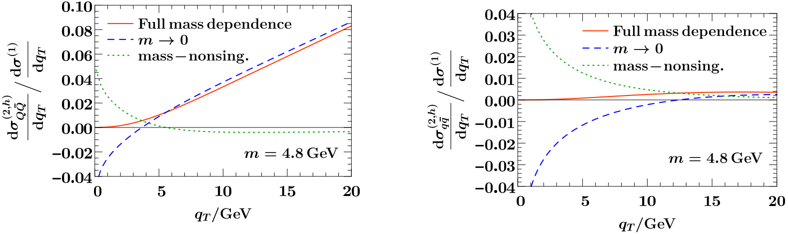
Fixed-flavour ($n_f = 5$) number scheme (FFN) backward evolution (left) vs variable-flavour (VFN) forward evolution (right) from S.Camarda .

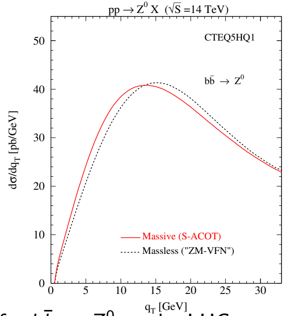
Heavy-quark mass effects in q_T resummation

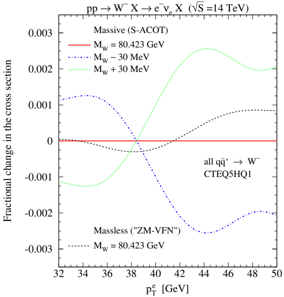


Primary (left) and secondary (right) massive bottom quark contributions for the Z q_T spectrum at fixed $\mathcal{O}(\alpha_S^2)$ order relative to the full $\mathcal{O}(\alpha_S)$ result [Pietrulewicz et al.('17)].

Heavy-quark mass effects in q_T resummation

$$\tilde{F}_{q_f/h}(x,b,M,m_Q) = \sum_a \int_x^1 \frac{dz}{z} \sqrt{S_q(M,b,,m_Q)} C_{q_f\,a}(z;\alpha_S(b_0^2/b^2),m_Q)\, f_{a/h}(x/z,b_0^2/b^2,m_Q)$$





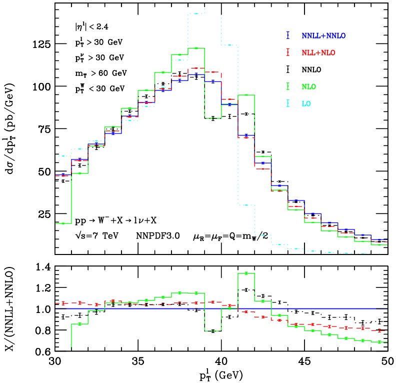
Left: $b\bar{b} \to Z^0$ at the LHC, massive "S-ACOT" scheme (red solid) vs massless "VFN" scheme (black dashed). Right: The fractional change in the distribution $d\sigma/dp_T^e$ for $pp \to W^- X \to e^- \nu_e X$ at the LHC, relative difference wrt massless "VFN" and S-ACOT schemes and effects by variations of M_W by ± 30 MeV (dotted green and dot-dashed blue) [Berge et al.('05)].

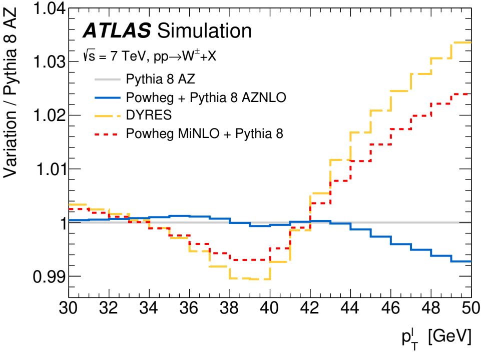
Conclusions

- Very precise experimental data have been collected by the LHC and much more data are forthcoming with the High-Luminosity (HL-LHC) upgrade.
- To fully exploit the information contained in the experimental data precise theoretical predictions of the SM cross sections are necessary $\Rightarrow$ computation of **higher-order and all-order pQCD corrections, non perturbative corrections and heavy-quark mass effects**.
- Discussed the formalism to perform all-order resummation for transverse-momentum distribution and preliminary studies on heavy-quark mass effects.

Back up slides

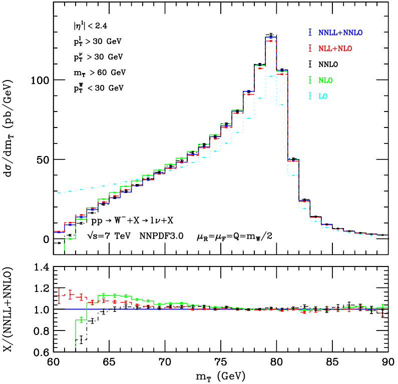
Lepton p_T distributions from W decay

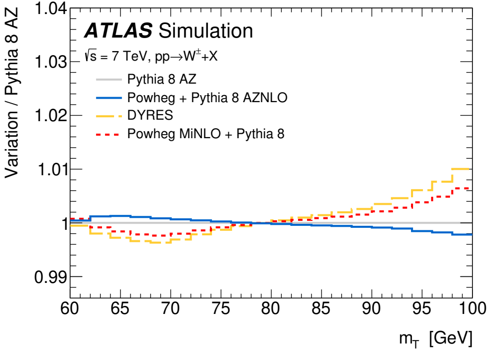




Ratios of the lepton p_T normalised distribution obtained using Powheg+Pythia 8 AZNLO, DYRES and Powheg MiNLO+Pythia 8 to the distribution obtained using PYTHIA 8 AZ.

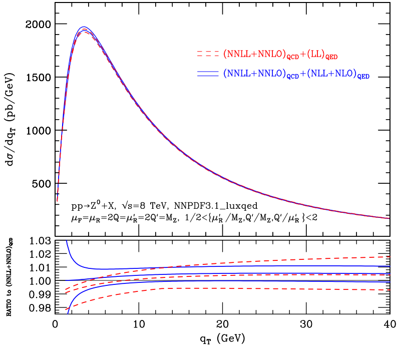
Transverse-mass distributions from W decay



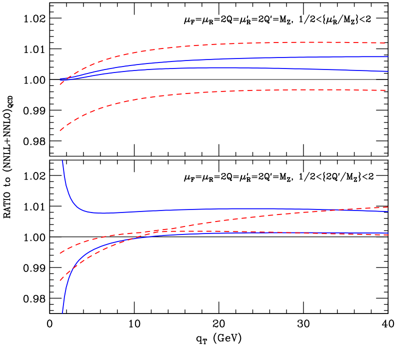


Ratios of the m_T normalised distribution obtained using Powheg+Pythia 8 AZNLO, DYRES and Powheg MiNLO+Pythia 8 to the distribution obtained using PYTHIA 8 AZ.

Combined QED and QCD q_T resummation for Z production at the LHC [Cieri,G.F.,Sborlini ('18)]

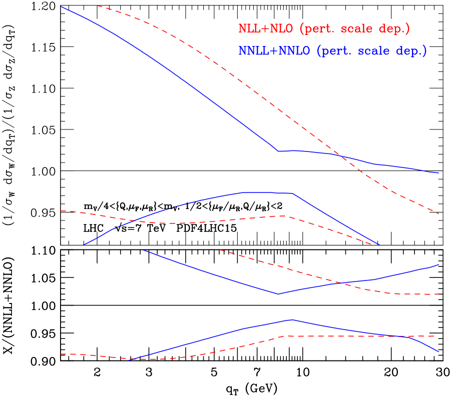


Z qT spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.

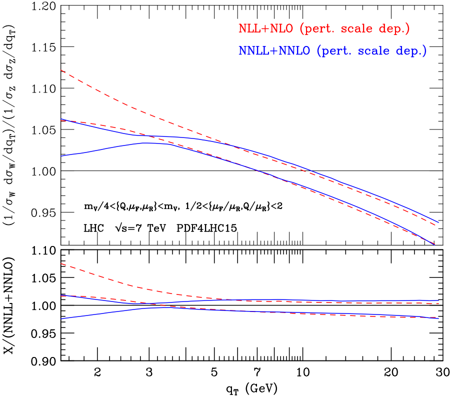


Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

W/Z ratio q_T spectrum: perturbative scale uncertainty

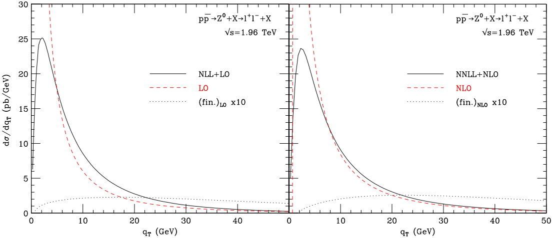


DYqT resummed predictions for the ratio of W/Z normalized q_T spectra. Uncorrelated perturbative scale variation band.



DYqT resummed predictions for the ratio of W/Z normalized q_T spectra. Correlated perturbative scale variation band.

Matching with fixed-order results



- To obtain a uniform accuracy over the range $q_T \ll M$ up to $q_T \sim M$, *resummed* and *fixed-order* components have to be consistently matched $\frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2}$,

$$\left[\frac{d\hat{\sigma}_{ab}^{(fin.)}}{dq_T^2}\right]_{f.o.} = \left[\frac{d\hat{\sigma}_{ab}}{dq_T^2}\right]_{f.o.} - \left[\frac{d\hat{\sigma}_{ab}^{(res.)}}{dq_T^2}\right]_{f.o.}$$

- Finite NLO component contribution is: $\lesssim 1\%$ near the peak, $\sim 8\%$ at $q_T \sim 20\ GeV$, $\sim 60\%$ at $q_T \sim 50\ GeV$.
- Integral of the matched curve reproduce the total cross section to better 1% (check of the code).

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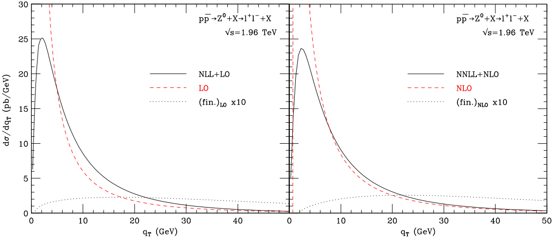

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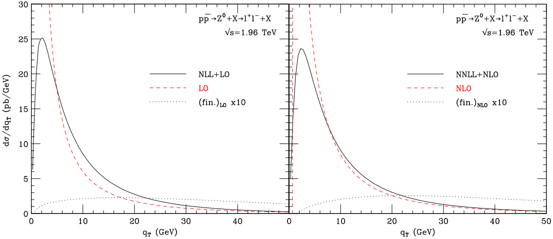

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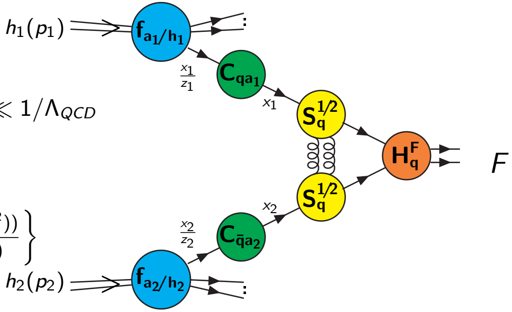
q_T resummation formula ($q\bar{q}$ annihilation)



$$M \gg \Lambda_{QCD} \; , \quad b \gg 1/M \; , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times \; \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \; \beta(\alpha_S(q^2)) \; \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T} \, dM^2 \, dy \, d\mathbf{\Omega}} = \frac{M^2}{s} \left[d\sigma^{(0)}_{q\bar{q},F} \right] H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \sum_{a_1, a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} \; e^{i\mathbf{b}\cdot\mathbf{q_T}} \; S_q(M, b)$$
$$\times \int_{x_1}^{1} \frac{dz_1}{z_1} \; C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^{1} \frac{dz_2}{z_2} \; C_{\bar{q}\, a_2}(z_2; \alpha_S(b_0^2/b^2)) \; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$S_q(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \; \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} \quad .$$

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- *All-order universal* factorization formula relates $H_c^F(\alpha_S)$ to the virtual amplitude

$$\mathcal{M}_{ab\to F} = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{M}_{ab\to F}^{(n)} \,,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \tilde{I}^{(n)}(\epsilon) \,,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{ab\to F} = \left[1 - \tilde{I}(\epsilon, M^2)\right] \mathcal{M}_{ab\to F} \,,$$

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$$H_q^F(\alpha_S) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}|^2}$$

- $\mathcal{H}^{F\,(2)}_{NNLO}$ coefficients calculated [Catani,Grazzini ('11)],[Catani,Cieri,de Florian, G.F.,Grazzini ('12)].
- Fully-differential NNLO QCD calculations implemented in public available Monte Carlo programs: DYNNLO, 2γNNLO [Catani,Cieri,de Florian,G.F.,Grazzini ('09),('12)], HVNNLO [G.F.,Grazzini,Tramontano ('11),('14),('15)].

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HqT/DYqT: q_T-resummation at NNLL

- We have performed the resummation up to NNLL+NNLO. It means that our complete formula includes:
  - NNLL logarithmic contributions to all orders (i.e. $\mathcal{O}(\alpha_S^n L^{n-1})$ in the exponent);
  - NNLO corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at small q_T;
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- NLO+PS generators (MC@NLO/POWHEG) reach NLL+NLO accuracy.
- The calculation of the resummed q_T spectrum are implemented in numerical codes HqT [Bozzi,Catani,de Florian,Grazzini('03,'06,'08)], [de Florian,G.F.,Grazzini,Tommasini('11)] and DYqT [Bozzi,Catani,de Florian,G.F.,Grazzini('08,'10)] (public versions of both codes are available).

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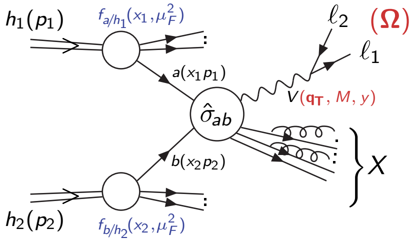
DYRes/HRes: q_T resummation and decay



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- Full kinematical dependence on decay products: possible to apply cuts.
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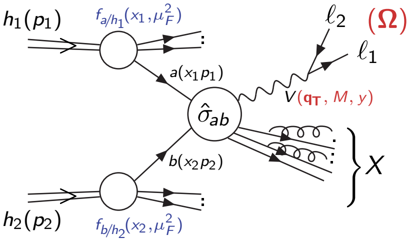

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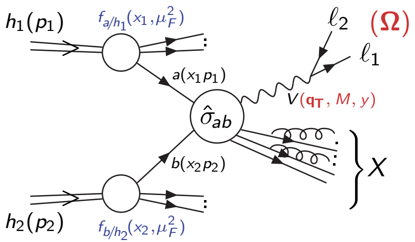

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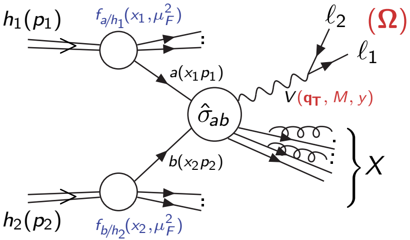

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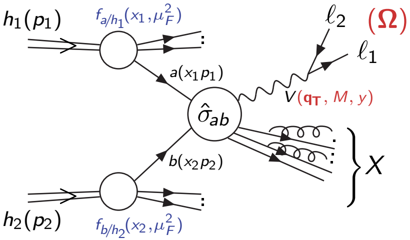

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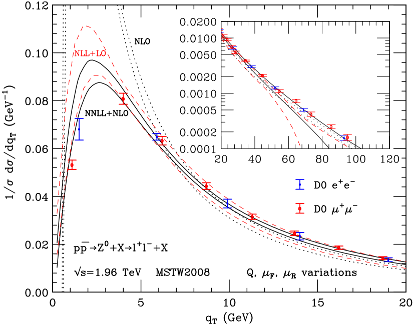
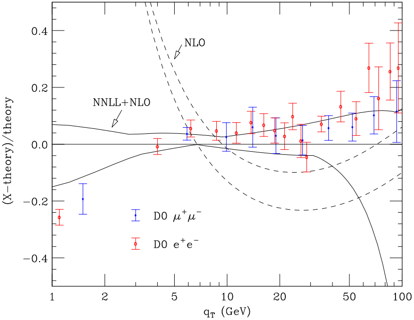
DYqT results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R, μ_F, Q independently:
$$\frac{1}{2} \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$$
- Significant reduction of scale dependence from NLL to NNLL for all q_T.
- Good convergence of resummed results: NNLL and NLL bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).
  The perturbative uncertainty of the NNLL results is comparable with the experimental errors.

DYqT **results:** q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10\,GeV$ and $\pm 12\%$ at $q_T = 50\,GeV$. For $q_T \geq 60\,GeV$ the resummed result looses predictivity.
- At large values of q_T, the NLO and NNLL bands overlap.
  At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data.
  In the small-q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL band.